On the sensitivity of energy-energy correlation to TMD dynamics

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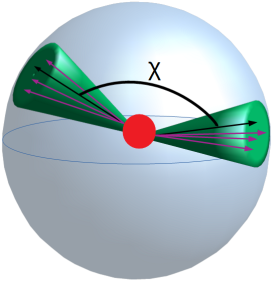
Outline

1. Introduction
2. ARTEMIDE implementation
3. Data
4. Results
   - Replica method
   - SCANs

Energy-Energy Correlators

(e^+e^- collisions)



$$\mathrm{EEC}(\chi) = \frac{\mathrm{d}\sigma}{\mathrm{d}\cos\chi} = \sum_{i,j} \int \mathrm{d}\sigma_{e^+e^- \to ij+X} \frac{E_i E_j}{Q^2} \delta(\cos\theta_{ij} - \cos\chi)$$

$$\chi \longleftrightarrow z = \frac{1-\cos(\chi)}{2}$$

Back-to-back limit

$z \to 1$ $(\chi \to 180^\circ)$

- Fixed order perturbative result: EEC $\sim \log(1-z)$ LARGE LOGARITHMS !!
- All orders resummation:

$$\underbrace{\lim_{z\to 1}\frac{\mathrm{d}\sigma}{\mathrm{d}z} \leftrightarrow \lim_{q_\perp \to 0}\frac{\mathrm{d}\sigma}{\mathrm{d}x_i \mathrm{d}x_j \mathrm{d}^2\vec{q}_\perp} \qquad \& \qquad \text{TMD factorization}}$$

$$\Downarrow$$

Factorization theorem

$$\frac{\mathrm{d}\sigma}{\mathrm{d}z} = \frac{\hat{\sigma}_0}{8}\sum_{f,\bar{f}} H_{f\bar{f}}(Q,\mu)\int_0^\infty \mathrm{d}(b_T Q)^2 J_0(b_T Q\sqrt{1-z}) J_f(b_T,\mu,\zeta) J_{\bar{f}}(b_T,\mu,\bar{\zeta})$$

* $\hat{\sigma}_0$ Born cross section
* $H_{f\bar{f}}(Q,\mu)$ hard function $\leftrightarrow$ [$\mathcal{O}(\alpha_s^4)$]
* Jet function $J_f(b_T,\mu,\zeta) \equiv \sum_h \int_0^1 \mathrm{d}z\, z^3 \overbrace{D_{f\to h}(z,b_T,\mu,\zeta)}^{\text{TMDFF}} \leftrightarrow$ N^{4}LL+$\mathcal{O}(\alpha_s^3)$

$$\mathrm{d}\sigma \sim \int \mathrm{Bessel}_J \times \mathrm{TMD} \times \mathrm{TMD} \Longrightarrow \boxed{\texttt{ARTEMIDE}}$$ [JHEP05(2024)036] [arxiv, 2503.11201]

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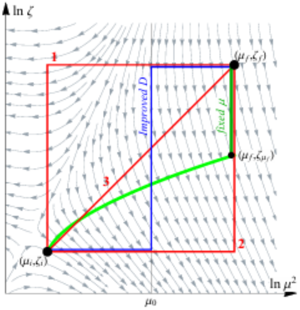
ζ-prescription

[I.Scimemi, A. Vladimirov]



$$\mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2} D_{f\to h}(z, b_T, \mu, \zeta) = -\frac{\gamma_D^f(\mu,\zeta)}{2} D_{f\to h}(z, b_T, \mu, \zeta)$$

$$\zeta \frac{\mathrm{d}}{\mathrm{d}\zeta} D_{f\to h}(z, b_T, \mu, \zeta) = -\mathcal{D}^f(\mu, b_T) D_{f\to h}(z, b_T, \mu, \zeta)$$

$*$ γ_D^f , $\mathcal{D}^f$ $\leftrightarrow$ TMDFF, rapidity anomalous dimensions

