Beyond Leading Power in SIDIS Jet Production using Soft Background Fields

arxiv 2507.03072 (M. Jaarsma, O. Del Rio, I. Scimemi and W. Waalewijn)

Resummation, Evolution and Factorization 2025

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PID2022-136510NB-C31

Power Corrections to TMD Factorization

- Cross sections of inelastic processes **factorize** into different blocks (when $Q^2 \gg \Lambda^2_{QCD}$ and $Q^2 \gg k_T^2$)

Hard TMD/Jet Distributions

$$\frac{d\sigma}{d[\ldots]dQdk_T} = \boxed{H(Q^2)} \otimes \boxed{F_1(x_1, k_T)} \otimes \boxed{F_1(x_2, k_T)}$$

LP (very well studied)

Power Corrections to TMD Factorization

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$$\frac{d\sigma}{d[\ldots]dQdk_T} = H(Q^2) \otimes F_1(x_1, k_T) \otimes F_1(x_2, k_T)$$

LP (very well studied)

$$+\frac{k_T}{Q} H_2(Q^2) \otimes F_1(x_1, k_T) \otimes F_2(x_3, k_T)$$

NLP (our focus)

- Motivations:
  - Study sub-leading power observables
  - Increase the theoretical precision and the applicability domain
  - Restore broken properties (EM gauge invariance)

Power Corrections to TMD Factorization

- Cross sections of inelastic processes **factorize** into different blocks (when $Q^2 \gg \Lambda_{QCD}^2$ and $Q^2 \gg k_T^2$)

$$\frac{d\sigma}{d[\ldots]dQdk_T} = H(Q^2) \otimes F_1(x_1, k_T) \otimes F_1(x_2, k_T) \qquad \text{LP (very well studied)}$$

$$+ \frac{k_T}{Q} H_2(Q^2) \otimes F_1(x_1, k_T) \otimes F_2(x_3, k_T) \qquad \text{NLP (our focus)}$$

- Different methods for sub-leading power contributions
  - Soft Collinear Effective Theory (SCET) [2112.07680]
  - Background Field Method (BFM) [2109.09771]

  ⋮

Apparent disagreement!?!

Soft Background Field Method

- Hadronic tensor as a functional integral

$$W^{\mu\nu}_{SIDIS} = \int \frac{d^4b}{(2\pi)^4} e^{i\,(b\cdot k)} \int [dq\,dA]\ \Psi^*_P\ J^{\mu\,\dagger}(b)\ \Psi_p\ \Psi^*_p\ J^{\nu}(0)\ \Psi_P\ e^{i\,S_{QCD}[q,A]}$$

$$\boxed{J^{\mu}(b) = \bar{q}(b)\,\gamma^{\mu}\,q(b)}$$

EM Current

Soft Background Field Method

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$$\boxed{J^\mu(b) = \bar{q}(b)\,\gamma^\mu\, q(b)}$$

EM Current

- **Split** QCD quark and gluon fields:

$$q(b) = \varphi(b) + q(b) + q(b) + q(b)$$

$$A^\mu(b) = \underbrace{B^\mu(b)}_{\text{Dynamical}} + \underbrace{A^\mu(b) + A^\mu(b) + A^\mu(b)}_{\text{Background}}$$

$$\longrightarrow$$

$$S_{QCD}[q,A] = S_{QCD}[\varphi,B] + S_{QCD}[q,A] + S_{QCD}[q,A] + S_{QCD}[q,A] + S_{int}$$

Soft Background Field Method

- **Scaling** of derivatives of the fields ($\lambda \sim k_T/Q$)

$$\underbrace{\begin{array}{c}(\partial^+, \partial^-, \partial_T)\, q \lesssim (\lambda^2, 1, \lambda)\, q \\ (\partial^+, \partial^-, \partial_T)\, A^\mu \lesssim (\lambda^2, 1, \lambda)\, A^\mu\end{array}}_{n\text{-Collinear}} \qquad \underbrace{\begin{array}{c}(\partial^+, \partial^-, \partial_T)\, q \lesssim (1, \lambda^2, \lambda)\, q \\ (\partial^+, \partial^-, \partial_T)\, A^\mu \lesssim (1, \lambda^2, \lambda)\, A^\mu\end{array}}_{\bar{n}\text{-Collinear}} \qquad \underbrace{\begin{array}{c}(\partial^+, \partial^-, \partial_T)\, q \lesssim (\lambda, \lambda, \lambda)\, q \\ (\partial^+, \partial^-, \partial_T)\, A^\mu \lesssim (\lambda, \lambda, \lambda)\, A^\mu\end{array}}_{\text{Soft}}$$

- EOMs give us the power counting of fields

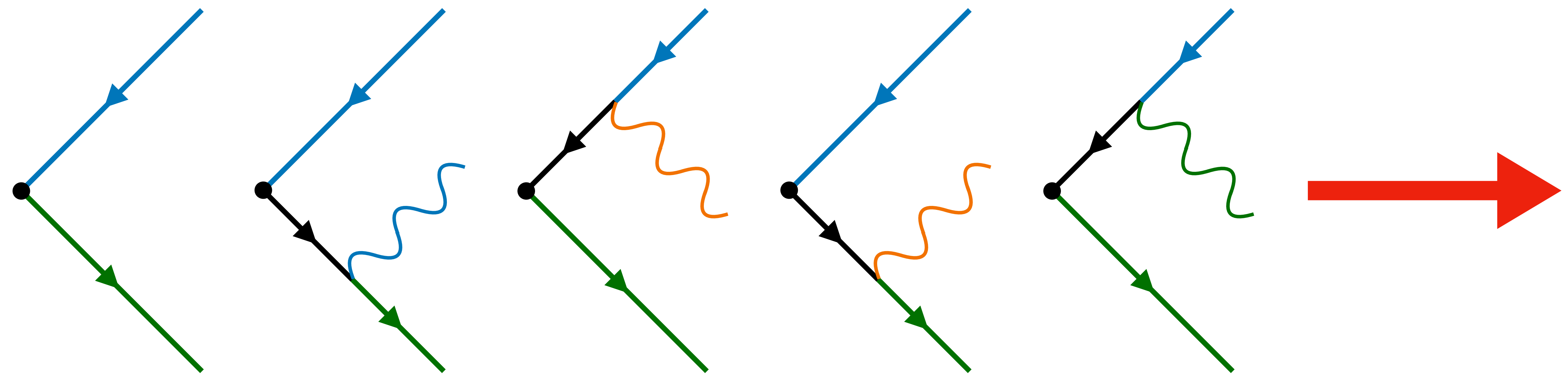
$$\xi = \frac{\gamma^+\gamma^-}{2} q \sim \lambda \qquad \xi = \frac{\gamma^-\gamma^+}{2} q \sim \lambda \qquad q \sim \lambda^{3/2}$$

$$A^\mu \sim (\lambda^2, 1, \lambda) \qquad A^\mu \sim (1, \lambda^2, \lambda) \qquad A^\mu \sim (\lambda, \lambda, \lambda)$$

