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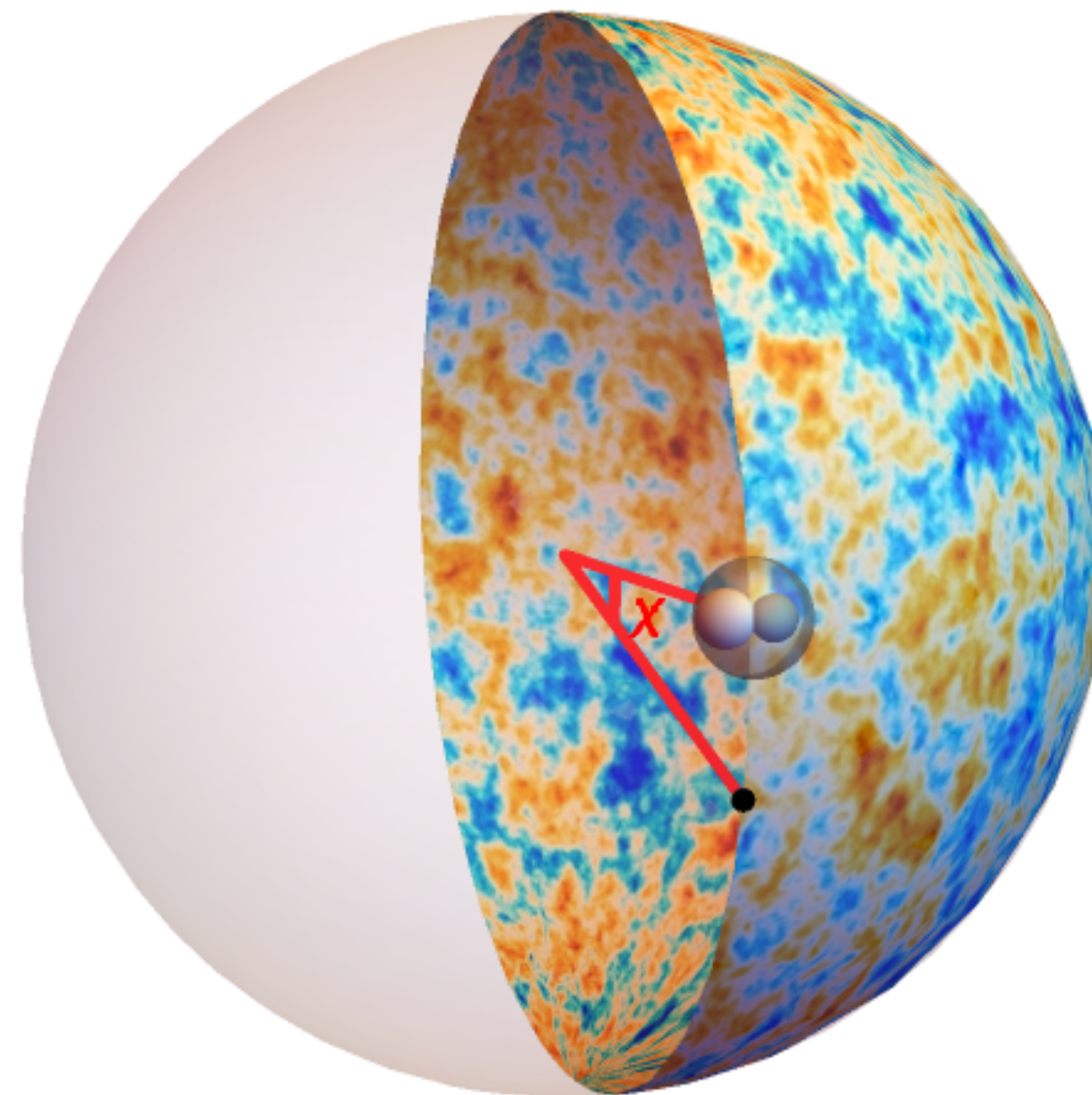
TMDs & Semi-Inclusive Energy Correlators

— Factorization, Resummation & High-Energy Evolution

Yü Jiao Zhu
REF@Milan 2025

Energy Correlators

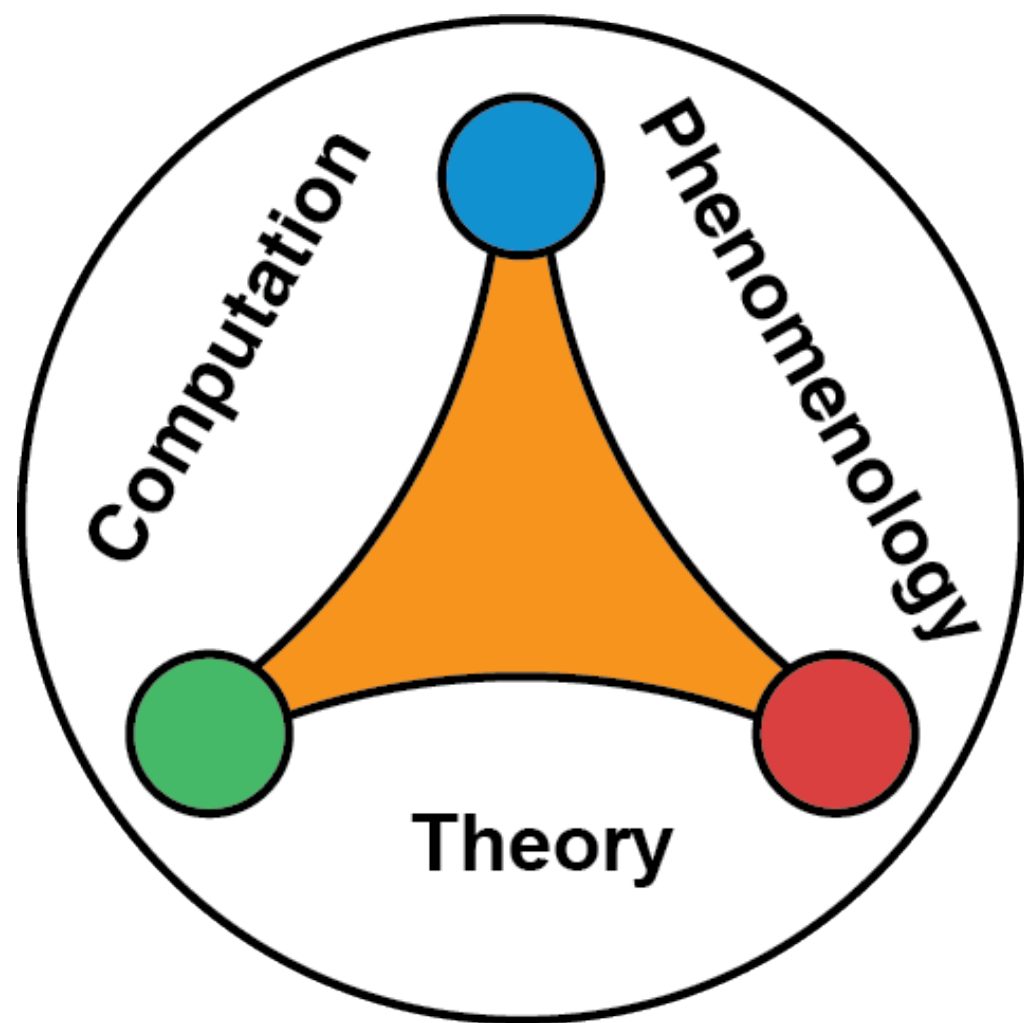
—mapping microscopic correlations among partons into macroscopic correlations among hadrons



Motivation: Understand the glue that binds us all

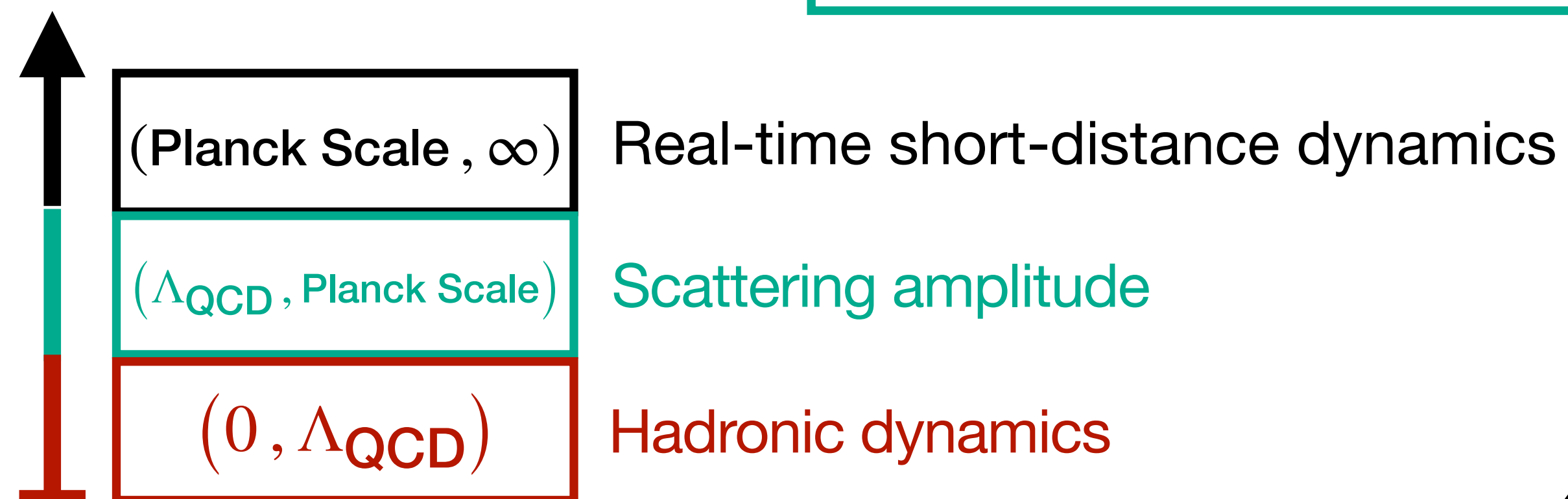
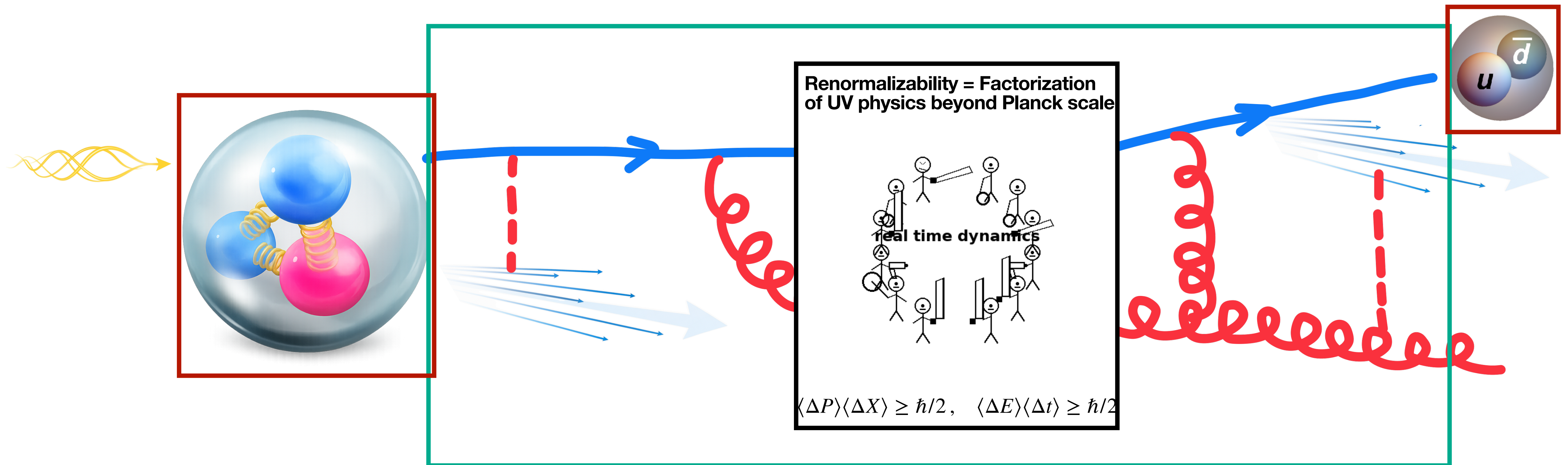
Physics Objectives

- Seeing confined motion
- Access hadronization
- Resolve spin structure
- BFKL and gluon saturation



	pQCD/Reggeon	Amplitudes/Correlators	Lattice
Theory			
Compute	<i>NNLO JET</i>	e-Collaboration	LPC
Pheno			

Parameters screened invisible physics, Physics theory are about **relations** between observables



the heart of factorization:
Observable = NP $\otimes$ P

The SIDIS Landscape

Kinematics of Current Region Fragmentation in Semi-Inclusive Deeply Inelastic Scattering

M. Boglione^a, J. Collins^b, L. Gamberg^c, J. O. Gonzalez-Hernandez^{d,e}, T. C. Rogers^{d,e}, N. Sato^e

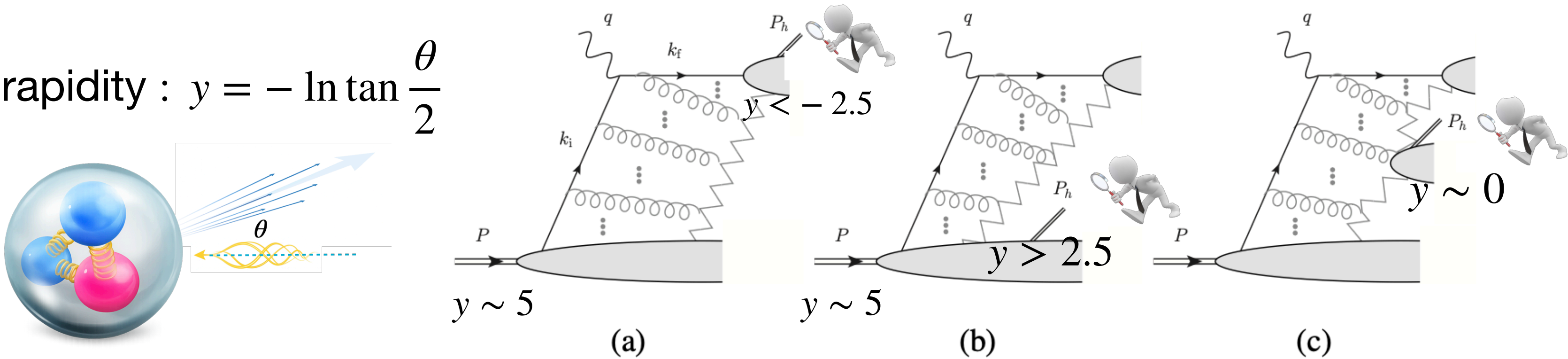


Figure 1: Lowest order SIDIS graphs corresponding to (a) the current region (b) the target region and (c) the central (soft) region. The faded zigzag lines represent non-perturbative and other interactions (e.g. hadronization) between the outgoing parton and the target jet.

Observable = NP ⊗ P

Ob	NP	P
DIS	PDFs	DIS coefficients
Spectator ECs	NECs	DIS coefficients
Back-to-back Jets	TMDs	Finite S matrix
Rapidity Gapped Jets	???	$C(\mu) = 1$

Finite S matrix as SCET Wilson coefficient

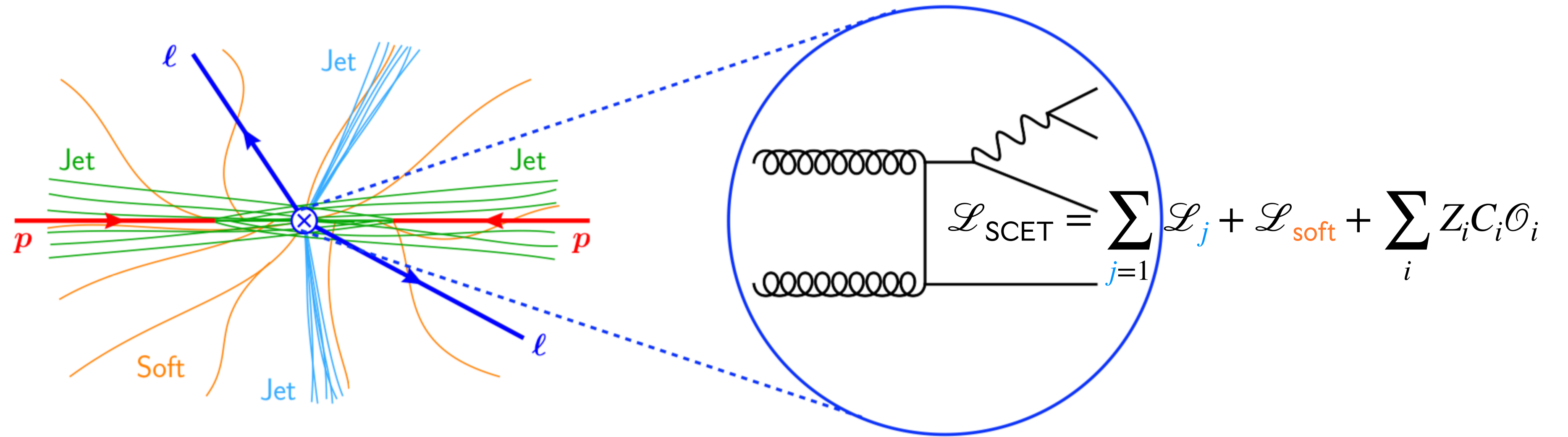


Figure 1 – Schematic LHC collision. The collinear (green and blue) and soft radiation (orange) are described dynamically in SCET. The hard scattering (zoomed in on the right) is encoded in matching coefficients.

