Phenomenology of heavy flavor jet angularities

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Based on 25XX.YYYYY

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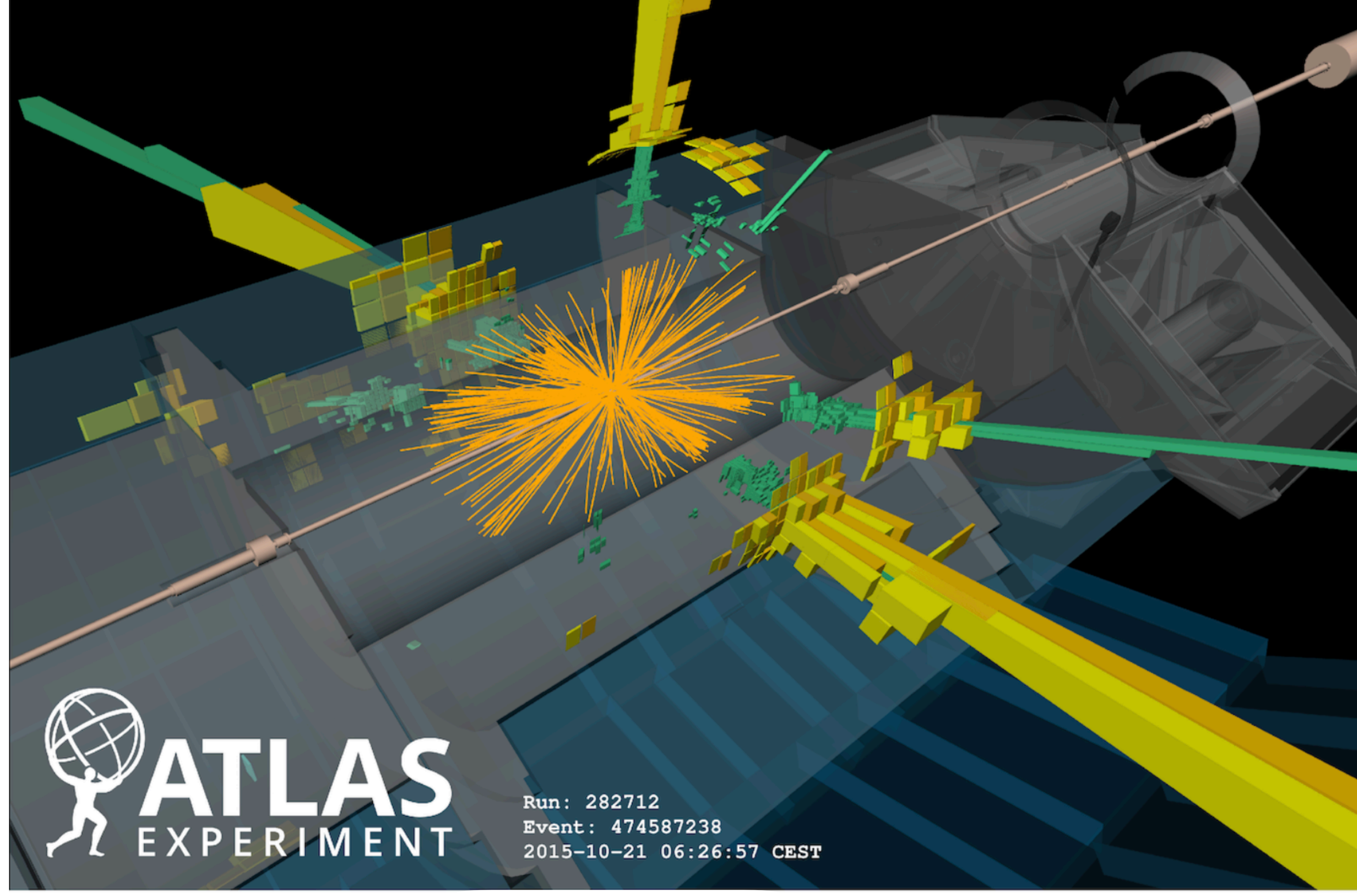
Introduction

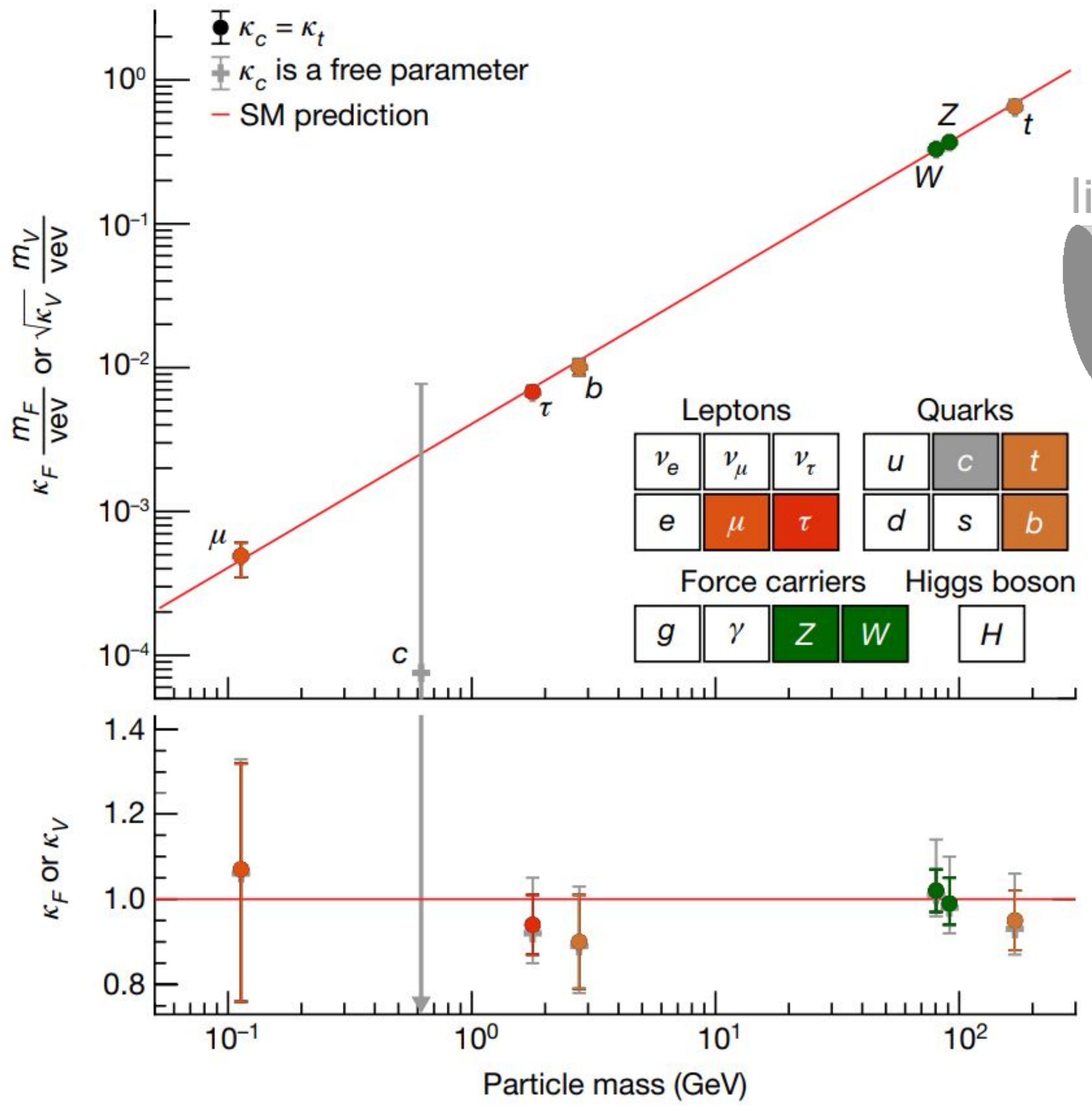


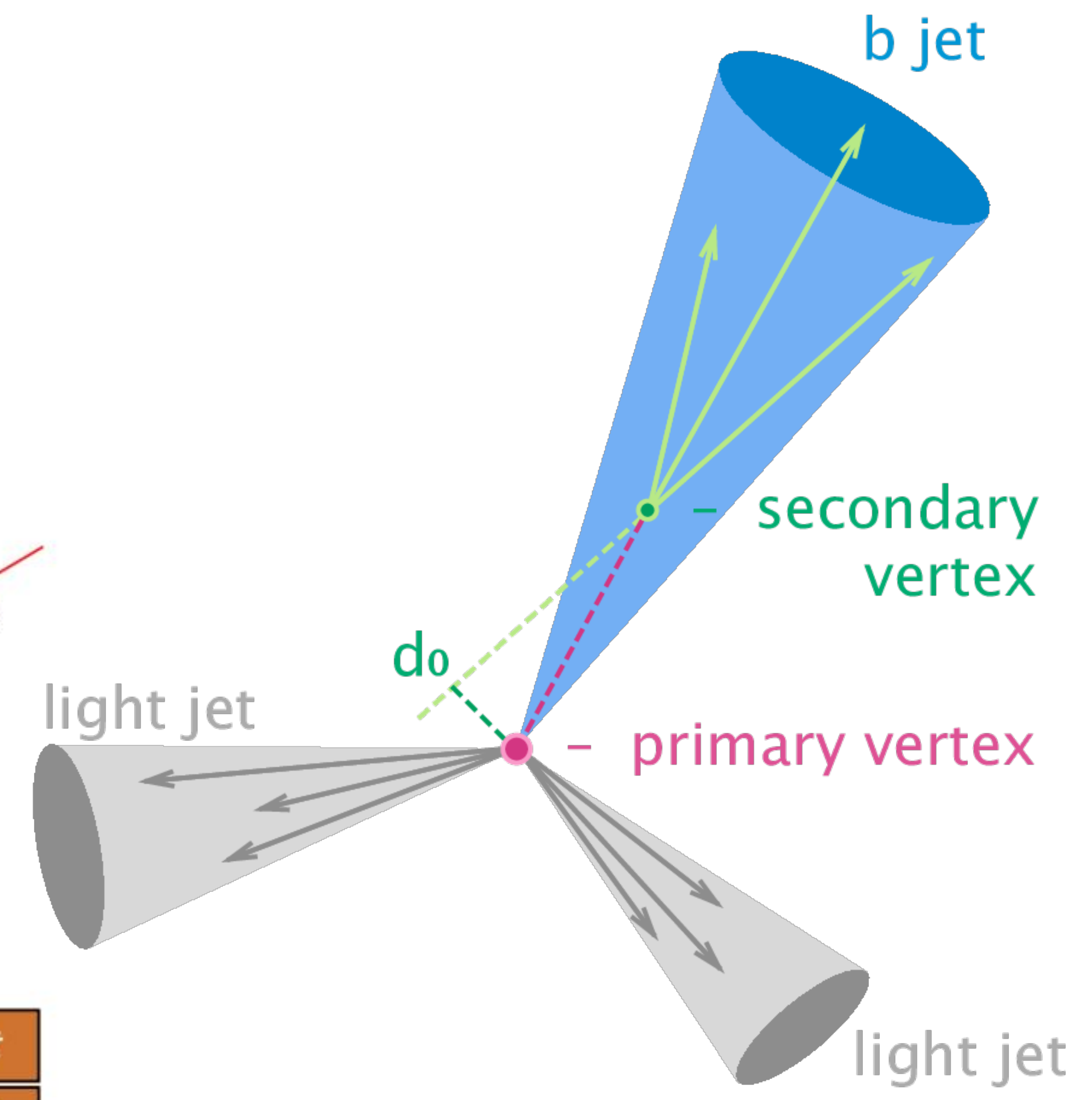
- **Jets**: collimated beams of hadrons abundantly produced in collider experiments

