

Quantum Scaling in Energy Correlators: Perturbative and Non-perturbative Aspects

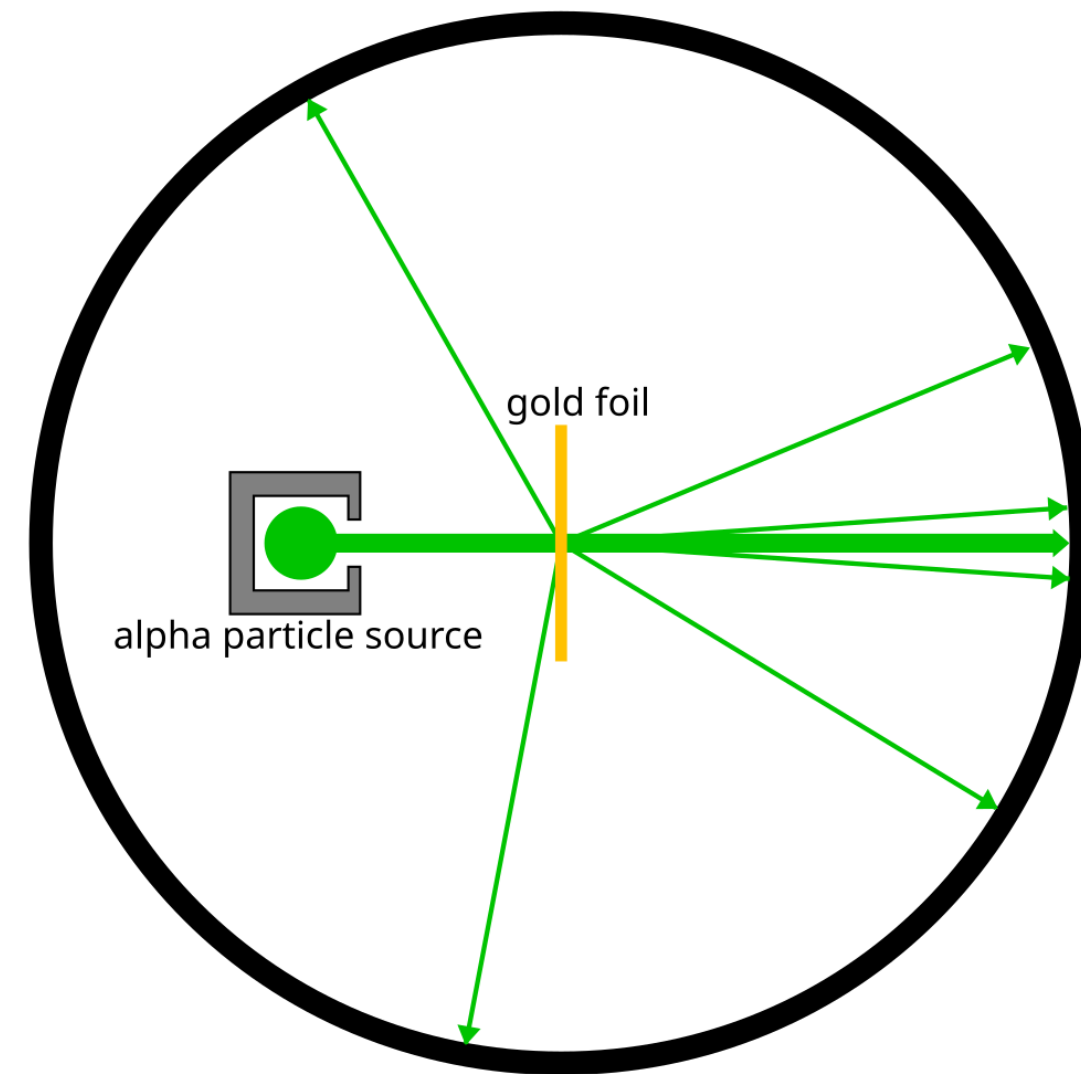
Hao Chen (MIT)

with Cyuan-Han Chang, Xiaohui Liu, David Simmons-Duffin, Feng Yuan, Hua Xing Zhu [2507.15923]
with Pier Monni, Zhen Xu, Hua Xing Zhu [2406.06668]

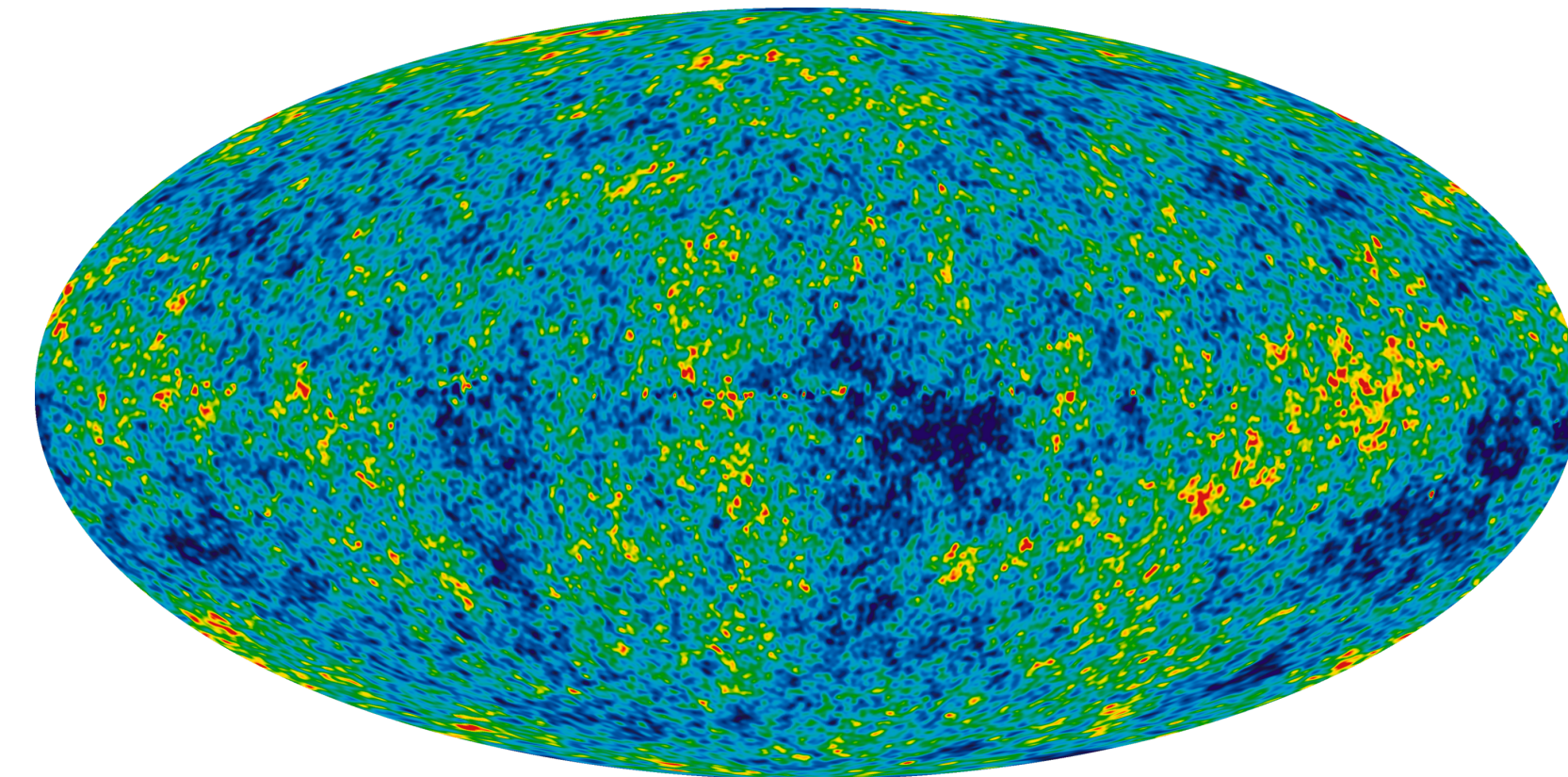
REF 2025, Milan University

October 13, 2025

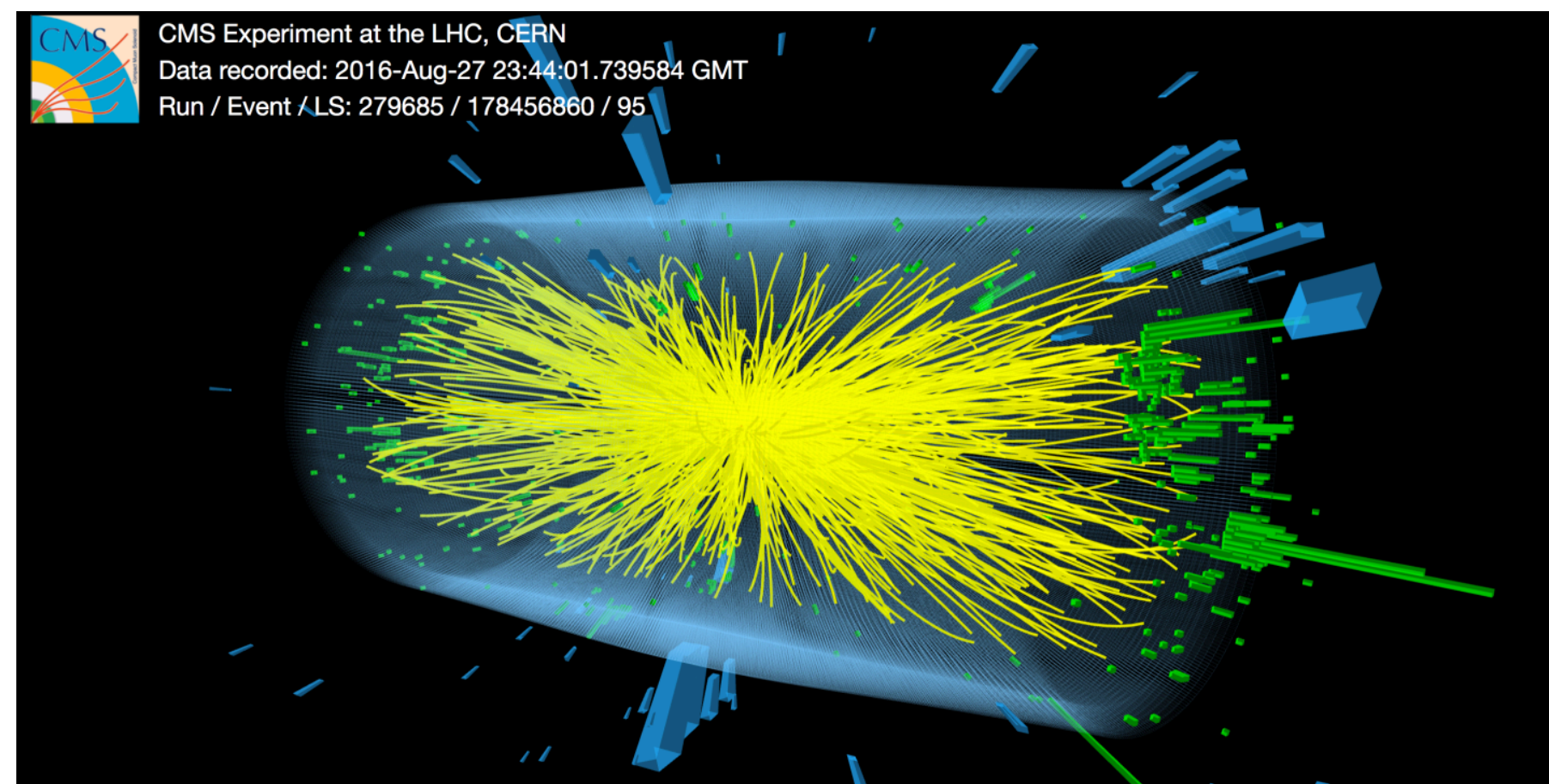
Energy/particle flux is a useful tool to understand microscopic/underlying physics.



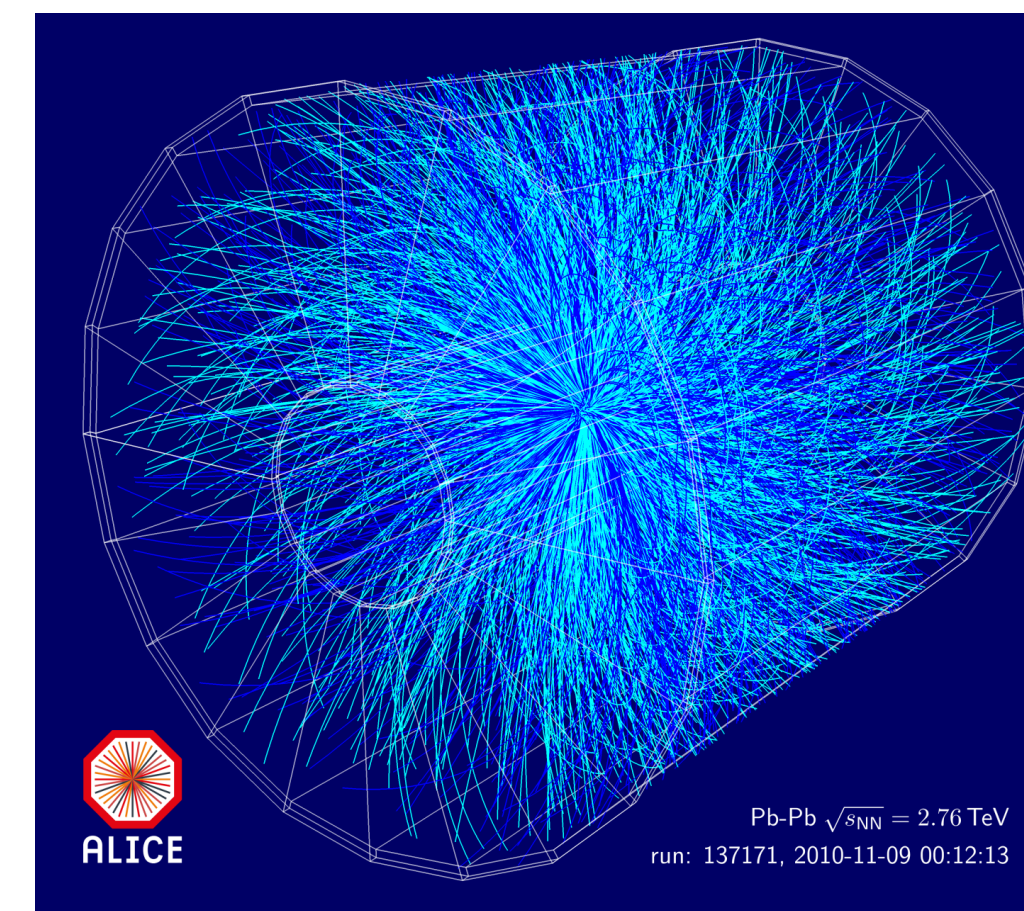
Rutherford Experiment



CMB



LHC collisions

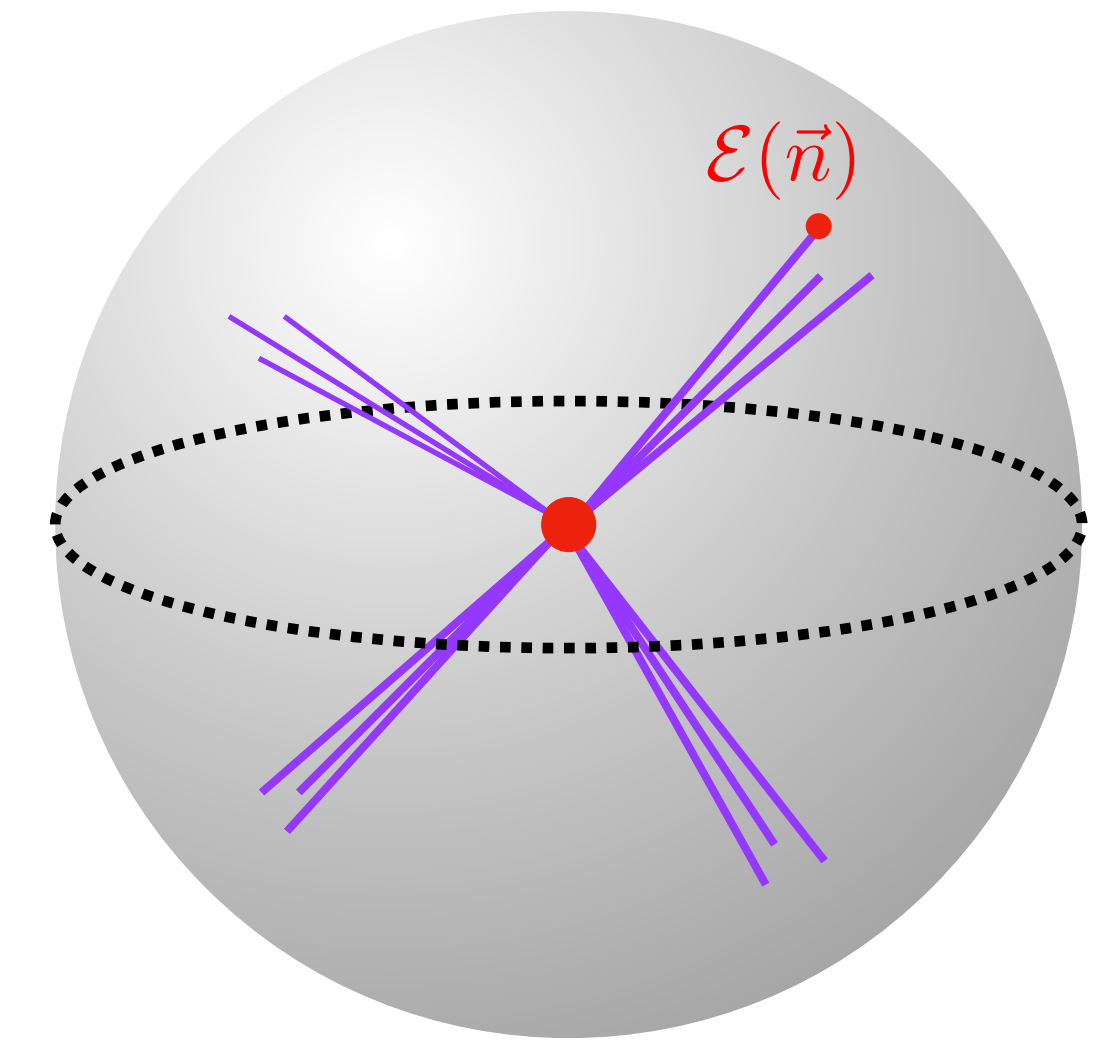


Energy Flux in Particle Physics and Field Theory

$\mathcal{E}(\vec{n})$ measures energy flux along specific angles:

$$\mathcal{E}(\vec{n})|X\rangle = \sum_{i \in X} E_i \delta(\hat{p}_i - \vec{n})|X\rangle \quad [\text{Sternman, 1975}]$$

50 Years of Energy Flux in QCD

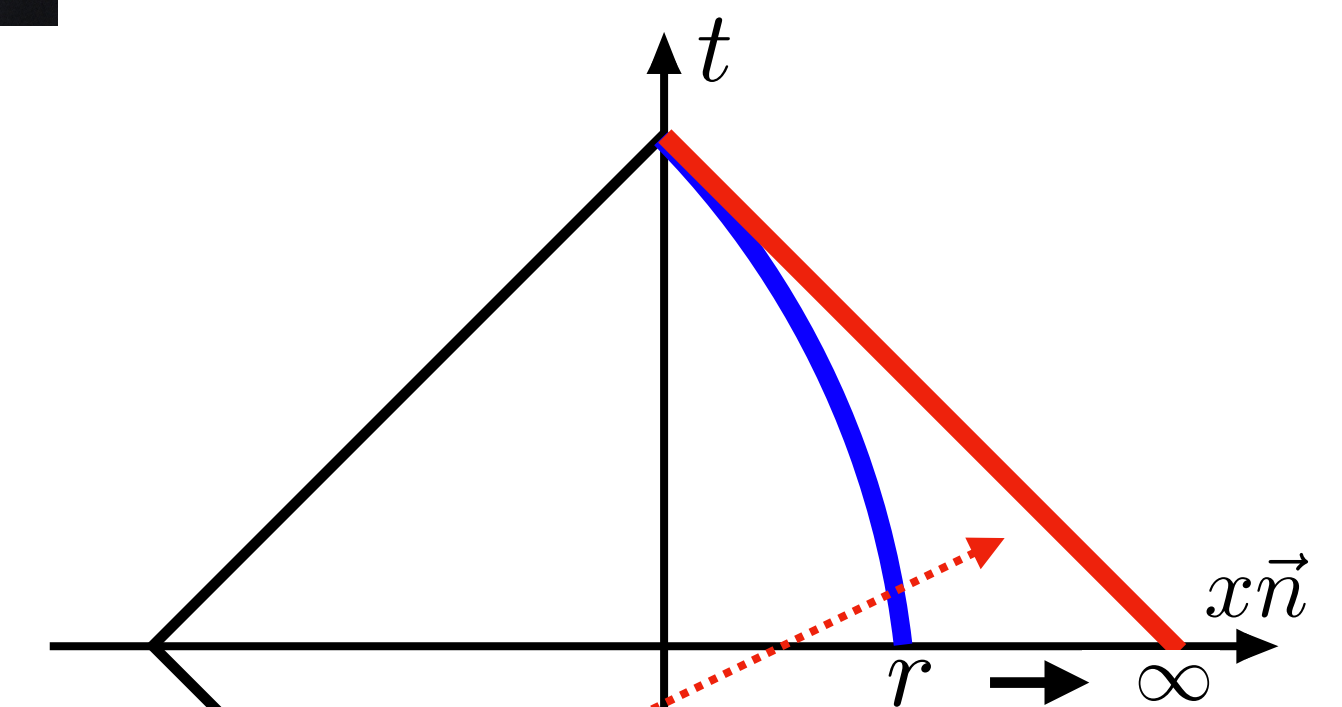


Energy detector is a light-ray operator.

$$\mathcal{E}(\vec{n}) = \lim_{r \rightarrow \infty} r^2 \int_0^\infty dt \, \vec{n}_i T^{0i}(t, r\vec{n})$$

[Sveshnikov, Tkachov, 1995;
Hofman, Maldacena, 2008;
Kravchuk, Simmons-Duffin, 2018]

light-ray operators $\sim$ (non-local) operators on a light ray



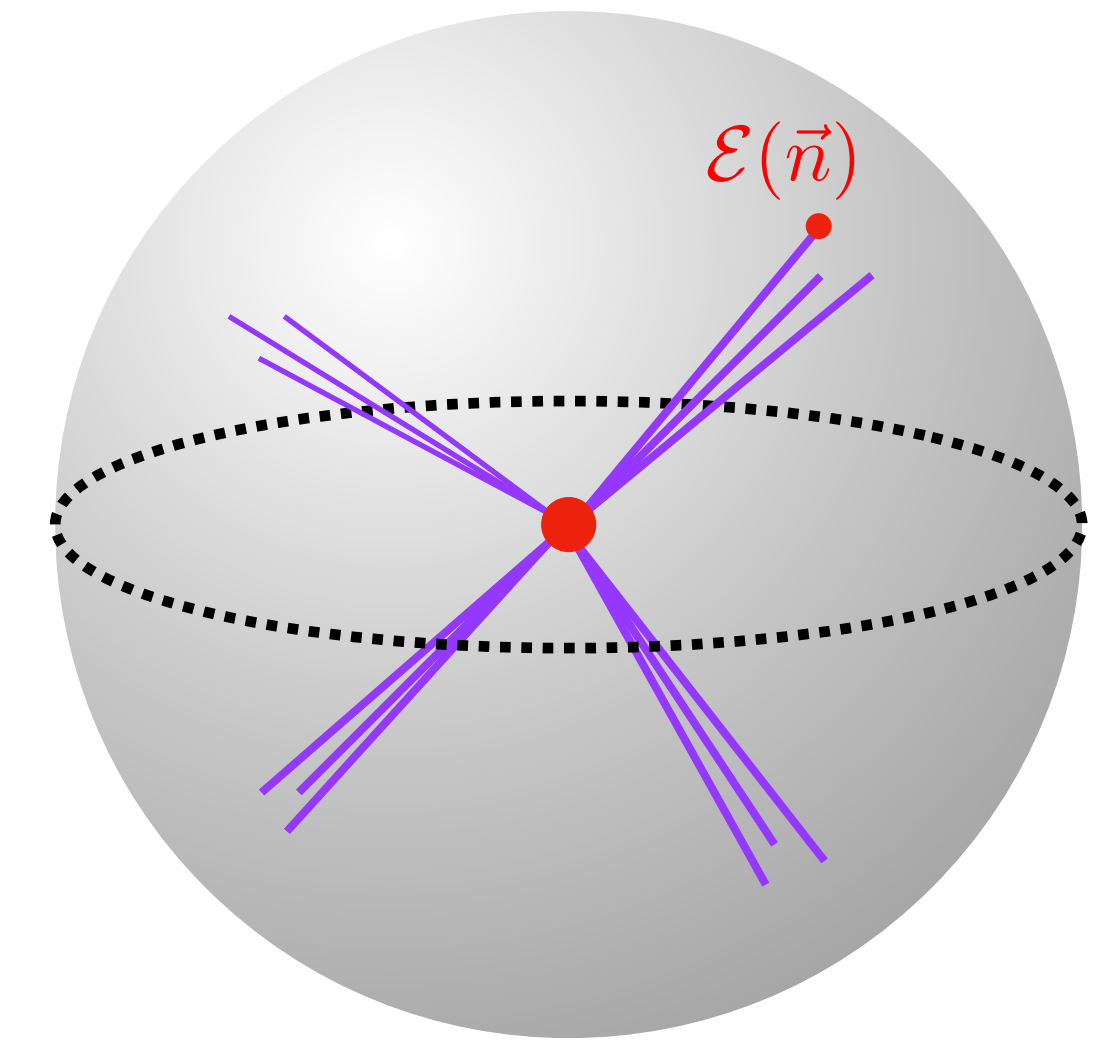
Penrose diagram

Energy Flux in Particle Physics and Field Theory

$\mathcal{E}(\vec{n})$ measures energy flux along specific angles:

Particles

$$\mathcal{E}(\vec{n})|X\rangle = \sum_{i \in X} E_i \delta(\hat{p}_i - \vec{n})|X\rangle \quad [\text{Sternman, 1975}]$$



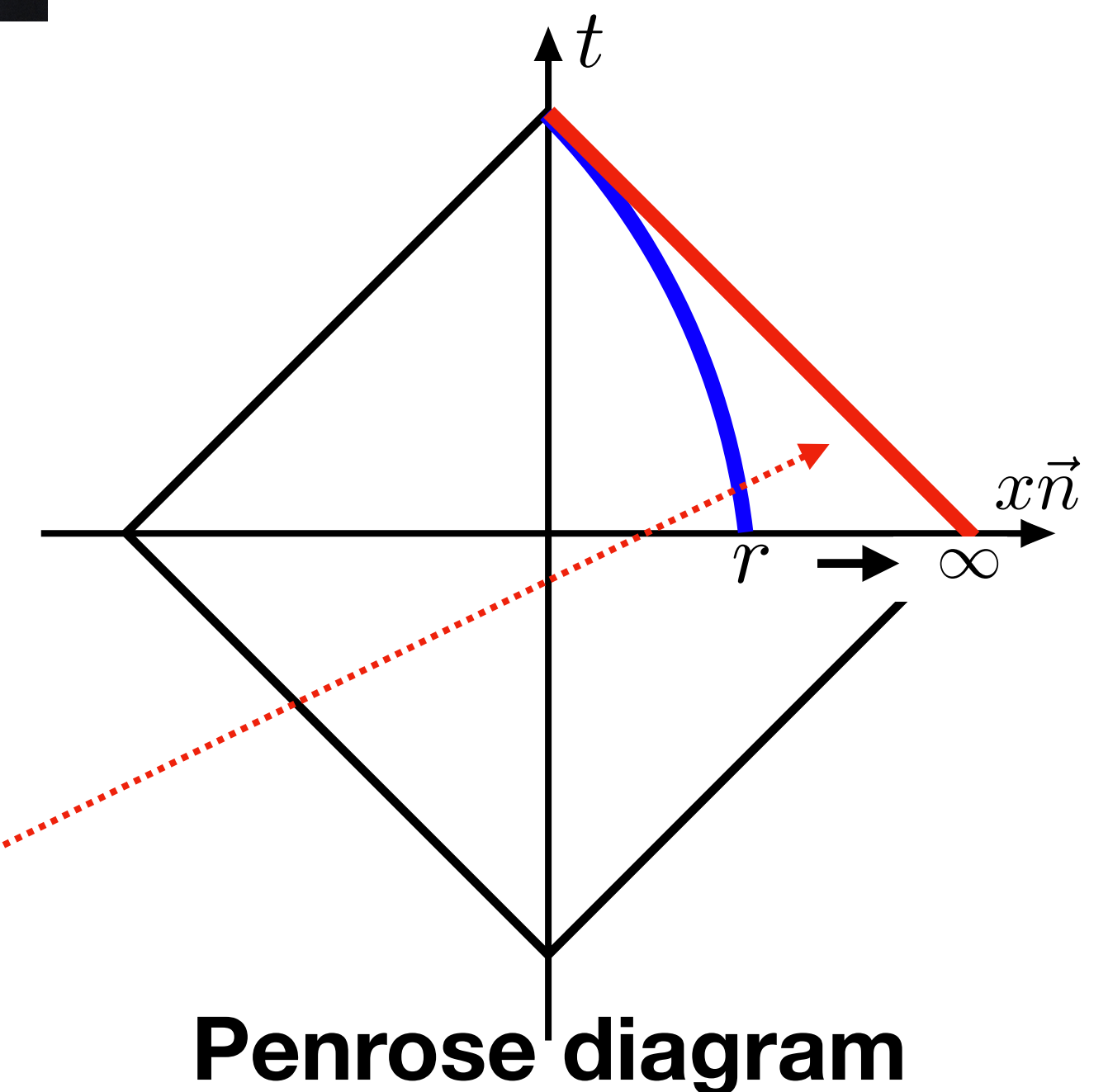
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Fields/Operators

[Sveshnikov, Tkachov, 1995;
Hofman, Maldacena, 2008;
Kravchuk, Simmons-Duffin, 2018]

light-ray operators $\sim$ (non-local) operators on a light ray



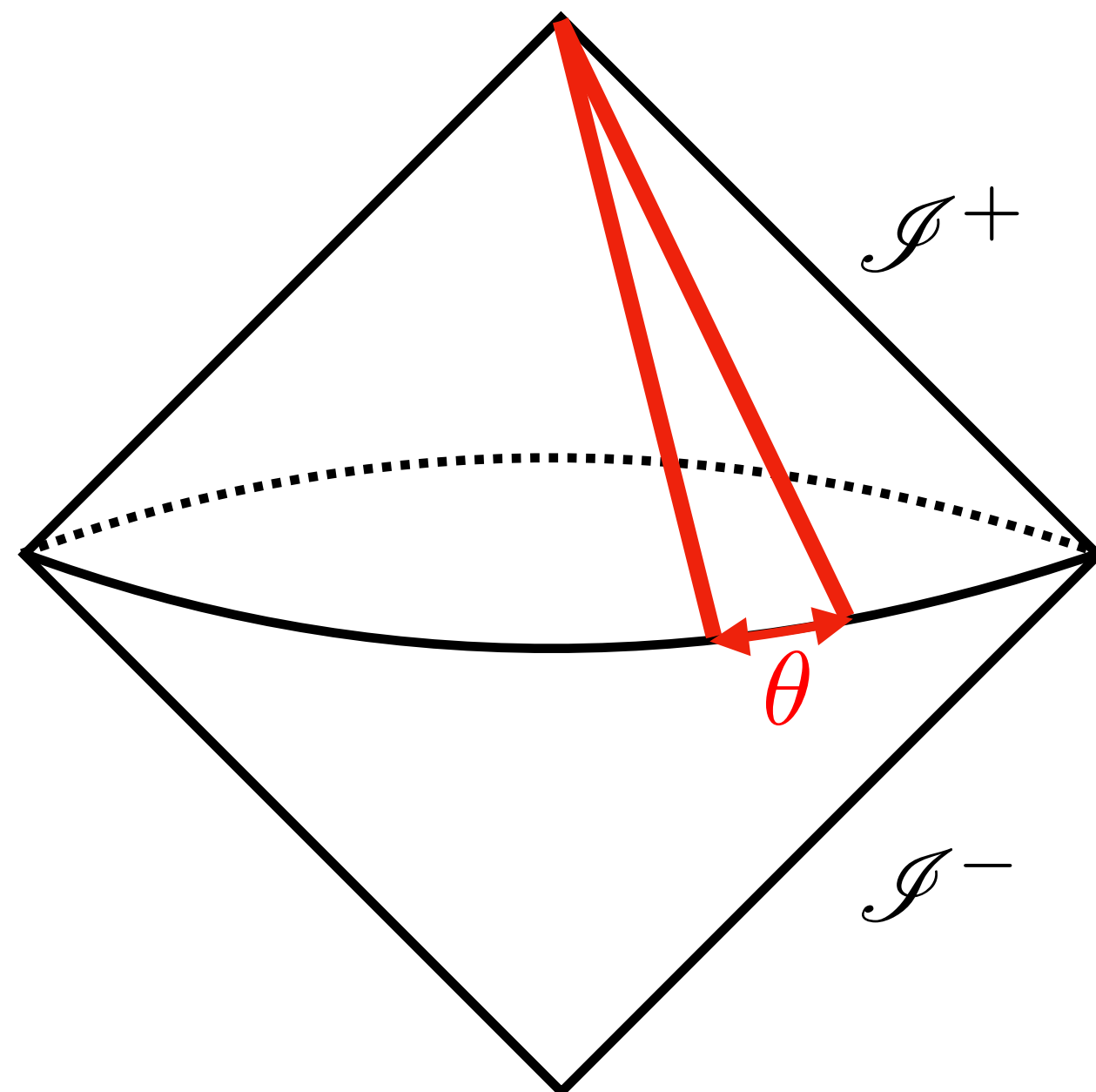
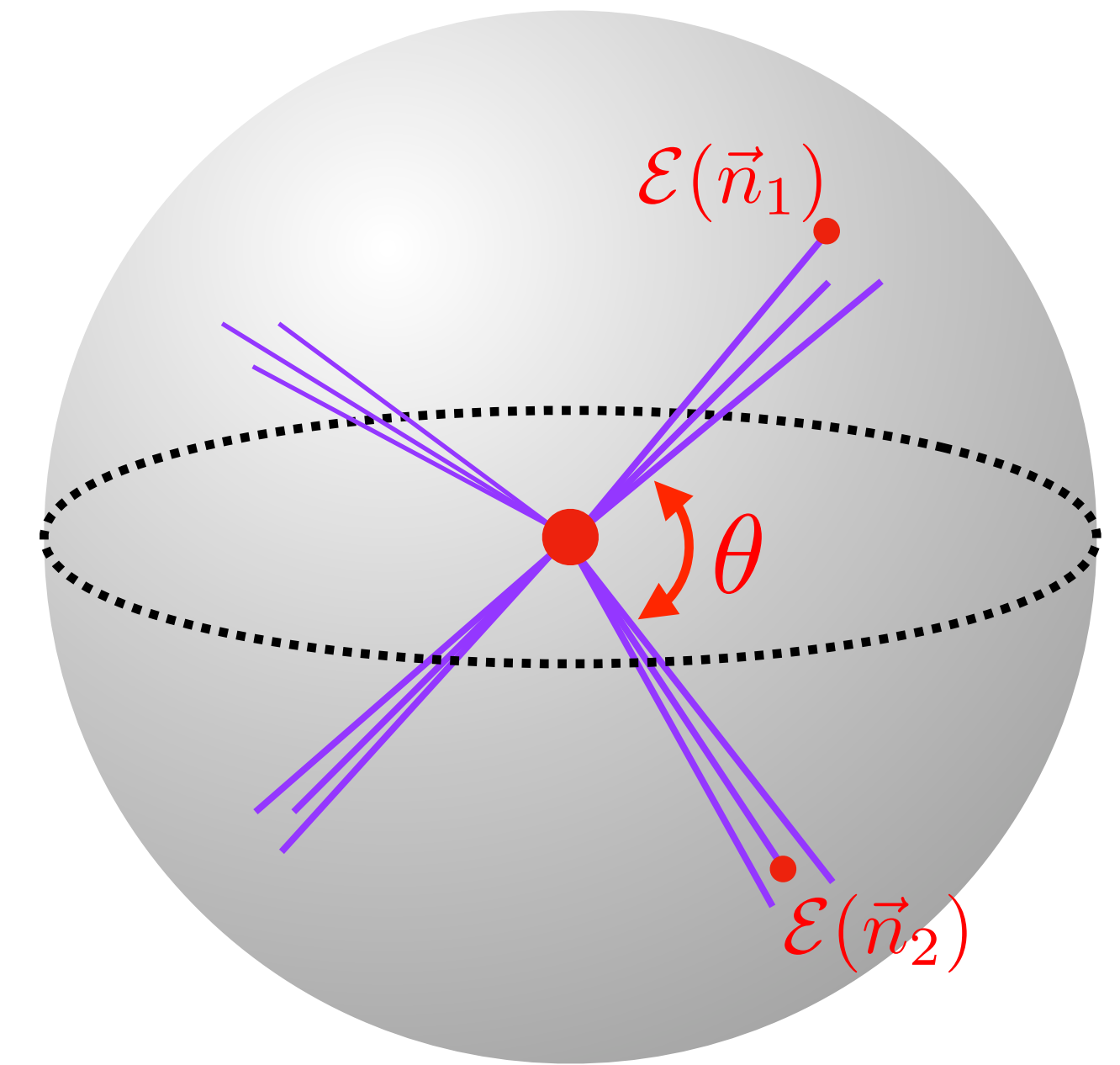
Penrose diagram

