Higher-twist corrections to exclusive vector meson production at high energies

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[Boussarie, M.F., Szymanowski, Wallon. PRL 134 (2025) 4]

[Boussarie, M.F., Szymanowski, Wallon. PRD 111 (2025) 1]

[Boussarie, Delle Rose, M.F., Papa, Szymanowski, Wallon ...]

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NATIONAL CENTRE FOR NUCLEAR RESEARCH ŚWIERK

NARODOWA AGENCJA WYMIANY AKADEMICKIEJ

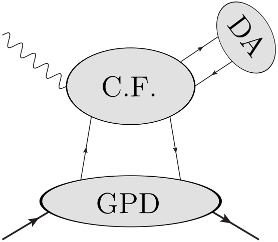
THE ULAM PROGRAMME

Deeply virtual meson production (DVMP)

- Exclusive vector meson leptoproduction

  $\gamma^{(*)}(p_\gamma)+P(p_0)\to\rho(p_\rho)+P(p_0')$

- Extensively studied at HERA



- Collinear factorization proven at all order for

$$\gamma^{(*)}(p_\gamma)+P(p_0)\to\rho_L(p_\rho)+P(p_0')$$

[Collins, Frankfurt, Strikman (1997)]
[Radyushkin (1997)]

- NLO corrections in Collinear Factorization

[D.Yu. Ivanov, L. Szymanowski, G. Krasnikov (2004)]

- NLO corrections to the production of a longitudinally polarized ρ-meson at small-x

[Ivanov, Kotsky, Papa (2004)]

[Boussarie, Grabovsky, Ivanov, Szymanowski, Wallon (2017)]

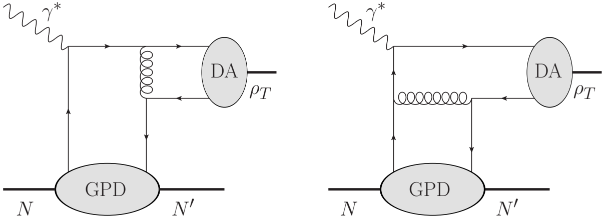
[Mäntysaari, Pentalla (2022)]

Transversely polarized ρ-meson production

- The leading DA (twist 2) of ρ_T is **chiral odd** ($\sigma^{\mu\nu}$ coupling)
- The amplitude for $\gamma^* N \to \rho_T N'$ is zero to all order in perturbation theory at the leading twist

[Diehl, Gousset, Pire (1999)] [Collins, Diehl (2000)]

- Lowest order diagrammatic argument:



$$\gamma^\alpha[\gamma^\mu, \gamma^\nu]\gamma_\alpha = 0$$

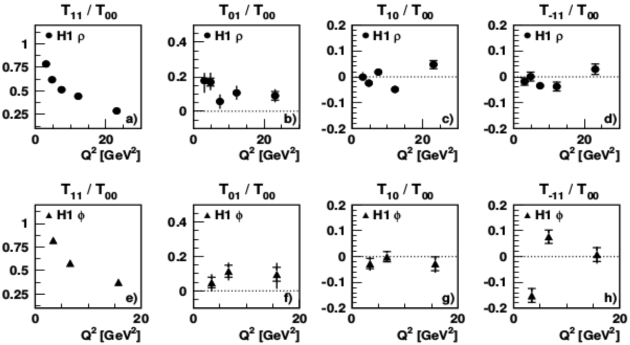
- Transversally polarized vector meson production start at the **twist-3**
- Collinear treatment at the twist-3 leads to **end point singularities**

[Mankiewicz, Piller (2000)] [Anikin, Teryaev (2002)]

Transversely polarized vector meson production

- HERA data for the ρ and ϕ meson **[F.D. Aaron et al. (2010)]**

$$\gamma^*(\lambda_\gamma)p \to V(\lambda_V)p \qquad \lambda_\gamma = 0, 1, -1 \quad \text{and} \quad \lambda_V = 0, 1, -1$$



"Leading twist description brought the knowledge that hadrons are made of quarks and gluons; if we want to learn *how are they made*, we have to understand higher-twist effects" [David Politzer]

Possible solutions

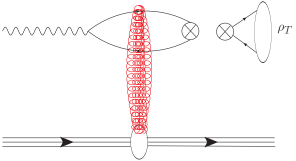
- To solve this issues one should go **beyond collinear factorization**
- One possibility is allowing for a relative transverse momentum between the outgoing quark and anti-quark **[Goloskokov, Kroll (2007)]**

  Distribution amplitude $\phi(x) \longrightarrow$ light-cone wavefunction $\phi(x, \vec{k}_T)$
- Alternative: moving to ***t*-channel transverse momentum factorization** ($\vec{k}_T$-factorization)

  [Anikin, Ivanov, B. Pire, L. Szymanowski, S. Wallon (2010)]

- Shockwave formalism

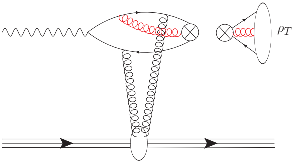
  [Balitsky (1996-2001)]



- It is an all-twist factorization in the t-channel

- Higher-twist collinear factorization

  [Ball, Braun, Koike, Tanaka (1998)]



- Systematic inclusion of higher-twist effects in the s-channel

- Most general description of the DVMP at high-energy

[Boussarie, M.F. , Szymanowski, Wallon (2025)]
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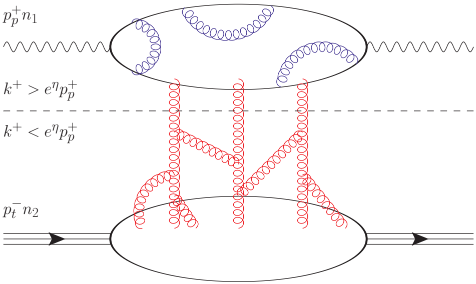
Shockwave approach

- High-energy approximation $s = (p_p + p_t)^2 \gg \{Q^2\}$



$$p_p = p_p^+ n_1 - \frac{Q^2}{2p_p^+} n_2$$

$$p_t = \frac{m_t^2}{2p_t^-} n_1 + p_t^- n_2$$

$$p_p^+ \sim p_t^- \sim \sqrt{\frac{s}{2}}$$

$$n_1^2 = n_2^2 = 0 \quad n_1 \cdot n_2 = 1$$

- Separation of the gluonic field into “fast" (quantum) part and “slow" (classical) part through a rapidity parameter $\eta < 0$
  [McLerran and Venugopalan (1994)] [Balitsky (1996-2001)]

$$\mathcal{A}^\mu(k^+, k^-, \vec{k}) = A^\mu(k^+ > e^\eta p_p^+, k^-, \vec{k}) + b^\mu(k^+ < e^\eta p_p^+, k^-, \vec{k}) \qquad e^\eta \ll 1$$

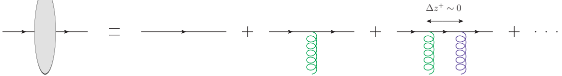
- Large longitudinal Boost: $\Lambda = \sqrt{(1+\beta)/(1-\beta)} \sim \sqrt{s}$

$$b^\mu(x^+, x^-, \vec{x}) = \delta(x^+)\mathbf{B}(\vec{x})n_2^\mu + \mathcal{O}\left(\Lambda^{-1}\right) \quad \leftarrow \text{ \textbf{Shockwave} approximation}$$

Shockwave approach

- Multiple interactions with the target $\rightarrow$ ***path-ordered Wilson lines***

$$V^{\eta}_{\vec{z}_i} = \mathcal{P}\exp\left[ig\int_{-\infty}^{+\infty} dz_i^+ b_\eta^-\left(z_i^+, \vec{z}_i\right)\right]$$



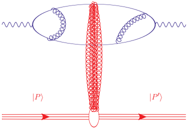
$$V_{\vec{z}_i} = 1 + ig\int_{-\infty}^{+\infty} dz_i^+ b_\eta^-\left(z_i^+, \vec{z}_i\right) + (ig)^2\int_{-\infty}^{+\infty} dz_i^+ dz_j^+ b_\eta^-\left(z_i^+, \vec{z}_i\right) b_\eta^-\left(z_j^+, \vec{z}_i\right)\theta\left(z_{ij}^+\right) + \cdots$$

- Factorization in the Shockwave approximation

$$\mathcal{M}^{\eta} = N_c\int d^d z_{1\perp} d^d z_{2\perp}\,\Phi^{\eta}(z_{1\perp}, z_{2\perp})\left\langle P'\left|\left[\frac{1}{N_c}\mathrm{Tr}\left(V^{\eta}_{\vec{z}_1} V^{\eta\dagger}_{\vec{z}_2}\right) - 1\right](\vec{z}_1, \vec{z}_2)\right| P\right\rangle$$

- ***Dipole operator***

$$\mathcal{U}^{\eta}_{ij} = \frac{1}{N_c}\mathrm{Tr}\left(V^{\eta}_{\vec{z}_i} V^{\eta\dagger}_{\vec{z}_j}\right) - 1$$