Null-evolution lines: $\zeta_\mu(b_T)$

$$\Rightarrow D_{f\to h}(z, b_T, \mu, \zeta) = \left(\frac{\zeta}{\zeta_\mu(b_T)}\right)^{-\mathcal{D}^f(\mu, b_T)} \underbrace{D_{f\to h}(z, b_T, \mu, \zeta_\mu(b_T))}_{D_{f\to h}(z, b_T)\ \text{OPTIMAL TMDFF}}$$

Non-Perturbative models

Collins-Soper kernel

$$\mathcal{D}(\mu, b_T) = \mathcal{D}_{\text{pert}}(b^*, \mu^*) + \int_{\mu^*}^{\mu} \frac{d\mu'}{\mu'} \Gamma_{\text{cusp}}(\mu') + \mathcal{D}_{\text{NP}}(b_T)$$

- b^*-prescription:

$$b^*(b_T) = \frac{b_T}{\sqrt{1 + \frac{b_T^2}{B_{\text{NP}}^2}}}, \qquad \mu^*(b_T) = \frac{2e^{-\gamma_E}}{b^*(b_T)}$$

  - $B_{\text{NP}} \equiv 1.5\text{GeV}^{-1}$

- Non-perturbative correction:

$$\mathcal{D}_{\text{NP}}(b_T) = b_T\, b^* \left[c_0 + c_1 \ln\left(\frac{b^*}{B_{\text{NP}}}\right)\right]$$

  - c_0 and c_1 free parameters

Non-Perturbative models

(Optimal) Jet function

→ Optimal Jet function:

$$J_f(b_T) \equiv \overbrace{\sum_h}^{\substack{\text{sum over}\\ \text{all hadrons}}} \int_0^1 \mathrm{d}z\, z^3 \overbrace{D_{f\to h}(z, b_T)}^{\text{Optimal TMDFF}}$$

→ Optimal TMD fragmentation function: LO OPE $\otimes$ model

$$D_{f\to h}(z,b) \equiv \frac{1}{z^2}\int_z^1 \frac{\mathrm{d}y}{y} \sum_{f'} y^2 \overbrace{\mathbb{C}_{f\to f'}(y, b, \mu_{\text{OPE}})}^{\text{perturbative coeff.}} \overbrace{d_{1,f'\to h}\left(\frac{z}{y}, \mu_{\text{OPE}}\right)}^{\text{Collinear FF (DATA)}} \overbrace{D^h_{\text{NP}}(z,b)}^{\text{NP model}}$$

✗ $J = \sum_h \int$ LO OPE $\otimes$ model

Collinear FF data sets do not satisfy the sum rule:

$$\sum_{h\in\text{Data}} \int_0^1 \mathrm{d}z\, z\, d_{f\to h}(z,\mu) \sim 0.5 \neq 1$$

Non-Perturbative models

(Optimal) Jet function

I) $J^{\text{pert.}}(b_T) \equiv \lim_{b_T\to 0} J_f\left(b_T\right)\big|_{f=q,\,\bar{q}}$

$$\left.\begin{array}{r}\lim_{b\to 0} D^h_{\rm NP}(z,b)=1\\ \sum_h \int_0^1 \mathrm{d}z\, z\, d_{f\to h}(z,\mu)=1\end{array}\right\} \Rightarrow J^{\text{pert.}}(b_T) = \sum_{f'}\int_0^1 \mathrm{d}y\, y^3 \mathbb{C}_{f\to f'}\left(y, b_T, \mu_{\text{OPE}}\right)\big|_{f=q,\,\bar{q}}$$