Energy-energy correlator (EEC)

Energy-energy correlation measures the correlation of two **energy detectors** at spatial infinity

[Basham, Brown, Ellis and Love, 1978]

$$\frac{d\Sigma}{d\zeta} = \sum_{i,j} \int d\sigma \frac{E_i E_j}{Q^2} \delta \left(\zeta - \frac{1 - \cos \theta_{ij}}{2} \right)$$



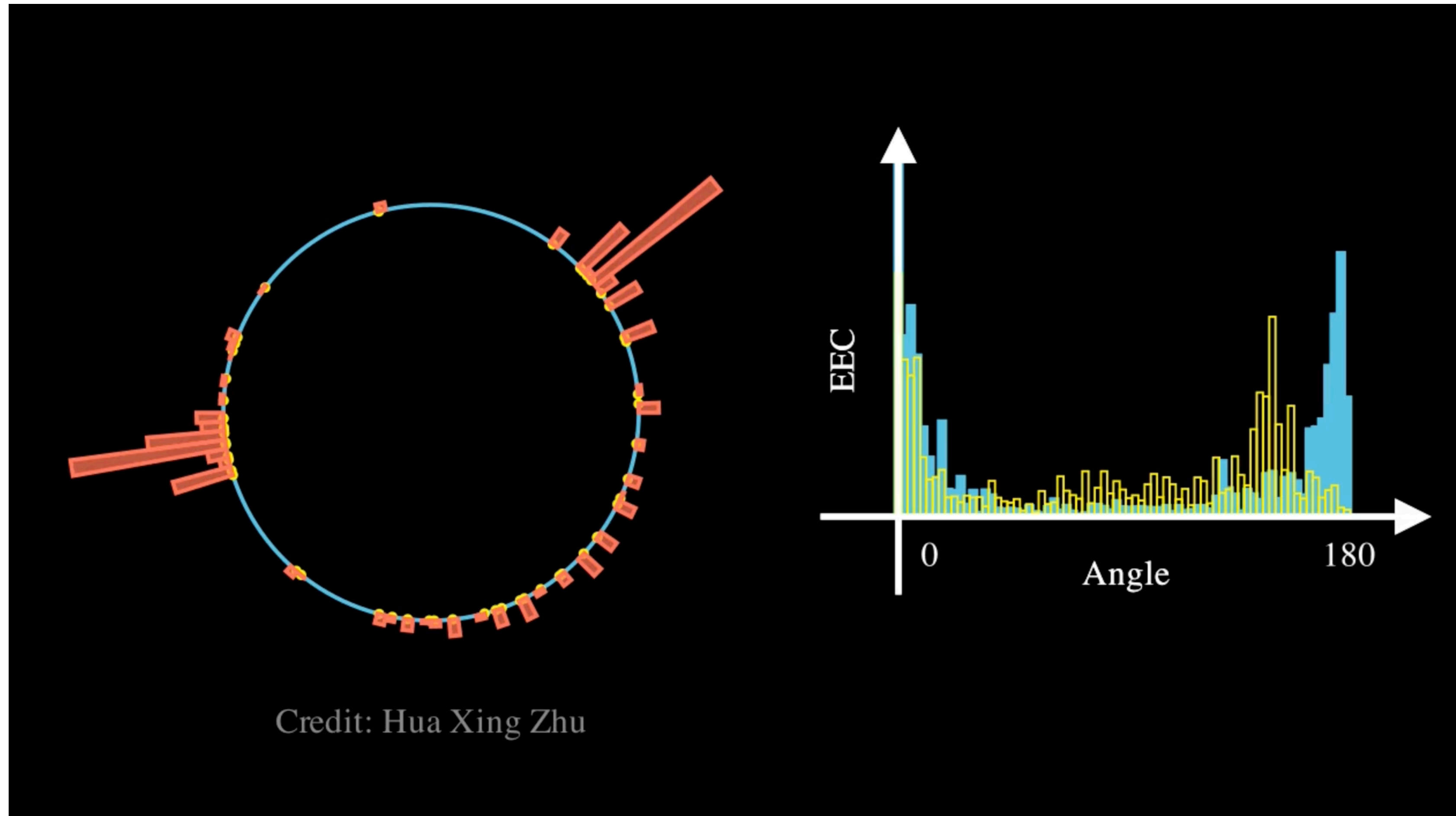
EEC and its multi-point generalization are correlation functions of **light-ray operators**

$$\langle O'(-q) | \mathcal{E}(\vec{n}_1) \mathcal{E}(\vec{n}_2) \dots | O(q) \rangle$$

$$q = (Q, 0, 0, 0)$$

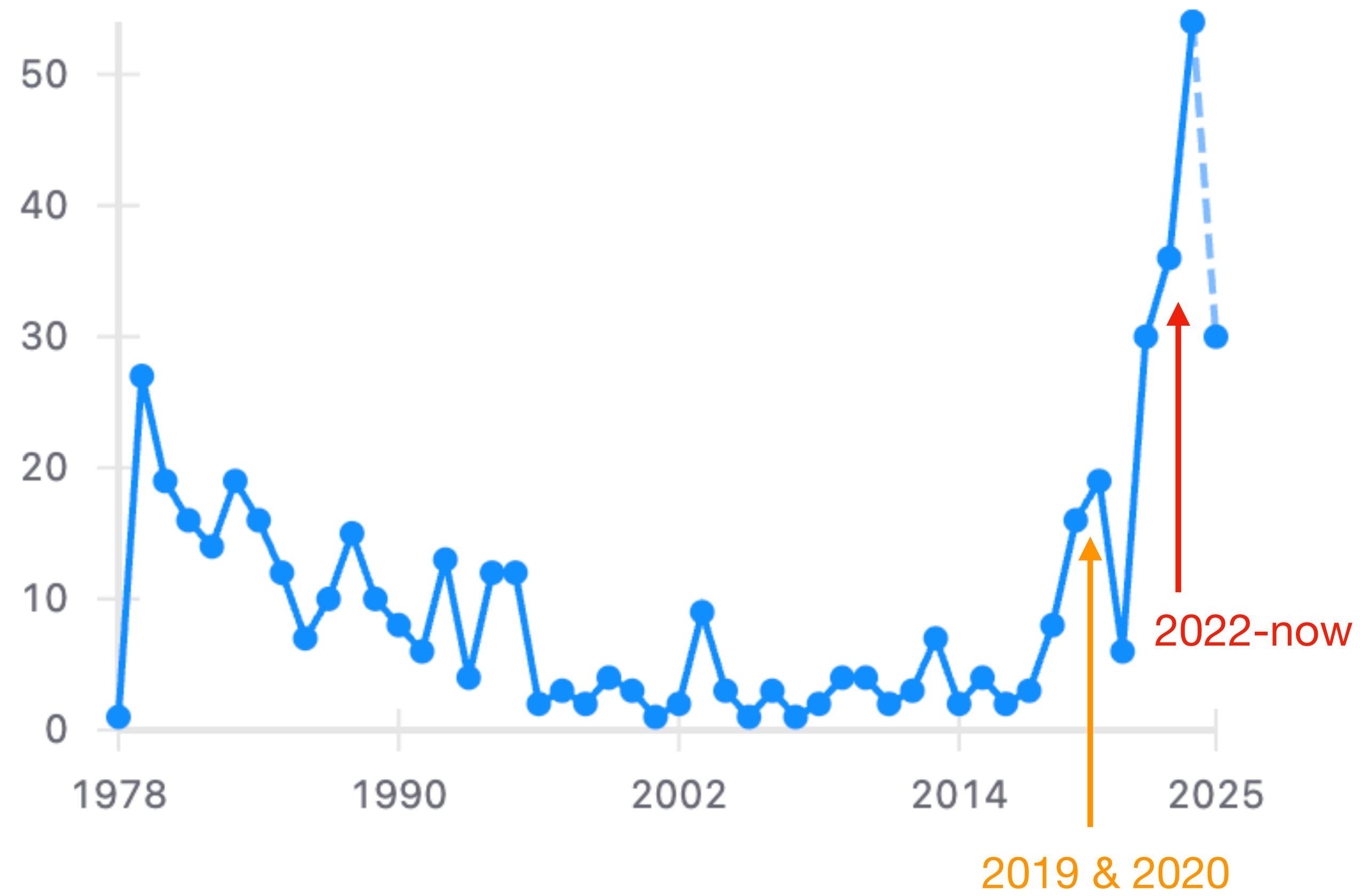
EEC is a weighted cross section.

Each event contributes a distribution to the ensemble average.



see animation https://indico.fis.ucm.es/event/20/contributions/545/attachments/356/640/EEC_visualization.mp4

Citations per year

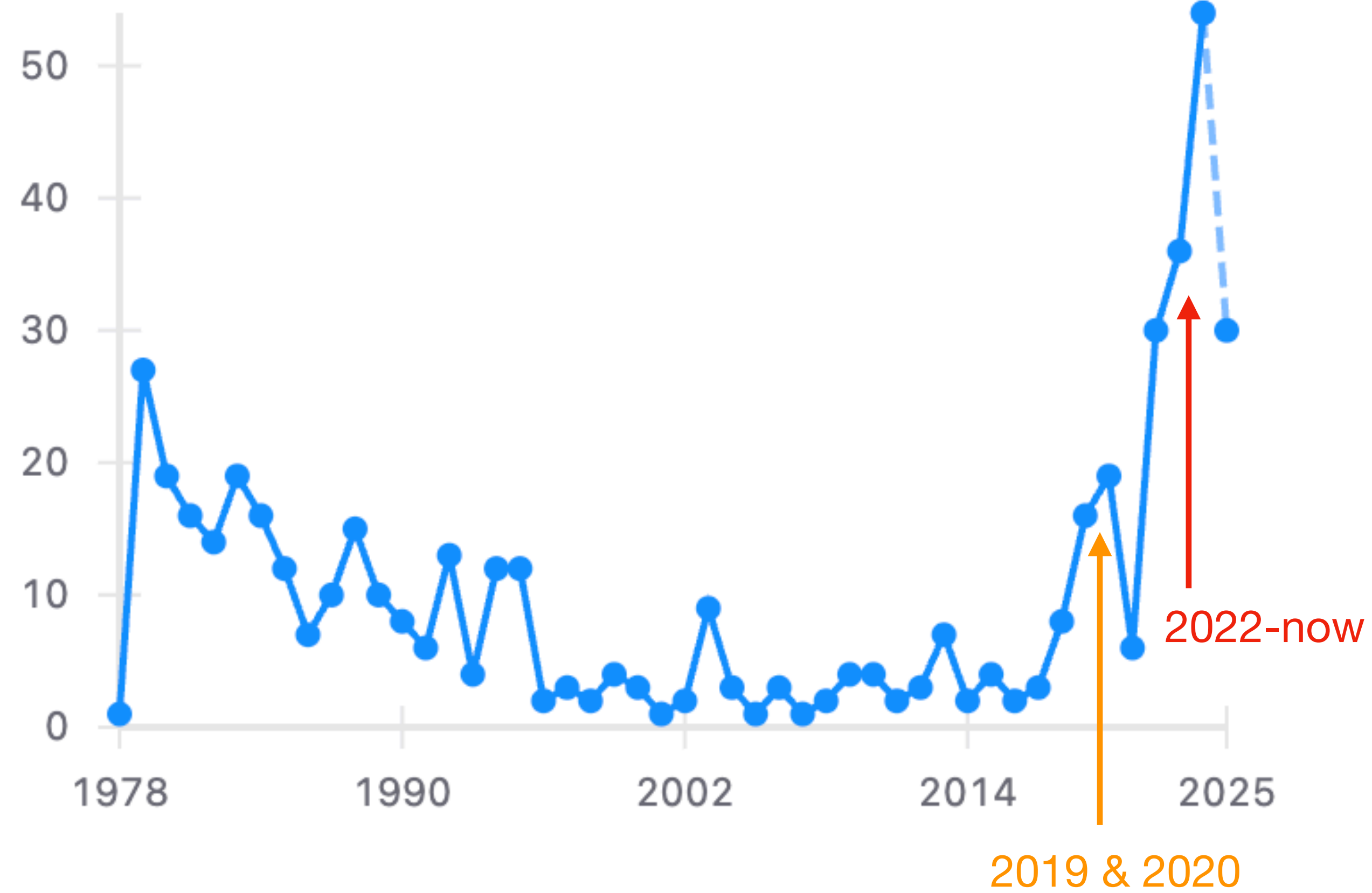


Citation history of the first EEC paper (1978)



Citations per year

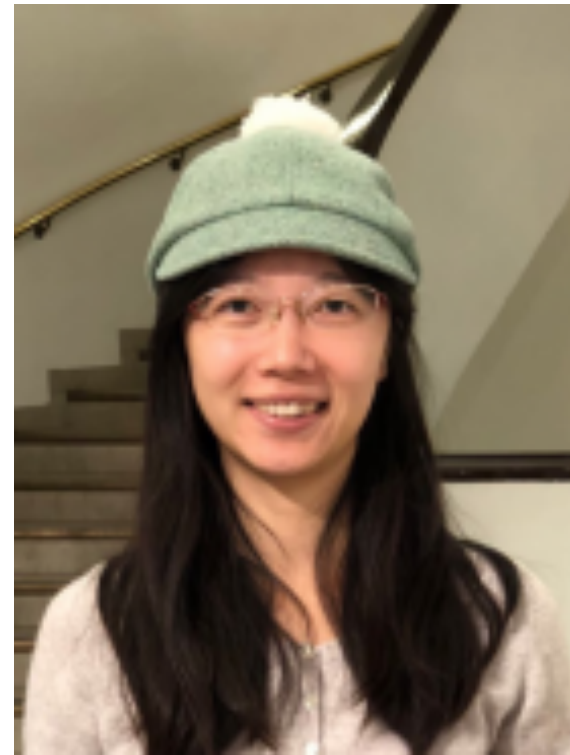
Experimentalists Theorists



Citation history of the first EEC paper (1978)

Energy Correlators “Renaissance”

[2307.07510]



Meng Xiao



Experimentalists Theorists

EEC inside jets @ CMS



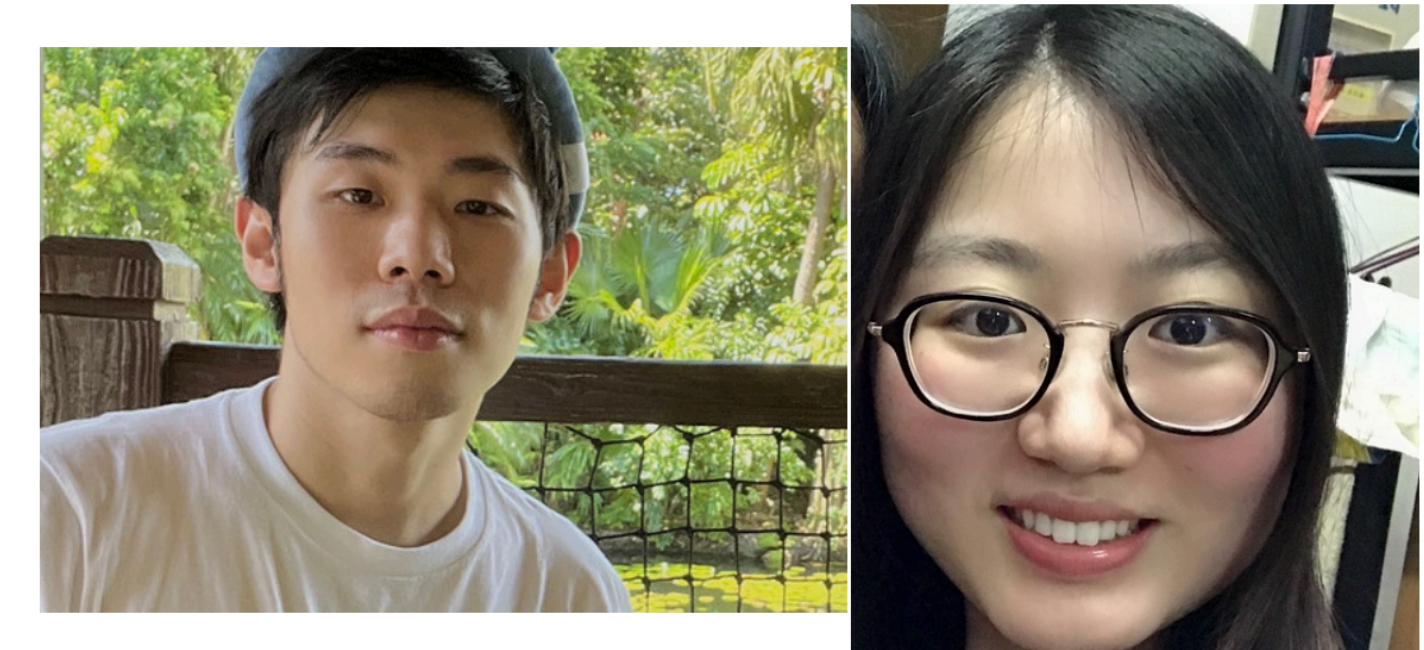
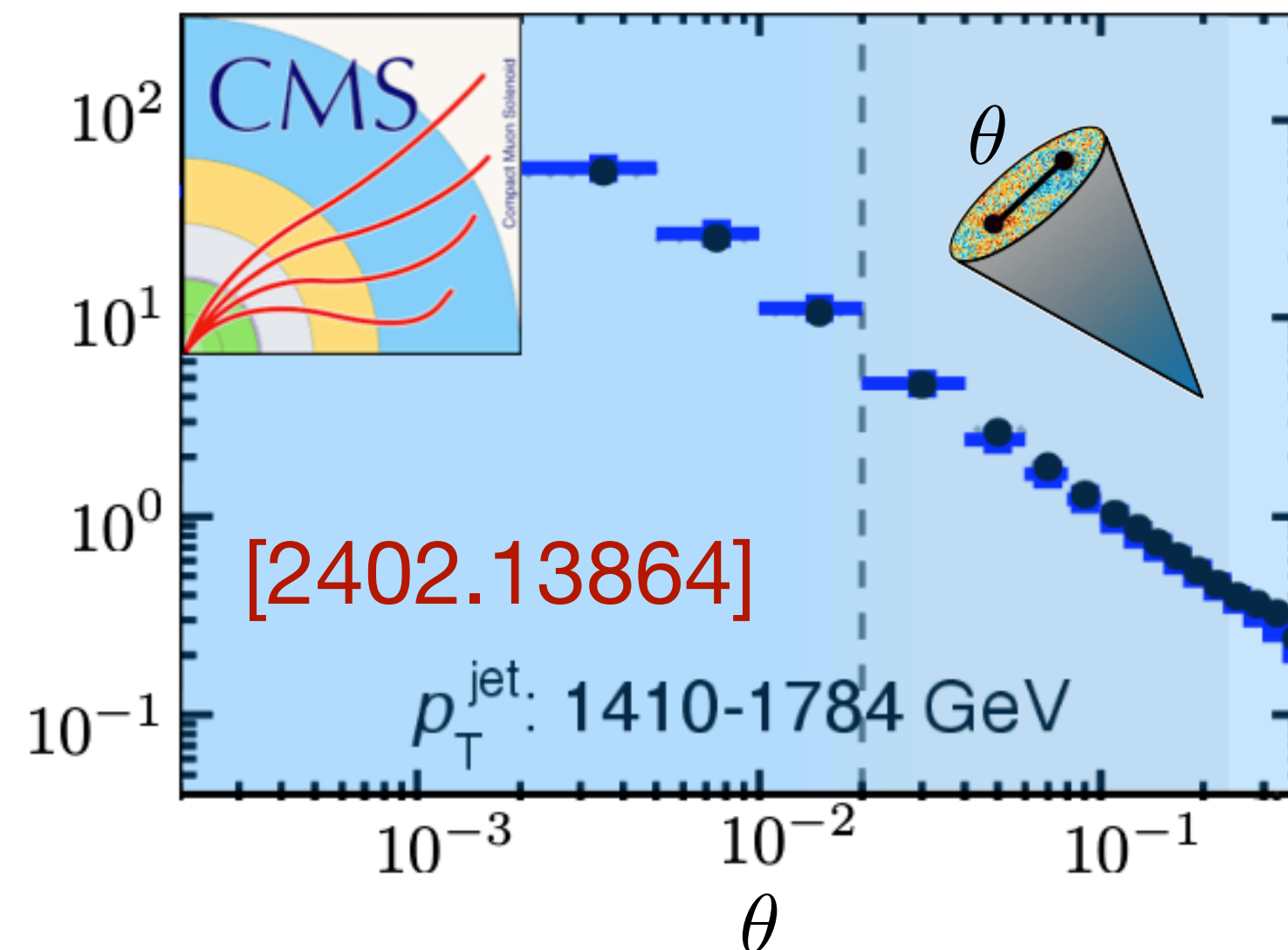
HuaXing Zhu



Zhen Lin

Chenfeng Lu

Yulei Ye



Xiaoyuan Zhang

Yibei Li

+Wen Chen, Jun Gao and Zhen Xu

(sorry for the limited space)

Most precise α_s measurement
from jet substructure

Perturbative scaling behavior

[Other related works by Diego Hofman, Juan Maldacena, Lance Dixon, Ian Mout, David Simmons-Duffin, Alexander Zhiboedov, Gregory Korchemsky, HC, Kyle Lee...]

Energy Correlators “Renaissance”



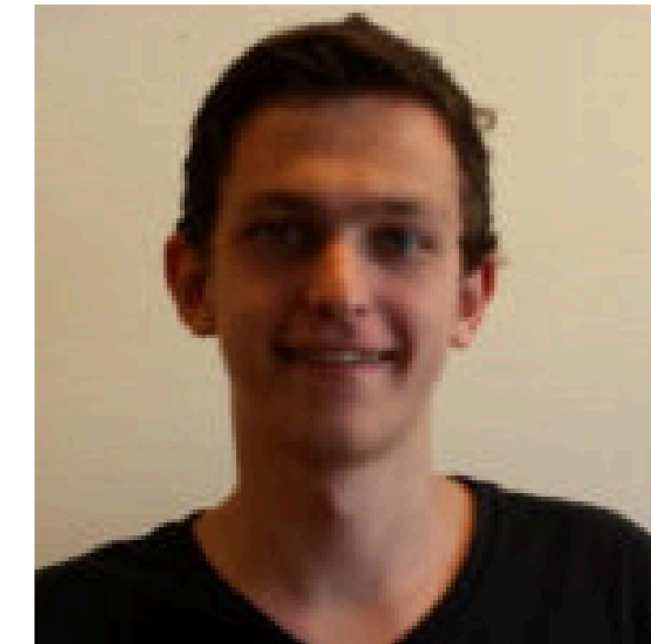
Hannah Bossi



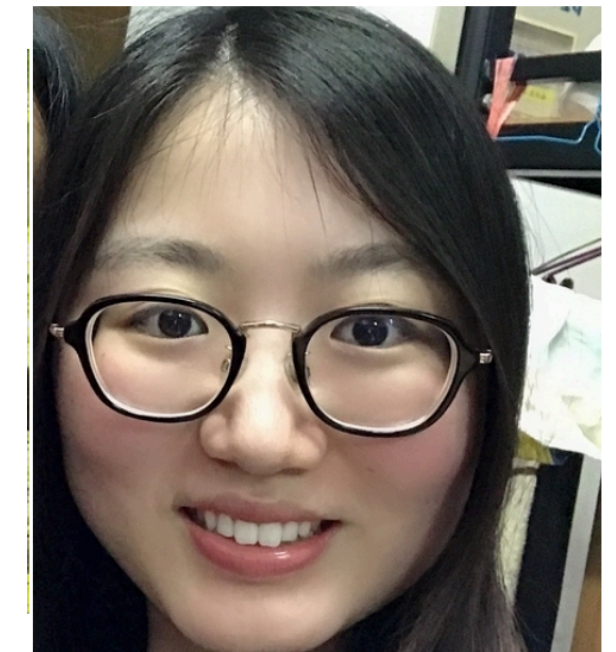
Yu-Chen Chen



Experimentalists Theorists



Max Jaarsma



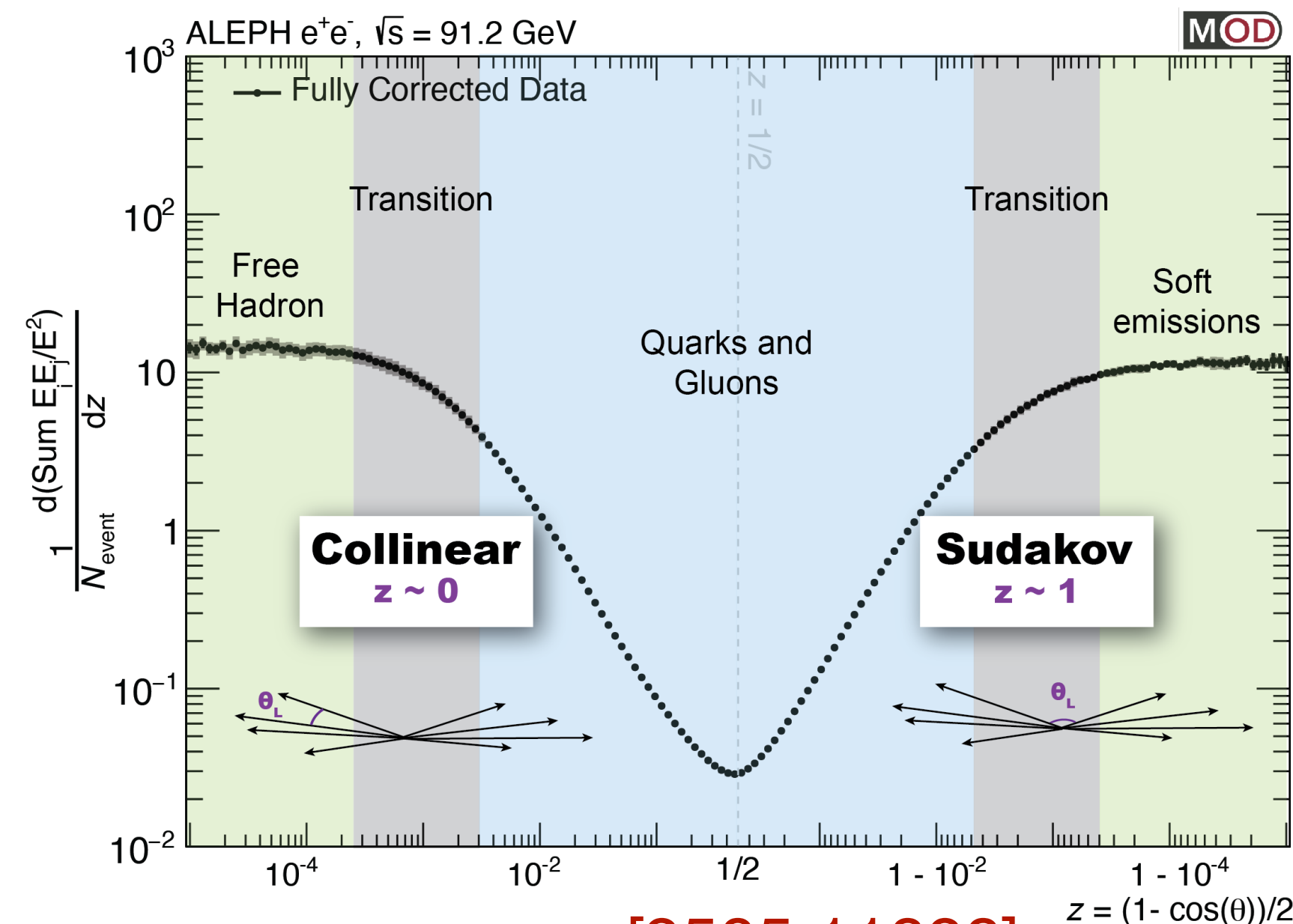
Yibei Li



Jingyu Zhang

Yi Chen

Track EEC from LEP Data



[2505.11828]



Ian Mout

Wouter Waalewijn HuaXing Zhu

[theory paper, to appear]

+ Innocenti,
Badea, Baty,
Maggi,
McGinn +

(sorry for the
limited space)



Yen-Jie Lee

Energy Correlators “Renaissance”

see [EEC review \[2506.09119\]](#) for more details

Energy Correlators: A Journey From Theory to Experiment

Ian Moutl^{1,*} and Hua Xing Zhu^{2,3,†}

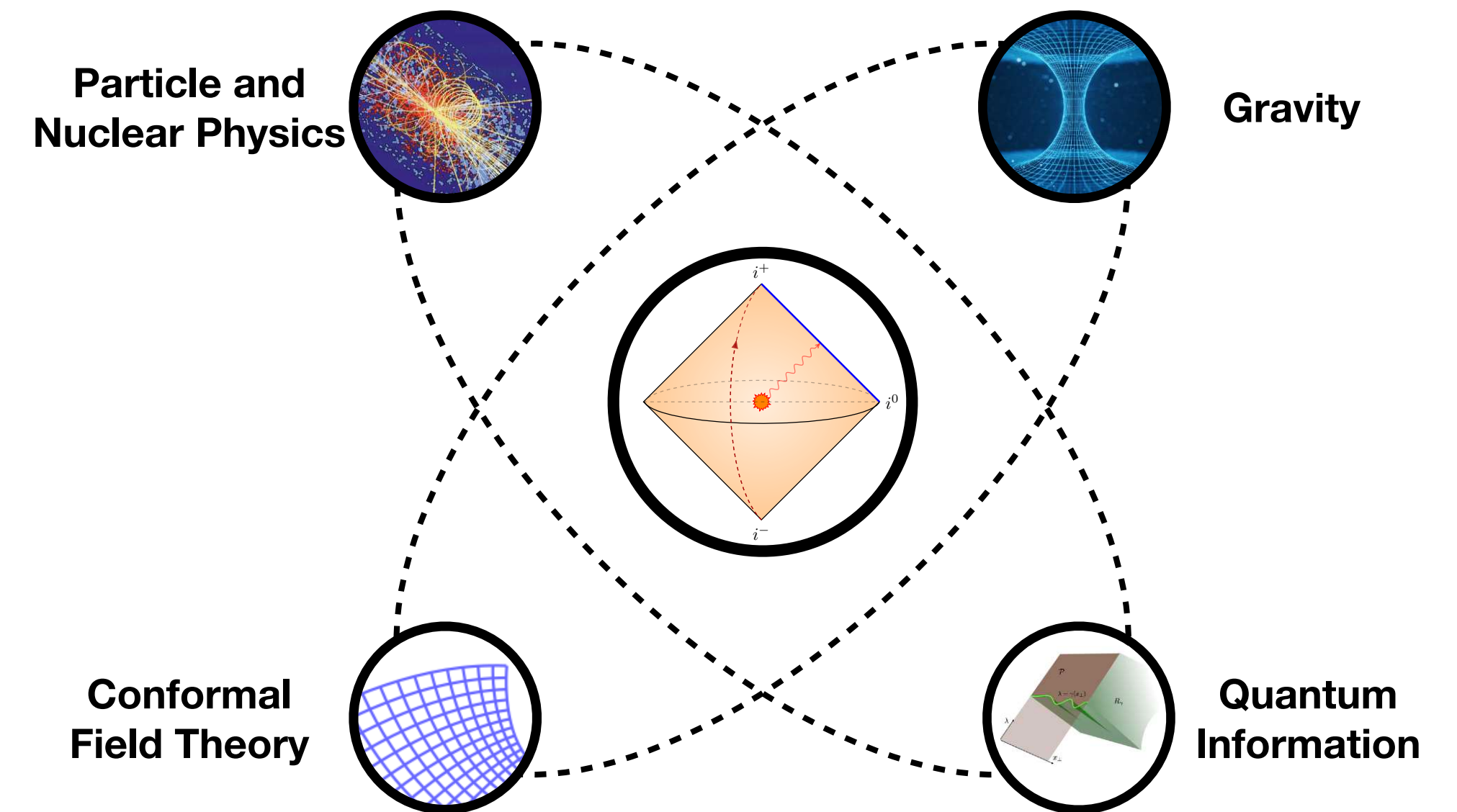
¹*Department of Physics,
Yale University, New Haven,
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²*School of Physics,
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China*

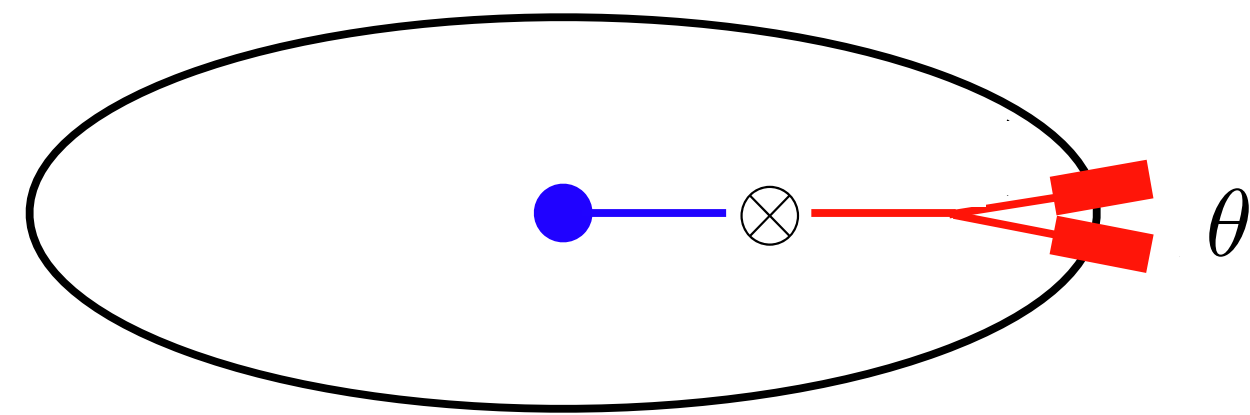
³*Center for High Energy Physics,
Peking University, Beijing 100871,
China*

Collider experiments offer a unique opportunity to explore the Standard Model (SM), and to search for new physics, new interactions, and new principles of nature. The theoretical abstraction of a collider, namely the study of correlations in asymptotic fluxes, provides one of the most basic examples of an observable in quantum field theory (QFT) and quantum gravity. Energy flux is described in QFT by energy flow operators, a particular example of light-ray operators. In addition to their central role in the theoretical description of collider physics, energy flow operators play an important role in diverse areas of formal QFT and gravity, providing a connection between real world collider phenomenology, and the deep underlying principles of QFT.