- Potential of **jet** observables measurements at LHC:
  - ✦ Search for New Physics
  - ✦ Test the fundamental properties of the SM
    - Determination of α_S (Britzger *et al.* 1712.00480)
    - Input to constrain PDFs (ATLAS, 2101.05095)

★ Heavy flavor (HF) **jets**: initiated by HF quarks, i.e. $m_q > \Lambda_{\text{QCD}} \sim 1$ GeV

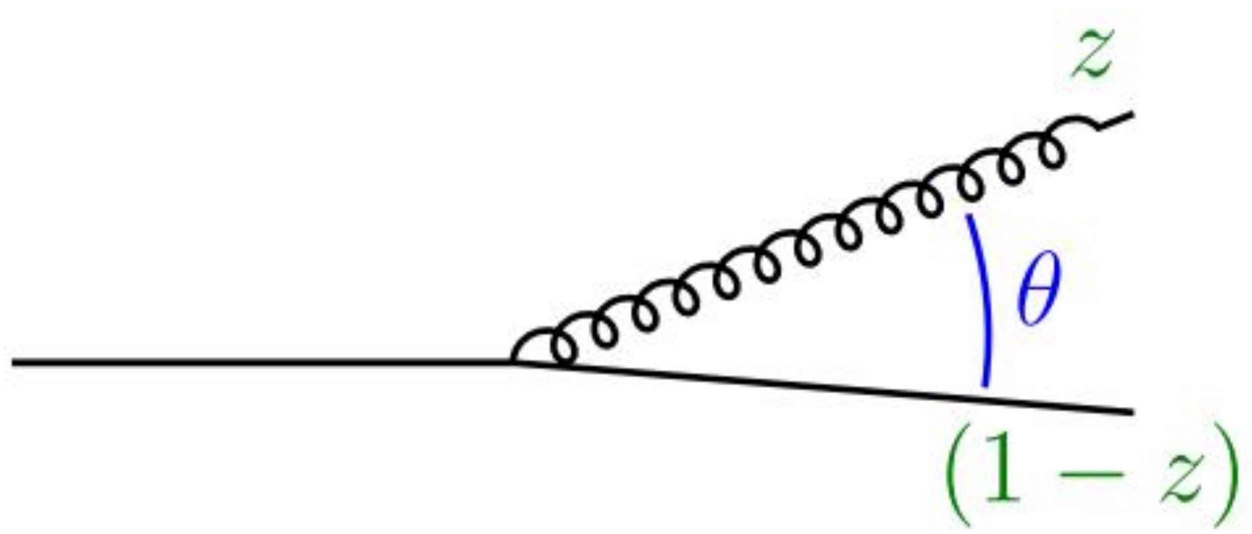
- ✦ Portal to non-perturbative physics (NP): B hadrons and D mesons easily detected
- ✦ Probe the Higgs sector of SM: $m_q \propto Y_q$





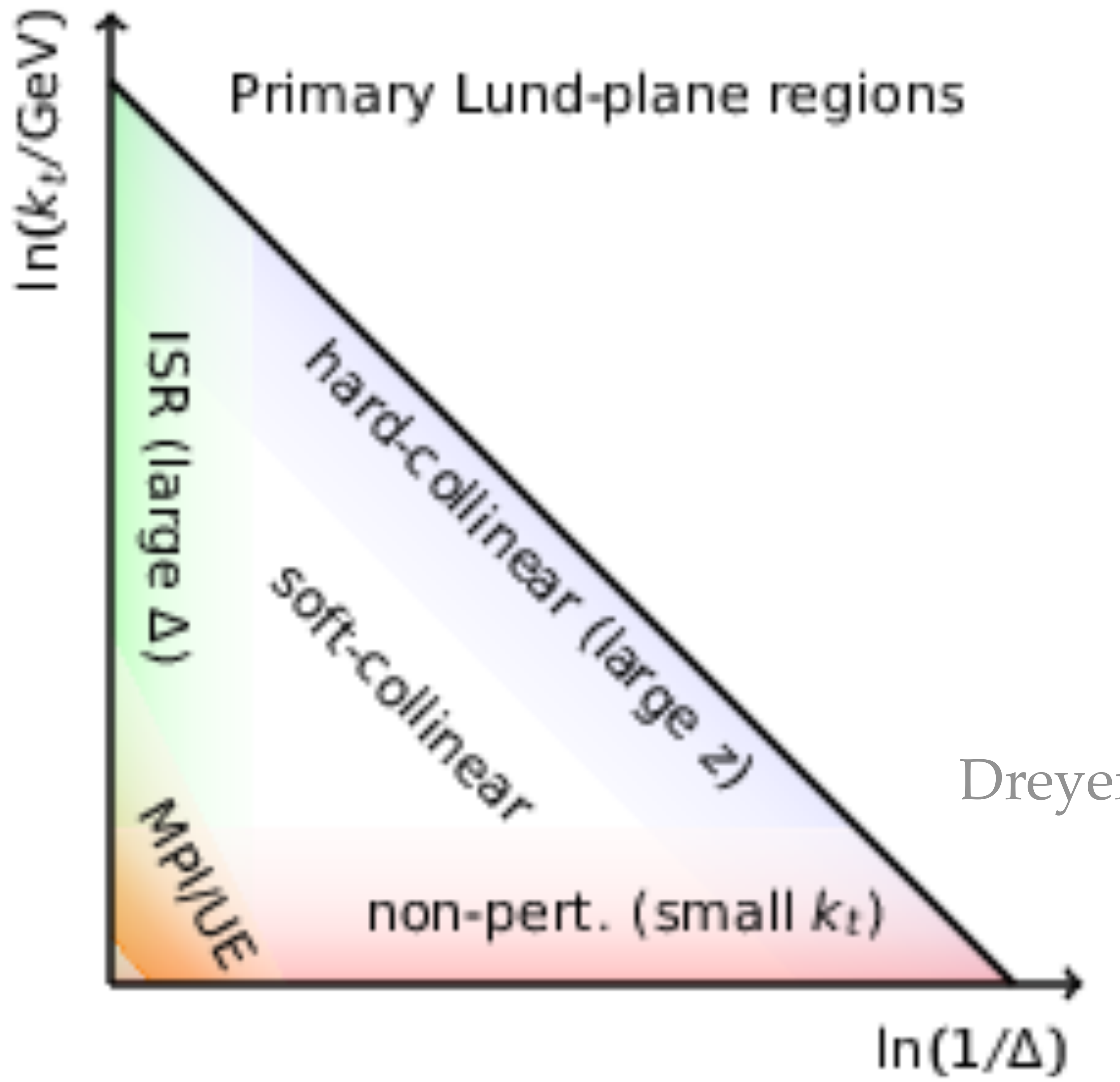
Soft and collinear limit in massless QCD

- At high energies the emission of partons is enhanced in the soft ($z \to 0$) and collinear ($\theta^2 \to 0$) limit



$$P(z,\theta^2)\,\mathrm{d}z\,\mathrm{d}\theta^2 = \frac{\alpha_s C_F}{2\pi}\frac{\mathrm{d}z}{z}\frac{\mathrm{d}\theta^2}{\theta^2}, \qquad z = \frac{E_g}{E_q + E_g}$$

- Soft and collinear emissions distributed in a log-log plane → Lund Plane



Dreyer *et al.* 1807.04758

Dead-cone effect

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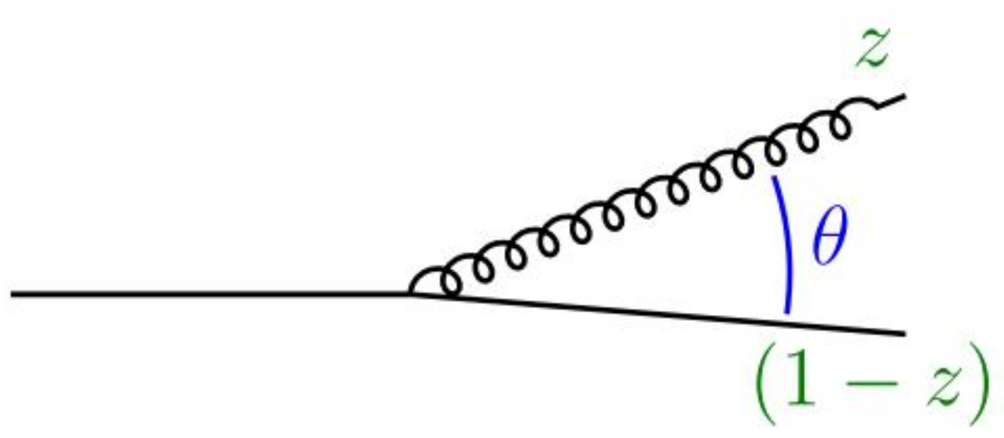