$$\begin{aligned}J^{\text{pert.}}(b_T) = {} & 1 + \left(\frac{16}{3} - 2\pi^2\right)\alpha_s + \frac{\alpha_s^2}{486}\left[2n_f\left(936\zeta_3 + 594\pi^2 - 989\right) + 3\left(2736\zeta_3 + 516\pi^4 - 9150\pi^2 + 10777\right)\right]\\ & + \frac{\alpha_s^3}{1377810}\Big\{-28n_f^2\left(1332720\zeta_3 + 3618\pi^4 + 157140\pi^2 - 114055\right)\\ & - 420n_f\left(-1846062\zeta_3 + 30\pi^2(2313\zeta_3 - 23912) - 372600\zeta_5 + 24222\pi^4 + 730687\right)\\ & - 15\Big[126\pi^2(1664467 - 383310\zeta_3) - 21(216\zeta_3(1577\zeta_3 - 20116) - 20741616\zeta_5 + 5925635)\\ & + 438352\pi^6 - 15926337\pi^4\Big]\Big\} + \mathcal{O}(\alpha_s^4) = J^{\text{pert.}}(b_T, \mu, \zeta_\mu)\end{aligned}$$

([*M.A.Ebert, B.Mistlberger, G.Vita*])

* with $\alpha_s \equiv \alpha_s\left(\frac{2e^{-\gamma_E}}{b_T}\right)$

Non-Perturbative models

(Optimal) Jet function

II) $J_f(b_T) \equiv J^{\text{pert.}}(b_J^*)\, J_f^{\text{NP}}(b_T)$

- b_J^*-prescription:

$$b_J^*(b_T) = \frac{b_T}{\sqrt{1 + \frac{b_T^2}{B_J^{*2}}}}$$

  - $B_J^* \equiv 0.22\text{GeV}^{-1} \Rightarrow \frac{2e^{-\gamma_E}}{b_J^*} > 5\text{GeV}$ (Avoid numerical fluctuations below bottom threshold m_b)

- Non-perturbative correction:
  - → $\lim_{b_T \to 0} J^{\text{NP}}(b_T) = 1$
  - → Numerical justification: [H.T. Li, Y. Makris , I. Vitev]

$$\left.\begin{array}{r} J_f^{\text{NP}}(b) \sim \sum_h \int_0^1 \mathrm{d}z\, z\, d_{f\to h}(z,\mu) D_{\text{NP}}^h(z,b)|_{\mu=\mu_0} \\ D_{\text{NP}}^h(z,b) \sim \text{Gaussian} \end{array}\right\} \Rightarrow \underset{(\sim f-\text{independent})}{J_f^{\text{NP}}(b_T) \sim e^{-a\, b_T}}$$

  - → Similar model used in [Z.-B. Kang, J. Penttala and C. Zhang]

Non-Perturbative models

(Optimal) Jet function

MODEL 1: Exponential model:

$$J^{\mathrm{NP}}(b_T) = e^{-a_1 b_T}$$

* One free parameter a_1
* Enough to describe shape of the data

MODEL 2:

$$J^{\mathrm{NP}}(b_T) = e^{-a_1 b_T}\,\frac{1 + a_2 b_T^2}{1 + a_3 b_T^2}$$

* Quadratic corrections a_2, a_3
* $a_3 > 0$

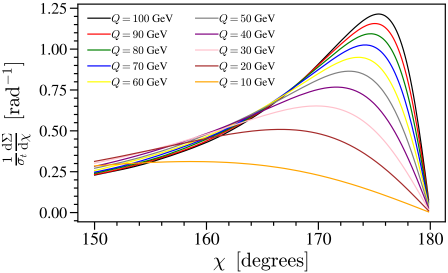
Outline

Experiment	Q (GeV)	References
OPAL	91.0, 91.2, 90.7-91.7	PLB [1990, 1992], [ZPC 1993]
SLD	91.2	[PRD 1995]
TOPAZ	53.3, 59.5	[PLB 1989]
MARKII	29	[PRD 1988]
TASSO	14, 22, 34.8, 43.5	[ZPC 1987]
MAC	29	[PRD 1985]
PLUTO	7.7, 9.4, 12, 13, 17, 22, 27.6, 27.6-31.6	[PLB 1981]
CELLO	22, 34	[ZPC 1982]
JADE	14, 22, 34	[ZPC 1984]
DELPHI	91.2	ZPC [1992, 1993]

