

UNIVERSITÄT BERN

AEC
ALBERT EINSTEIN CENTER
FOR FUNDAMENTAL PHYSIC

Low-energy theory of jet processes & PDF factorization

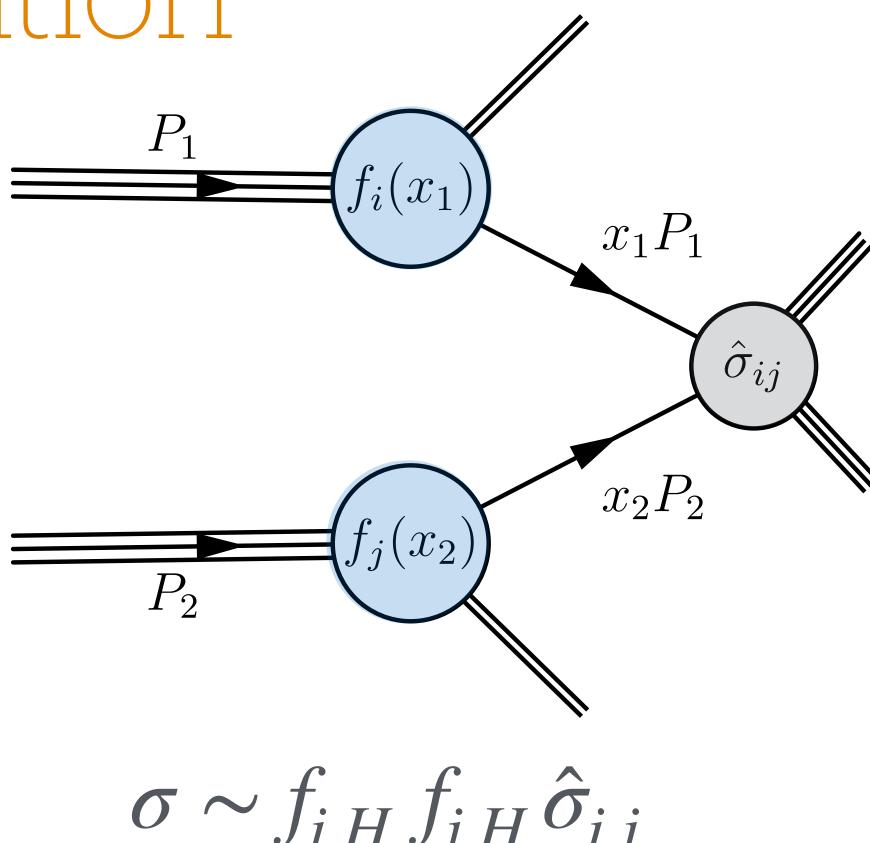
Dominik Schwienbacher

University of Bern

Based on 2408.10308, Phys.Rev.Lett. 134 (2025) 6, 061901 & 2509.07082

with Thomas Becher, Patrick Hager, Sebastian Jaskiewicz & Matthias Neubert

PDF factorization



$$\sigma \sim f_{i,H} f_{j,H} \hat{\sigma}_{i,j}$$

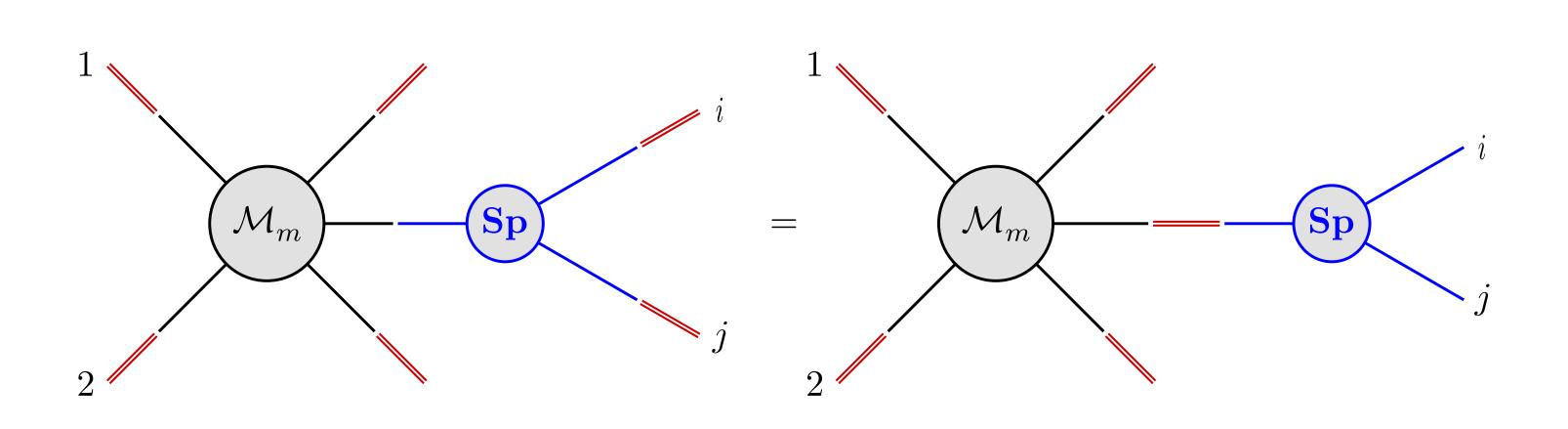
- Factorization of long- and short-range physics
- Only proof for inclusive Drell-Yan CSS, '85/'88
- Crucially, Glauber interactions cancel in this specific case

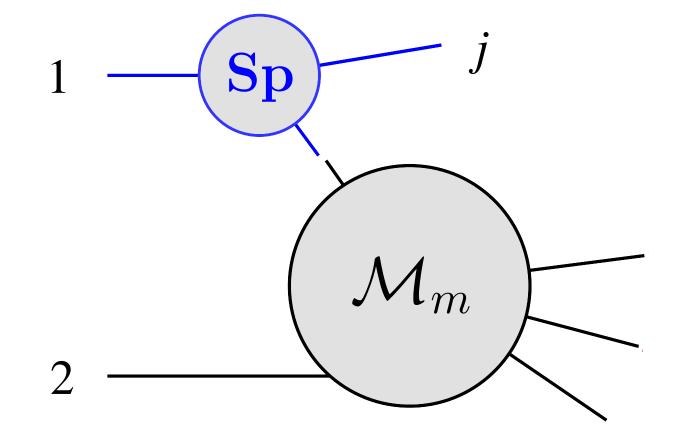
Collinear factorization

See also Daniel's and Prasanna's talks!

time-like splitting

space-like splitting





factorization works!

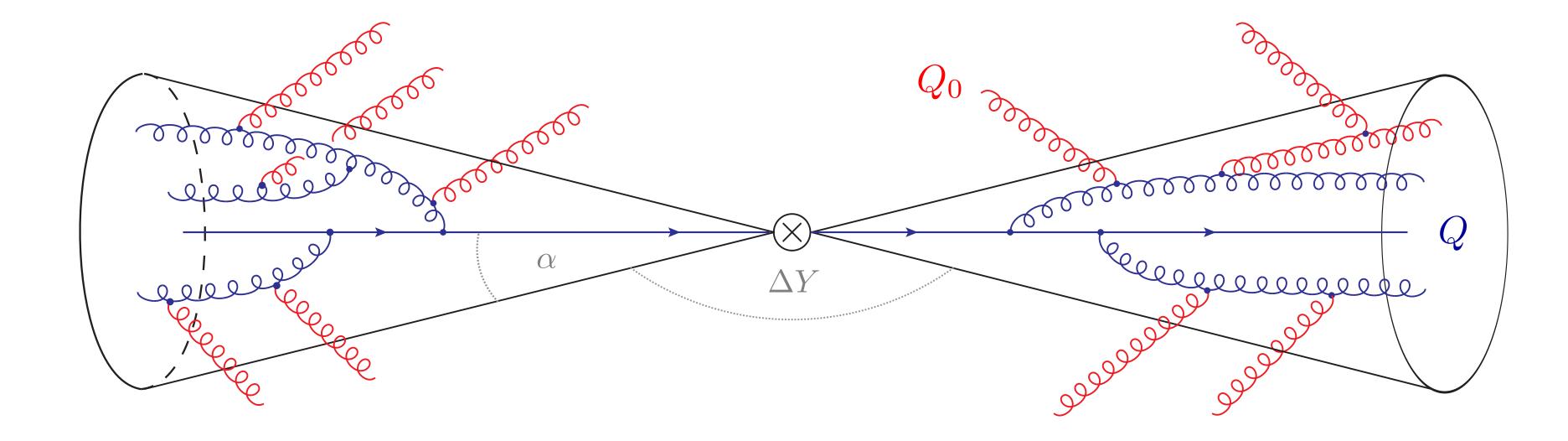
factorization violated!