EMPLOYING HELICITY AMPLITUDES FOR RESUMMATION IN SCET

Ian Moutl,¹ Iain W. Stewart,¹ Frank J. Tackmann,² Wouter J. Waalewijn,^{3,4 a}

¹*Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, MA 02139, USA*

²*Theory Group, Deutsches Elektronen-Synchrotron (DESY), D-22607 Hamburg, Germany*

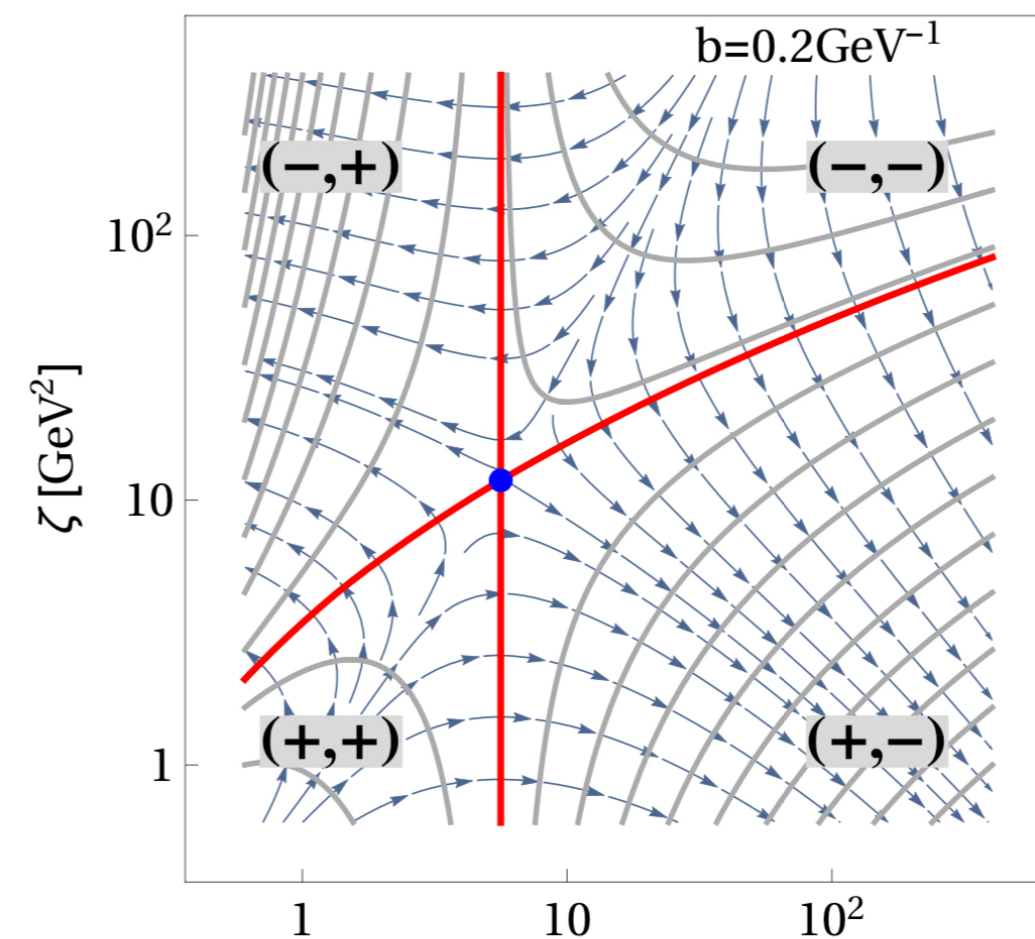
³*ITFA, University of Amsterdam, Science Park 904, 1018 XE, Amsterdam, The Netherlands*

⁴*Nikhef, Theory Group, Science Park 105, 1098 XG, Amsterdam, The Netherlands*

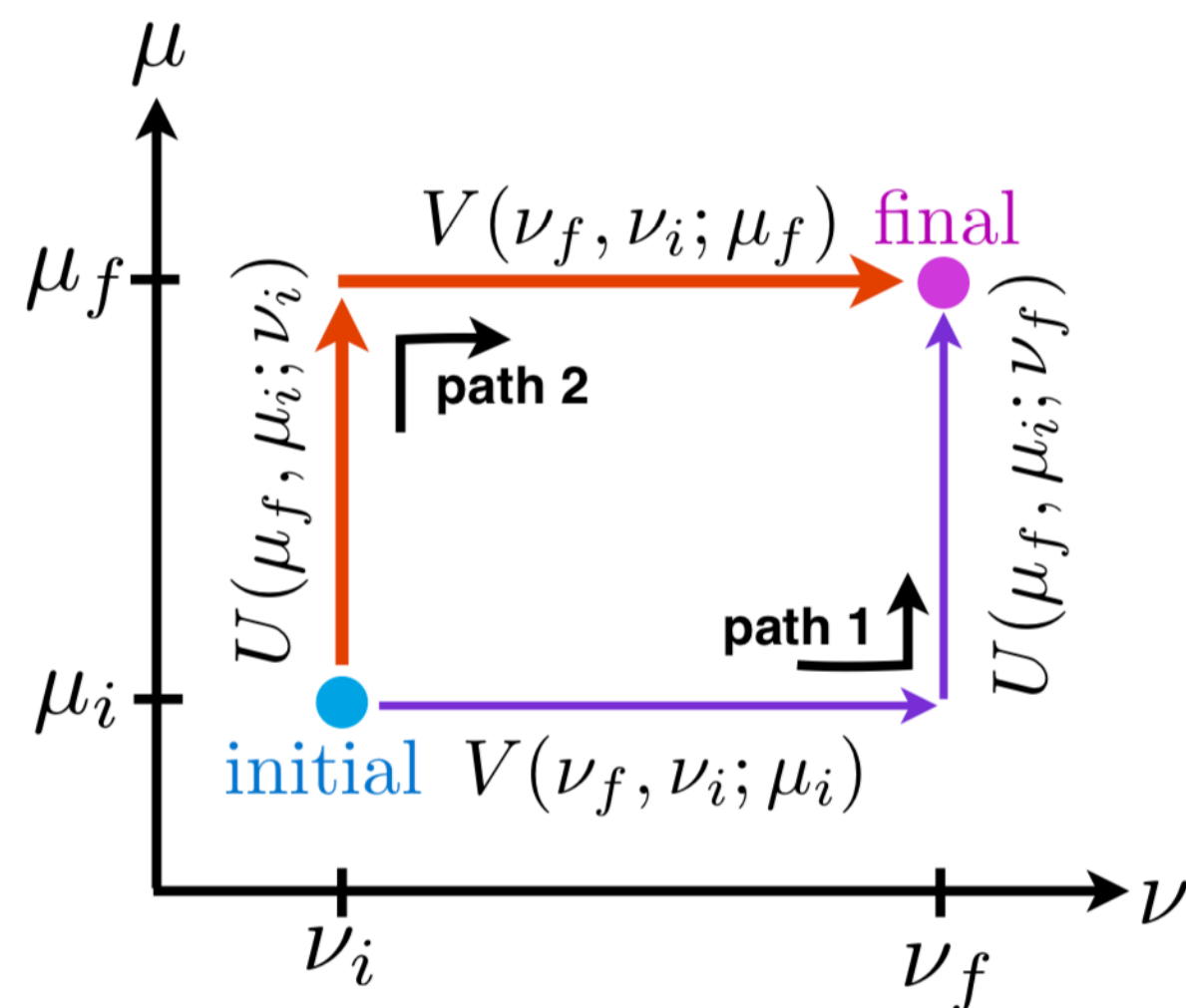
Rapidity Renormalization Group I

4-loop CS Kernel [I.Moult, H.X.Zhu, Y.J.Zhu]
[C.Duhr, B. Mistlberger, G. Vita]

CSS RG
 (μ^2, ζ)



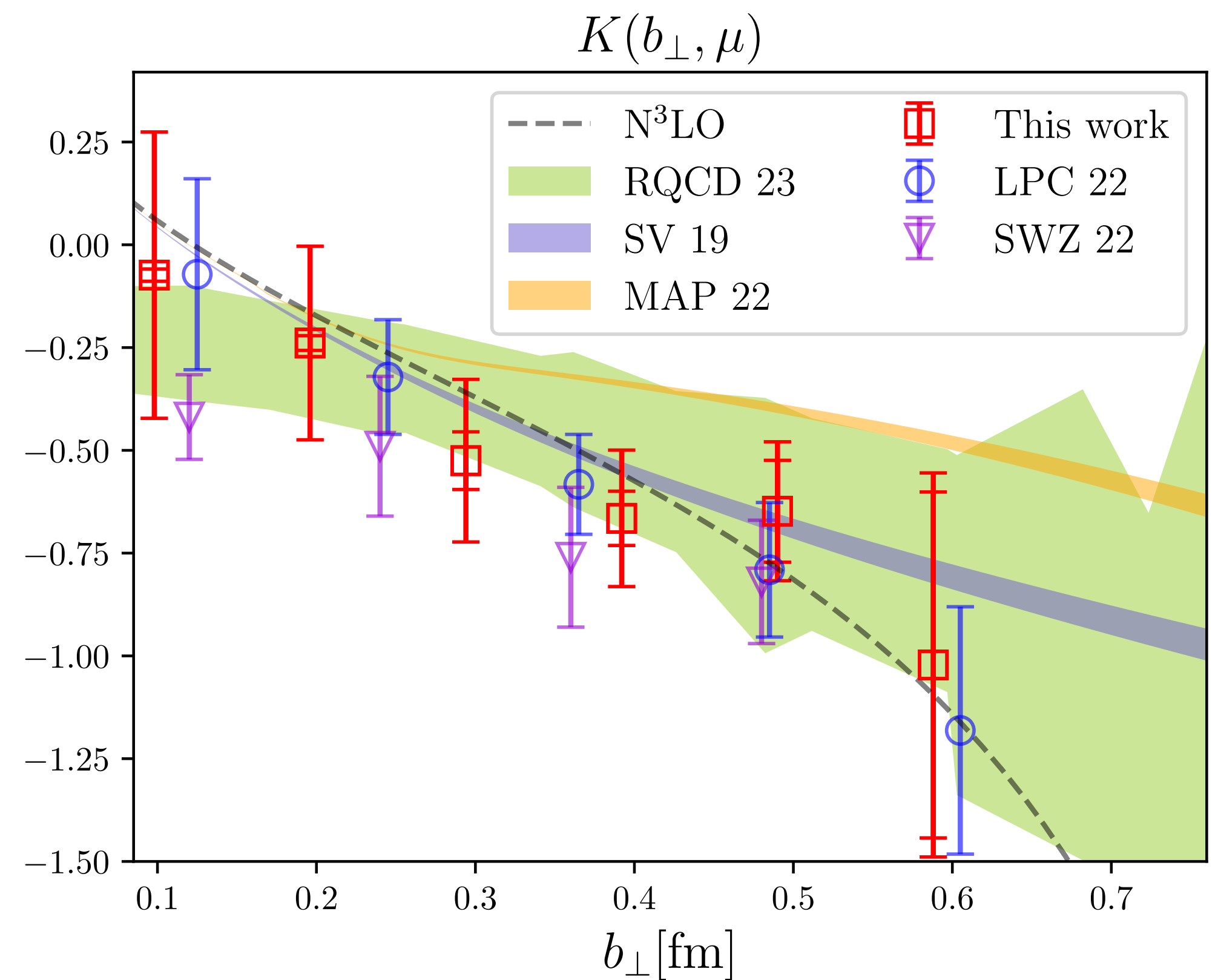
SCETII RG
 (μ, ν)



[J.Y.Chiu, A.Jain, D.Neil, I.Z.Rothstein]

the Collins-Soper Kernel

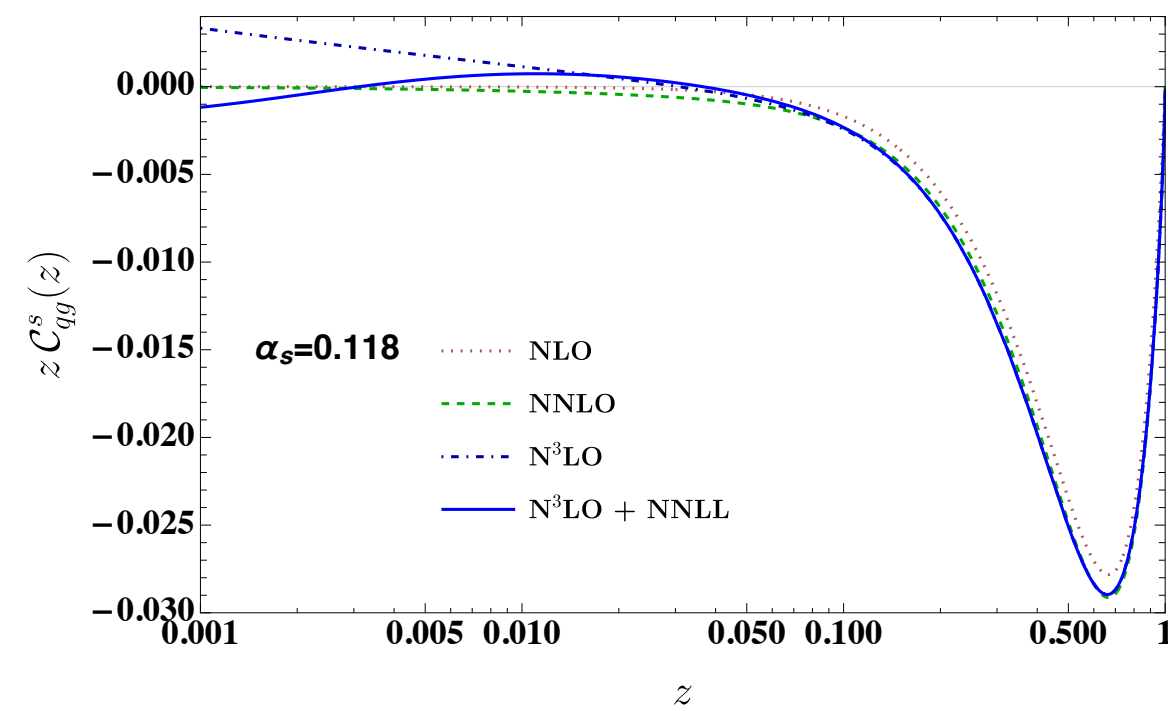
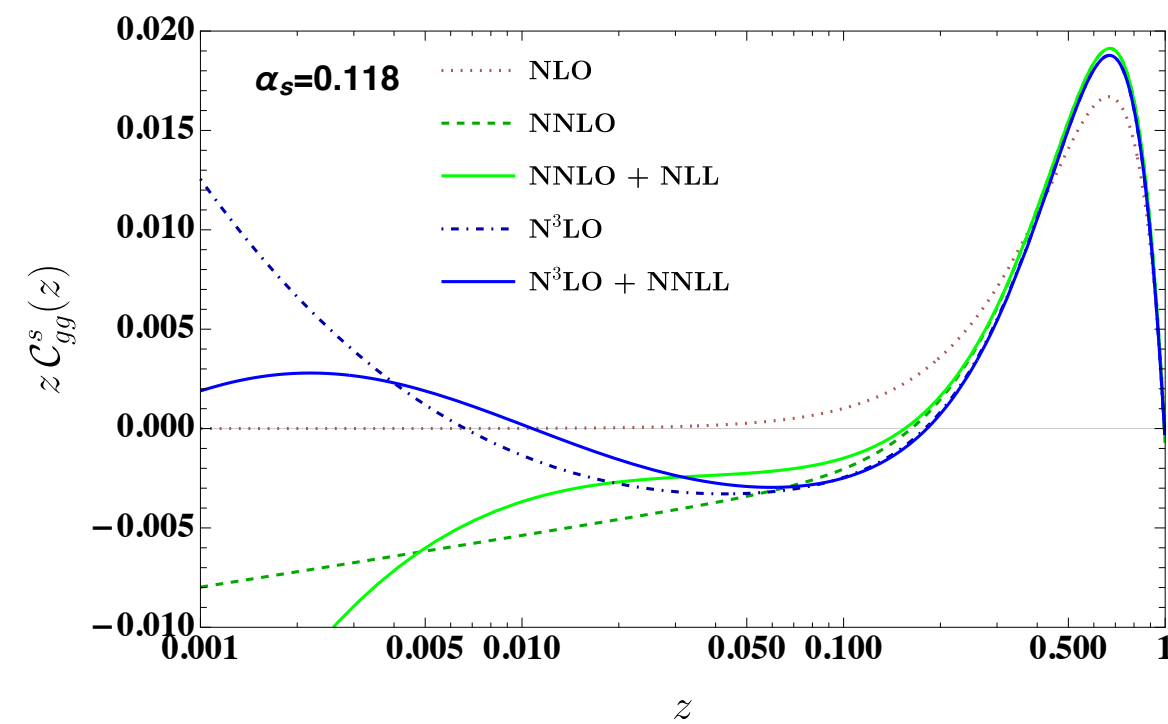
$$K(b_\perp, \mu) = -2A_{\text{cusp}}^q(\mu, \mu_b) + \gamma_R^q(\mu_b)$$



LPC

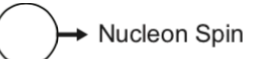
Polarized N3LO twist-2 matchings + NNLO DGLAP

Small-x resummation of Linearly Polarized Gluon TMDFFs



[M.A.Ebert, B.Mistlberger and G.Vita]

[M.x.Luo, T.Z.Yang, H.X.Zhu and Y.J.Zhu]

Leading Quark TMDPDFs  Nucleon Spin  Quark Spin

		Quark Polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1 = \text{Unpolarized}$		$h_1^\perp = \text{Boer-Mulders}$
	L		$g_1 = \text{Helicity}$	$h_{1L}^\perp = \text{Worm-gear}$
	T	$f_{1T}^\perp = \text{Sivers}$	$g_{1T}^\perp = \text{Worm-gear}$	$h_1 = \text{Transversity}$ $h_{1T}^\perp = \text{Pretzelosity}$

Figure 2.5: Leading power quark parton distribution functions for the proton or a spin-1/2 hadron.

Leading Quark TMDFFs  Hadron Spin  Quark Spin

		Quark Polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Unpolarized (or Spin 0) Hadrons		$D_1 = \text{Unpolarized}$		$H_1^\perp = \text{Collins}$
Polarized Hadrons	L		$G_1 = \text{Helicity}$	$H_{1L}^\perp$
	T	$D_{1T}^\perp = \text{Polarizing FF}$	$G_{1T}^\perp$	$H_1 = \text{Transversity}$ $H_{1T}^\perp$

Figure 2.6: Leading power quark TMD fragmentation functions for a spin-1/2 (or for an unpolarized or spin 0) hadron.

Leading Gluon TMDPDFs  Nucleon Spin  Gluon Operator Helicities

		Gluon Operator Polarization		
		Un-Polarized	Helicity 0 antisymmetric	Helicity 2
Nucleon Polarization	U	$f_1^g = \text{Unpolarized}$		$h_1^{\perp g} = \text{Linearly Polarized}$
	L		$g_{1L}^g = \text{Helicity}$	$h_{1L}^{\perp g}$
	T	$f_{1T}^{\perp g}$	$g_{1T}^{\perp g}$	$h_{1T}^g = \text{Transversity}$ $h_{1T}^{\perp g}$

Figure 2.7: Leading power gluon TMD parton distribution functions for a spin-1/2 hadron (or unpolarized hadron).