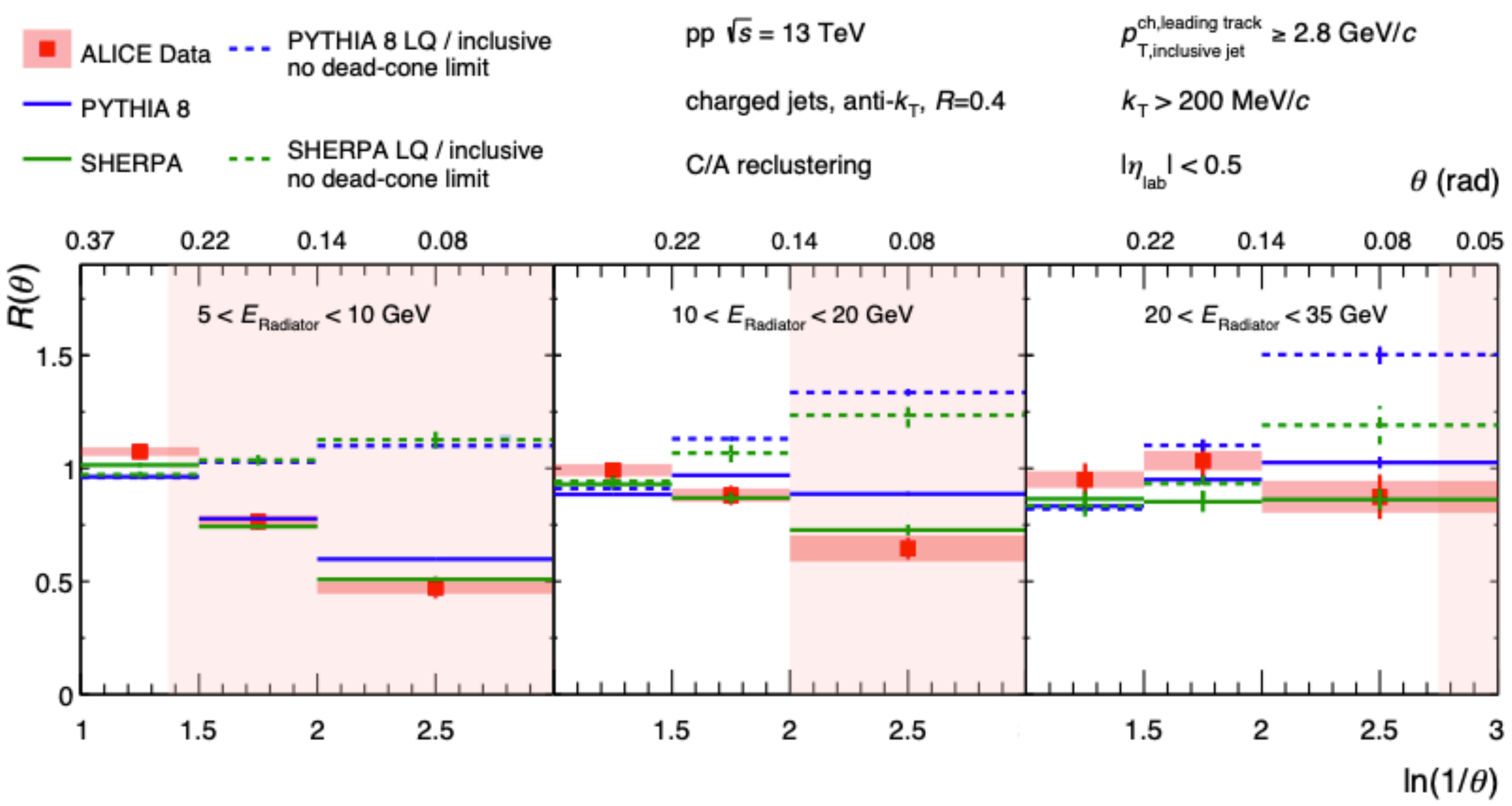
$$P(z,\theta^2)\,\mathrm{d}z\,\mathrm{d}\theta^2 = \frac{\alpha_s C_F}{2\pi}\frac{\mathrm{d}z}{z}\frac{\mathrm{d}\theta^2}{\theta^2}, \quad z = \frac{E_g}{E_q + E_g}$$

- For **jets** initiated by an HF the corresponding mass shields the collinear singularity

$$P(z,\theta^2)\,\mathrm{d}z\,\mathrm{d}\theta^2 = \frac{\alpha_s C_F}{2\pi}\frac{\mathrm{d}z}{z}\frac{\mathrm{d}\theta^2}{\theta^2 + \frac{m^2}{E^2}}$$

- ***Dead-cone*** effect: the radiation emitted off an HF is suppressed inside a cone of opening $\theta_{dead} \sim m/E$

- Great interest by the entire experimental community (ATLAS, ALICE, CMS, LHCb) in HF **jet** substructure observables: need for theoretical prediction!



ALICE (2022)

Jet angularities

- Goal: study substructure observables sensitive to ***dead-cone*** effect

- We focus on ***jet angularities***, which probe both the angular and the transverse momentum distribution of particles within a **jet**

$$\lambda_\alpha^\kappa = \sum_{i \in \mathrm{jet}} \left(\frac{p_{T_i}}{p_{T_{\mathrm{jet}}}} \right)^\kappa \left(\frac{\Delta R_i}{R_0} \right)^\alpha$$

$$\Delta R_i = \sqrt{(y_i - y_{\mathrm{jet}})^2 + (\phi_i - \phi_{\mathrm{jet}})^2}$$

- Restriction to the infrared and collinear safe (IRC) case $\kappa = 1$

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- Others definitions for ***jet angularities*** are possible with HF **jets**

$$\dot{\lambda}_\alpha = \sum_{i \in \text{jet}} \left(\frac{p_{T_i}}{p_{T_{\text{jet}}}} \right)^\kappa \left(\frac{2p_i \cdot n}{p_{Ti} R_0} \right)^{\frac{\alpha}{2}}, \qquad \mathring{\lambda}_\alpha = \sum_{i \in \text{jet}} \left(\frac{p_{T_i}}{p_{T_{\text{jet}}}} \right)^\kappa \left(\frac{2p_i \cdot n_0}{p_{Ti} R_0} \right)^{\frac{\alpha}{2}}$$

$n(n_0)$ is a massive (massless) 4 vector aligned with WTA axis

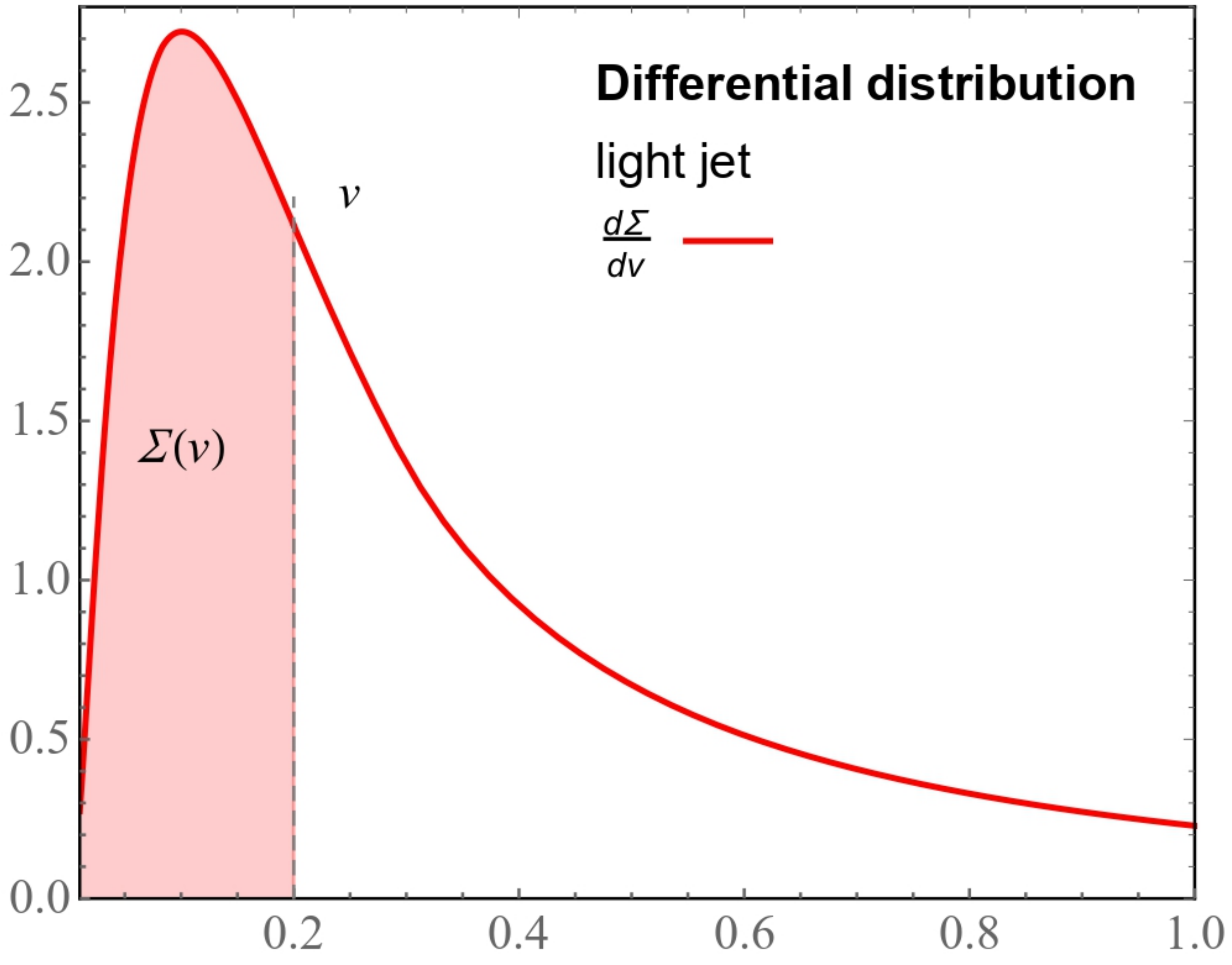
but λ_α^κ is the most suitable to expose the ***dead-cone*** effect (Dhani *et al.* 2410.05415)

All-orders resummation

- HF **jets** characterized by a variety of scales:
  - ✦ hard scale Q (e.g. partonic c.o.m. energy $\hat{s}$, **jet** p_T, …)
  - ✦ HF mass m
  - ✦ scale v set by the observable
  - ✦ …
- Large logs involving ratios of such scales will (certainly) appear and spoil the convergence of perturbative expansions
- E.g. for the (normalized) cumulative distribution

$$\Sigma(v) = \frac{1}{\sigma_0}\int_0^v \mathrm{d}v' \frac{\mathrm{d}\sigma}{\mathrm{d}v'} \propto \sum_n^{\infty} \alpha_S^n \sum_k^{2n} c_k \log^k v$$

$\Rightarrow$ (multiple) Resummations become necessary!



The CAESAR formalism

- HF **jets** characterized by a variety of scales: hard scale Q ($\hat{s}, p_T, \ldots$), HF mass m, scale v set by the observable,… ⇒ large logs will (certainly) appear and (multiple) resummations become necessary!

- Resummed predictions for HF ***jet angularities*** at **NLO + NLL** from SHERPA implementation of the CAESAR resummation formalism

  Banfi *et al.* hep-ph/0407286

- SHERPA plugin
  - Automated framework for resummation and matching — Gerwick *et al.* 1411.7325
  - Resummed predictions at **NLO+NLL** for multijet resolution scales in e^+e^- — Baberuxki *et al.* 1912.09396
  - **NLO+NLL** predictions for SOFTDROP event shapes — Baron *et al.* 2012.09574
  - Massless ***jet angularities*** at **NLO+NLL(+NP)** — Caletti *et al.* 2104.06920, Reichelt *et al.* 2112.09545
  - Inclusion of of mass effects for b-quark in ***jet angularities*** (this work) — Ghira *et al.* 25XX.YYYY

- CAESAR: general approach to perform soft-gluon resummation at **NLL** accuracy for a large class of "*suitable*" observables

- What does "*suitable*" means?