Recently it has become possible to measure correlation functions of energy flow operators in a wide variety of collider experiments, providing an exciting new connection between collider physics and formal theory. In this review, we provide a survey of recent progress in our understanding of energy operators and their correlators, highlighting their importance in both formal theory and collider phenomenology, and in particular, their great potential for bridging these areas to provide new ways to understand the real world. We intend this article as a resource for both formal theorists interested in understanding how light-ray operators are being applied in particle and nuclear physics, as well as for experimentalists interested in the theoretical motivation for these observables. Most importantly, we aim to stimulate further interaction between the formal, phenomenological and experimental communities through the common lens of energy correlators.



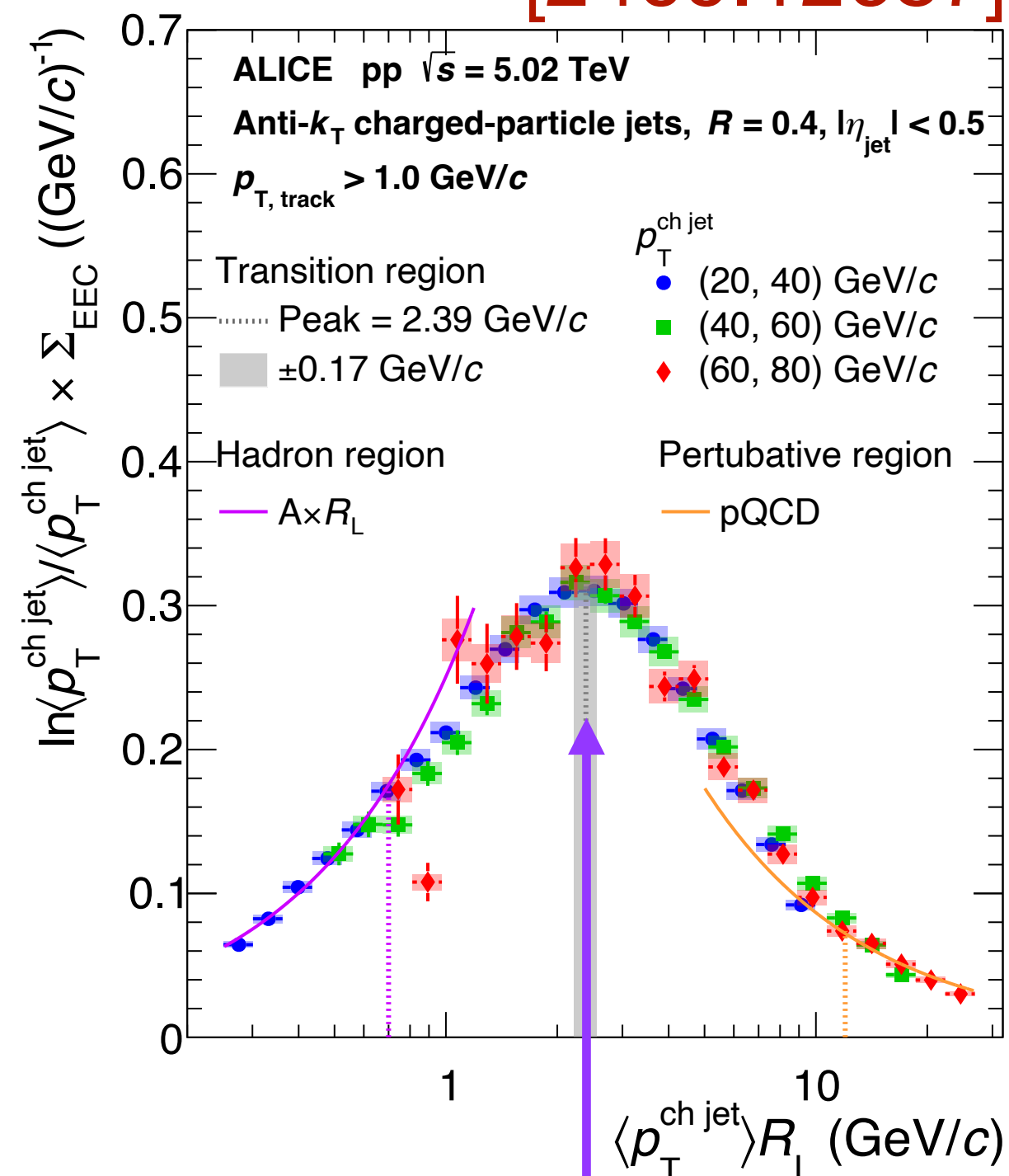
A Single Lens for Multiple Scales



probing the scale θE_{jet} in the collinear limit

QCD transition

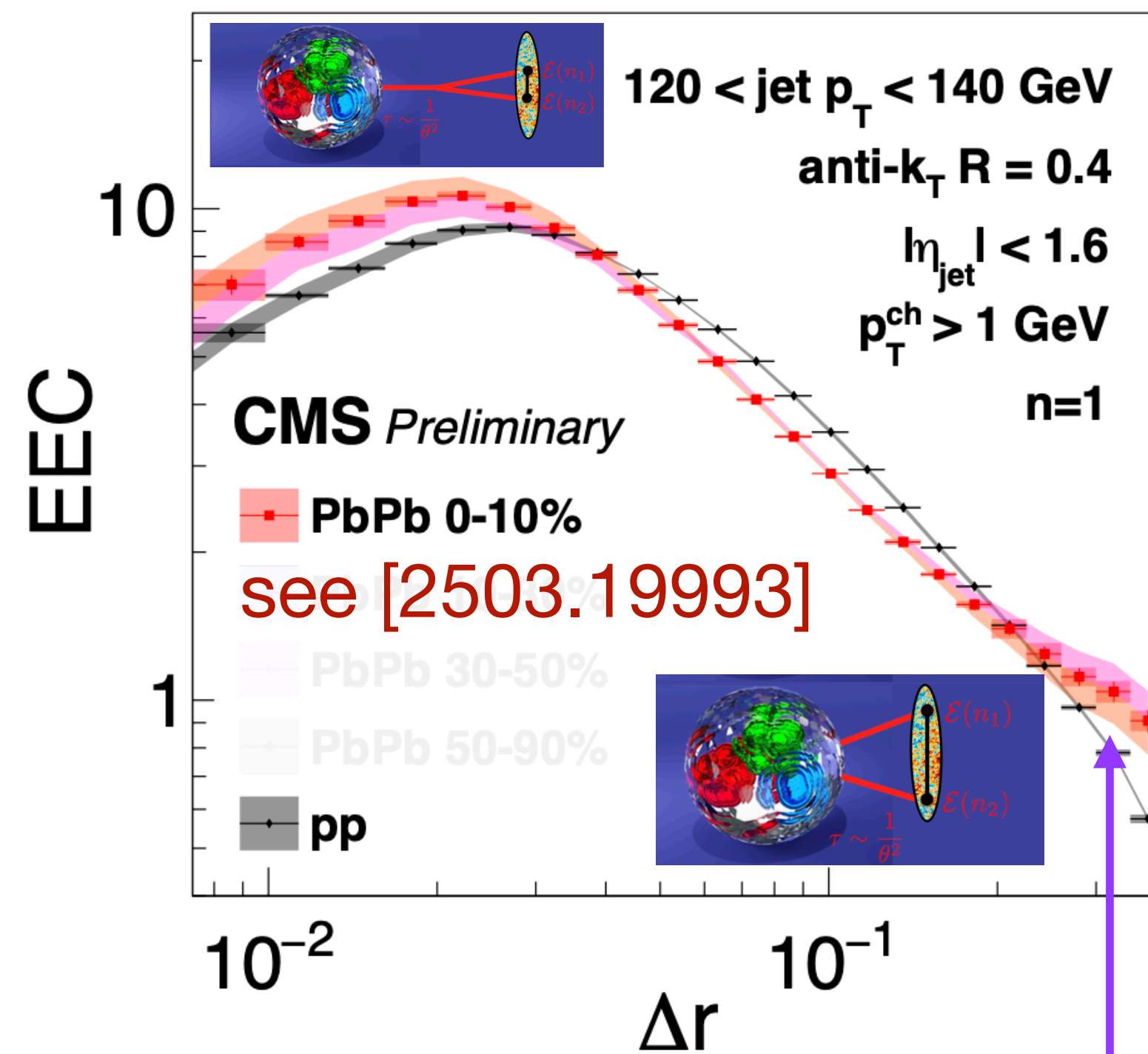
[2409.12687]



$\propto \Lambda_{\text{QCD}}$

QGP effect

1.70 nb⁻¹ PbPb (5.02 TeV) + 302 pb⁻¹ pp (5.02 TeV)

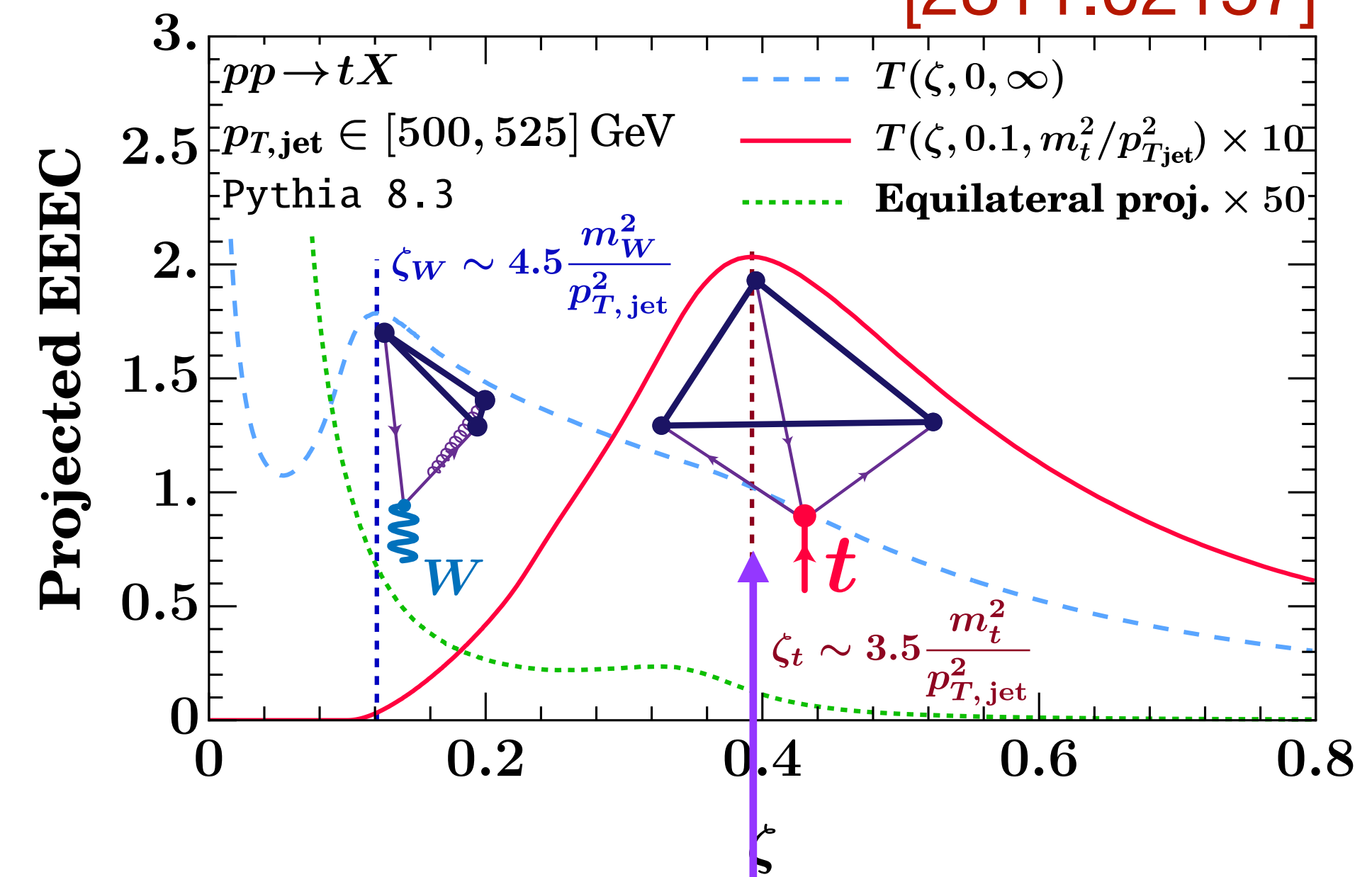


[Adapted from Ian Moul's slides]

$\propto \Lambda_{\text{QGP}}$

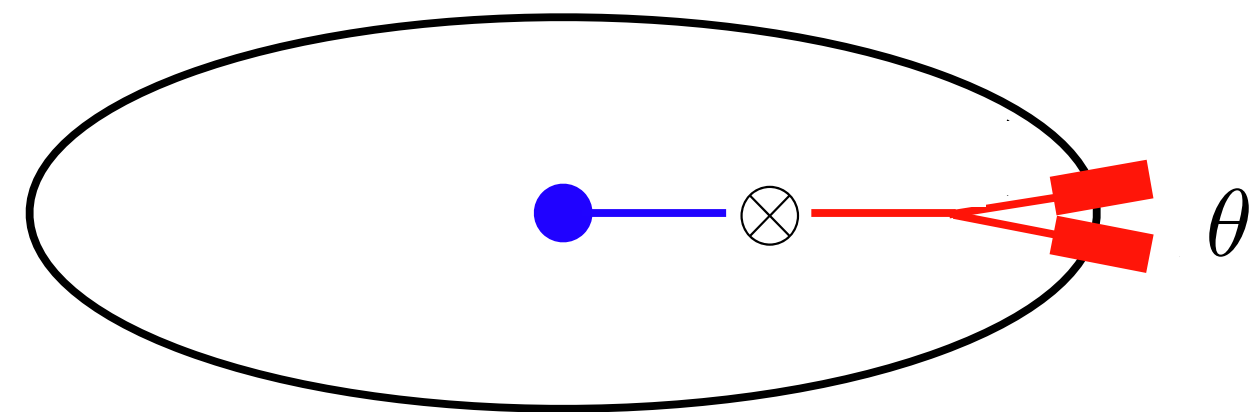
Top quark decay

[2311.02157]



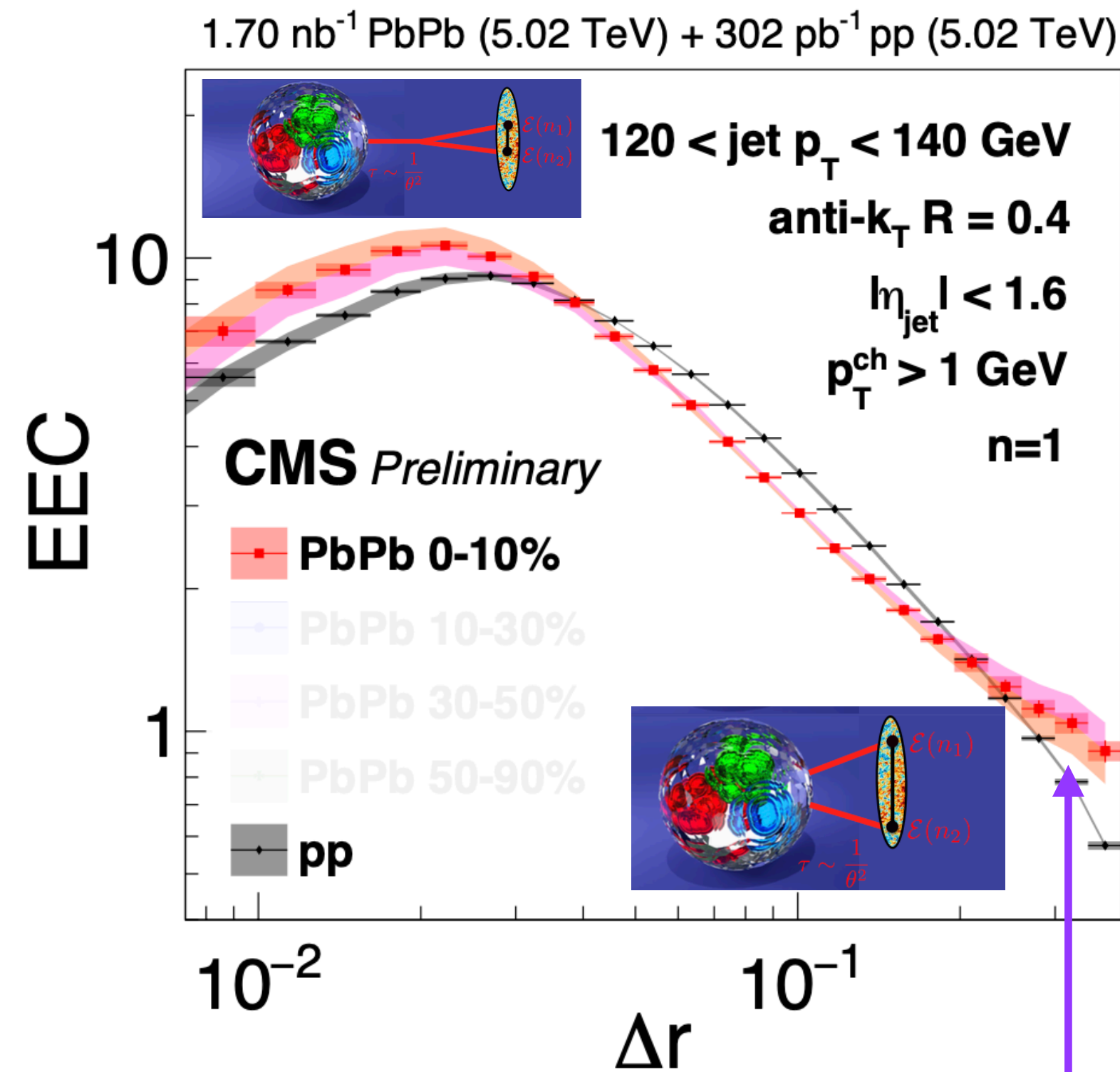
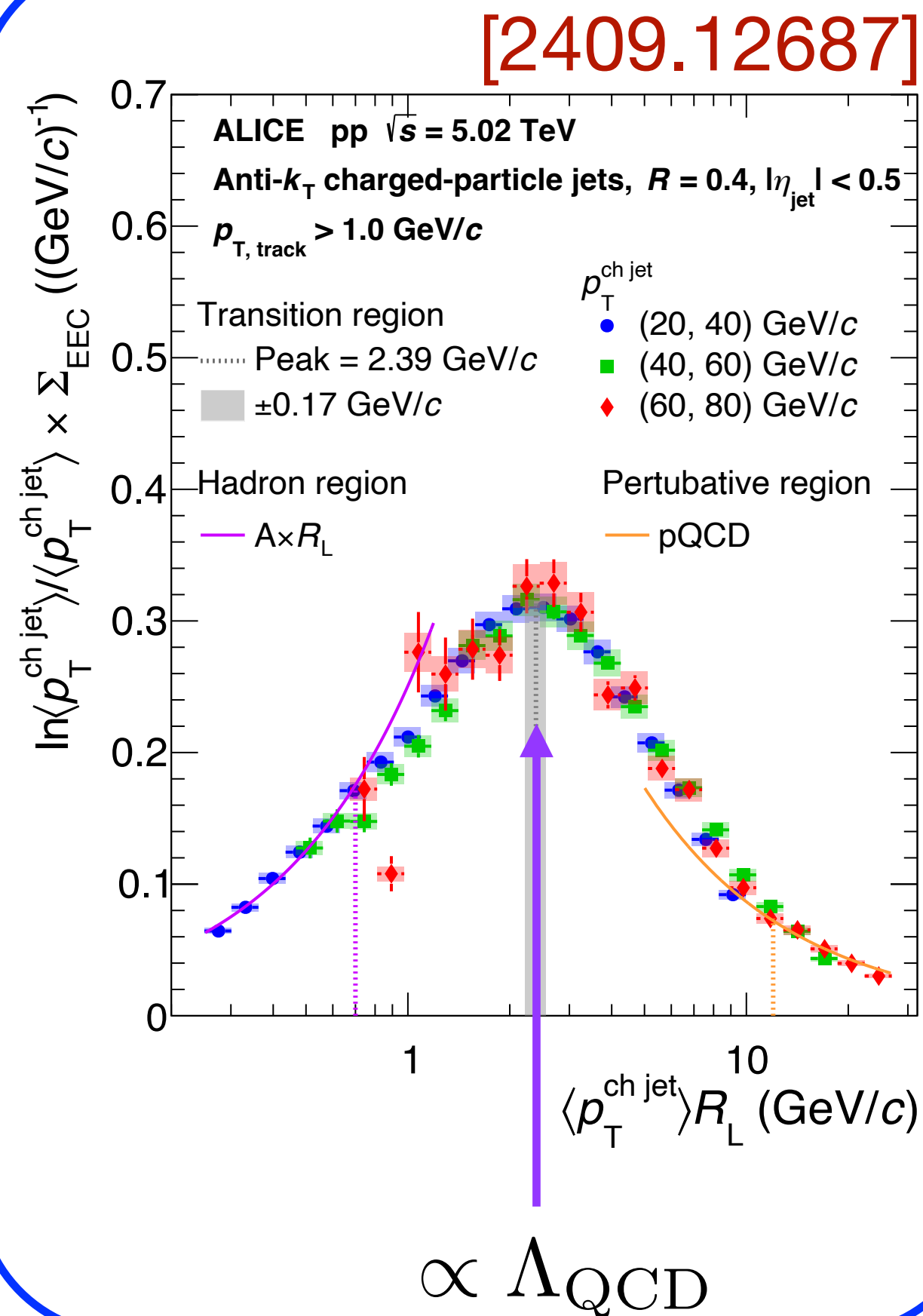
$\propto m_t$

A Single Lens for Multiple Scales



probing the scale θE_{jet} in the collinear limit

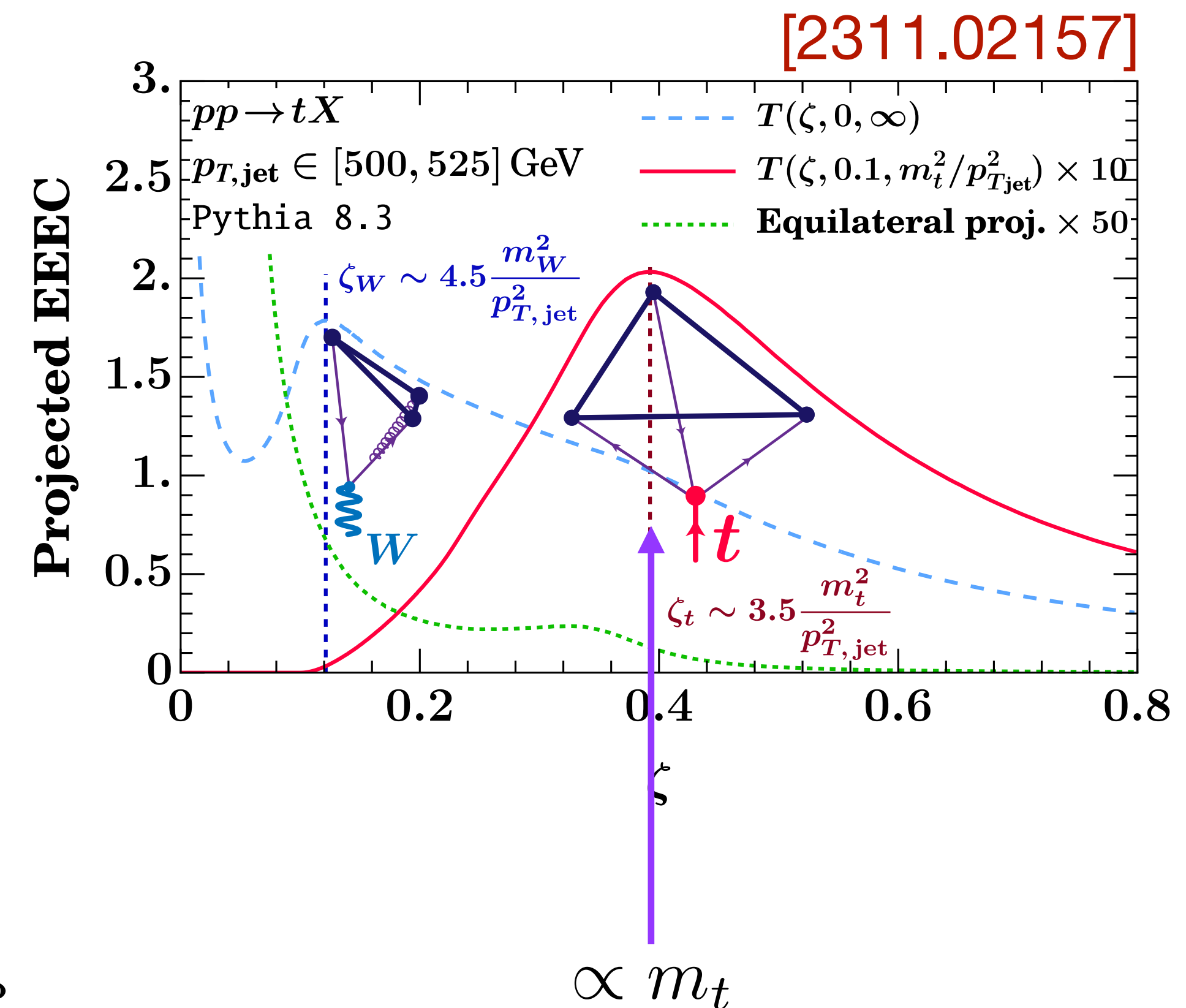
This talk

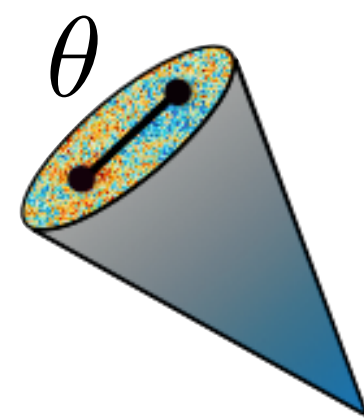


[Adapted from Ian Moul's slides]

13

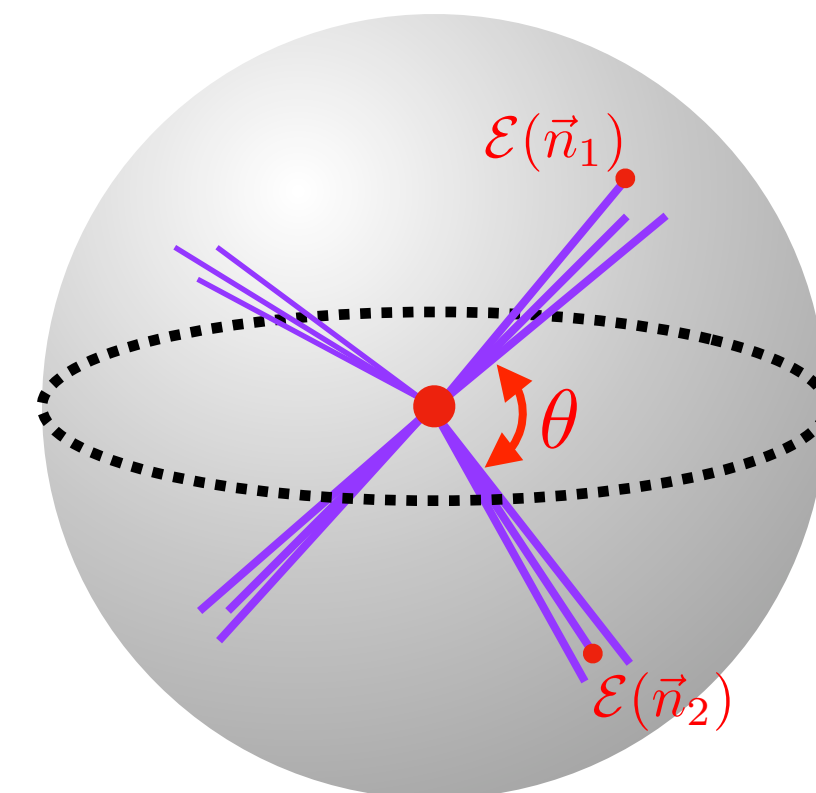
$\propto \Lambda_{\text{QGP}}$





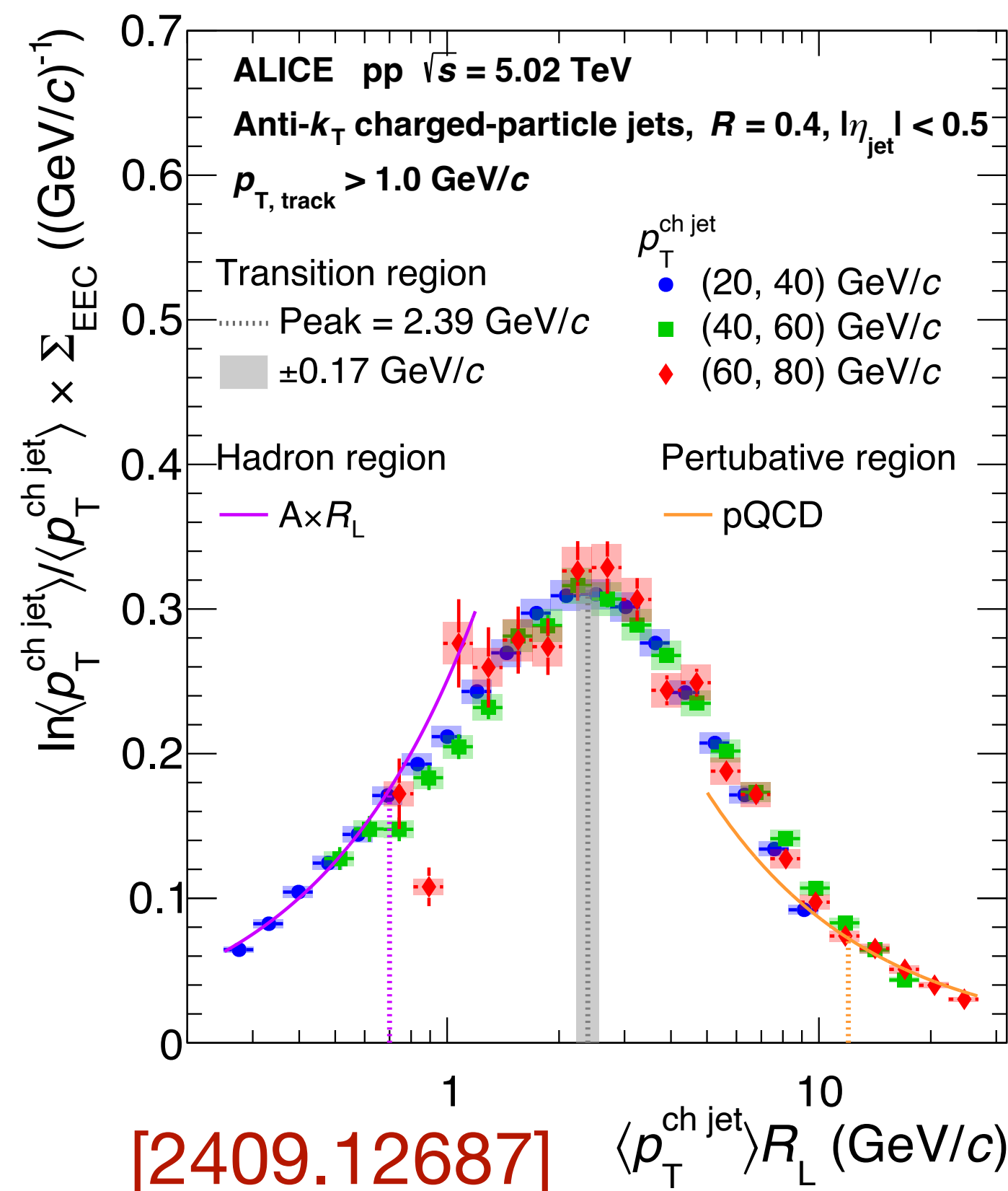
$$\frac{d\Sigma}{d\zeta} = \sum_{i,j} \int d\sigma \frac{E_i E_j}{Q^2} \delta \left(\zeta - \frac{1 - \cos \theta_{ij}}{2} \right)$$