Leading Gluon TMDFFs  Hadron Spin  Gluon Operator Helicities

		Gluon Operator Polarization		
		Un-Polarized	Helicity 0 antisymmetric	Helicity 2
Unpolarized (or Spin 0) Hadrons		$D_1^g = \text{Unpolarized}$		$H_1^{\perp g} = \text{Linearly Polarized}$
Polarized Hadrons	L		$G_{1L}^g = \text{Helicity}$	$H_{1L}^{\perp g}$
	T	$D_{1T}^{\perp g}$	$G_{1T}^{\perp g}$	$H_{1T}^g = \text{Transversity}$ $H_{1T}^{\perp g}$

Figure 2.8: Leading power gluon TMD fragmentation functions for an unpolarized or spin-0 hadron or a polarized spin-1/2 hadron.

Lattice QCD Milestone

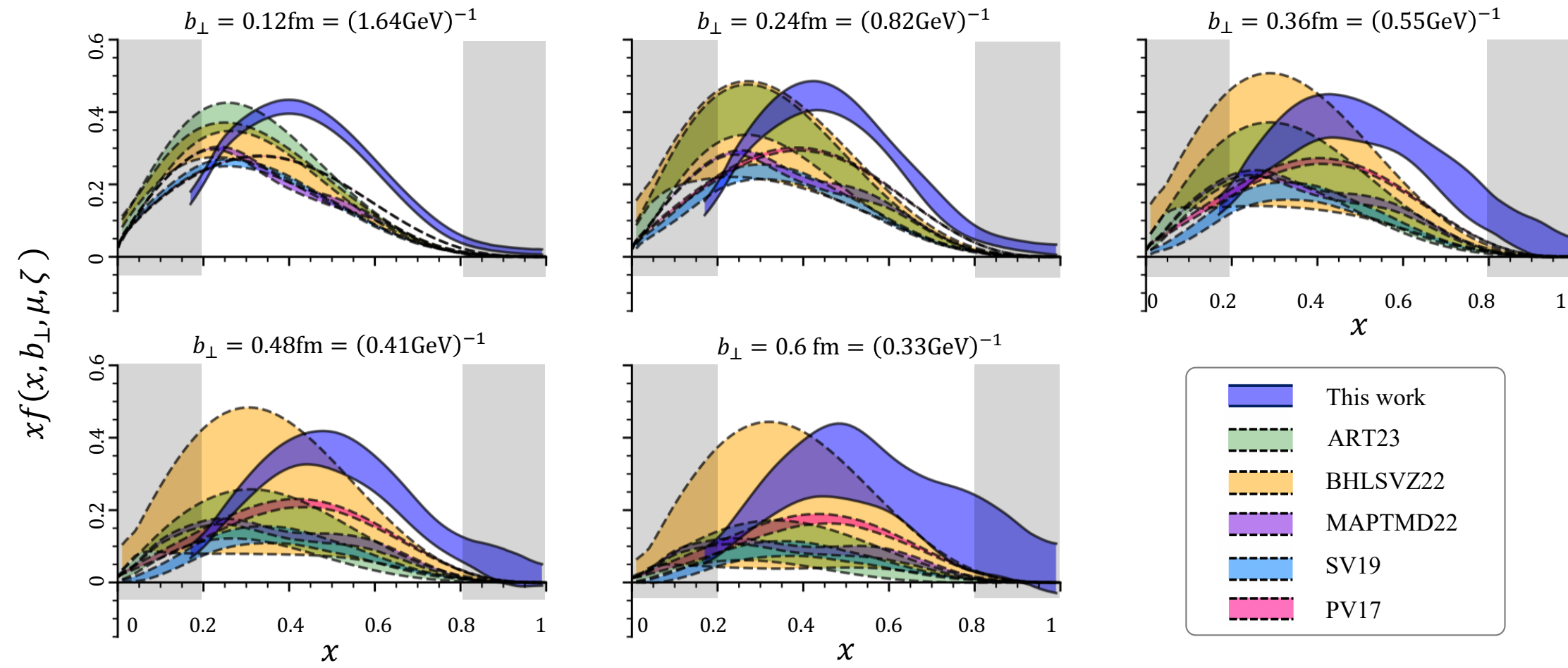
PHYSICAL REVIEW D **109**, 114513 (2024)



Unpolarized transverse momentum dependent parton distributions of the nucleon from lattice QCD

Jin-Chen He,^{1,2,3} Min-Huan Chu,^{1,2} Jun Hua,^{4,5} Xiangdong Ji,³ Andreas Schäfer,⁶ Yushan Su,³ Wei Wang,^{2,7,*} Yi-Bo Yang,^{8,9,10,11} Jian-Hui Zhang,^{12,13} and Qi-An Zhang^{14,†}

(Lattice Parton Collaboration (LPC))



PHYSICAL REVIEW LETTERS **125**, 192001 (2020)

Lattice QCD Calculations of Transverse-Momentum-Dependent Soft Function through Large-Momentum Effective Theory

Qi-An Zhang,¹ Jun Hua,² Yikai Huo,^{2,3} Xiangdong Ji,^{1,4} Yizhuang Liu,¹ Yu-Sheng Liu,¹ Maximilian Schlemmer,⁵ Andreas Schäfer,⁵ Peng Sun,⁶ Wei Wang,^{2,*} and Yi-Bo Yang^{7,8,9,†}



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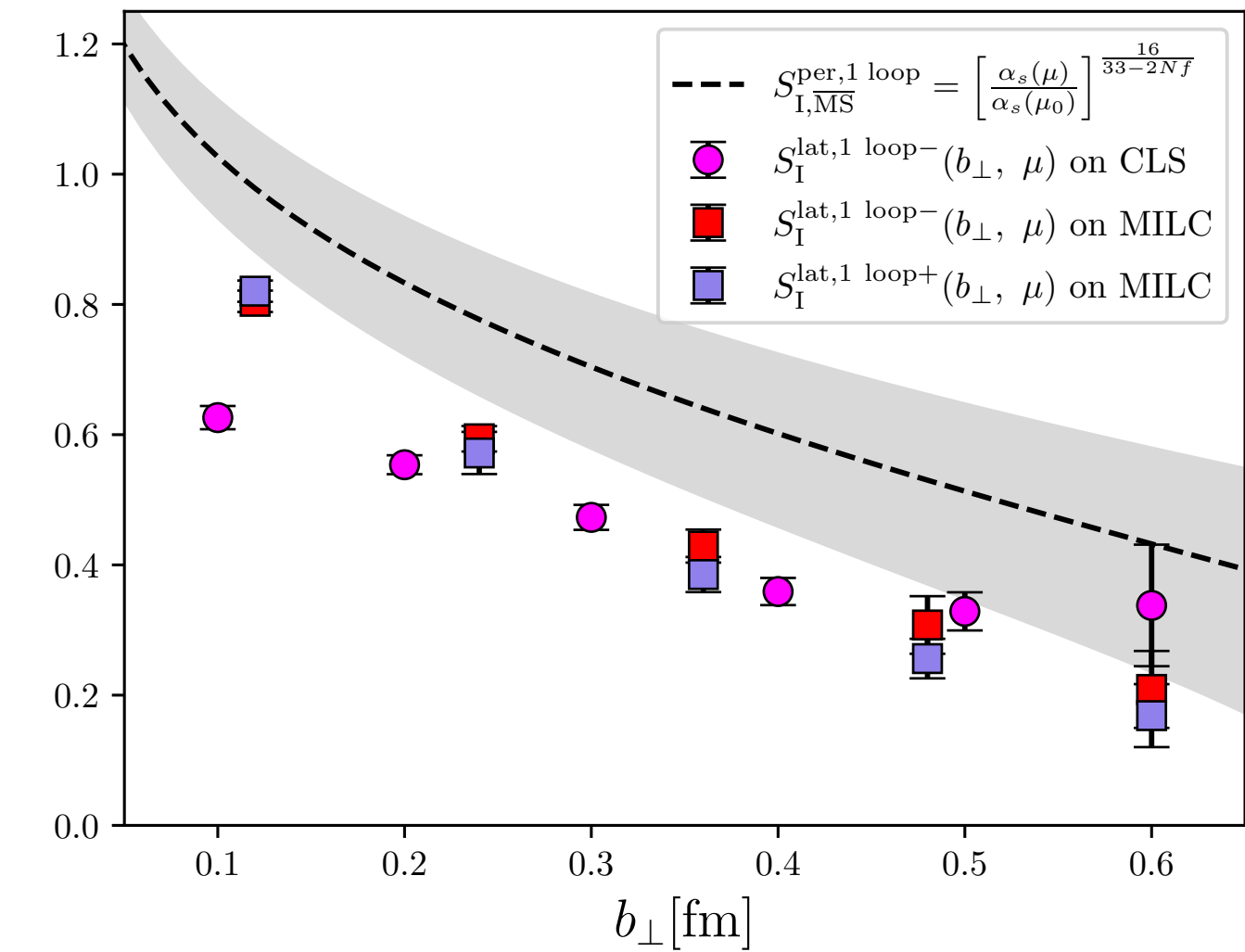
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Lattice calculation of the intrinsic soft function and the Collins-Soper kernel



The Lattice Parton Collaboration (LPC)

Min-Huan Chu,^{a,b} Jin-Chen He,^{a,c} Jun Hua,^{d,e,f} Jian Liang,^{d,e,f} Xiangdong Ji,^c Andreas Schäfer,^g Hai-Tao Shu,^g Yushan Su,^c Lisa Walter,^g Wei Wang,^{a,h} Ji-Hao Wang,^{i,j} Yi-Bo Yang,^{i,j,k,l} Jun Zeng^a and Qi-An Zhang^m



Why Energy Correlators?

EEC: model-independent probe, final-state EEC jet perturbative N4LL, no jet algorithm!

- SIDIS qT

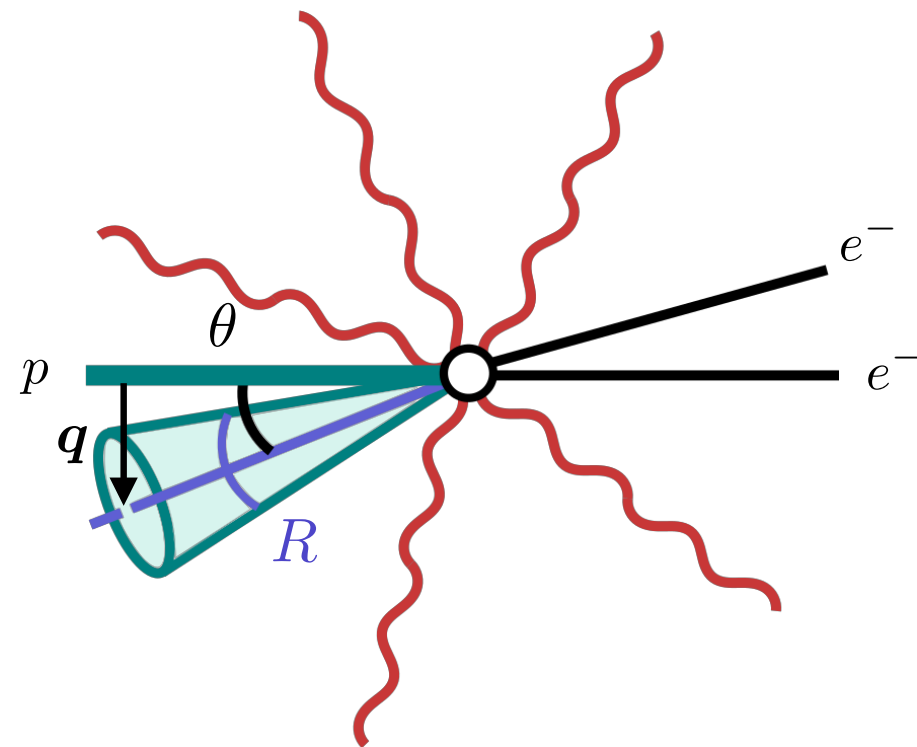
$$\sigma_{\text{SIDIS}} \propto \left| \text{Diagram 1} \right|^2 \approx \left| \text{Diagram 2} \right|^2 \otimes \left| \text{Diagram 3} \right|^2 \otimes \left| \text{Diagram 4} \right|^2$$

1. Convolution in q_T space, needs simultaneous fit of both initial-state and final-state TMDs

2. Needs empirical models

$$d\sigma(q_T) = \sigma_0 H(Q, \mu) f_1^q(x, \zeta_n, k_T, \mu) \otimes D_1^q(z, \zeta_{\bar{n}}, q_T - k_T, \mu)$$

- Jet based



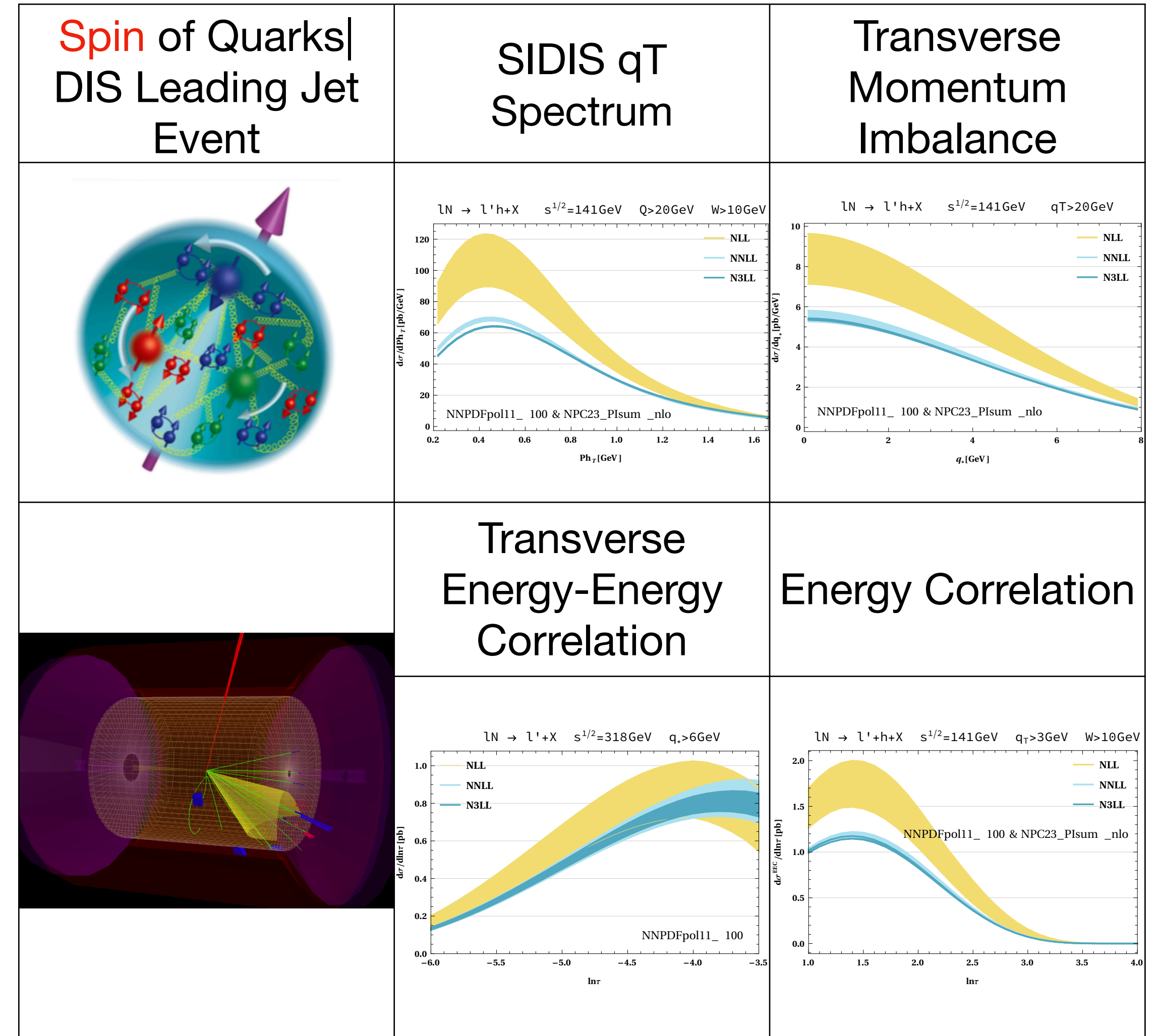
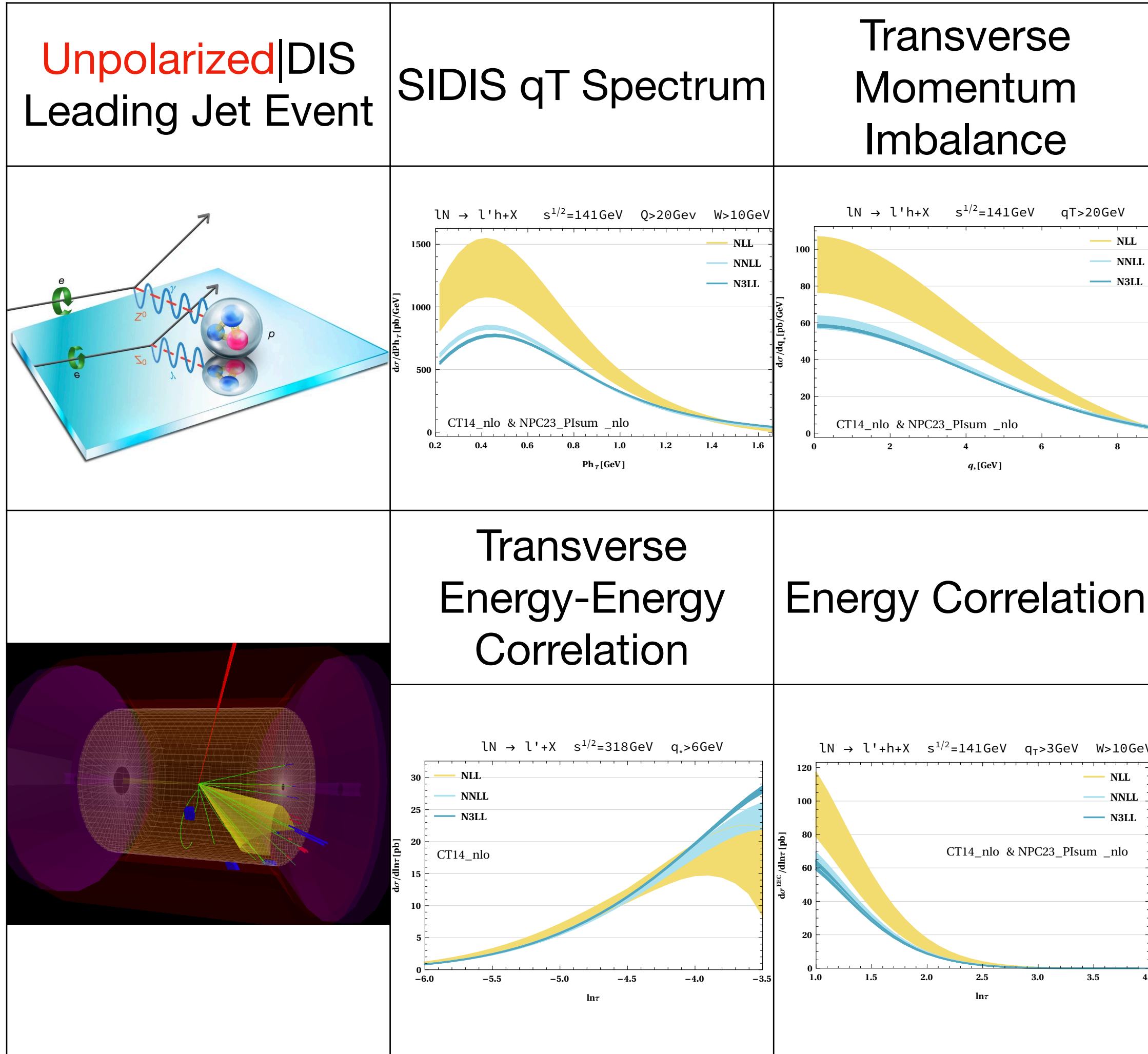
$$d\sigma(q_T) = \sigma_0 H(Q, \mu) f_1^q(x, \zeta_n, k_T, \mu) \otimes \mathcal{J}^q \left(\zeta_{\bar{n}}, \frac{QR}{2}, q_T - k_T, \mu \right)$$

