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Double quarkonium production in hadronic collisions: quark polarization and TMD effects

Carlo Flore

Università di Cagliari & INFN - Sezione di Cagliari

Resummation, Evolution, Factorization 2025
Milano
October 15th, 2025

CF, C. Pisano,
arxiv:2508:15482 [hep-ph]

Introduction

- Quarkonia as key QCD probes:
  heavy-quark hierarchy $M_Q \gg \Lambda_{\mathrm{QCD}}$, clean experimental signature

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  3. NRQCD and Color Octet Mechanism
     [G. T. Bodwin, E. Braaten, P. Lepage, PRD 51 (1995) 1125-1171; P. L. Cho, A. K. Leibovich, PRD 53 (1996) 150-162 & PRD 53 (1996) 6203-6217]
     - double expansion in v and α_s
     - higher Fock states of the mesons taken into account: $Q\bar{Q}$ can be produced in octet states with different quantum number as the meson

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Introduction - inclusive production

- Single quarkonium production
  - NLO computations available for CSM and NRQCD
  - NNLO* available for CSM, full NNLO not available in NRQCD
    [J.-P. Lansberg, EPJC (2009) 61, 693-703 & Phys. Rept. 889 (2020) 1-106]
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  - role of $q\bar{q}$ channel?
  - recent COMPASS and LHCb data motivate complementary studies for future fixed-target experiments

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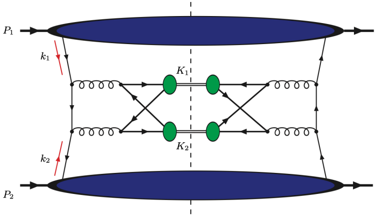
Di-onium production



$$h_1(P_1, S_1) + h_2(P_2, S_2) \to \mathcal{Q}(K_1) + \mathcal{Q}(K_2) + X$$

- We consider

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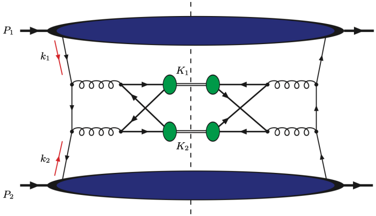

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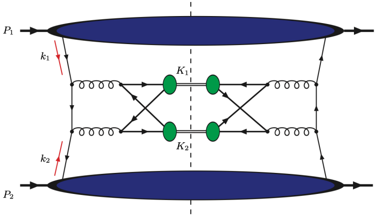

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- unpolarized partonic amplitude (integrated over azimuthal angles) agrees with

  V. G. Kartvelishvili, Sh. M. Esakiya, Yad. Fiz., 38:722–726, 1983

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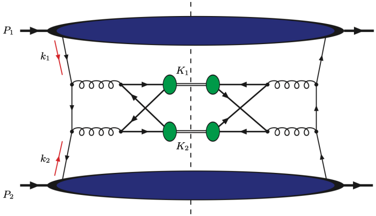

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- TMD cross section:

$$\frac{d\sigma}{dy_1\, dy_2\, d^2\boldsymbol{K}_\perp\, d^2\boldsymbol{q}_T} = \frac{1}{16\pi^2 s^2} \int d^2k_{1T}\, d^2k_{2T}\, \delta^2(\boldsymbol{k}_{1T} + \boldsymbol{k}_{2T} - \boldsymbol{q}_T)$$

$$\sum_q \Phi^q(x_1, \boldsymbol{k}_{1T}) \otimes \overline{\Phi}^q(x_2, \boldsymbol{k}_{2T}) \otimes |\mathcal{M}_{q\,\bar{q}\to\mathcal{Q}\,\mathcal{Q}}|^2 + \{\Phi^q \leftrightarrow \overline{\Phi}^q\}$$

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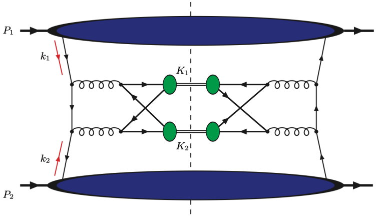

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- Final results expressed in terms of

$$z = \frac{K_1 \cdot k_1}{k_1 \cdot k_2} = \frac{1}{1 + e^{y_1 - y_2}}, \quad M^2_{\mathcal{Q}\mathcal{Q}} = \frac{M^2_\perp}{z\,(1-z)}, \quad Y_{\mathcal{Q}\mathcal{Q}} = \frac{1}{2} \ln\left(\frac{x_1}{x_2}\right) \simeq y_1 + \frac{1}{2} \ln\left(\frac{z}{1-z}\right) = \frac{y_1 + y_2}{2}$$

