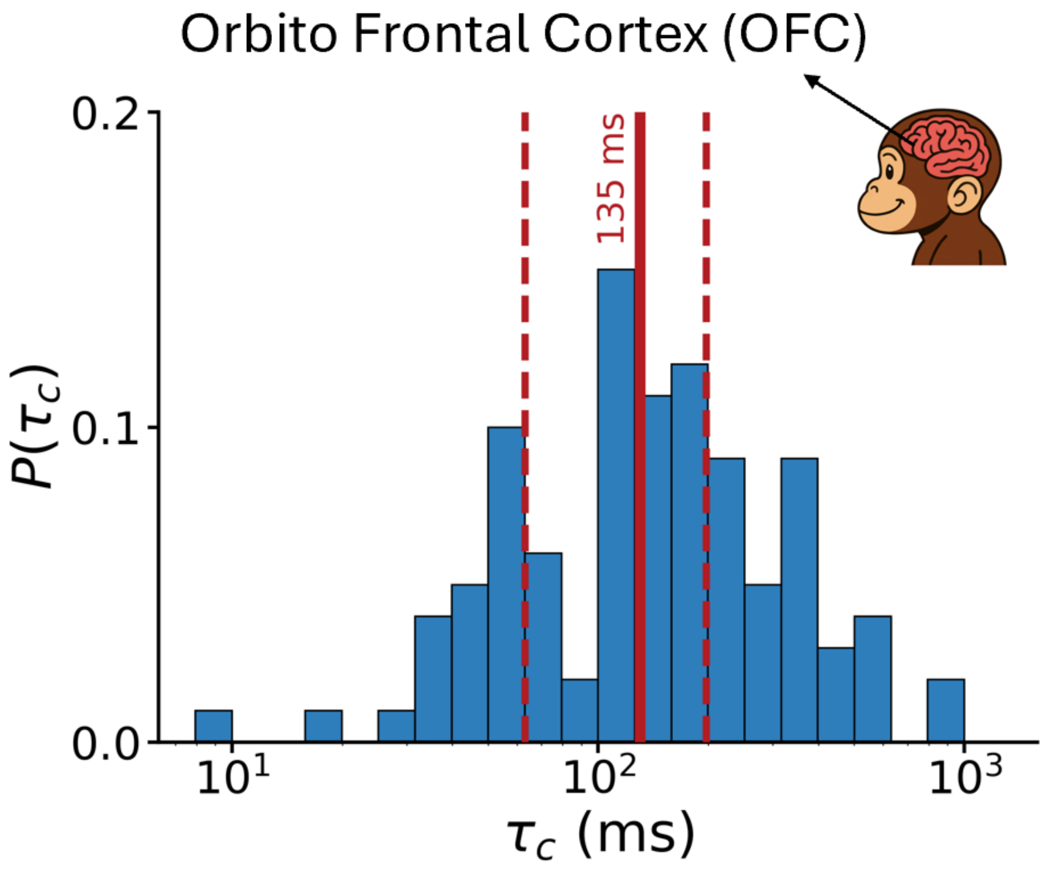
Emergence of Heterogeneity of Intrinsic Timescales in Random Neural Networks

Luca Taffarello

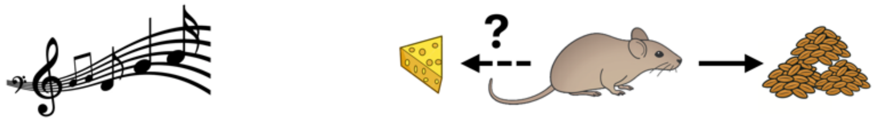
17/09/2025

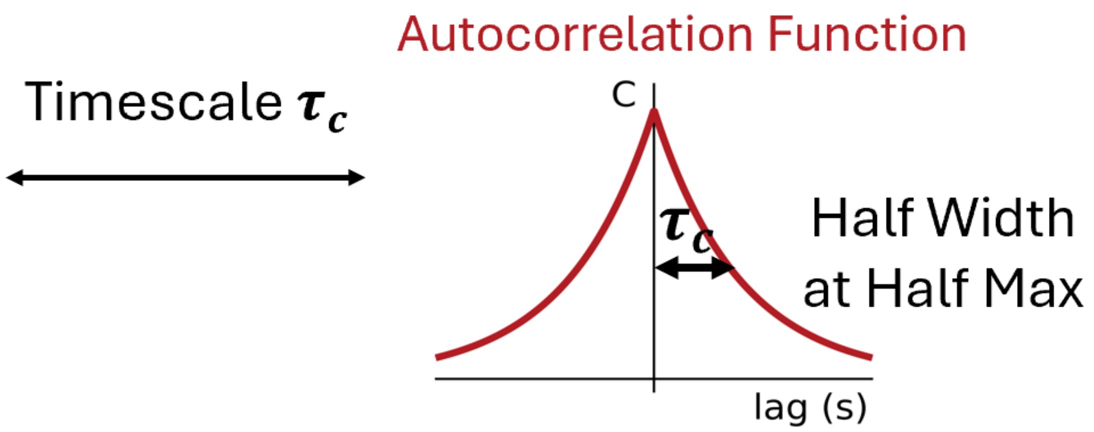
Timescales in Neural Circuits



Cavanagh et al. (2016)

- Cortical neurons exhibit a **broad range of intrinsic timescales**, both *within* and *across* areas, in resting and task-evoked activity
- Leveraging computations at multiple timescales enables **complex tasks** such as memory, learning and decision-making





What are the **neural mechanisms** underlying the emergence of multiple timescales?

From Neural to Network Dynamics

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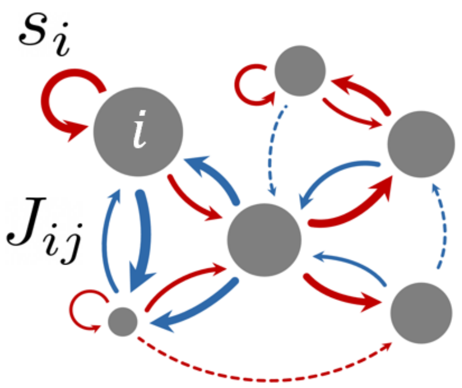
Previous models attribute the emergence of multiple timescales to **neural assemblies** of **heterogeneous sizes:**