1. Need clustering algorithms

2. Hindered by NGLs (WTA)

3. In low-energy machine?

Quark TMDs from Event Shapes



Gluon TMDs from Dijet Transverse Energy-Energy Correlations

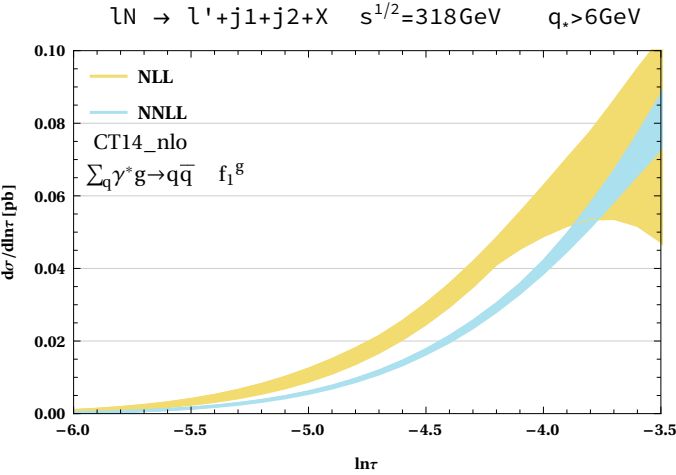
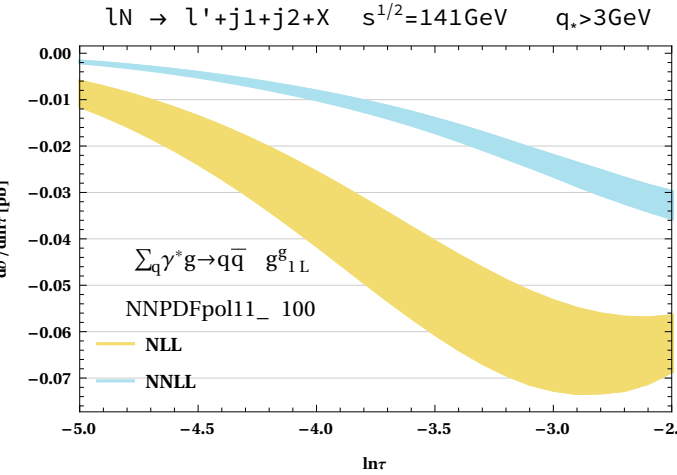
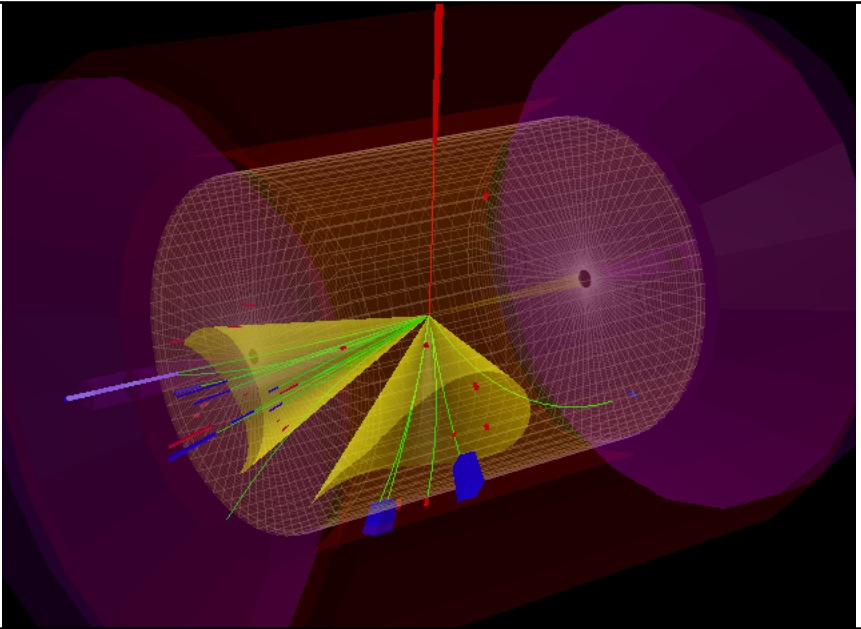
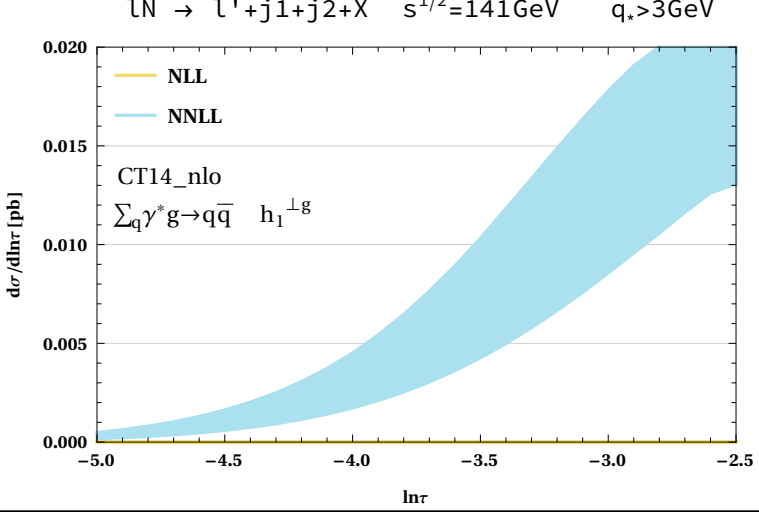
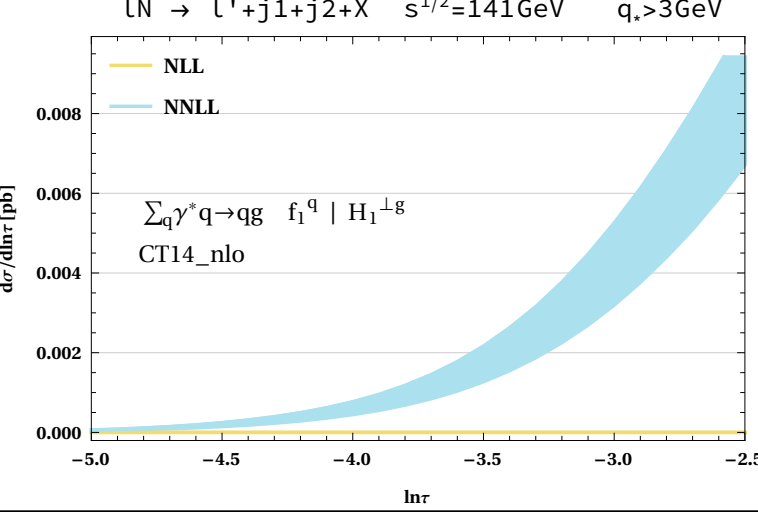
where $K_{\perp}$ is the intrinsic transverse momentum of the gluons. However, for the contribution of linearly-polarized gluons, direct terms identically vanish

$$\left(\frac{g_{\perp}^{\mu\nu}}{d-2}+\frac{\widetilde{K}_{\perp}^{\mu}\widetilde{K}_{\perp}^{\nu}}{\widetilde{K}_T^2}\right)\varepsilon_{\mp\mu}(n_J,\bar{n}_J)\varepsilon_{\pm\nu}(n_J,\bar{n}_J)=0\,,\tag{2.28}$$

while crossed terms with helicity-flipped gluons survive, and contributions from different operator bases are added coherently, e.g., for $\gamma^*\bar{q}\rightarrow\bar{q}g$ we have contributions from linearly-polarized gluons as

$$\begin{aligned} |\sum_{\pm}\langle\ell\ell'\pm X_{23}^{\pm}X_4^{\pm}X_s|0\rangle|^2\simeq&..+\frac{1}{|\omega_2||\omega_3|}\Big\langle(Y_{\textcolor{red}{2}}^{\dagger}\textcolor{violet}{y}_4Y_{\textcolor{red}{3}})^{\dagger}(Y_{\textcolor{red}{2}}^{\dagger}\textcolor{violet}{y}_4Y_{\textcolor{red}{3}})\Big\rangle\\&\times\Big[\langle\mathcal{C}(\textcolor{green}{\pm};\textcolor{blue}{\pm};-)|\mathcal{C}(\textcolor{green}{\pm};\textcolor{blue}{\pm};+)\rangle\Big\langle\mathcal{A}_{4\perp}^{+}\mathcal{A}_{4\perp}^{+}\Big\rangle+\langle\mathcal{C}(\textcolor{green}{\pm};\textcolor{blue}{\pm};+)|\mathcal{C}(\textcolor{green}{\pm};\textcolor{blue}{\pm};-)\rangle\Big\langle\mathcal{A}_{4\perp}^{-}\mathcal{A}_{4\perp}^{-}\Big\rangle\Big]\\&\times\Big\langle\bar{\chi}_{\textcolor{blue}{3}}\frac{\not{\boldsymbol{\phi}}_3}{2}\chi_{\textcolor{blue}{3}}\left(\bar{\chi}_{\textcolor{blue}{2}}\frac{\not{\boldsymbol{\phi}}_2}{2}\chi_{\textcolor{blue}{2}}\pm\bar{\chi}_{\textcolor{blue}{2}}\frac{\not{\boldsymbol{\phi}}_2}{2}\gamma_5\chi_{\textcolor{blue}{2}}\right)\Big\rangle.\end{aligned}\tag{2.29}$$

[Jun, Hai Tao, Yu Jiao (to appear)]

Spin of Gluons DIS Dijet Event		Unpolarized Gluon	Gluon Helicity Distribution
			
		Linearly Polarized Gluon Beam	Linearly Polarized Gluon Jet
			

The SIDIS Landscape

Kinematics of Current Region Fragmentation in Semi-Inclusive Deeply Inelastic Scattering

M. Boglione^a, J. Collins^b, L. Gamberg^c, J. O. Gonzalez-Hernandez^{d,e}, T. C. Rogers^{d,e}, N. Sato^e

rapidity : $y = -\ln \tan \frac{\theta}{2}$

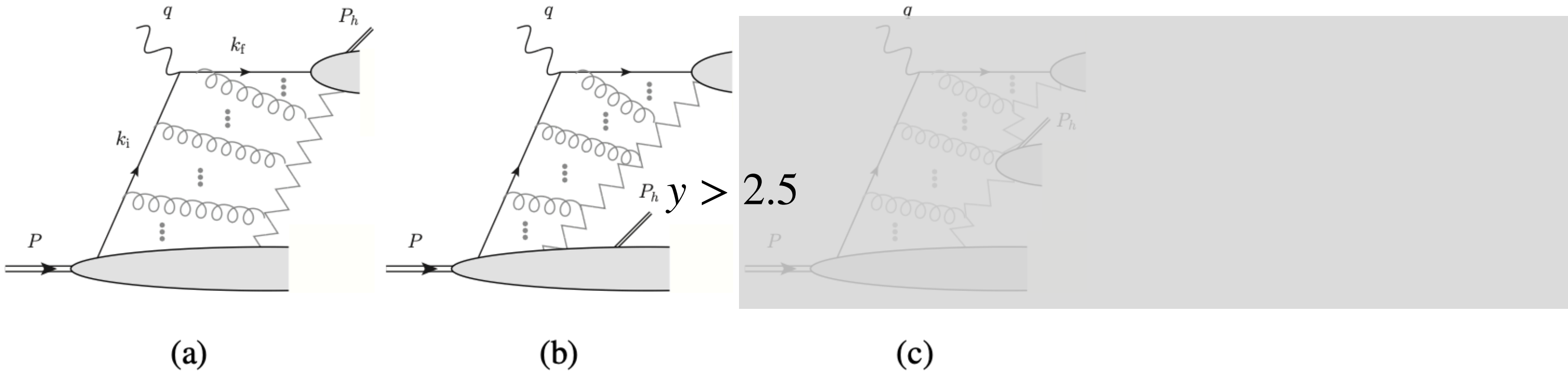


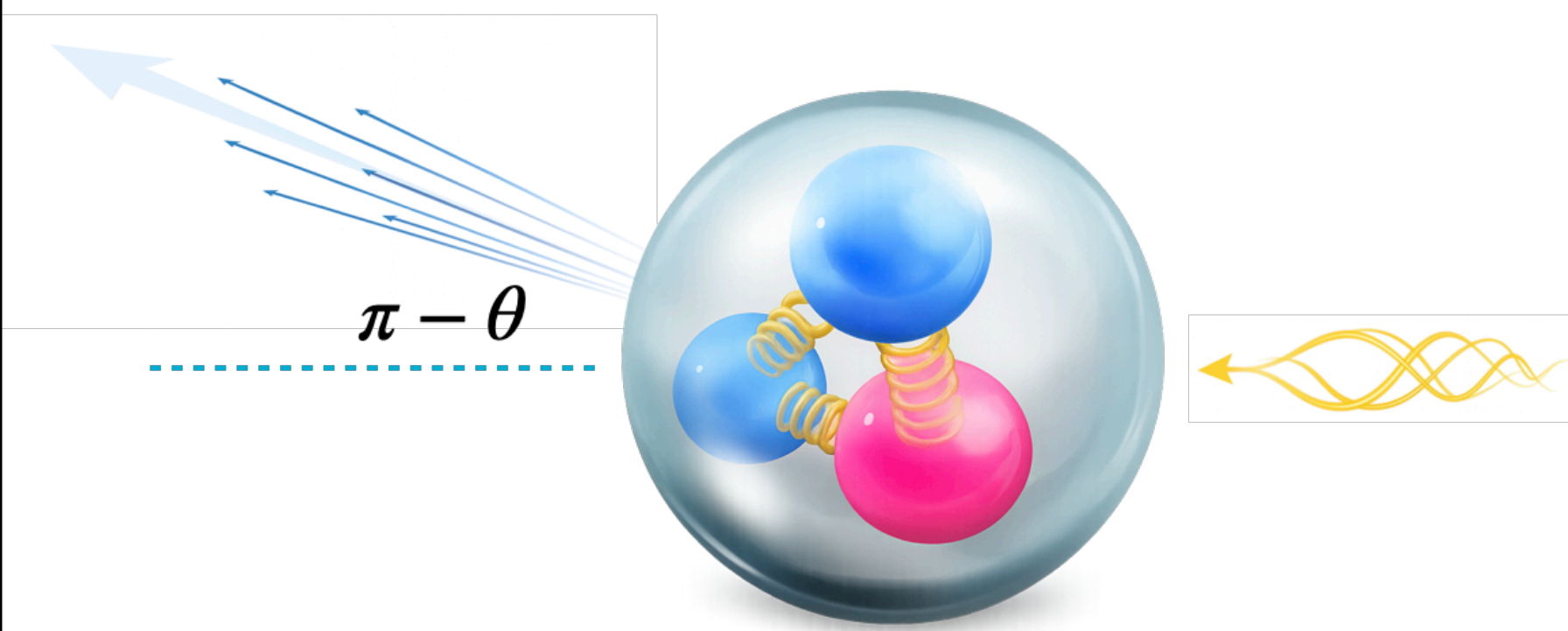
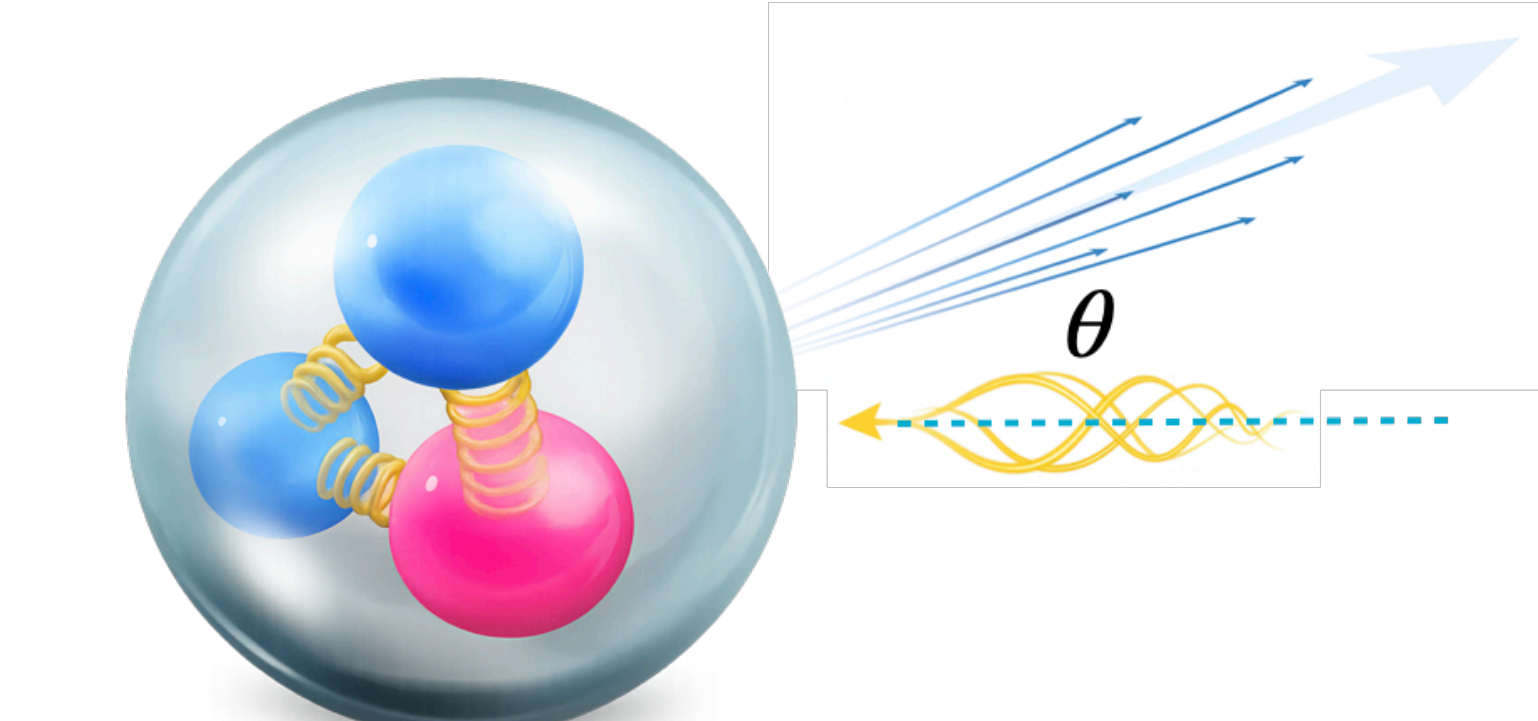
Figure 1: Lowest order SIDIS graphs corresponding to (a) the current region (b) the target region and (c) the central (soft) region. The faded zigzag lines represent non-perturbative and other interactions (e.g. hadronization) between the outgoing parton and the target jet.