- Renormalization group equation of $\mathcal{U}^{\eta}_{ij}$ $\rightarrow$ **B-JIMWLK (BK) equations**

Theoretical framework

- **Effective background field operators**

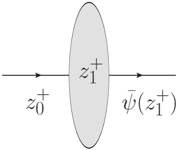
[Boussarie, M.F., Szymanowski, Wallon (2025)]

$$\left[\psi_{\text{eff}}\ (z_0)\right]_{z_0^+<0} = \psi\left(z_0\right) - \int \mathrm{d}^D z_2 G_0\left(z_{02}\right)\left(V^\dagger_{\boldsymbol{z}_2} - 1\right)\gamma^+\psi\left(z_2\right)\delta(z_2^+)$$

$$\left[\bar{\psi}_{\text{eff}}\ (z_0)\right]_{z_0^+<0} = \bar{\psi}\left(z_0\right) + \int \mathrm{d}^D z_1 \bar{\psi}\left(z_1\right)\gamma^+\left(V_{\boldsymbol{z}_1} - 1\right)G_0\left(z_{10}\right)\delta(z_1^+)$$

$$\left[A^{\mu a}_{\text{eff}}\ (z_0)\right]_{z_0^+<0} = A^{\mu a}\left(z_0\right) + 2i\int \mathrm{d}^D z_3 \delta(z_3^+) F^b_{-\sigma}\left(z_3\right) G^{\mu\sigma_\perp}\left(z_{30}\right)\left(U^{ab}_{\boldsymbol{z}_3} - \delta^{ab}\right)$$

- Example: Antiquark effective operator



  - A fermionic line starts at the light-cone time $z_0^+ < 0$
  - Freely propagates to $z_1^+ = 0$
  - It interacts eikonally at z_1^+ with the background shockwave field

Theoretical framework

- Such operators serve to construct amplitudes involving non-perturbative matrix elements of general off light-cone correlators, i.e. **without any reference to the twist-expansion**
- Proof by induction
- Shockwave effective Feynman rules are easily reproduced

$$\left[v_{\alpha}^{ij}\left(p_{\bar{q}}, z_0\right)\right]_{z_0^+<0} \equiv \left[\psi_{\mathrm{eff},\alpha}^{j}\left(z_0\right)\right]_{z_0^+<0} |i, p_{\bar{q}}\rangle = -\frac{(-i)^{d/2}}{2(2\pi)^{d/2}} \left(\frac{p_{\bar{q}}^+}{-z_0^+}\right)^{d/2} \theta(p_{\bar{q}}^+)\theta(-z_0^+)$$

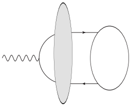
$$\times \int d^d z_2 V_{\vec{z}_2}^{ij\dagger} \frac{-z_0^+\gamma^- + \hat{z}_{20\perp}}{-z_0^+}\gamma^+ \frac{v(p_{\bar{q}})}{\sqrt{2p_{\bar{q}}^+}} \exp\left\{ip_{\bar{q}}^+\left(z_0^- - \frac{\vec{z}_{20}^{\,2}}{2z_0^+} + i0\right) - i\vec{p}_{\bar{q}}\cdot\vec{z}_{20}\right\}$$

$$G_{ij}\left(z_2, z_0\right)\big|_{z_2^+>0>z_0^+} \equiv \psi_i(z_2)\left[\bar{\psi}_{\mathrm{eff},j}\left(z_0\right)\right]_{z_0^+<0}$$

$$= \frac{i\Gamma(d+1)}{4(2\pi)^{d+1}} \int d^d\vec{z}_1 V_{ij}\left(\vec{z}_1\right) \frac{\left(z_2^+\gamma^- + \hat{z}_{21\perp}\right)\gamma^+\left(-z_0^+\gamma^- + \hat{z}_{10\perp}\right)}{\left(-z_0^+z_2^+\right)^{\frac{D}{2}}\left(-z_{20}^- + \frac{\vec{z}_{21}^2}{2z_2^+} - \frac{\vec{z}_{10}^2}{2z_0^+} + i\varepsilon\right)^{d+1}}\theta(z_2^+)\theta(-z_0^+)$$

ρ_T-meson production: diagrams

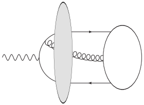
- **Two-body contribution**



i. Dependence of the leading Fock state wave function – with a minimal number of (valence) partons – on **transverse momentum**

$$\mathcal{A}_2 = -ie_q \int \mathrm{d}^D z_0 \theta(-z_0^+) \left\langle P\left(p'\right) M\left(p_M\right) \left| \overline{\psi}_{\mathrm{eff}}\left(z_0\right) \hat{\varepsilon}_q \mathrm{e}^{-i(q\cdot z_0)} \psi_{\mathrm{eff}}\left(z_0\right) \right| P\left(p\right) \right\rangle$$

- **Three-body contribution**



i. Distributions with a **non-minimal parton configuration**

$$\mathcal{A}_{3,q} = -ie_q \int \mathrm{d}^D z_4 \mathrm{d}^D z_0 \theta(-z_4^+) \theta(-z_0^+)$$

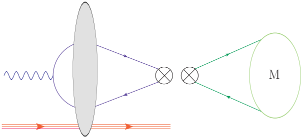
$$\times \left\langle P\left(p'\right) M\left(p_M\right) \left| \overline{\psi}_{\mathrm{eff}}\left(z_4\right) \gamma_\mu A^{\mu a}_{\mathrm{eff}}(z_4) i g t^a G(z_{40}) \hat{\varepsilon}_q \mathrm{e}^{-i(q\cdot z_0)} \psi_{\mathrm{eff}}\left(z_0\right) \right| P\left(p\right) \right\rangle$$

- Diagrams where the gluon is attached to the external quark legs are absent in the coefficient functions of the Operator Product Expansion (OPE)

[Politzer (1980)]

ρ_T-meson production: factorization

- **Two-body contribution**

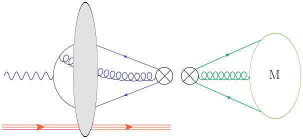


$$\mathcal{A}_2 = ie_q \int \mathrm{d}^D z_0 \int \mathrm{d}^D z_1 \int \mathrm{d}^D z_2 \theta(-z_0^+)\delta(z_1^+)\delta(z_2^+)\left\langle M\left(p_M\right)\left|\overline{\psi}\left(z_1\right)\Gamma^\lambda \psi\left(z_2\right)\right|0\right\rangle$$

$$\times\left\langle P\left(p'\right)\left|1-\frac{1}{N_c}\mathrm{tr}\left(V_{\boldsymbol{z}_1}V_{\boldsymbol{z}_2}^\dagger\right)\right|P\left(p\right)\right\rangle \frac{1}{4}\mathrm{tr}_D\left[\gamma^+ G_0\left(z_{10}\right)\hat{\varepsilon}_q \mathrm{e}^{-i(q\cdot z_0)}G_0\left(z_{02}\right)\gamma^+\Gamma_\lambda\right]$$

Hard part

- **Three-body contribution**



$$\mathcal{A}_{q3} = -ie_q \int \mathrm{d}^D z_4 \mathrm{d}^D z_3 \mathrm{d}^D z_2 \mathrm{d}^D z_1 \mathrm{d}^D z_0 \theta(-z_4^+)\delta(z_3^+)\delta(z_2^+)\delta(z_1^+)\theta\left(-z_0^+\right)\mathrm{e}^{-i(q\cdot z_0)}$$

$$\times\left\langle M\left(p_M\right)\left|\overline{\psi}\left(z_1\right)\Gamma^\lambda g F_{-\sigma}\left(z_3\right)\psi\left(z_2\right)\right|0\right\rangle\left\langle P\left(p'\right)\left|\mathrm{tr}\left(V_{\boldsymbol{z}_1}t^a V_{\boldsymbol{z}_2}^\dagger t^b U_{\boldsymbol{z}_3}^{ab}\right)\right|P\left(p\right)\right\rangle$$

$$\times\frac{1}{N_c^2-1}\mathrm{tr}_D\left[\gamma^+ G_0\left(z_{14}\right)\gamma_\mu G^{\mu\sigma_\perp}\left(z_{34}\right)G_0\left(z_{40}\right)\hat{\varepsilon}_q G_0\left(z_{02}\right)\gamma^+\Gamma_\lambda\right] - \text{n.i.}$$

Results: two-body contribution

- **Dipole amplitude**

$$\mathcal{A}_2 = \int_0^1 \mathrm{d}x \int \mathrm{d}^2\boldsymbol{r}\Psi\left(x,\boldsymbol{r}\right) \int \mathrm{d}^d\boldsymbol{b}\, \mathrm{e}^{i(\boldsymbol{q}-\boldsymbol{p}_M)\cdot\boldsymbol{b}} \left\langle P\left(p'\right) \left| 1 - \frac{1}{N_c}\mathrm{tr}\left(V_{\boldsymbol{b}+\bar{x}\boldsymbol{r}} V^\dagger_{\boldsymbol{b}-x\boldsymbol{r}}\right) \right| P\left(p\right) \right\rangle$$