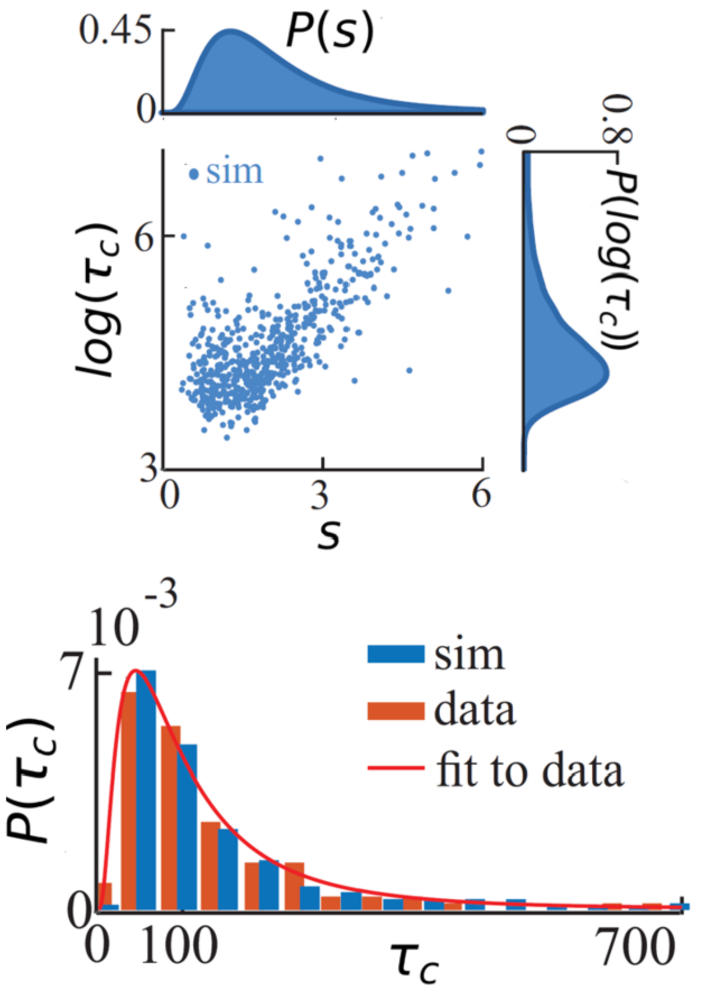


$$\frac{dx_i}{dt} = -x_i + \underbrace{s_i\phi(x_i)}_{\text{CLUSTER SELF-INTERACTION}} + \underbrace{\sum_j J_{ij}\phi(x_j)}_{\text{NETWORK CONTRIBUTION}}$$

Lognormal distribution of self-couplings $\Rightarrow$ Lognormal distribution of timescales

STRUCTURE-FUNCTION RELATION

- Timescale heterogeneity is a **network-level** phenomenon
- **Non-uniform connectivity** generate activity with diverse timescales



Stern, Istrate, Mazzucato (2023)

Empirical Features of Cortical Connectivity

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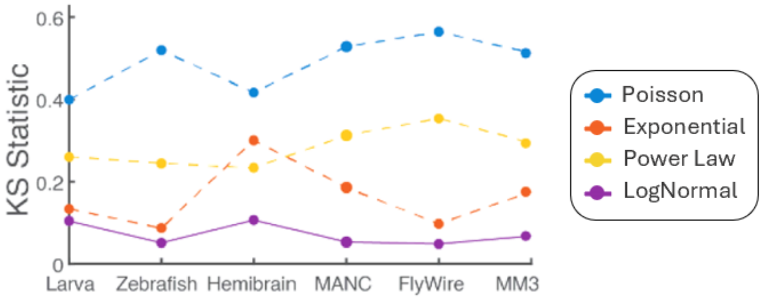
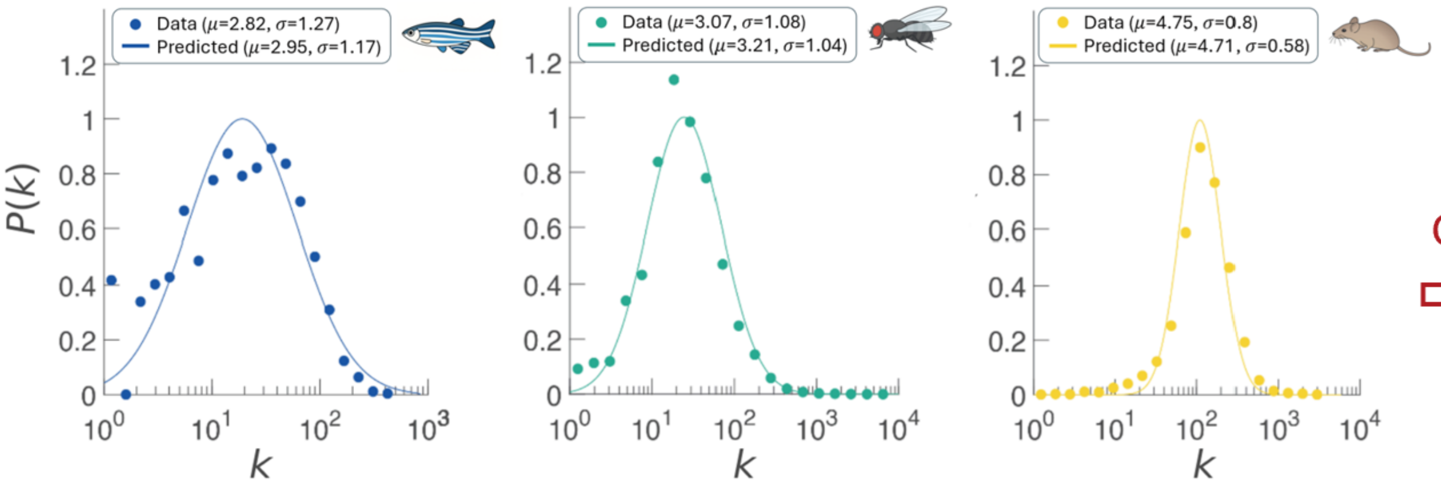
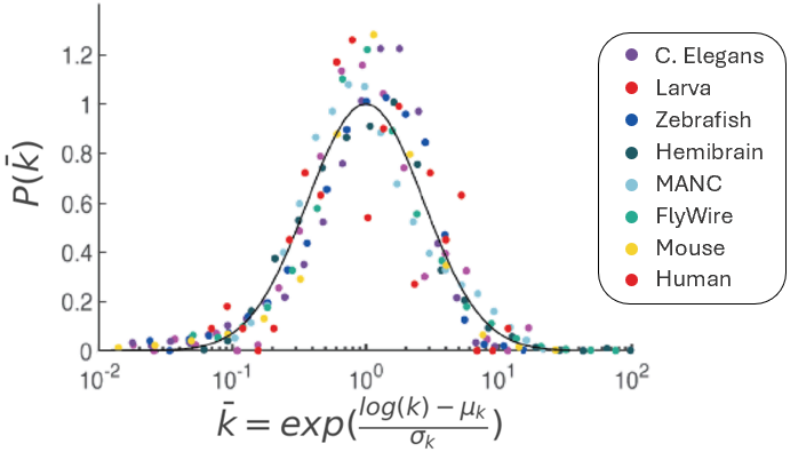
- **Degree heterogeneity:** Experimentally mapped connectomes exhibit **lognormal degree distributions** (consistent with neuronal size being governed by multiplicative processes)



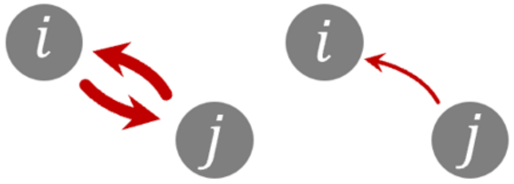


Data Collapse



Piazza, D. L. Barabasi, Castro, Menichetti, A. Barabasi (2025)

- **Partially symmetric connectivity**: Overrepresentation of reciprocal connections, which are on average stronger than unidirectional ones