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Angular structure of the cross section

$$
\begin{aligned}
d\sigma = \frac{131072}{243}\,\frac{\alpha_s^4}{M_{\mathcal{Q}}^2\, M_\perp^6\, s}\,|R_{\mathcal{Q}}(0)|^4\, z^4(1-z)^4 \Big\{ & F_{UU} + F_{UU}^{\cos 2(\phi_T-\phi_\perp)} \cos 2\,(\phi_T-\phi_\perp) \\
& + \Big[S_{1L}\, F_{LU}^{\sin 2(\phi_T-\phi_\perp)} + S_{2L}\, F_{UL}^{\sin 2(\phi_T-\phi_\perp)} \Big] \sin 2(\phi_T-\phi_\perp) + |\boldsymbol{S}_{1T}| \Big[F_{TU}^{\sin(\phi_T-\phi_{S_1})} \sin(\phi_T-\phi_{S_1}) \\
& + F_{TU}^{\sin(\phi_T+\phi_{S_1}-2\phi_\perp)} \sin(\phi_T+\phi_{S_1}-2\phi_\perp) + F_{TU}^{\sin(3\phi_T-\phi_{S_1}-2\phi_\perp)} \sin(3\phi_T-\phi_{S_1}-2\phi_\perp) \Big] \\
& + |\boldsymbol{S}_{2T}| \Big[F_{UT}^{\sin(\phi_T-\phi_{S_2})} \sin(\phi_T-\phi_{S_2}) + F_{UT}^{\sin(\phi_T+\phi_{S_2}-2\phi_\perp)} \sin(\phi_T+\phi_{S_2}-2\phi_\perp) \\
& + F_{UT}^{\sin(3\phi_T-\phi_{S_2}-2\phi_\perp)} \sin(3\phi_T-\phi_{S_2}-2\phi_\perp) \Big] \\
& + [\text{double polarized azimuthal modulations}]
\end{aligned}
$$

- all the structure functions can be expressed as products of hard factors and TMD convolutions

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$$

- all the structure functions can be expressed as products of hard factors and TMD convolutions
- only two hard factors appear

$$
H\left(z,\frac{M_{\mathcal{Q}}^2}{M_\perp^2}\right) = 5 - 12\,z(1-z)\left(1-\frac{M_{\mathcal{Q}}^2}{M_\perp^2}\right) - \frac{M_{\mathcal{Q}}^2}{M_\perp^2}\,, \qquad \Delta H\left(z,\frac{M_{\mathcal{Q}}^2}{M_\perp^2}\right) = -\left(1-\frac{M_{\mathcal{Q}}^2}{M_\perp^2}\right)[1-12\,z(1-z)]
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$$

- same azimuthal modulation as for LO channel $q\bar{q}\to\gamma^*\to\ell^+\ell^-$ of the Drell-Yan process **[S. Arnold, A. Metz, M. Schlegel, PRD 79, 034005, 2009]**

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Phenomenology

- We present predictions for the unpolarized cross section and for the following azimuthal asymmetries:

$$\langle \cos 2(\phi_T - \phi_\perp) \rangle = \frac{F_{UU}^{\cos 2(\phi_T - \phi_\perp)}}{F_{UU}} = \frac{\Delta H}{H} \frac{\mathcal{C}[w_{UU}\, h_1^{\perp\, q}\, h_1^{\perp\, \bar{q}}]}{\mathcal{C}[f_1^q\, f_1^{\bar{q}}]}$$

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- SPS only; $|R_{\mathcal{Q}}(0)|^2$ from a power-law type potential; $M_{\mathcal{Q}\mathcal{Q}}$ as hard scale of the process

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- SPS only; $|R_{\mathcal{Q}}(0)|^2$ from a power-law type potential; $M_{\mathcal{Q}\mathcal{Q}}$ as hard scale of the process
- NLL TMDs: MAP22 for π^- and PV17 (+PV20_Sivers) and MAP22 for p TMDs; h_1^q from **M. Boglione *et al.*, PLB 854 (2024) 138712**

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- SPS only; $|R_{\mathcal{Q}}(0)|^2$ from a power-law type potential; $M_{\mathcal{Q}\mathcal{Q}}$ as hard scale of the process
- NLL TMDs: MAP22 for π^- and PV17 (+PV20_Sivers) and MAP22 for p TMDs; h_1^q from **M. Boglione *et al.*, PLB 854 (2024) 138712**
- all predictions involving the Boer-Mulders functions are done saturating the corresponding bound:

$$|h_1^{\perp q}(x, \boldsymbol{k}_T)| \leq \frac{M_h}{|\boldsymbol{k}_T|} f_1^q(x, \boldsymbol{k}_T)$$

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Phenomenology

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- comparison with COMPASS data and predictions for SMOG/LHCspin

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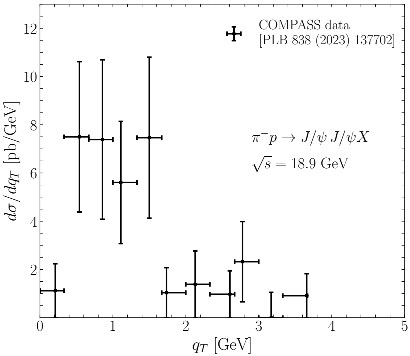
Di-J/ψ at COMPASS



- COMPASS data for $\pi^- p \to J/\psi\, J/\psi\, X$ at $\sqrt{s} \simeq 18.9$ GeV
- data given as a function of $M_{\psi\psi}$, q_T, and

$$x_\parallel^{\psi\psi} = \frac{p_L^{\psi\psi}}{p_{\rm beam}}, \quad |\Delta x_\parallel^{\psi\psi}| = |x_\parallel^{\psi_1} - x_\parallel^{\psi_2}|, \quad x_\parallel^{\psi_i} = \frac{p_L^{\psi_i}}{p_{\rm beam}}$$