Observable = NP ⊗ P

Ob	NP	P
DIS	PDFs	DIS coefficients
Spectator ECs	NECs	DIS coefficients
Back-to-back Jets	TMDs	Finite S matrix
Rapidity Gapped Jets	???	$C(\mu) = 1$

Energy Correlators Resolving Proton Spin

$$\langle \hat{\mathbf{E}}(\hat{n}) \rangle_N = \int d^4x e^{iq \cdot x} \langle N, S_{\parallel} | J(x) \hat{\mathbf{E}}(\hat{n}) J(0) | N, S_{\parallel} \rangle$$

	TMD PDFs	NECs
DIS		

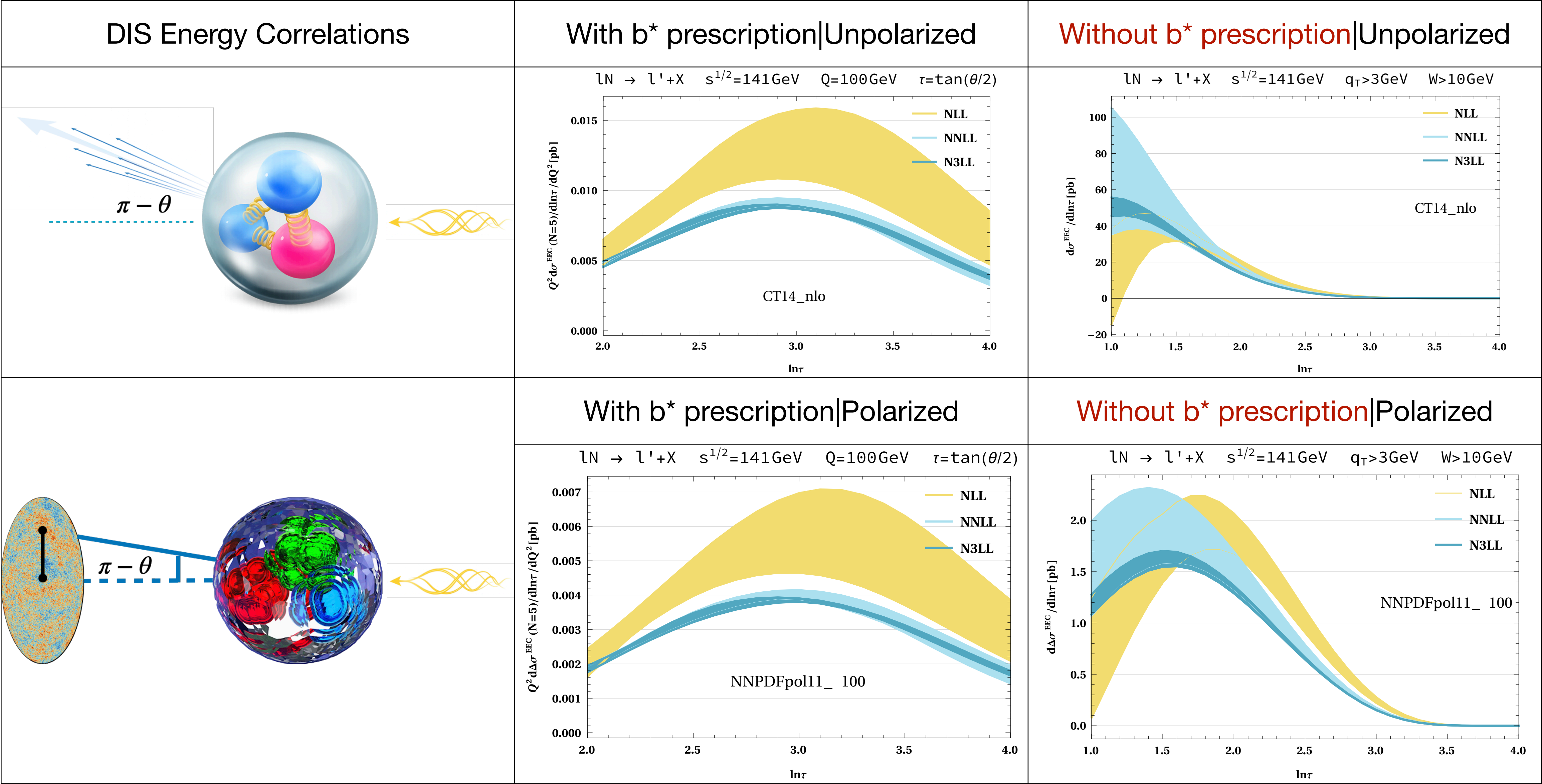
Motivation

- Spin decomposition, Ji sum rule (frame independent), Jaffe–Manohar (infinite-momentum frame):

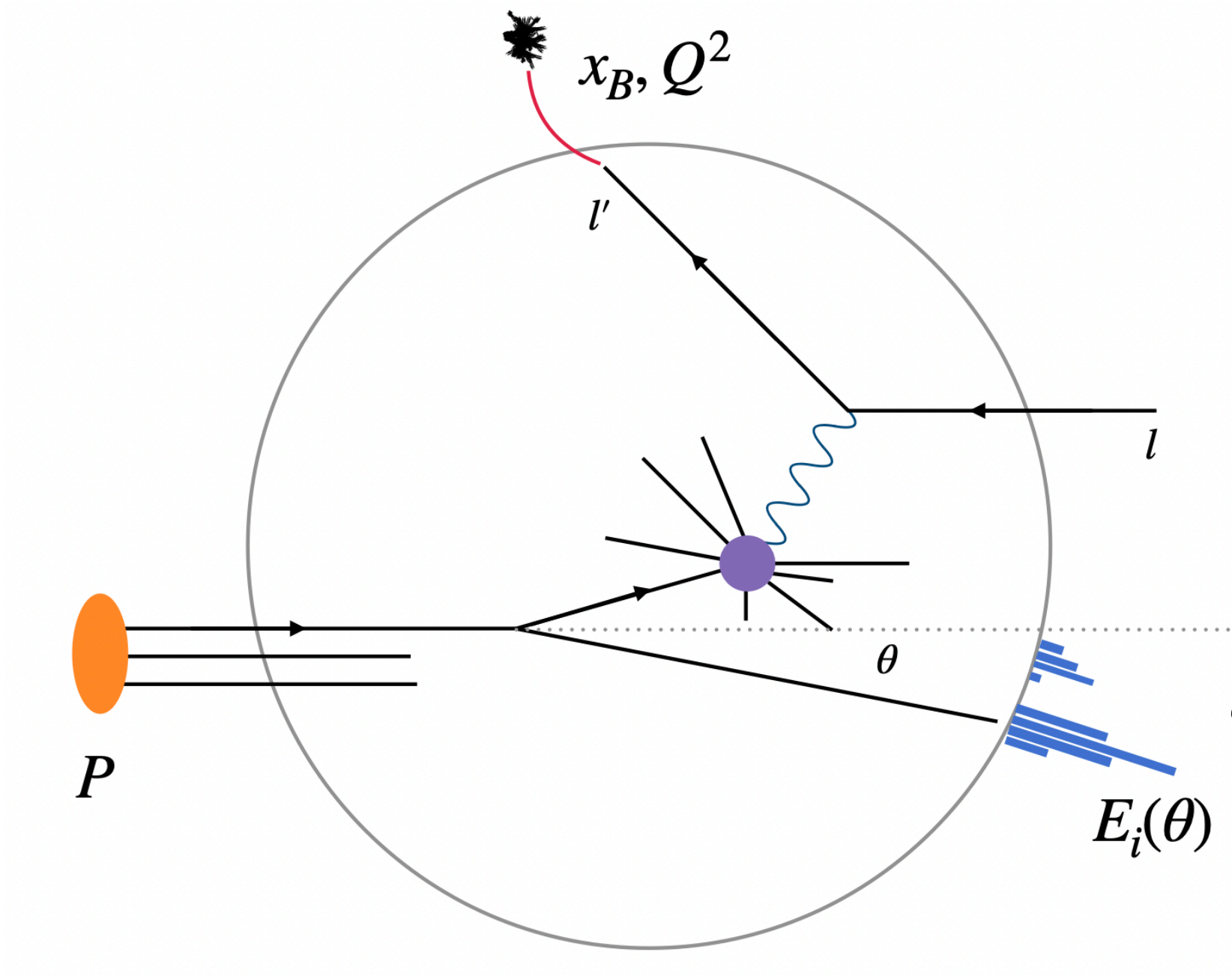
$$\frac{\hbar}{2} = \frac{1}{2} \Delta \Sigma + \Delta G + \ell_q + \ell_g$$

$\Downarrow$
14 pretzelosity $h_{1T}^{\perp q}$

Current Fragmentation



Target Fragmentation



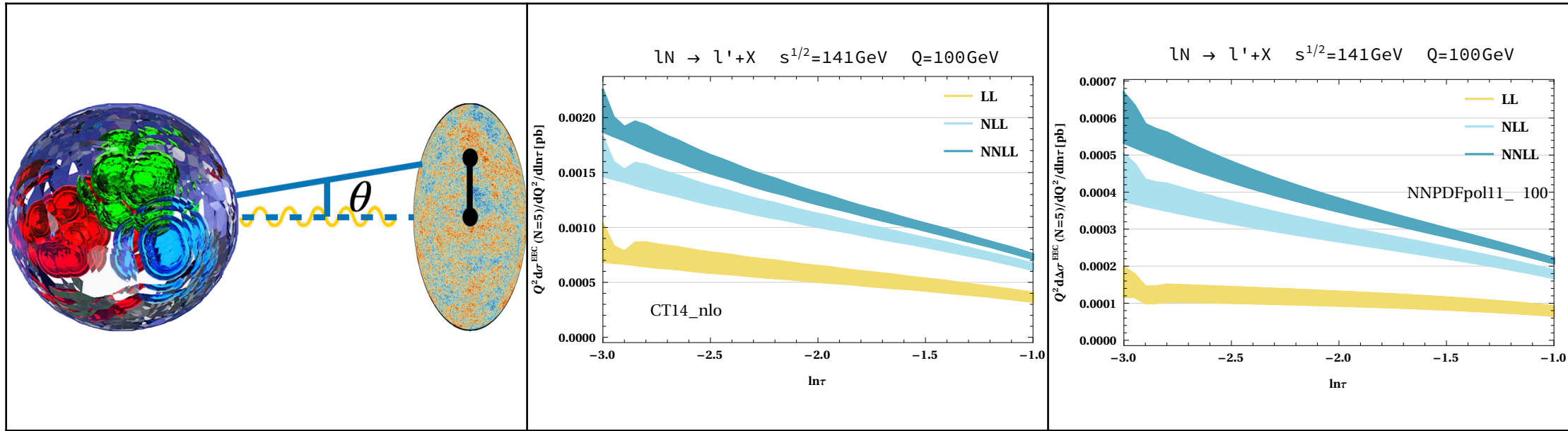
Observable = NP⊗P

Ob	NP	P
Spectator energy correlation	Nucleon Energy Correlators	DIS coefficients

Nucleon Energy Correlators [Xiaohui, H.X.Zhu]

$$O_n^{i;\Gamma}(\omega_{\pm},\theta)=\frac{1}{\omega_+}\left[\bar{\chi}_{n,\omega_1}^{(i)}\Gamma\hat{\mathbf{E}}(\theta)\chi_{n,\omega_2}^{(i)}\right]$$

$$O_n^{g;\Gamma}(\omega_{\pm},\theta)=\frac{-1}{\omega_+^2}\left[\mathcal{B}_{n,\omega_1}^{\perp\mu}\Gamma_{\mu\nu}\hat{\mathbf{E}}(\theta)\mathcal{B}_{n,\omega_2}^{\perp\nu}\right]$$



$$F_{i/N}^{\text{EC}} \sim \langle P, S_{\parallel} | O_n^{i;\Gamma} | P, S_{\parallel} \rangle$$

↓ Twist-2 matching

$$\mathcal{F}_{i/N}^{\text{EC}}\left(z,\ln\frac{E_a\sin\theta}{\mu},\mu\right)=f_i(z,\mu)-\int_z^1\frac{d\xi}{\xi}\mathcal{I}_{ij}\left(\frac{z}{\xi},\ln\frac{E_a\sin\theta}{\mu},\alpha_s(\mu)\right)\xi f_j(\xi,\mu)+\mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{q_T}\right)$$

Crossing initial-state target to the final-state identified hadron

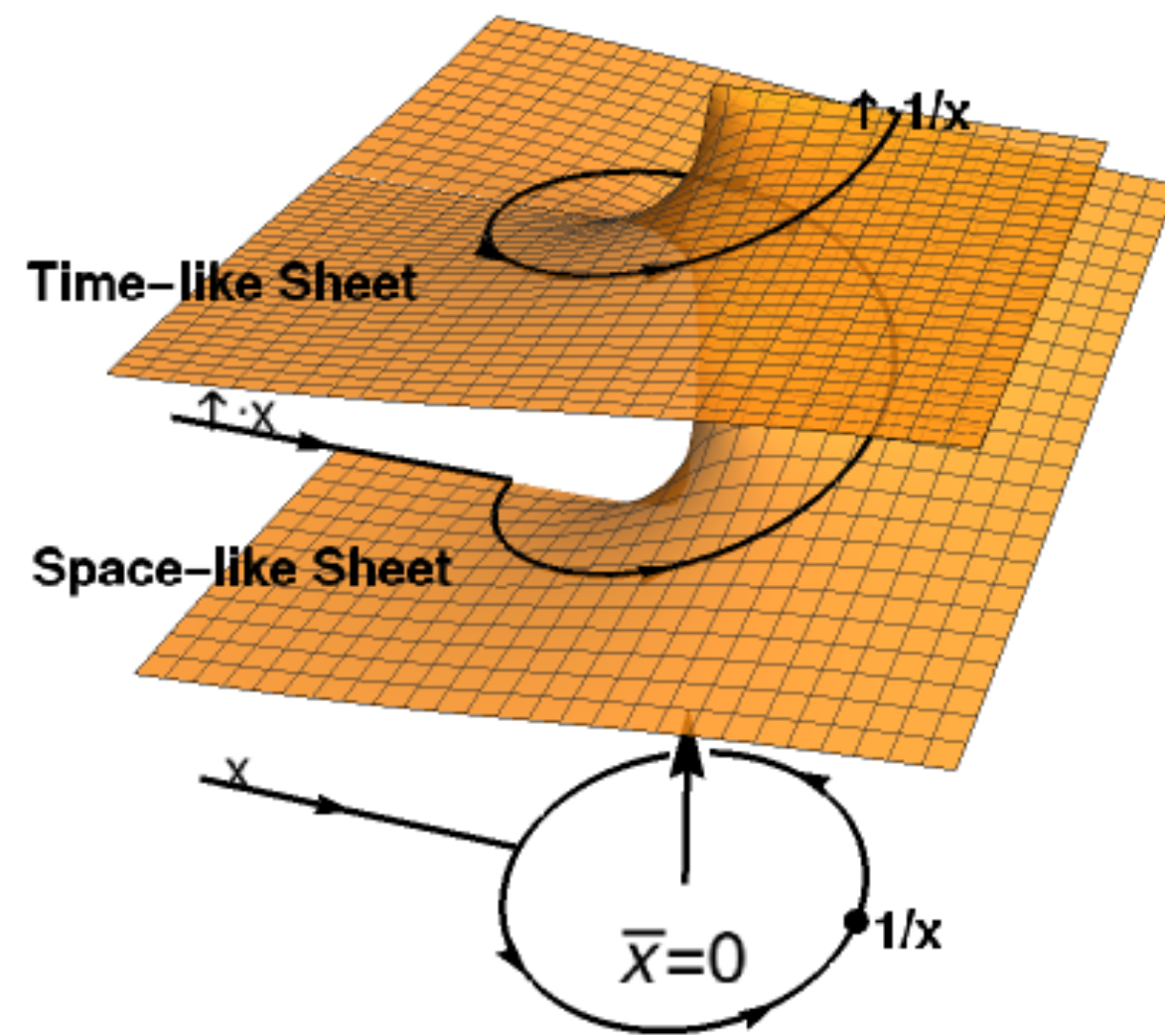
Motivation: probe hadronization, quark transversity

$$\langle \hat{\mathbf{E}}(\hat{n}) \rangle_h = \int d^4x e^{iq \cdot x} \langle 0 | J(x) \hat{\mathbf{E}}(\hat{n}) a_h^\dagger a_h J(0) | 0 \rangle$$