The CAESAR formalism

- What does *"suitable"* means?
- The observable has to be:
  - Recursively IRC (rIRC) safe
  - The scaling behavior of the observable in the soft and collinear limit with respect to any external momentum is assumed to be of the general form

$$V(\mathcal{B}, k) = \left(\frac{k_t^{(l)}}{\mu_Q}\right)^{a_l} e^{-b_l \eta^{(l)}} d_l(\theta) g_l(\phi)$$

◉ μ_Q: resummation scale, of the order of hard scale

  - Additional condition: globalness

- *Jet angularities* correspond to $V(\mathcal{B}, k)$ for:

$$a_l = 1, \qquad b_l = \alpha - 1, \qquad d_l g_l = \left(2\frac{\cosh y_{\text{jet}}}{R_0}\right)^{\alpha-1} \frac{\mu_Q}{p_{T_{\text{jet}}} R_0}$$

- <u>However</u>: by definition *jet angularities* are ***<u>non-global</u>*** observables ⇒ appearance of non-global logs (NGLs) (Dasgupta & Salam, <u>hep-ph/0104277</u>)

The CAESAR master formula

- For such observables, the cumulative distribution at **NLL** accuracy is derived through the generic master formula

$$\Sigma_{\mathrm{res}}(v) = \sum_{\delta} \Sigma_{\mathrm{res}}^{\delta}(v),$$

$$\Sigma_{\mathrm{res}}^{\delta}(v) = \int \mathrm{d}\mathcal{B}_{\delta}\, \frac{\mathrm{d}\sigma_{\delta}}{\mathrm{d}\mathcal{B}_{\delta}}\, \exp\left[-\sum_{l\in\delta} R_l^{\mathcal{B}_\delta}(L)\right] \mathcal{S}^{\mathcal{B}_\delta}(L)\, \mathcal{P}^{\mathcal{B}_\delta}(L)\, \mathcal{F}^{\mathcal{B}_\delta}(L)\, \mathcal{H}^{\delta}(\mathcal{B}_\delta)$$

- δ = partonic channel
- $L \equiv \log(v)$

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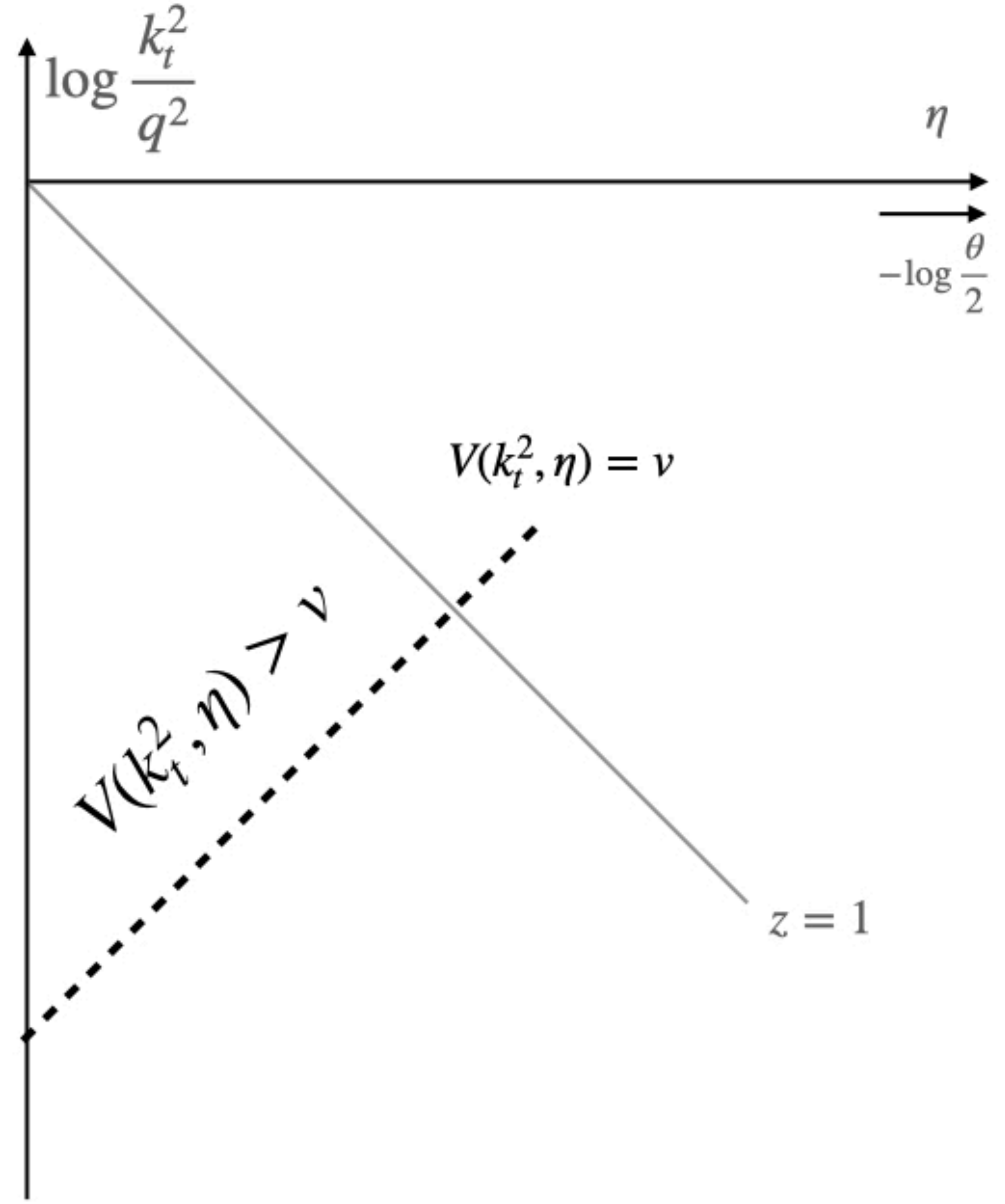

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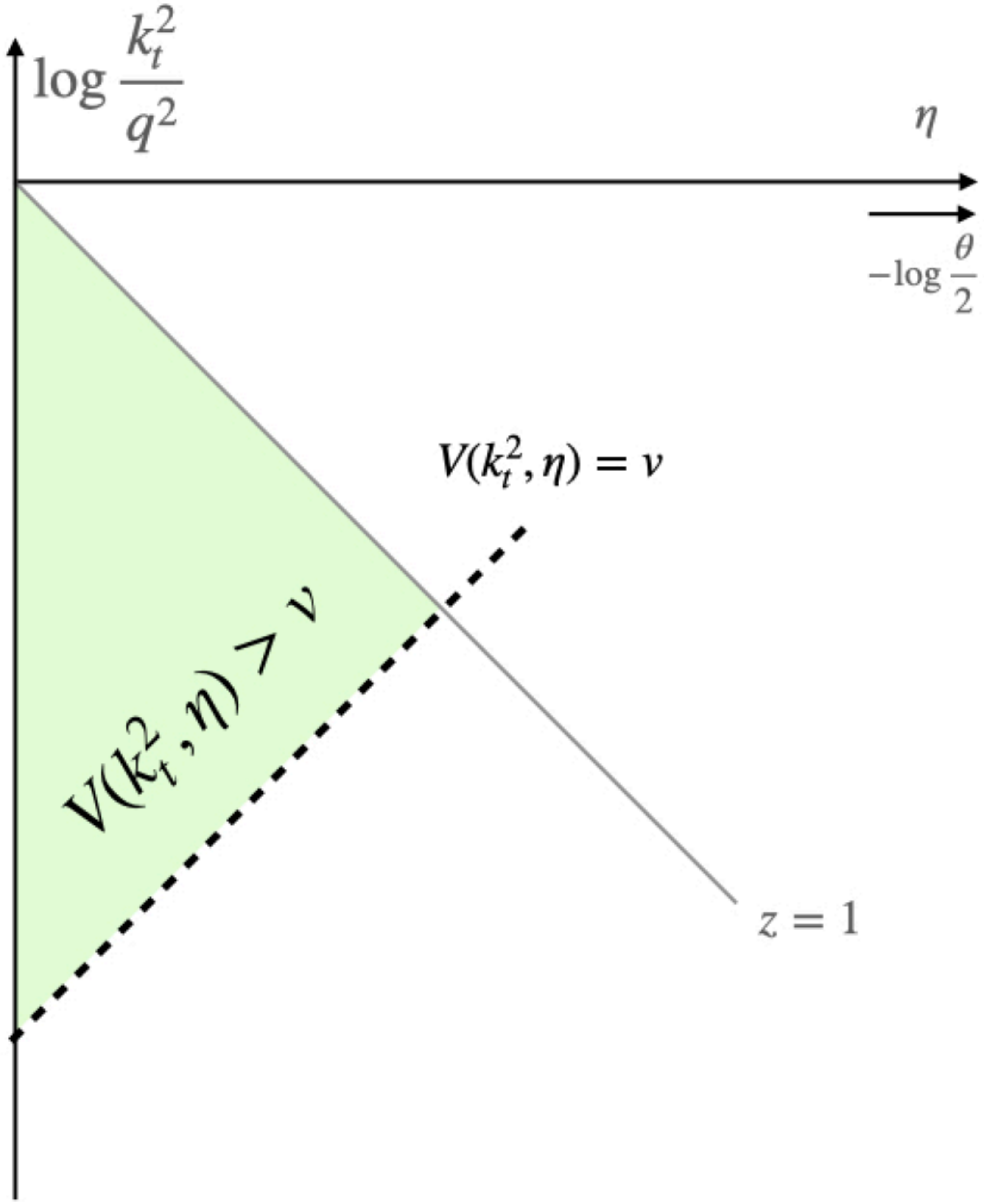

✦ Collinear radiators ($\forall$ leg l).
Resummed exponent R is the Sudakov form factor (shaded green area)