- **Coordinate-space impact factor**

$$\Psi_2\left(x,\boldsymbol{r}\right) = e_q \delta \left(1 - \frac{p_M^+}{q^+}\right)\left(\varepsilon_{q\mu} - \frac{\varepsilon_q^+}{q^+} q_\mu\right)$$

$$\times\left[\phi_{\gamma^+}\left(x,\boldsymbol{r}\right)\left(2x\bar{x}q^\mu - i(x-\bar{x})\frac{\partial}{\partial r_{\perp\mu}}\right) + \epsilon^{\mu\nu+-}\phi_{\gamma^+\gamma^5}\left(x,\boldsymbol{r}\right)\frac{\partial}{\partial r_\perp^\nu}\right] K_0\left(\sqrt{x\bar{x}Q^2\boldsymbol{r}^2}\right)$$

- Two-body vacuum to meson matrix elements

$$\phi_{\gamma^+}\left(x,\boldsymbol{r}\right) = \frac{1}{2\pi}\int_{-\infty}^{\infty} dr^- e^{ixp_M^+ r^-} \left\langle M\left(p_M\right)\left|\overline{\psi}\left(r\right)\gamma^+\psi\left(0\right)\right| 0\right\rangle_{r^+=0}$$

$$\phi_{\gamma^+\gamma^5}\left(x,\boldsymbol{r}\right) = \frac{1}{2\pi}\int_{-\infty}^{\infty} dr^- e^{ixp_M^+ r^-} \left\langle M\left(p_M\right)\left|\overline{\psi}\left(r\right)\gamma^+\gamma^5\psi\left(0\right)\right| 0\right\rangle_{r^+=0}$$

At this stage, r^2 is arbitrary, in principle off the light-cone.

Results: three-body contribution

- Three-body amplitude: involves **dipole** and **double dipole** contributions

$$\mathcal{A}_3 = \left(\prod_{i=1}^{3}\int dx_i\theta(x_i)\right)\delta(1-x_1-x_2-x_3)\int d^2\boldsymbol{z}_1 d^2\boldsymbol{z}_2 d^2\boldsymbol{z}_3 e^{i\boldsymbol{q}(x_1\boldsymbol{z}_1+x_2\boldsymbol{z}_2+x_3\boldsymbol{z}_3)}$$

$$\times\Psi_3\left(x_1,x_2,x_3,\boldsymbol{z}_1,\boldsymbol{z}_2,\boldsymbol{z}_3\right)\left\langle P\left(p'\right)\middle|\mathcal{U}_{\boldsymbol{z}_1\boldsymbol{z}_3}\mathcal{U}_{\boldsymbol{z}_3\boldsymbol{z}_2}-\mathcal{U}_{\boldsymbol{z}_1\boldsymbol{z}_3}-\mathcal{U}_{\boldsymbol{z}_3\boldsymbol{z}_2}+\frac{1}{N_c^2}\mathcal{U}_{\boldsymbol{z}_1\boldsymbol{z}_2}\middle|P\left(p\right)\right\rangle$$

- **Coordinate-space impact factor** (with $Z=\sqrt{x_1x_2\boldsymbol{z}_{12}^2+x_2x_3\boldsymbol{z}_{23}^2+x_1x_3\boldsymbol{z}_{31}^2}$)

$$\Psi_3\left(x_1,x_2,x_3,\boldsymbol{z}_1,\boldsymbol{z}_2,\boldsymbol{z}_3\right)=\frac{e_q q^+}{2(4\pi)}\frac{N_c^2}{N_c^2-1}\left(\varepsilon_{q\rho}-\frac{\varepsilon_q^+}{q^+}q_\rho\right)$$

$$\times\left\{\chi_{\gamma^+\sigma}\left[\left(4ig_{\perp\perp}^{\rho\sigma}\frac{x_1x_2}{1-x_2}\frac{Q}{Z}K_1(QZ)+T_1^{\sigma\rho\nu}(x_1,x_2,x_3)\frac{z_{23\perp\nu}}{\boldsymbol{z}_{23}^2}K_0(QZ)\right)-(1\leftrightarrow 2)\right]\right.$$

$$\left.-\chi_{\gamma^+\gamma^5\sigma}\left[\left(4\epsilon^{\sigma\rho+-}\frac{x_1x_2}{1-x_2}\frac{Q}{Z}K_1(QZ)+T_2^{\sigma\rho\nu}(x_1,x_2,x_3)\frac{z_{23\perp\nu}}{\boldsymbol{z}_{23}^2}K_0(QZ)\right)+(1\leftrightarrow 2)\right]\right\}$$

- Three-body vacuum to meson matrix elements

$$\chi_{\Gamma^\lambda,\sigma}\equiv\chi_{\Gamma^\lambda,\sigma}(x_1,x_2,x_3,\boldsymbol{z}_1,\boldsymbol{z}_2,\boldsymbol{z}_3)=$$

$$\int_{-\infty}^{\infty}\frac{\mathrm{d}z_1^-}{2\pi}\frac{\mathrm{d}z_2^-}{2\pi}\frac{\mathrm{d}z_3^-}{2\pi}\mathrm{e}^{-ix_1q^+z_1^--ix_2q^+z_2^--ix_3q^+z_3^-}\left\langle M\left(p_M\right)\middle|\overline{\psi}\left(z_1\right)\Gamma^\lambda gF_{-\sigma}\left(z_3\right)\psi\left(z_2\right)\middle|0\right\rangle_{z_{1,2,3}^+=0}$$

At this stage, $z_{13}^2, z_{32}^2, z_{21}^2$ are arbitrary, in principle off the light-cone.

Covariant collinear factorization

- Covariant collinear factorization is based on the non-local operator product expansion (OPE)

[Balitsky, Braun (1989)]

  - Expansion in powers of the hard scale = expansion of **string operators** in powers of deviation from the light-cone
  - Each coefficient of this OPE expansion = finite sum of on-light-cone **non-local operator**

- For each term in this Taylor expansion: vacuum-to-meson matrix elements contribute to different kinematic twist
- Up to twist 3: only the first term in the Taylor expansion of the off-light-cone matrix elements survives (next one is r^2 suppressed, i.e. twist 4)
- Example: parametrization of the 2-body vector matrix element

$$\left\langle M(p_M)\left|\overline{\psi}\,(r)\,\gamma^{\mu}[r,0]\psi\,(0)\right|0\right\rangle\Big|_{r^2=0}$$

$$\sim f_M m_M \int_0^1 \mathrm{d}x e^{ix(p_M\cdot r)}\left[\underbrace{p_M^{\mu}\frac{(\varepsilon_M^*\cdot r)}{(p_M\cdot r)}\phi_{\parallel}(x)}_{\substack{\sim Q \\ \text{twist 2}}}+\underbrace{\varepsilon_{M,T}^{*\mu}\,g_{\perp}^{(v)}(x)}_{\substack{\sim 1 \\ \text{twist 3}}}-\underbrace{\frac{1}{2}r^{\mu}\frac{(\varepsilon_M^*\cdot r)}{(p_M\cdot r)^2}m_\rho^2 g_3(x)}_{\substack{\sim 1/Q \\ \text{twist 4}}}\right]$$

Covariant collinear factorization

- **Covariant collinear factorization**

[Braun, Filyanov (1990)]
[Ball, Braun, Koike, Tanaka (1998)]

  i. Minimal basis of *independent distributions* (twist-3 collinear DAs)
  ii. *Minimal numbers of non-perturbative parameters*
  iii. Easy to perform the calculation directly in coordinate space

- 2- and 3-body operators in gauge invariant form, on the light-cone $z^2 = 0$

$$\boxed{\begin{gathered}\langle M(p_M)|\overline{\psi}(z)\Gamma_\lambda\,[z,0]\,\psi(0)|0\rangle \\ \langle M(p_M)|\overline{\psi}(z)\gamma_\lambda\,[z,tz]\,gF^{\mu\nu}(tz)\,[tz,0]\,\psi(0)|0\rangle \\ \langle M(p_M)|\overline{\psi}(z)\gamma_\lambda\gamma^5\,[z,tz]\,g\tilde{F}^{\mu\nu}(tz)\,[tz,0]\,\psi(0)|0\rangle\end{gathered}}$$

where

$$[z,0] = \mathcal{P}_{\exp}\left[ig\int_0^1 dt A^\mu\,(tz)\,z_\mu\right]$$

- Gauge invariant matrix elements can be related to the non-gauge invariant one within a fixed twist

$$\left\langle M\,(p_M)\left|\overline{\psi}\,(r)\,[r,0]\Gamma^\lambda\psi\,(0)\right|0\right\rangle_{r^+=0} \longleftrightarrow \left\langle M\,(p_M)\left|\overline{\psi}\,(r)\,\Gamma^\lambda\psi\,(0)\right|0\right\rangle_{r^+=0}$$