[Basham, Brown, Ellis and Love, 1978]



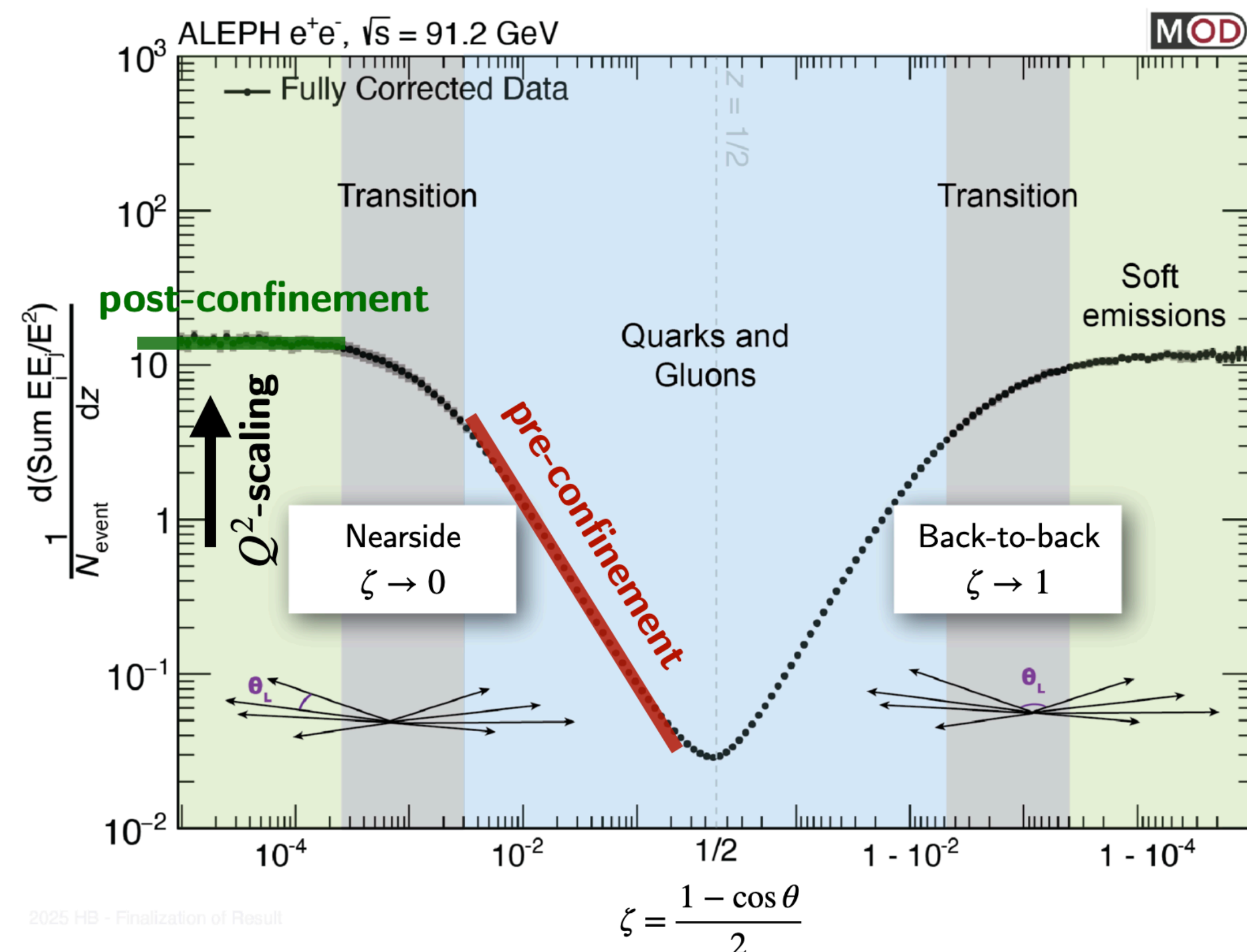
collinear EEC @ LHC

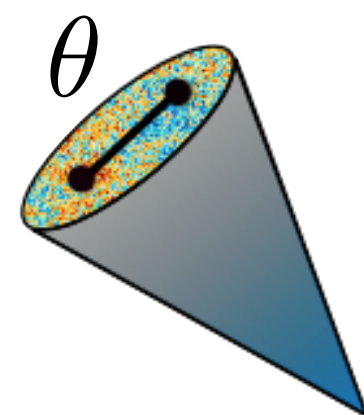
full-range EEC @ LEP



change variable

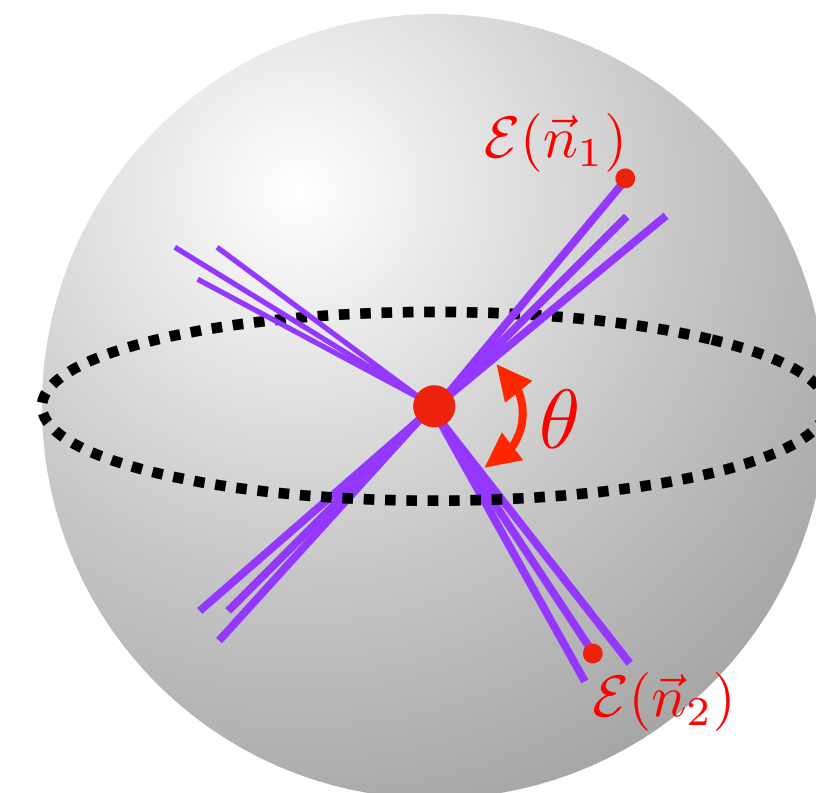
$$\theta \rightarrow \zeta$$



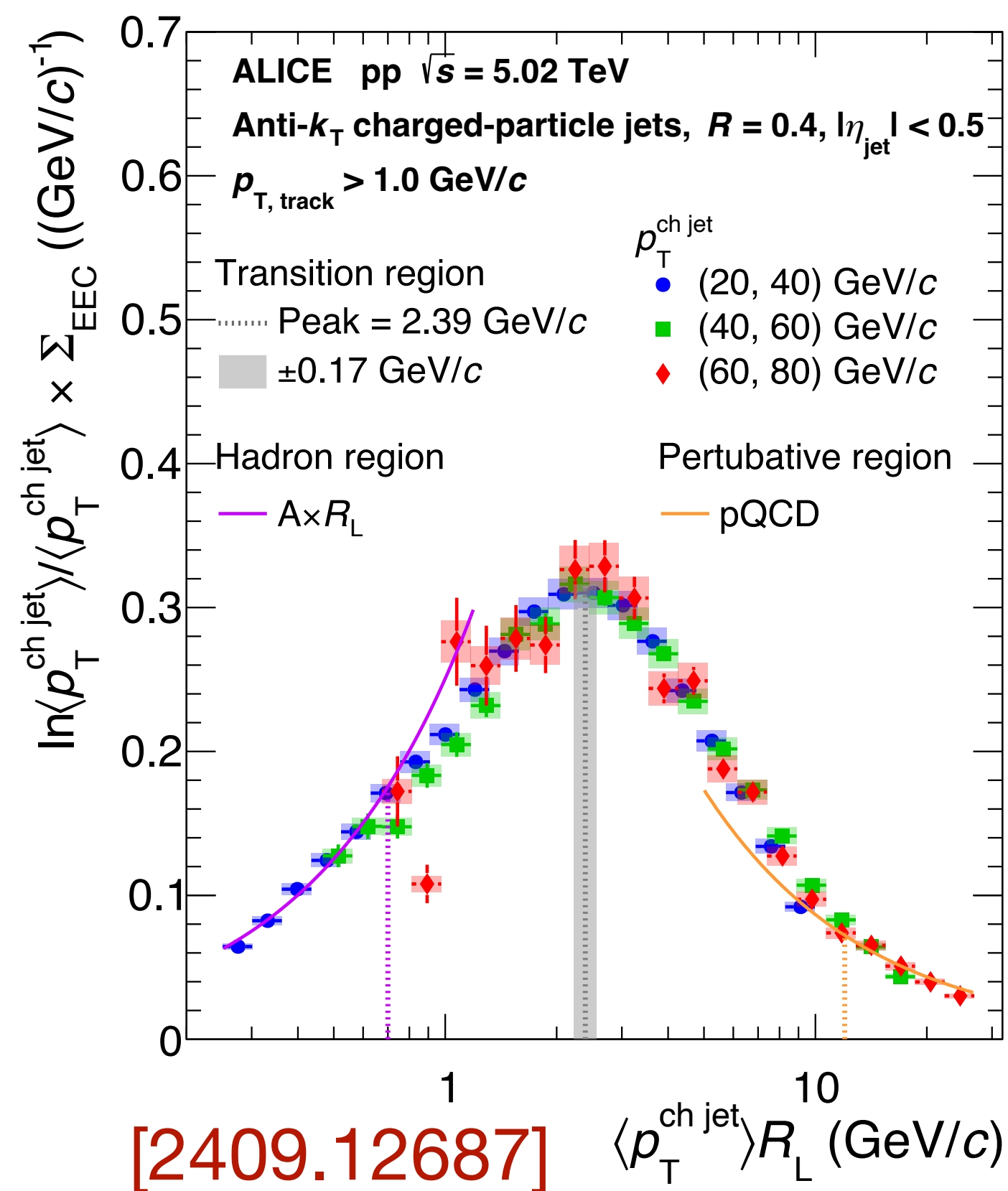


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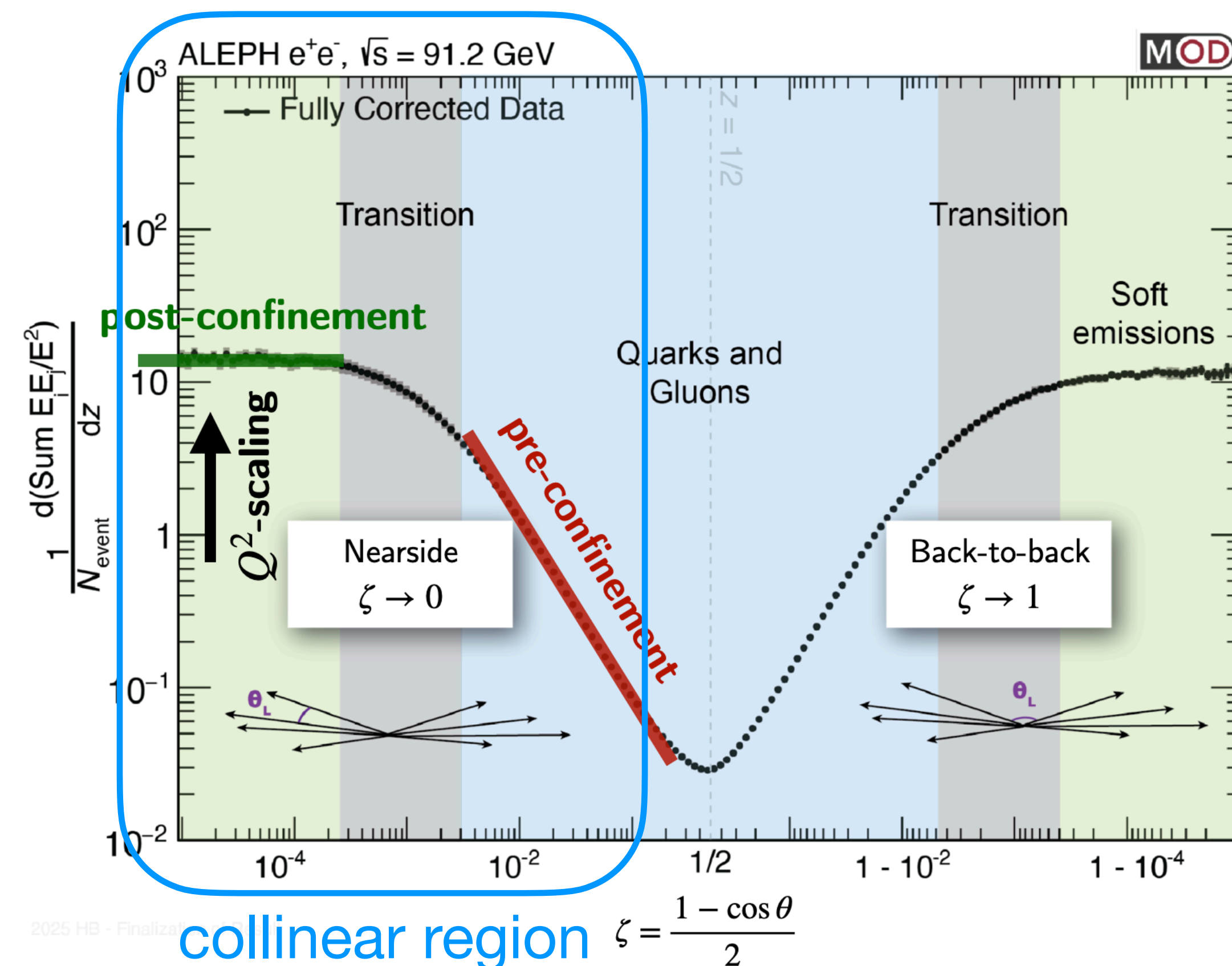
collinear EEC @ LHC

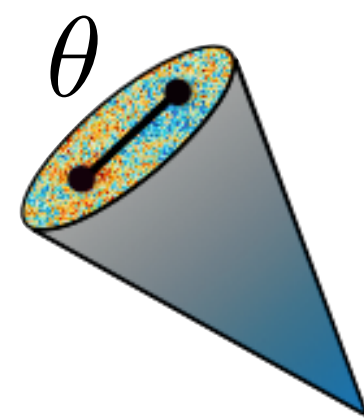


change variable

$$\theta \rightarrow \zeta$$

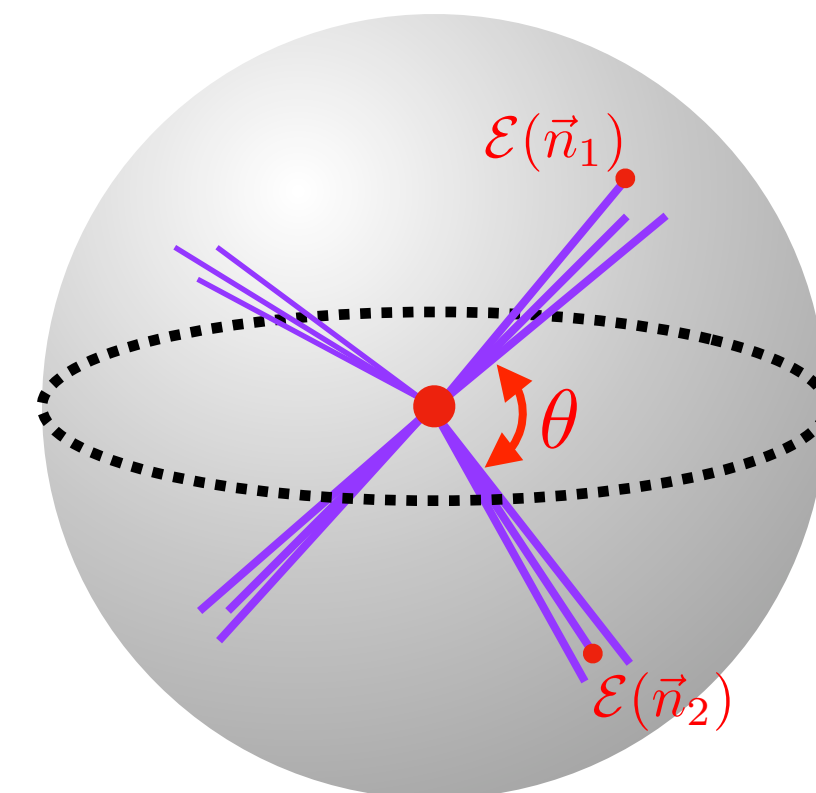
full-range EEC @ LEP





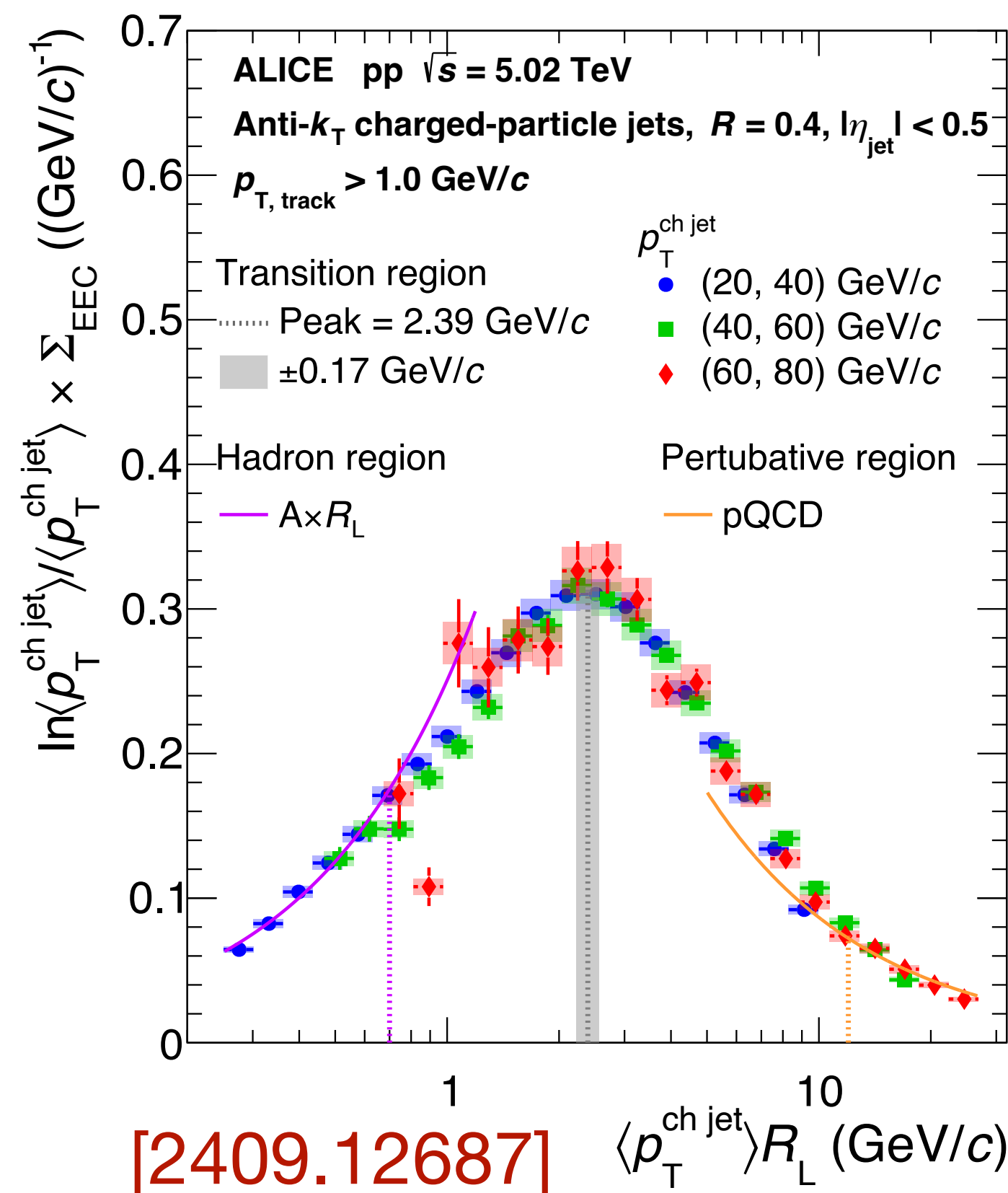
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[Basham, Brown, Ellis and Love, 1978]



collinear EEC @ LHC

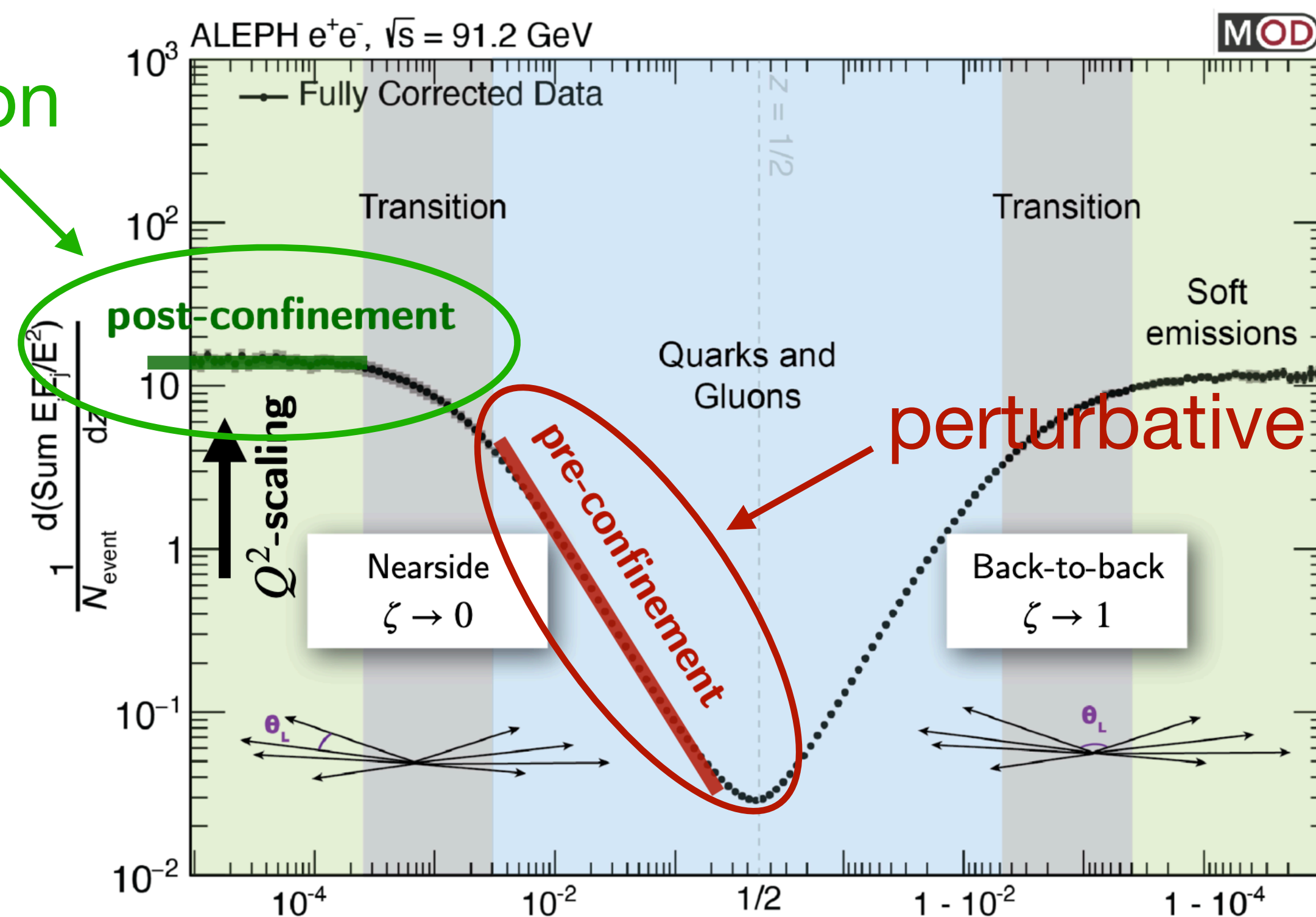
full-range EEC @ LEP



free hadron

change variable

$$\theta \rightarrow \zeta$$



see discussion with CMS Open Data
[Komiske, Moul, Thaler, Zhu, 2022]

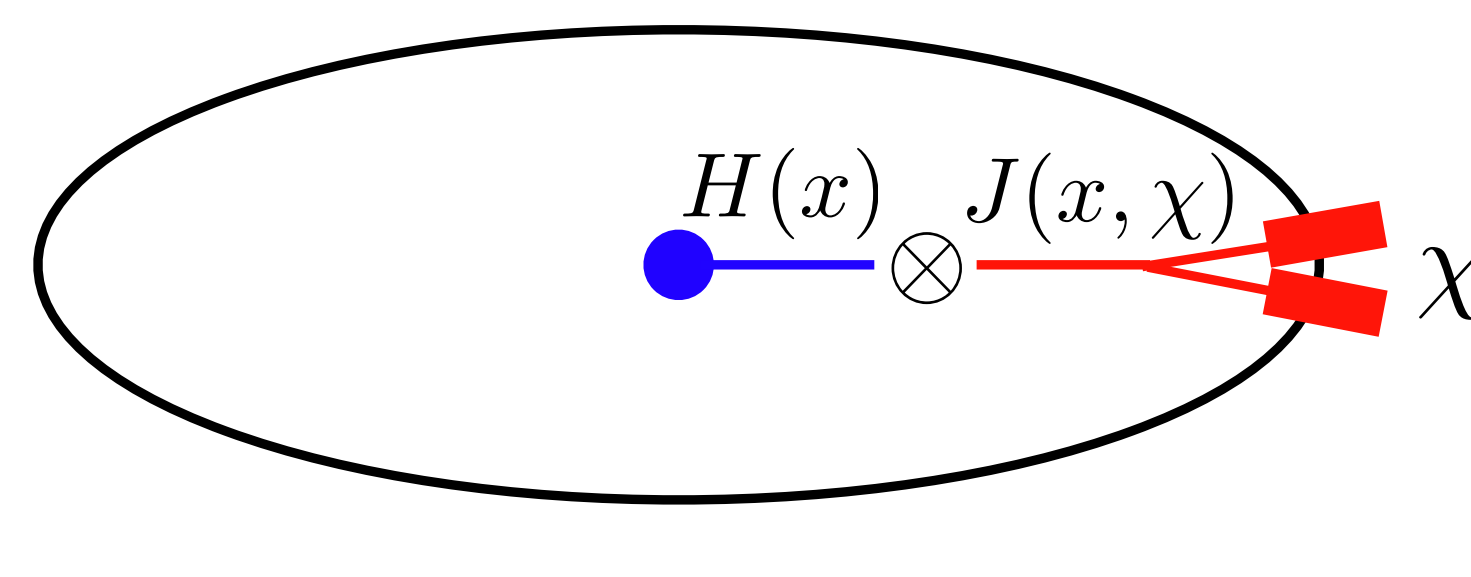
Outline

- Perturbative factorization
- Post-confinement/free hadron region
- Pre-confinement/perturbative region: power corrections

Perturbative Factorization

Theoretic Toolkits in the Collinear Limit

EEC exhibits collinear universality in pQCD, described by the **factorization formula**



$$\Sigma(\zeta, \log \frac{Q^2}{\mu^2}, \mu) = \int_0^1 dx \, x^2 \vec{J}(\log \frac{\zeta x^2 Q^2}{\mu^2}, \mu) \cdot \vec{H}(x, \frac{Q^2}{\mu^2}, \mu)$$

Jet function **Hard function**

convolution in momentum fraction x

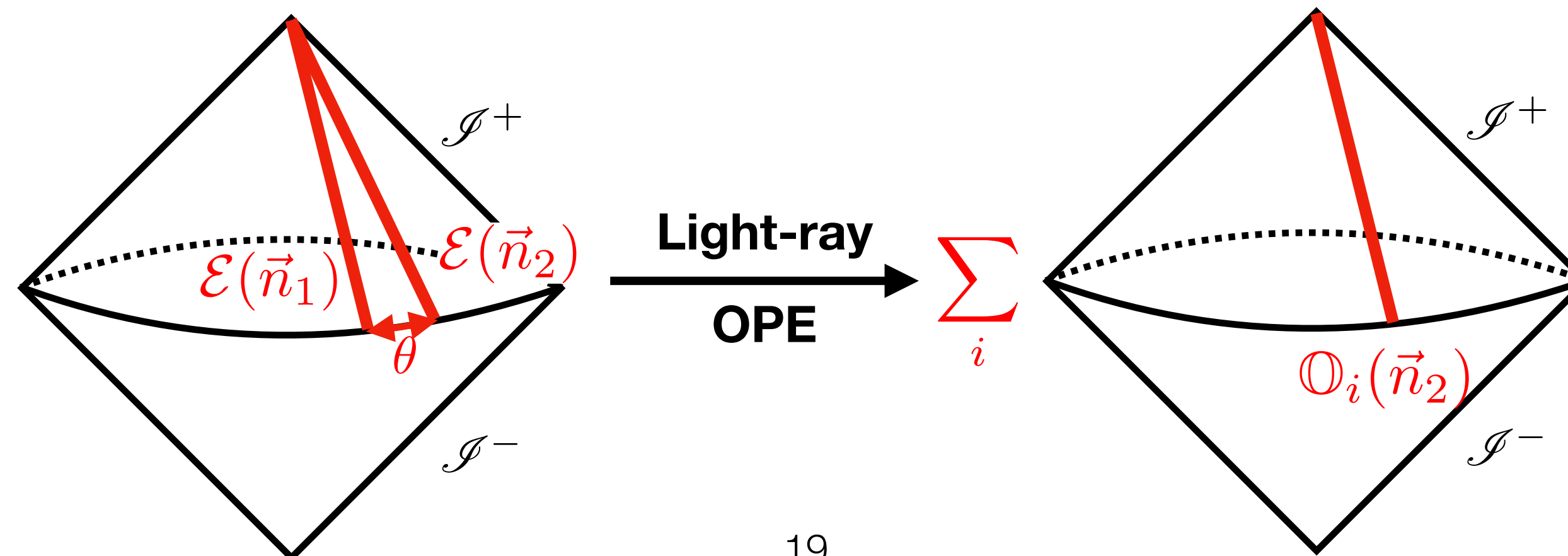
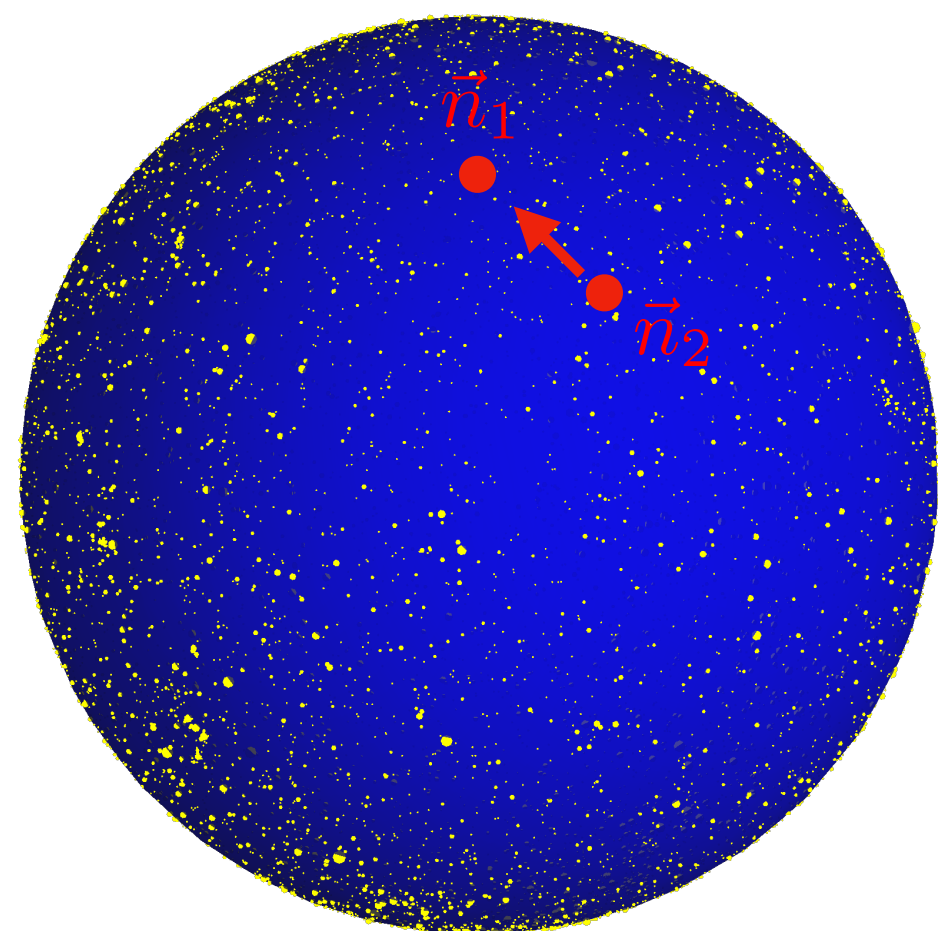
[Dixon, Moul, Zhu, 2019]

As a correlation function of light-ray operators, collinear limit has the interpretation of **light-ray OPE**.