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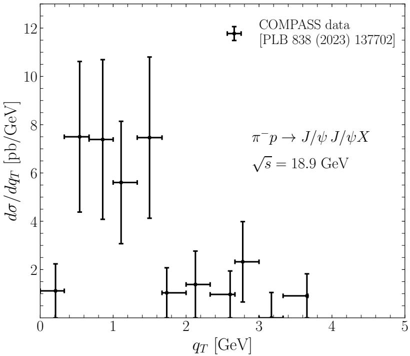

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- DPS negligible; only $\sim$ 25 events recorded

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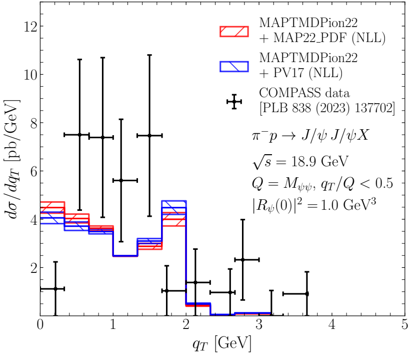
Di-J/ψ at COMPASS



- predictions based on MAP extractions (PV17 as baseline for Sivers asymmetry)

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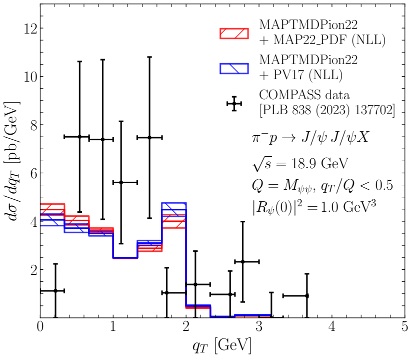
Di-J/ψ at COMPASS



- predictions based on MAP extractions (PV17 as baseline for Sivers asymmetry)
- *gg* channel negligible ($\mathcal{O}(10^{-3})$ pb)

$\Rightarrow$ computation based on **[J. Bor, PhD Thesis]**

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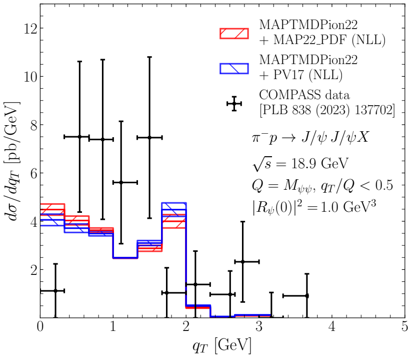

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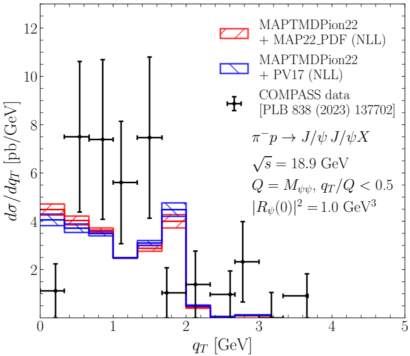

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- NLO QCD corrections and inclusion of feed-down effects can improve our description

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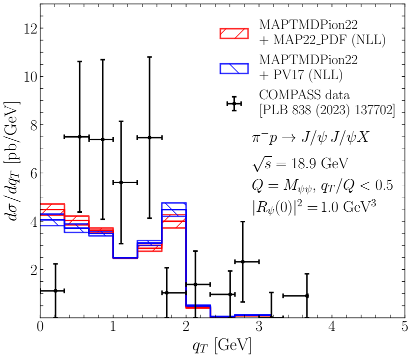

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- fairly good agreement with measured data
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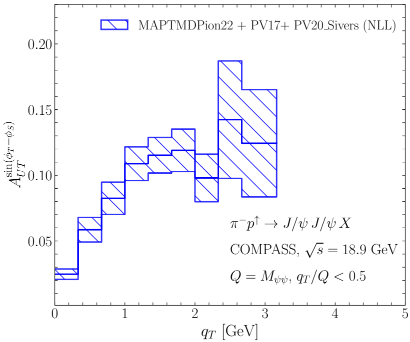
- yet, good baseline for estimating azimuthal asymmetries

Di-J/ψ at COMPASS - azimuthal asymmetries (I)



- predictions made using same kinematics and q_T-binning of the unpolarized cross section

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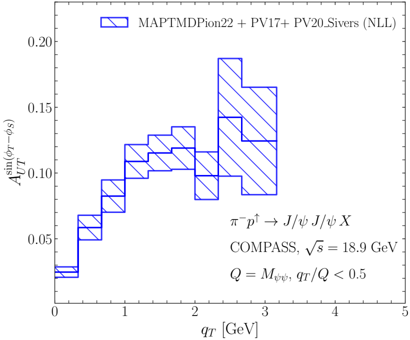

- predictions made using same kinematics and q_T-binning of the unpolarized cross section
- large Sivers asymmetry (sign change included)