- Hard modes $(\varphi\,, B) \to$ Integrate out

Effective Current Operator at LP

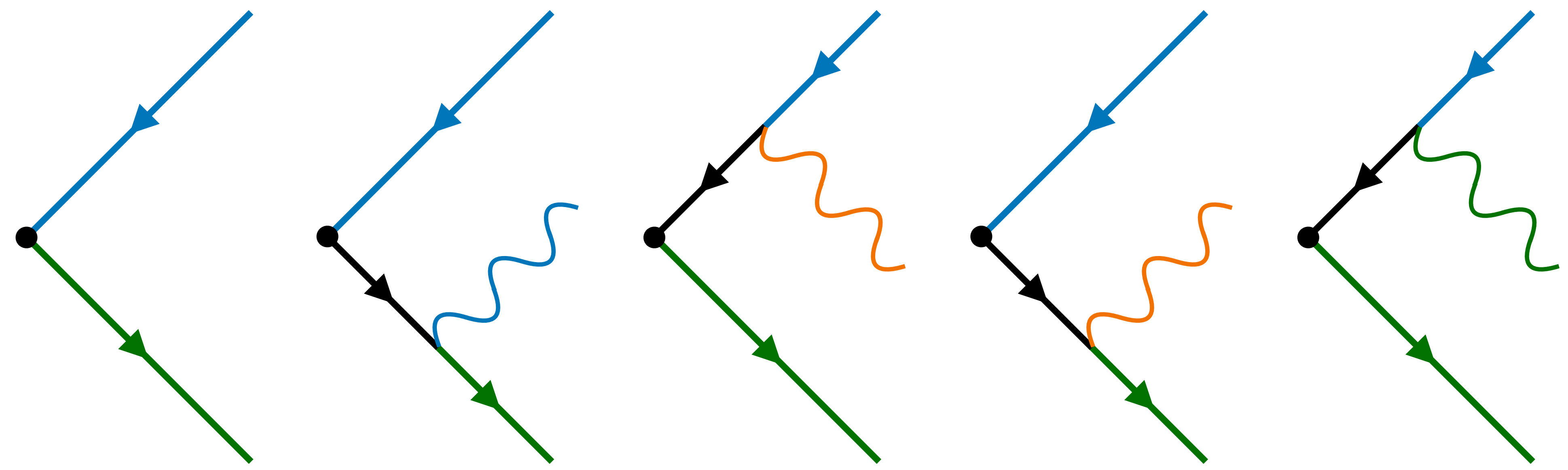
$$J^{\mu}_{eff}(b) = \int [d\bar{\varphi}\, d\varphi\, dB]\Big(\bar{\varphi}(b)+\bar{q}(b)+\bar{q}(b)+\bar{q}(b)\Big)\gamma^{\mu}\Big(\varphi(b)+q(b)+q(b)+q(b)\Big)\, e^{i\,S_{QCD}[\bar{\varphi},\varphi,B]}\, e^{i\,S_{int}}$$



$$J^{\mu}_{eff,\,LP} = \bar{\xi}\,\gamma^{\mu}_{T}\left(1 - g\frac{1}{i\partial^{-}}A^{-}+g\frac{1}{i\partial^{+}}A^{+} - g\frac{1}{i\partial^{-}}A^{-}+g\frac{1}{i\partial^{+}}A^{+}\right)\xi + h.c. + \mathcal{O}\left(g^{2}\right)$$

Effective Current Operator at LP

$$J^{\mu}_{eff}(b) = \int [d\bar{\varphi}\, d\varphi\, dB]\left(\bar{\varphi}(b)+\bar{q}(b)+\bar{q}(b)+\bar{q}(b)\right)\gamma^{\mu}\left(\varphi(b)+q(b)+q(b)+q(b)\right)\, e^{i\,S_{QCD}[\bar{\varphi},\varphi,B]}\, e^{i\,S_{int}}$$

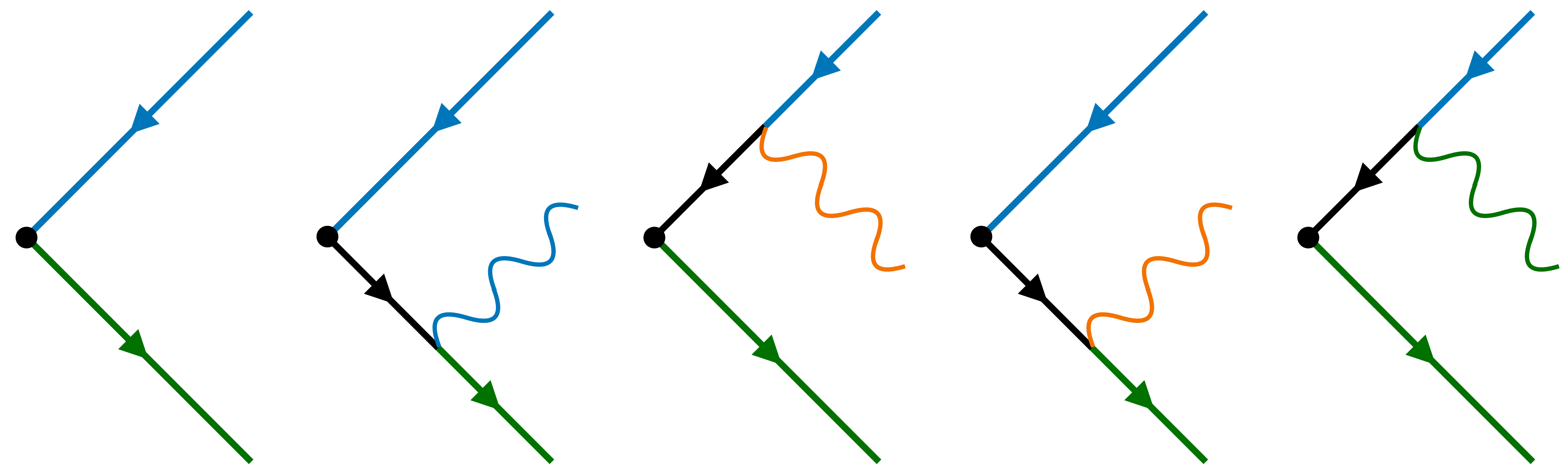


$$\boxed{J^{\mu}_{eff,\,LP} = \bar{\xi}\,\gamma^{\mu}_{T}\left(1 - g\frac{1}{i\partial^{-}}A^{-} + g\frac{1}{i\partial^{+}}A^{+} - g\frac{1}{i\partial^{-}}A^{-} + g\frac{1}{i\partial^{+}}A^{+}\right)\xi + h.c. + \mathcal{O}\left(g^{2}\right)}$$

- Wilson lines W, W, S_n and $S_{\bar{n}}$ $(\chi = W^{\dagger}\xi)$ $\xrightarrow{\text{Gauge Invariant}}$ $\boxed{J^{\mu}_{eff,\,LP} = \bar{\chi}\,\gamma^{\mu}_{T}\, S_n\, S_{\bar{n}}\, \chi + h.c. + \mathcal{O}\left(g^{2}\right)}$

Effective Current Operator at LP

$$J^{\mu}_{eff}(b) = \int [d\bar{\varphi}\, d\varphi\, dB]\big(\bar{\varphi}(b)+\bar{q}(b)+\bar{q}(b)+\bar{q}(b)\big)\gamma^{\mu}\big(\varphi(b)+q(b)+q(b)+q(b)\big)\, e^{i\,S_{QCD}[\bar{\varphi},\varphi,B]}\, e^{i\,S_{int}}$$



$$\boxed{J^{\mu}_{eff,\,LP} = \bar{\xi}\,\gamma^{\mu}_{T}\left(1 - g\frac{1}{i\partial^{-}}A^{-} + g\frac{1}{i\partial^{+}}A^{+} - g\frac{1}{i\partial^{-}}A^{-} + g\frac{1}{i\partial^{+}}A^{+}\right)\xi + h.c. + \mathcal{O}(g^{2})}$$

- Wilson lines W, W, S_n and $S_{\bar{n}}$ $(\chi = W^{\dagger}\xi)$ $\xrightarrow{\text{Gauge Invariant}}$ $\boxed{J^{\mu}_{eff,\,LP} = \bar{\chi}\,\gamma^{\mu}_{T}\, S_n\, S_{\bar{n}}\,\chi + h.c. + \mathcal{O}(g^{2})}$

- Wilson Coefficients $\xrightarrow{\text{All Orders}}$ $\boxed{J^{\mu}_{eff,\,LP}(0) = \int dy^{+}dy^{-}C_1(y^{+},y^{-})\bar{\chi}(y^{+}\bar{n})\,\gamma^{\mu}_{T}\, S^{\dagger}_{n}\, S_{\bar{n}}\,\chi(y^{-}n) + h.c.}$