Martí, Brunel, Ostojic (2018)

Model: Beyond Fully-Connected RNNs

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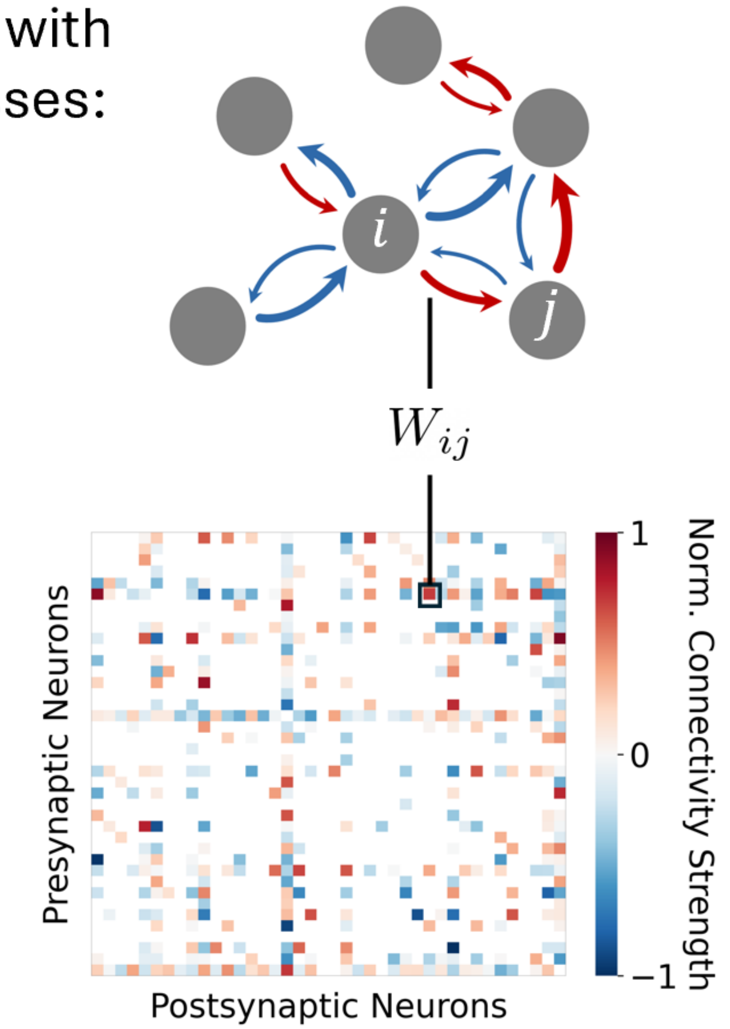
Cortical circuits are modelled as RNNs of firing-rate units, with synaptic couplings determined by two independent random processes:

$$\frac{dx_i}{dt} = -x_i + \sum_j W_{ij}\phi(x_j) \qquad \phi(\cdot) = \tanh(\cdot)$$

$$W_{ij} = (A \odot J)_{ij} = A_{ij}J_{ij}$$



Adjacency Matrix A
(**structure** of interaction)

$$A_{ij} \sim \text{Bernoulli}(p_{ij})$$

$$p_{ij} = \frac{k_i k_j}{N\langle k\rangle} \qquad \boxed{\{k_1, \dots, k_N\} \sim P(k)}$$

DEGREE DISTRIBUTION
(lognormal)

Quenched Disorder J
(**strength** of interactions)

$$J_{ij} \sim \mathcal{N}\left(0, \frac{g^2}{\langle k\rangle}\right)$$

$$\boxed{\text{corr}(J_{ij}, J_{ji}) = \gamma}$$

PARTIAL SYMMETRY

Dynamical Mean Field Theory (DMFT)

DMFT allows us to derive a **self-consistent effective dynamics** for **neurons of degree k**, by averaging over the ensemble of matrices A and J:

$$\dot{x}_k(t) = -x_k(t) + \underbrace{g\sqrt{\frac{k}{\langle k \rangle}}\eta(t)}_{\text{NETWORK INTERACTION}} + \underbrace{\gamma g^2 \frac{k}{\langle k \rangle}\int_0^t \mathrm{d}t' G(t,t')\phi(x_k(t'))}_{\text{NON-MARKOVIAN MEMORY TERM}}$$

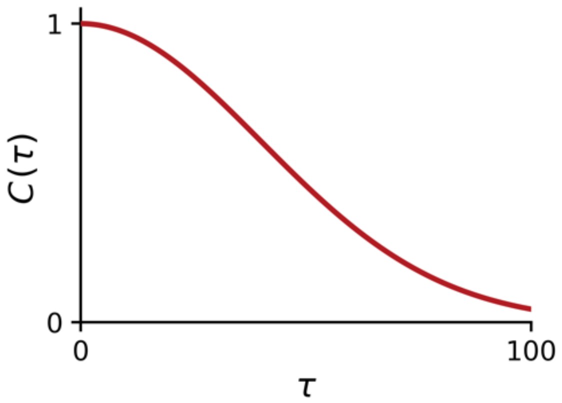
Self-consistent equations:

$$\langle \eta(t) \rangle_\eta = 0$$

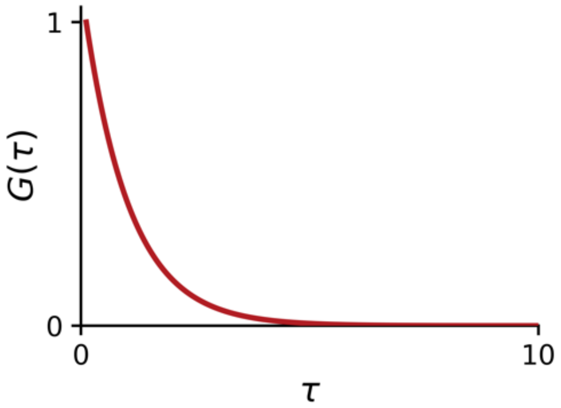
$$\langle \eta(t)\eta(t') \rangle_\eta = \sum_k P(k)\frac{k}{\langle k \rangle}\langle \phi(x_k(t))\phi(x_k(t')) \rangle_\eta$$

$$G(t,t') = \sum_k P(k)\frac{k}{\langle k \rangle}\left\langle \frac{\partial \phi(x_k(t))}{\partial \eta(t')} \right\rangle_\eta$$

Autocorrelation function



Response function



Sompolinsky, Crisanti, Sommers (1988)

J. I. Park, D. S. Lee, S. H. Lee, H. J. Park (2024)

Linear Stability Analysis

Stationary equation:

$$x_k^* - \gamma g^2 \frac{k}{\langle k \rangle} \chi \phi(x_k^*) = g \sqrt{\frac{k}{\langle k \rangle}} \eta^* \qquad \chi = \int \mathrm{d}\tau G(\tau)$$

Critical condition:

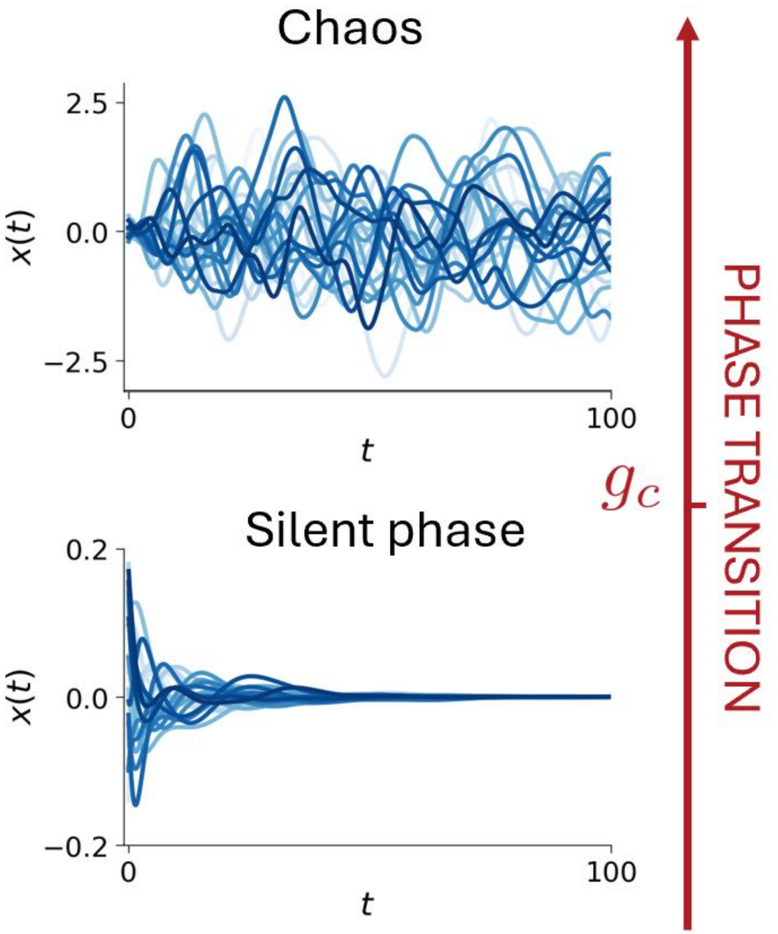
$$1 = g^2 \sum P(k) \frac{k^2}{\langle k \rangle^2} \left\langle (\phi'(x_k^*))^2 \left(\frac{1}{1 - \gamma g^2 \chi \frac{k}{\langle k \rangle} \phi'(x_k^*)} \right)^2 \right\rangle_{\eta^*}$$

$\phi(\cdot) = \tanh(\cdot) \;\Rightarrow\;$ not analytic $P(x^*)$

Stationary distribution (**trivial fixed point** $x_k^* = 0$):

$$P(x_k^*) = \delta(x_k^*) \qquad P(x^*) = \sum_k P(k) P(x_k^*) = \delta(x^*)$$

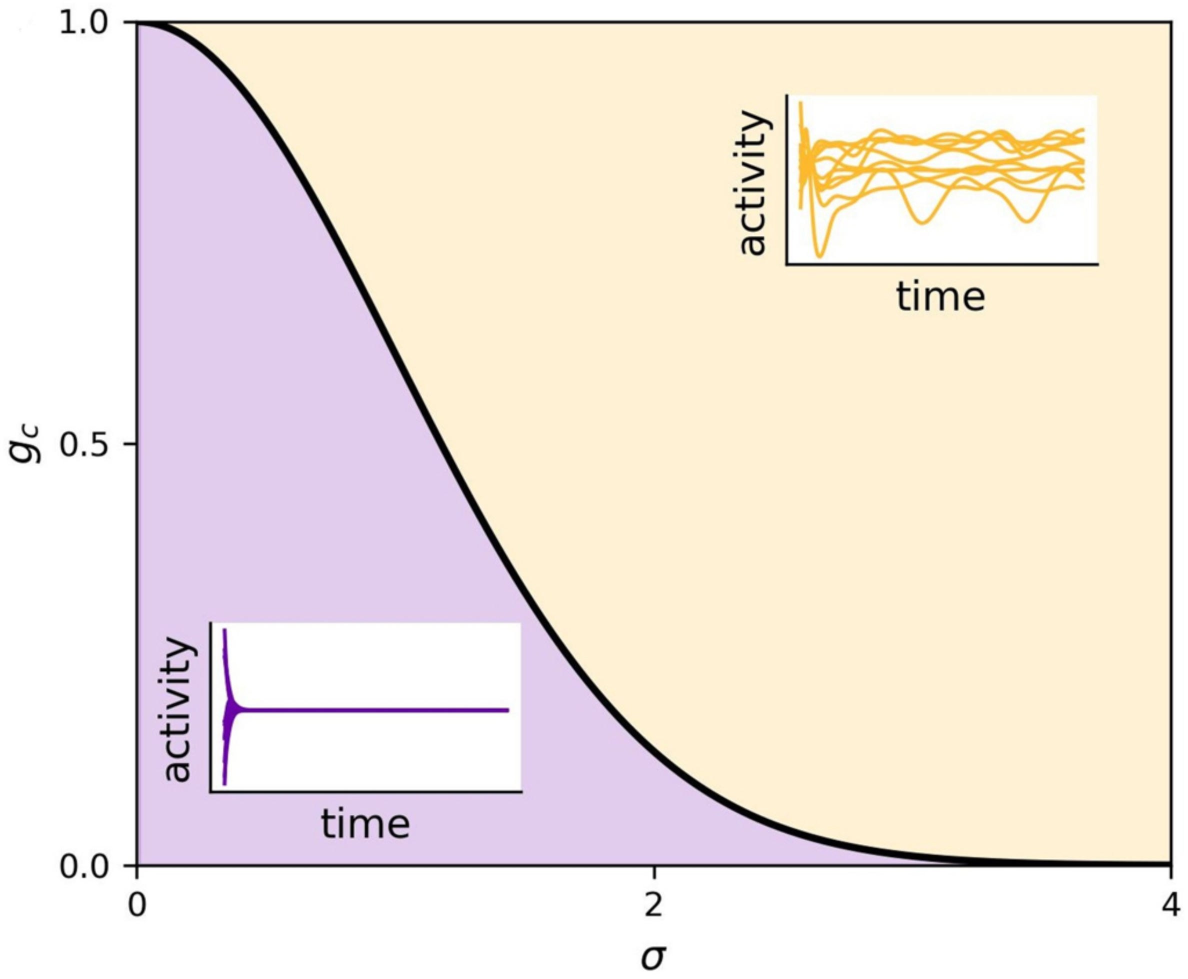
$$1 = g^2 \sum P(k) \frac{k^2}{\langle k \rangle^2} \left(\frac{1}{1 - \gamma g^2 \chi \frac{k}{\langle k \rangle}} \right)^2 \xrightarrow{\gamma = 0} \boxed{g_c = \frac{\langle k \rangle}{\sqrt{\langle k^2 \rangle}}}$$