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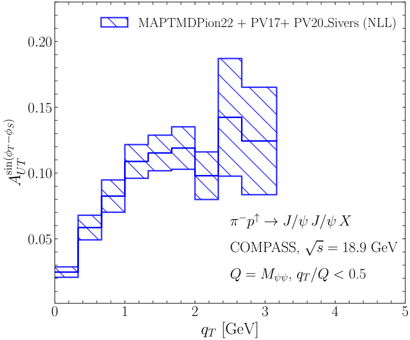

- predictions made using same kinematics and q_T-binning of the unpolarized cross section
- large Sivers asymmetry (sign change included)
- sign of $A_{UT}^{\sin(\phi_T - \phi_S)}$ driven by dominant $\bar{u}_\pi u_p$ channel

$\Rightarrow$ valence region explored in both π^- and p

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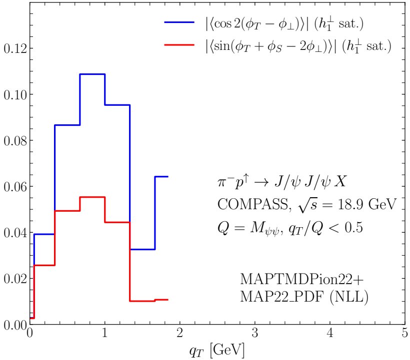
Di-J/ψ at COMPASS - azimuthal asymmetries (II)



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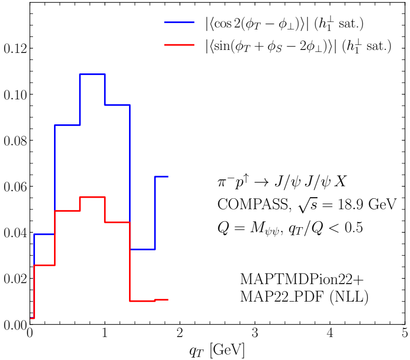

- predictions made using same kinematics and q_T-binning of the unpolarized cross section
- maximum modulation of 5-10% and 2-5% for $|\langle\cos 2(\phi_T - \phi_\perp)\rangle|$, $|\langle\sin(\phi_T + \phi_S - 2\phi_\perp)\rangle|$ respectively

$\Rightarrow$ $|\langle\cos 2(\phi_T - \phi_\perp)\rangle|$ magnitude similar to DY case

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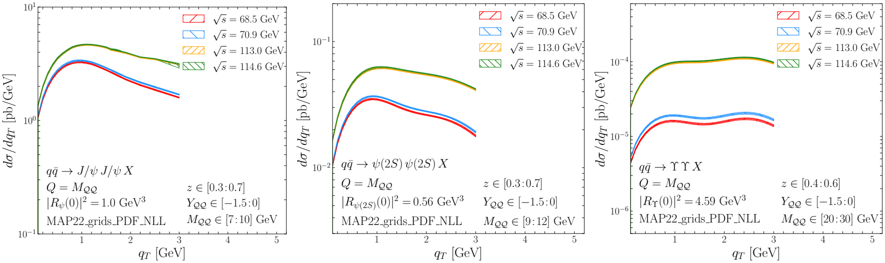
SMOG & LHCspin



- gg channel no longer negligible: 30/40% of the $q\bar{q}$ one at low $\sqrt{s}$, 2$\times$ $q\bar{q}$ channel at higher energies

$\Rightarrow$ further studies on gluon TMDs needed to confirm our estimates

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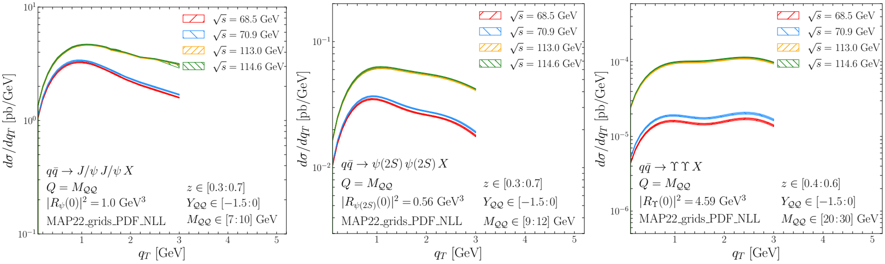

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- di-J/ψ yield measurable at LHCb in the fixed-target mode thanks to LHC upgrade for LHC Run 3

  similar expectation for di-$\psi(2S)$, di-Υ

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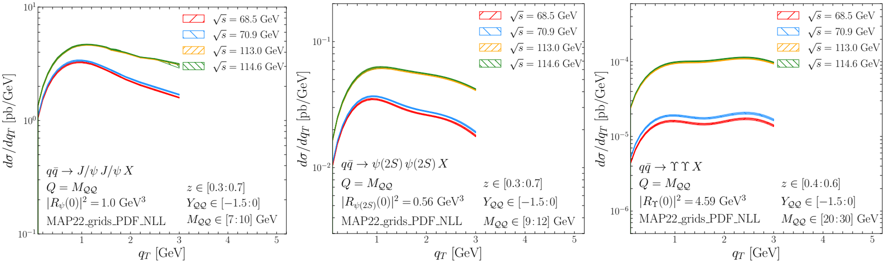