Twist-expanded results: two-body contribution

- Twist-expanded results: two-body contribution

$$\Psi_2(x,\boldsymbol{r}) = e_q m_M f_M \delta \left(1 - \frac{p_M^+}{q^+}\right)\left(\varepsilon_{q\mu} - \frac{\varepsilon_q^+}{q^+} q_\mu\right)\left(\varepsilon^*_{M\alpha} - \frac{\varepsilon_M^{*+}}{p_M^+} p_{M\alpha}\right)$$

$$\times \left[-i r_\perp^\alpha (h(x) - \tilde{h}(x))\left(2x\bar{x}q^\mu + (x-\bar{x})\frac{-i\partial}{\partial r_{\perp\mu}}\right) + \right.$$

$$\left. \epsilon^{\mu\nu+-}\epsilon^{+\alpha-\delta} r_{\perp\delta}\left(\frac{g_\perp^{(a)}(x) - \tilde{g}_\perp^{(a)}(x)}{4}\right)\frac{\partial}{\partial r_\perp^\nu}\right] K_0\left(\sqrt{x\bar{x}Q^2\boldsymbol{r}^2}\right)$$

- Collinear ingredients:

$$h(x) = \text{kinematic twist-3 vector DA}$$

$$g_\perp^{(a)}(u) = \text{kinematic twist-3 axial DA}$$

- Collinear ingredients: gauge link related terms

$$\widetilde{h}(x) = \frac{f_{3M}^V}{f_M}\int_0^x \mathrm{d}x_q \int_0^{1-x} \mathrm{d}x_{\overline{q}} \, \frac{V(x_q, x_{\overline{q}})}{(1 - x_q - x_{\overline{q}})^2}$$

$$\widetilde{g}_\perp^{(a)}(x) = 4\frac{f_{3M}^A}{f_M}\int_0^x \mathrm{d}x_q \int_0^{1-x} \mathrm{d}x_{\overline{q}} \, \frac{A(x_q, x_{\overline{q}})}{(1 - x_q - x_{\overline{q}} + i\epsilon)^2}$$

f_M = vector decay constant constant

Dilute regime

- **Reggeon** definition **[Caron-Huot (2013)]** $\quad R^a(\boldsymbol{z}) \equiv \dfrac{f^{abc}}{gC_A} \ln\left(U^{bc}_{\boldsymbol{z}}\right)$
- Expansion of the *Wilson line* in Reggeized gluons

$$V_{\boldsymbol{z}_1} = 1 + ig\boldsymbol{t}^a R^a\left(\boldsymbol{z}_1\right) - \frac{1}{2}g^2 \boldsymbol{t}^a \boldsymbol{t}^b R^a\left(\boldsymbol{z}_1\right) R^b\left(\boldsymbol{z}_1\right) + O\left(g^3\right)$$

- **BFKL $\boldsymbol{k}_{\mathrm{T}}$-factorization**

$$\mathcal{A}_2^{\text{dilute}} = \frac{g^2}{4N_c}(2\pi)^d \delta^d(\boldsymbol{q} - \boldsymbol{p}_M - \boldsymbol{\Delta}) \int \frac{\mathrm{d}^d \ell}{(2\pi)^d} \mathcal{U}(\ell) \int_0^1 \mathrm{d}x$$
$$\times \underbrace{\left[\Phi_2\left(x, \boldsymbol{\ell} - \frac{x - \bar{x}}{2}\boldsymbol{\Delta}\right) + \Phi_2\left(x, -\boldsymbol{\ell} - \frac{x - \bar{x}}{2}\boldsymbol{\Delta}\right) - \Phi_2(x, \bar{x}\boldsymbol{\Delta}) - \Phi_2(x, -x\boldsymbol{\Delta})\right]}_{\Phi_{2,\mathrm{BFKL}}(x, \boldsymbol{l}, \boldsymbol{\Delta})}$$

- $\mathcal{U}(\boldsymbol{l}) \to k_T$-unintegrated gluon density (UGD) in the BFKL sense

$$\mathcal{U}(\boldsymbol{\ell}) \equiv \int \mathrm{d}^d \boldsymbol{v} \mathrm{e}^{-i(\boldsymbol{\ell}\cdot\boldsymbol{v})} \left\langle P\left(p'\right) \middle| R^a\left(\frac{\boldsymbol{v}}{2}\right) R^a\left(-\frac{\boldsymbol{v}}{2}\right) \middle| P(p) \right\rangle$$

- The 3-body BFKL impact factor is a combination of 12 BK impact factors
  (dilute, $\boldsymbol{\Delta} = 0$, $T \to T$) **[Anikin, Ivanov, Pire, Szymanowski, Wallon (2009)]**
  - **Momentum space** BFKL calculation → *Reggeized gluons*
  - Twist-expansion → *Light-Cone Collinear Factorization*
    [Ellis, Furmanwski, Petronzio (1982)]

Spin-density matrix

- Expansion around the forward direction ($\boldsymbol{\Delta} = 0$)

$$A^{L,L} \sim \frac{1}{Q} \qquad A^{T,T}|_{\text{no flip}} \sim \frac{1}{Q}\frac{m_M}{Q} \qquad A^{T,L} \sim \frac{1}{Q}\frac{|\boldsymbol{\Delta}|}{Q}$$

$$A^{L,T} \sim \frac{1}{Q}\frac{m_M}{Q}\frac{|\boldsymbol{\Delta}|}{Q} \qquad A^{T,T}|_{\text{flip}} \sim \frac{1}{Q}\frac{m_M}{Q}\frac{|\boldsymbol{\Delta}|^2}{Q^2}$$

- Hierarchy observed at HERA **[F. Aaron et al., H1 coll. (2009)]**

$$A^{L,L} \gg A^{T,T}|_{\text{no flip}} \gg A^{T,L} \gg A^{L,T}, A^{T,T}|_{\text{flip}}$$

- Initial condition $\rightarrow$ **MV model** **[McLerran, Venugopalan (1994)]**

$$\mathcal{U}^{\eta=0}(\boldsymbol{r}) = 1 - \exp^{-\frac{\boldsymbol{r}^2 Q_0^2}{4}\ln\left(\frac{1}{|\boldsymbol{r}|\Lambda}+e\right)}$$

- Numerical LO BK evolution

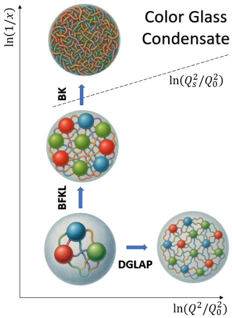
$$\frac{\partial \mathcal{U}_{12}^{\eta}}{\partial \eta} = \frac{\alpha_s N_c}{2\pi^2}\int d^2\vec{z}_3 \left(\frac{\vec{z}_{12}^{\,2}}{\vec{z}_{23}^{\,2}\vec{z}_{31}^{\,2}}\right)\left[\underbrace{\mathcal{U}_{13}^{\eta} + \mathcal{U}_{32}^{\eta} - \mathcal{U}_{12}^{\eta}}_{\text{BFKL}} - \delta\, \mathcal{U}_{13}^{\eta}\mathcal{U}_{32}^{\eta}\right]$$

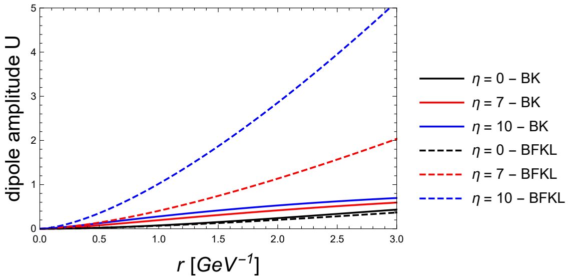
- $\delta = 1$ for BK and $\delta = 0$ to investigate the dilute (BFKL) regime
- We also implement NLO running coupling effects

  [Ducloue, Iancu, Mueller, Soyez, Triantafyllopoulos (2019)]

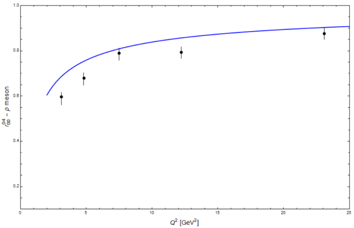
Phenomenological perspectives

- Non-linear vs BFKL evolution





- $r_{00}^{04} \simeq \sigma_L/(\sigma_L + \sigma_T)$
- $W = 100$ GeV
- Dipole approximation



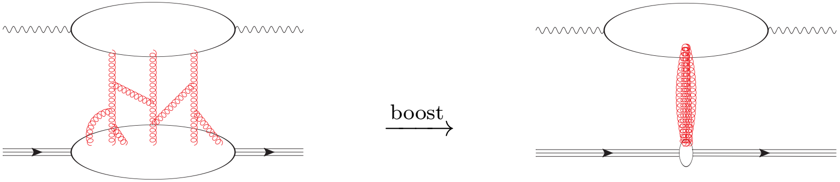
Thanks for your attention

Backup

Shockwave approach

- Large longitudinal boost: $\Lambda = \sqrt{\frac{1+\beta}{1-\beta}} \sim \frac{\sqrt{s}}{m_t}$

$$\begin{cases} b^+(x^+, x^-, \vec{x}) & = \Lambda^{-1} b_0^+(\Lambda x^+, \Lambda^{-1} x^-, \vec{x}) \\ b^-(x^+, x^-, \vec{x}) & = \Lambda b_0^-(\Lambda x^+, \Lambda^{-1} x^-, \vec{x}) \\ b^i(x^+, x^-, \vec{x}) & = b_0^i(\Lambda x^+, \Lambda^{-1} x^-, \vec{x}) \end{cases}$$