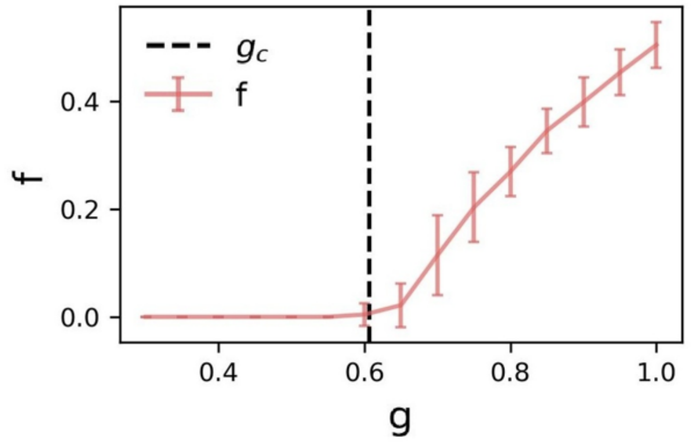
Higher structural heterogeneity $\Rightarrow$ Lower critical gain for transition to chaos

Phase Diagram

$\gamma = 0$ $\quad g_c = \exp\left(-\frac{\sigma^2}{2}\right)$



Theory VS Simulations

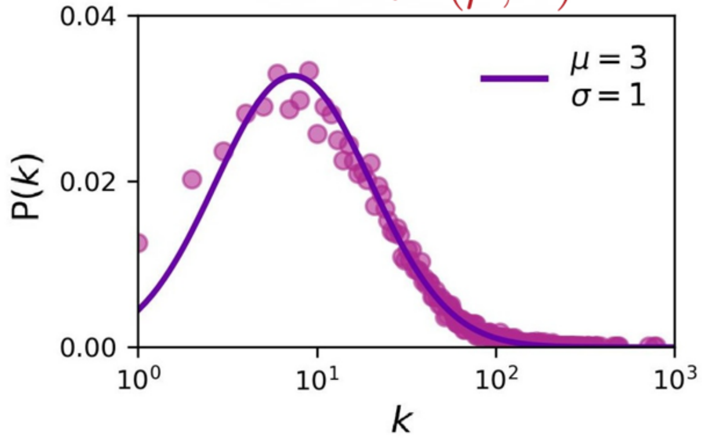


$$f = \left\langle \langle x^2 \rangle - \langle x \rangle^2 \right\rangle_T$$

$f = 0 \quad \Rightarrow$ silence phase

$f > 0 \quad \Rightarrow$ unstable phase

$k \sim \mathcal{LN}(\mu, \sigma)$



Lognormal degree distribution matching the one observed in cortical circuits

Jacobian Spectrum

Stability matrix: $$M_{ij} = -\delta_{ij} + W_{ij}\phi'(x_j^*)\big|_{x_j^*=0} = -\delta_{ij} + W_{ij}$$ $\gamma = 0$

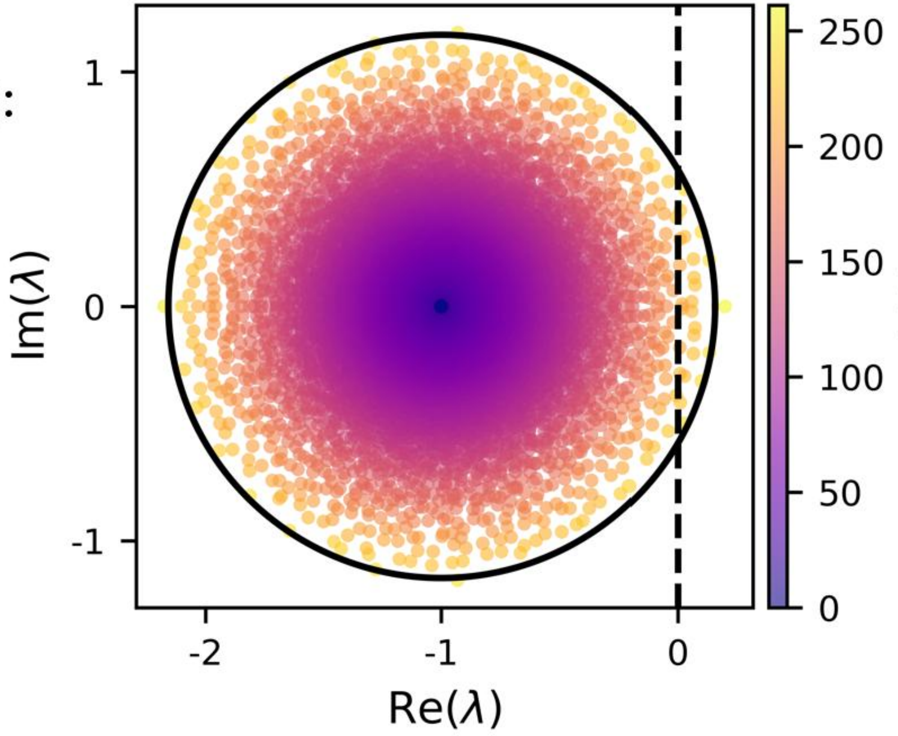
- most **unstable modes** localized on **high-degree nodes**
- **non-uniform spectral density** due to structural heterogeneity

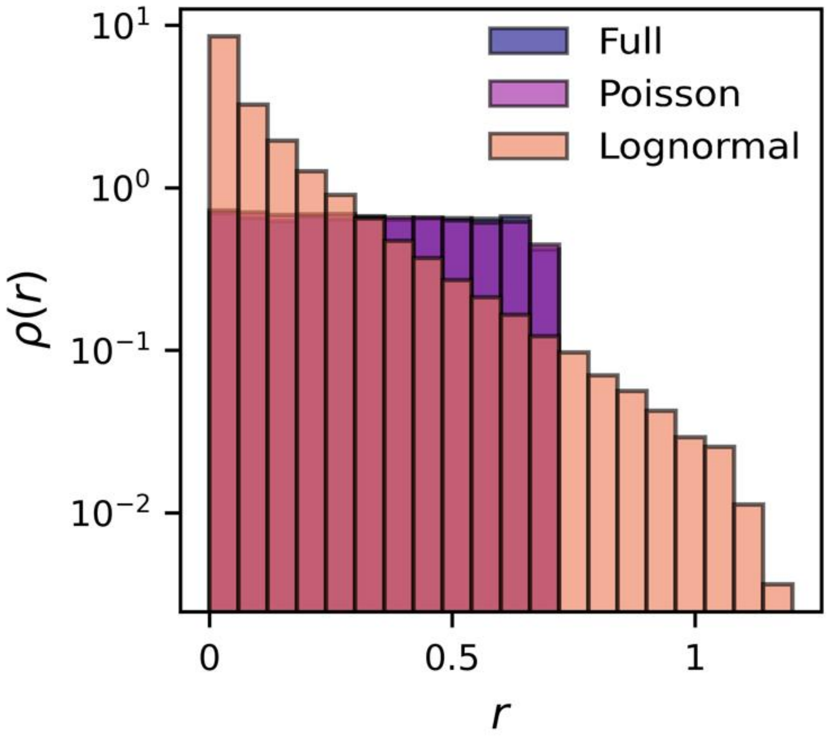
Spectral Radius (Random Matrix Theory):

$$\mathcal{R} = g\frac{\sqrt{\langle k^2\rangle}}{\langle k\rangle}$$

Degree-Weighted Localization:

$$f_k(\lambda_j) = \frac{\sum_i k_i |\psi_i^{(j)}|^2}{\sum_i |\psi_i^{(j)}|^2}$$





Harris, Meffin, Burkitt, Peterson (2023)