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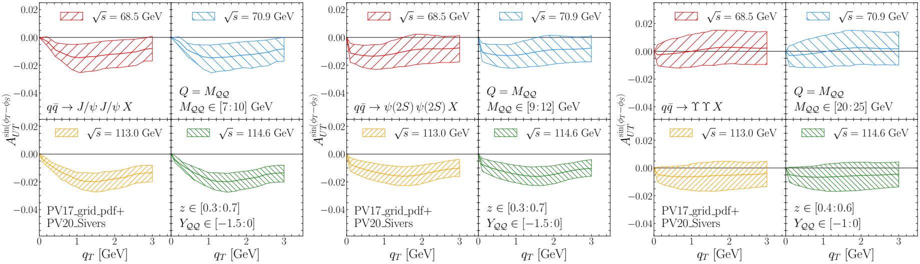
  similar expectation for di-$\psi(2S)$, di-Υ

- q_T-range for di-Υ could be extended, but would probe unconstrained large-x behavior of unpolarized TMDs

  $\Rightarrow$ di-Υ production a tool for constraining unpolarized TMDs at large x

SMOG & LHCspin - Sivers asymmetry



- again, sign change for $f_{1T}^{\perp}$

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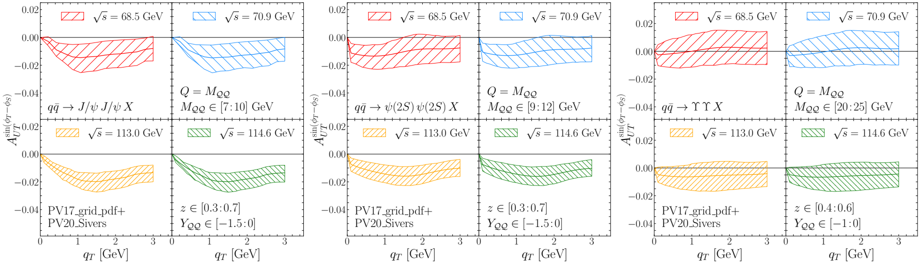
SMOG & LHCspin - Sivers asymmetry



- again, sign change for $f_{1T}^{\perp}$
- negative sign of $A_{UT}^{\sin(\phi_T-\phi_S)}$ (cfr. w/ COMPASS) due to different flavor composition of π^- and p

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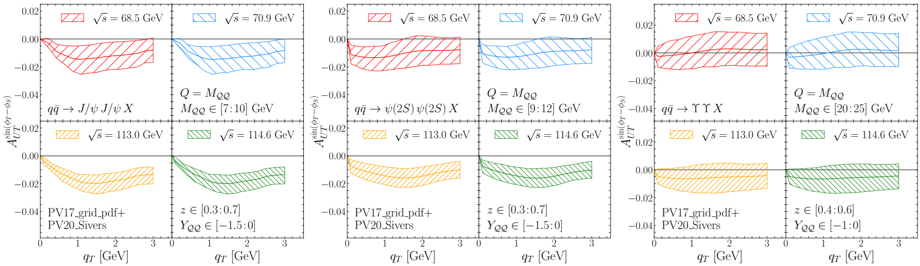
SMOG & LHCspin - Sivers asymmetry



- again, sign change for $f_{1T}^{\perp}$
- negative sign of $A_{UT}^{\sin(\phi_T - \phi_S)}$ (cfr. w/ COMPASS) due to different flavor composition of π^- and p
- magnitude of 1-2% expected for di-J/ψ and di-$\psi(2S)$; di-Υ suppressed due to probed x_2 range

$\Rightarrow$ larger asymmetry would signal nonzero gluon Sivers function

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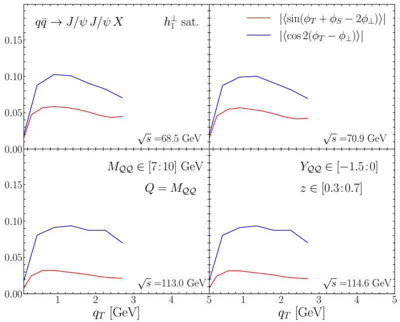
SMOG & LHCspin - azimuthal asymmetries



- similarly to COMPASS, maximized $|\langle \cos 2(\phi_T - \phi_\perp)\rangle|$ asymmetry larger, $\mathcal{O}(10\%)$, than $|\langle \sin(\phi_T + \phi_S - 2\phi_\perp)\rangle|$ ($\sim 5\%$)

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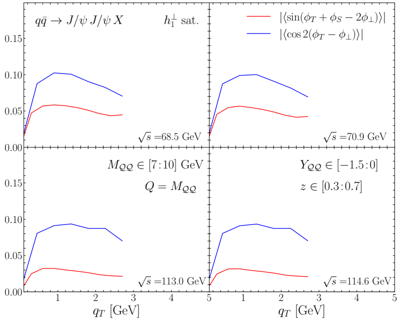
SMOG & LHCspin - azimuthal asymmetries



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- slight increase (decrease) of $|\langle \cos 2(\phi_T - \phi_\perp)\rangle|$ ($|\langle \sin(\phi_T + \phi_S - 2\phi_\perp)\rangle|$) asymmetry at larger $\sqrt{s}$