Effective Current Operator at NLP

$$
\begin{aligned}
J^{\mu}_{eff,NLP}(0) = &-\int dy^+ dy^- \, C_1(y^+,y^-)\,\bar{\chi}(y^+)\, S_n^\dagger\, S_{\bar{n}} \left(n^\mu \frac{\partial\!\!\!/_T}{\partial^+}\chi(y^-)\right) \\
&-\int dy^+ dy^- \, C_1(y^+,y^-)\,\bar{\chi}(y^+)\, S_n^\dagger\, S_{\bar{n}}\, \mathtt{A}^\rho_{T,\bar{n}}\,\gamma_\rho \left(\frac{n^\mu}{i\partial^+}\chi(y^-)\right) \\
&-\int dy^+ dy^- \, C_1(y^+,y^-)\,\bar{\chi}(y^+)\, S_n^\dagger\, S_{\bar{n}} \left(\frac{1}{i\partial^-}\mathtt{A}^\rho_{T,\bar{n}}\right)\gamma^\mu_T \left(\frac{\partial_\rho}{\partial^+}\chi(y^-)\right) \\
&+\int dy_1^+ dy_2^+ dy^- \, C_2(y_1^+,y_2^+,y^-)\,\bar{\chi}(y_1^+)\,\mathtt{A}^\rho_T(y_2^+)\left(\frac{\bar{n}^\mu}{i\overleftarrow{\partial}^-} - \frac{n^\mu}{i\overrightarrow{\partial}^+}\right)\gamma_\rho\, S_n^\dagger\, S_{\bar{n}}\,\chi(y^-) \\
&-\frac{1}{2}\int dy^+ dz^+ dy^- \, C_3(y^+,z^+,y^-)\,\bar{\chi}(y^+)\, S_n^\dagger S_{\bar{n}} \left(\frac{\partial^\rho_T}{\partial^-}\mathtt{A}^\sigma_{T,\bar{n}}(z^+)\right)\gamma^\mu_T\,\gamma_\rho\,\gamma_\sigma \left(\frac{1}{i\partial^+}\chi(y^-)\right) \\
&-\frac{1}{2}\int dy^+ dz^+ dy^- \, C_4(y^+,z^+,y^-)\,\bar{\chi}(y^+)\, S_n^\dagger S_{\bar{n}}\,\mathtt{A}^\rho_T(y^-)\,\gamma^\mu_T\,\gamma^-\,\gamma_\rho \left(\frac{1}{i\partial^-}\chi_{\bar{n}}(z^+)\right) \\
&-\frac{1}{2}\int dy_1^+ dy_2^+ dy^- \, C_5(y^+,z^+,y^-)\,\bar{\chi}(y_1^+)\, S_n^\dagger S_{\bar{n}}\,\mathtt{A}^\rho_T(y^-)\,\gamma^\mu_T\,\gamma^-\,\gamma_\rho\, S_{\bar{n}}^\dagger S_n \left(\frac{1}{i\partial^-}\chi(y_2^+)\right) + (n \leftrightarrow \bar{n}) + h.c.
\end{aligned}
$$

- **7 operator structures**
- **5 independent coefficients**

$\left(\mathtt{A}^\rho_T = W^\dagger[iD^\rho_T, W]\right)$

Effective Current Operator at NLP

$$
\begin{aligned}
J^{\mu}_{eff,NLP}(0) = &-\int dy^+ dy^- C_1(y^+,y^-)\, \bar{\chi}(y^+)\, S_n^\dagger\, S_{\bar{n}} \left(n^\mu \frac{\not{\partial}_T}{\partial^+} \chi(y^-) \right) \\
&-\int dy^+ dy^- C_1(y^+,y^-)\, \bar{\chi}(y^+)\, S_n^\dagger\, S_{\bar{n}}\, \mathtt{A}^\rho_{T,\bar{n}}\, \gamma_\rho \left(\frac{n^\mu}{i\partial^+} \chi(y^-) \right) \\
&-\int dy^+ dy^- C_1(y^+,y^-)\, \bar{\chi}(y^+)\, S_n^\dagger\, S_{\bar{n}} \left(\frac{1}{i\partial^-} \mathtt{A}^\rho_{T,\bar{n}} \right) \gamma_T^\mu \left(\frac{\partial_\rho}{\partial^+} \chi(y^-) \right)
\end{aligned}
$$

Coefficients constrained by symmetries
(Current conservation, RPI, CPT,...)

Effective Current Operator at NLP

$$J^{\mu}_{eff,NLP}(0) = -\int dy^+ dy^- \, C_1(y^+, y^-) \, \bar{\chi}(y^+) \, S_n^\dagger \, S_{\bar{n}} \left(n^\mu \frac{\not{\partial}_T}{\partial^+} \chi(y^-) \right)$$

Kinematic power correction

Genuine power correction

$$+\int dy_1^+ dy_2^+ dy^- \, C_2(y_1^+, y_2^+, y^-) \, \bar{\chi}(y_1^+) \, \mathcal{A}_T^\rho(y_2^+) \left(\frac{\bar{n}^\mu}{i\overleftarrow{\partial}^-} - \frac{n^\mu}{i\overrightarrow{\partial}^+} \right) \gamma_\rho \, S_n^\dagger \, S_{\bar{n}} \, \chi(y^-)$$

Agree with previous methods :)

Effective Current Operator at NLP

$J^{\mu}_{eff,\,NLP}(0) =$

Also agrees
Contributes
$W^{\mu\nu}$ at NNLP

Collinear q-$\bar{q}$ contribution

$$-\frac{1}{2}\int dy_1^+ dy_2^+ dy^-\, C_5(y^+, z^+, y^-)\, \bar{\chi}(y_1^+)\, S_n^\dagger S_{\bar{n}}\, \mathrm{A}_T^\rho(y^-)\, \gamma_T^\mu \gamma^- \gamma_\rho\, S_{\bar{n}}^\dagger S_n \left(\frac{1}{i\partial^-}\chi(y_2^+)\right)$$

Effective Current Operator at NLP

$J^{\mu}_{eff,NLP}(0) =$

Soft fields contributions

$$-\int dy^+dy^-\, C_1(y^+,y^-)\,\bar{\chi}(y^+)\, S_n^\dagger\, S_{\bar{n}}\, \mathtt{A}^{\rho}_{T,\bar{n}}\, \gamma_\rho \left(\frac{n^\mu}{i\partial^+}\chi(y^-)\right)$$

$$-\int dy^+dy^-\, C_1(y^+,y^-)\,\bar{\chi}(y^+)\, S_n^\dagger\, S_{\bar{n}} \left(\frac{1}{i\partial^-}\mathtt{A}^{\rho}_{T,\bar{n}}\right) \gamma_T^\mu \left(\frac{\partial_\rho}{\partial^+}\chi(y^-)\right)$$

$$-\frac{1}{2}\int dy^+dz^+dy^-\, C_3(y^+,z^+,y^-)\,\bar{\chi}(y^+)\, S_n^\dagger S_{\bar{n}} \left(\frac{\partial_T^\rho}{\partial^-}\mathtt{A}^{\sigma}_{T,\bar{n}}(z^+)\right) \gamma_T^\mu \gamma_\rho \gamma_\sigma \left(\frac{1}{i\partial^+}\chi(y^-)\right)$$

$$-\frac{1}{2}\int dy^+dz^+dy^-\, C_4(y^+,z^+,y^-)\,\bar{\chi}(y^+)\, S_n^\dagger S_{\bar{n}}\, \mathtt{A}^{\rho}_{T}(y^-)\, \gamma_T^\mu \gamma^- \gamma_\rho \left(\frac{1}{i\partial^-}\chi_{\bar{n}}(z^+)\right)$$

Effective Current Operator at NLP

$$J^{\mu}_{eff,NLP}(0) =$$

$$-\int dy^+ dy^- \, C_1(y^+, y^-) \, \bar{\chi}(y^+) \, S_n^\dagger \, S_{\bar{n}} \, \mathtt{A}^{\rho}_{T,\bar{n}} \, \gamma_\rho \left(\frac{n^\mu}{i\partial^+} \chi(y^-) \right)$$

Soft fields contributions

Appears in [2112.07680] with different coefficient

Effective Current Operator at NLP

$J^{\mu}_{eff,NLP}(0) =$

Soft fields contributions

$$-\int dy^+ dy^- \, C_1(y^+, y^-) \, \bar{\chi}(y^+) \, S_n^{\dagger} \, S_{\bar{n}} \left(\frac{1}{i\partial^-} \mathtt{A}^{\rho}_{T,\bar{n}} \right) \gamma^{\mu}_T \left(\frac{\partial_\rho}{\partial^+} \chi(y^-) \right)$$

Inverse derivatives of soft fields are new (vacuum matrix elements vanish)

$$-\frac{1}{2} \int dy^+ dz^+ dy^- \, C_3(y^+, z^+, y^-) \, \bar{\chi}(y^+) \, S_n^{\dagger} S_{\bar{n}} \left(\frac{\partial^{\rho}_T}{\partial^-} \mathtt{A}^{\sigma}_{T,\bar{n}}(z^+) \right) \gamma^{\mu}_T \gamma_\rho \gamma_\sigma \left(\frac{1}{i\partial^+} \chi(y^-) \right)$$

$$-\frac{1}{2} \int dy^+ dz^+ dy^- \, C_4(y^+, z^+, y^-) \, \bar{\chi}(y^+) \, S_n^{\dagger} S_{\bar{n}} \, \mathtt{A}^{\rho}_T(y^-) \, \gamma^{\mu}_T \gamma^- \gamma_\rho \left(\frac{1}{i\partial^-} \chi_{\bar{n}}(z^+) \right)$$