Catani, de Florian, Rodrigo, '12

Observable

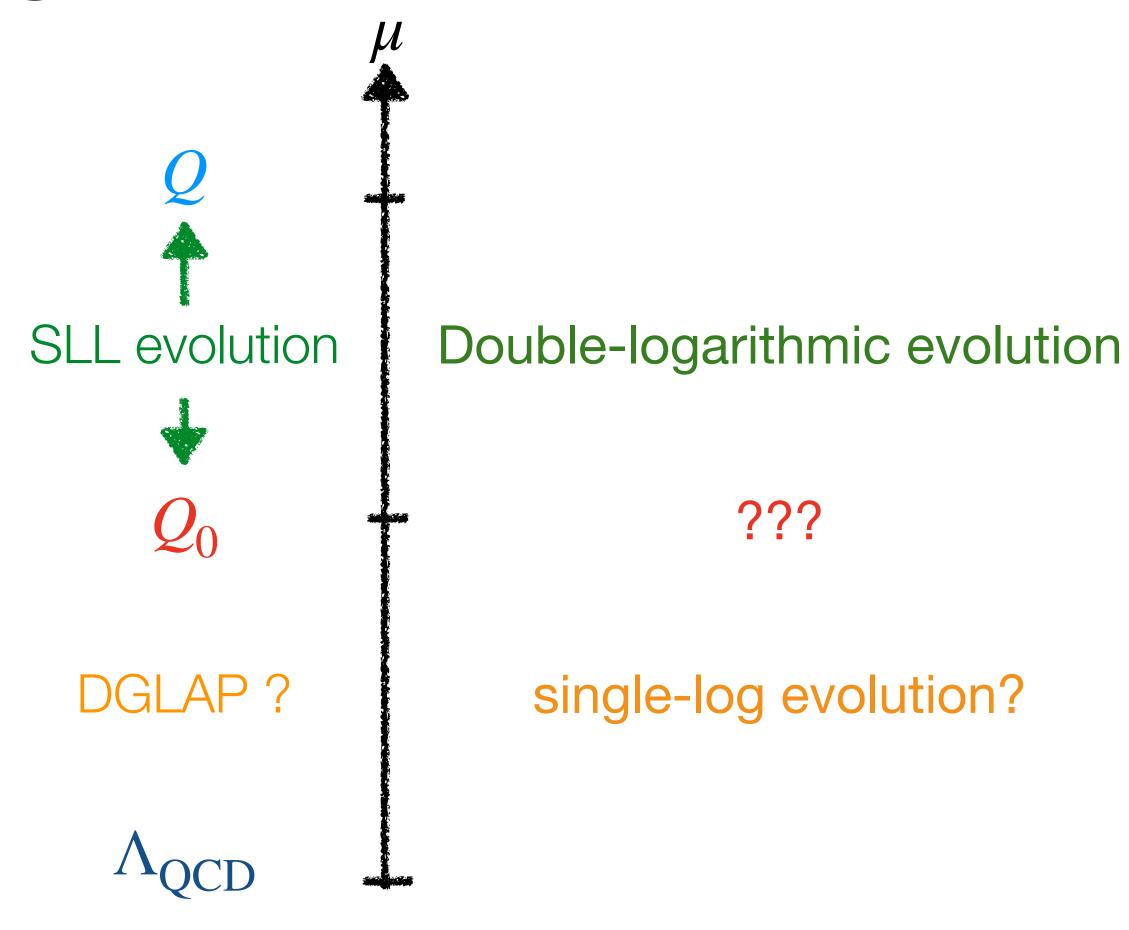
Factorization sensitive observable!

- ullet Look at gap-in-between-jets cross section with veto scale Q_0
- ullet Jet scale ${\cal Q}$ and gap ΔY



$$\sigma \sim \sigma_B + \alpha_s \ln((Q/Q_0)) + \alpha_s^2 \ln^2(Q/Q_0) + ... + \alpha_s^4 \ln^5(Q/Q_0) + ...$$

Evolution



Requires highly non-trivial interplay for consistency with DGLAP!

Upshot of the talk

Collinear factorization breaking at $\mu = Q$



soft-collinear factorization breaking by Glauber modes at $\mu=Q_0$

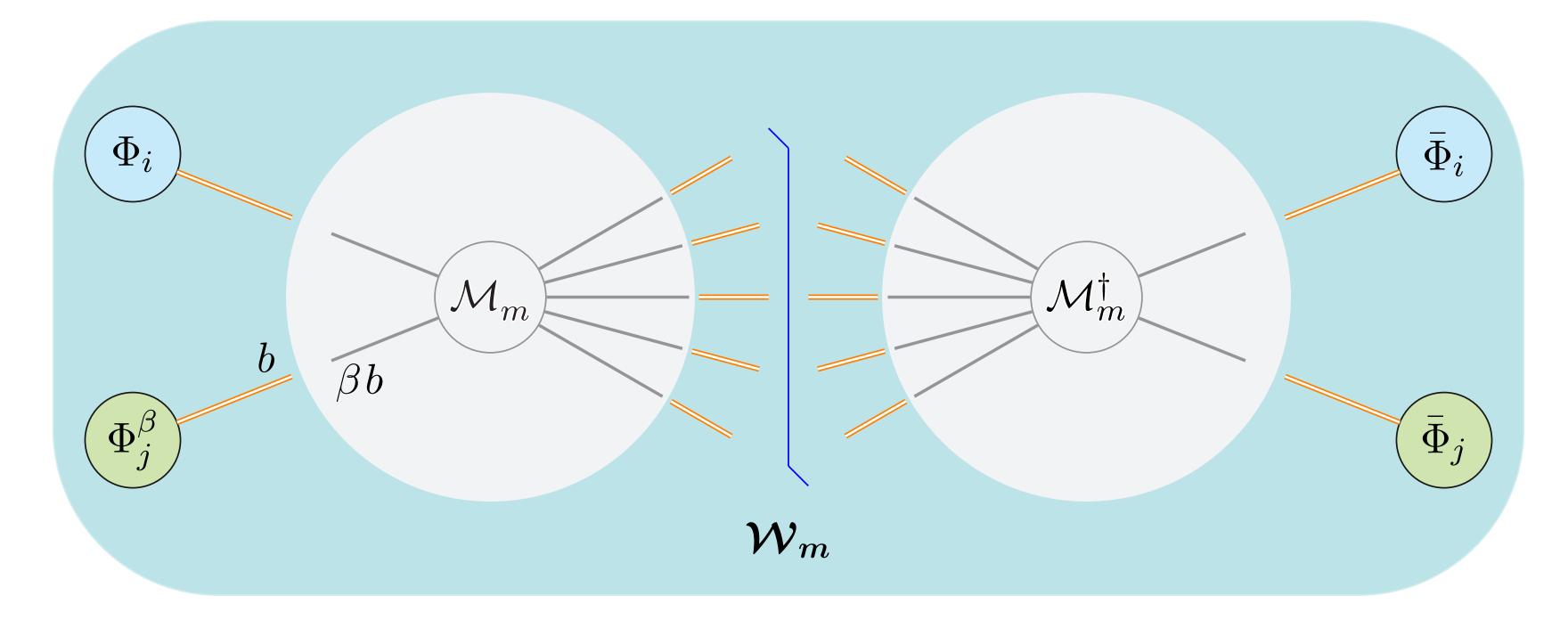
$$= \begin{array}{c} \text{PDF factorization} \\ \text{for } \mu < Q_0 \end{array}$$