[Hofman, Maldacena, 2008; Kologlu, Kravchuk, Simmons-Duffin, Zhiboedov, 2019]

- Lorentz Symmetry
- RG invariance
- Dimensional Analysis

[HC, 2023]



Various Operator Product Expansion



Short distance **scaling behavior** is determined by **local** OPE.

Local Operator OPE

$$\mathcal{O}_1(x_1) \mathcal{O}_2(x_2) \sim \sum \mathcal{O}(x_1)$$

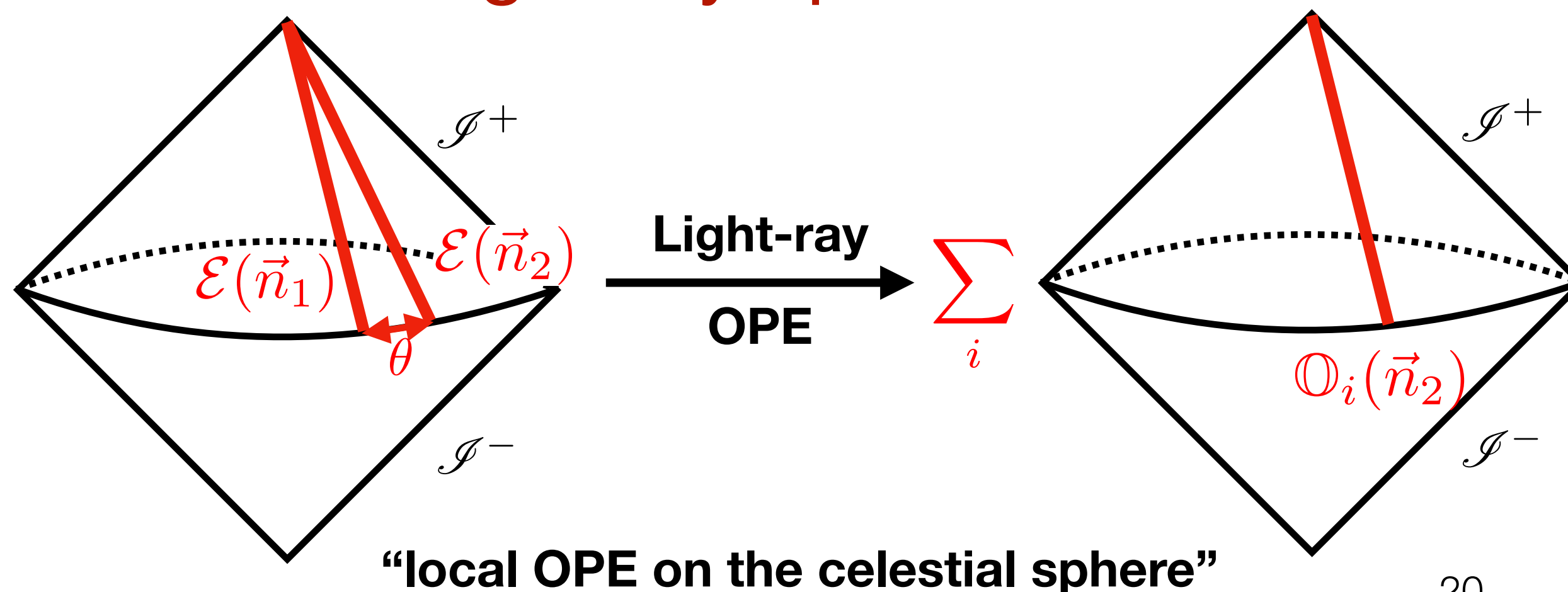
Euclidean distance OPE

is determined by operator dimension

Lightcone OPE

is controlled by twist expansion

Small angle behavior is controlled by the **OPE** of **light-ray operators**.



Light-ray OPE is a fixed-spin twist expansion in CFT

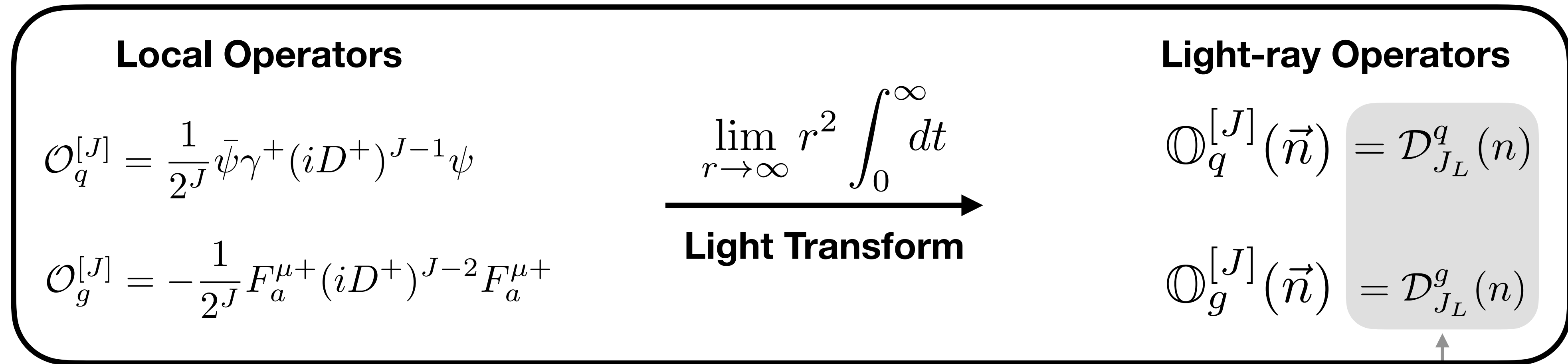
$$\mathcal{E}(\vec{n}_1) \mathcal{E}(\vec{n}_2) \sim \sum_i c_i \theta^{\tau_i - 4} \mathbb{O}_i^{[J=3]}(\vec{n}_2)$$

spin-3 light-ray operators

[Hofman, Maldacena, 2008; Kologlu, Kravchuk, Simmons-Duffin, Zhiboedov, 2019]

Leading-Twist (DGLAP) Operators in QCD

For unpolarized cases, there are only two kinds of twist-2 operators in QCD



J_L is the spin of light-ray operator, which is more useful than label J in the mixing problem. For bare DGLAP detectors, $J_L = -1 - J$.

The analytic continuation of **even spin** branch is

Physics Interpretation

[in free theory]

Measuring E^{J-1}

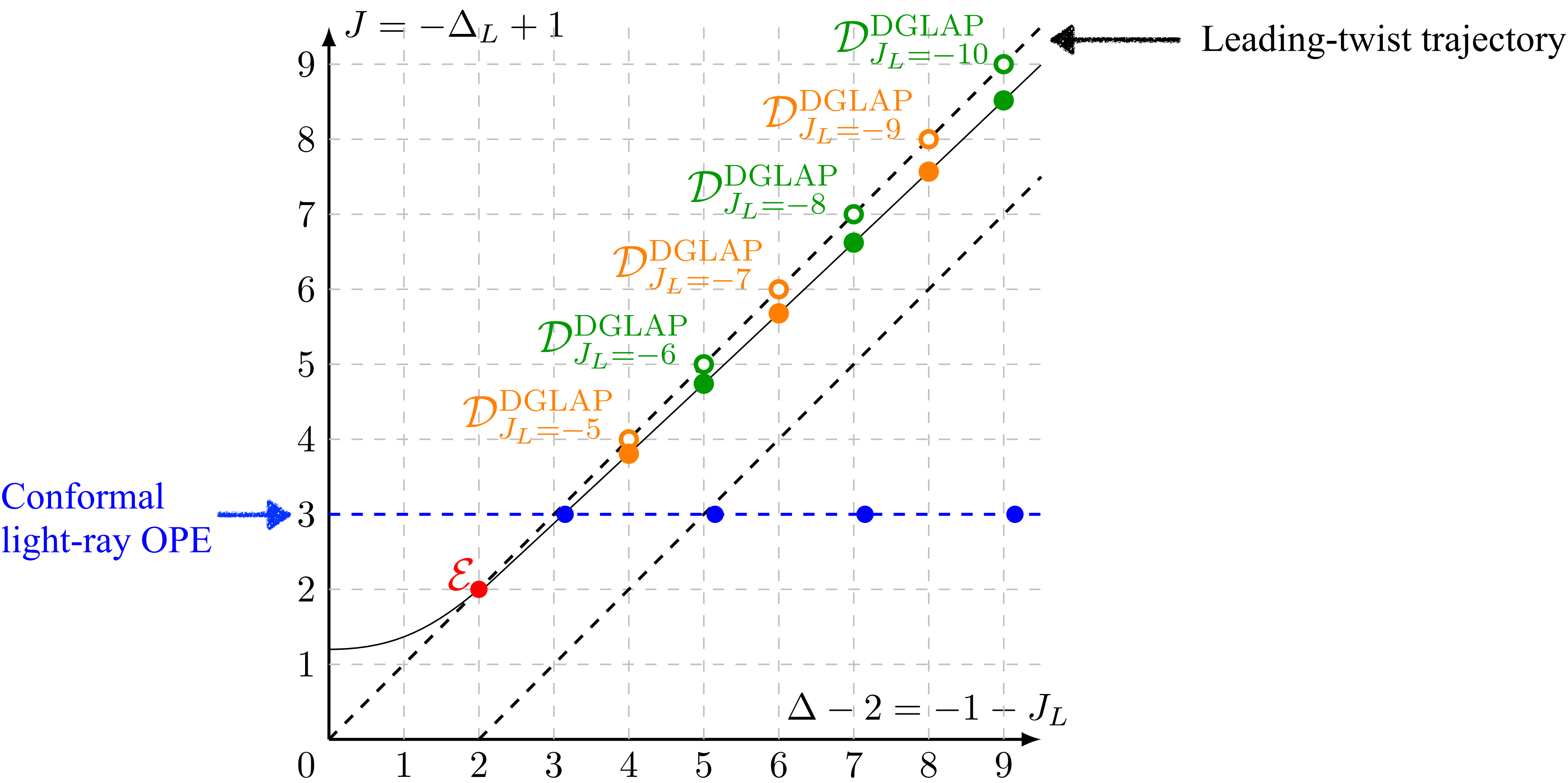
$$\mathbb{O}_q^{[J]}(\vec{n}) = \sum_s \int \frac{d^3 p}{(2\pi)^3 2E} \delta^{(2)}(\hat{p} - \vec{n}) E^{J-1} (b_{\vec{p},s}^\dagger b_{\vec{p},s} + d_{\vec{p},s}^\dagger d_{\vec{p},s})$$

$$\mathbb{O}_g^{[J]}(\vec{n}) = \sum_{\lambda,c} \int \frac{d^3 p}{(2\pi)^3 2E} \delta^{(2)}(\hat{p} - \vec{n}) E^{J-1} a_{\vec{p},\lambda,c}^\dagger a_{\vec{p},\lambda,c}$$

Not IR-safe measurement

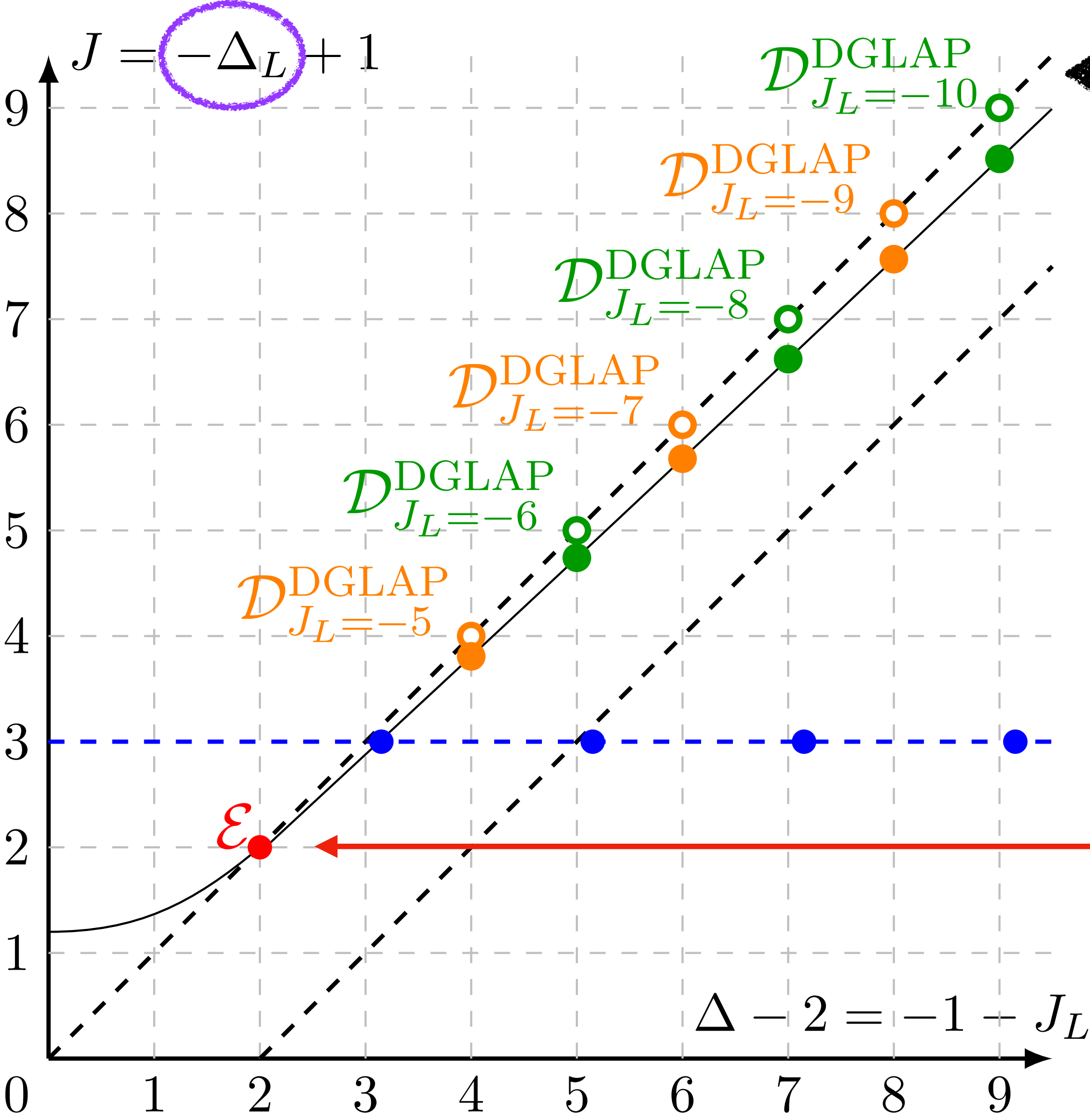
[HC, Mout, Zhu, 2021]

Operators on Chew-Frautschi Plot



Operators on Chew-Frautschi Plot

dim. of light-ray operators



Leading-twist trajectory

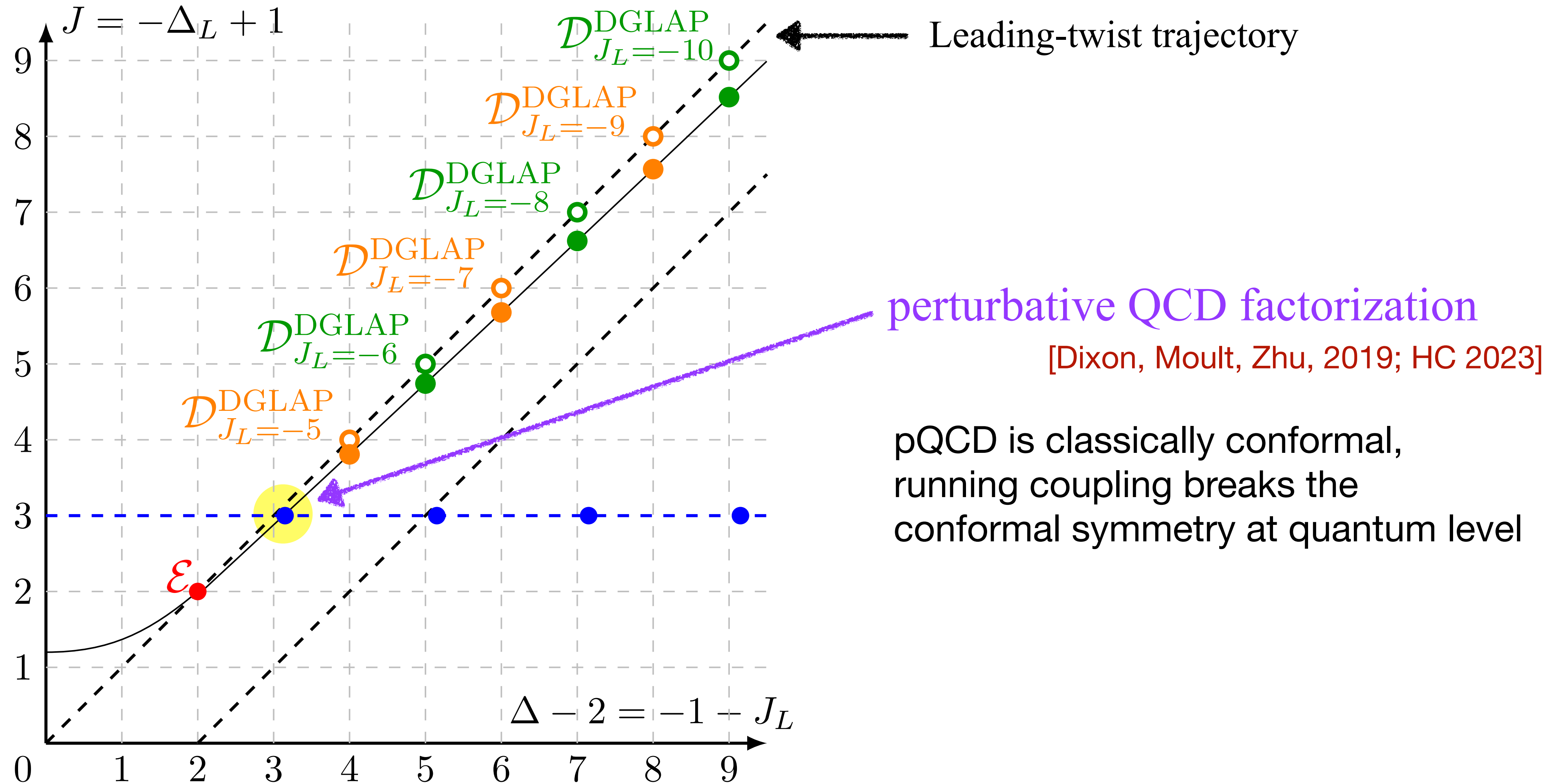
[Hofman, Maldacena, 2008; Kologlu, Kravchuk, Simmons-Duffin, Zhiboedov, 2019]

Conformal light-ray OPE

EEC dimension=2

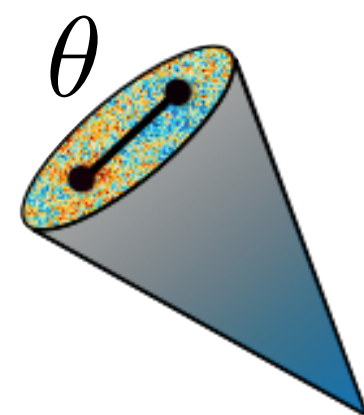
energy detector dimension=1

Operators on Chew-Frautschi Plot



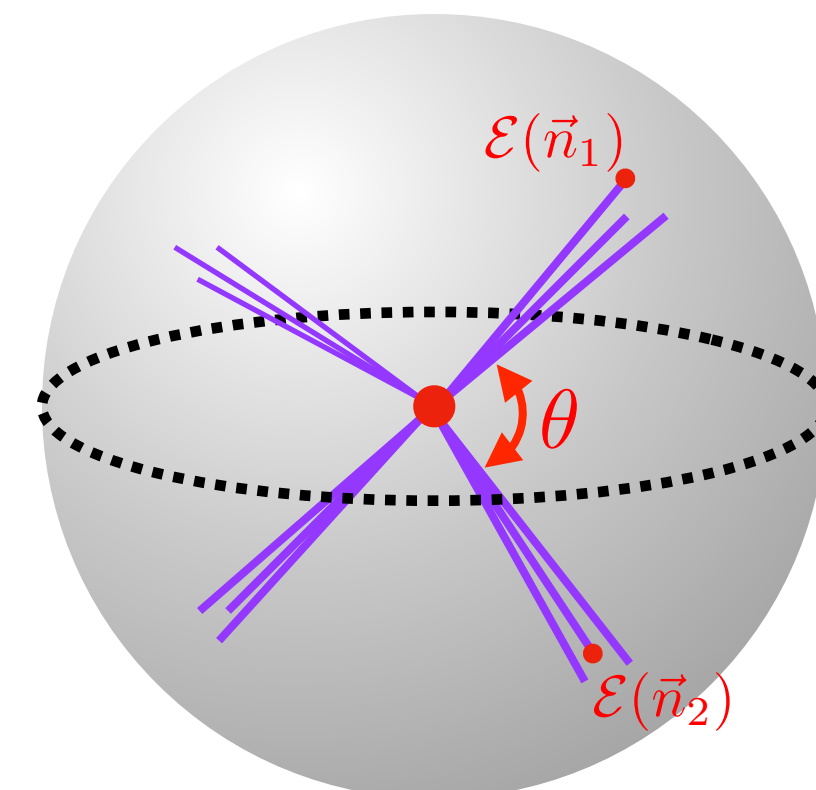
Post-confinement regime/free hadron regime

[Chang, HC, Liu, Simmons-Duffin, Yuan, Zhu, 2507.15923]



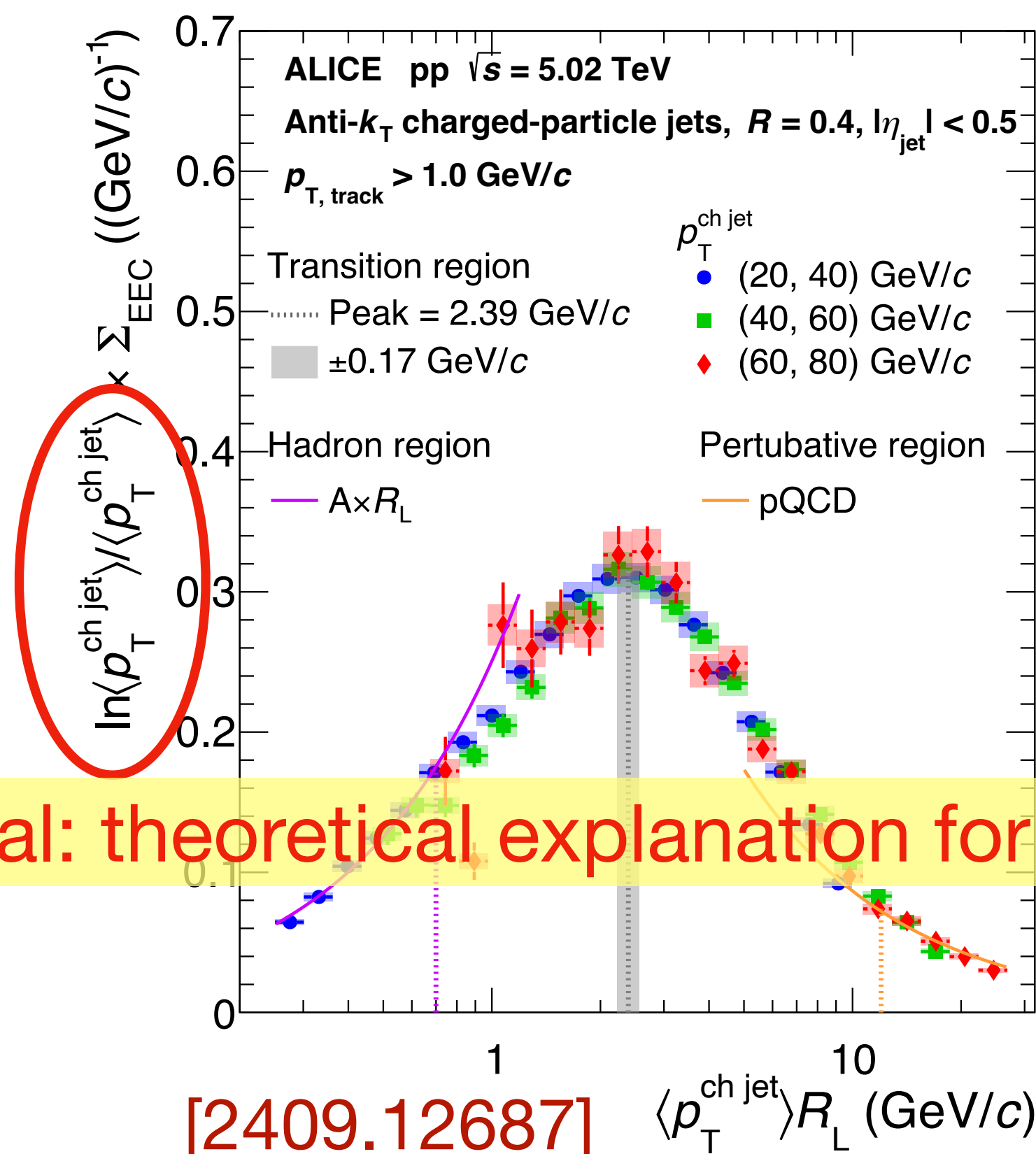
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[Basham, Brown, Ellis and Love, 1978]