Data

Comments

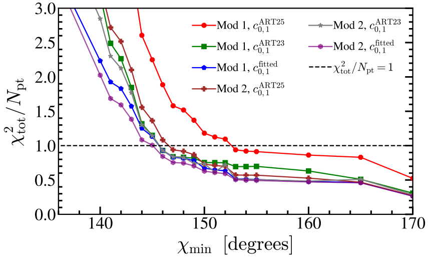
- Publication dates: 1981-1995 $\Rightarrow$ Data from $>$30 years ago
- Intended for α_s extractions from central region
  (EEC used in conjunction with results for event shapes)
- Few details on error estimation $\Rightarrow$ Uncorrelated uncertainties
- Focus on shape $\Rightarrow$ include set-by-set normalization (best χ^2)



- Fit range: (validity of back-to-back factorization theorem)

$$\boxed{\chi_{\min} \equiv 150^\circ}$$

$$\left(\sqrt{1-z} \sim \frac{q_T}{Q} \sim 0.25 \implies \chi \sim 150^\circ\right)$$



- Total number of points 243

Fits

→ 3 different fits for each J^{NP}:

I. CS kernel fixed to ART25 values (obtained from DY+SIDIS data)
II. CS kernel fixed to ART23 values (obtained from DY data)
III. CS kernel fitted

→ 500 replicas of data for parameters error. For error propagation in ART kernels, variation:

- EEC data replica
- c_0, c_1 results from ART replicas

Outline

Parameters Fit

Model 1

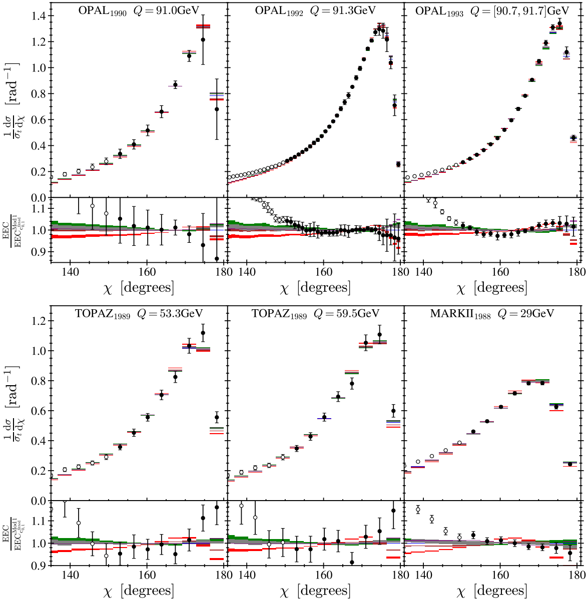
	$c_{0,1}^{\text{ART25}}$	$c_{0,1}^{\text{ART23}}$	$c_{0,1}^{\text{fitted}}$
$c_0[\text{GeV}^{-2}]$	$8.59^{+0.23}_{-0.17} \cdot 10^{-2}$	$3.69^{+0.65}_{-0.61} \cdot 10^{-2}$	$3.17^{+0.04}_{-0.03} \cdot 10^{-2}$
$c_1[\text{GeV}^{-2}]$	$3.03^{+0.38}_{-0.41} \cdot 10^{-2}$	$5.82^{+0.64}_{-0.88} \cdot 10^{-2}$	$2.37^{+0.07}_{-0.1} \cdot 10^{-2}$
$a_1[\text{GeV}]$	$0.879^{+0.014}_{-0.015}$	$1.002^{+0.009}_{-0.016}$	$0.941^{+0.002}_{-0.003}$
$\chi^2_{\text{tot}}/N_{\text{pt}}$	1.18	0.75	0.67