$b_0^\mu(x)$

$b^\mu(x^+, x^-, \vec{x}) = \delta(x^+)\mathbf{B}(\vec{x}) n_2^\mu + \mathcal{O}\left(\Lambda^{-1}\right)$

Shockwave approximation

- Light-cone gauge $A \cdot n_2 = 0$

$$A \cdot b = 0 \implies \textit{Simple effective Lagrangian}$$

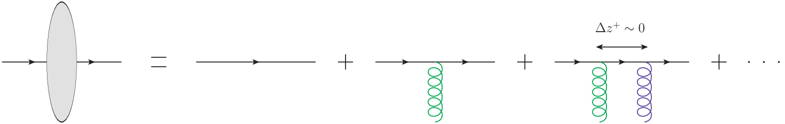
Shockwave approach

- Interactions with the simple shockwave field

  i. Independence from x^- $\implies$ conservation of p^+ (**eikonal** approx.)

  ii. $\delta(x^+)$ $\implies$ interactions at a **single transverse coordinate**.

- Quark line through the shockwave



$$V_{\vec{z}_i} = 1 + ig\int_{-\infty}^{+\infty} dz_i^+ b_\eta^- \left(z_i^+, \vec{z}_i\right) + (ig)^2 \int_{-\infty}^{+\infty} dz_i^+ dz_j^+ b_\eta^- \left(z_i^+, \vec{z}_i\right) b_\eta^- \left(z_j^+, \vec{z}_i\right) \theta\left(z_{ij}^+\right) + \cdots$$

- Multiple interactions with the target $\rightarrow$ ***path-ordered Wilson lines***

$$V_{\vec{z}}^\eta = \mathcal{P}\exp\left[ig\int_{-\infty}^{+\infty} dz_i^+ b_\eta^- \left(z_i^+, \vec{z}\right)\right]$$

Balitsky-JIMWLK evolution equations

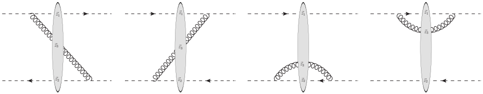
- ***Balitsky-JIMWLK evolution equations*** for the dipole
  [Balitsky — Jalilian-Marian, Iancu, McLerran, Weigert, Kovner, Leonidov]

$$\frac{\partial \mathcal{U}_{12}^{\eta}}{\partial \eta} = \frac{\alpha_s N_c}{2\pi^2} \int d^2\vec{z}_3 \left(\frac{\vec{z}_{12}^{\,2}}{\vec{z}_{23}^{\,2}\vec{z}_{31}^{\,2}} \right) \left[\underbrace{\mathcal{U}_{13}^{\eta} + \mathcal{U}_{32}^{\eta} - \mathcal{U}_{12}^{\eta}}_{\text{BFKL}} - \mathcal{U}_{13}^{\eta}\mathcal{U}_{32}^{\eta} \right]$$

$$\frac{\partial \mathcal{U}_{13}^{\eta}\mathcal{U}_{32}^{\eta}}{\partial \eta} = \cdots \qquad \longleftarrow \quad \textit{Balitsky hierarchy}$$

$$\vdots$$

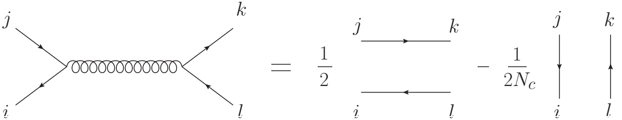
- Double dipole contribution and Dipole contribution



- Dipole contribution

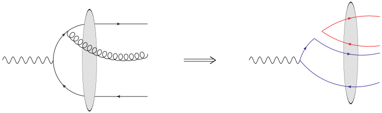
Balitsky-Kovchegov evolution equation

- Large-N_c limit **[G. 't Hooft (1974)]**



$$\boxed{t^a_{ij} t^a_{kl} = \frac{1}{2}\left(\delta_{il}\delta_{jk} - \frac{1}{N_c}\delta_{ij}\delta_{kl}\right)}$$

- Double dipole $\rightarrow$ Dipole $\times$ dipole



$$\langle \mathcal{U}^{\eta}_{13}\mathcal{U}^{\eta}_{32}\rangle \rightarrow \langle \mathcal{U}^{\eta}_{13}\rangle\,\langle \mathcal{U}^{\eta}_{32}\rangle$$

- Hierarchy of equations broken $\rightarrow$ closed non-linear **BK-equation**
[I. I. Balitsky (1995)] [Y. V. Kovchegov (1999)]

$$\frac{\partial \langle \mathcal{U}^{\eta}_{12}\rangle}{\partial \eta} = \frac{\alpha_s N_c}{2\pi^2}\int d^2\vec{z}_3 \left(\frac{\vec{z}^{\,2}_{12}}{\vec{z}^{\,2}_{23}\vec{z}^{\,2}_{31}}\right)\left[\langle \mathcal{U}^{\eta}_{13}\rangle + \langle \mathcal{U}^{\eta}_{32}\rangle - \langle \mathcal{U}^{\eta}_{12}\rangle - \langle \mathcal{U}^{\eta}_{13}\rangle\langle \mathcal{U}^{\eta}_{32}\rangle\right]$$

with $\langle \mathcal{U}^{\eta}_{12}\rangle \equiv \langle P'|\mathcal{U}^{\eta}_{12}|P\rangle$

Covariant collinear factorization

- Before twist expansion, our result does not contain gauge links between fields
- Three-body matrix element

$$\left\langle M(p)\left|\overline{\psi}\left(z_q\right)\gamma^{\lambda}gF^{\mu\nu}\left(z_g\right)\psi\left(z_{\overline{q}}\right)\right|0\right\rangle$$
$$\simeq\left\langle M(p)\left|\overline{\psi}(z_q)[z_q,z_g]\gamma^{\lambda}gF^{\mu\nu}(z_g)[z_g,z_{\overline{q}}]\psi(z_{\overline{q}})\right|0\right\rangle_{z_i^+=0}+\text{twist four}$$

- Gauge link effects do not contribute to the 3-body result within twist 3
- Two-body matrix element

$$\left\langle M\left(p_M\right)\left|\overline{\psi}\left(r\right)[r,0]\Gamma^{\lambda}\psi\left(0\right)\right|0\right\rangle_{r^+=0}\longleftrightarrow\left\langle M\left(p_M\right)\left|\overline{\psi}\left(r\right)\Gamma^{\lambda}\psi\left(0\right)\right|0\right\rangle_{r^+=0}$$

- Expansion of the gauge link

$$[z,0]=\mathcal{P}\exp\left[ig\int_0^1 dtA^{\mu}(tz)z_{\mu}\right]\simeq 1+ig\int_0^1 dtA^{\mu}\left(tz\right)z_{\mu}+\text{higher twist}$$

- In a given n light-cone gauge

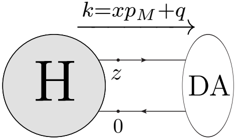
$$A^{\mu}(z)=\int_0^{\infty}\mathrm{d}\sigma\,\mathrm{e}^{-\epsilon\sigma}n_{\nu}F^{\mu\nu}(z+\sigma n)$$

- Gauge link effect does contribute to the 2-body twist-3 result

Light-cone collinear factorization

- Most general two-body amplitude

$$\mathcal{A}_2 = \int \frac{d^4k}{(2\pi)^4} \int d^4z e^{-ik\cdot z} \langle M(p_M)|\overline{\psi}_\alpha^{\,i}(z)\,\psi_\beta^{\,j}(0)\,|0\rangle H_{2,\alpha\beta}^{ij}$$



- The separation is choosen as $z^\mu = \lambda n_2^\mu \implies$ no Wilson line in the n_2 **light-cone gauge**
- Sudakov decomposition: $k = k^+ n_1 + q = xp_M + q$ and small q expansion
- Two-body amplitude after Fierz decomposition

$$\mathcal{A}_2 = \frac{1}{4N_c} p_M^+ \int \frac{\mathrm{d}x}{2\pi} \int \frac{\mathrm{d}q^-}{2\pi} \int \frac{\mathrm{d}^d \boldsymbol{q}}{(2\pi)^d} \int \mathrm{d}^D z\, \mathrm{e}^{-ixp_M^+ z^- - iq^- z^+ + i(\boldsymbol{q}\cdot\boldsymbol{z})}$$
$$\times \left\langle M\left(p_M\right) \middle| \overline{\psi}\left(z\right)\Gamma_\lambda \psi\left(0\right) \middle| 0 \right\rangle \mathrm{tr}\left[H_2\left(xp_M + q\right)\Gamma^\lambda\right]$$