The Hadronic Tensor

$$W^{\mu\nu}_{SIDIS} = \int \frac{d^4b}{(2\pi)^4} e^{i(b\cdot k)} \langle P | J^{\mu\,\dagger}_{eff}(b) | p, X \rangle \langle p, X | J^{\nu}_{eff}(0) | P \rangle$$

- **Unsubtracted** leading-power hadronic tensor

$$W^{\mu\nu}_{uns.,LP} = \int \frac{d^4b}{(2\pi)^4} e^{i(b\cdot k)} \int dy^+ dy^- C_1(y^+, y^-) \int dz^+ dz^- C_1^*(z^+, z^-) (\gamma_T^\mu)_{ij} (\gamma_T^\nu)_{kl} \langle 0 | S^\dagger_{\bar{n}} S_n(b) | X \rangle \langle X | S^\dagger_n S_{\bar{n}}(0) | 0 \rangle$$

$$\times \langle P | \bar{\chi}_i(b_T + b^- n + y^- n) | X \rangle \langle X | \chi_l(z^- n) | P \rangle \langle 0 | \chi_j(b_T + b^+ \bar{n} + y^+ \bar{n}) | p, X \rangle \langle p, X | \bar{\chi}_k(z^+ \bar{n}) | 0 \rangle + h.c.$$

- At NLP some matrix elements vanish (fermion number violation, boost invariance,...)

$$\langle 0 | \bar{\chi}_i(b_T + b^+ \bar{n} + y_1^+ \bar{n}) \frac{1}{i\partial^-} \chi_j(b_T + b^+ \bar{n} + y_2^+ \bar{n}) | p, X \rangle \langle p, X | \bar{\chi}_k(z^+ \bar{n}) | 0 \rangle = 0$$

$$\langle P | \mathcal{A}^\rho_T(b_T + y^- n) | X \rangle \langle X | \chi_l(z^- n) | P \rangle = 0 \qquad \langle 0 | \left(\frac{1}{i\partial^-} \mathcal{A}^\rho_{T,\bar{n}}(b_T) \right) S^\dagger_{\bar{n}} S_n(b_T) | X \rangle \langle X | S^\dagger_n S_{\bar{n}}(0) | 0 \rangle = 0 \quad \cdots$$

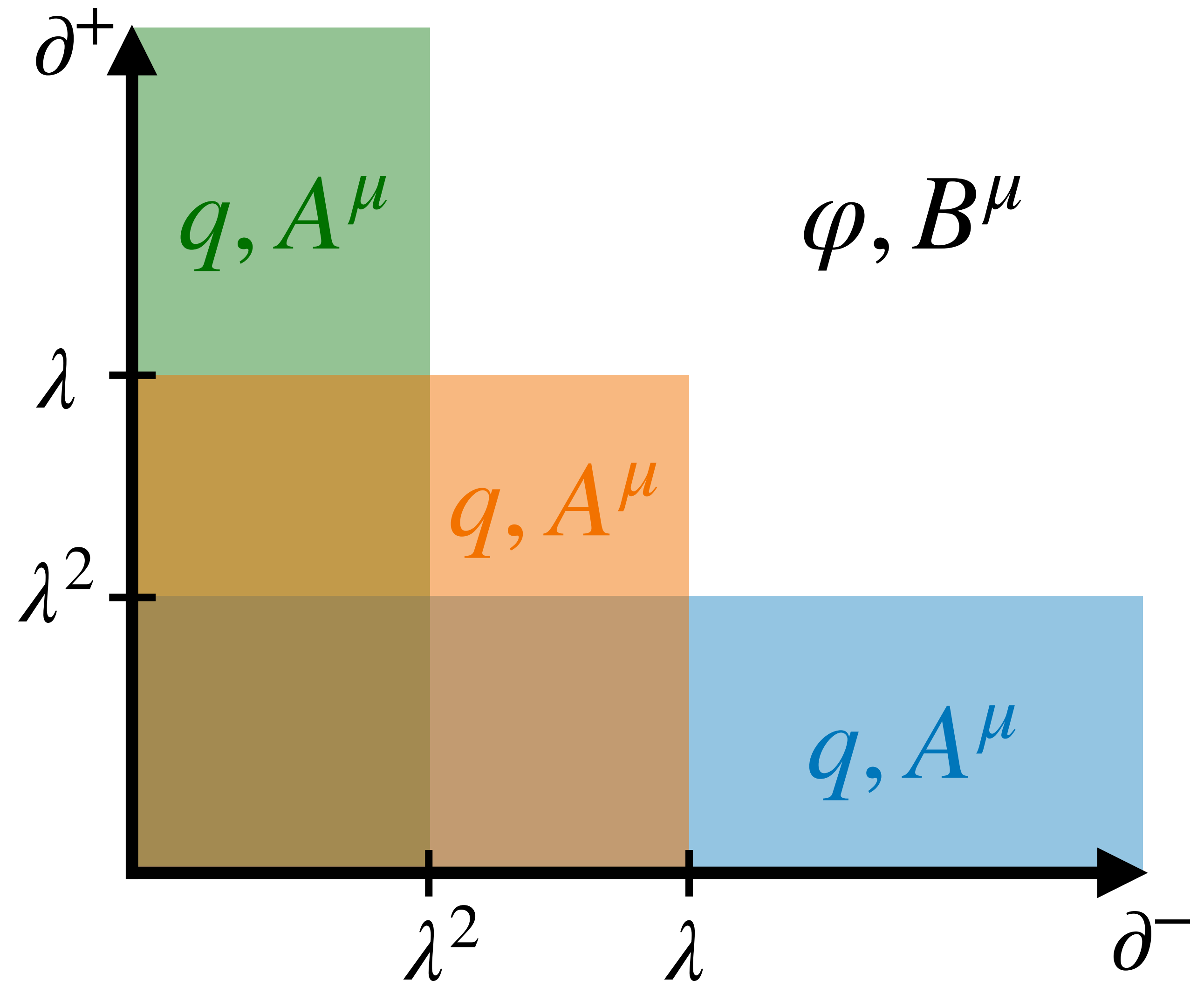
The Zero-bin Subtraction

- **Overlap** of soft and collinear fields → Rapidity divergences

- Split and redefine collinear fields

$$\chi \to S_{\bar{n}}^{\dagger} S_n \, \chi + \chi_{\bar{n}} \qquad\qquad \chi \to S_n^{\dagger} S_{\bar{n}} \, \chi + \chi_n$$

$$\mathtt{A}_T^{\mu} \to S_{\bar{n}}^{\dagger} S_n \; \mathtt{A}_T^{\mu} \; S_n^{\dagger} S_{\bar{n}} + \mathtt{A}_{T,\bar{n}}^{\mu} \qquad\qquad \mathtt{A}_T^{\mu} \to S_n^{\dagger} S_{\bar{n}} \; \mathtt{A}_T^{\mu} \; S_{\bar{n}}^{\dagger} S_n + \mathtt{A}_{T,n}^{\mu}$$



- Use transformations to obtain **pure-collinear matrix elements**

$$\langle 0 \,|\, \chi_j(b_T + b^+ \bar{n}) \,|\, p, X \rangle \langle p, X \,|\, \bar{\chi}_k(0) \,|\, 0 \rangle_{sub} = \frac{\langle 0 \,|\, \chi_j(b_T + b^+ \bar{n}) \,|\, p, X \rangle \langle p, X \,|\, \bar{\chi}_k(0) \,|\, 0 \rangle}{\langle 0 \,|\, S_{\bar{n}}^{\dagger} S_n(b_T) \,|\, X \rangle \langle X \,|\, S_n^{\dagger} S_{\bar{n}}(0) \,|\, 0 \rangle}$$

$$\langle P \,|\, \bar{\chi}_i(b_T + b^- n) \,|\, X \rangle \langle X \,|\, \chi_l(0) \,|\, P \rangle_{sub} = \frac{\langle P \,|\, \bar{\chi}_i(b_T + b^- n) \,|\, X \rangle \langle X \,|\, \chi_l(0) \,|\, P \rangle}{\langle 0 \,|\, S_{\bar{n}}^{\dagger} S_n(b_T) \,|\, X \rangle \langle X \,|\, S_n^{\dagger} S_{\bar{n}}(0) \,|\, 0 \rangle}$$