"factorization restoration"

Outline

- Factorization theorem and appearance of SLLs
- RG-evolution & implications
- RG-consistency for the low-energy matrix element
 - Explicit 3-loop check
- Outlook

Resumnation

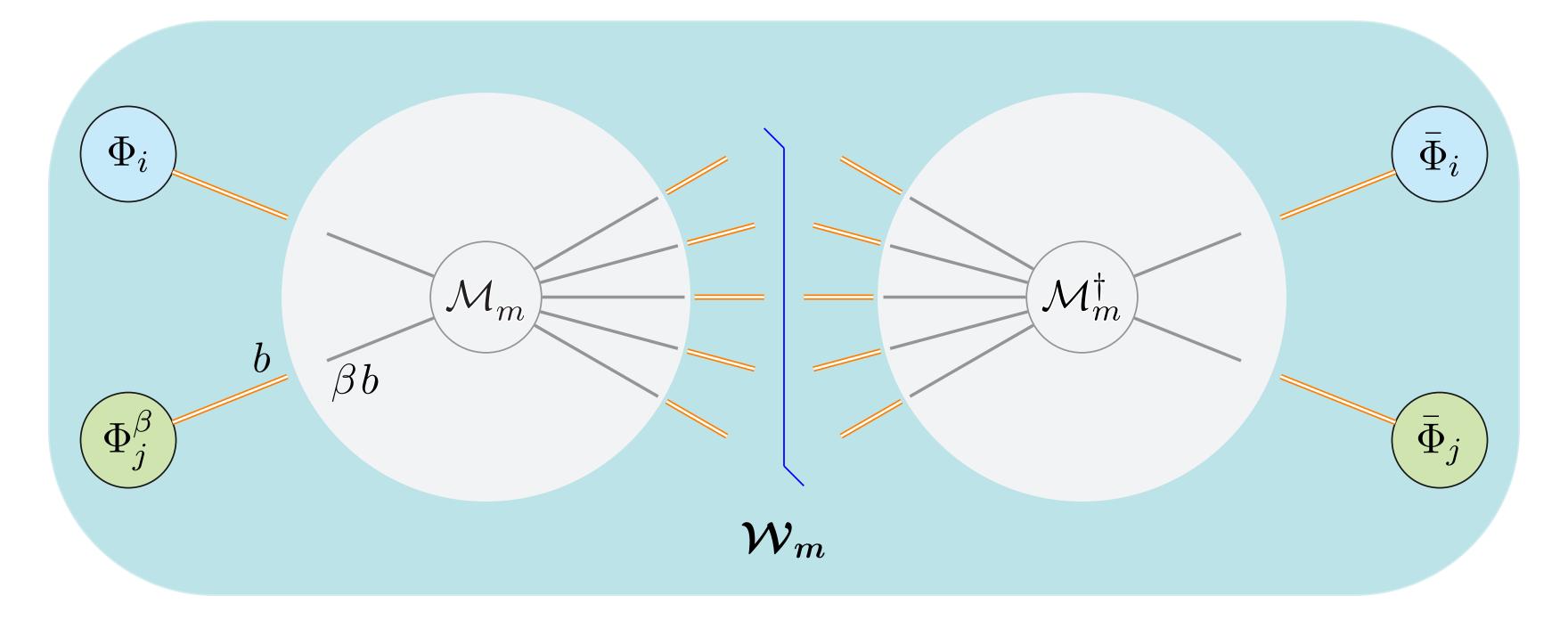


• For $Q_0 \ll Q$ we can derive

$$\sigma_{2\to M} = \sum_{m=M}^{\infty} \int d\Pi_m \int dx_1 \, dx_2 \, \langle \langle [\mathcal{H}_{ij\to m}]^{ab\,|\,\bar{a}\bar{b}}_{\alpha\beta\,|\,\bar{\alpha}\bar{\beta}} (\{\underline{p}\}, Q, \mu) \, [\mathcal{W}_{ij\to m}]^{ab\,|\,\bar{a}\bar{b}}_{\alpha\beta\,|\,\bar{\alpha}\bar{\beta}} (\{\underline{n}\}, Q_0, x_1, x_2, \mu) \rangle \rangle ,$$

Factorization between hard and soft-collinear physics

Resumnation



• For $Q_0 \ll Q$ we can derive

$$\sigma_{2\to M}(Q_0) = \int dx_1 \int dx_2 \sum_{m=2+M}^{\infty} \left\langle \mathcal{H}_m(\{\underline{n}\}, s, x_1, x_2, \mu) \otimes \mathcal{W}_m(\{\underline{n}\}, Q_0, x_1, x_2, \mu) \right\rangle$$

Becher, Neubert, Shao, '21+Stillger'23

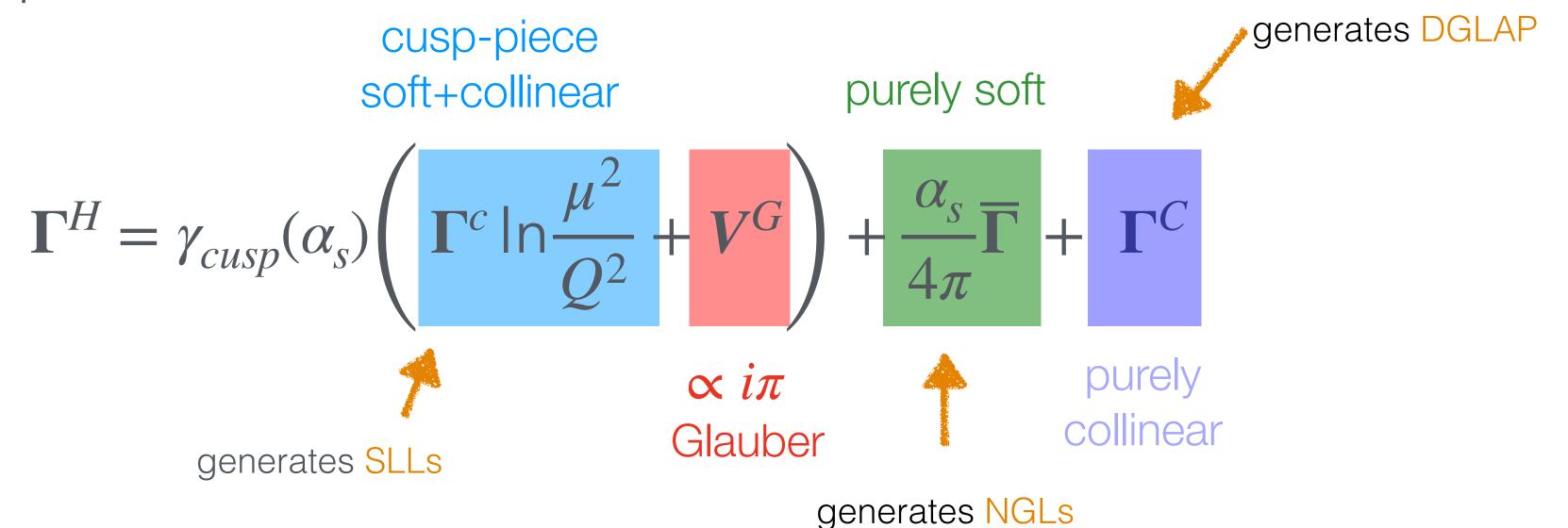
Factorization between hard and soft-collinear physics

RG-evolution

Renormalized hard functions fulfill RG equation

$$\frac{d}{d\ln\mu}\mathcal{H}_m = -\sum_{l=m_0}^m \mathcal{H}_l \frac{\Gamma_{lm}^H}{\Gamma_{lm}}$$
 matrix in multiplicity and color space

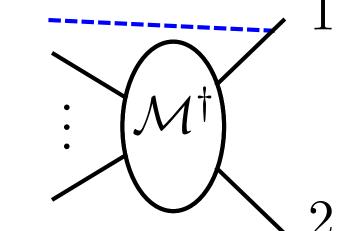
One-loop hard anomalous dimension:



Cusp terms

$$\mathcal{H}_m \, oldsymbol{R}_1^c = \left(oldsymbol{\mathcal{M}} \right) : \left(oldsymbol{\mathcal{M}}^\dagger \right) = \left(oldsymbol{\mathcal{M}} \right) : \left(oldsymbol{\mathcal{M}}^\dagger \right) = \left(oldsymbol{\mathcal{M}} \right) : \left(oldsymbol{\mathcal{M}}^\dagger \right) : \left(oldsymbol{\mathcal{M}} \right) : \left(oldsy$$

$$egin{aligned} oldsymbol{R}_i^c &= -4oldsymbol{T}_{i,L} \circ oldsymbol{T}_{i,R} \, \delta(n_{m+1} - n_i) \ oldsymbol{V}_i^c &= 4C_i \, oldsymbol{1} \end{aligned}$$



SLLs are directly connected to factorization violation!