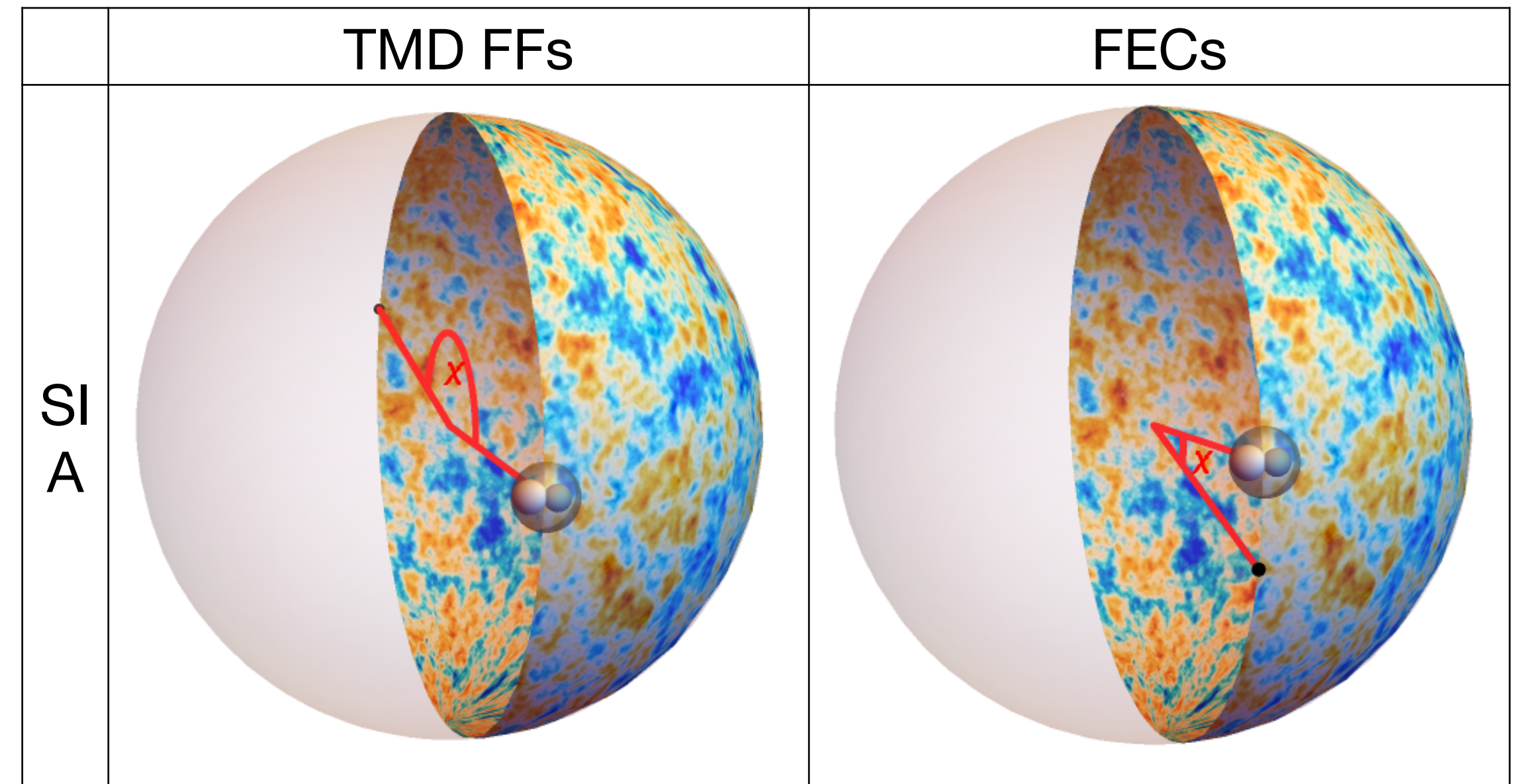
Crossing

$$x_B \rightarrow 1/x_B$$

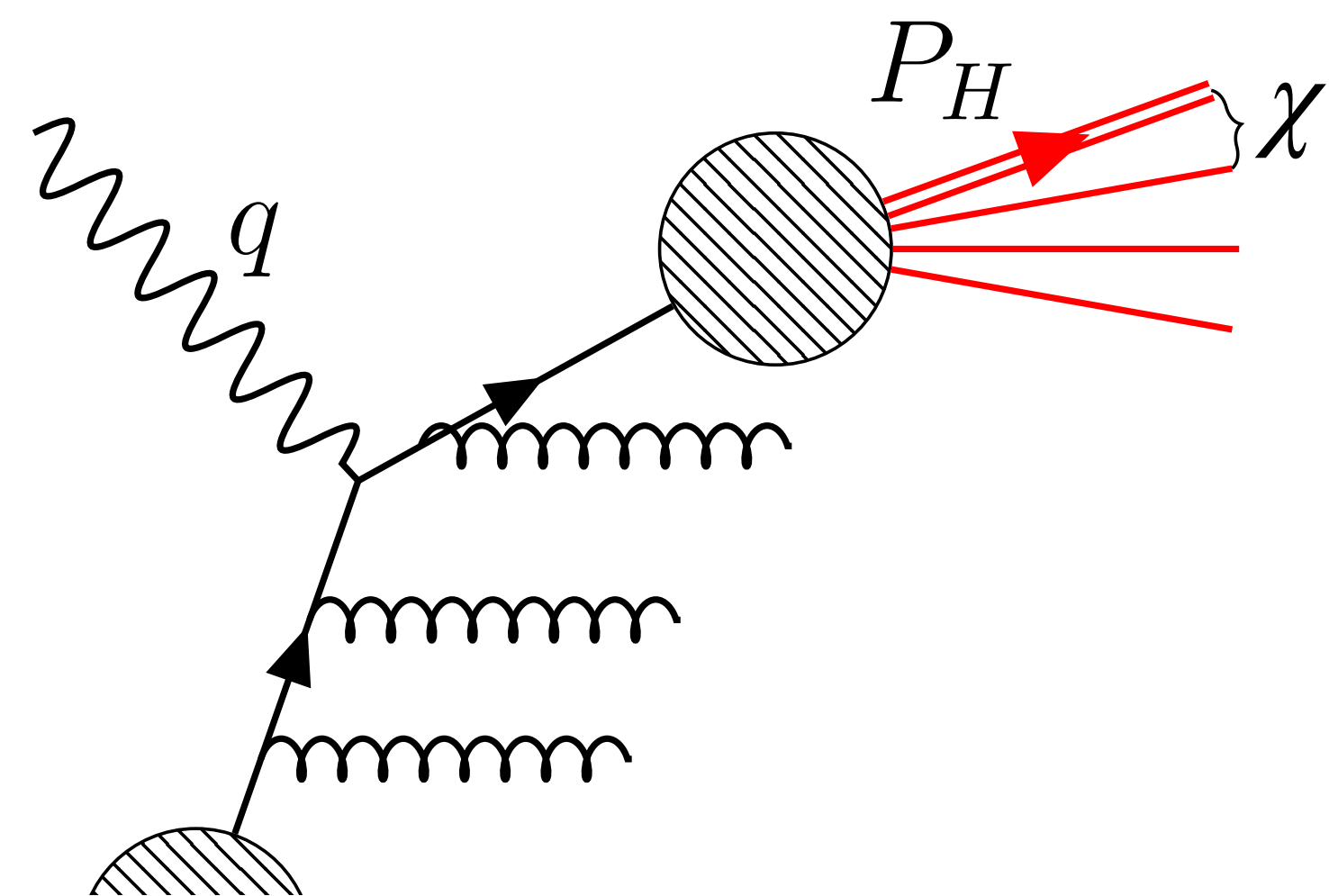
$$\langle \hat{\mathbf{E}}(\hat{n}) \rangle_h = \int d^4x e^{iq \cdot x} \langle h, S_{\parallel} | J(x) \hat{\mathbf{E}}(\hat{n}) J(0) | h, S_{\parallel} \rangle$$



Inversion relation between Bjorken variable x_B and Feynman variable $z_F = 1/x_B$



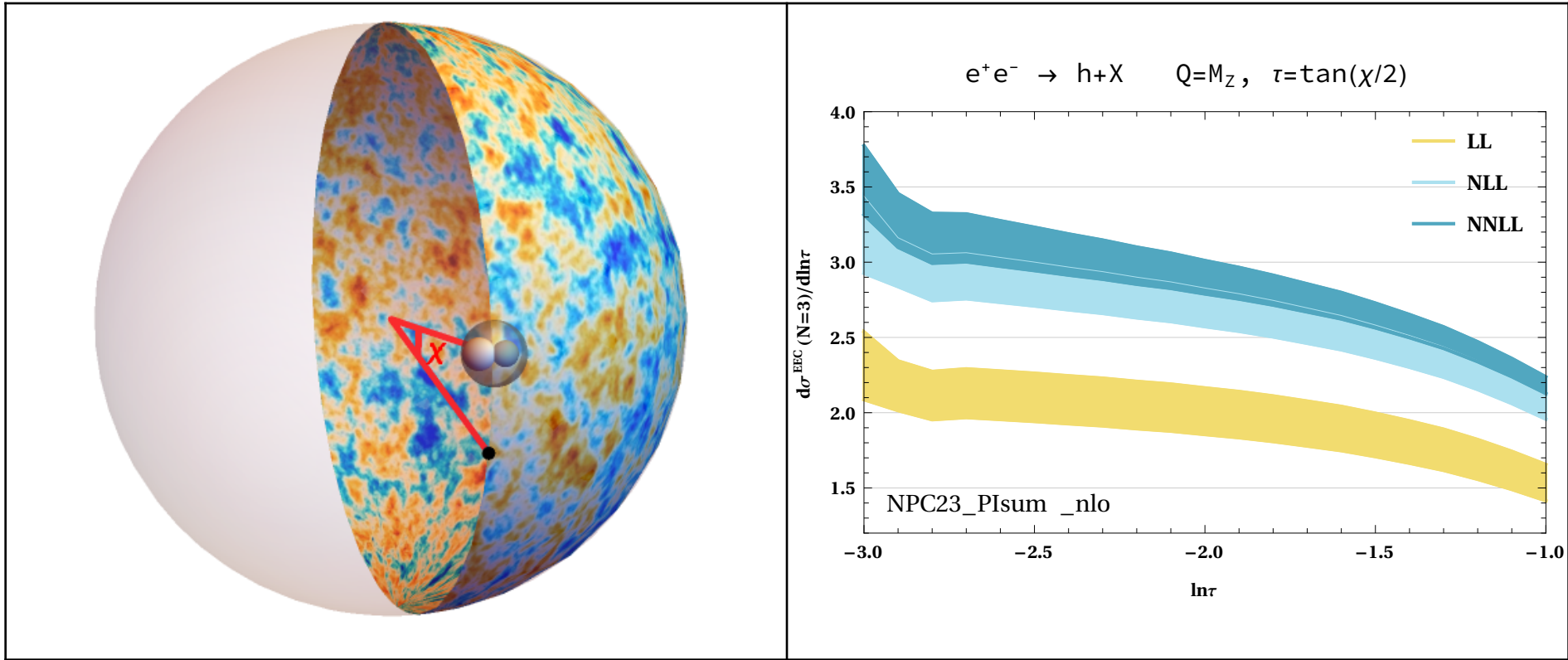
Collinear limit



Observable = NP⊗P

Ob	NP	P
Spectator energy correlation	Fragmentation Energy Correlators	SIA coefficients

Fragmentation Energy Correlators [Xiaohui Liu, H.X.Zhu]



$$\left[D_{h/q}^{\text{FEC}}\right]^{ij}\left(z,\ln \frac{E_h \sin \chi}{\mu}, \mu\right) \equiv z \int \frac{d t^{-}}{2 \pi} e^{i t^{-} \frac{P_h^{+}}{z}}\left\langle 0\left|W\left(\infty, t^{-}\right) \Psi_i\left(t^{-}\right) \hat{\mathbf{E}}(\chi)\right|h ; X\right\rangle\left\langle h ; X\left|\bar{\Psi}_j W\left(0, \infty\right)\right| 0\right\rangle$$

$$\left[D_{h/g}^{\text{FEC}}\right]^{\mu \nu}\left(z,\ln \frac{E_h \sin \chi}{\mu}, \mu\right) \equiv \frac{z^2}{2 P_h^{+}} \int \frac{d t^{-}}{2 \pi} e^{i t^{-} \frac{P_h^{+}}{z}}\left\langle 0\left|\mathcal{W}\left(\infty, t^{-}\right) F^{+\mu}\left(t^{-}\right) \hat{\mathbf{E}}(\chi)\right|h ; X\right\rangle\left\langle h ; X\left|F^{+\nu} \mathcal{W}\left(0, \infty\right)\right| 0\right\rangle$$

Twist-2 Matching $E_a=E_h / z$

$$\mathcal{D}_{h l i}^{\text{FEC}}\left(z,\ln \frac{E_a \sin \chi}{\mu}, \mu\right)=-d_i(z, \mu)+\int_z^1 \frac{d \xi}{\xi} \frac{1}{\xi} d_j(\xi, \mu) C_{j i}\left(\frac{z}{\xi}, \ln \frac{E_a \sin \chi}{\mu}, \alpha_s(\mu)\right)+\mathcal{O}\left(\frac{\Lambda_{\text {QCD }}}{q_T}\right)$$