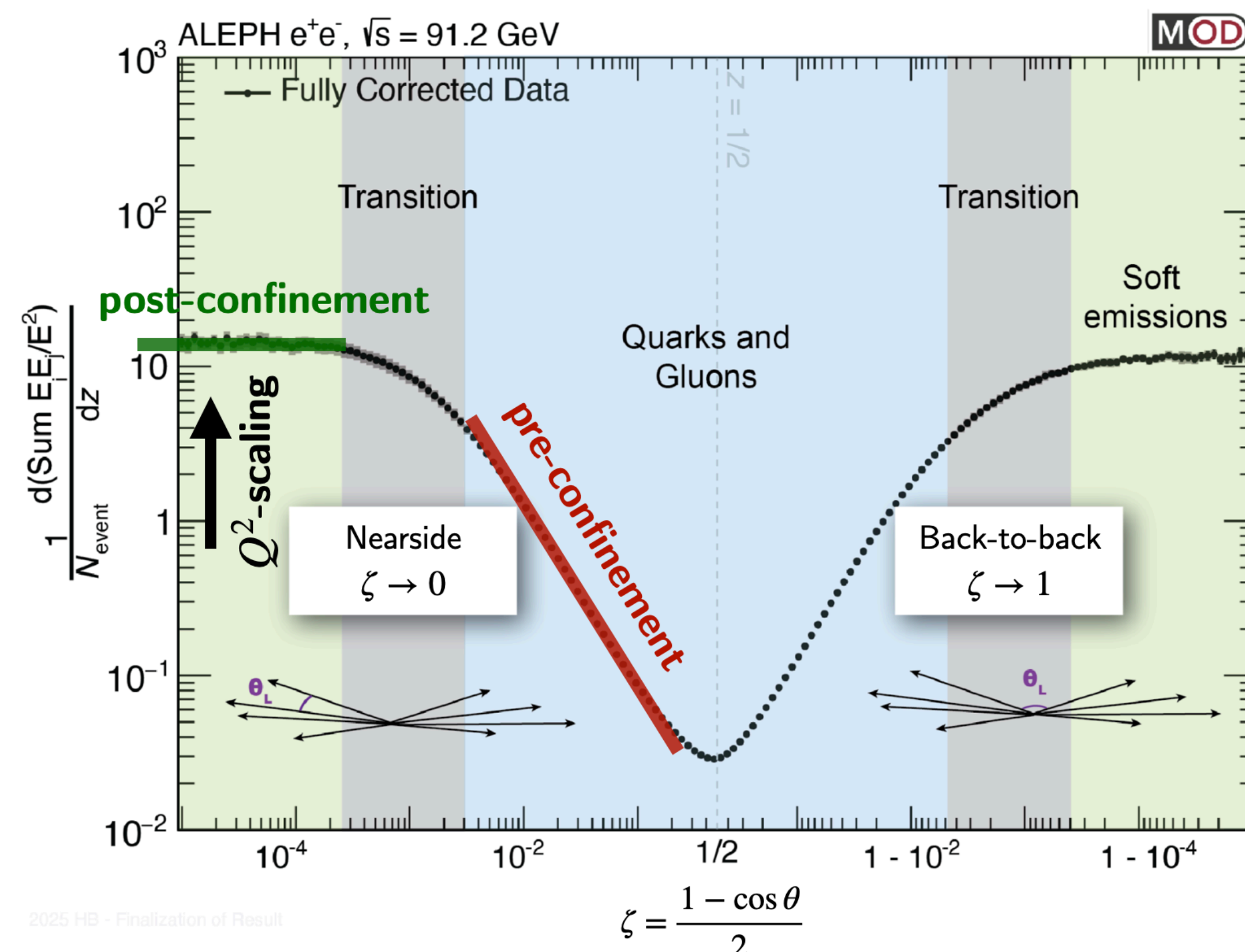
collinear EEC @ LHC

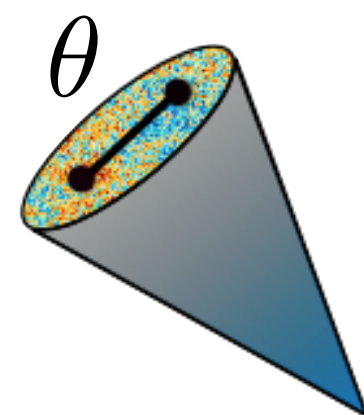
full-range EEC @ LEP



change variable

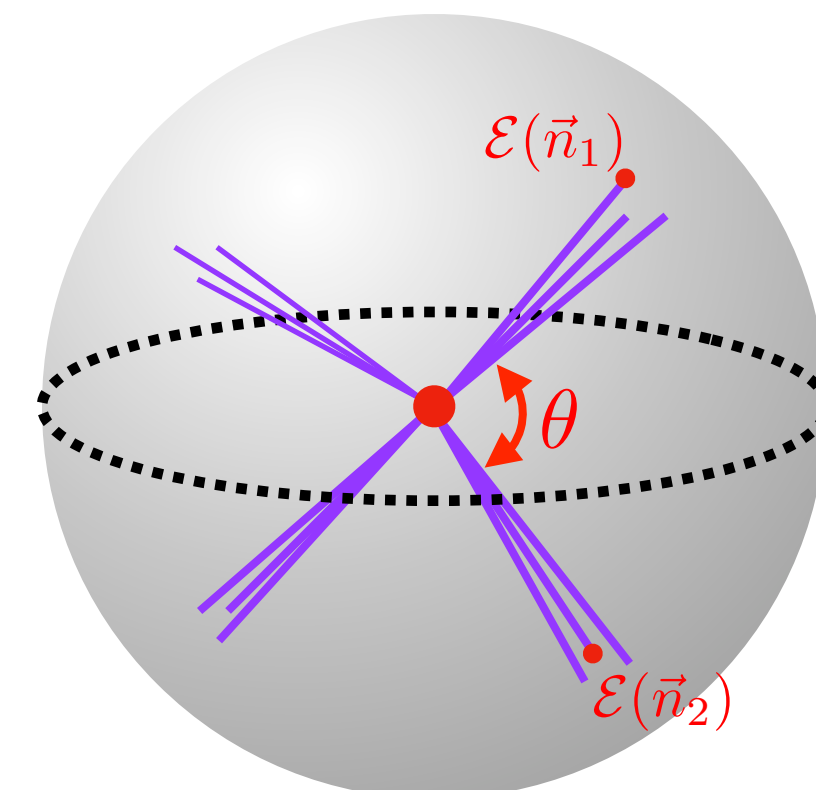
$$\theta \rightarrow \zeta$$





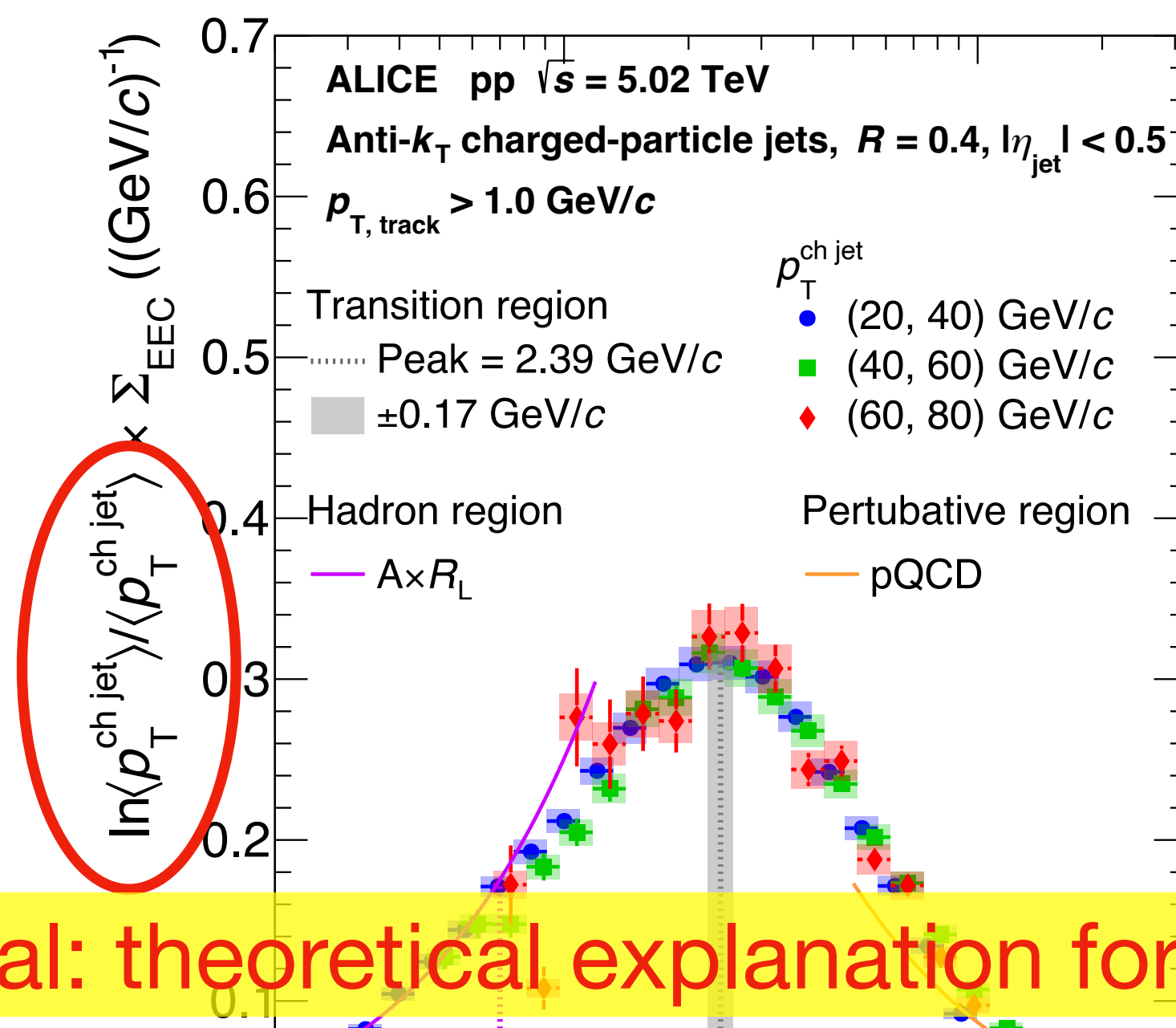
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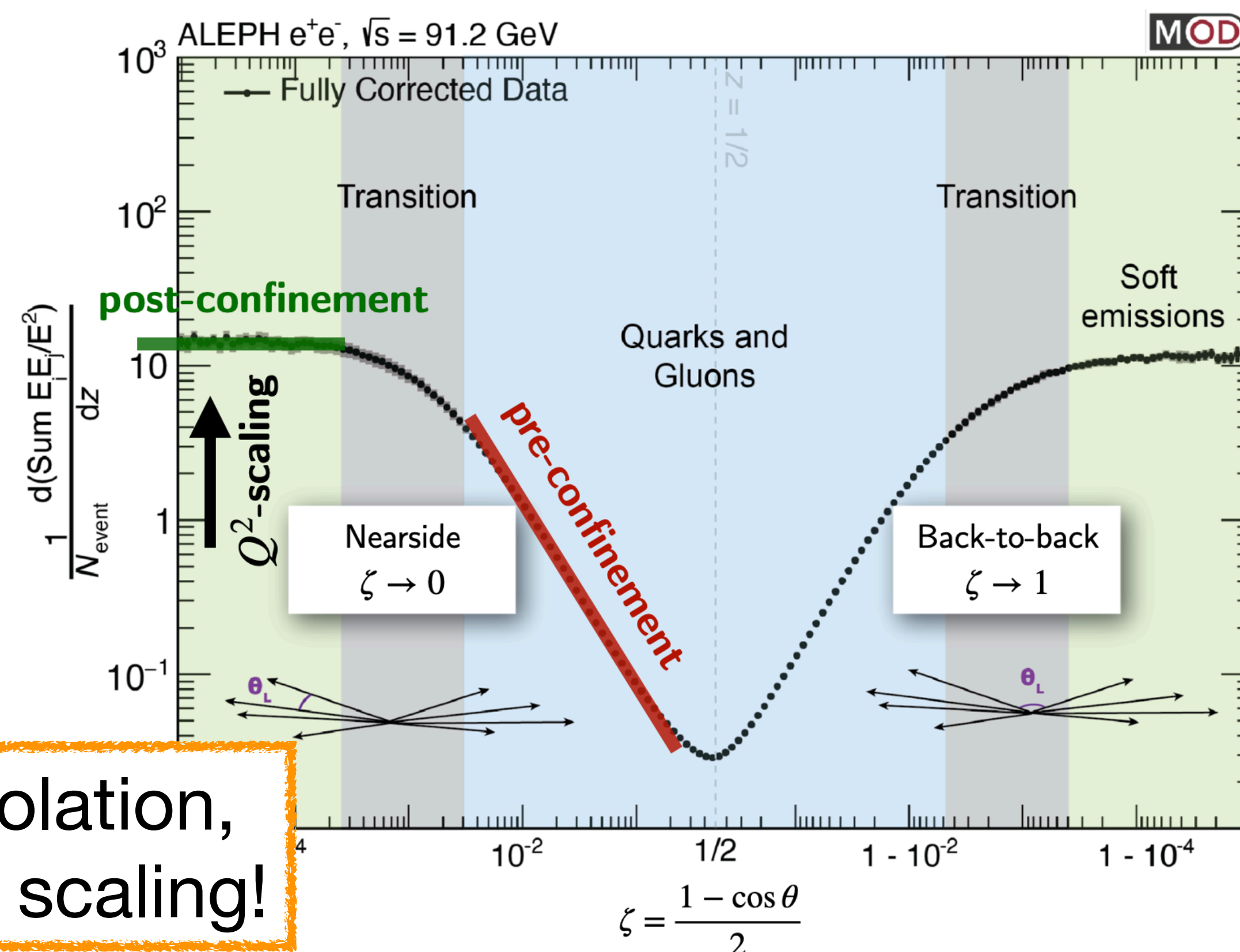


collinear EEC @ LHC

full-range EEC @ LEP



change variable
 $\theta \rightarrow \zeta$



Goal: theoretical explanation for this prefactor

If you are familiar with Bjorken scaling and violation, this is a parallel story — **classical** + **quantum** scaling!

Light-ray OPE in Post-confinement Region

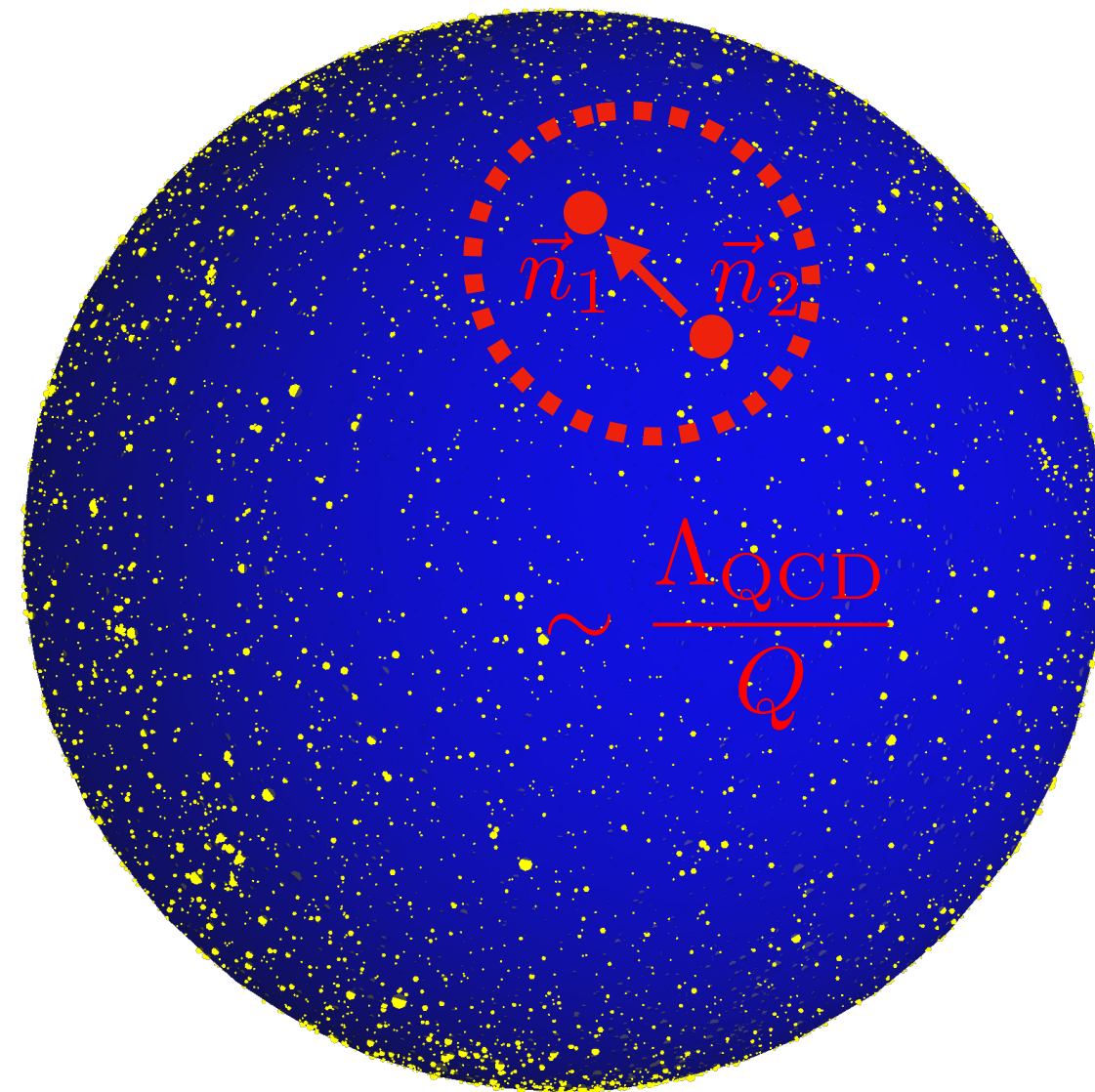
Post-confinement region — free hadron approx.

$$\zeta Q^2 \ll \Lambda_{\text{QCD}}$$

$$\mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2) \sim \zeta^0[\mathcal{E}\mathcal{E}] + \dots$$

regular behavior
(non-interacting hadrons)

hadronic operators



Compare with perturbative region

$$\mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2) \sim \frac{1}{\zeta} \mathcal{D}_{J_L=-4}^{\text{DGLAP}} + \dots$$

singular behavior
($1/\theta^2$ collinear divergence)

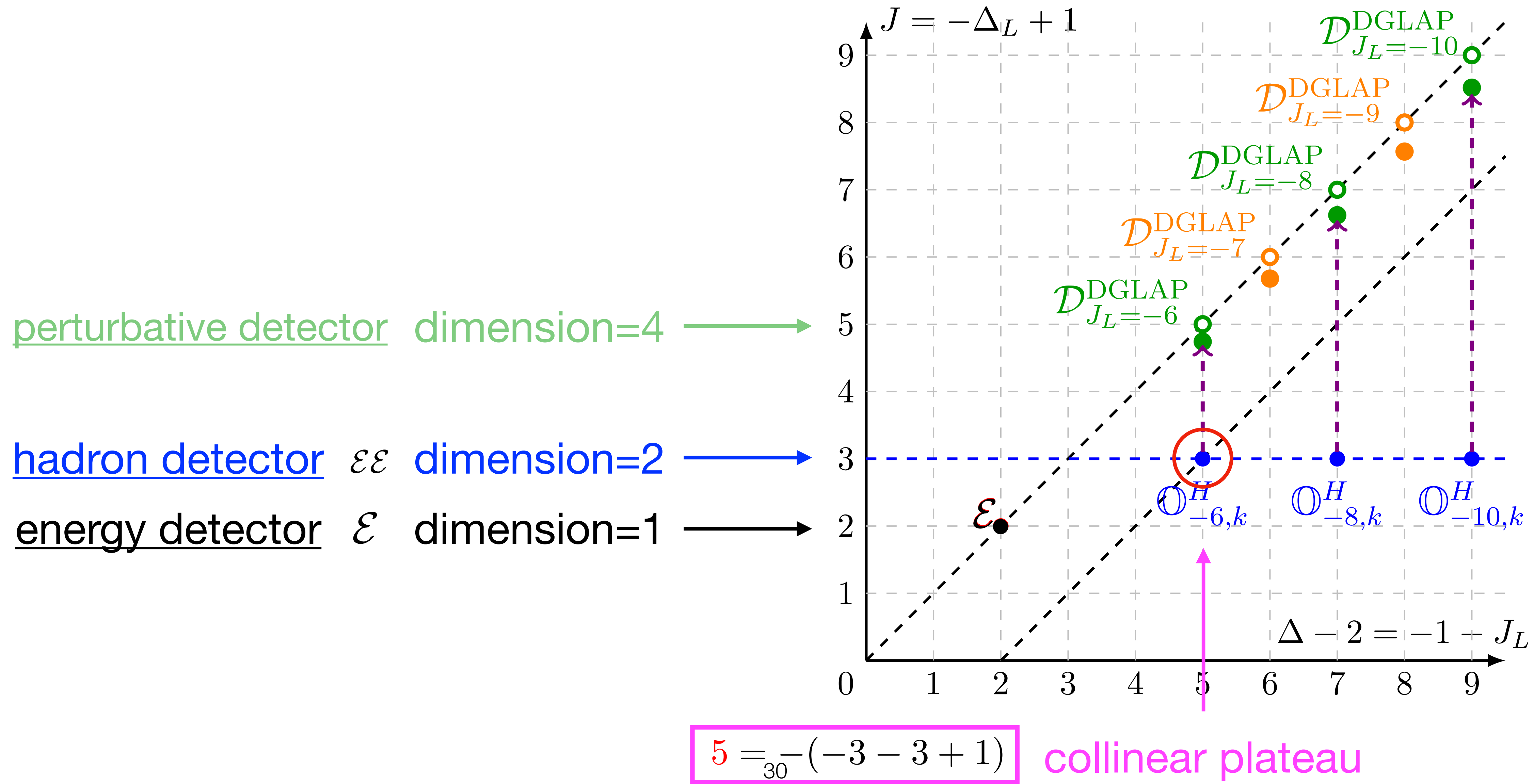
perturbative operators

magic of kindergarten math

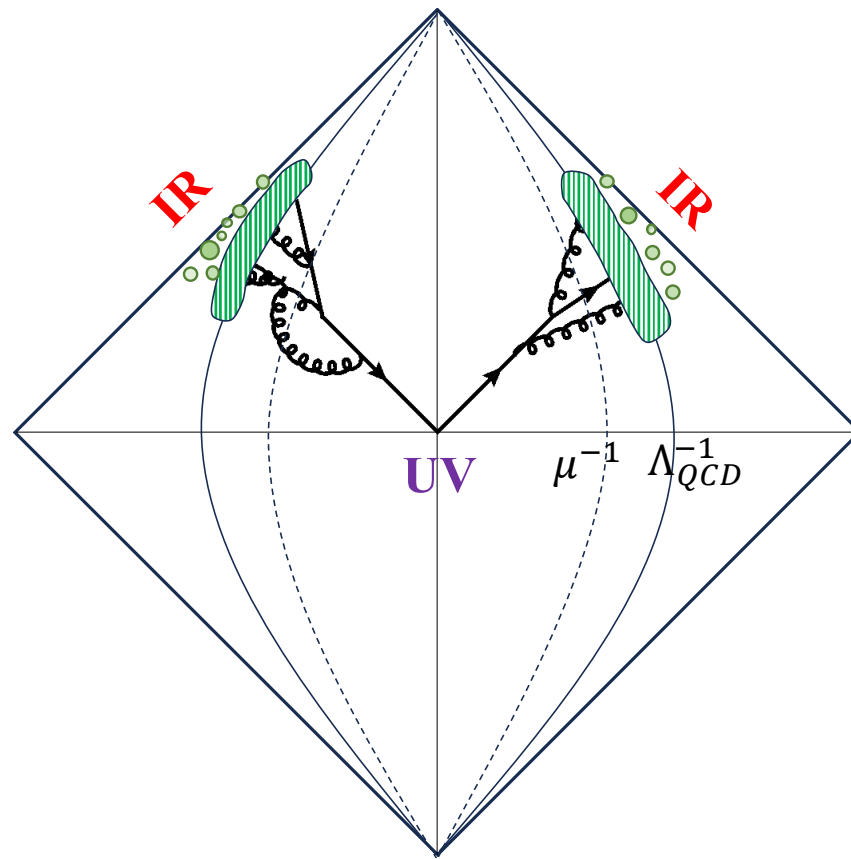
$$5 = -(-3 - 3 + 1)$$

$$2 = 5 - (1 + 1 + 1)$$

Operator Spectrum (Chew-Frautschi Plot)



Magic of CF plots (Classical)



perturbative detector dimension=4 →

↑ detector matching ★

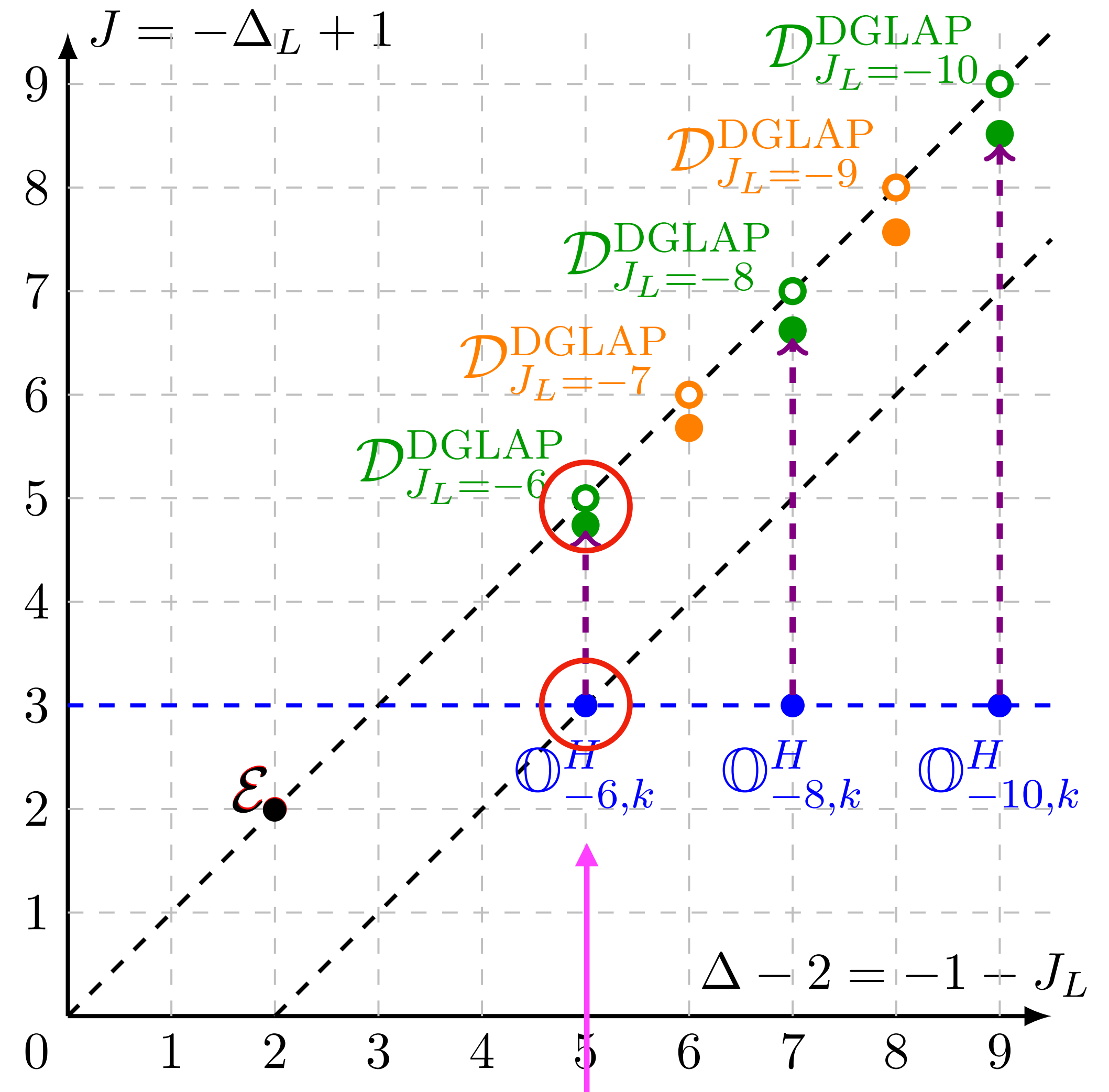
hadron detector $\varepsilon\varepsilon$ dimension=2 →