Light-cone collinear factorization

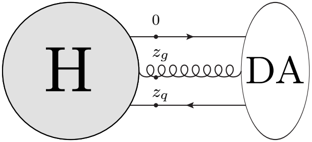
- **Taylor expansion** of the hard part

$$H_2\left(xp_M+q\right)=H_2\left(xp_M\right)+q_{\perp\mu}\left[\frac{\partial}{\partial q_{\perp\mu}}H_2\left(xp_M+q\right)\right]_{k=xp_M}+\text{h.t.}$$

- Two-body amplitude factorized form up to the **twist-3**

$$\mathcal{A}_2=\frac{1}{4N_c}\int \mathrm{d}x\, p_M^+\int\frac{\mathrm{d}z^-}{2\pi}\mathrm{e}^{-ixp_M^+z^-}\left\{\left\langle M\left(p\right)\left|\overline{\psi}\left(z^-\right)\Gamma_\lambda\psi\left(0\right)\right|0\right\rangle \mathrm{tr}\left[H_2\left(xp_M\right)\Gamma^\lambda\right]\right.$$
$$\left.+i\left\langle M\left(p_M\right)\left|\overline{\psi}\left(z^-\right)\overleftrightarrow{\partial}_{\perp\mu}\Gamma_\lambda\psi\left(0\right)\right|0\right\rangle \mathrm{tr}\left[\partial_\perp^\mu H_2\left(xp_M\right)\Gamma^\lambda\right]\right\}$$

- *Gauge invariance* is broken $\Longrightarrow$ need to include a **3-body contribution**



- Three-body contribution factorized

$$\mathcal{A}_3=\frac{1}{2(N_c^2-1)}\int \mathrm{d}x_q\mathrm{d}x_g\left(p_M^+\right)^2\int\frac{\mathrm{d}z_q^-}{2\pi}\frac{\mathrm{d}z_g^-}{2\pi}\mathrm{e}^{-ix_qp_M^+z_q^- -ix_gp_M^+z_g^-}$$
$$\times\left\langle M\left(p\right)\left|\overline{\psi}\left(z_q^-\right)\Gamma_\lambda gA_\mu\left(z_g^-\right)\psi\left(0\right)\right|0\right\rangle \mathrm{tr}\left[t^bH_3^{\mu,b}\left(x_qp_M,x_gp_M\right)\Gamma^\lambda\right]$$

Light-cone collinear factorization

- *Light-cone collinear factorization*

  [Ellis, Furmanwski, Petronzio (1982)] [Anikin, Teryaev (2002)]
- Factorization around the dominant **light-cone direction** is naturally implemented in **momentum space**
- **Overcomplete set of distributions** must be reduced exploiting QCD equations of motion

$$\langle i(\hat{D}(0)\psi(0))_\alpha \bar{\psi}_\beta(z)\rangle = 0 \qquad \langle i\psi_\alpha(0)(\bar{\psi}(z)\overleftarrow{\hat{D}}(z))_\beta\rangle = 0$$

- Invariance of the amplitude under **rotation on the light-cone**

  [Anikin, Ivanov, Pire, Szymanowski, Wallon (2009)]

  i. Independence of the amplitude from the choice of n

  ii. Given a “natural" choice n_0, we can define

$$n^\mu = \alpha p^\mu + \beta n_0^\mu + n_\perp^\mu$$

  iii. Imposing $p \cdot n = 1$ and $n^2 = 0 \longrightarrow \beta = 1\,, \alpha = -n_\perp^2/2$

  iiii. The freedom is parametrized in terms of the transverse component

$$\frac{\partial \mathcal{A}}{\partial n_\perp^\mu} = 0$$

- Twist-expanded results: three-body contribution

$$
\begin{aligned}
&\Psi_3\left(x_1, x_2, x_3, \boldsymbol{z}_1, \boldsymbol{z}_2, \boldsymbol{z}_3\right) \\
&= \frac{e_q m_M c_f}{8\pi} \delta \left(1 - \frac{p_M^+}{q^+}\right) \left(\varepsilon_{q\rho} - \frac{\varepsilon_q^+}{q^+} q_\rho\right) \left(\varepsilon_M^{*\mu} - \frac{p_M^\mu}{p_M^+} \varepsilon_M^{*+}\right) \left(\prod_{j=1}^{3} \theta(x_j)\theta(1-x_j) e^{-i x_j \boldsymbol{p}_M \boldsymbol{z}_j}\right) \\
&\times \Bigg\{ - i f_{3M}^V g_{\sigma\mu} V(x_1, x_2) \left[\left(4 i g_\perp^{\rho\sigma} \frac{x_1 x_2}{1 - x_2} \frac{Q}{Z} K_1(QZ) + T_1^{\sigma\rho\nu}(\{x_i\}) \frac{z_{23\perp\nu}}{\boldsymbol{z}_{23}^2} K_0(QZ)\right) - (1 \leftrightarrow 2)\right] \\
&- \epsilon_{-+\sigma\beta} f_{3M}^A g_{\perp\mu}^\beta A(x_1, x_2) \left[\left(4 \epsilon^{\sigma\rho+-} \frac{x_1 x_2}{1 - x_2} \frac{Q}{Z} K_1(QZ) + T_2^{\sigma\rho\nu}(\{x_i\}) \frac{z_{23\perp\nu}}{\boldsymbol{z}_{23}^2} K_0(QZ)\right) + (1 \leftrightarrow 2)\right]\Bigg\}
\end{aligned}
$$

- Collinear ingredients:

$$V(x_1, x_2) = \text{genuine twist-3 vector DA}$$

$$A(x_1, x_2) = \text{genuine twist-3 axial DA}$$

$$f_{3M}^V \text{ and } f_{3M}^A = \text{renormalization constant}$$

Explicit two-body term in the dilute and $\Delta = 0$ limit

- **BK impact factor**

$$\Phi_{2,\boldsymbol{\Delta}=0}\left(x,\boldsymbol{l}\right) = 2\pi m_M f_M e_q \delta(1 - p_M^+/q^+)$$
$$\times \left[\frac{2\boldsymbol{l}^2}{\left[\boldsymbol{l}^2 + x\bar{x}Q^2\right]^2} T_{\text{f.}}\, \phi_{2,\text{f.}}(x) - \frac{x\bar{x}Q^2}{\left[\boldsymbol{l}^2 + x\bar{x}Q^2\right]^2} T_{\text{n.f.}}\, \phi_{2,\text{n.f.}}(x)\right]$$

- Helicity structures and DAs combinations

$$T_{\text{n.f.}} = \boldsymbol{\varepsilon}_q \cdot \boldsymbol{\varepsilon}_M^* \qquad \phi_{2,\text{n.f.}}(x) = (2x-1)(h(x) - \tilde{h}(x)) + \frac{g_\perp^{(a)}(x) - \tilde{g}_\perp^{(a)}(x)}{4}$$

$$T_{\text{f.}} = \frac{(\boldsymbol{\varepsilon}_q \cdot \boldsymbol{l})(\boldsymbol{\varepsilon}_M^* \cdot \boldsymbol{l})}{\boldsymbol{l}^2} - \frac{\boldsymbol{\varepsilon}_q \cdot \boldsymbol{\varepsilon}_M^*}{2} \qquad \phi_{2,\text{f.}}(x) = (2x-1)(h(x) - \tilde{h}(x)) - \frac{g_\perp^{(a)}(x) - \tilde{g}_\perp^{(a)}(x)}{4}$$

- **Forward limit matching**

$$\Phi_{2,\boldsymbol{\Delta}=0}^{\text{BFKL}}\left(x,\boldsymbol{l}\right) = 2\left(\Phi_{2,\boldsymbol{\Delta}=0}\left(x,\boldsymbol{l}\right) - \Phi_{2,\boldsymbol{\Delta}=0}\left(x,\boldsymbol{0}\right)\right)$$

- **BFKL impact factor**

$$\Phi_{2,\boldsymbol{\Delta}=0}^{\text{BFKL}}\left(x,\boldsymbol{l}\right) = 4\pi m_M f_M e_q \delta(1 - p_M^+/q^+)$$
$$\times \left[\frac{2\boldsymbol{l}^2}{\left[\boldsymbol{l}^2 + x\bar{x}Q^2\right]^2} T_{\text{f.}}\, \phi_{\text{f.}}(x) + \frac{\boldsymbol{l}^2(\boldsymbol{l}^2 + 2x\bar{x}Q^2)}{x\bar{x}Q^2\left[\boldsymbol{l}^2 + x\bar{x}Q^2\right]^2} T_{\text{n.f.}}\, \phi_{\text{n.f.}}(x)\right]$$

- From BFKL to BK $\longrightarrow$ subtraction of the $|\boldsymbol{l}| \to \infty$ limit