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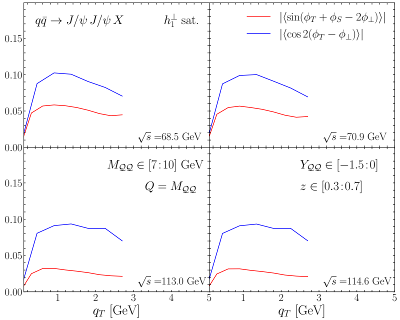

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- slight increase (decrease) of $|\langle \cos 2(\phi_T - \phi_\perp)\rangle|$ ($|\langle \sin(\phi_T + \phi_S - 2\phi_\perp)\rangle|$) asymmetry at larger $\sqrt{s}$
- larger asymmetries, $\mathcal{O}(10-20\%)$, (not shown) with larger M_{QQ} within same Y_{QQ} and z ranges, and for di-$\psi(2S)$ and di-Υ production

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Conclusions

- We studied double quarkonium production in hadronic collisions at fixed-target experiments within the TMD framework

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Thank you

Backup

The Color Singlet Model

C.-H. Chang, NPB172, 425 (1980); R. Baier, R. Rückl, Z. Phys. C 19, 251 (1983)

The CSM supposes two factorizations:

1. *collinear*, in which the hadronic cross section can be written as the convolution of the PDF(s) with the partonic cross section
2. *between the hard part* (a perturbative amplitude, which describes the $Q\bar{Q}$ pair production) *and the soft part* (a non-perturbative, long-distance matrix element – LDME –, describing hadronization)

- Perturbative creation of two heavy quarks, Q and $\bar{Q}$
  - on-shell
  - in a color singlet state
  - with a vanishing relative momentum
  - in a $^3S_1$ state (for J/ψ, ψ' and Υ)
- Non-perturbative binding of quarks
  $\rightarrow$ Schrödinger wave function at $r = 0$

CSM: the Taylor series expansion of the amplitude in the $Q\bar{Q}$ relative momentum (v) to the first non-vanishing (Leading-v NRQCD) term.

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Quark-quark TMD correlator

- $q - q$ correlator for a spin-1/2 hadron:

$$\Phi^{q\,[\mathcal{U}]}(x,\boldsymbol{k}_T) = \int \frac{d(\lambda\cdot p)\, d^2\lambda_T}{(2\pi)^3}\, e^{ik\cdot\lambda}\, \langle P,S|\, \overline{\psi}(0)\, \mathcal{U}_{[0,\lambda]}\, \psi(\lambda)\, |P,S\rangle\, \big|_{\rm LF}$$

- decomposed as

$$\Phi^q(x,\boldsymbol{k}_T) = \Phi^q_U(x,\boldsymbol{k}_T) + \Phi^q_L(x,\boldsymbol{k}_T) + \Phi^q_T(x,\boldsymbol{k}_T)$$

where

$$\Phi^q_U(x,\boldsymbol{k}_T) = \frac{1}{2}\left\{ f_1^q(x,\boldsymbol{k}_T^2)\,\not{P} + i h_1^{\perp\,q}(x,\boldsymbol{k}_T^2)\,\frac{[\not{k}_T,\not{P}]}{2M_h}\right\},$$

$$\Phi^q_L(x,\boldsymbol{k}_T) = \frac{1}{2} S_L \left\{ g_{1L}^q(x,\boldsymbol{k}_T^2)\gamma^5\not{P} + h_{1L}^{\perp q}(x,\boldsymbol{k}_T^2)\,\gamma^5\,\frac{[\not{k}_T,\not{P}]}{2M_h}\right\},$$

$$\Phi^q_T(x,\boldsymbol{k}_T) = \frac{1}{2}\left\{ -\frac{\epsilon^{k_T S_T}}{M_h} f_{1T}^{\perp\,q}(x,\boldsymbol{k}_T^2)\,\not{P} - \frac{k_T\cdot S_T}{M_h}\, g_{1T}^{\perp\,q}(x,\boldsymbol{k}_T^2)\,\gamma^5\not{P} + h_{1T}^q(x,\boldsymbol{k}_T^2)\,\gamma^5\,\frac{[\not{S}_T,\not{P}]}{2} \right.$$
$$\left. - \frac{k_T\cdot S_T}{M_h}\, h_{1T}^{\perp\,q}(x,\boldsymbol{k}_T^2)\,\gamma^5\,\frac{[\not{k}_T,\not{P}]}{2M_h}\right\}$$