energy detector $\mathcal{E}$ dimension=1 →

$$2 = 5 - (1 + 1 + 1)$$

$$\sim \Lambda_{\text{QCD}}^{-2} \longrightarrow Q^2 / \Lambda_{\text{QCD}}^2$$

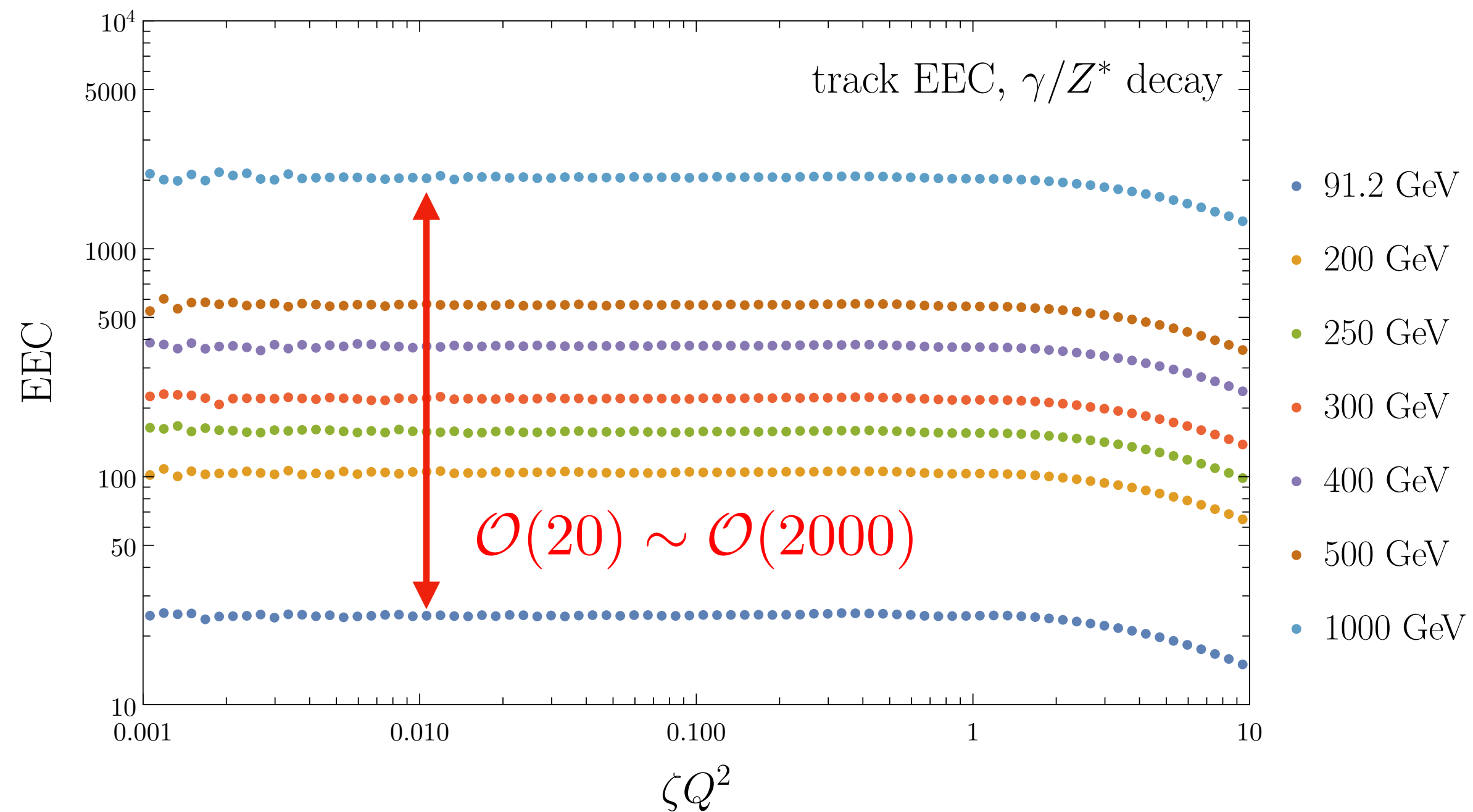
classical scaling



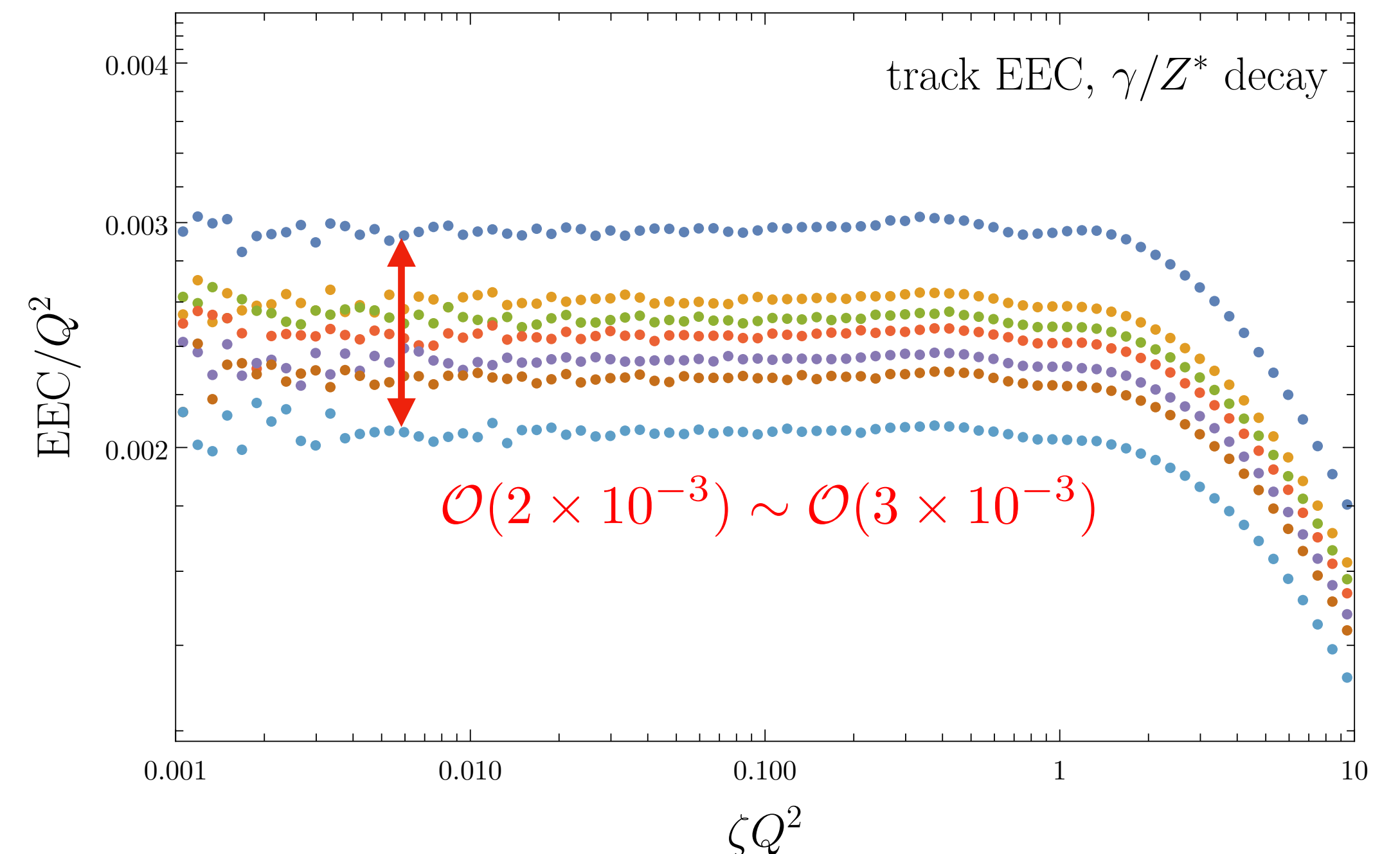
collinear plateau

Verification of Classical Scaling

Original EEC

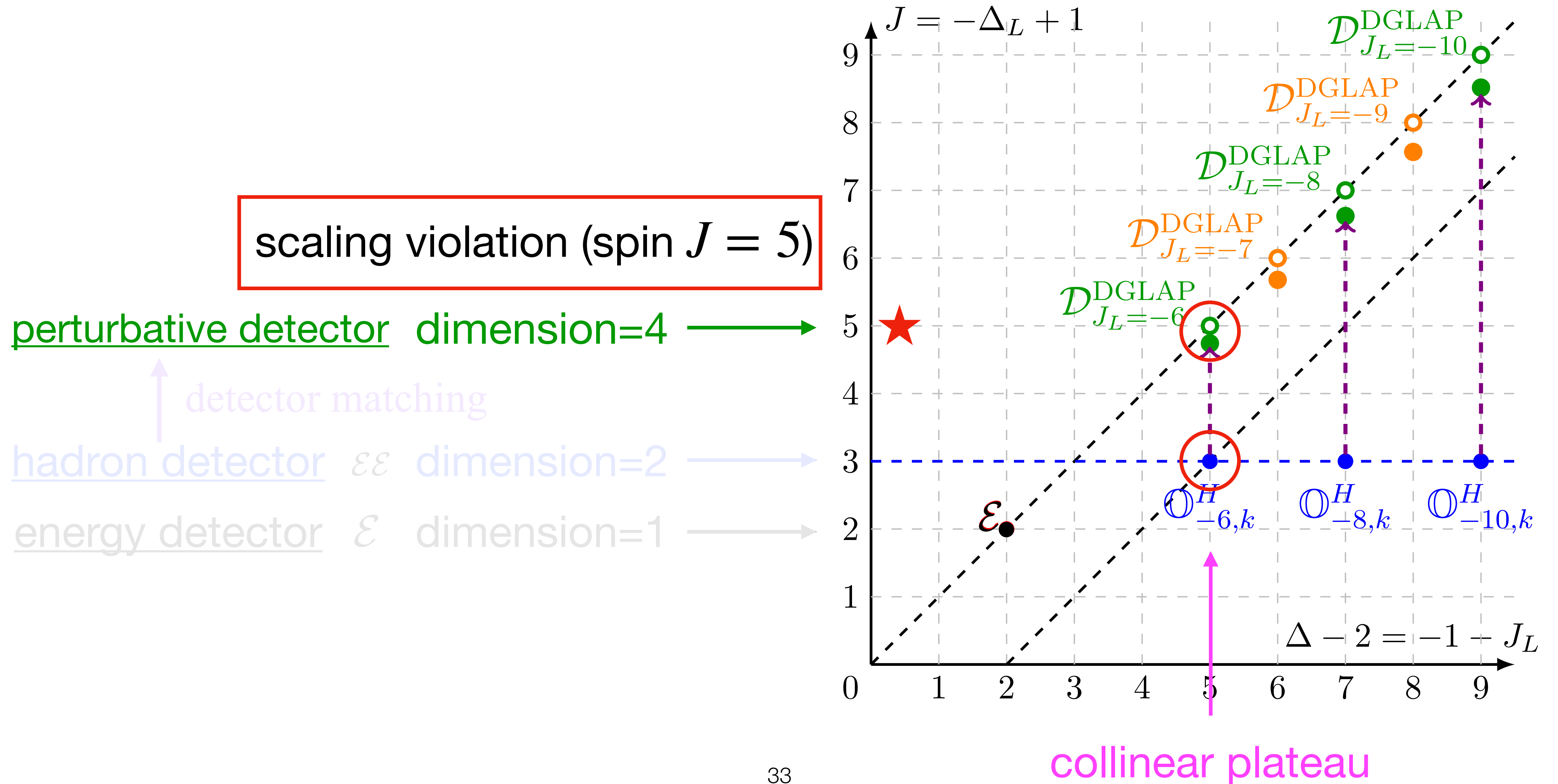


Classical Scaling Stripped EEC



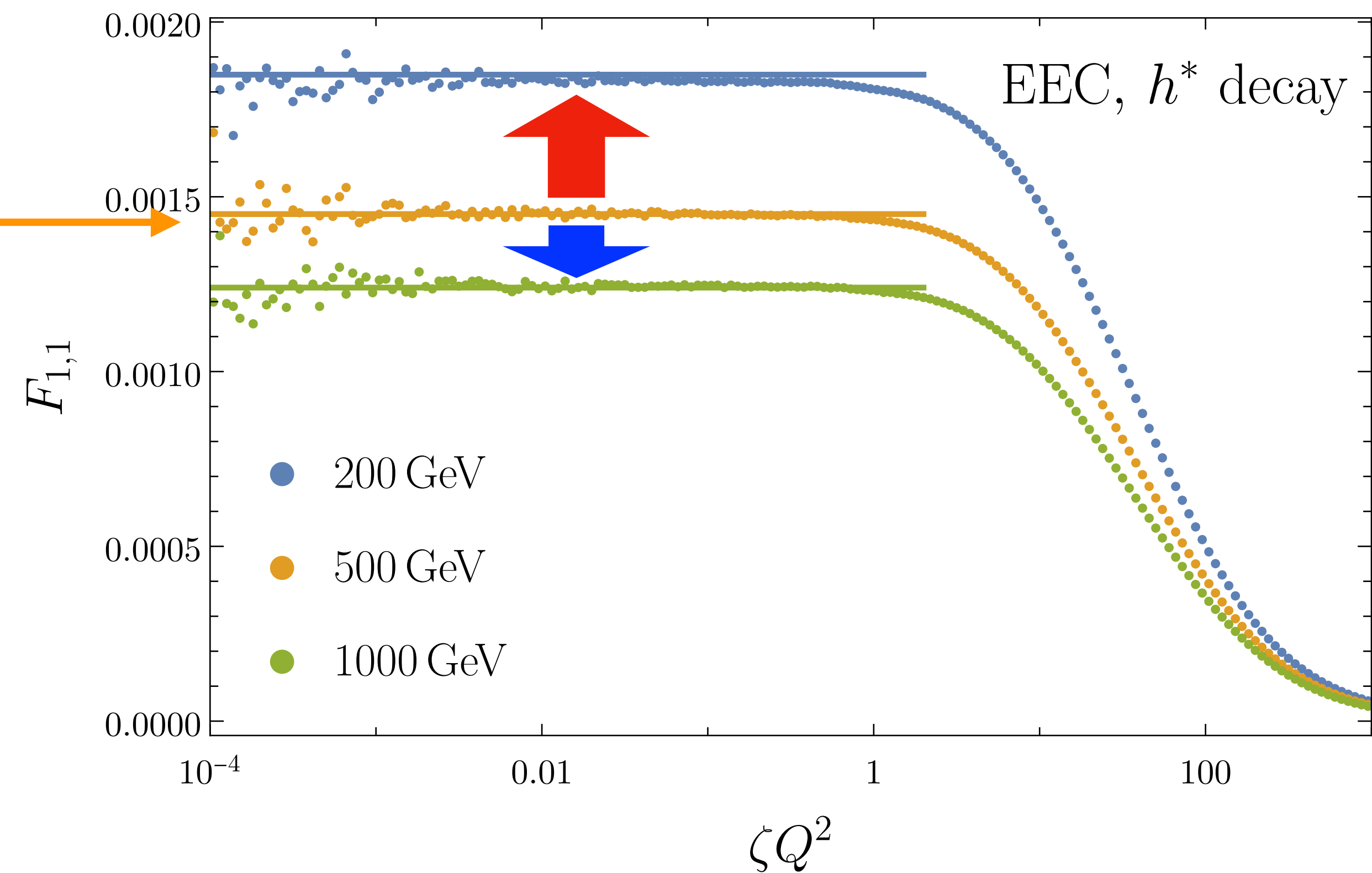
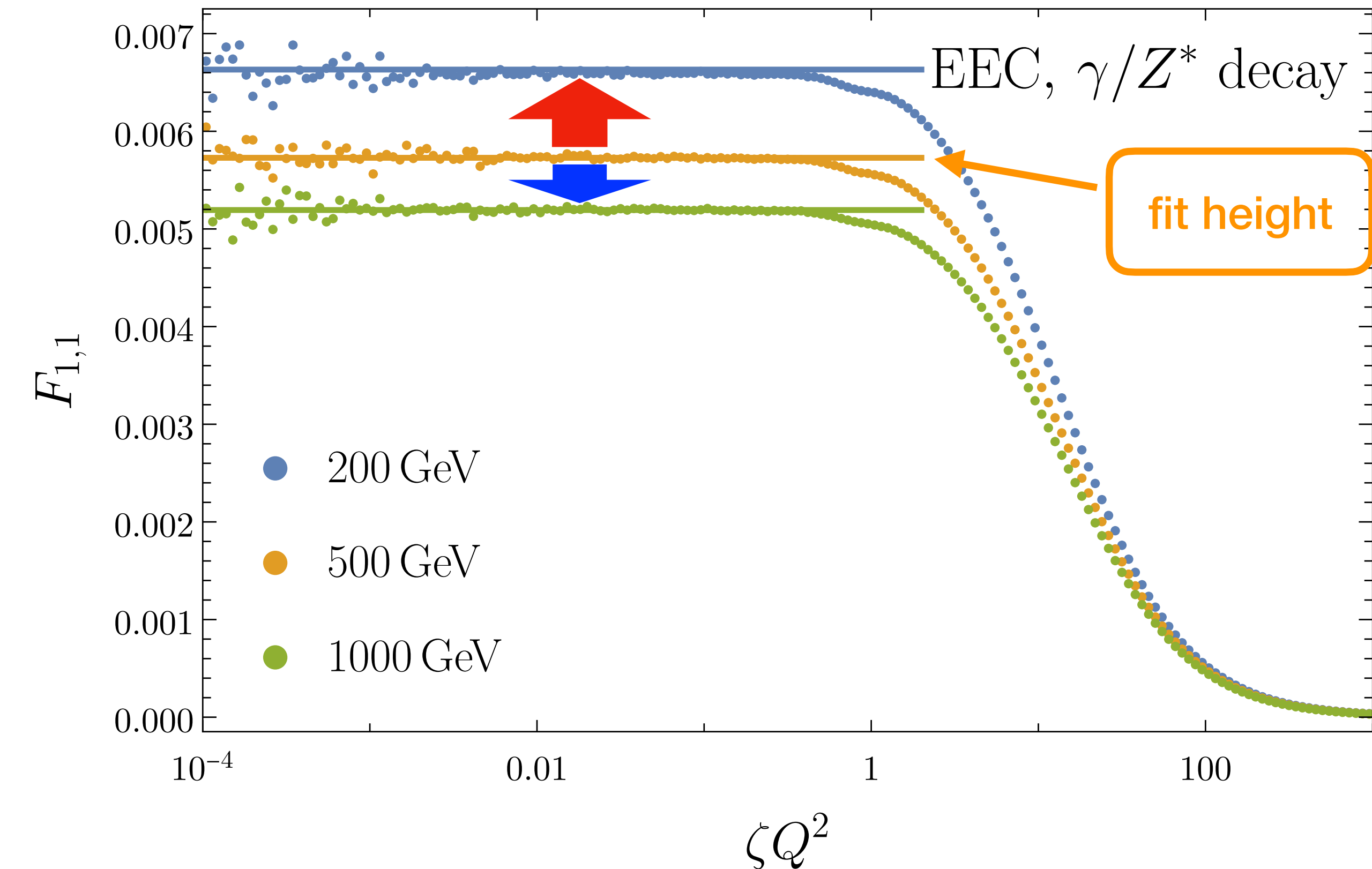
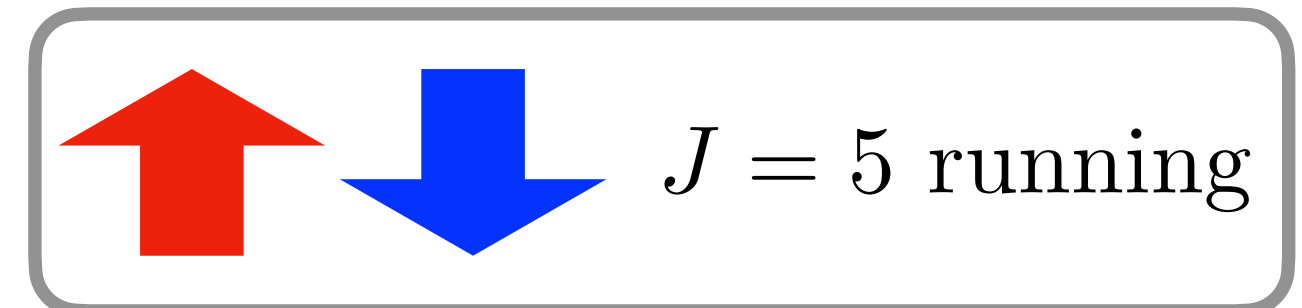
(small) anomalous scaling remains

Magic of CF plots (Quantum)



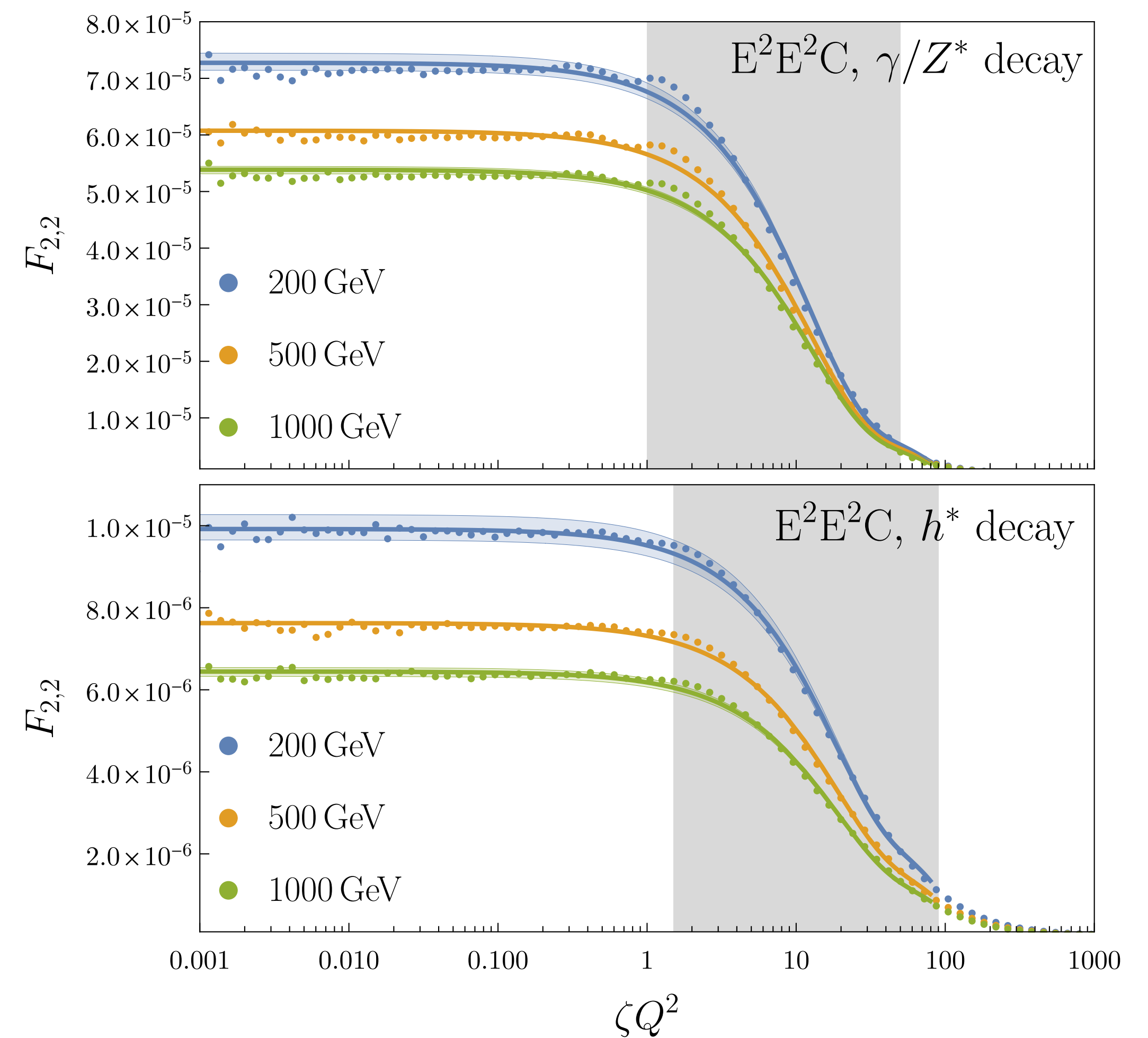
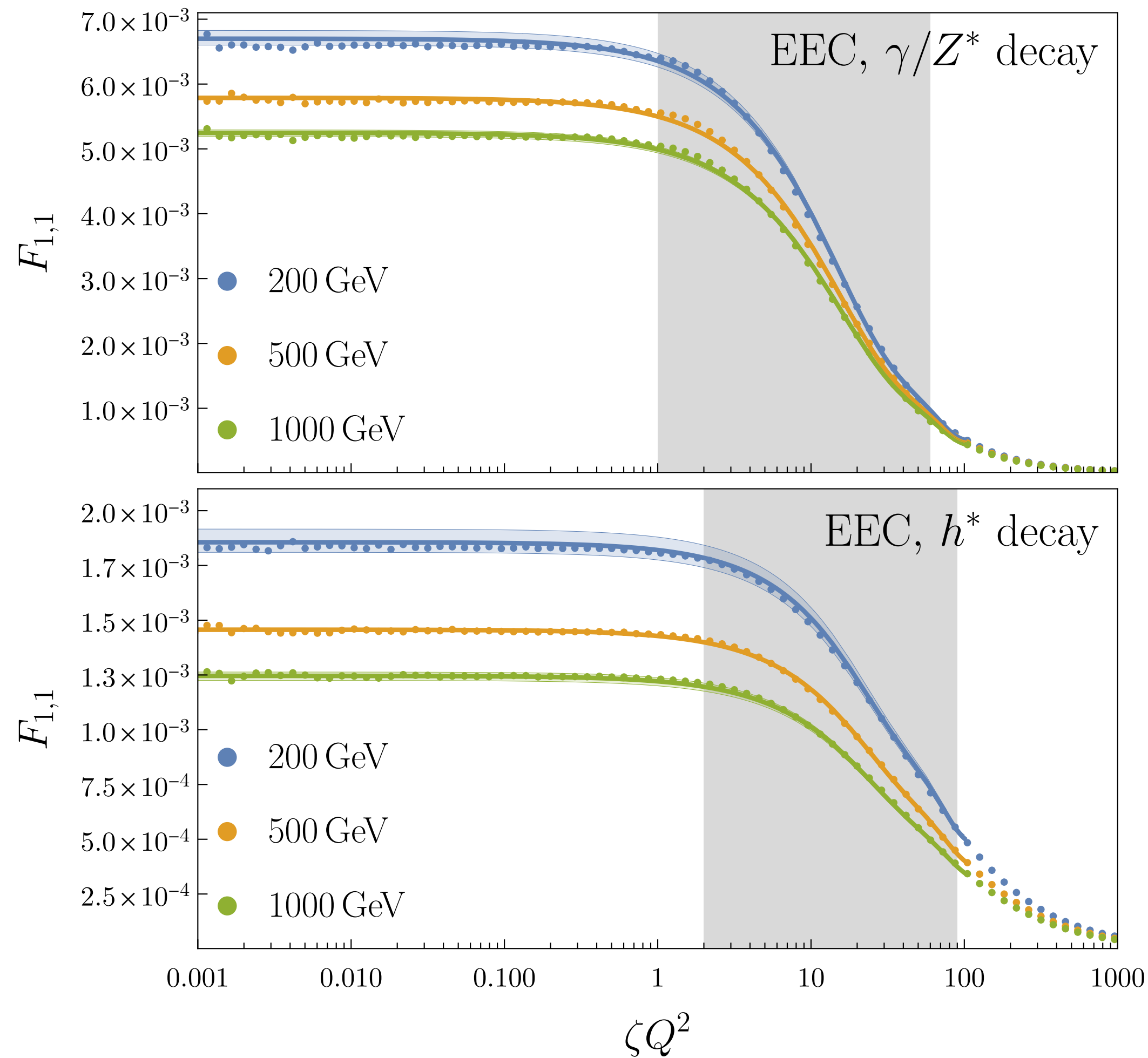
Comparison with Pythia Simulation

Fit 500 GeV (orange) curves and predict other curves using scale evolution.



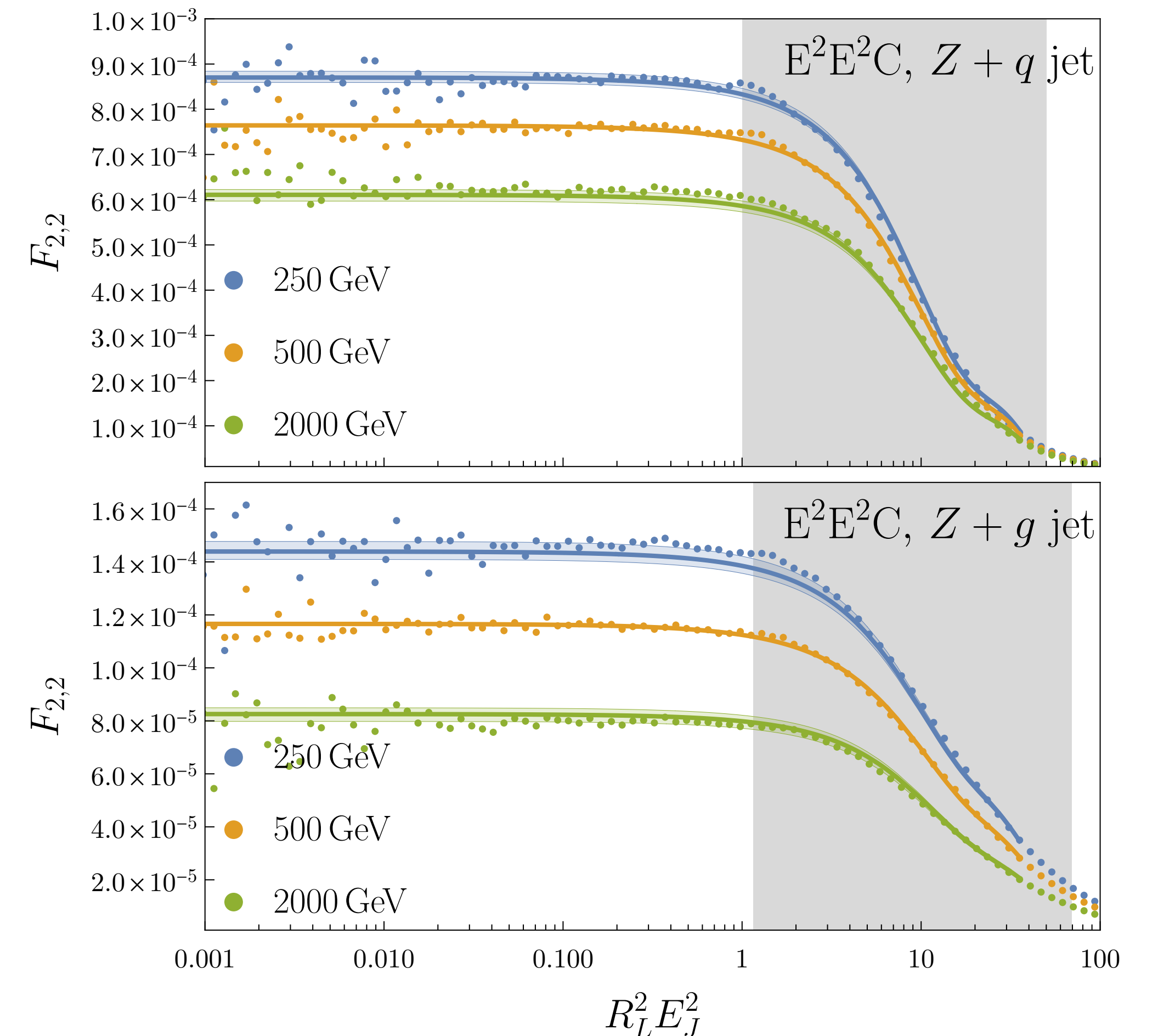
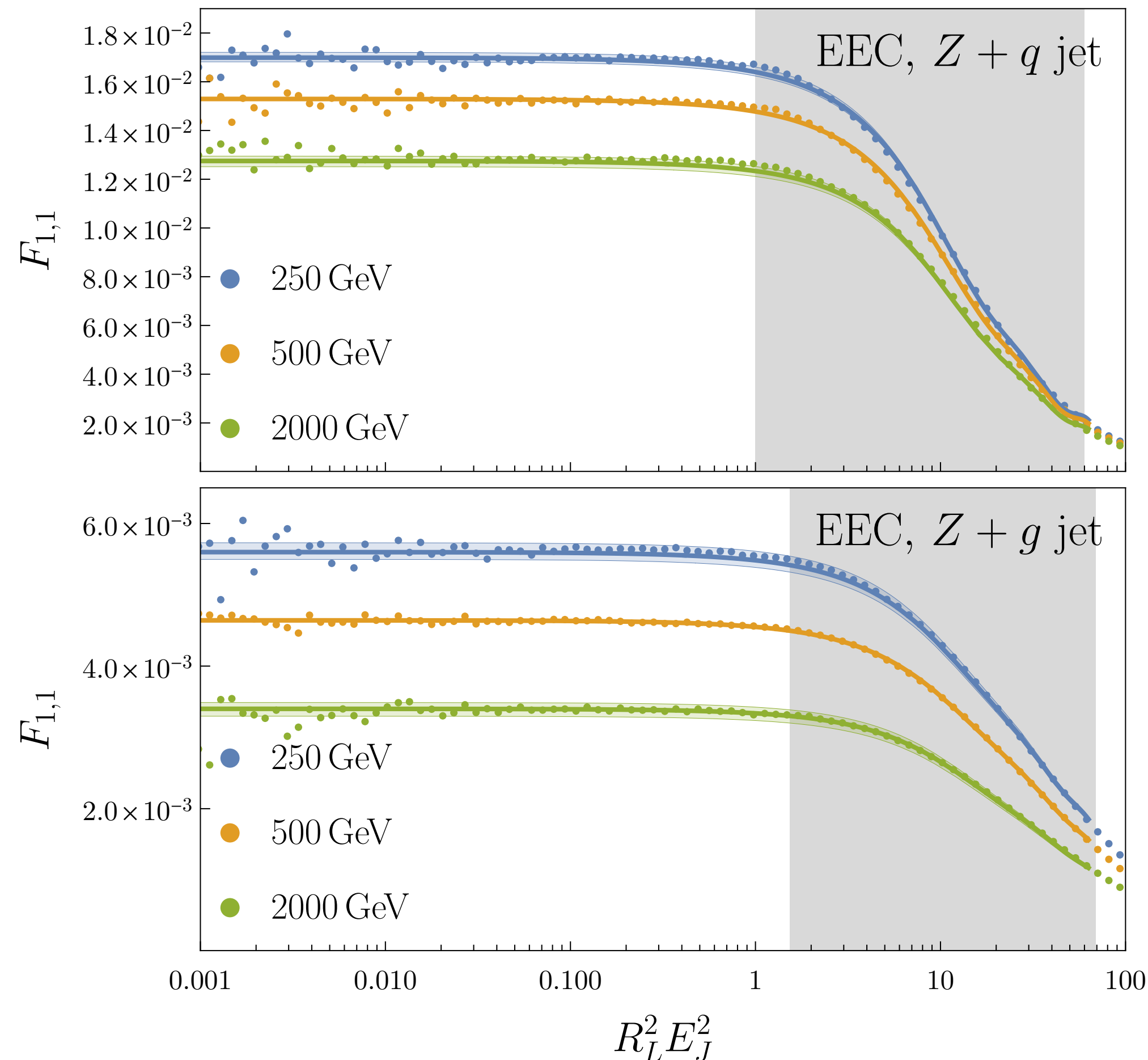
Adding more terms

Fit **500 GeV (orange) curves** and predict other curves using scale evolution.



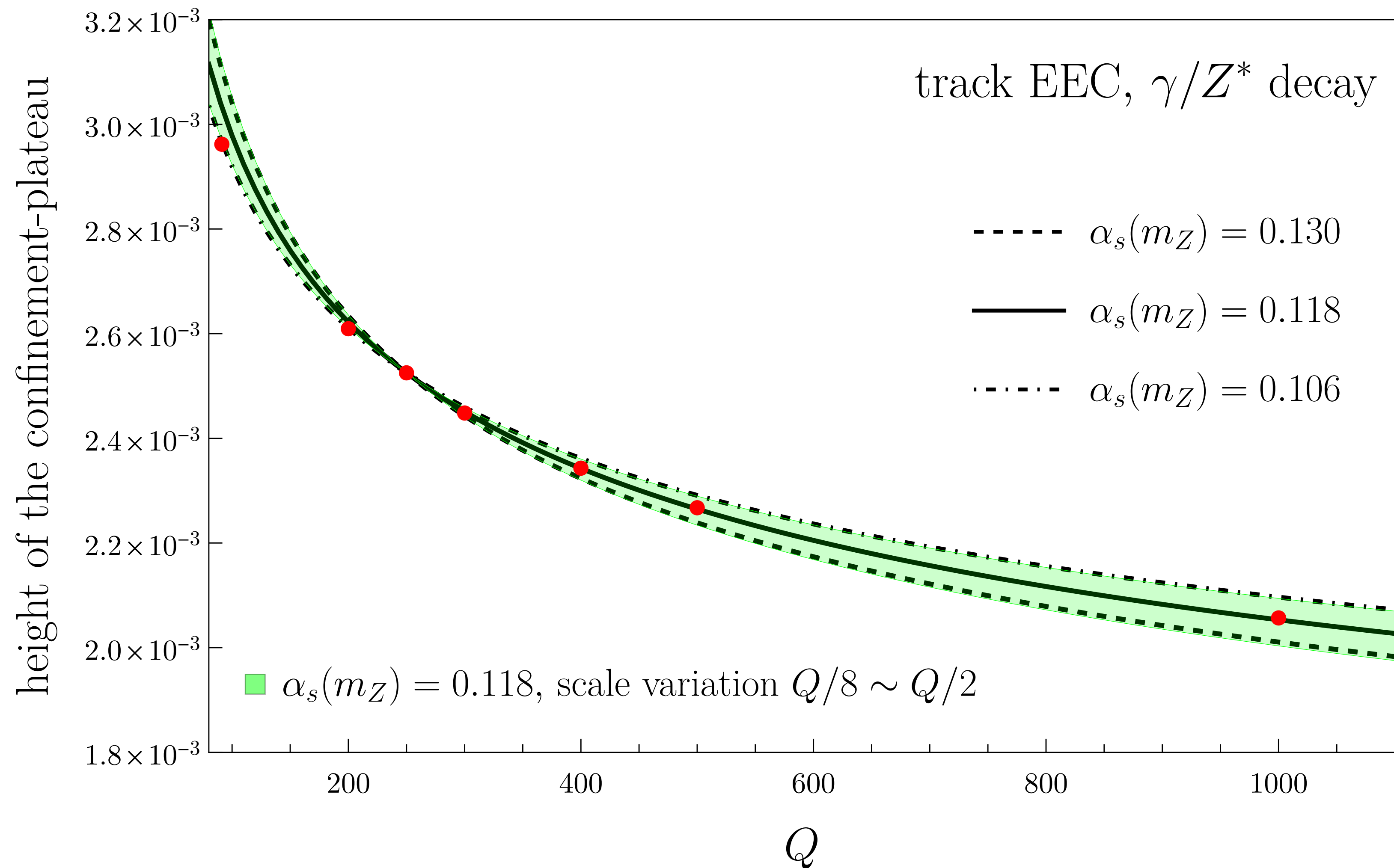
LHC jets in Pythia

Fit 500 GeV (orange) curves and predict other curves using scale evolution.



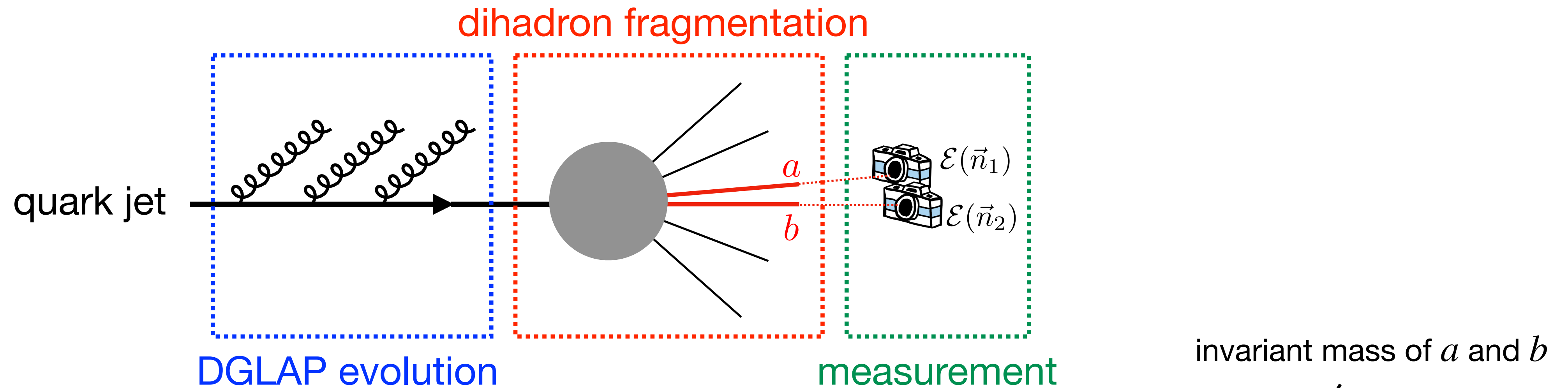
“Running” Plateau and Running Coupling

[Height]



Dihadron Fragmentation Picture

The same physics can be understood with dihadron fragmentation



$$\text{EEC}(\zeta, Q) = \int_0^1 dx x^2 \vec{H}\left(x, \frac{Q^2}{\mu^2}\right) \int_0^\infty dm^2 \delta\left(\zeta - \frac{m^2}{4x^2\xi_1\xi_2Q^2}\right) \cdot \sum_{(a,b)} \int d\xi_1 d\xi_2 \frac{\xi_1\xi_2}{4} \vec{D}_{h_a h_b}(\xi_1, \xi_2; m^2; \mu^2; \Lambda_{\text{QCD}}^2)$$

Four other papers
appearing in the
same week

also see [Lee, Stewart, 2507.11495; Kang, Metz, Pitonyak, Zhang, 2507.17444; Herrmann, Kang, Penttala, Zhang, 2507.17704; Guo, Yuan, Zhao, 2507.15820]

Perturbative Region: Power Corrections

[HC, Pier Monni, Zhen Xu, Hua Xing Zhu, 2406.06668]

Light-ray OPE for Leading Power Correction

In the perturbative region, we expect

$$\text{EEC} = \text{perturbative QCD prediction} + \text{power corrections in } \frac{\Lambda_{QCD}}{Q}$$

Previous studies on hadronization effects of EEC show universal feature in the collinear limit

consider the region

$\Lambda_{QCD} \ll \theta Q \ll Q$

$$\sim \frac{1}{\theta^3 Q}$$

Fragmentation model: Basham, Brown, Ellis, Love, 1979;
Soft gluon approximation: Korchemsky and Sterman, 1999;
Renormalon analysis: Schindler, Stewart, Sun, 2023

Understanding the θ scaling from the light-ray OPE (controlled by twist expansion)

Perturbative QCD

$\mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2) \sim \frac{C}{\theta^2} \mathcal{D}_{J_L=-4}^{\text{DGLAP}}$

Hadronization effects

$\frac{\text{dim}=1}{\Lambda_{QCD}} \frac{B}{\theta^3} \mathcal{D}_{J_L=-3}^{\text{DGLAP}}$

fixed by boost symmetry

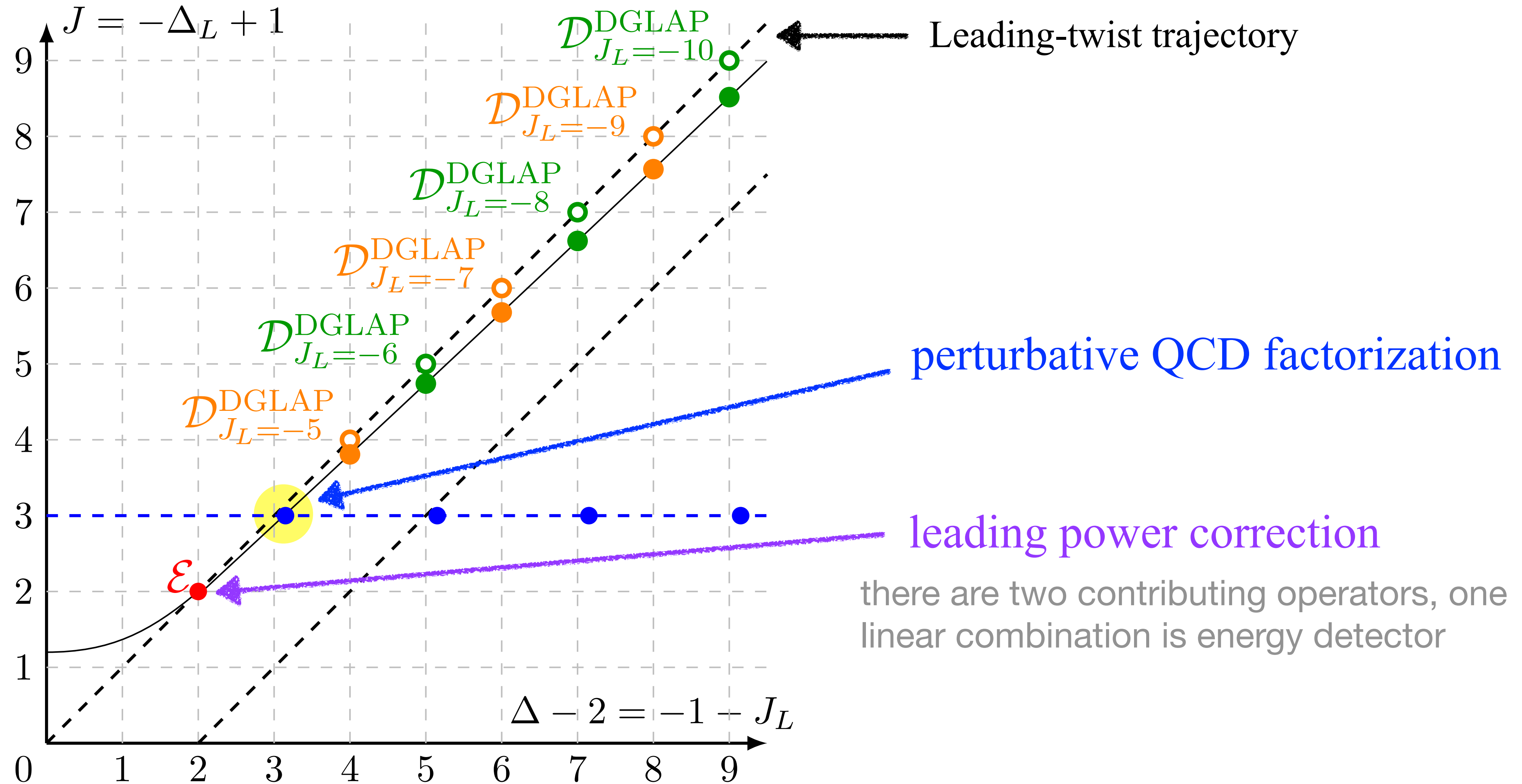
fixed by dimensional analysis

[HC, Monni, Xu, Zhu, 2024]

...