Dilute regime: three-body contribution

- Linearization in the three-body case → combination of more impact factors

$$\mathcal{A}_3^{\text{dilute}} = \left(\prod_{i=1}^{3}\int dx_i\theta(x_i)\right)\delta(1-x_1-x_2-x_3)\frac{-g^2}{4N_c}\int\frac{d^d\boldsymbol{l}}{(2\pi)^d}\mathcal{U}(\ell)$$

$$\Big\{\Phi_3'\left(\{x\},\bar{x}_1\boldsymbol{\Delta},-x_2\boldsymbol{\Delta},-x_3\boldsymbol{\Delta}\right)-\Phi_3'\left(\{x\},\left(\frac{1-2x_1}{2}\right)\boldsymbol{\Delta}-\boldsymbol{l},-x_2\boldsymbol{\Delta},\left(\frac{1-2x_3}{2}\right)\boldsymbol{\Delta}+\boldsymbol{l}\right)$$

$$-\Phi_3'\left(\{x\},\left(\frac{1-2x_1}{2}\right)\boldsymbol{\Delta}+\boldsymbol{l},-x_2\boldsymbol{\Delta},\left(\frac{1-2x_3}{2}\right)\boldsymbol{\Delta}-\boldsymbol{l}\right)+\Phi_3'\left(\{x\},-x_1\boldsymbol{\Delta},-x_2\boldsymbol{\Delta},\bar{x}_3\boldsymbol{\Delta}\right)$$

$$+\Phi_3'\left(\{x\},-x_1\boldsymbol{\Delta},-x_2\boldsymbol{\Delta},\bar{x}_3\boldsymbol{\Delta}\right)-\Phi_3'\left(\{x\},-x_1\boldsymbol{\Delta},\left(\frac{1-2x_2}{2}\right)\boldsymbol{\Delta}+\boldsymbol{l},\left(\frac{1-2x_3}{2}\right)\boldsymbol{\Delta}-\boldsymbol{l}\right)$$

$$-\Phi_3'\left(\{x\},-x_1\boldsymbol{\Delta},\left(\frac{1-2x_2}{2}\right)\boldsymbol{\Delta}-\boldsymbol{l},\left(\frac{1-2x_3}{2}\right)\boldsymbol{\Delta}+\boldsymbol{l}\right)+\Phi_3'\left(\{x\},-x_1\boldsymbol{\Delta},\bar{x}_2\boldsymbol{\Delta},-x_3\boldsymbol{\Delta}\right)$$

$$-\frac{1}{N_c^2}\Big[\Phi_3'\left(\{x\},\bar{x}_1\boldsymbol{\Delta},-x_2\boldsymbol{\Delta},-x_3\boldsymbol{\Delta}\right)-\Phi_3'\left(\{x\},\left(\frac{1-2x_1}{2}\right)\boldsymbol{\Delta}-\boldsymbol{l},\left(\frac{1-2x_2}{2}\right)\boldsymbol{\Delta}+\boldsymbol{l},-x_3\boldsymbol{\Delta}\right)$$

$$-\Phi_3'\left(\{x\},\left(\frac{1-2x_1}{2}\right)\boldsymbol{\Delta}+\boldsymbol{l},\left(\frac{1-2x_2}{2}\right)\boldsymbol{\Delta}-\boldsymbol{l},-x_3\boldsymbol{\Delta}\right)+\Phi_3'\left(\{x\},-x_1\boldsymbol{\Delta},\bar{x}_2\boldsymbol{\Delta},-x_3\boldsymbol{\Delta}\right)\Big]\Big\}$$

where

$$\Phi_3'(\{x\},\{\boldsymbol{p}+x\boldsymbol{p}_M\})\equiv\Phi_3(\{x\},\{\boldsymbol{p}\})$$

Dilute regime: three-body contribution

- Fourier transform

$$\Phi_3\left(\{x\},\{\boldsymbol{p}\}\right)=\left(\prod_{j=1}^{3}\int d^2\boldsymbol{z}_j e^{-i\boldsymbol{z}_j\boldsymbol{p}_j}\right)\Psi_3\left(\{x\},\{\boldsymbol{z}\}\right)$$

- **Momentum space impact factor** (after twist expansion)

$$\Phi_3\left(\{x\},\{\boldsymbol{p}\}\right)=\frac{e_q m_M}{4}c_f\left(\varepsilon_{q\rho}-\frac{\varepsilon_q^+}{q^+}q_\rho\right)\left(\varepsilon_M^{*\beta}-\frac{p_M^\beta}{p_M^+}\varepsilon_M^{*+}\right)\delta\left(1-\frac{p_M^+}{q^+}\right)$$
$$\times\left(\prod_{j=1}^{3}\frac{\theta(1-x_j)\theta(x_j)}{x_j}\right)\frac{(2\pi)^3\delta^{(2)}\left(\sum_{i=1}^3\boldsymbol{p}_i+x_i\boldsymbol{p}_M\right)}{\left[Q^2+\sum_{i=1}^3(\boldsymbol{p}_i+x_i\boldsymbol{p}_M)^2/x_i\right]}\left\{g_{\beta\sigma}f_{3M}^V V(x_1,x_2)\left(4g_{\perp\perp}^{\rho\sigma}\frac{x_1x_2}{1-x_2}\right.\right.$$
$$\left.+\tilde{T}_1^{\sigma\rho\nu}(\{x\})\big|_{\boldsymbol{k}_i=-x_i\boldsymbol{p}_M}\frac{x_1x_2(p_3+x_3p_M)_{\perp\nu}-x_1x_3(p_2+x_2p_M)_{\perp\nu}}{(\boldsymbol{p}_1+x_1\boldsymbol{p}_M)^2+x_1(1-x_1)Q^2}\right)-\epsilon_{-+\sigma\beta}f_{3M}^A A(x_1,x_2)$$
$$\left.\times\left(4\frac{x_1x_2}{1-x_2}\epsilon^{\sigma\rho+-}+i\tilde{T}_2^{\sigma\rho\nu}(\{x\})\big|_{\boldsymbol{k}_i=-x_i\boldsymbol{p}_M}\frac{x_1x_2(p_3+x_3p_M)_{\perp\nu}-x_1x_3(p_2+x_2p_M)_{\perp\nu}}{(\boldsymbol{p}_1+x_1\boldsymbol{p}_M)^2+x_1(1-x_1)Q^2}\right)\right\}$$
$$+(1\leftrightarrow 2)$$

Explicit three-body term in the dilute and $\Delta = 0$ limit

- The 3-body BFKL impact factor is a combination of 12 BK impact factors

$$\Phi_3\left(\{x\},\{\boldsymbol{p}\}\right) = \left(\prod_{j=1}^{3}\int d^2\boldsymbol{z}_j e^{-i\boldsymbol{z}_j\boldsymbol{p}_j}\right)\Psi_3\left(\{x\},\{\boldsymbol{z}\}\right)$$

- Transverse to transverse transition in the **forward** and **dilute** limit

$$\begin{aligned}
\mathcal{A}_{3T,\boldsymbol{\Delta}=\mathbf{0}}^{\text{dilute}} &= e_q m_M \frac{g^2}{N_c}(2\pi)\delta\left(1-\frac{p_M^+}{q^+}\right)(2\pi)^2\delta^2\left(\boldsymbol{q}-\boldsymbol{p}_M\right)\int\frac{\mathrm{d}^d\boldsymbol{\ell}}{(2\pi)^d}\mathcal{U}(\boldsymbol{\ell}) \\
&\times\left(\prod_{i=1}^{3}\int_0^1\frac{dx_i}{x_i}\right)\frac{\delta(1-x_1-x_2-x_3)}{x_3}\frac{\boldsymbol{\ell}^2}{Q^2}\Big\{T_{\text{f.}}\left[f_{3M}^V V\left(x_1,x_2\right)-f_{3M}^A A\left(x_1,x_2\right)\right] \\
&\times 2x_1\left(\frac{x_3 c_f}{\boldsymbol{\ell}^2+\frac{x_2x_3}{x_2+x_3}Q^2}+\frac{x_3 c_f}{\boldsymbol{\ell}^2+\frac{x_1x_3}{x_1+x_3}Q^2}-\frac{\bar{x}_3\left(1-c_f\right)}{\boldsymbol{\ell}^2+\frac{x_1x_2}{x_1+x_2}Q^2}+\frac{x_2-\bar{x}_1c_f}{\boldsymbol{\ell}^2+x_1\bar{x}_1Q^2}+\frac{x_1-\bar{x}_2c_f}{\boldsymbol{\ell}^2+x_2\bar{x}_2Q^2}\right) \\
&-T_{\text{n.f.}}\left[f_{3M}^V V\left(x_1,x_2\right)+f_{3M}^A A\left(x_1,x_2\right)\right] \\
&\times\left(\frac{\left(1-c_f\right)x_1\bar{x}_3}{\bar{x}_3\boldsymbol{\ell}^2+x_1x_2Q^2}-\frac{c_f x_3^2}{\bar{x}_1\boldsymbol{\ell}^2+x_2x_3Q^2}-\frac{\left(x_2-\bar{x}_1c_f\right)x_1x_2}{\bar{x}_1\left(\boldsymbol{\ell}^2+x_1\bar{x}_1Q^2\right)}-\frac{\left(x_1-\bar{x}_2c_f\right)\bar{x}_2}{\left(\boldsymbol{\ell}^2+x_2\bar{x}_2Q^2\right)}\right)\Big\}
\end{aligned}$$