Model 2

	$c_{0,1}^{\text{ART25}}$	$c_{0,1}^{\text{ART23}}$	$c_{0,1}^{\text{fitted}}$
$c_0[\text{GeV}^{-2}]$	$8.59^{+0.23}_{-0.17} \cdot 10^{-2}$	$3.69^{+0.65}_{-0.61} \cdot 10^{-2}$	$1.41^{+0.22}_{-0.19} \cdot 10^{-2}$
$c_1[\text{GeV}^{-2}]$	$3.03^{+0.38}_{-0.41} \cdot 10^{-2}$	$5.82^{+0.64}_{-0.88} \cdot 10^{-2}$	$5.31^{+0.34}_{-0.23} \cdot 10^{-2}$
$a_1[\text{GeV}]$	$0.976^{+0.013}_{-0.013}$	$0.949^{+0.033}_{-0.030}$	$0.917^{+0.003}_{-0.003}$
$a_2[\text{GeV}^2]$	$0.246^{+0.017}_{-0.018}$	$0.83^{+0.95}_{-0.85}$	$0.88^{+0.04}_{-0.04}$
$a_3[\text{GeV}^2]$	$(5.^{+10.}_{-4.7}) \times 10^{-7}$	$1.0^{+1.1}_{-1.0}$	$1.2^{+0.1}_{-0.1}$
$\chi^2_{\text{tot}}/N_{\text{pt}}$	0.73	0.73	0.63

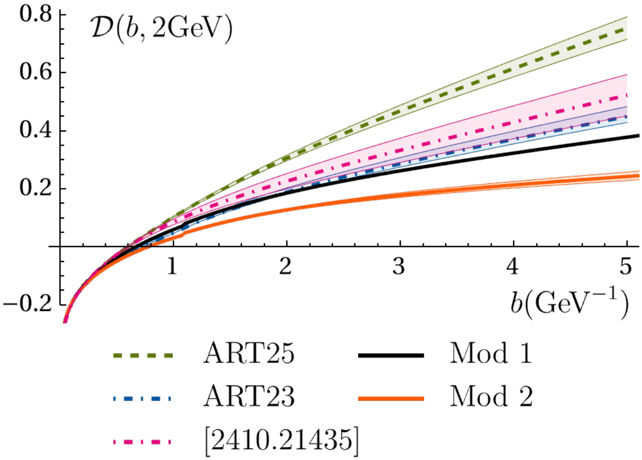
Cross Section

Replica method

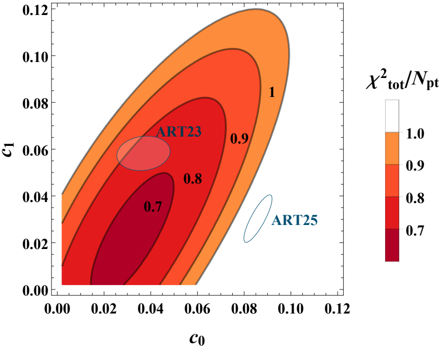


Collins-Soper kernel

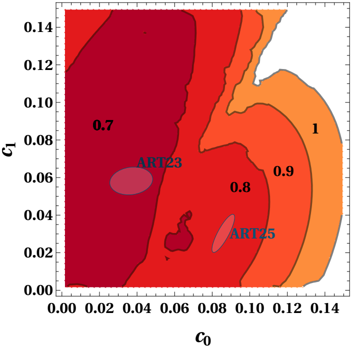
Replica method



SCANs



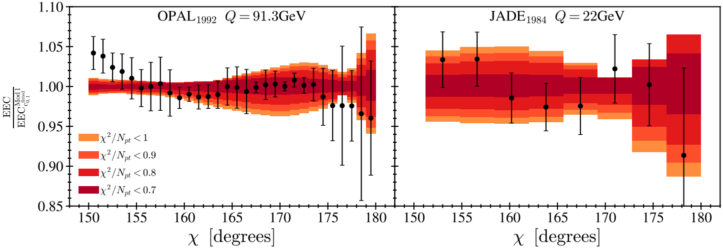
Model 1 (a_1 fitted)