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Full angular structure of the cross section

$$
\begin{aligned}
\mathrm{d}\sigma = \frac{131072}{243}\,\frac{\alpha_s^4}{M_{\mathcal{Q}}^2\,M_\perp^6\,s}\,|R_{\mathcal{Q}}(0)|^4\,z^4(1-z)^4 \Big\{ & F_{UU} + F_{UU}^{\cos 2(\phi_T-\phi_\perp)}\cos 2\,(\phi_T-\phi_\perp) \\
& + \Big[S_{1L}\,F_{LU}^{\sin 2(\phi_T-\phi_\perp)} + S_{2L}\,F_{UL}^{\sin 2(\phi_T-\phi_\perp)}\Big]\sin 2(\phi_T-\phi_\perp) + |\boldsymbol{S}_{1T}|\Big[F_{TU}^{\sin(\phi_T-\phi_{S_1})}\sin(\phi_T-\phi_{S_1}) \\
& + F_{TU}^{\sin(\phi_T+\phi_{S_1}-2\phi_\perp)}\sin(\phi_T+\phi_{S_1}-2\phi_\perp) + F_{TU}^{\sin(3\phi_T-\phi_{S_1}-2\phi_\perp)}\sin(3\phi_T-\phi_{S_1}-2\phi_\perp)\Big] \\
& + |\boldsymbol{S}_{2T}|\Big[F_{UT}^{\sin(\phi_T-\phi_{S_2})}\sin(\phi_T-\phi_{S_2}) + F_{UT}^{\sin(\phi_T+\phi_{S_2}-2\phi_\perp)}\sin(\phi_T+\phi_{S_2}-2\phi_\perp) \\
& + F_{UT}^{\sin(3\phi_T-\phi_{S_2}-2\phi_\perp)}\sin(3\phi_T-\phi_{S_2}-2\phi_\perp)\Big] + S_{1L}\,S_{2L}\Big[F_{LL} + F_{LL}^{\cos 2(\phi_T-\phi_\perp)}\cos 2(\phi_T-\phi_\perp)\Big] \\
& + S_{1L}|\boldsymbol{S}_{2T}|\Big[F_{LT}^{\cos(\phi_T-\phi_{S_2})}\cos(\phi_T-\phi_{S_2}) + F_{LT}^{\cos(\phi_T+\phi_{S_2}-2\phi_\perp)}\cos(\phi_T+\phi_{S_2}-2\phi_\perp) \\
& + F_{LT}^{\cos(3\phi_T-\phi_{S_2}-2\phi_\perp)}\cos(3\phi_T-\phi_{S_2}-2\phi_\perp)\Big] + |\boldsymbol{S}_{1T}|\,S_{2L}\Big[F_{TL}^{\cos(\phi_T-\phi_{S_1})}\cos(\phi_T-\phi_{S_1}) \\
& + F_{TL}^{\cos(\phi_T+\phi_{S_1}-2\phi_\perp)}\cos(\phi_T+\phi_{S_1}-2\phi_\perp) + F_{TL}^{\cos(3\phi_T-\phi_{S_1}-2\phi_\perp)}\cos(3\phi_T-\phi_{S_1}-2\phi_\perp)\Big] \\
& + |\boldsymbol{S}_{1T}||\boldsymbol{S}_{2T}|\Big[F_{TT}^{\cos(\phi_{S_1}-\phi_{S_2})}\cos(\phi_{S_1}-\phi_{S_2}) + F_{TT}^{\cos(2\phi_T-\phi_{S_1}-\phi_{S_2})}\cos(2\phi_T-\phi_{S_1}-\phi_{S_2}) \\
& + F_{TT}^{\cos(\phi_{S_1}+\phi_{S_2}-2\phi_\perp)}\cos(\phi_{S_1}+\phi_{S_2}-2\phi_\perp) + F_{TT}^{\cos(2\phi_T+\phi_{S_1}-\phi_{S_2}-2\phi_\perp)}\cos(2\phi_T+\phi_{S_1}-\phi_{S_2}-2\phi_\perp) \\
& + F_{TT}^{\cos(2\phi_T-\phi_{S_1}+\phi_{S_2}-2\phi_\perp)}\cos(2\phi_T-\phi_{S_1}+\phi_{S_2}-2\phi_\perp) + F_{TT}^{\cos(4\phi_T-\phi_{S_1}-\phi_{S_2}-2\phi_\perp)}\cos(4\phi_T-\phi_{S_1}-\phi_{S_2}-2\phi_\perp)\Big]
\end{aligned}
$$

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Di-J/ψ production - v-scaling in NRQCD

Table: Scaling with p_T and v of $d\sigma/dp_T^2$ for $gg \to c\bar{c}(m)c\bar{c}(n)$ times the respective LDMEs and branching fractions for the relevant pairings (m,n) of $c\bar{c}$ Fock states. Note that ${}^3P_J^{[1]}$ are counted separately for $J = 0, 1, 2$.

(m,n)	${}^3S_1^{[1]}$	${}^3S_1^{[8]}$	${}^1S_0^{[8]}$	${}^3P_J^{[8]}$	${}^3P_J^{[1]}$
${}^3S_1^{[1]}$	$1/p_T^8$	v^4/p_T^8	v^3/p_T^8	v^4/p_T^8	0^*
${}^3S_1^{[8]}$	$\cdots$	v^8/p_T^4	v^7/p_T^6	v^8/p_T^6	v^8/p_T^6
${}^1S_0^{[8]}$	$\cdots$	$\cdots$	v^6/p_T^8	v^7/p_T^8	v^7/p_T^8
${}^3P_J^{[8]}$	$\cdots$	$\cdots$	$\cdots$	v^8/p_T^8	v^8/p_T^8
${}^3P_J^{[1]}$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	v^8/p_T^8