Effective Self-Couplings

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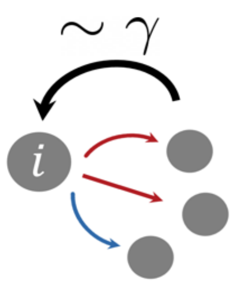
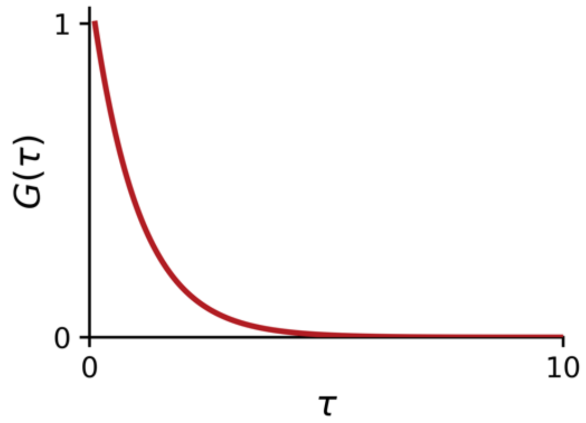
$$\dot{x}_k(t) = -x_k(t) + g\sqrt{\frac{k}{\langle k \rangle}}\eta(t) + \gamma g^2 \frac{k}{\langle k \rangle}\int_0^t \mathrm{d}t' G(t,t')\phi(x_k(t'))$$

$$\Big\downarrow \; G(\tau) \simeq \chi\delta(\tau)$$

$$\dot{x}_k(t) = -x_k(t) + g\sqrt{\frac{k}{\langle k \rangle}}\eta(t) + s_k\phi(x_k(t)) \qquad \boxed{s_k = \gamma g^2 \frac{k}{\langle k \rangle}\chi}$$



EFFECTIVE SELF-COUPLINGS



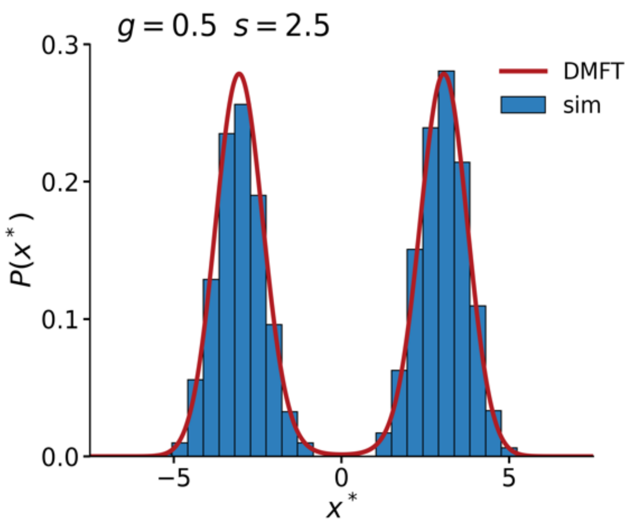
Stationary distribution (**UCNA** approx.):

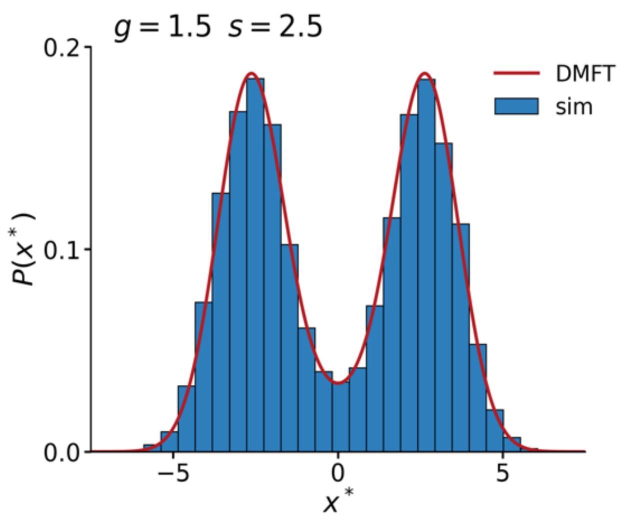
$$P(x_k^*) \propto \frac{1}{1 + \tau_\zeta U''(x_k^*)} \exp\left(-\frac{2U(x_k^*)}{D}\right)$$

$$U(x) = \frac{1}{2}x^2 - s_k \ln\left(\cosh(x)\right)$$

Stern, Istrate, Mazzucato (2023)