Towards High-Energy Diffractive Energy Correlators

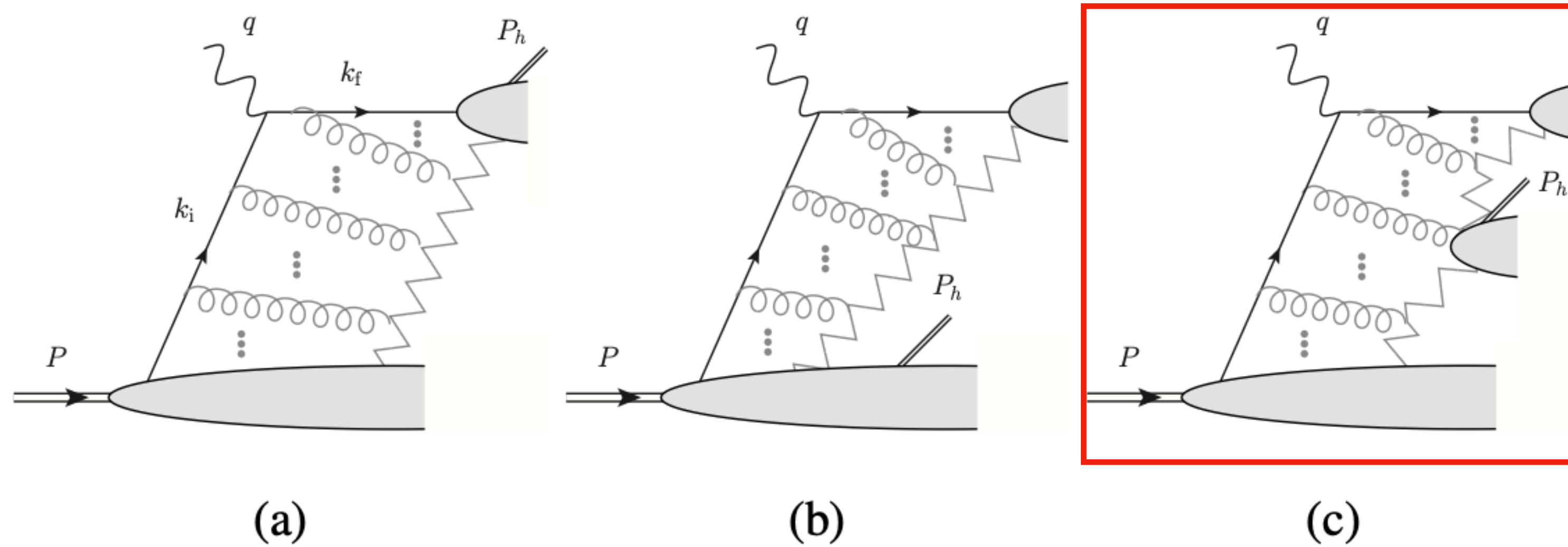
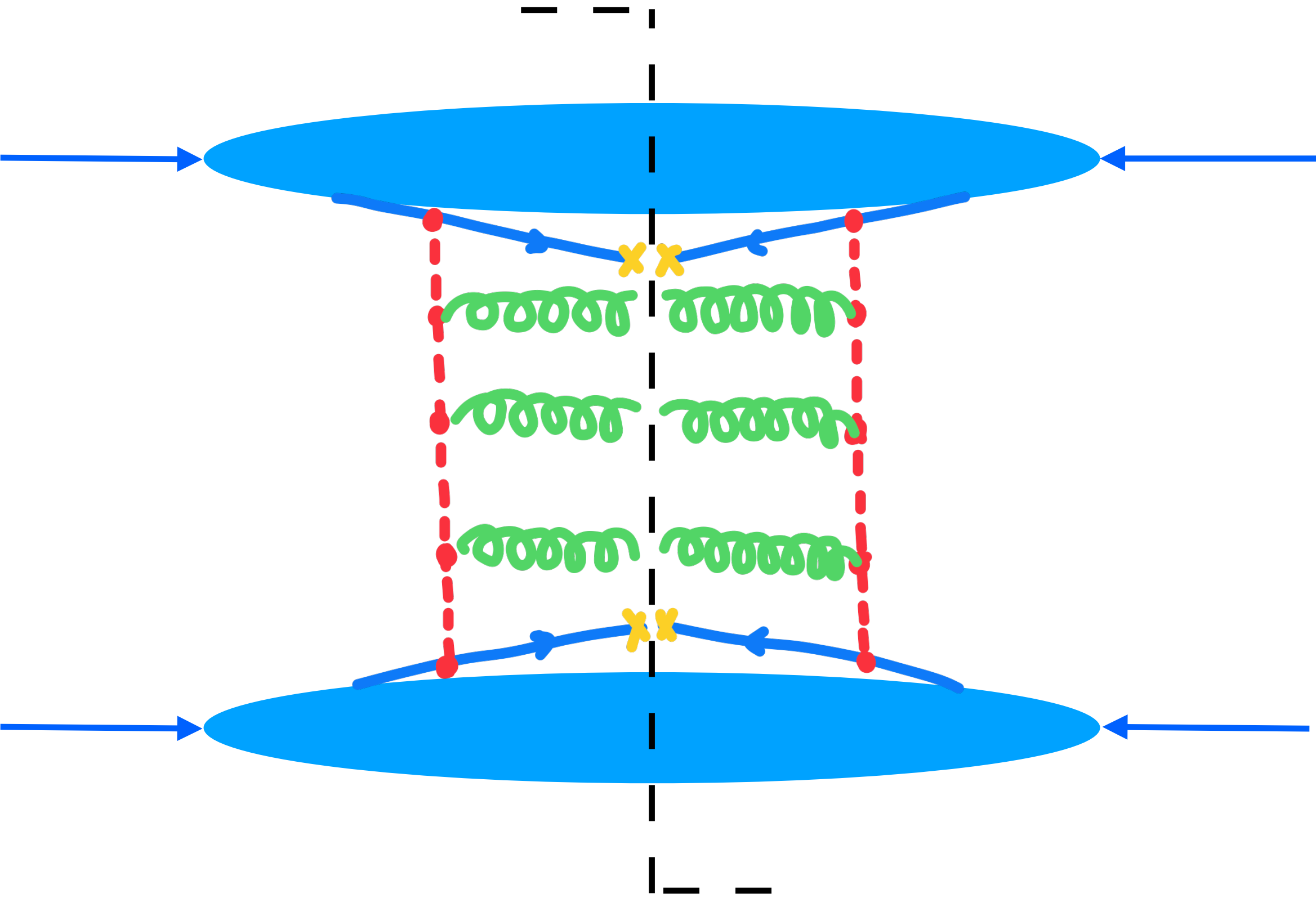
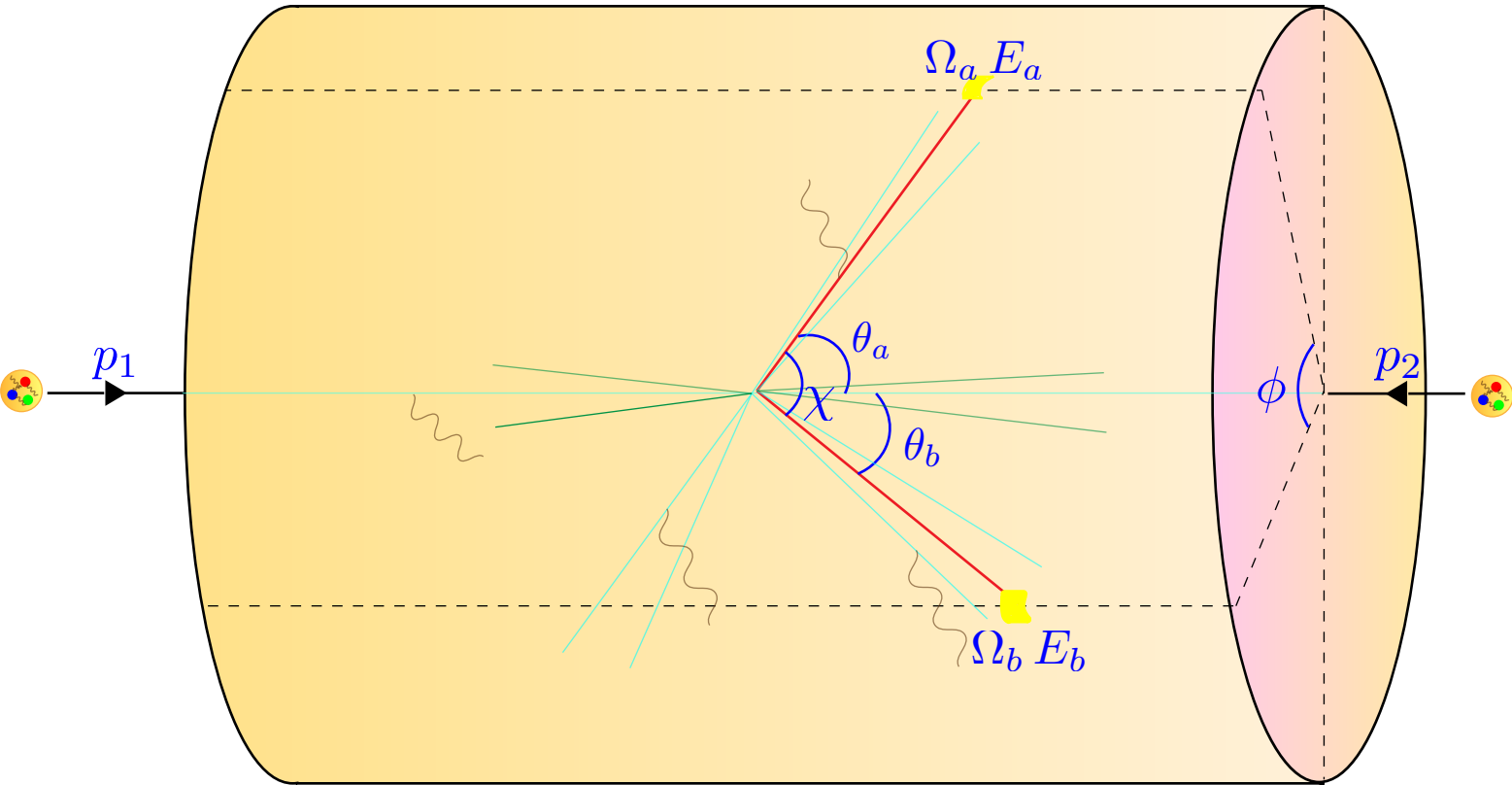


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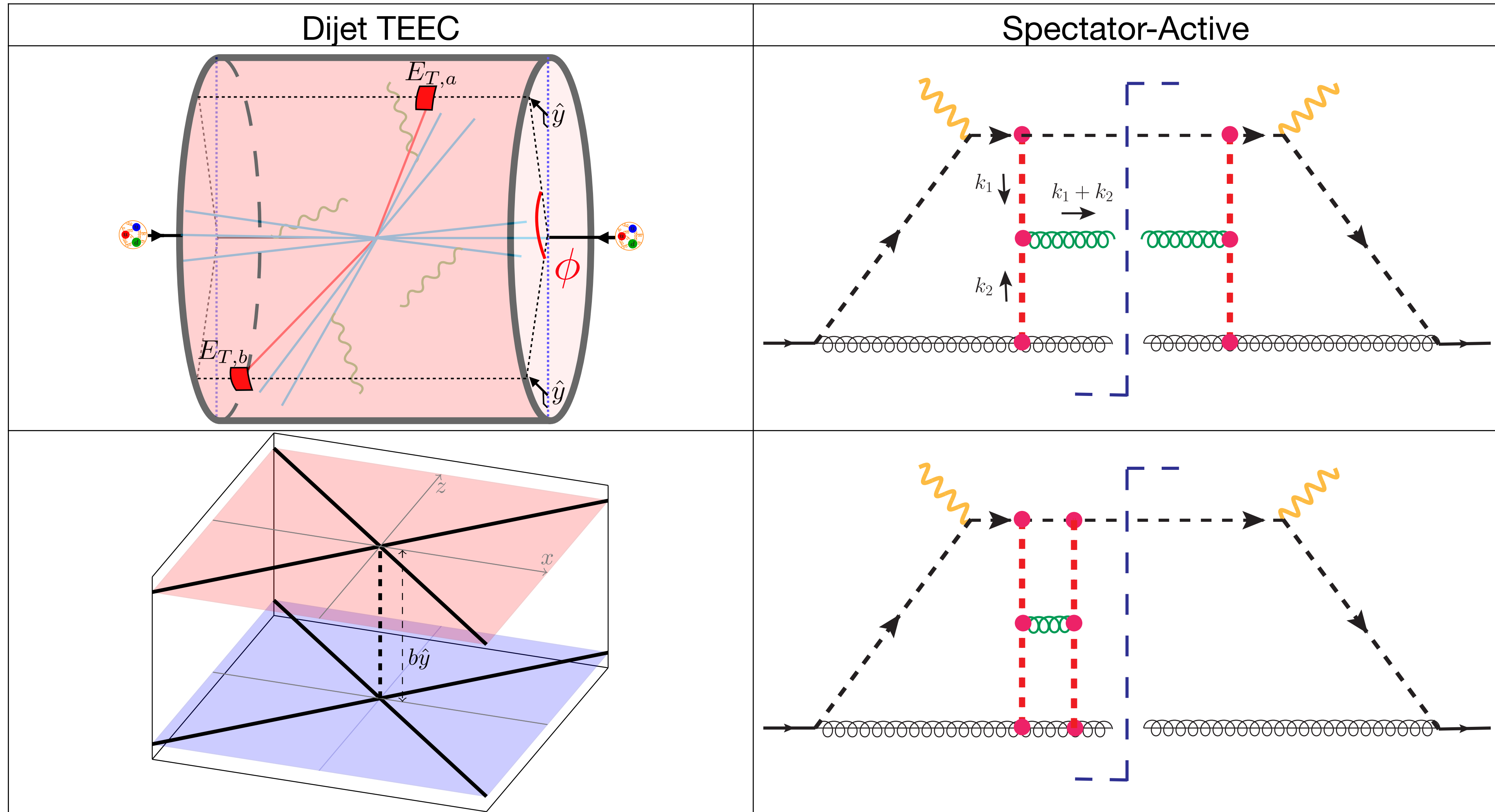
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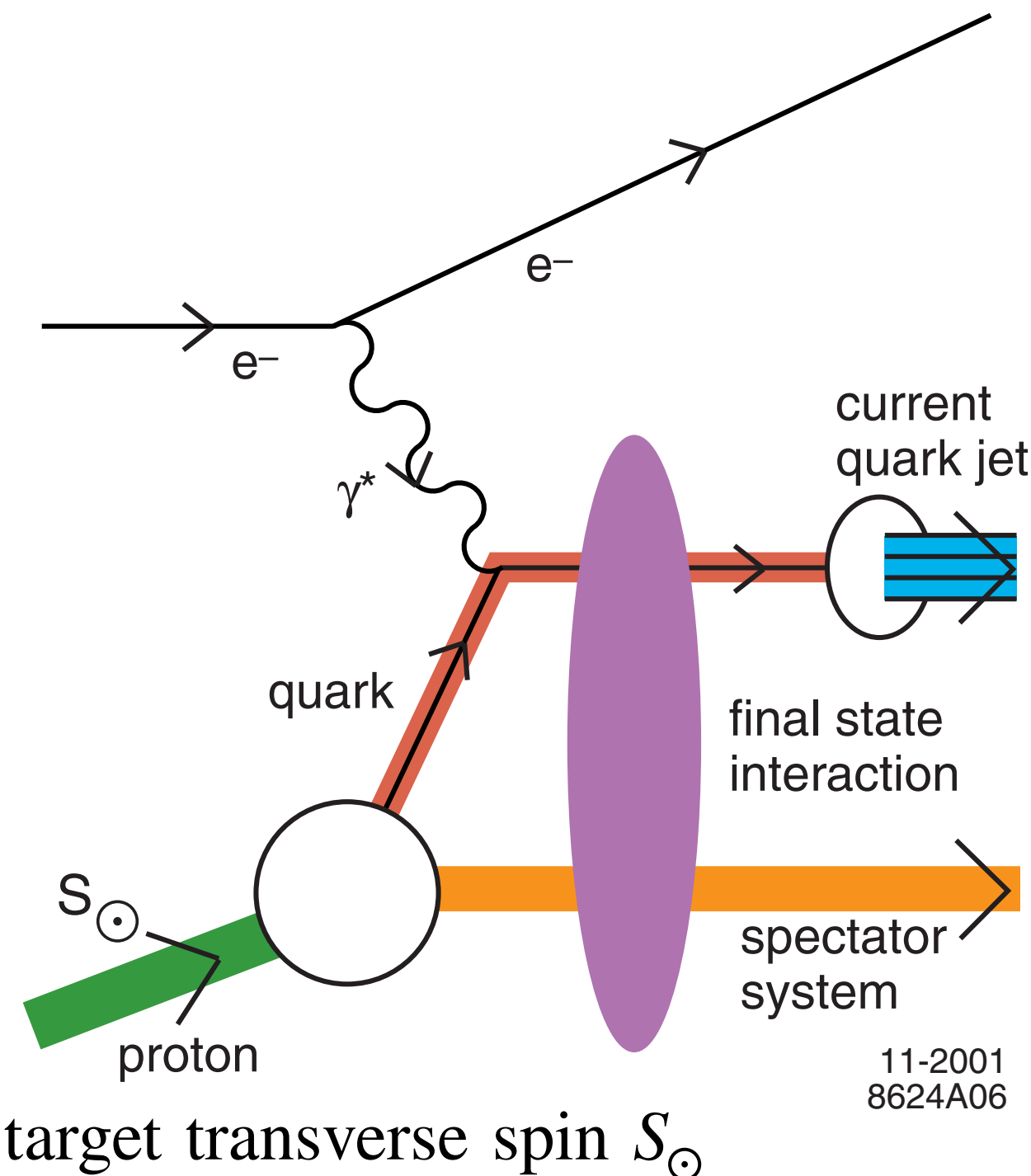
EEC of Mueller-Navelet Jets



Factorization and Factorization Violation



Process-dependence of TMDs & NECs



We will see that, in the Drell-Yan process, proving factorization requires that the TMD parton densities have past-pointing Wilson lines. Thus the Siverson function reverses sign between the two processes:

$$f_{1T,\text{SIDIS}}^\perp(x, k_T) = -f_{1T,\text{DY}}^\perp(x, k_T), \quad (13.124)$$

SSA given by interference between $\pm$ helicity :

$$\delta\sigma = |\mathcal{A}(S_\odot)|^2 - |\mathcal{A}(-S_\odot)|^2 \propto \text{Im} [\mathcal{A}(+) \mathcal{A}^*(-)]$$

$$\mathcal{A}(\pm) = |\mathcal{A}(\pm)| e^{i\chi_\pm} \implies \delta\sigma \propto \sin(\chi_+ - \chi_-)$$

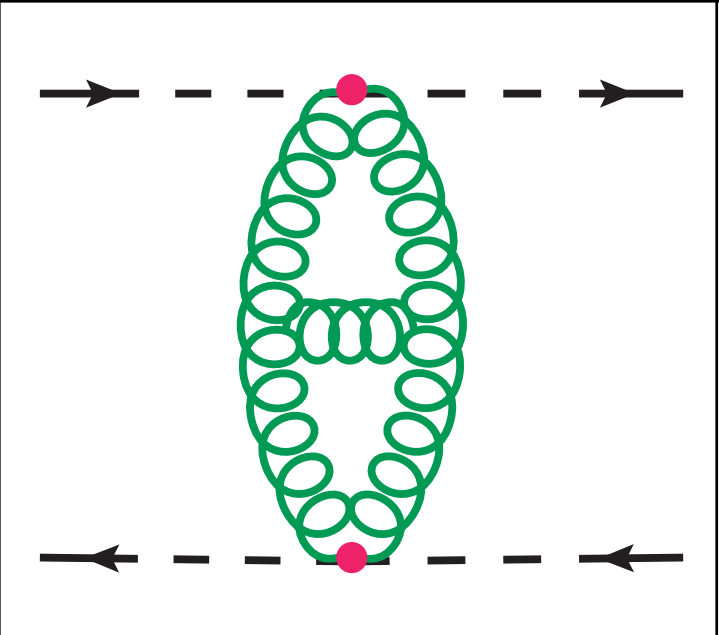
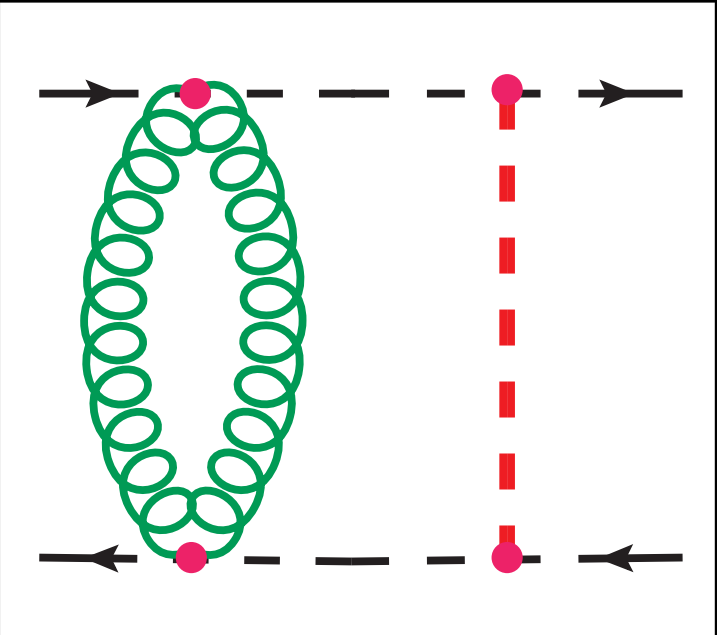
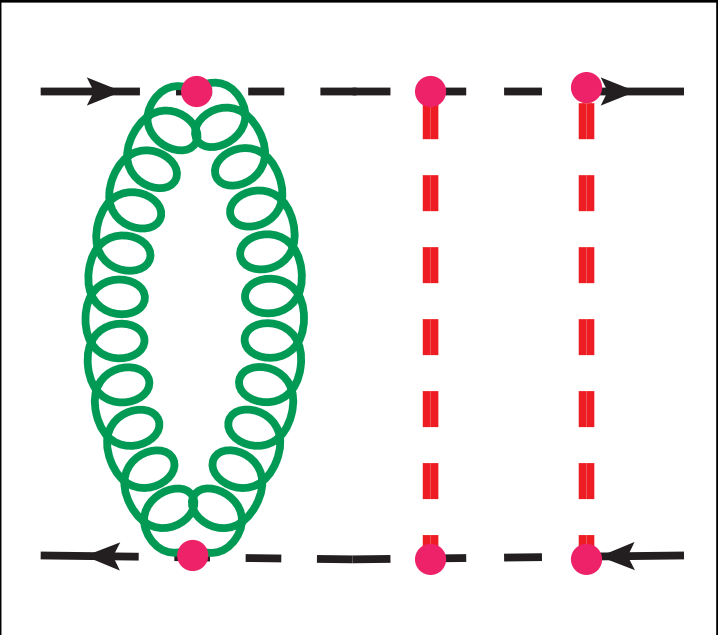
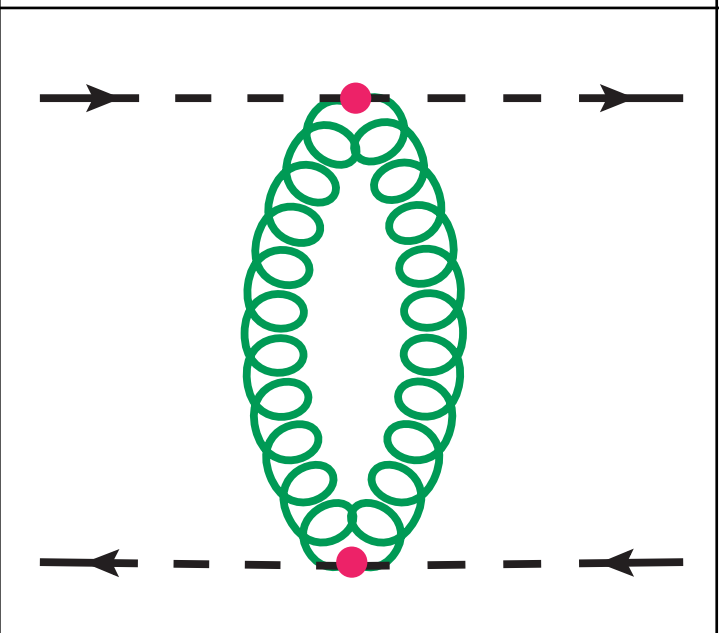
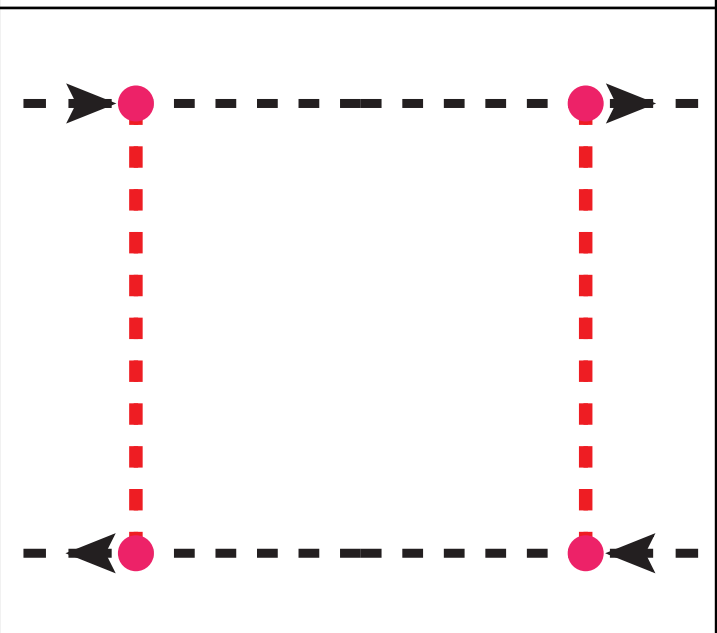
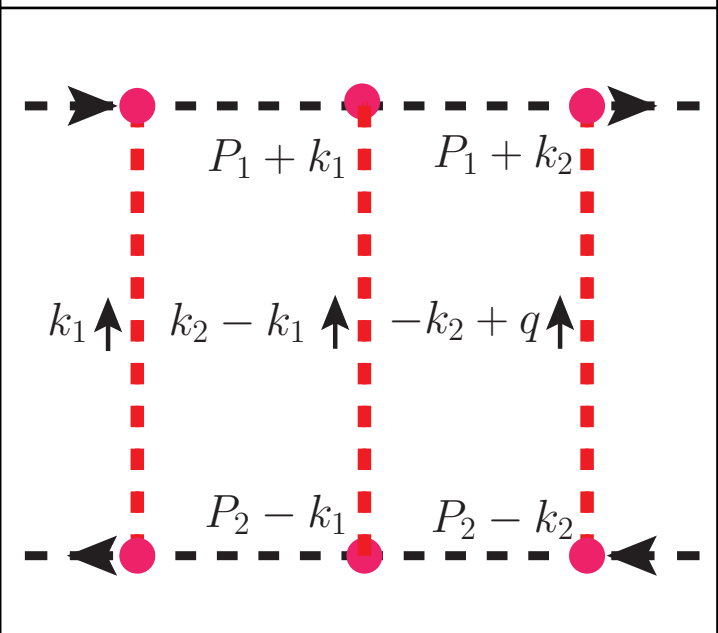
[Brodsky, Hwang and Schmidt (2002)]