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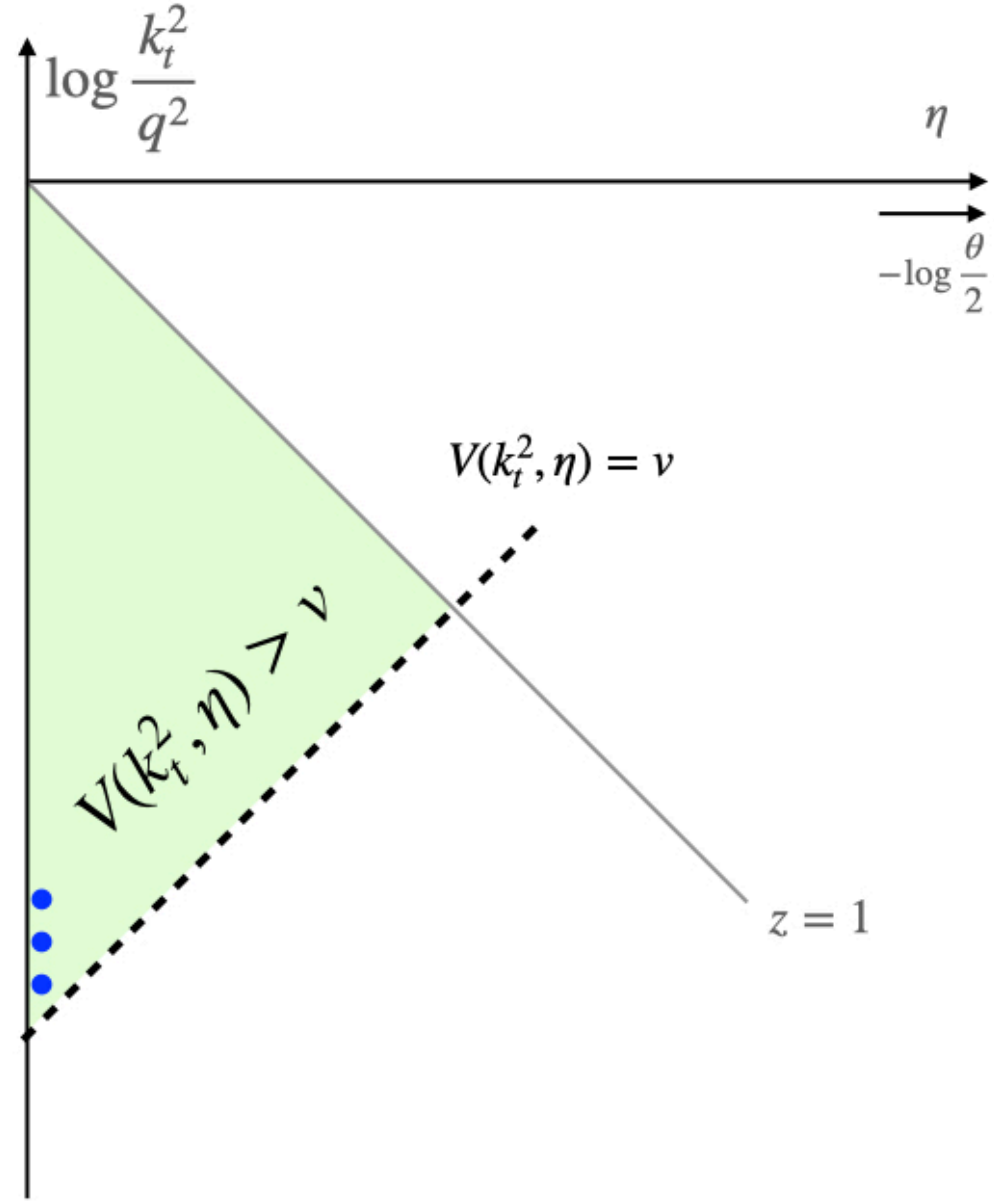

✦ Soft function: $S^{\mathcal{B}_\delta}(L) = S^{\mathcal{B}_\delta}_{\rm global}(L)\, S^{\mathcal{B}_\delta}_{\rm non\text{-}global}(L)$

- **Global**: for jet-shapes, ∝ to power corrections in R_0 and m
- **Non global:** → Computed numerically in the large-N_c limit (Dasgupta & Salam, hep-ph/0104277)

↓

NLL resummation same as massless case (Dhani *et al*, 2410.05415)

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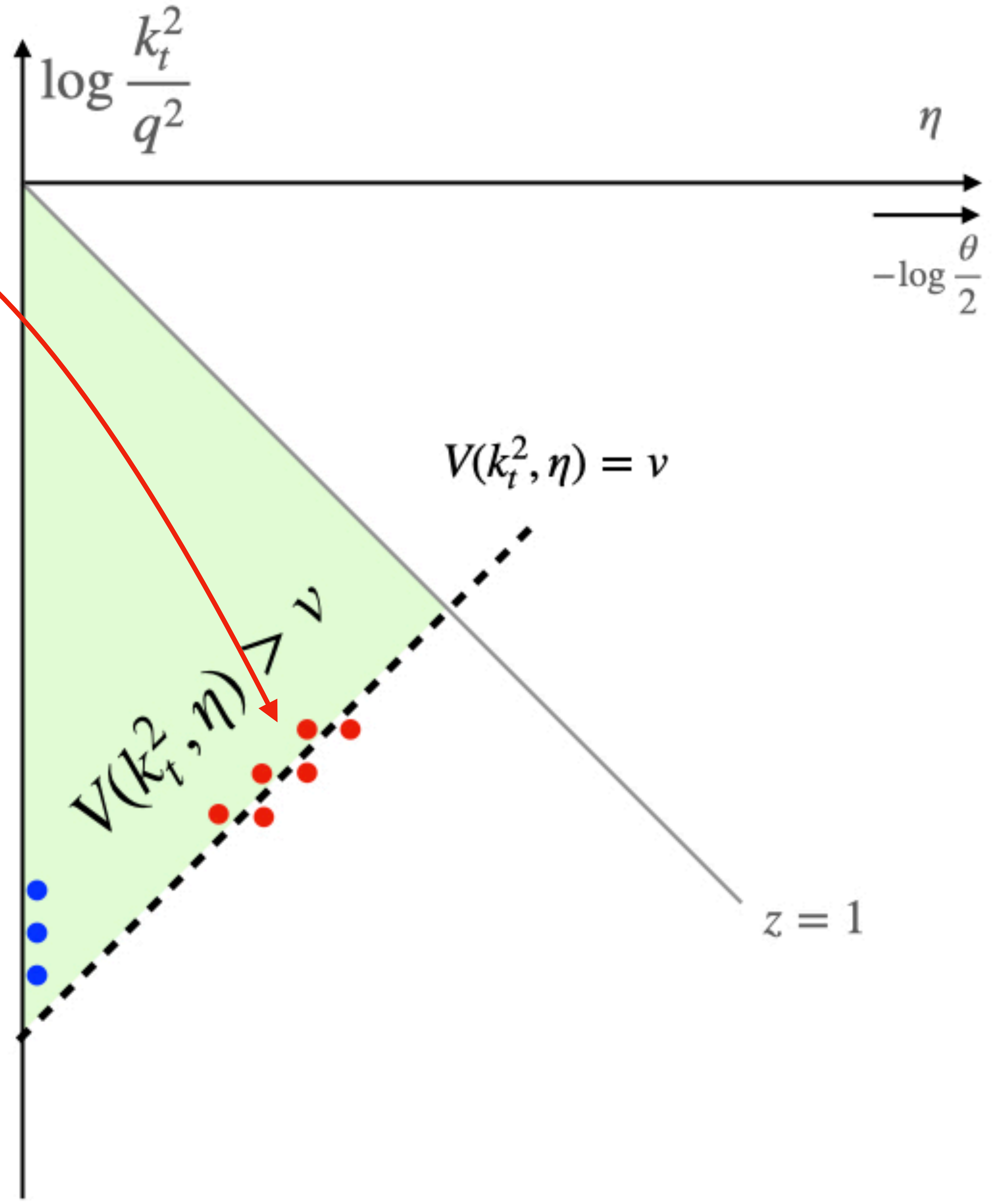

✦ Multiple emission function.
For additive observables as ***angularities***

$$\mathcal{F}^{\mathcal{B}_\delta} = \frac{e^{-\gamma_E R'}}{\Gamma(1+R')}, \quad R' = \partial R/\partial L, \quad R = \sum_{l\in\delta} R_l(L)$$

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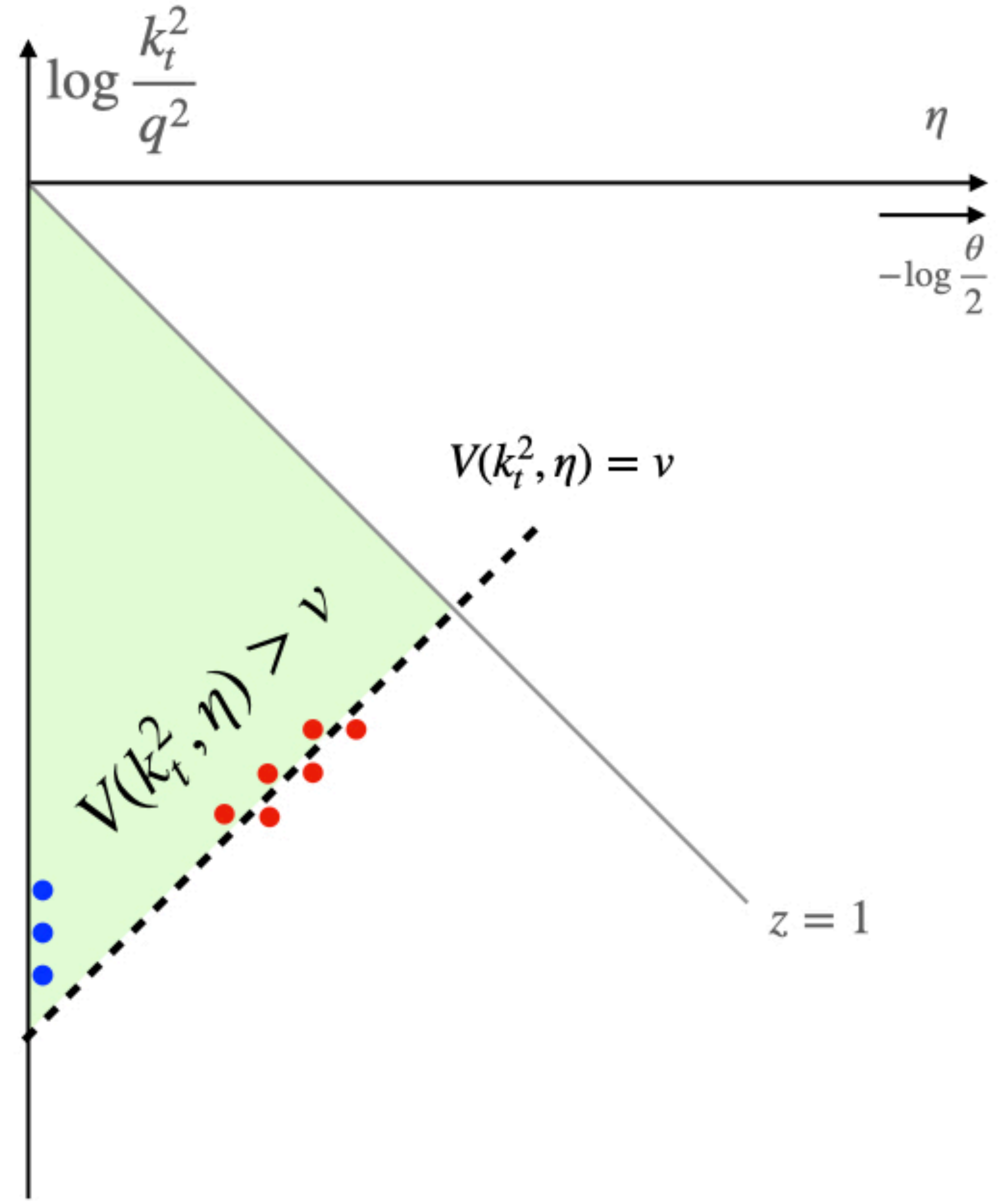


✦ Ratio of PDFs: includes corrections for PDF evolution for initial state radiation

$$\mathcal{P}^{\mathcal{B}_\delta}(L) = \prod_{l=1}^{2} \frac{f_l\left(x_l^{\mathcal{B}_\delta},\, e^{-L/(a_l+b_l)}\mu_{\text{F}}\right)}{f_l\left(x_l^{\mathcal{B}_\delta},\, \mu_{\text{F}}\right)}$$