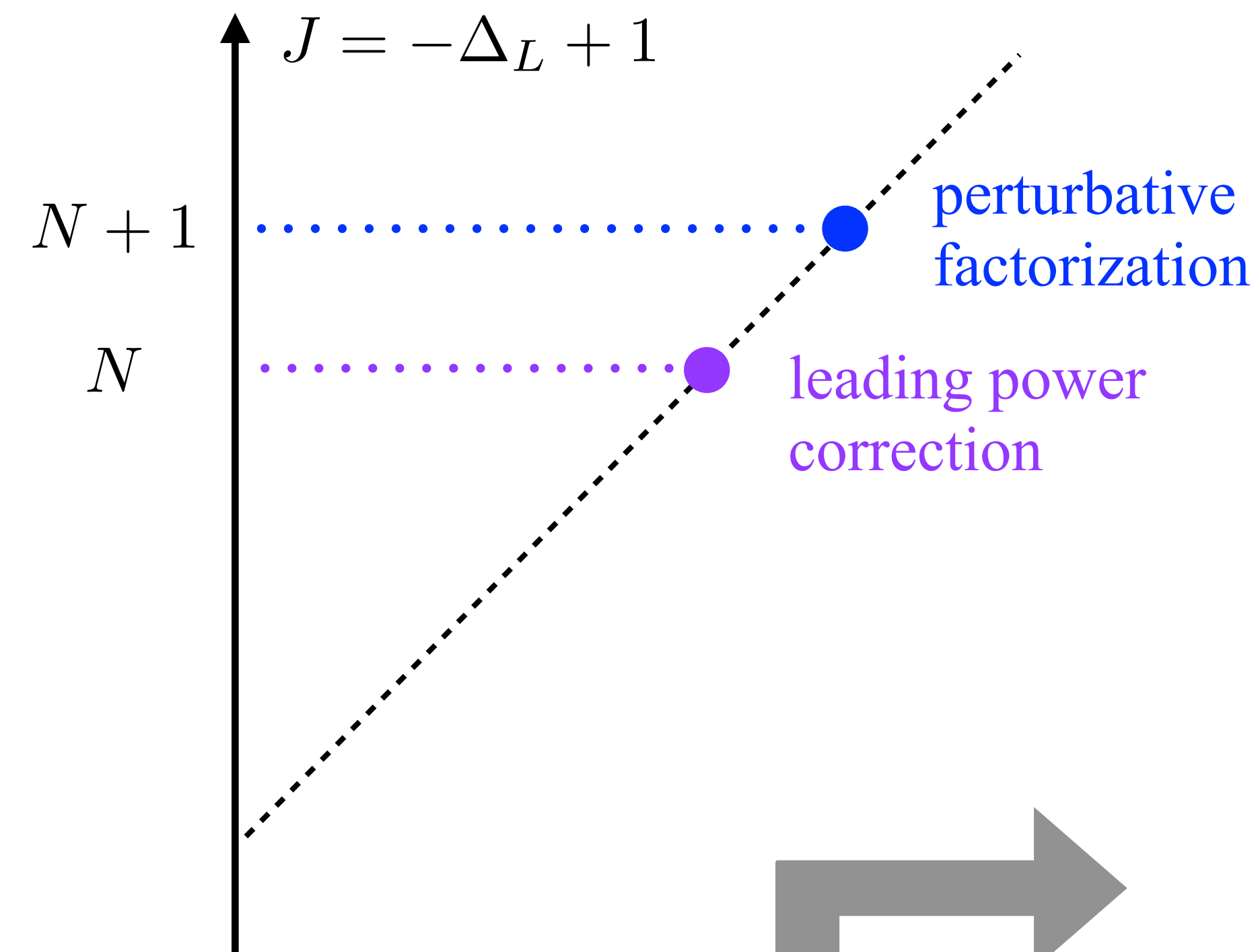
also see [Lee, Pathak, Stewart, Sun, 2024] for equivalent result 40

Leading Power Correction on CF Plot

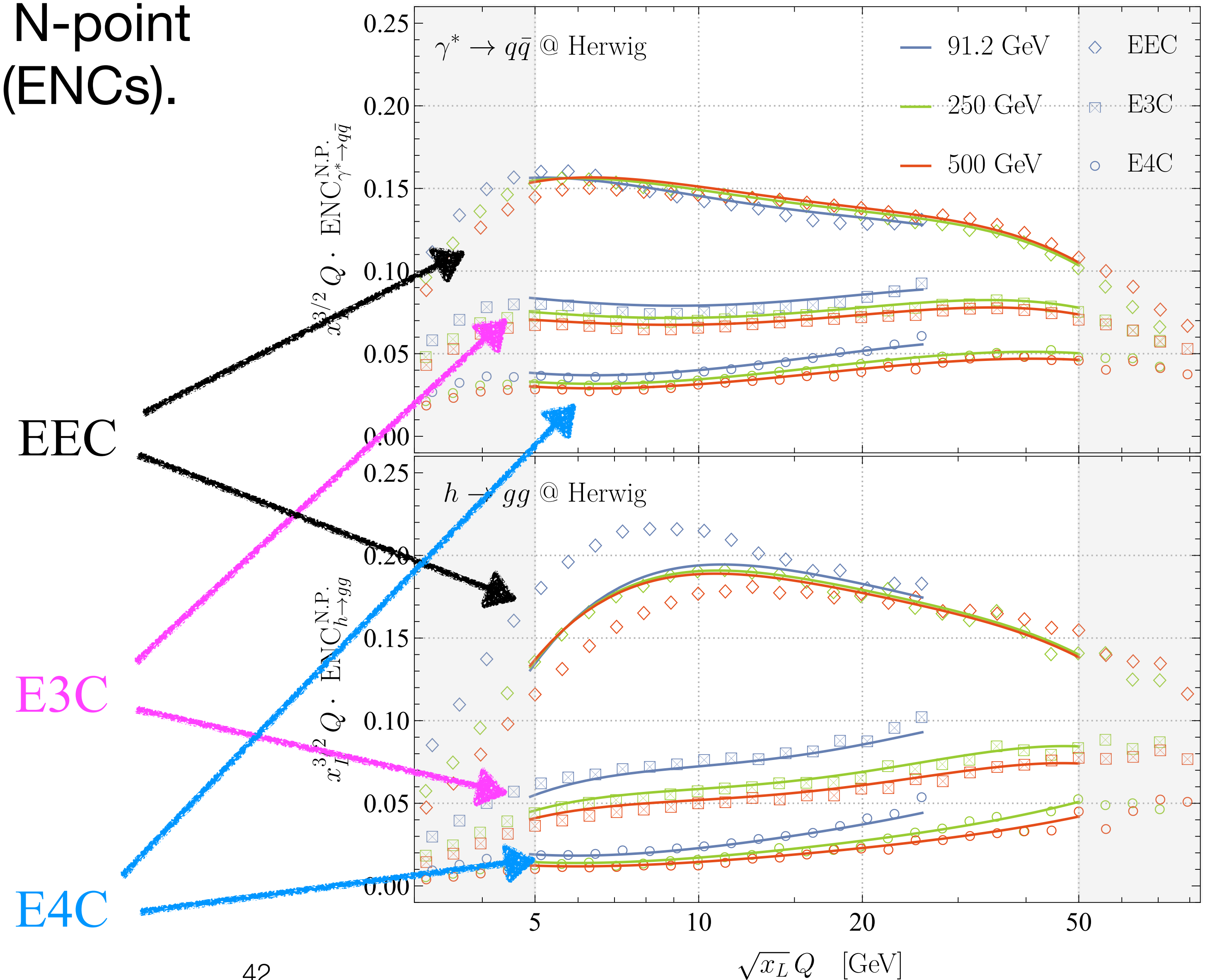


Scale Violation and Comparison to MC

This can also be extended to N-point projective energy correlators (ENCs).

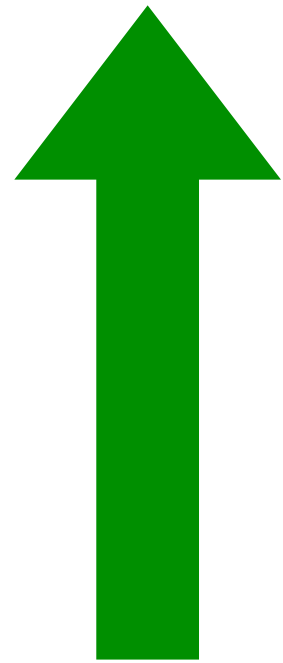


1. Extract the difference between hadron-level and parton level result
2. Strip off classical behavior

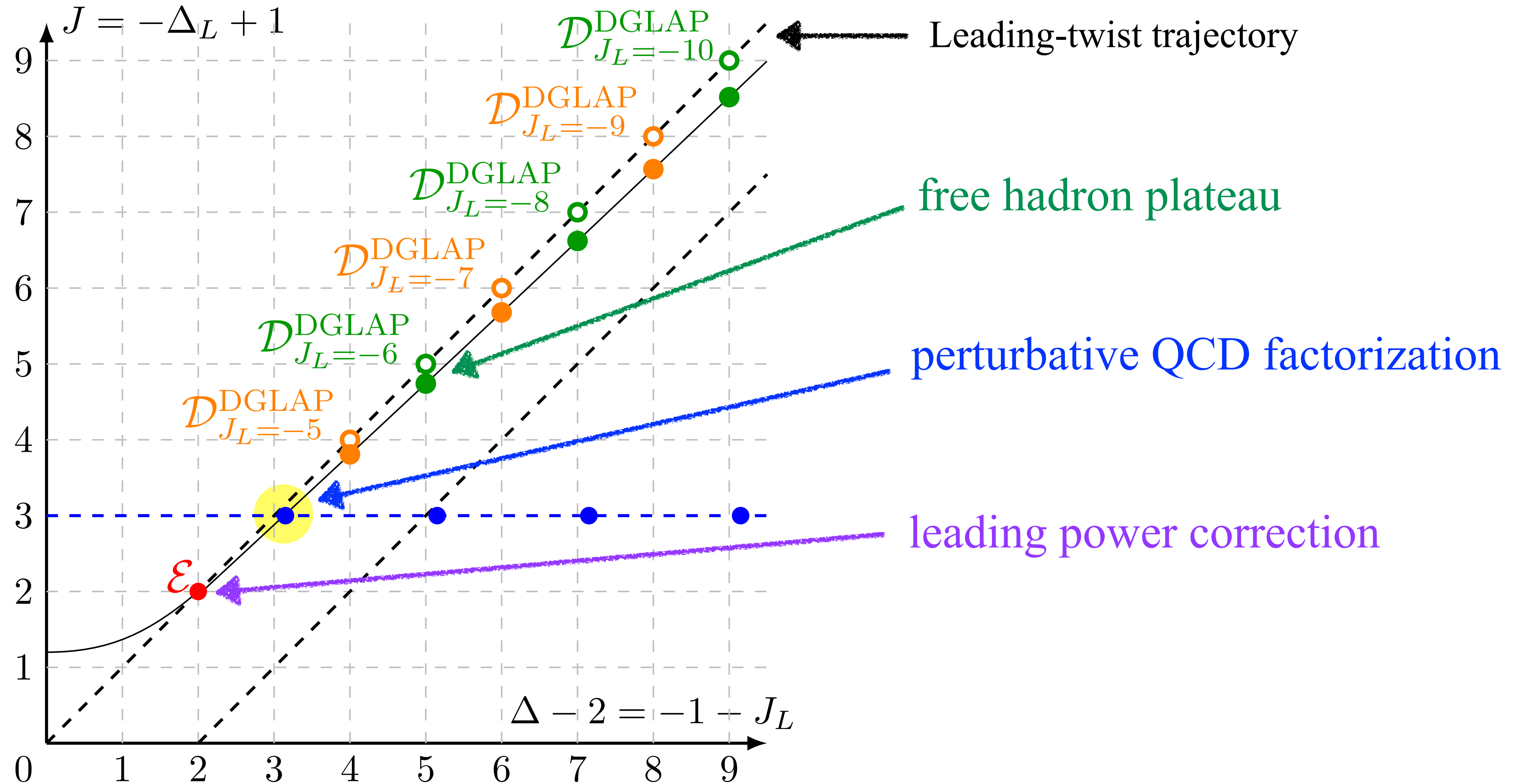
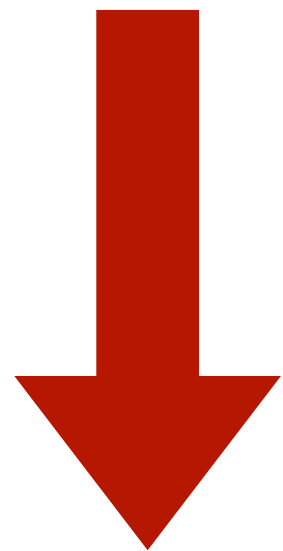


Collinear EEC on CF Plot

expansion in
plateau region

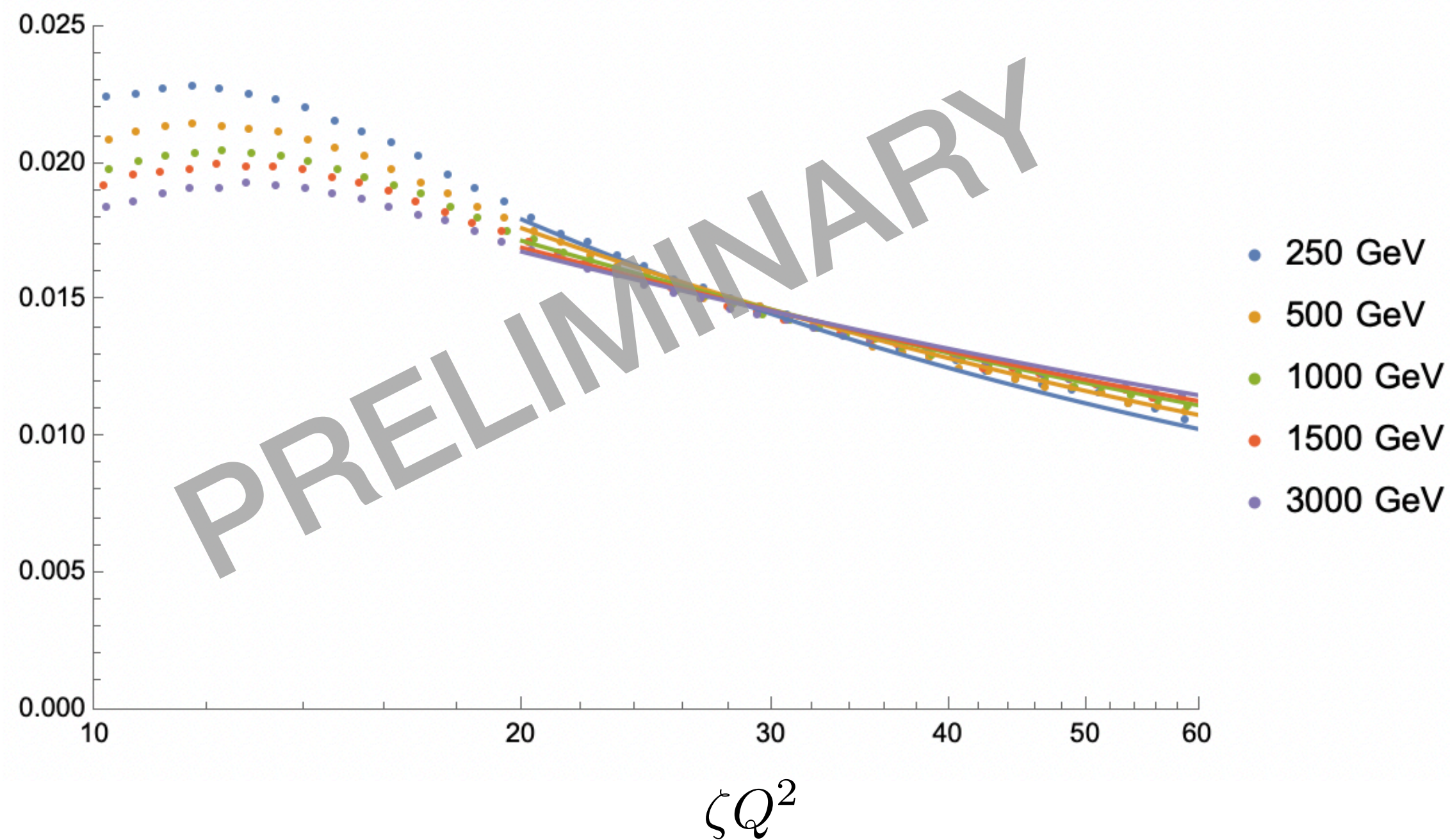


expansion in
pert. region

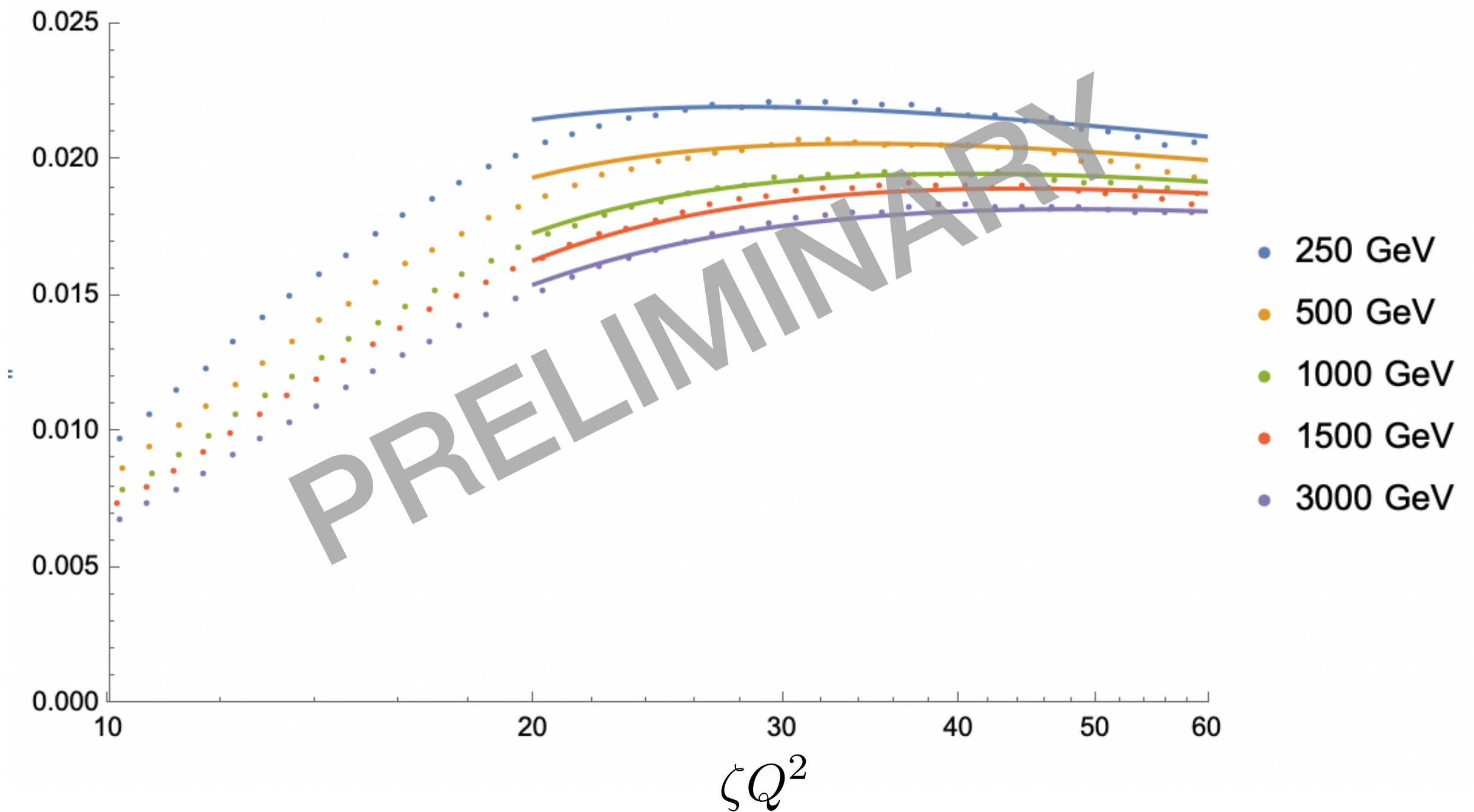


Adding Next-to-leading power parametrization

Pythia γ^*/Z^* -decay



Pythia h^* -decay

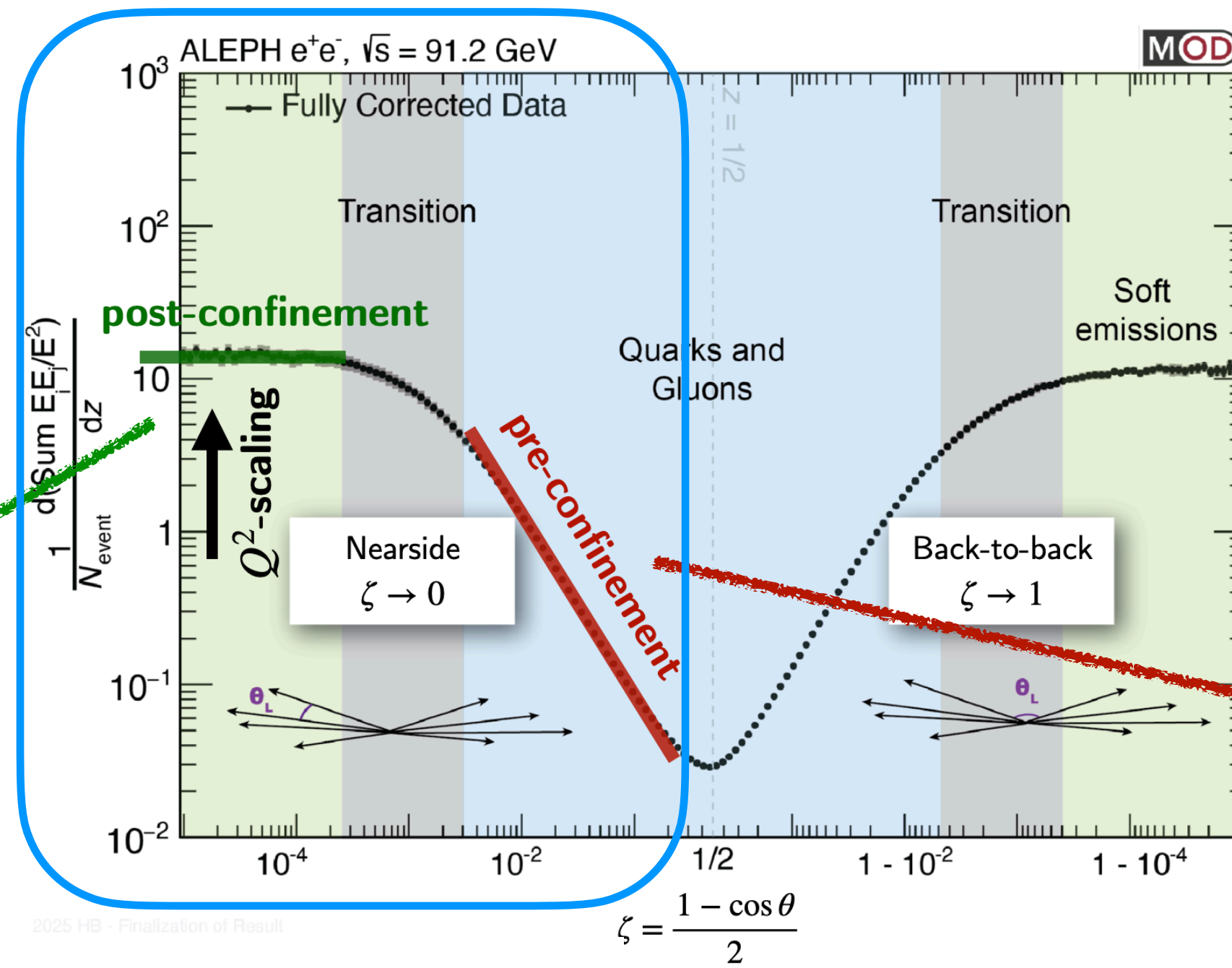


Collinear Factorization Formula

$$E^\alpha E^\beta C(\zeta, Q) = \frac{1}{\zeta} \int_0^1 dx x^{\alpha+\beta} \vec{J}_{\alpha,\beta} \left(\frac{\zeta x^2 Q^2}{\Lambda_{\text{QCD}}^2}, \frac{\Lambda_{\text{QCD}}^2}{\mu^2}; \mu^2 \right) \cdot \vec{H} \left(x, \frac{Q^2}{\mu^2}; \mu^2 \right) + \dots$$

↓ or
 $E^N C$

↓ or
 $\vec{J}_N$



Different expansions of jet function

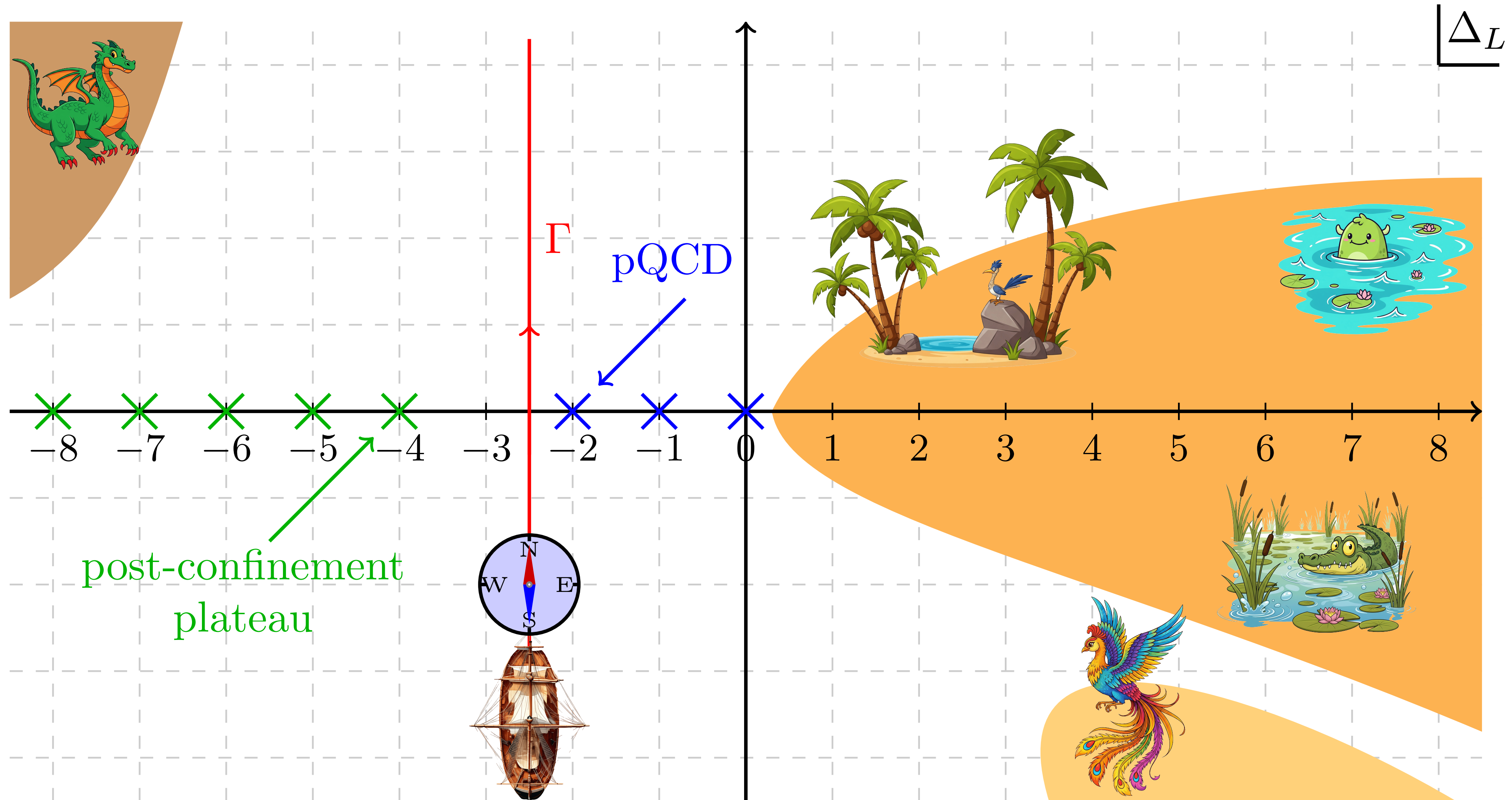
$$\vec{J}_{\alpha,\beta} \left(\frac{\zeta Q^2}{\Lambda_{\text{QCD}}^2}, \frac{\Lambda_{\text{QCD}}^2}{\mu^2}; \mu^2 \right)$$

$$= \sum_{n \geq 0} (\zeta Q^2)^{n+1} \vec{J}_{\alpha,\beta}^{(n)} = \sum_{n \geq 0} (\zeta Q^2)^{n+1} \vec{R}_n^{\alpha,\beta}(\mu^2; \zeta Q^2, \Lambda_{\text{QCD}}^2)$$

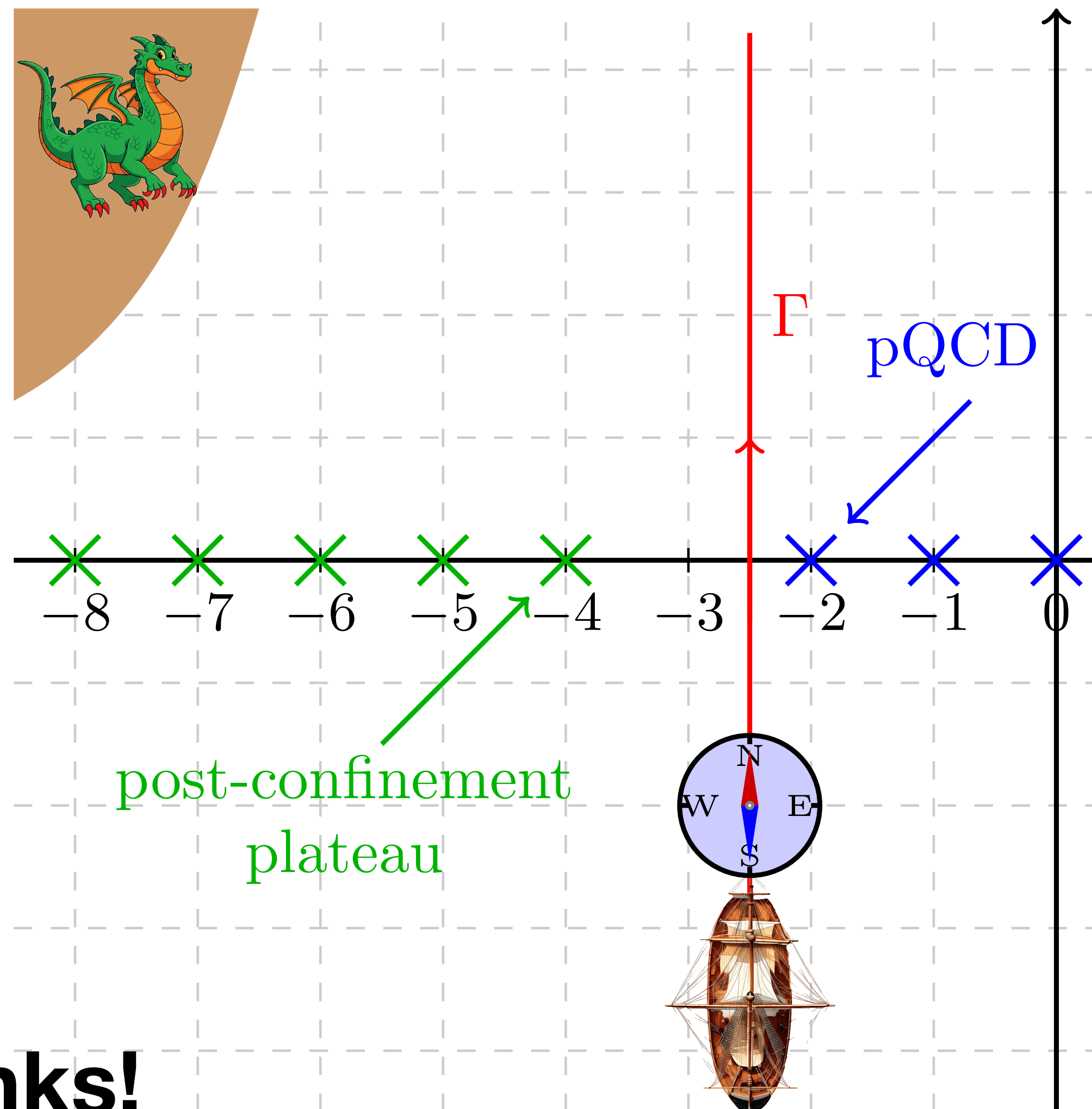
$$\vec{J}_N \left(\frac{\zeta x^2 Q^2}{\mu^2}, \frac{\Lambda_{\text{QCD}}^2}{\mu^2}; \mu^2 \right)$$

$$= \vec{J}_N^{\text{P.T.}} \left(\frac{\zeta x^2 Q^2}{\mu^2}; \alpha_s(\mu^2) \right) + \frac{\Lambda_{\text{QCD}}}{\sqrt{\zeta x Q}} \vec{J}_N^{(1)} \left(\frac{\zeta x^2 Q^2}{\mu^2}, \frac{\Lambda_{\text{QCD}}^2}{\zeta x^2 Q^2}; \mu^2 \right) + \dots$$

Conjectured “Marine Chart” for Light-ray OPE in QCD



Conjectured “Marine Chart” for Light-ray OPE in QCD



Summary

Collinear factorization and scale evolution for energy-energy correlator

- perturbative region ($J \leq 3$)
- post-confinement region ($J \geq 5$)

Unified picture of light-ray OPE and collinear factorization

- Light-ray OPE: manifest symmetry properties
- Collinear factorization: intuitive

Thanks!