- **The forward and dilute limit matches the previous result**
  [Anikin, Ivanov, Pire, Szymanowski, Wallon (2009)]

BFKL approach + twist-expansion via **light-cone collinear factorization**

Higher-twist ρ-meson DAs

- Twist and chiral classification of the ρ-meson distribution amplitudes

[Ball, Braun, Koike, Tanaka (1998)]

- Two body distribution amplitudes

Twist	2	3	4
	$O(1)$	$O(1/Q)$	$O(1/Q^2)$
$e_{\parallel}$	$\phi_{\parallel}$	$h_{\parallel}^{(t)}$, $h_{\parallel}^{(s)}$	g_3
$e_{\perp}$	$\underline{\phi_{\perp}}$	$\overline{g_{\perp}^{(v)}}$, $\overline{g_{\perp}^{(a)}}$	$\underline{h_3}$

- $g_{\perp}^{(v)}$ and $g_{\perp}^{(a)}$ are the vector and axial two-body twist-3 DA

$$\langle M(p_M)|\overline{\psi}(z)\Gamma_\lambda\,[z,0]\,\psi(0)|0\rangle$$

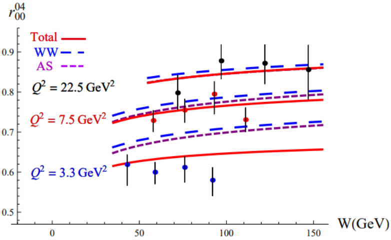
- Vector and axial three-body twist-3 DA

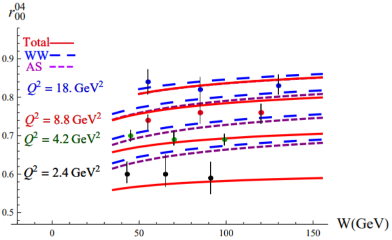
$$\langle M(p_M)|\overline{\psi}(z)\gamma_\lambda\,[z,tz]\,gF^{\mu\nu}(tz)\,[tz,0]\,\psi(0)|0\rangle$$

$$\langle M(p_M)|\overline{\psi}(z)\gamma_\lambda\,[z,tz]\,g\tilde{F}^{\mu\nu}(tz)\,[tz,0]\,\psi(0)|0\rangle$$

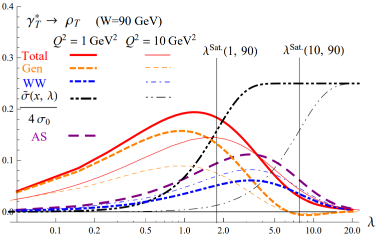
Spin density matrix element r_{00}^{04} and radial wavefunction

- $r_{00}^{04} \simeq \sigma_L/(\sigma_L + \sigma_T)$





- Rescaled radial wavefunction $P(\lambda = rQ, \mu_F(Q^2))/Q$



Result for the Wandzura-Wilczek part

- Two-body amplitude

$$\mathcal{A}_2^{\lambda_\gamma,\lambda_V} = (2\pi)^4 q^+ \delta^{(4)}(q + p_t - p_M - p_{t'}) \int d^2\boldsymbol{r}\, \mathcal{U}(\boldsymbol{r}) \int_0^1 dz\, e^{iz\boldsymbol{\Delta}\cdot\boldsymbol{r}}\, \psi_2^{\lambda_\gamma,\lambda_V}(z,\boldsymbol{r})$$

- Different transition amplitudes

$$\psi_2^{L,L}(z,\boldsymbol{r}) = \frac{e_q m_M f_M}{2\pi} 2z\bar{z}Q^2 K_0\left(\sqrt{z\bar{z}Q^2\boldsymbol{r}^2}\right)\left[\frac{\varepsilon_\gamma^+}{q^+}\frac{\varepsilon_M^{+*}}{p_M^+}\right]\varphi_\parallel(z)$$

$$\psi_2^{T,L}(z,\boldsymbol{r}) = \frac{e_q m_M f_M}{2\pi}(2z-1)\sqrt{\frac{z\bar{z}Q^2}{\boldsymbol{r}^2}} K_1\left(\sqrt{z\bar{z}Q^2\boldsymbol{r}^2}\right)\left[i\frac{\varepsilon_M^{+*}}{p_M^+}(\boldsymbol{r}\cdot\boldsymbol{\varepsilon}_\gamma)\right]\varphi_\parallel(z)$$

$$\psi_2^{L,T}(z,\boldsymbol{r}) = \frac{e_q m_M f_M}{2\pi} 2z\bar{z}Q^2 K_0\left(\sqrt{z\bar{z}Q^2\boldsymbol{r}^2}\right)\left[i\frac{\varepsilon_\gamma^+}{q^+}(\boldsymbol{r}\cdot\boldsymbol{\varepsilon}_M^*)\right]\varphi_1^T(z)$$

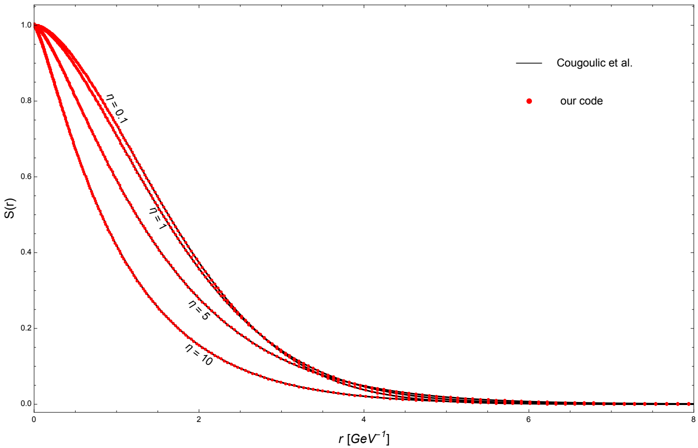
$$\psi_2^{T,T}(z,\boldsymbol{r}) = -\frac{e_q m_M f_M}{2\pi}\sqrt{z\bar{z}Q^2\boldsymbol{r}^2} K_1\left(\sqrt{z\bar{z}Q^2\boldsymbol{r}^2}\right)\left[T_{\text{f.}}(\boldsymbol{r})\,\phi_{2,\text{f.}}(z) + \frac{T_{\text{n.f.}}}{2}\,\phi_{2,\text{n.f.}}(z)\right]$$

- Helicity structures

$$T_{\text{f.}}(\boldsymbol{r}) = \frac{(\boldsymbol{\varepsilon}_\gamma\cdot\boldsymbol{r})\left(\boldsymbol{\varepsilon}_M^*\cdot\boldsymbol{r}\right)}{\boldsymbol{r}^2} - \frac{\boldsymbol{\varepsilon}_\gamma\cdot\boldsymbol{\varepsilon}_M^*}{2} \qquad T_{\text{n.f.}} = \boldsymbol{\varepsilon}_\gamma\cdot\boldsymbol{\varepsilon}_M^*$$

Numerical solution of running coupling BK

- BK evolution of $S(r)$



One-loop corrections at operator level

- One-loop corrections in the operator form ($\Gamma^0 = \{\gamma^0, \gamma^0\gamma^5\}$)

[Radyushkin (2018,2019)]

$$\delta\mathcal{O}^0\left(z_3\right) = -\frac{\alpha_s}{2\pi} C_F \int_0^1 du \int_0^{1-u} dv \bar{\psi}\left(uz\right) \Gamma^0 \psi\left(\bar{v}z\right)$$
$$\times \left\{ \left(\delta(v) \left[\frac{\bar{u}}{u}\right]_+ + \delta(u) \left[\frac{\bar{v}}{v}\right]_+ + 1 \right) \left(\ln\left(\frac{-z^2\mu^2}{4e^{-2\gamma_E - 1}} \right) + \frac{1}{\epsilon_{\mathrm{IR}}} \right) \right.$$
$$\left. +2 \left(\delta(v) \left[\frac{\ln u}{u}\right]_+ + \delta(u) \left[\frac{\ln v}{v}\right]_+ - 1 \right) + Z'\left(z^2\right) \delta(u)\delta(v) \right\}$$

$\downarrow$ What does this **single equation** contain?

- Its logarithmic dependence on z^2 reflects the UV-behavior of the operator
- **Renormalization group equations** of QCD distributions (**ERBL**, **DGLAP**, and **GPD evolution** equations) are determined by deriving with respect to $\ln(-z^2)$ **[Balitsky, Braun (1987)] [Radyushkin (2018,2019)]**
- One can get the **matching kernel** for quark **PDFs**, $\langle p|\mathcal{O}^0\left(z\right)|p\rangle$, **DAs**, $\langle M|\mathcal{O}^0\left(z\right)|0\rangle$, **GPDs**, $\langle p'|\mathcal{O}^0\left(z\right)|p\rangle$
- $Z'(z^2)$ collects the UV-singular part and the contributions of the quark self-energy diagram $\propto \delta(u)\,\delta(v)$