- These are only present for initial states i=1,2, for final states they cancel
- They are multiplied by $\ln \frac{\mu^2}{\Omega^2}$ and give rise to double-logarithmic running!

SLLs from RG evolution

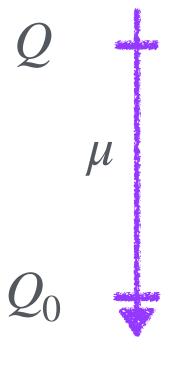
• Evolve hard function from $\mu_h \sim Q$ to $\mu_s \sim Q_0$

$$\frac{d}{d\ln\mu}\mathcal{H}_m = -\sum_{l=m_0}^m \mathcal{H}_l \frac{\mathbf{\Gamma}_{lm}^H}{\mathbf{\Gamma}_{lm}^H}$$

$$\sigma(Q, Q_0) = \sum_{m,l=m_0}^{\infty} \int d\xi_1 d\xi_2 \left\langle \mathcal{H}_m(Q, \mu_h) U_{ml}(\mu_h, \mu_s) \otimes \mathcal{W}_l(Q_0, \mu_s) \right\rangle$$

$$U(\mu_h, \mu_s) = \mathbf{P} \exp \left[\int_{\mu_s}^{\mu_h} \frac{d\mu}{\mu} \mathbf{\Gamma}^H(\mu) \right]$$

$$= \mathbf{1} + \int_{\mu_s}^{\mu_h} \frac{d\mu_1}{\mu_1} \mathbf{\Gamma}^H(\mu_1) + \int_{\mu_s}^{\mu_h} \frac{d\mu_1}{\mu_1} \int_{\mu_1}^{\mu_h} \frac{d\mu_2}{\mu_2} \mathbf{\Gamma}^H(\mu_1) \mathbf{\Gamma}^H(\mu_2)$$

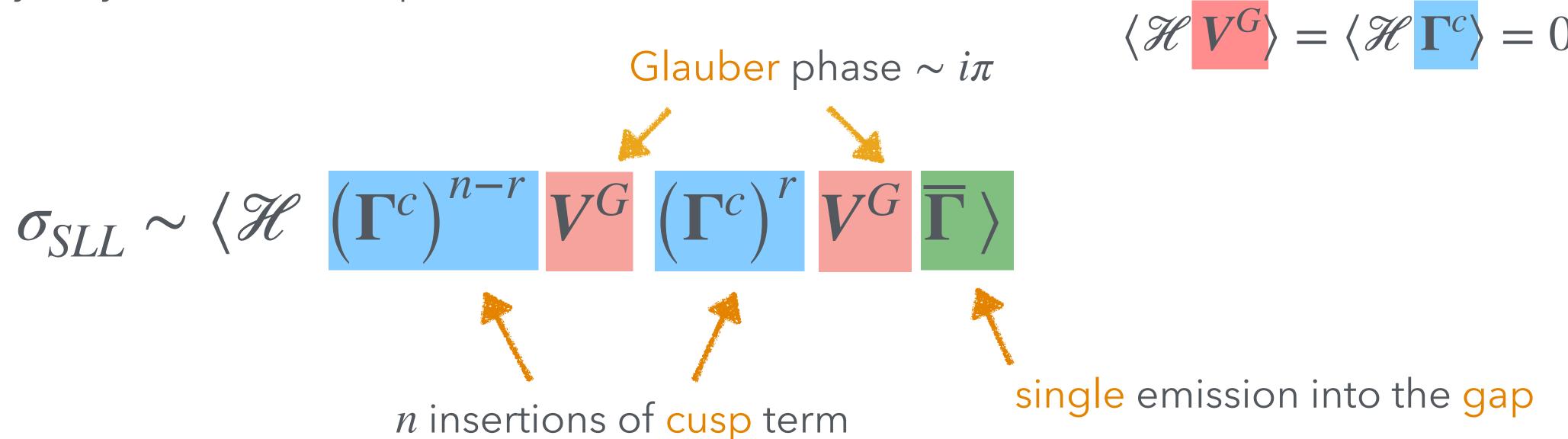


"tower" of anomalous dimensions

 $U(\mu_h, \mu_s)$ achieves resummation of logarithms

SLLs

- ullet For finite N_c Glauber phases spoil collinear cancellations
 - Appearance of super-leading logarithms
 - Only very few structures possible



Outline

Factorization theorem and appearance of SLL & formalism



- RG-consistency check for the low-energy matrix element
 - Know that $\langle \mathcal{H}_m(\{\underline{n}\}, s, x_1, x_2, \mu) \otimes \mathcal{W}_m(\{\underline{n}\}, Q_0, x_1, x_2, \mu) \rangle$ must be finite!
 - Use

$$\langle \mathcal{H}_{m}^{\text{bare}} \mathcal{W}_{m}^{\text{bare}} \rangle = \langle (\mathcal{H}_{m}^{\text{bare}} \mathbf{Z}^{-1}) (\mathbf{Z} \mathcal{W}_{m}^{\text{bare}} \rangle$$

$$\mathcal{W}^{\text{bare}} = \mathcal{I}^{\text{bare}} * f^{\text{bare}} f^{\text{bare}} = \mathcal{I}^{\text{bare}} * Z_{\text{PDF}}^{-1} * f f$$

$$\text{finite} \qquad \text{finite}$$

RG-consistency

ullet Renormalization factor Z given by

$$\mathbf{Z} = \mathbf{1} + \frac{\alpha_s}{4\pi} \left(-\frac{\mathbf{\Gamma}_0'}{4\varepsilon^2} - \frac{\mathbf{\Gamma}_0}{2\varepsilon} \right) + \left(\frac{\alpha_s}{4\pi} \right)^2 \left(\frac{\mathbf{\Gamma}_0'\mathbf{\Gamma}_0}{32\varepsilon^3} + \frac{\mathbf{\Gamma}_0^2}{8\varepsilon^2} + \dots \right) + \left(\frac{\alpha_s}{4\pi} \right)^3 \left(-\frac{\mathbf{\Gamma}_0'\mathbf{\Gamma}_0^2}{288\varepsilon^4} - \frac{\mathbf{\Gamma}_0^3}{48\varepsilon^3} + \dots \right) + \mathcal{O}(\alpha_s^4)$$