* 0 means it starts at $\mathcal{O}(\alpha_s^5)$

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COMPASS kinematics

- COMPASS is a fixed-target experiment, therefore $P_1^+ \approx \sqrt{2}\, p_{\rm beam}$ and $P_2^- = M_p/\sqrt{2}$, and we have

$$x_\parallel^{\psi\psi} = \frac{K_1^+ + K_2^+}{P_1^+}\left[1 - \frac{M_\perp^2}{2\,K_1^+ K_2^+}\right] = \frac{\mathcal{K}_+}{P_1^+}\left(1 - \frac{M_{\psi\psi}^2}{2\,\mathcal{K}_+^2}\right),\ \Delta x_\parallel^{\psi\psi} = \frac{K_1^+ - K_2^+}{P_1^+}\left[1 - \frac{M_\perp^2}{2\,K_1^+ K_2^+}\right] = \frac{\mathcal{K}_-}{P_1^+}\left(1 - \frac{M_{\psi\psi}^2}{2\,\mathcal{K}_+^2}\right)$$

  with $\mathcal{K}_+ \equiv K_1^+ + K_2^+$ and $\mathcal{K}_- \equiv K_1^+ - K_2^+$
- Jacobian:

$$\left|\frac{\partial(\mathcal{K}_+, \mathcal{K}_-)}{\partial(x_\parallel^{\psi\psi}, \Delta x_\parallel^{\psi\psi})}\right| = \frac{4\,(P_1^+)^2\,\mathcal{K}_+^4}{[4\,\mathcal{K}_+^4 - M_{\psi\psi}^4]} = \frac{\mathcal{K}_+^2}{(x_\parallel^{\psi\psi})^2\left[1 + \dfrac{M_{\psi\psi}^2}{x_\parallel^{\psi\psi}\, x_1\, (P_1^+)^2}\right]}$$

- invariant phase-space:

$$\frac{d^3K_1}{2E_1}\,\frac{d^3K_2}{2E_2} = \frac{\pi}{8\,(x_\parallel^{\psi\psi})^2}\left[1 + \frac{M_{\psi\psi}^2}{2\,x_\parallel^{\psi\psi}\, x_1\, p_{\rm beam}^2}\right]^{-1} dx_\parallel^{\psi\psi}\, d\Delta x_\parallel^{\psi\psi}\, dM_{\psi\psi}^2\, d^2\boldsymbol{q}_T$$

- cross section:

$$\frac{d\sigma}{dx_\parallel^{\psi\psi}\, d|\Delta x_\parallel^{\psi\psi}|\, dM_{\psi\psi}^2\, d^2\boldsymbol{q}_T} = \frac{1}{s^2}\,\frac{1}{32\pi}\,\frac{1}{(x_\parallel^{\psi\psi})^2}\left[1 + \frac{M_{\psi\psi}^2}{2\,x_\parallel^{\psi\psi}\, x_1\, p_{\rm beam}^2}\right]^{-1}\int d^2k_{1T}\, d^2k_{2T}\, \delta^2(\boldsymbol{k}_{1T} + \boldsymbol{k}_{2T} - \boldsymbol{q}_T)$$
$$\times \sum_q \Phi^q(x_1, \boldsymbol{k}_{1T}) \otimes \overline{\Phi}^q(x_2, \boldsymbol{k}_{2T}) \otimes |\mathcal{M}_{q\bar{q}\to J/\psi J/\psi}|^2 + \{\Phi^q \leftrightarrow \overline{\Phi}^q\}$$

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LHCb kinematics

- Jacobian for the transformation $(y_1, y_2, \boldsymbol{K}_\perp^2) \mapsto (z, Y_{\mathcal{QQ}}, M_{\mathcal{QQ}}^2)$. We have $d^2\boldsymbol{K}_\perp = d\phi_\perp dM_\perp^2$, thus we can write

$$z = \frac{1}{1+e^{y_1-y_2}} \equiv \frac{1}{1+e^{\Delta y}}, \qquad Y_{\mathcal{QQ}} = \frac{y_1+y_2}{2}, \qquad M_{\mathcal{QQ}}^2 = M_\perp^2 \frac{(1+e^{\Delta y})^2}{e^{\Delta y}},$$

with $\Delta y \equiv y_1 - y_2$

- the Jacobian is

$$J = \left| \frac{\partial z \, \partial Y_{\mathcal{QQ}} \, \partial M_{\mathcal{QQ}}^2}{\partial y_1 \, \partial y_2 \, \partial M_\perp^2} \right| = \begin{vmatrix} -\frac{e^{\Delta y}}{(1+e^{\Delta y})^2} & \frac{e^{\Delta y}}{(1+e^{\Delta y})^2} & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \\ 2M_\perp^2 \sinh \Delta y & -2M_\perp^2 \sinh \Delta y & \frac{(1+e^{\Delta y})^2}{e^{\Delta y}} \end{vmatrix}$$

- Using the expression above for z, one has

$$\frac{e^{\Delta y}}{(1+e^{\Delta y})^2} = z(1-z)$$

and substituting in J we get $J = 1$

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