The Subtracted Hadronic Tensor

- The overlap subtraction gives us the usual structure for the cancellation of rapidity divergences (we employ $\delta^{\pm}$ regulators and $(\zeta, \bar{\zeta})$ as rapidity scales)

$$\langle 0 | S_{\bar{n}}^{\dagger} S_n(b_T) | X \rangle \langle X | S_n^{\dagger} S_{\bar{n}}(0) | 0 \rangle = \sqrt{S(b_T, \zeta(\delta^+/k^+)^2)} \sqrt{S(b_T, \bar{\zeta}(\delta^-/k^-)^2)}$$

Soft Factors

- At leading-power

$$W_{LP}^{\mu\nu} = \frac{(\gamma_T^{\mu})_{ij} (\gamma_T^{\nu})_{kl}}{N_c} \int \frac{d^2 b_T}{(2\pi)^2} e^{i(b_T \cdot k_T)} \int dy^+ dy^- e^{iy^+k^-} e^{iy^-k^+} C_1(y^+, y^-) \int dz^+ dz^- e^{-iz^+k^-} e^{-iz^-k^+} C_1^*(z^+, z^-)$$

$$\times \int \frac{db^-}{2\pi} e^{ib^-k^+} \frac{\langle P | \bar{\chi}_i(b_T + b^- n) | X \rangle \langle X | \chi_l(0) | P \rangle}{\sqrt{S(b_T, \zeta(\delta^+/k^+)^2)}} \int \frac{db^+}{2\pi} e^{ib^+k^-} \frac{\langle 0 | \chi_j(b_T + b^+ \bar{n}) | p, X \rangle \langle p, X | \bar{\chi}_k(0) | 0 \rangle}{\sqrt{S(b_T, \bar{\zeta}(\delta^-/k^-)^2)}}$$

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- At leading-power

$$W_{LP}^{\mu\nu} = \frac{(\gamma_T^{\mu})_{ij} (\gamma_T^{\nu})_{kl}}{N_c} \int \frac{d^2 b_T}{(2\pi)^2} e^{i(b_T \cdot k_T)} \boxed{\int dy^+ dy^- e^{iy^+k^-} e^{iy^-k^+} C_1(y^+, y^-) \int dz^+ dz^- e^{-iz^+k^-} e^{-iz^-k^+} C_1^*(z^+, z^-)}$$

$H_1(Q^2)$ **Hard Function**

$$\times \boxed{\int \frac{db^-}{2\pi} e^{ib^-k^+} \frac{\langle P | \bar{\chi}_i(b_T + b^- n) | X \rangle \langle X | \chi_l(0) | P \rangle}{\sqrt{S(b_T, \zeta(\delta^+/k^+)^2)}}} \boxed{\int \frac{db^+}{2\pi} e^{ib^+k^-} \frac{\langle 0 | \chi_j(b_T + b^+ \bar{n}) | p, X \rangle \langle p, X | \bar{\chi}_k(0) | 0 \rangle}{\sqrt{S(b_T, \bar{\zeta}(\delta^-/k^-)^2)}}}$$

$[F_{q,11}(x, b_T; \zeta)]_{li}$ **Physical TMDs** $[D_{q,11}(z, b_T; \bar{\zeta})]_{jk}$

The Subtracted Hadronic Tensor

- The overlap subtraction gives us the usual structure for the cancellation of rapidity divergences (we employ $\delta^{\pm}$ regulators and $(\zeta, \bar{\zeta})$ as rapidity scales)

$$\langle 0 | S^{\dagger}_{\bar{n}} S_n(b_T) | X \rangle \langle X | S^{\dagger}_{n} S_{\bar{n}}(0) | 0 \rangle = \sqrt{S(b_T, \zeta(\delta^+/k^+)^2)}\,\sqrt{S(b_T, \bar{\zeta}(\delta^-/k^-)^2)}$$

Soft Factors

- At leading-power

$$W^{\mu\nu}_{LP} = \frac{(\gamma^{\mu}_T)_{ij}(\gamma^{\nu}_T)_{kl}}{N_c} H_1(Q^2) \int \frac{d^2 b_T}{(2\pi)^2} e^{i\,(b_T \cdot k_T)} \sum_q \left\{ [F_{q,11}]_{li} [D_{q,11}]_{jk} + [F_{\bar{q},11}]_{jk} [D_{\bar{q},11}]_{li} \right\}$$

The Subtracted Hadronic Tensor

- The overlap subtraction gives us the usual structure for the cancellation of rapidity divergences (we employ $\delta^{\pm}$ regulators and $(\zeta, \bar{\zeta})$ as rapidity scales)

$$\langle 0 | S^{\dagger}_{\bar{n}} S_n(b_T) | X \rangle \langle X | S^{\dagger}_n S_{\bar{n}}(0) | 0 \rangle = \sqrt{S(b_T, \zeta(\delta^+/k^+)^2)} \sqrt{S(b_T, \bar{\zeta}(\delta^-/k^-)^2)}$$

Soft Factors

- At leading-power

$$W^{\mu\nu}_{LP} = \frac{(\gamma^{\mu}_T)_{ij} (\gamma^{\nu}_T)_{kl}}{N_c} H_1(Q^2) \int \frac{d^2 b_T}{(2\pi)^2} e^{i\,(b_T \cdot k_T)} \sum_q \left\{ [F_{q,11}]_{li} [D_{q,11}]_{jk} + [F_{\bar{q},11}]_{jk} [D_{\bar{q},11}]_{li} \right\}$$

- At NLP we have two types of corrections

$$W^{\mu\nu}_{NLP} = W^{\mu\nu}_{kNLP} + W^{\mu\nu}_{gNLP}$$

The Subtracted Hadronic Tensor at NLP

- The overlap subtraction also helps define rapidity-free derivatives of the LP TMDs

Same Lorentz structure and coefficient

$$\langle \partial_T^\rho \chi_j \rangle \langle \bar{\chi}_k \rangle \langle S_{\bar{n}}^\dagger S_n \rangle \langle S_n^\dagger S_{\bar{n}} \rangle$$

$$\langle \chi_j \rangle \langle \bar{\chi}_k \rangle \langle S_{\bar{n}}^\dagger S_n \, \mathtt{A}_{\mathtt{T,n}}^\rho \rangle \langle S_n^\dagger S_{\bar{n}} \rangle$$

$$\longrightarrow \frac{\langle \partial_T^\rho \chi_j \rangle \langle \bar{\chi}_k \rangle}{\langle S_{\bar{n}}^\dagger S_n \rangle \langle S_n^\dagger S_{\bar{n}} \rangle} - \frac{1}{2} \langle \chi_j \rangle \langle \bar{\chi}_k \rangle \frac{\langle \partial_T^\rho (S_{\bar{n}}^\dagger S_n) \rangle \langle S_n^\dagger S_{\bar{n}} \rangle}{\langle S_{\bar{n}}^\dagger S_n \rangle \langle S_n^\dagger S_{\bar{n}} \rangle}$$

The Subtracted Hadronic Tensor at NLP

- The overlap subtraction also helps define rapidity-free derivatives of the LP TMDs

Same Lorentz structure and coefficient

$\langle \partial_T^\rho \chi_j \rangle \langle \bar{\chi}_k \rangle \langle S_{\bar{n}}^\dagger S_n \rangle \langle S_n^\dagger S_{\bar{n}} \rangle$

$\langle \chi_j \rangle \langle \bar{\chi}_k \rangle \langle S_{\bar{n}}^\dagger S_n \, \mathtt{A}_{\mathtt{T,n}}^\rho \rangle \langle S_n^\dagger S_{\bar{n}} \rangle$

$$\dashrightarrow \frac{b_T^\rho}{b_T^2} [D'_{q,11}]_{jk} = \left[\partial_T^\rho - \frac{1}{2} (\partial_T^\rho \boxed{K}) \ln\left(\frac{\zeta}{\bar{\zeta}}\right) \right] [D_{q,11}]_{jk}$$