Becher, Neubert'09

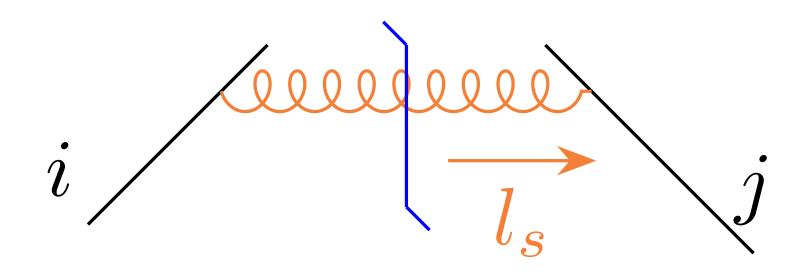
• Soft-collinear matrix element has to be rendered finite $\mathcal{W}_m(\mu) = Z \cdot \mathcal{W}_m^{\mathrm{bare}}$ such that

$$\mathcal{F}_{m}^{\text{bare}} = 1 + \frac{\alpha_{s}}{4\pi} \frac{\overline{\Gamma}}{2\varepsilon} + \left(\frac{\alpha_{s}}{4\pi}\right)^{2} \left(\frac{V^{G}\overline{\Gamma}}{2\varepsilon^{2}} + \dots\right) + \left(\frac{\alpha_{s}}{4\pi}\right)^{3} \left[\frac{\Gamma^{c}V^{G}\overline{\Gamma}}{3\varepsilon^{3}} \left(\frac{11}{6\varepsilon} + \ln\frac{\mu_{s}^{2}}{Q^{2}} + \frac{9}{2}\ln\frac{\mu_{s}^{2}}{Q_{0}^{2}}\right) + \frac{V^{G}V^{G}\overline{\Gamma}}{3\varepsilon^{3}} + \frac{\left[\Gamma^{C}, V^{G}\right]\overline{\Gamma}}{12\varepsilon^{3}} + \dots\right]$$

 $+\mathcal{O}(\alpha_s^4)$

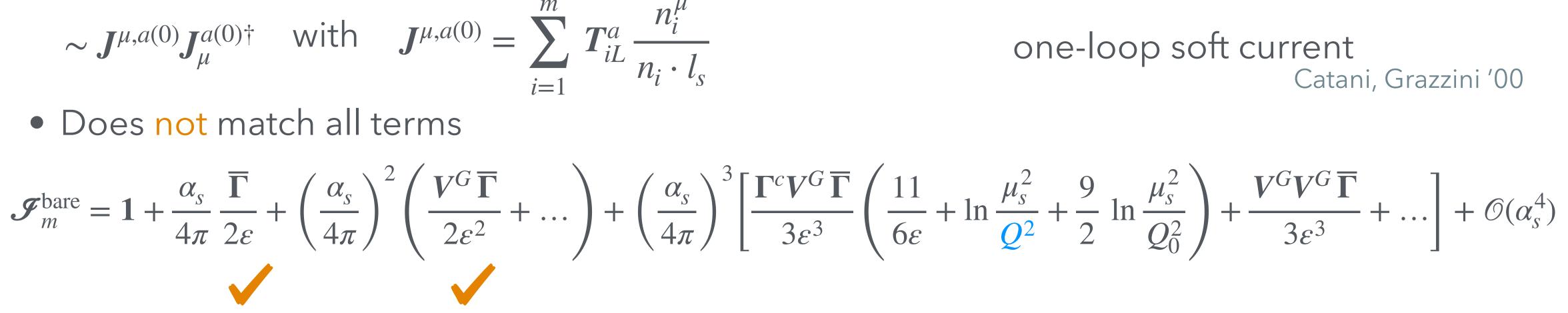
RG-consistency

Look at tree-level, one-loop & two-loop level



eikonal factor

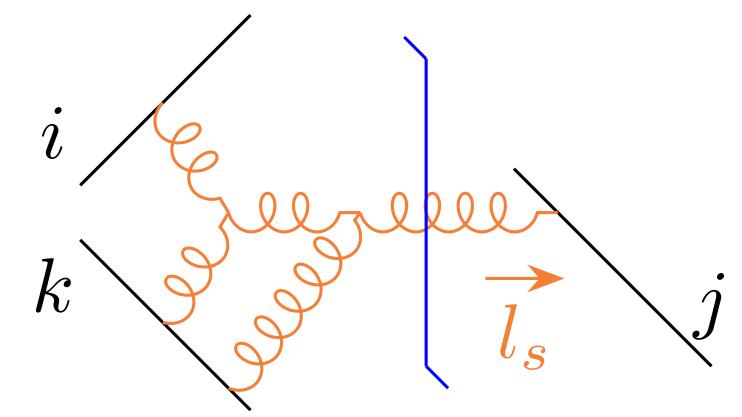
$$\sim J^{\mu,a(0)}J_{\mu}^{a(0)\dagger}$$
 with $J^{\mu,a(0)}=\sum_{i=1}^{m}T_{iL}^{a}\frac{n_{i}^{\mu}}{n_{i}\cdot l_{s}}$