✦ *Jet angularities* not sensitive to radiation collinear to the initial state legs $\Rightarrow \mathcal{P}^{\mathcal{B}_\delta} = 1$

Quasi-collinear limit

- With HF employ ***quasi-collinear*** limit:
  - ✦ Heavy quark mass m and k_t of emitted gluon small compared to the hard scale
  - ✦ Ratio m/k_t kept fixed
  - ✦ Interpolates between ***collinear factorization*** ($m \ll k_t$) and ***dead-cone*** ($k_t \ll m$)
- QCD amplitudes with massive partons factorize in the ***quasi-collinear*** limit:

$$|\mathcal{M}|^2 \simeq |\mathcal{M}_0|^2 \frac{8\pi\alpha_S z(1-z)}{k_t^2 + z^2 m^2} P_{gb}(z, k_t^2)$$

with

$$P_{gb}(z, k_t^2) = C_F \left(\frac{1+(1-z)^2}{z} - \frac{2m^2 z(1-z)}{k_t^2 + z^2 m^2} \right)$$

the LO time-like (massive) splitting function

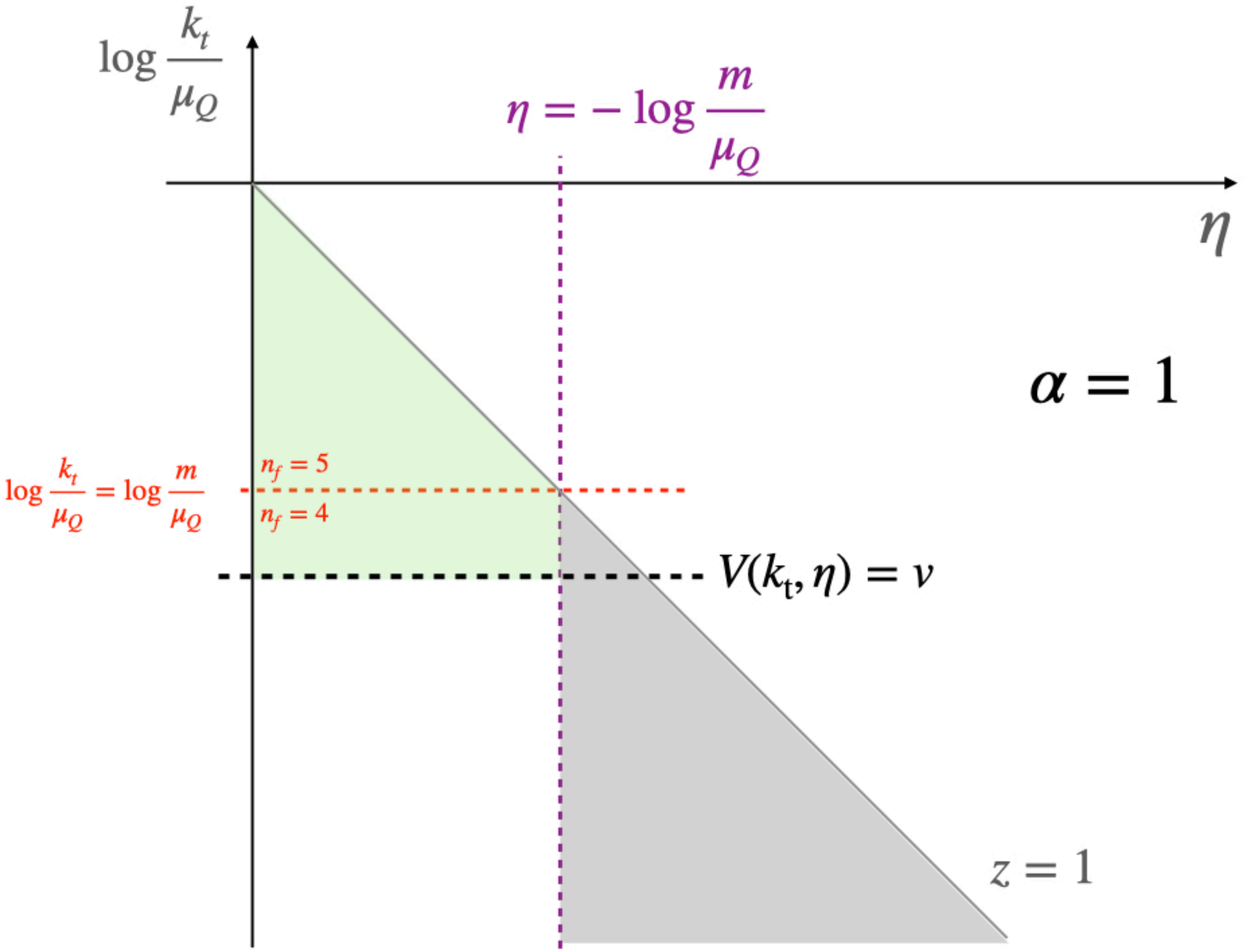
b-quark radiator

- The radiator associated to a b-quark in ***quasi-collinear*** limit for ***jet angularities*** reads

$$R_b = \int_0^1 \mathrm{d}z \int_0^{\mu_Q^2} \frac{\mathrm{d}k_t^2}{k_t^2 + z^2 m^2} \frac{\alpha_S^{\mathrm{CMW}}(k_t^2)}{2\pi} P_{gb}(z, k_t^2) \Theta\left(V(k_t) - \lambda_\alpha\right)$$

Dhani *et al.* 2410.05415

- ✦ On Lund plane (for emission off a b-quark):
  - ✦ Sudakov form factor : green shaded area given by R_b
  - ✦ Upper bound from collinear momentum conservation: $z < 1$
  - ✦ *Dead-cone* (grey region) condition: $k_t > zm$



- ✦ Additional feature: running coupling accounted in the CMW scheme (Catani *et al.* 298129)

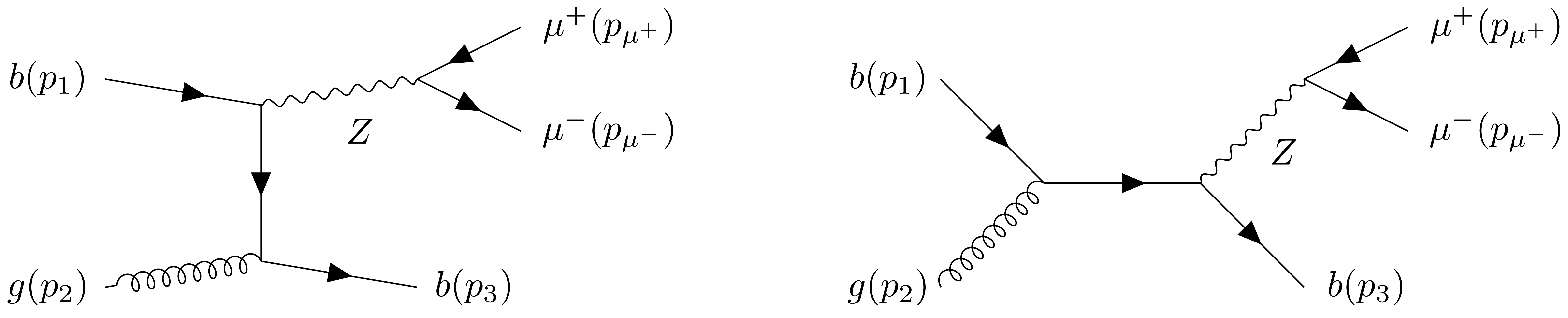
$$\alpha_S^{\mathrm{CMW}}(k_t^2) = \alpha_S(k_t^2)\left(1 + \alpha_S(k_t^2)\frac{K^{(n)}}{2\pi}\right), \qquad K^{(n)} = C_A\left(\frac{67}{18} - \frac{\pi^2}{6}\right) - \frac{5}{9}n$$

combined with decoupling scheme to take into account all mass effects

$$\alpha_S(k_t^2) = \alpha_S^{(5)}(k_t^2)\Theta(k_t^2 - m_b^2) + \alpha_S^{(4)}(k_t^2)\Theta(m_b^2 - k_t^2)$$

NLL resummation in $pp \to Z\,(\mu^+\mu^-) + b$

- We focus on the process $Z(\mu^+\mu^-) + b$

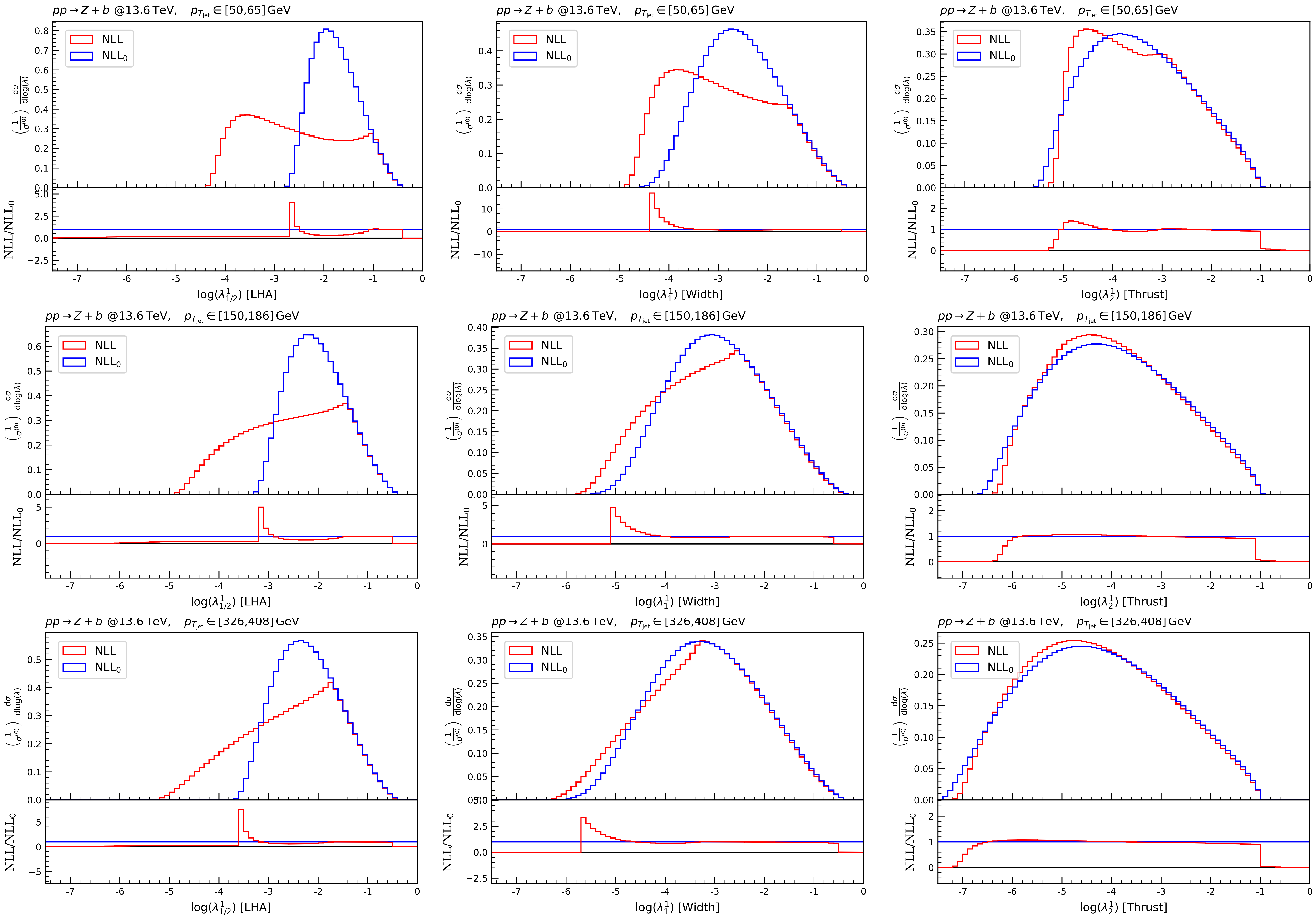


- ***Jet angularities*** not sensitive to radiation collinear to initial state legs:
  - ✦ PDFs ratio $\to \mathcal{P} = 1$
  - ✦ Radiators $\to R_1 = R_2 = 0,\ R_3 = R_b \neq 0$
  - ✦ Global part of Soft function: $\propto$ power corrections in R_0 and m
  - ★ Additionally: hard scale $\mu_Q = p_T R_0$