Collins-Soper kernel

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Collins-Soper kernel

- These terms give rise to the **kinematic power correction** of the NLP hadronic tensor

$$W^{\mu\nu}_{kNLP} = -\frac{i}{N_c} H_1(Q^2) \int \frac{d^2 b_T}{(2\pi)^2} e^{i(b_T \cdot k_T)} \frac{b_\rho}{b_T^2} \left\{ \frac{1}{k^-} \left[\bar{n}^\mu (\gamma_T^\rho)_{ij} (\gamma_T^\nu)_{kl} + \bar{n}^\nu (\gamma_T^\mu)_{ij} (\gamma_T^\rho)_{kl}\right] \sum_{q,\bar{q}} \left[F_{q,11}\right]_{li} \left[D'_{q,11}\right]_{jk} \right.$$
$$\left. + \frac{1}{k^+} \left[n^\mu (\gamma_T^\rho)_{ij} (\gamma_T^\nu)_{kl} + n^\nu (\gamma_T^\mu)_{ij} (\gamma_T^\rho)_{kl}\right] \sum_{q,\bar{q}} \left[F'_{q,11}\right]_{li} \left[D_{q,11}\right]_{jk} \right\}$$

The Subtracted Hadronic Tensor at NLP

- Twist-3 operator definitions with subtracted rapidity

$$\langle [\mathtt{A}_T^\rho \chi]_j \rangle \langle \bar{\chi}_k \rangle \;\longrightarrow\; \frac{\langle [\mathtt{A}_T^\rho \chi]_j \rangle \langle \bar{\chi}_k \rangle - \langle \chi_j \rangle \langle \bar{\chi}_k \rangle \langle \mathtt{A}_{T,\bar{n}}^\rho S_{\bar{n}}^\dagger S_n \rangle \langle S_n^\dagger S_{\bar{n}} \rangle}{\langle S_{\bar{n}}^\dagger S_n \rangle \langle S_n^\dagger S_{\bar{n}} \rangle}$$

The Subtracted Hadronic Tensor at NLP

- Twist-3 operator definitions with subtracted rapidity

$$\left[D_{q,21}(z,\xi,b_T;\bar{\zeta})\right]_{jk} = iq^- \int \frac{db_1^+}{2\pi} e^{i\bar{\xi}b_1^+k^-} \int \frac{db_2^+}{2\pi} e^{i\xi b_2^+k^-} \left\{ \frac{\langle 0 \,|\, [\cancel{A}_T(b_T + b_2^+\bar{n}))\chi(b_T + b_1^+\bar{n})]_j \,|\, p, X\rangle\langle p, X \,|\, \bar{\chi}_k(0) \,|\, 0\rangle}{\sqrt{S(b_T, \bar{\zeta}(\delta^-/k^-)^2)}} \right.$$

$$\left. - \frac{\langle 0 \,|\, [\gamma_T^\rho \chi(b_T + b_1^+\bar{n})]_j \,|\, p, X\rangle\langle p, X \,|\, \bar{\chi}_k(0) \,|\, 0\rangle}{\sqrt{S(b_T, \bar{\zeta}(\delta^-/k^-)^2)}} \langle 0 \,|\, \mathbb{A}^\rho_{T,\bar{n}}(b_T + b_2^+\bar{n})\, S_{\bar{n}}^\dagger S_n(b_T) \,|\, X\rangle\langle X \,|\, S_n^\dagger S_{\bar{n}}(0) \,|\, 0\rangle \right\}$$

- Also free of endpoint divergences

$$\lim_{\xi\to 0} \xi \left[D_{q,21}(z,\xi,b_T;\bar{\zeta})\right]_{jk} = 0$$

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$$- \frac{\langle 0 | [\gamma_T^\rho \chi(b_T+b_1^+\bar{n})]_j | p,X\rangle\langle p,X|\bar{\chi}_k(0)|0\rangle}{\sqrt{S(b_T,\bar{\zeta}(\delta^-/k^-)^2)}} \langle 0 | \mathbb{A}^\rho_{T,\bar{n}}(b_T+b_2^+\bar{n})\, S_{\bar{n}}^\dagger S_n(b_T) | X\rangle\langle X | S_n^\dagger S_{\bar{n}}(0)|0\rangle \Bigg\}$$

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The Subtracted Hadronic Tensor at NLP

- The **genuine higher-twist correction**

$$
\begin{aligned}
W^{\mu\nu}_{gNLP} = \frac{i}{N_c}\int d\xi\, H_2(Q^2,\xi)\int \frac{d^2b_T}{(2\pi)^2} e^{i\,(b_T\cdot k_T)} \Bigg\{ \left[\frac{\bar{n}^\mu}{k^-} - \frac{n^\mu}{k^+}\right] (\mathbb{1})_{ij}\, (\gamma_T^\nu)_{kl} \sum_q \Big([F_{q,11}]_{li}[D_{q,21}]_{jk} \\
- [F_{\bar{q},11}]_{jk}[D_{\bar{q},21}]_{li} + [F_{q,21}]_{li}[D_{q,11}]_{jk} - [F_{\bar{q},21}]_{jk}[D_{\bar{q},11}]_{li} \Big) \\
+ \left[\frac{\bar{n}^\nu}{k^-} - \frac{n^\nu}{k^+}\right] (\gamma_T^\mu)_{ij}\, (\mathbb{1})_{kl} \sum_q \Big([F_{q,11}]_{li}[D_{q,12}]_{jk} - [F_{\bar{q},11}]_{jk}[D_{\bar{q},12}]_{li} + [F_{q,12}]_{li}[D_{q,11}]_{jk} - [F_{\bar{q},12}]_{jk}[D_{\bar{q},11}]_{li} \Big) \Bigg\}
\end{aligned}
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& - [F_{\bar{q},11}]_{jk}[D_{\bar{q},21}]_{li} + [F_{q,21}]_{li}[D_{q,11}]_{jk} - [F_{\bar{q},21}]_{jk}[D_{\bar{q},11}]_{li} \Big) \\
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\end{aligned}
$$

- To agree with previous SCET results we found a condition of agreement

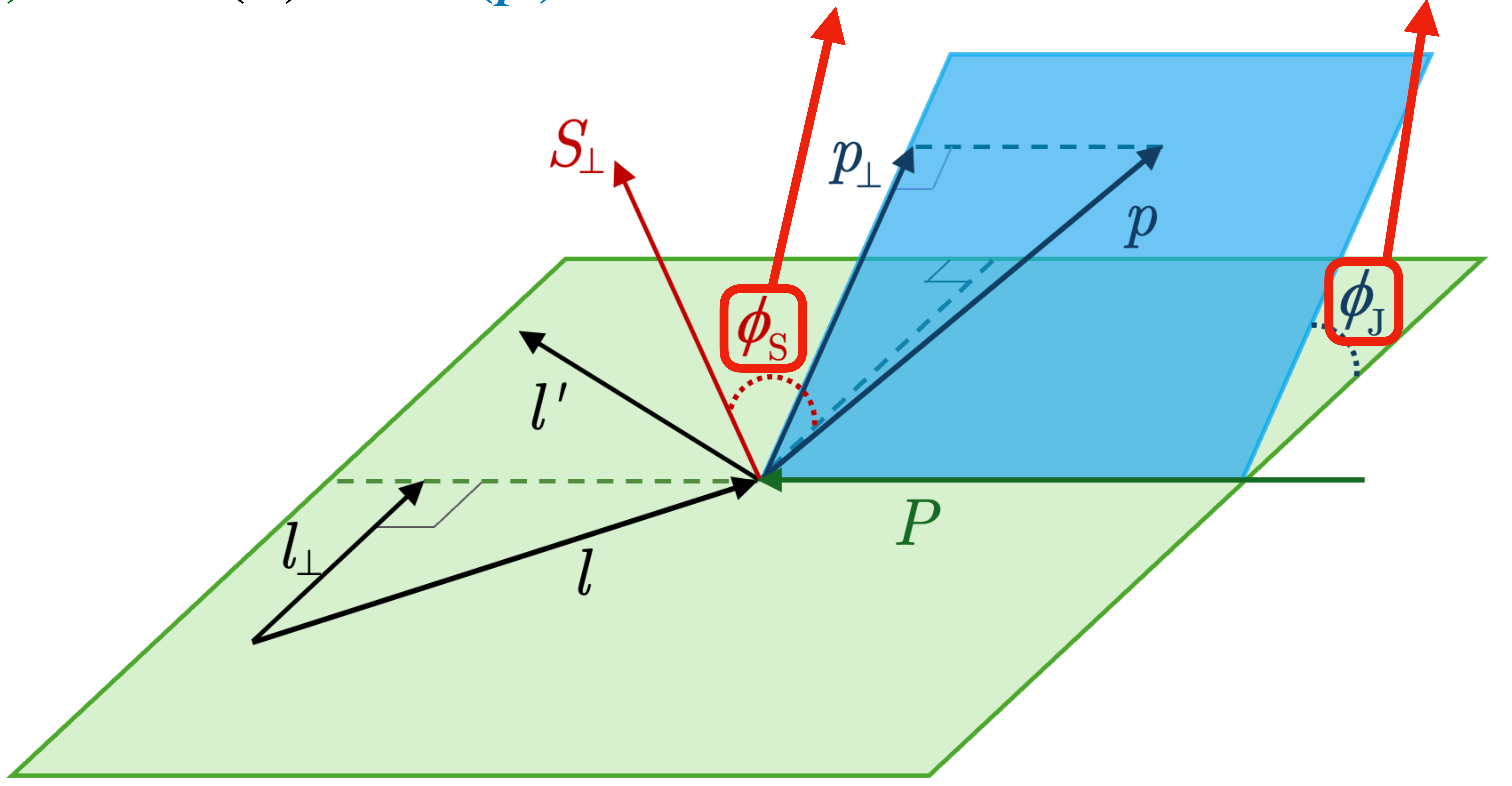
$$
H_2^{SCET}(Q^2,\xi) = H_1(Q^2) + [H_2 \otimes \boxed{H_C}](Q^2,\xi)
$$