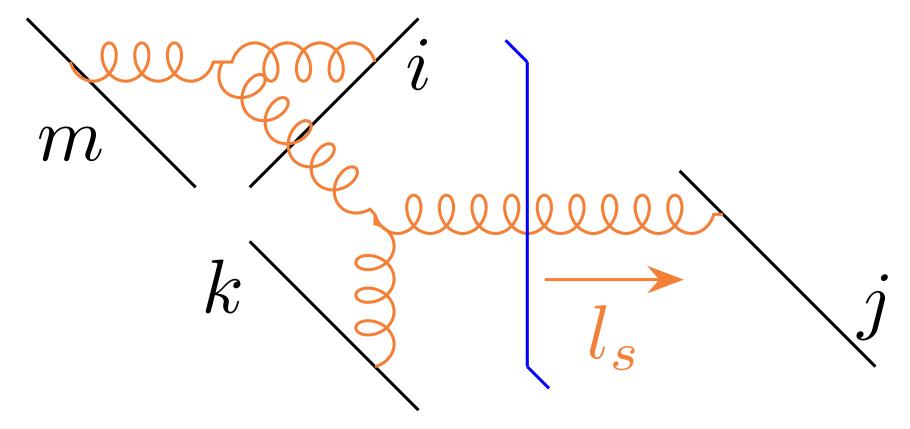


RG-consistency

Now, go one loop further

Duhr, Gehrmann '13 / Dixon, Herrmann, Yan, Zhu '20





dipole terms

Large logarithm

tripole terms

Color-aware DGLAP

Does not match all terms

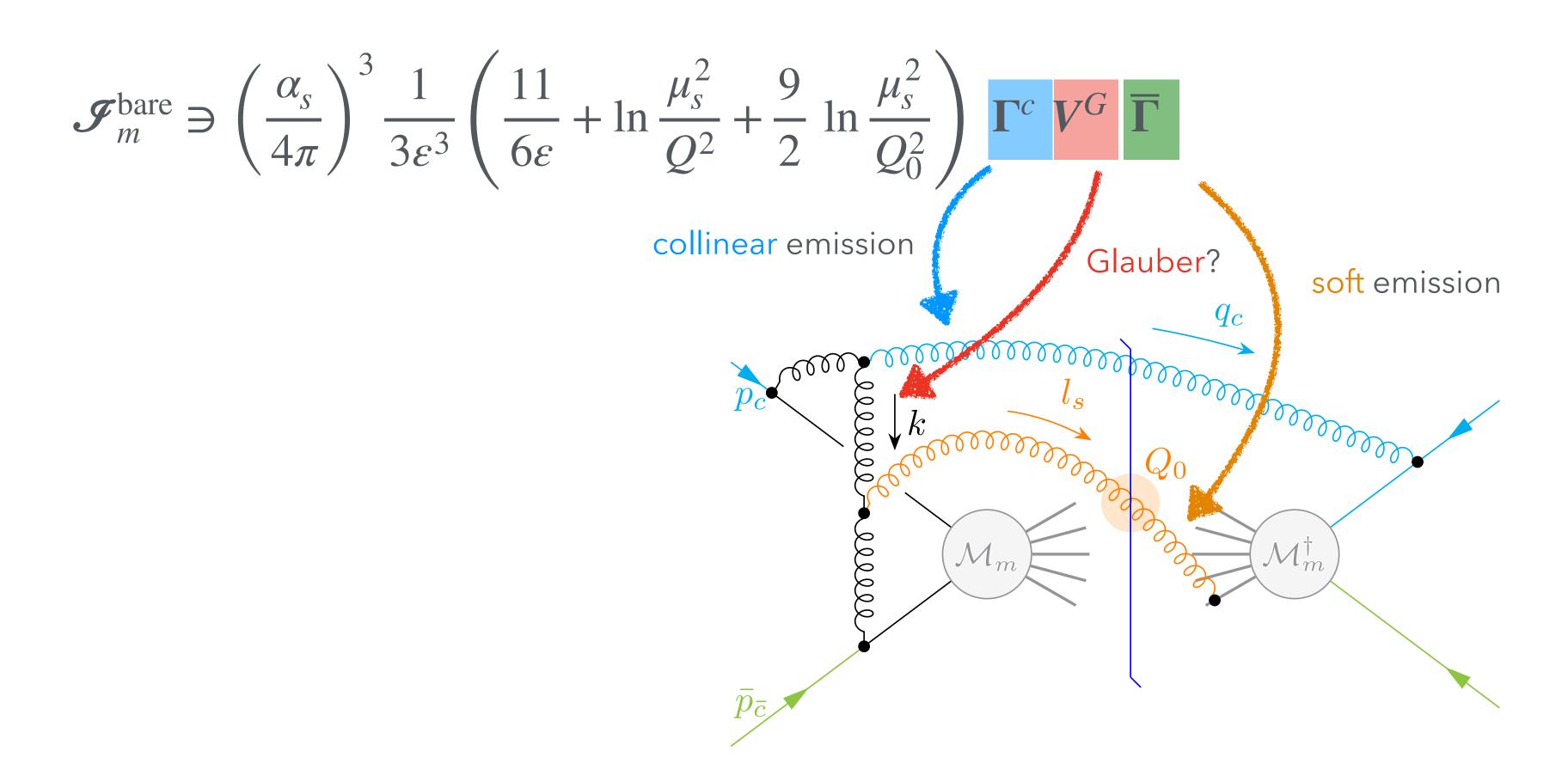
$$\mathcal{J}_{m}^{\text{bare}} = \mathbf{1} + \frac{\alpha_{s}}{4\pi} \frac{\overline{\Gamma}}{2\varepsilon} + \left(\frac{\alpha_{s}}{4\pi}\right)^{2} \left(\frac{V^{G}\overline{\Gamma}}{2\varepsilon^{2}} + \dots\right) + \left(\frac{\alpha_{s}}{4\pi}\right)^{3} \left[\frac{\Gamma^{c}V^{G}\overline{\Gamma}}{3\varepsilon^{3}} \left(\frac{11}{6\varepsilon} + \ln\frac{\mu_{s}^{2}}{Q^{2}} + \frac{9}{2} \ln\frac{\mu_{s}^{2}}{Q^{2}}\right) + \frac{V^{G}V^{G}\overline{\Gamma}}{3\varepsilon^{3}} + \frac{\left[\Gamma^{C}, V^{G}\right]\overline{\Gamma}}{12\varepsilon^{3}} + \dots\right] + \mathcal{O}(\alpha_{s}^{4})$$

Three options to get ln Q

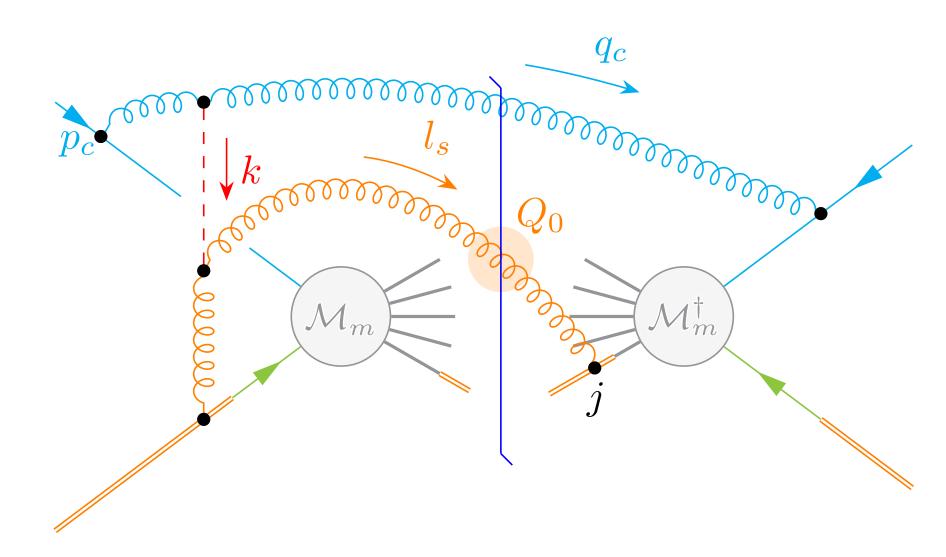
- 1. Perturbative on-shell modes with virtuality below Q_0
 - e.g. Ultra-soft modes or soft-collinear modes
- 2. A collinear anomaly inducing rapidity logarithms
 - In our case, the collinear alone is scaleless Glauber is needed
- 3. Non-Perturbative low-energy interactions among incoming hadrons
 - Complete breaking of PDF factorization! Non perturbative two-nucleon matrix elements

Glauber from method of regions

Look at terms we do not match



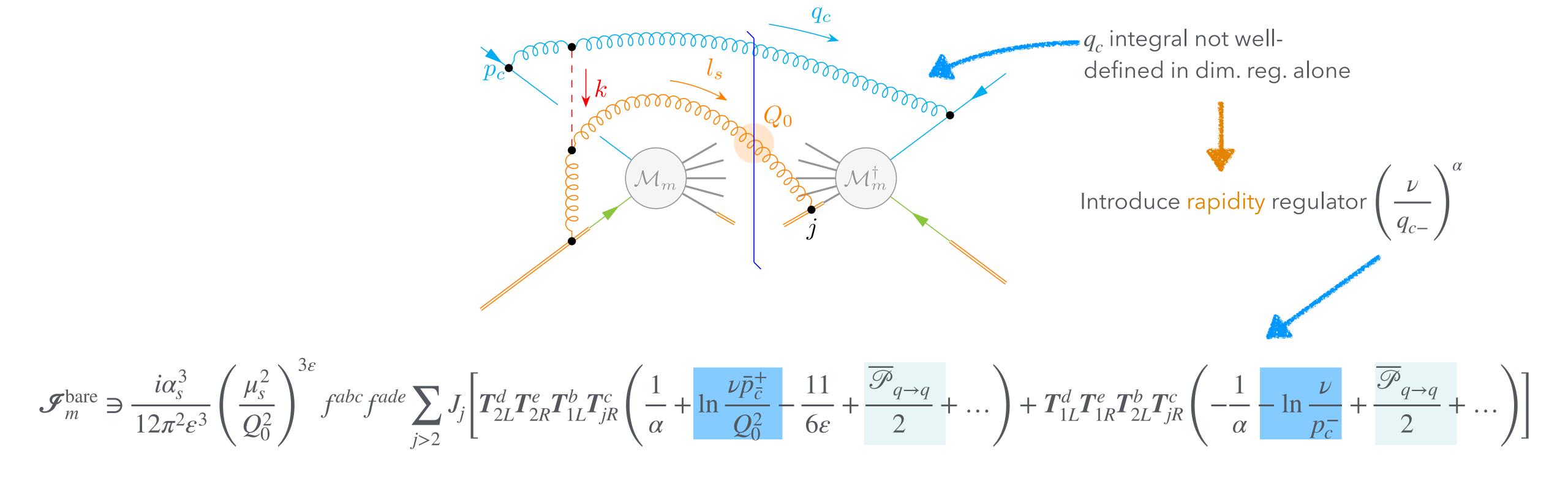
Effective Glauber vertex



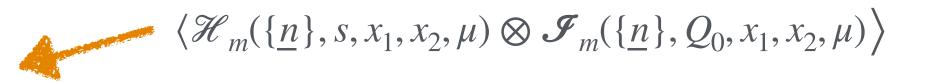
- Soft-collinear mode decouples
- Only contribution due to Glauber exchange
 - Either use expanded QCD or effective vertex using Glauber SCET