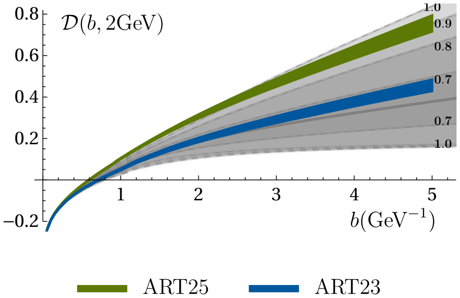


Model 2 ($a_{1,2,3}$ fitted)

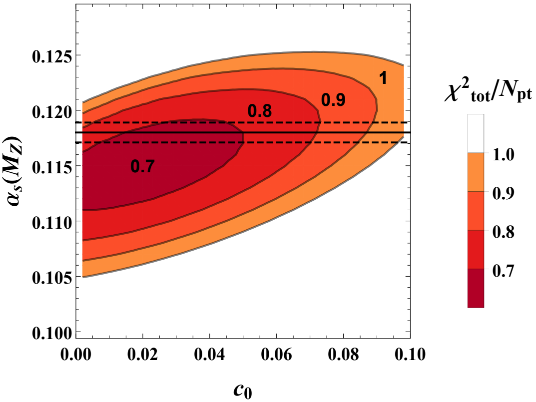
SCANs

Uncertainty bands





α_s extraction



$\chi^2/N_{pt} < 1.0$: $\alpha_s(M_Z) \in [0.105, 0.125]$

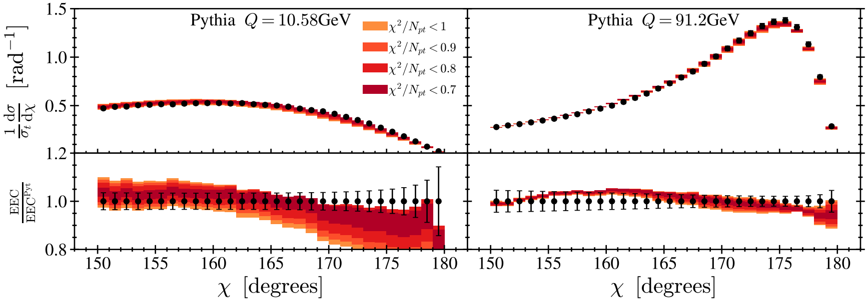
$\chi^2/N_{pt} < 0.9$: $\alpha_s(M_Z) \in [0.106, 0.124]$

$\chi^2/N_{pt} < 0.8$: $\alpha_s(M_Z) \in [0.108, 0.120]$

$\chi^2/N_{pt} < 0.7$: $\alpha_s(M_Z) \in [0.111, 0.119]$

Predictions

Comparison with [Pythia 8.3 (ffbar2gmZ)]



Conclusions

- → We implemented the back-to-back EEC factorization theorem at N^{4}LL+$\mathcal{O}(\alpha_s^3)$ order within **ARTEMIDE**'s framework.
- → Two different models for the non-perturbative part of the jet functions were proposed: one-parameter exponential and including corrections to this function.
- → The free parameters were fitted using, to the best of our knowledge, all data available (being the most recent set with more than 30 years).
- → The replica method underestimates errors when having very low χ^2 fit as a consequence of highly correlated data without specified correlation matrices.
- → Through SCANs in parameter space we obtained the corresponding range of values that can describe the data and we estimated the actual size of uncertainties.
- → The current EEC data in the TMD-sensitive region are not sufficient to impose meaningful constraints on either the Collins–Soper kernel or the strong coupling constant.
- → Predictions were given for *Belle* (at $Q = 10.58$GeV) and for experiments at $Q = M_Z$.

Thank you