Hard-Collinear
Matching Coefficient

Jet production in SIDIS

- Apply result to an specific process: $e^-(l)+h(P) \to e^-(l')+\text{Jet}(p)+X$

Azimuthal angles with jet and hadron's spin



- Kinematics of jet production in SIDIS

$$M^2 = P^2 \qquad k = l - l' \qquad Q^2 = -k^2$$

$$x = \frac{Q^2}{2k\cdot P} \qquad y = \frac{k\cdot P}{l\cdot P}$$

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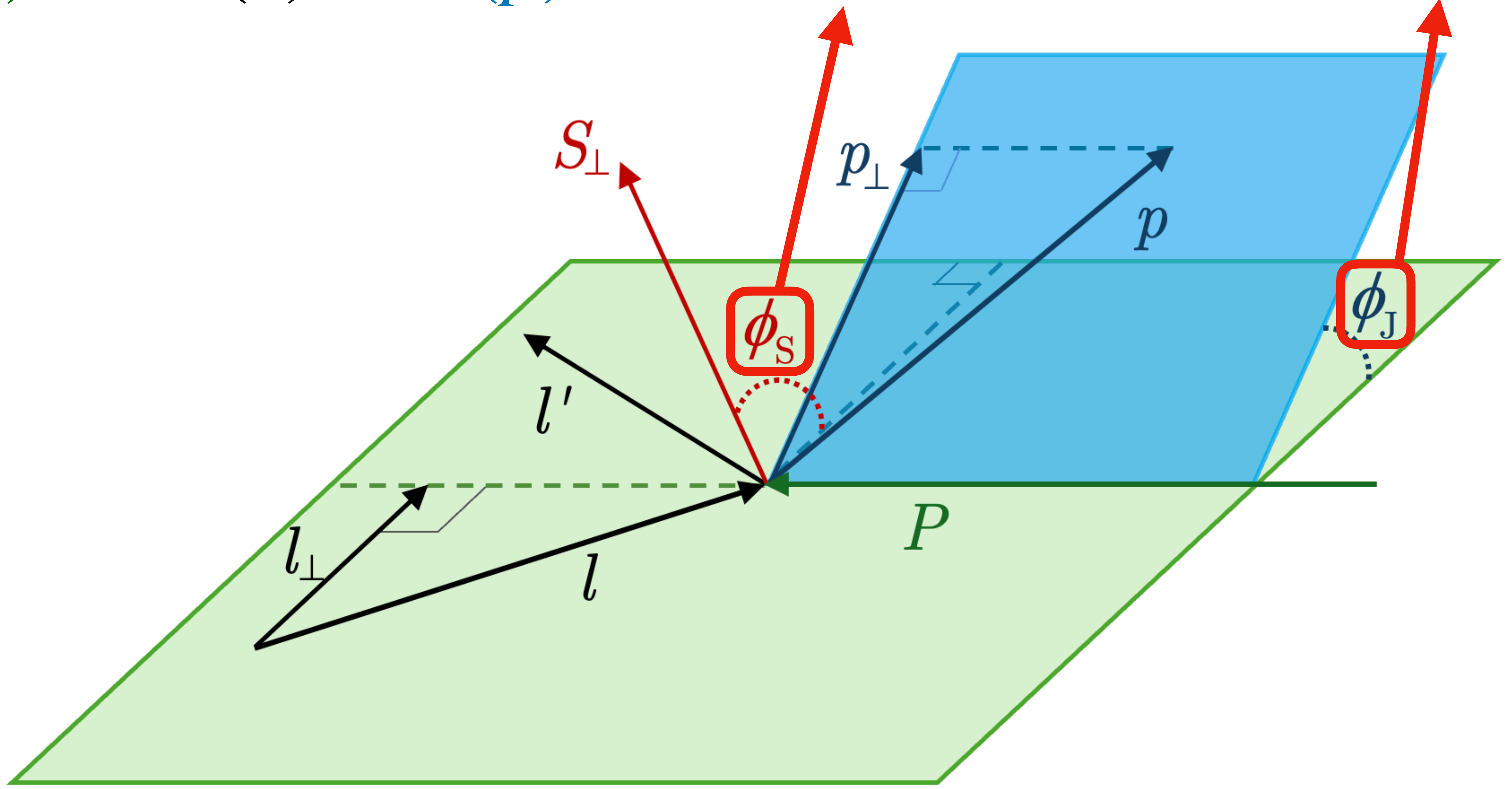

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$$x = \frac{Q^2}{2k \cdot P} \qquad y = \frac{k \cdot P}{l \cdot P}$$

- Cross section in the limit $Q^2 \gg M^2$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}\phi_J\,\mathrm{d}\phi_S\,\mathrm{d}\boldsymbol{k}^2} = \frac{\alpha_{em}^2 y}{8Q^4} \boxed{L_{\mu\nu}} W^{\mu\nu}$$

$$L_{\mu\nu} = 2\left(l_\mu l'_\nu + l'_\mu l_\nu - (l \cdot l')g_{\mu\nu}\right) + 2i\lambda_e \epsilon_{\mu\nu\rho\sigma} l^\rho l'^\sigma$$

Leptonic Tensor

Jet production in SIDIS at NLP

- **Fierz transformations** of Lorentz structures

$$4(\mathbb{1})_{ij}(\gamma_T^{\nu})_{kl} = i\epsilon_T^{\nu\alpha}[(\gamma^-)_{kj}(i\sigma^{\alpha+\gamma^5})_{il} + \dots$$

- Jet and TMDPDF definitions with no open spinor indices

$$(\gamma^-)_{kj}\big[D_{11}(b_T)\big]_{jk} = 2N_c J_{11}(b_T)$$

$$\epsilon_T^{\nu\alpha}(\sigma^{\alpha+}\gamma^5)_{il}\big[F_{21}(x,\xi,b_T)\big]_{li} = \mathtt{F}_{21}^{\nu}(x,\xi,b_T)$$

Jet production in SIDIS at NLP

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$$(\gamma^-)_{kj}\big[D_{11}(b_T)\big]_{jk} = 2N_c\, J_{11}(b_T)$$

$$\epsilon_T^{\nu\alpha}(\sigma^{\alpha+}\gamma^5)_{il}\big[F_{21}(x,\xi,b_T)\big]_{li} = \mathrm{F}_{21}^{\nu}(x,\xi,b_T)$$

- **Parametrization** of twist-3 TMDPDF

$$\mathrm{F}_{21}^{\nu} = i\frac{b_T^{\nu}}{b^2}f_2^{\perp} + S_L\frac{i\epsilon_T^{\nu\alpha}b_\alpha}{b^2}\,g_{2L}^{\perp} + \epsilon_T^{\nu\alpha}S_{T,\alpha}M f_{2T} + \epsilon_T^{\nu\alpha}S_T^{\beta}\left(\frac{g_{T,\alpha\beta}}{2} - \frac{b_\alpha b_\beta}{b^2}\right)M f_{2T}^{\perp}$$