The phase arising from the initial- and final-state interactions in QCD is analogous to the Coulomb phase of Abelian QED amplitudes. The Coulomb phase depends on the product of charges and relative velocity of each ingoing and outgoing charged pair [7]. Thus the sign of the phase in DY and DIS are opposite because of the different color charge of the ingoing $\bar{3}_C$ antiquark in DY and the outgoing 3_C quark in DIS. In order to check the sign, we will carry out the DY calculation explicitly in

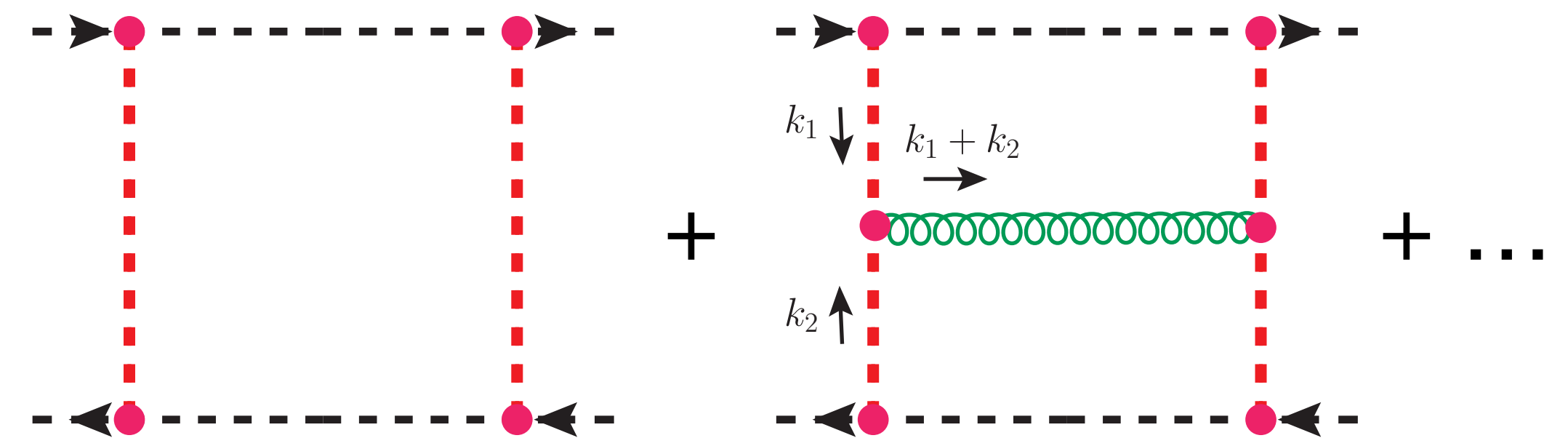
Rapidity Renormalization Group II

Regge Trajectory BFKL & Beyond

SCET_G

LL	NLL	NNLL	...
$a_s^m \ln^m \frac{-t}{s}$	$a_s^m (i\pi) \ln^{m-1} \frac{-t}{s}$	$\alpha_s^m (i\pi)^2 \ln^{m-2} \frac{-t}{s}$	...
			...
			...

Regge Trajectory $\left(\frac{s}{-t}\right)^{1+\alpha(t)} \frac{1}{\nu} \frac{d}{d\nu} \mathbf{S}_8 = -2\gamma_{(1,1)} \mathbf{S}_8$



BFKL Pomeron

LL Pomeron (singlet) RG:

$$\frac{1}{\nu} \frac{d}{d\nu} \mathbf{S}_1 = -\Gamma_{(2,2)} \otimes \mathbf{S}_1 - \mathbf{S}_1 \otimes \Gamma_{(2,2)}$$

ν : rapidity cut-off

Reggeon Field Theory vs. Glauber SCET

Gauge invariant effective action
for high energy processes in QCD

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Petersburg Institute of Nuclear Physics, Gatchina, 188350, St. Petersburg, Russia

and

Deutsches Elektronen-Synchrotron DESY, Hamburg

**An Effective Field Theory for Forward Scattering and
Factorization Violation**

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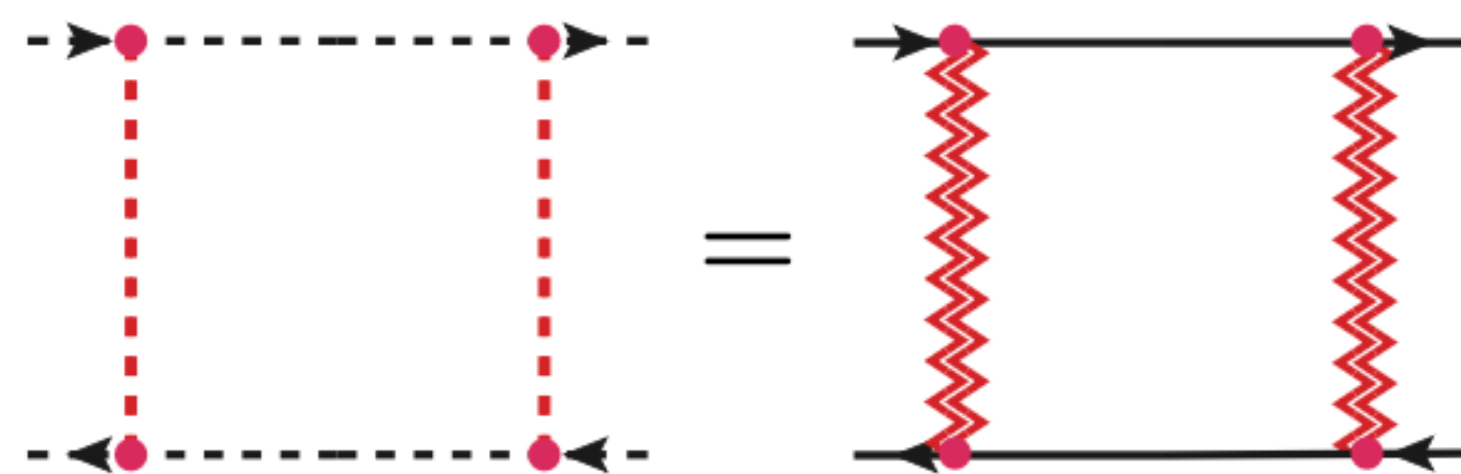
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Non-Renormalization Theorem

$$\text{SCET}_G = \text{EFT}_{\text{Reggeon}} = \text{QCD}_{|t| \ll s}$$

$$\begin{aligned} \text{Re}[\text{SCET}_G] = \text{Re}[\text{EFT}_{\text{Reggeon}}] = & \left(\frac{\tilde{\alpha}_s}{4\pi} \right) \mathcal{S}_8^{q\bar{q}} \left[2C_F \frac{2 - \epsilon + 2\epsilon^2}{\epsilon(-1 + 2\epsilon)} + 2N_f \frac{-1 + \epsilon}{(3 - 2\epsilon)(1 - 2\epsilon)} \right. \\ & \left. + C_A \left(\frac{12 - 21\epsilon + 15\epsilon^2 - 4\epsilon^3}{\epsilon(3 - 2\epsilon)(1 - 2\epsilon)} + 2 \ln \left(\frac{s}{-t} \right) + c_s(\epsilon) - c_n(\epsilon) \right) \right]. \end{aligned} \quad (2.25)$$



The diagram shows two Feynman diagrams separated by an equals sign. The left diagram is a box diagram with dashed lines and red dots at the vertices. The right diagram is a box diagram with wavy lines and red dots at the vertices. The diagrams are equated to a mathematical expression.

$$= \left(\frac{\tilde{\alpha}_s}{4\pi} \right) \frac{-4i\pi}{\epsilon} \left[\mathcal{S}_8^{q\bar{q}} \left(\frac{N_c}{2} - \frac{1}{N_c} \right) + \mathcal{S}_1^{q\bar{q}} \left(\frac{N_c^2 - 1}{4} \right) \right]$$

Virtual Vs. Cut

$$\begin{aligned}
 & \text{Diagram 1} + \text{Diagram 2} + (1, 2) \longleftrightarrow (3, 4) = \\
 & \frac{1}{2} \left(\text{Diagram 1} + \text{Diagram 2} + (1, 2) \longleftrightarrow (3, 4) \right)
 \end{aligned}$$

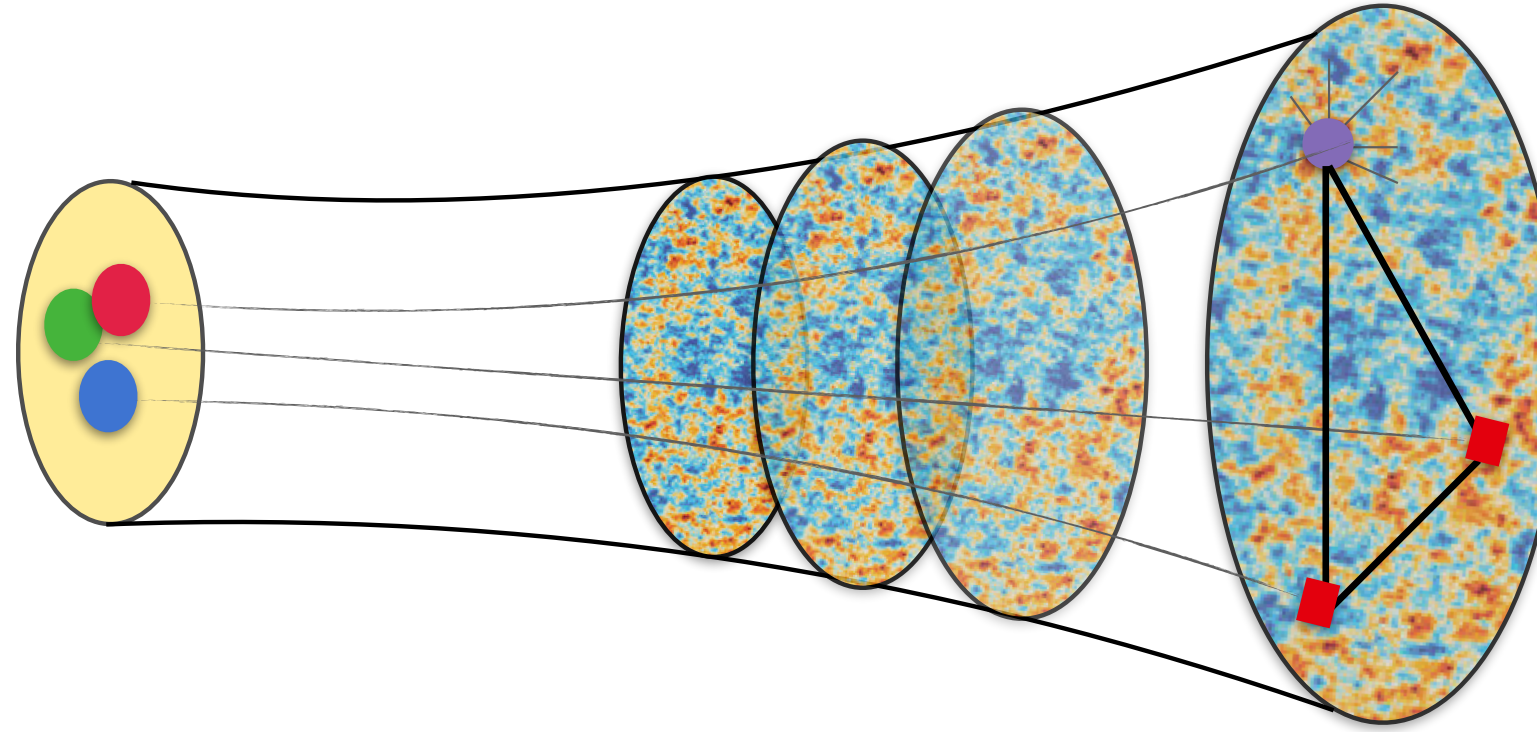
The diagrams show a rectangular loop with a horizontal gluon line. The left side of the loop is a vertical dashed line with two horizontal dashed lines extending from its top and bottom vertices. The right side is a vertical dashed line with two horizontal dashed lines extending from its top and bottom vertices. The top and bottom horizontal lines are dashed. The left vertical line is dashed. The right vertical line is dashed. The horizontal line connecting the two vertical lines is a gluon line (represented by a green wavy line). The top and bottom horizontal lines are dashed. The left vertical line is dashed. The right vertical line is dashed. The horizontal line connecting the two vertical lines is a gluon line (represented by a green wavy line). The top and bottom horizontal lines are dashed. The left vertical line is dashed. The right vertical line is dashed. The horizontal line connecting the two vertical lines is a gluon line (represented by a green wavy line).

The Reggeon & Glauber gluons should be defined in a regulator-independent way!

$$\text{Diagram 1} = \frac{1}{2} \times \text{Diagram 2}$$

The diagrams show a rectangular loop with a horizontal gluon line. The left side of the loop is a vertical dashed line with two horizontal dashed lines extending from its top and bottom vertices. The right side is a vertical dashed line with two horizontal dashed lines extending from its top and bottom vertices. The top and bottom horizontal lines are dashed. The left vertical line is dashed. The right vertical line is dashed. The horizontal line connecting the two vertical lines is a gluon line (represented by a green wavy line). The top and bottom horizontal lines are dashed. The left vertical line is dashed. The right vertical line is dashed. The horizontal line connecting the two vertical lines is a gluon line (represented by a green wavy line). The top and bottom horizontal lines are dashed. The left vertical line is dashed. The right vertical line is dashed. The horizontal line connecting the two vertical lines is a gluon line (represented by a green wavy line).

Why Energy Correlators?



- **a single observable** that embraces (1) hard radiation (2) current fragmentation (3) target fragmentation (4) multi-regge kinematics
- **collinear-safe**, allows consistent matching between perturbative and non-perturbative regions, thus can describe QCD phase transition from free parton gas to non-perturbative confinement
- **strict operator definition** in terms of correlation functions of local operators, which offers a promising avenue to probe the nonperturbative spin structure of the proton from first principles

OutLook

Tame Confined Motion with Energy Correlators

- Lattice QCD computation of Nucleon Energy Correlators
- Energy Correlators Resolving Quark Transversity & Sivers Effects
- Energy Correlators for high-energy diffractive processes

BFKL Program

- Two-loop Non-renormalization for Gluon and Quark Reggeization
- Small-x Resummation: a) sudakov region b) target fragmentation region

Impact of BLNY18 at perturbative region