- General setup, with cuts based on CMS measurement in 2109.03340:
  - ✦ $\alpha \in [0.5, 1, 2]$
  - ✦ $S = 13.6$ TeV
  - ✦ *Anti-kt* with $R_0 = 0.4$
  - ✦ $p_{\mathrm{T},\mu^\pm} > 26\,\mathrm{GeV}, \quad |\eta_{\mu^\pm}| < 2.4$
  - ✦ $m_{\mu^+\mu^-} \in [70, 110]\ \mathrm{GeV}$
  - ✦ $|y_{\mathrm{jet}}| < 1.7$

Caletti *et al.* 2104.06920
Reichelt *et al.* 2112.09545

NLL vs NLL_0



Transition from massive to massless occuring at

$$\tilde{\lambda}_\alpha \sim (m/p_T R_0)^\alpha$$

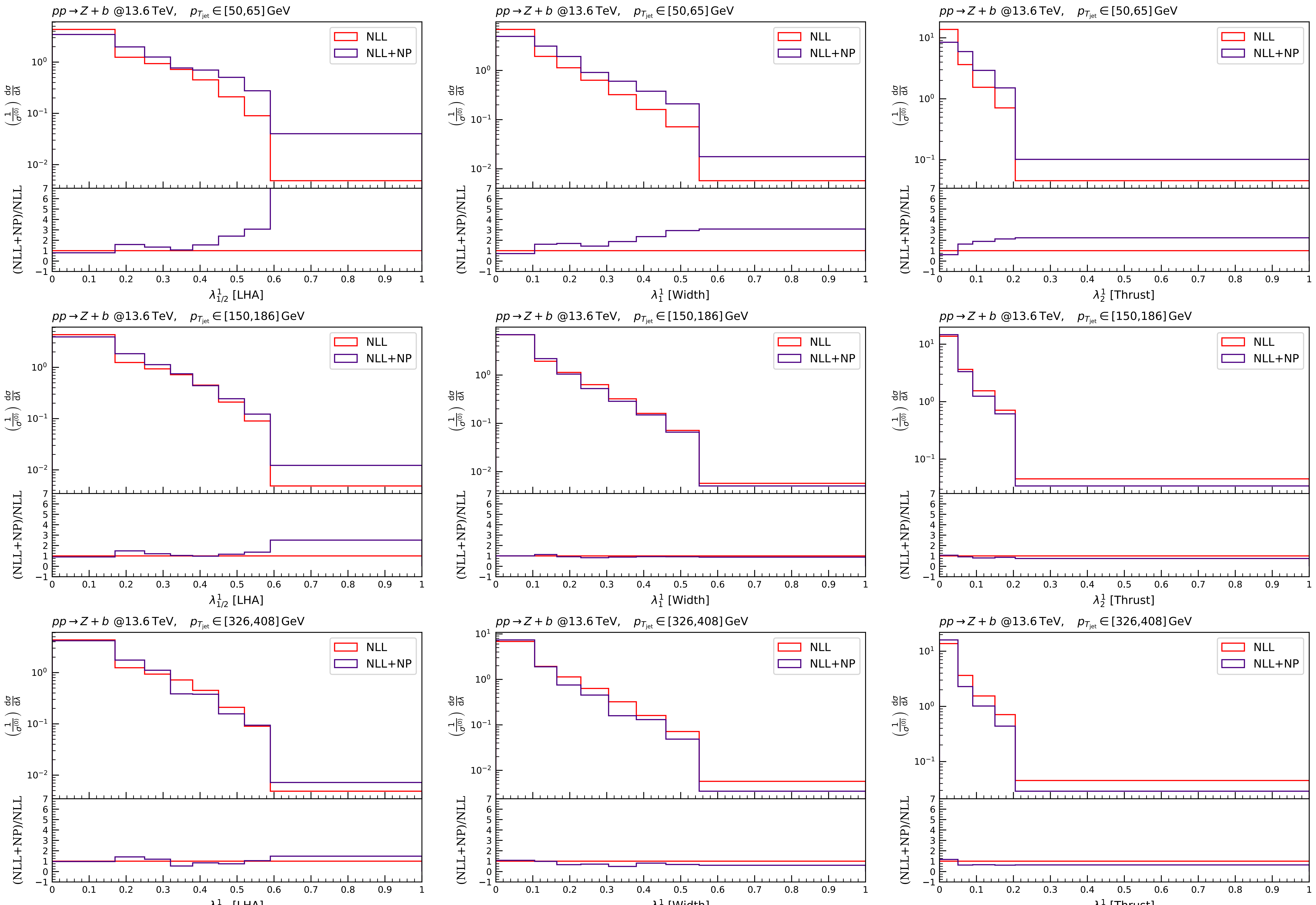
- $\forall \alpha$:
  $\lambda_\alpha \geq \tilde{\lambda}_\alpha$ then $\mathrm{NLL} \to \mathrm{NLL}_0$

- Higher $\alpha \Rightarrow$ smaller $\tilde{\lambda}_\alpha$.
  NLL and NLL_0 similar even at low p_T for sufficiently high α

- $\alpha \leq 1$:
  R_b suppressed wrt R_0 due to *dead-cone*
  $\Downarrow$
  NLL $\neq 0$ for smaller λ_α wrt NLL_0

PRELIMINARY RESULTS!

NLL vs NLL+NP



Large NP corrections in low-p_T bins $\forall\, \alpha \Rightarrow$ SOFTDROP will help in reducing them

Still not negligible NP corrections in higher -p_T bins for $\alpha < 1$: more sensible to NP effects since lower bound is on smaller k_t

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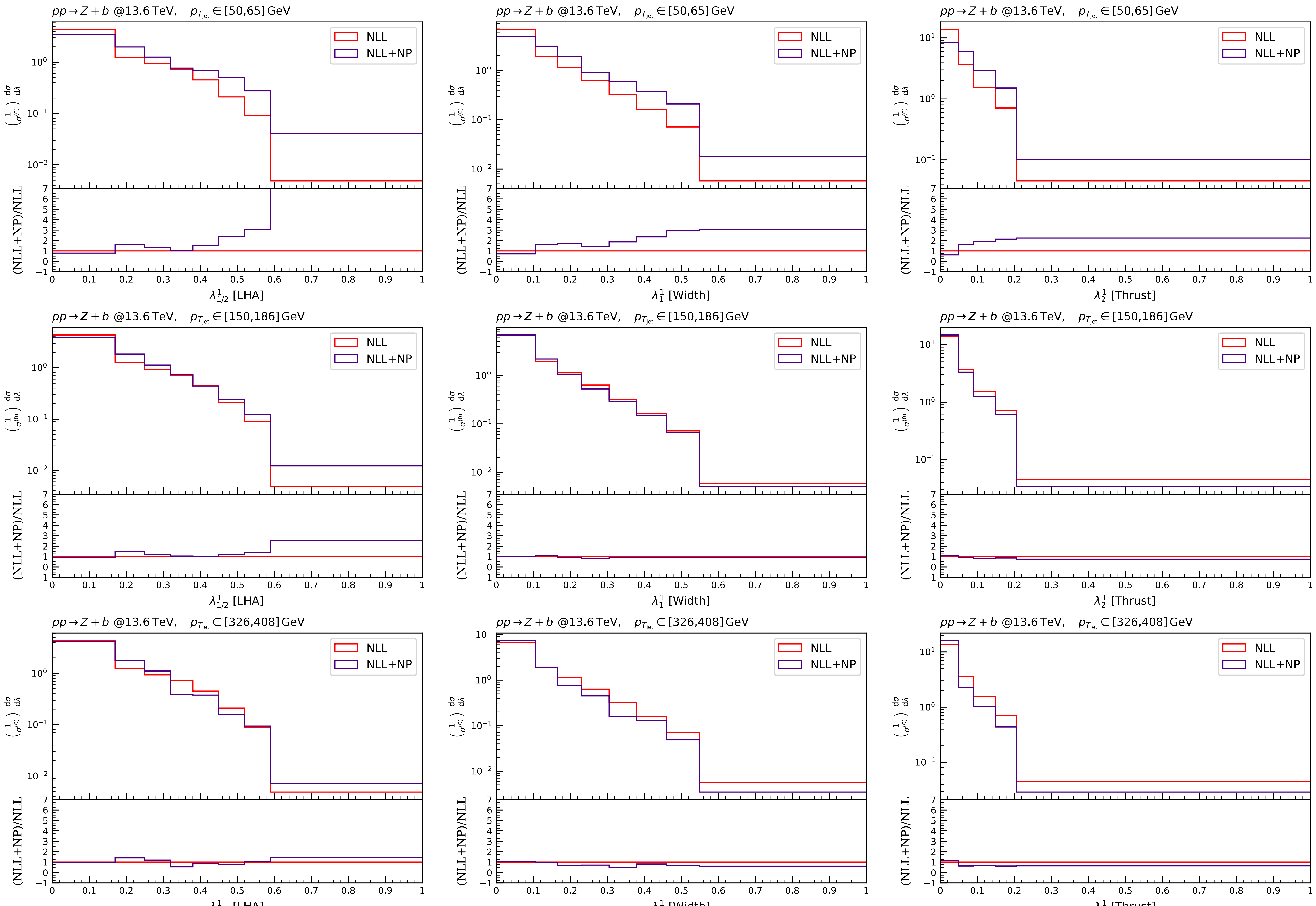