4 Independent Distributions

- Also 4 independent twist-2 TMDPDFs (f_1 , f_{1T} , g_1 , g_{1T}) and 2 independent jets (J_1 , J_2)

Jet production in SIDIS at NLP

- After contracting with leptonic tensor we obtain the most general **form factors**

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}\phi_J\,\mathrm{d}\phi_S\,\mathrm{d}\boldsymbol{k}^2} = \frac{\alpha_{em}^2}{8Q^2}\Bigg\{\frac{2-2y+y^2}{y}F_{UU,T}+\frac{2(2-y)\sqrt{1-y}}{y}\cos\phi_J\,F_{UU}^{\cos\phi_J}+\lambda_e 2\sqrt{1-y}\sin\phi_J\,F_{LU}^{\sin\phi_J}$$

$$+S_\parallel\frac{2(2-y)\sqrt{1-y}}{y}\sin\phi_J\,F_{UL}^{\sin\phi_J}+\lambda_e S_\parallel\left[(2-y)\,F_{LL}+2\sqrt{1-y}\cos\phi_J\,F_{LL}^{\cos\phi_J}\right]$$

$$+|S_\perp|\left[\frac{2-2y+y^2}{y}\sin(\phi_J-\phi_S)\,F_{UT,T}^{\sin(\phi_J-\phi_S)}+\frac{2(2-y)\sqrt{1-y}}{y}\left(\sin(\phi_S)\,F_{UT}^{\sin(\phi_S)}+\sin(2\phi_J-\phi_S)\,F_{UT}^{\sin(2\phi_J-\phi_S)}\right)\right]$$

$$+\lambda_e|S_\perp|\left[(2-y)\cos(\phi_J-\phi_S)\,F_{LT}^{\cos(\phi_J-\phi_S)}+2\sqrt{1-y}\left(\cos(\phi_S)\,F_{LT}^{\cos(\phi_S)}+\cos(2\phi_J-\phi_S)\,F_{LT}^{\cos(2\phi_J-\phi_S)}\right)\right]\Bigg\}$$

Jet production in SIDIS at NLP

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$$\frac{\mathrm{d}\sigma}{\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}\phi_J\,\mathrm{d}\phi_S\,\mathrm{d}\boldsymbol{k}^2} = \frac{\alpha_{em}^2}{8Q^2}$$

$$S_{\parallel}\frac{2(2-y)\sqrt{1-y}}{y}\sin\phi_J \boxed{F_{UL}^{\sin\phi_J}} \Rightarrow F_{UL}^{\sin\phi_J} = -\frac{2|\boldsymbol{k}|}{Q}\mathrm{Im}\Bigg(\sum_{q,\bar{q}}\int \mathrm{d}\xi\, H_2\left(x,\xi,Q^2\right)\int_0^\infty \mathrm{d}|\boldsymbol{b}|^2\,\frac{J_1(|\boldsymbol{b}||\boldsymbol{k}|)}{4\pi|\boldsymbol{b}||\boldsymbol{k}|}$$

$$\times\left\{J_1\left(\boldsymbol{b}^2;\bar{\zeta}\right)g_{2L}^{\perp,q}\left(x,\xi,\boldsymbol{b}^2;\zeta\right)+J_2\left(\xi,\boldsymbol{b}^2;\bar{\zeta}\right)g_1^q\left(x,\boldsymbol{b}^2;\zeta\right)\right\}\Bigg)$$

Jet production in SIDIS at NLP

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$$\times \left\{ J_1(\boldsymbol{b}^2;\bar{\zeta}) g_{2L}^{\perp,q}(x,\xi,\boldsymbol{b}^2;\zeta) + J_2(\xi,\boldsymbol{b}^2;\bar{\zeta}) g_1^q(x,\boldsymbol{b}^2;\zeta) \right\} \Bigg)$$

$$\Rightarrow \quad d\Sigma|_{\sin(\phi_J)} = \int_0^{2\pi} d\phi_J \frac{\sin(\phi_J)}{\pi} \int_0^{2\pi} d\phi_S \int_{0.01}^{0.95} dy \frac{d\sigma}{dx\, dy\, d\phi_J\, d\phi_S\, d\boldsymbol{k}^2}\Bigg|_{\lambda_e,|S_\perp|=0}$$

Jet production in SIDIS at NLP

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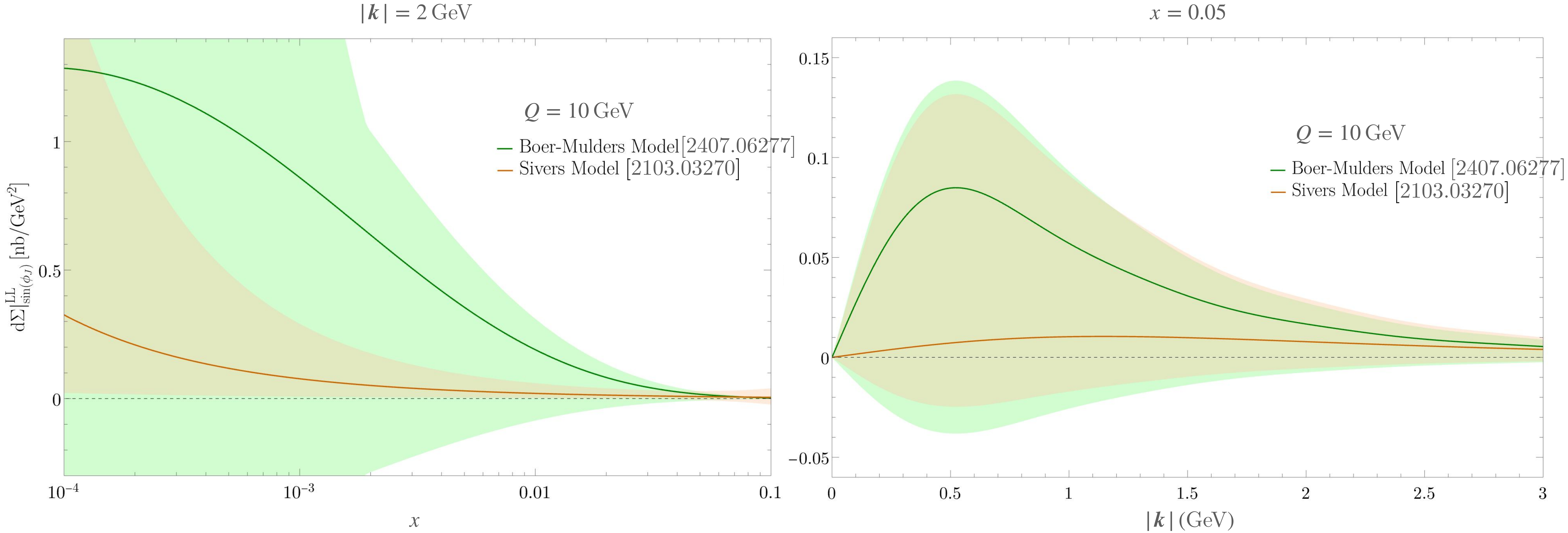
$$\times\Big\{J_1(\boldsymbol{b}^2;\bar{\zeta})g_{2L}^{\perp,q}(x,\xi,\boldsymbol{b}^2;\zeta)+J_2(\xi,\boldsymbol{b}^2;\bar{\zeta})g_1^q(x,\boldsymbol{b}^2;\zeta)\Big\}\Bigg)$$

$$\Rightarrow\; d\Sigma|_{\sin(\phi_J)} = \frac{\pi\alpha_{em}^2}{2Q^2} S_\parallel \int_{0.01}^{0.95} dy \frac{(2-y)\sqrt{1-y}}{y} F_{UL}^{\sin\phi_J}$$

Phenomenological Results for $F_{UL}^{\sin\phi_J}$

- Plot $\mathrm{d}\Sigma|_{\sin(\phi_J)}$ at leading logarithm accuracy using different TMD models for $g_{2L}^{\perp,q}$ (only contribution at this order)



Conclusions

- Construction of an effective current operator at next-to-leading power
  - Modified version of the BFM including a background field for the soft mode
- Hadronic tensor at NLP
  - Definitions for *physical* TMDs (free of rapidity and endpoint divergences)
  - Comparison with two previous approaches (further investigation needed)
- Most general form factors for the cross-section of a single jet production in SIDIS at NLP
  - Phenomenological results for purely NLP asymmetry $F_{UL}^{\sin\phi_J}$

Thank you for your attention!