Bistable Units





Emergence of timescale

Partially symmetric coupligs

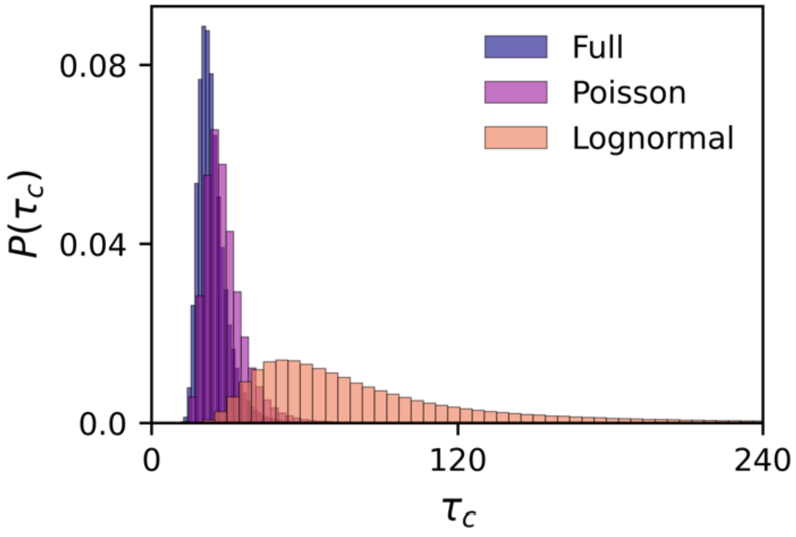
+

Heterogeneous degree distribution

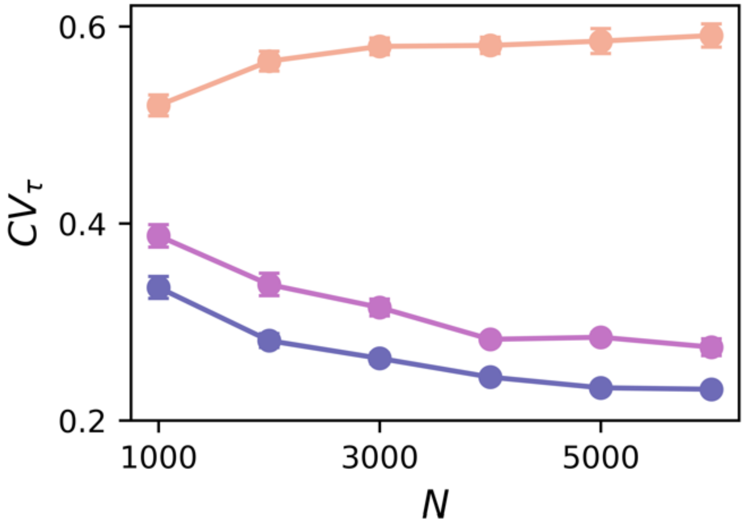
⇓

Emergence of **heterogeneous intrinsic timescales**

Timescale distribution



Scaling with N

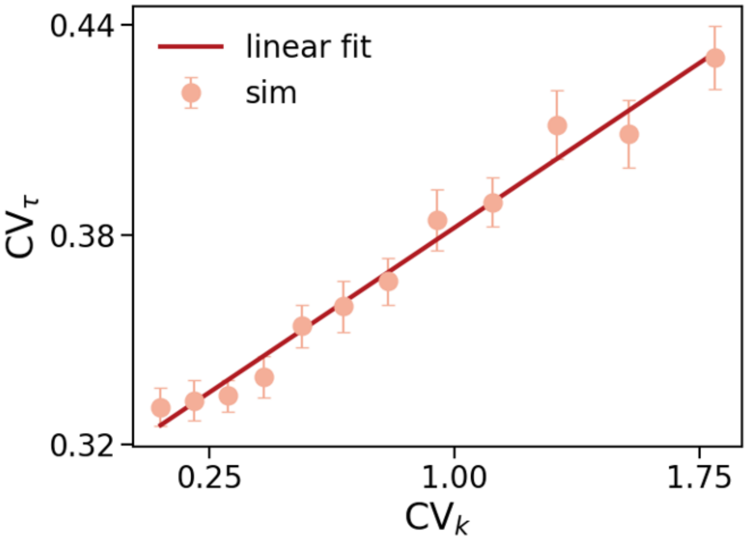


Nodes with higher degree ⇒ Slower dynamics (longer timescale)

Higher structural heterogeneity ⇒ Broader timescale distribution

$$CV_\tau = \frac{\sigma_\tau}{\mu_\tau}$$

$$CV_k = \sqrt{e^{\sigma^2} - 1}$$

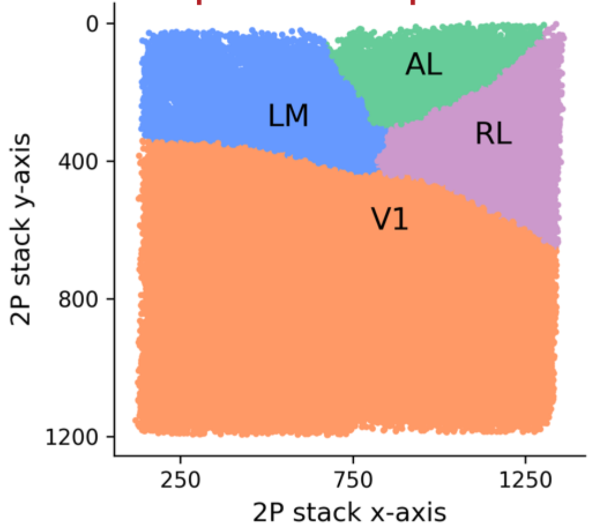


MICrONS Dataset

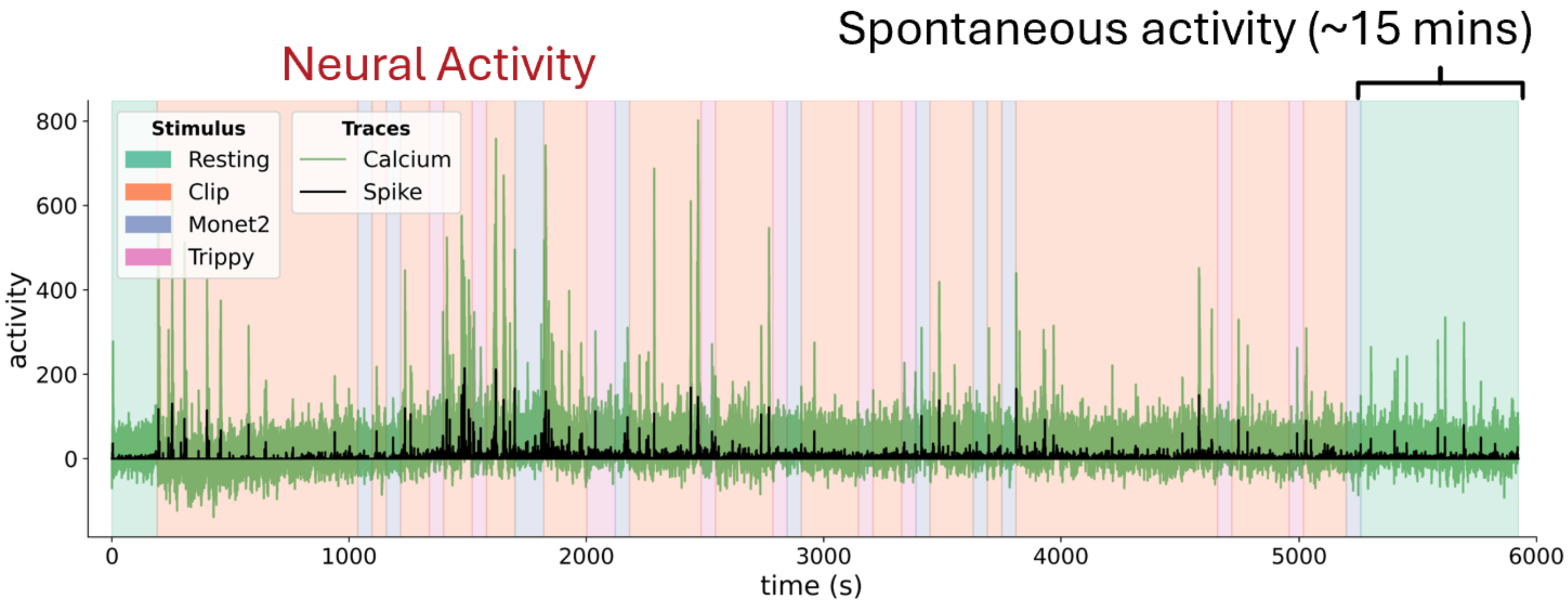
- ~75,000 functionally recorded neurons in **mouse visual cortex** (V1, RL, LM, AL) with 2-photon calcium imaging
- ~200,000 structurally reconstructed neurons with electron microscopy (~500M synapses)

⇒ **Key subset: ~15,000 matched neurons** (~2,000 proofread) with both **functional activity** and **structural connectivity**