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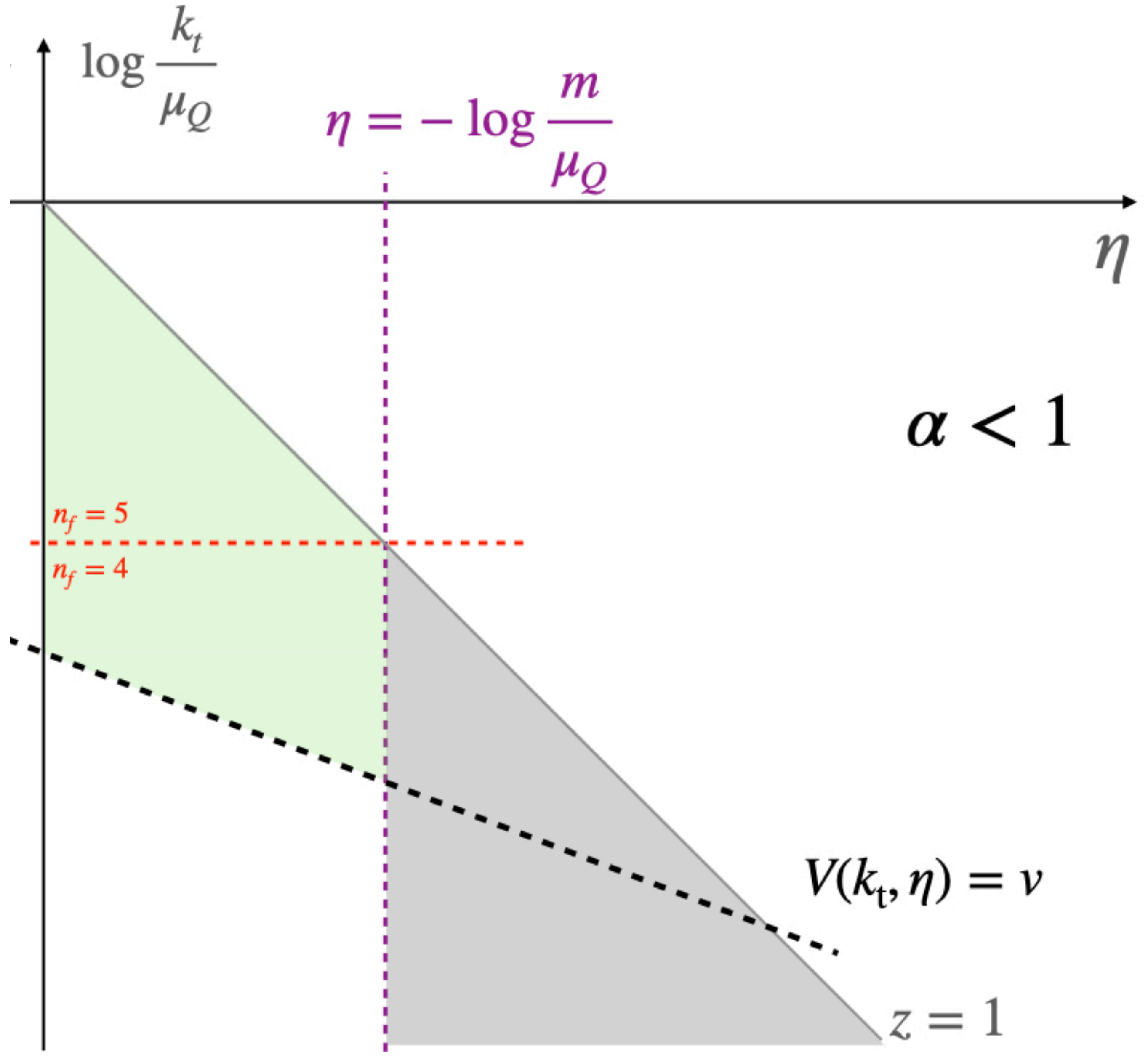


Conclusion and outlook

- *Jet angularities* represent a powerful class of observable to investigate the inner structure of **jets** and probe the *dead-cone* effect

- Despite being a ***non-global*** observable they are well suited to the CAESAR formalism ⇒ NGLs handled numerically and not too complicated with few external legs

- We extended previous works on resummed predictions for *jet angularities* through the SHERPA implementation of CAESAR by including mass effects from HF

- These results are still preliminary but will further increase the accuracy of theoretical prediction to be compared with data for processes involving **jets** initiated by HF

- Outlook:
  - Conclude this analysis (matching) and extend implementation with SOFTDROP
  - Analyze other processes of phenomenological relevance (e.g. $pp \to Zb\bar{b}$)
  - Generalize algorithm to c-quarks (conceptually easy, but many more hierarchies to take into account)

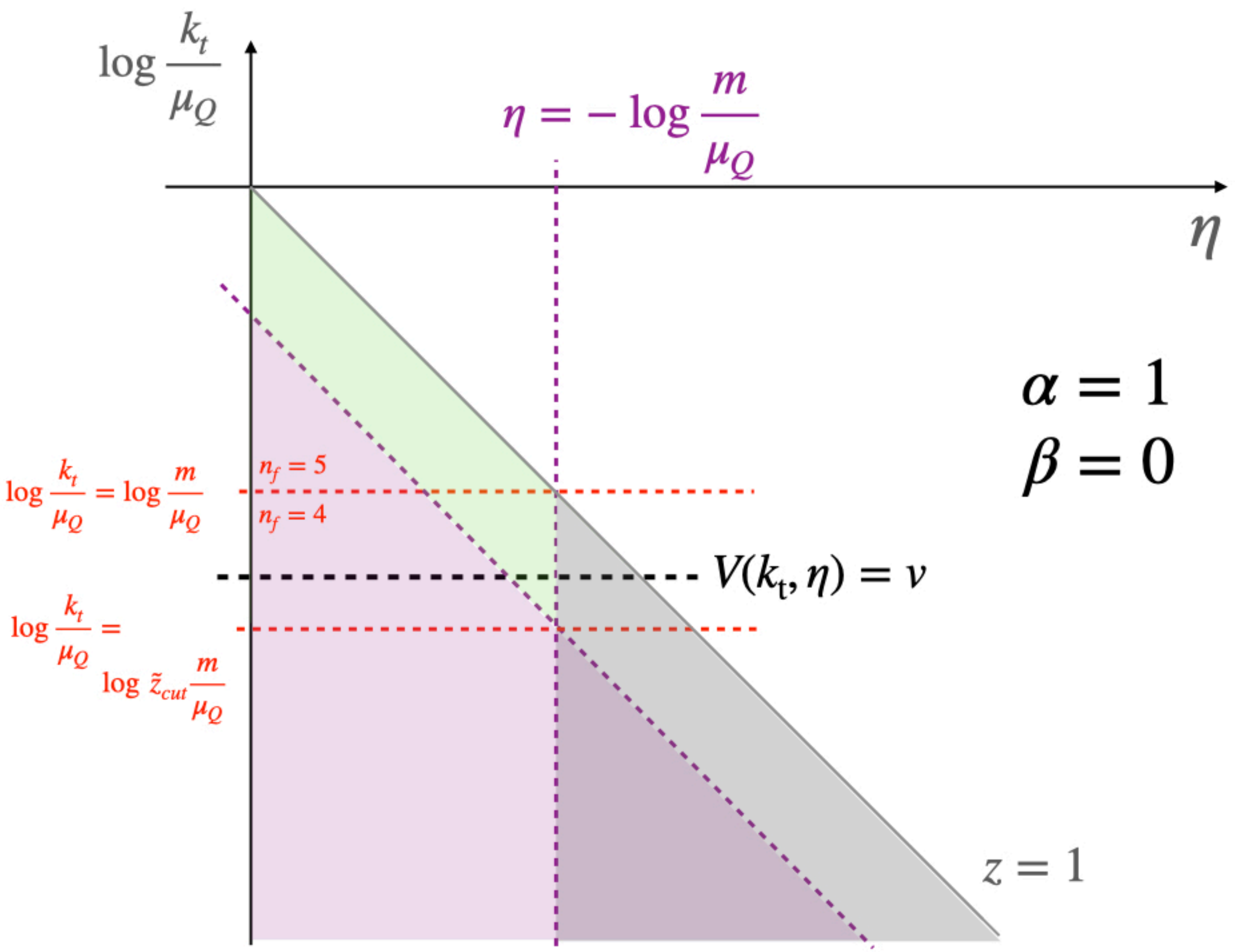
Backup

SoftDrop in a nutshell

- The structure of all-order resummed results is simplified with groomers such as SoftDrop

Larkoski *et al.* 1402.2657

- Groomers are JSS algorithms which “clean up” a **jet** by removing the constituents of soft origin:
    - ✦ Non-perturbative physics effects from hadronization and underlying event are reduced
    - ✦ Log-enhancement from soft-wide angles gluons is removed
    - ✦ Removal of NGLs by converting logs of the observable into logs of an external parameter



- The SoftDrop condition is

$$z \geq z_{\mathrm{cut}} \left(\frac{\Delta R_k}{R_0} \right)^{\beta}$$

with the parameter z_{cut} chosen such that:

- ✦ small enough to neglect its power-corrections
- ✦ $\log z_{\mathrm{cut}}$ not too large, to avoid its resummation

Soft function

- Soft function for jet-shapes

$$S^{\mathcal{B}_\delta}(L) = S^{\mathcal{B}_\delta}_{\text{global}}(L)\, S^{\mathcal{B}_\delta}_{\text{non-global}}(L)$$

- **Global** contribution:

$$S^{\mathcal{B}_\delta}_{\text{global}}(t) = \text{Tr}\left[H e^{-t\left(\mathbf{\Gamma}^{\mathcal{B}_\delta}\right)^\dagger} c\, e^{t\mathbf{\Gamma}^{\mathcal{B}_\delta}}\right] \frac{1}{\text{Tr}\left[cH\right]}, \qquad \mathbf{\Gamma}^{\mathcal{B}_\delta} = \sum_{i>j\in\mathcal{B}_\delta} \mathbf{T}_i \cdot \mathbf{T}_j \; I_{ij}$$

where c is the color metric and Γ is written in color space as sum over dipoles (only (1,2), (1,3), (2,3) for $Z+b$)

- Color conservation: $\mathbf{T}_i \cdot \mathbf{T}_j$ rewritten in terms of SU(3) Casimirs C_F, C_A $\Rightarrow$ matrix structure disappears

- I_{ij} obtained by integrating the matrix elements for each dipole ij over the appropriate phase space

- These coefficients are purely due to soft wide-angle radiation $\Rightarrow$ depend only on (power corrections of) R_0 (and m for HF) and not from angularity exponent α :

$$I_{1,2} \sim \frac{\alpha_{\text{S}}}{2\pi} R_0^2 \left[1 - e^{y_3}\frac{m^2}{s x_1^2} + \dots\right] \qquad I_{1,3} \sim \frac{\alpha_{\text{S}}}{2\pi}\left[-1 + \frac{R_0^2}{4} + \frac{m^2}{p_{\text{T}}^2 R_0^2} + \dots\right] \qquad I_{2,3} \sim \frac{\alpha_{\text{S}}}{2\pi}\left[\frac{R_0^2}{4} + 2\frac{m^2}{p_{\text{T}}^2 R_0^2} + \dots\right]$$