Rothstein, Stewart '16

Calculate the diagram & matching



- ullet lpha-poles cancel in between collinear and anti-collinear sector
- For gluonic contributions new structures $\propto n_{i\perp}^{\mu} n_{i\perp}^{\bar{\mu}}$



Under color trace we arrive at

$$\mathcal{J}_{m}^{\text{bare}} \ni \frac{\left(\frac{\alpha_{s}}{4\pi}\right)^{3} \frac{1}{12} \langle \mathcal{H}\left[\mathbf{\Gamma}^{C}, \mathbf{V}^{G}\right] \overline{\mathbf{\Gamma}} \rangle = \frac{\alpha_{s}^{3}}{24} \overline{\mathcal{P}}_{i \to i} \pi N_{c} f^{abc} \sum_{j>2}^{'} J_{j} \langle \mathcal{H} \mathbf{T}_{1}^{a} \mathbf{T}_{2}^{b} \mathbf{T}_{j}^{c} \rangle}{\left(\frac{\alpha_{s}}{4\pi}\right)^{3} \frac{1}{3\varepsilon^{3}} \left(\frac{11}{6\varepsilon} + \ln \frac{\mu_{s}^{2}}{Q^{2}} + \frac{9}{2} \ln \frac{\mu_{s}^{2}}{Q_{0}^{2}}\right) \mathbf{\Gamma}^{c} \mathbf{V}^{G} \overline{\mathbf{\Gamma}}}$$

Perturbative Glauber contribution yields

- Correct ln Q term
- Correct $\frac{1}{\varepsilon^4}$ pole
- Correct DGLAP

John Consistent with both SLL and DGLAP evolution!

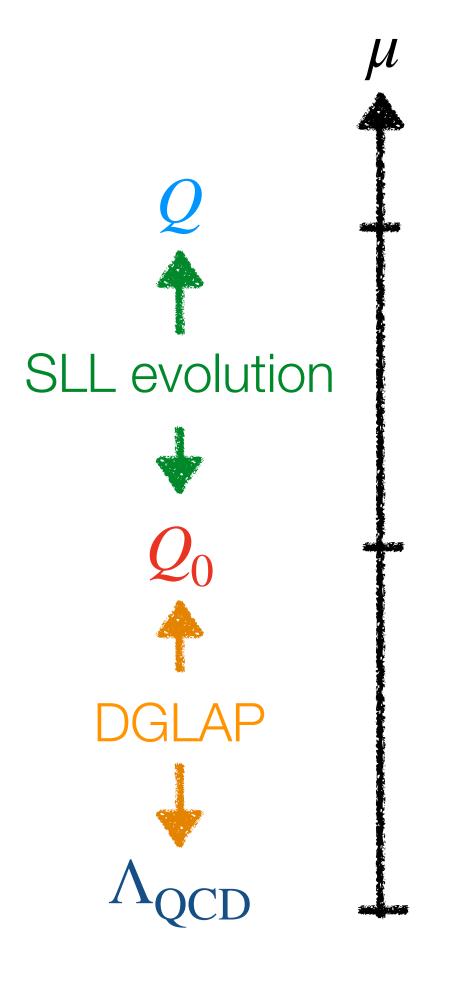
What happens below Q_0 ?

- Soft-collinear mode cancelled
 - ullet No modes probing physics below Q_0
- Match onto collinear fields $k^2 \sim \Lambda_{\rm OCD}^2$, along with their Wilson lines

Bauer, Fleming, Pirjol, Rothstein and Stewart '02

- ullet Below Q_0 the process becomes inclusive
 - Soft Wilson lines cancel
 - Collinear fields match onto PDFs
 - Operators & effective theory same as for Drell-Yan
 - Glauber modes cancel from CSS arguments

Conclusion



phase factors soft+collinear contributions double-log evolution

soft-collinear interaction Glauber contribution

"factorization restoration"

single-log evolution

Outlook

- Showed consistency of PDF factorization at least up to 3 loops
 - All elements of factorization breaking are present but cancel in exactly the right way
- Look at higher loops e.g. $\mathcal{F}_m^{(4)}$?

- All-order structure of Glauber terms
 - Proof of factorization?
 - Implications for other observables e.g. global event shapes & factorization theorems?

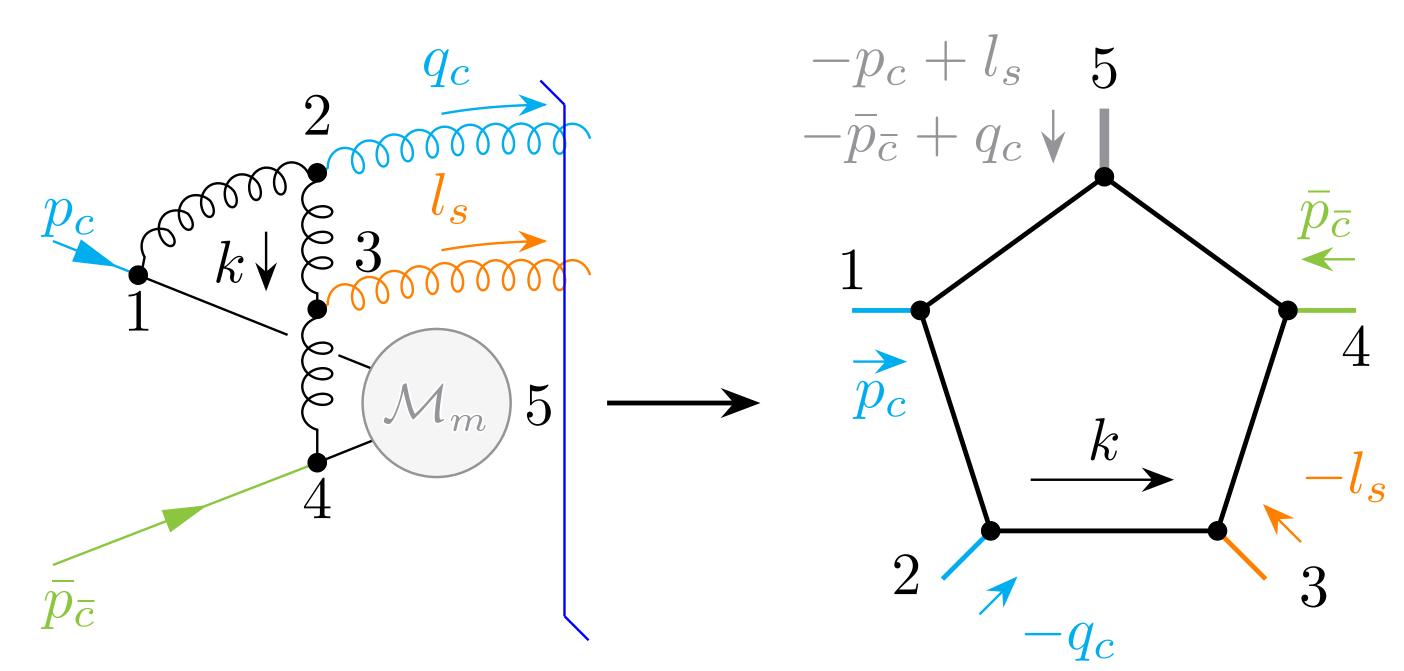
Backup

One loop soft current

$$\boldsymbol{J}^{\mu,a(1)} = -\frac{1}{(4\pi)^2} \frac{\Gamma^3(1-\varepsilon) \Gamma^2(\varepsilon)}{\Gamma(1-2\varepsilon)} \times i f^{abc} \sum_{i \neq j} \boldsymbol{T}_{iL}^b \boldsymbol{T}_{jL}^c \left(\frac{n_i^{\mu}}{n_i \cdot l_s} - \frac{n_j^{\mu}}{n_j \cdot l_s} \right) \left[\frac{2\pi \ n_i \cdot n_j \ e^{-i\lambda_{ij}\pi}}{n_i \cdot l_s \ n_j \cdot l_s \ e^{-i\lambda_{il}\pi} \ e^{-i\lambda_{jl}\pi}} \right]^{\varepsilon}$$