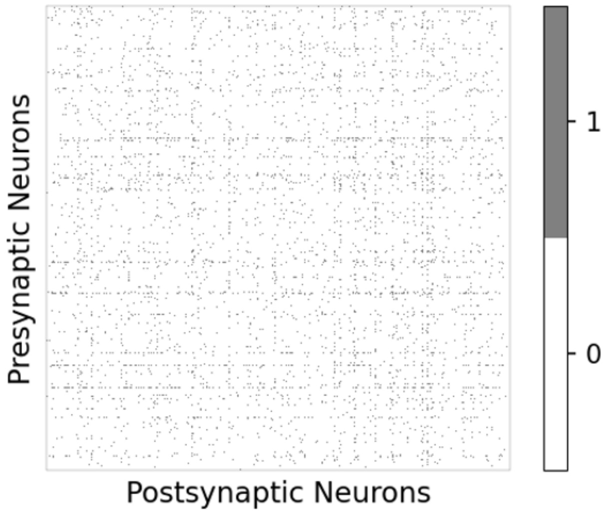
Spatial Map



Neural Activity



Adjacency Matrix



Timescale vs Degree: MICrONS Data

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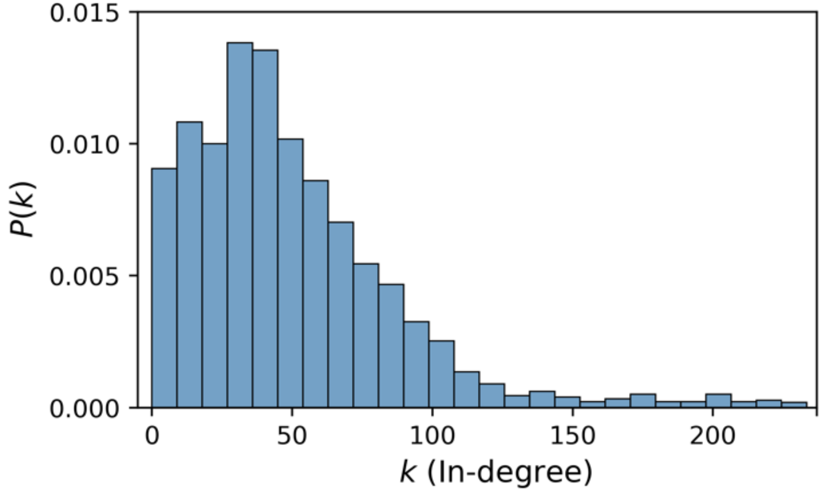
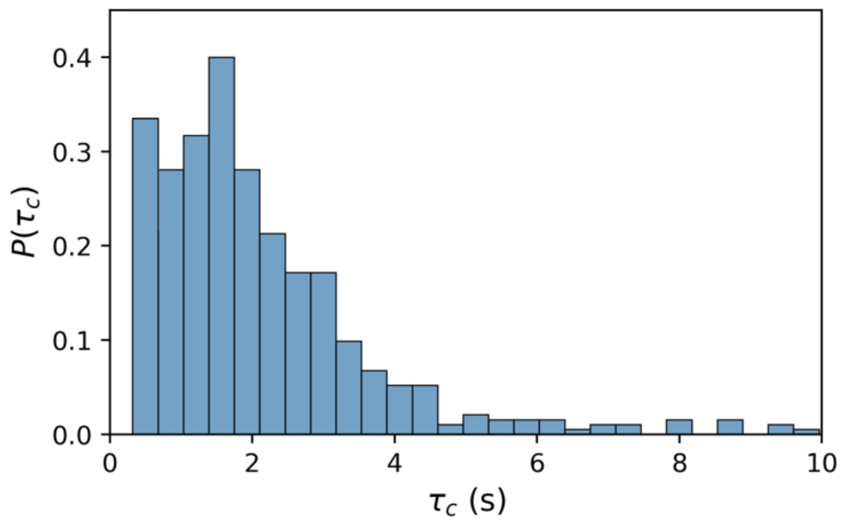
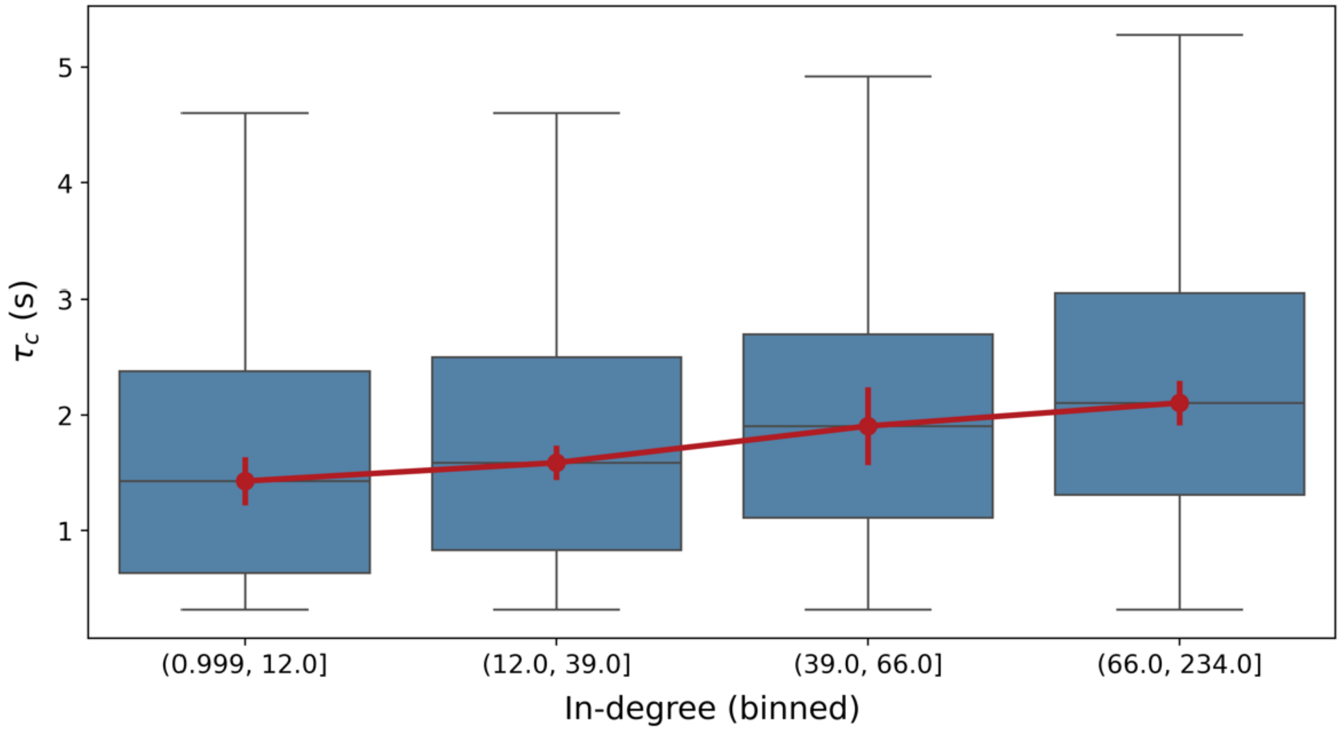
Degree distribution



Timescale distribution



Neurons with higher degree $\Rightarrow$ Slower activity (longer timescale)



Summary

- **Degree heterogeneity** and **partial symmetry** provide biologically plausible ingredients for the emergence of heterogeneous timescales
- **Heterogeneous DMFT** offers a powerful analytic framework to study dynamics in structured random networks
- Reciprocal connections induce **effective self-couplings** that scale with degree, making hubs the units with longer timescales
- Experimental data reveal a **relation between degree and timescale** in line with the model's predictions
- This mechanism may represent a **general principle** linking network structure to the emergence of a **hierarchy of timescales** across cortical areas

Thank to...

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