MoR analysis

- Make certain we are not missing any regions
 - Map to well-known scalar pentagon result
 Bern, Dixon, Kosower '94
 - Expand pentagon result in λ
 - Ascertain that full result is recovered using all possible regions



Glauber contribution

- In Euclidean region $s_{ij}=(p_i+p_j)^2<0, p_5^2<0$ only soft-collinear region with $k\sim(\lambda^2,\lambda,\lambda^{3/2})$
 - Cancels after q_c integration!
 - Leads to a "hidden" region with $k \sim (\lambda^2, \lambda, \lambda)$ for physical scattering region
 - Couples soft and collinear sectors collinear factorization breaking
- Perform k_+ and k_- integral via residues
 - Well-defined without additional regulators

$$I^{g} = i(4\pi)^{2-\varepsilon} \int \frac{d^{d}k}{(2\pi)^{d}} \frac{1}{-k_{T}^{2}} \frac{1}{k^{+}q_{c}^{-} - k_{T}^{2} - 2k_{T} \cdot q_{cT}}$$

$$\times \frac{1}{\left[-k^{+}(p_{c}^{-} - q_{c}^{-}) - q_{c}^{+}p_{c}^{-} - k_{T}^{2} - 2k_{T} \cdot q_{cT}\right]}$$

$$\times \frac{1}{\bar{p}_{\bar{c}}^{+}(k^{-} - l_{s}^{-})} \frac{1}{-l_{s}^{+}k^{-} - k_{T}^{2} + 2k_{T} \cdot l_{sT}}$$

Euclidean of-shell triangle in $d-2\varepsilon$

Glauber region in parameter space

Can perform region analysis in Schwinger or Lee-Pomeransky parameter space (like **Asy** and **PySecDec**)

$$(\overline{x}_1, x_2, x_3, x_4, x_5) \sim (\lambda^{-2}, \lambda^{-2}, \lambda^{-2}, \lambda^{-1}, \lambda^{-2})$$

$$\mathcal{F} = -\underbrace{x_1 x_3 s_{23}}_{\lambda^{-3}} - \underbrace{x_1 x_4 s_{51}}_{\lambda^{-3}} - \underbrace{x_3 x_5 s_{45}}_{\lambda^{-3}}$$
$$- \underbrace{x_4 x_5 m^2}_{\lambda^{-3}} - \underbrace{x_2 x_4 s_{34}}_{\lambda^{-2}} - \underbrace{x_2 x_5 s_{12}}_{\lambda^{-2}}$$

The Glauber region corresponds to a pinch due to cancellations in the ${\mathcal F}$ polynomial

$$\mathcal{F} = \underbrace{\left(-q_c^- x_1 + (p_c^- - q_c^-) x_5\right)}_{\lambda^{-2}} \underbrace{\left(l_s^+ x_3 - \bar{p}_{\bar{c}}^+ x_4\right)}_{\lambda^{-1}}$$

Euclidean region

Introduce kinematics

$$s_{12} = -p_c^- q_c^+, \quad s_{23} = q_c^- l_s^+, \quad s_{45} = -(p_c^- - q_c^-) l_s^+,$$

 $s_{34} = -\bar{p}_{\bar{c}}^+ l_s^-, \quad s_{51} = -q_c^- \bar{p}_{\bar{c}}^+, \quad p_5^2 = (p_c^- - q_c^-) \bar{p}_{\bar{c}}^+$

- In Euclidean region $s_{ij}=(p_i+p_j)^2<0, p_5^2<0$ only soft-collinear region with $k\sim(\lambda^2,\lambda,\lambda^{3/2})$
 - Also found by Asy2.1 & pySecDec
 - Compatible with option 1)
 - But decouples completely after q_c integration

Physical region

• For physical region extra terms due to cancellation

soft-collinear

$$k \sim (\lambda^2, \lambda, \lambda^{3/2})$$

$$\underbrace{s_{45}s_{51}}_{\lambda} - \underbrace{p_5^2s_{23}}_{\lambda} = \underbrace{p_c^- \bar{p}_{\bar{c}}^+ (q_{cT} + l_{sT})^2}_{\lambda^2} > 0$$

• Terms (proportional to a prefactor) arise

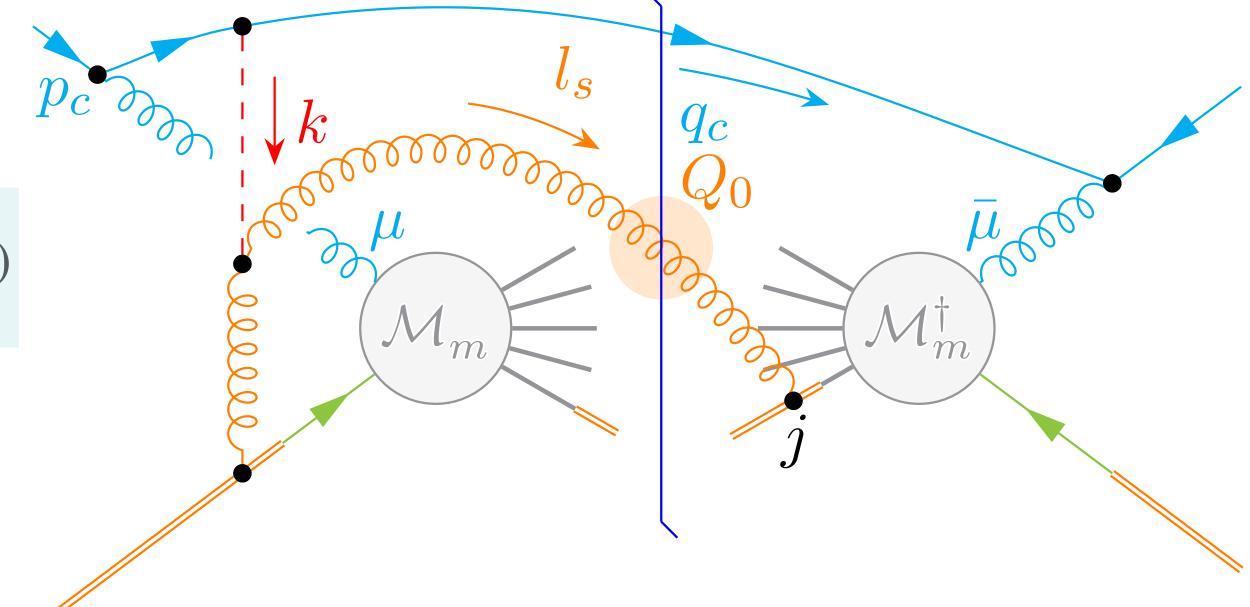
$$P = \underbrace{\frac{s_{45}s_{51}}{s_{45}s_{51} - p_5^2s_{23}}}_{\lambda^{-1}} \left[1 - e^{i\pi\varepsilon\Theta} \left(1 + \underbrace{\frac{p_5^2s_{23} - s_{45}s_{51}}{s_{45}s_{51}}} \right)^{-\varepsilon} \right]$$

$$P \sim \begin{cases} 1 & \text{for } \Theta = 0\\ \lambda^{-1} & \text{for } \Theta \neq 0 \end{cases}$$

Power enhancement in physical region, due to complex phase!

Off-diagonal splitting

$$\mathcal{J} \ni \frac{16}{3 \epsilon^{3}} \left(\frac{\alpha_{s}}{4\pi}\right)^{3} \pi f^{dbc} \sum_{j>2} \operatorname{Tr}\left(t^{\overline{a}} t^{d} t^{a}\right) T_{2}^{b} T_{j}^{c} J_{j} \overline{\mathcal{P}}_{q \to g}(z)$$



- Reproduces color-aware DGLAP
- @ leading pole simple Lorentz structure $\propto g^{\mu\bar{\mu}}$
- @ sub-leading poles new structures $\propto n_{j\perp}^{\mu} n_{j\perp}^{\bar{\mu}}$
 - Generalized factorization theorem