Unraveling the temporal dependence of ecological interaction measures

Finanziato dall'Unione europea
NextGenerationEU

Ministero dell'Università e della Ricerca

Italiadomani
PIANO NAZIONALE DI RIPRESA E RESILIENZA

Università degli Studi di Padova

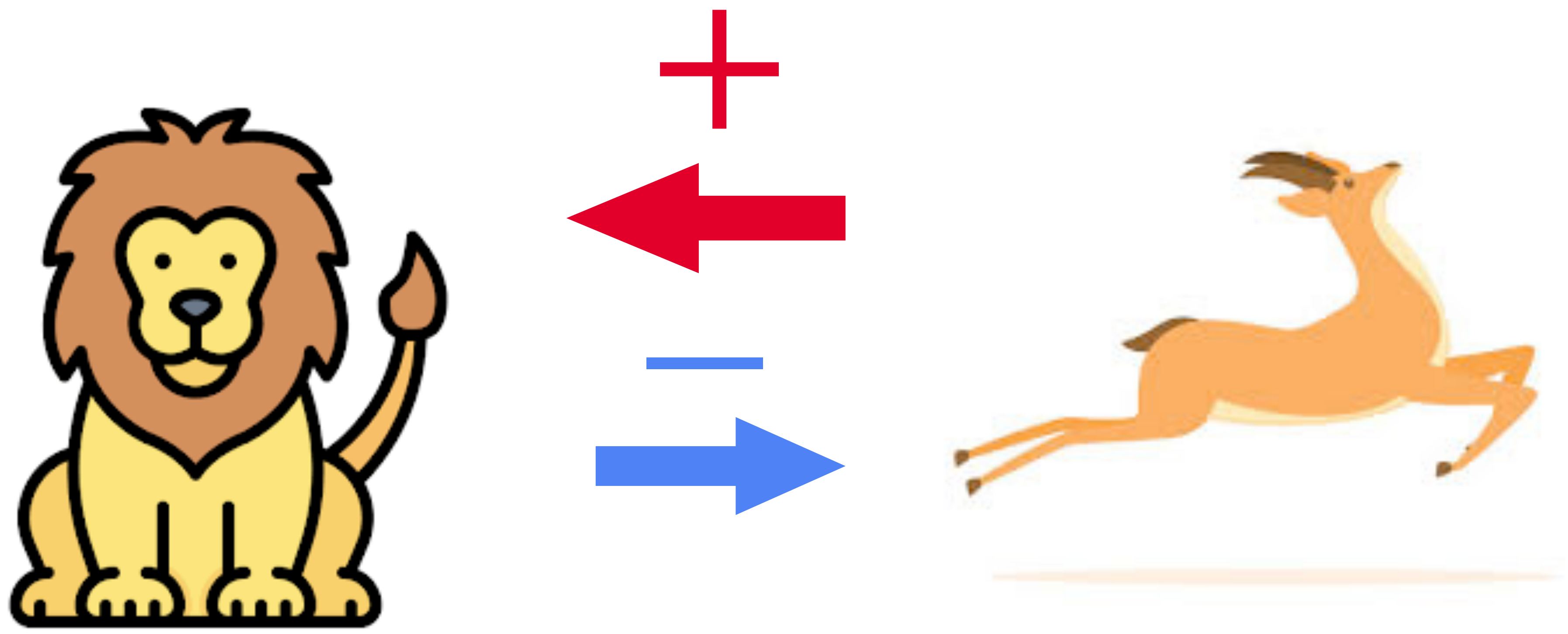
Laboratory of Interdisciplinary Physics

Interactions in ecology:

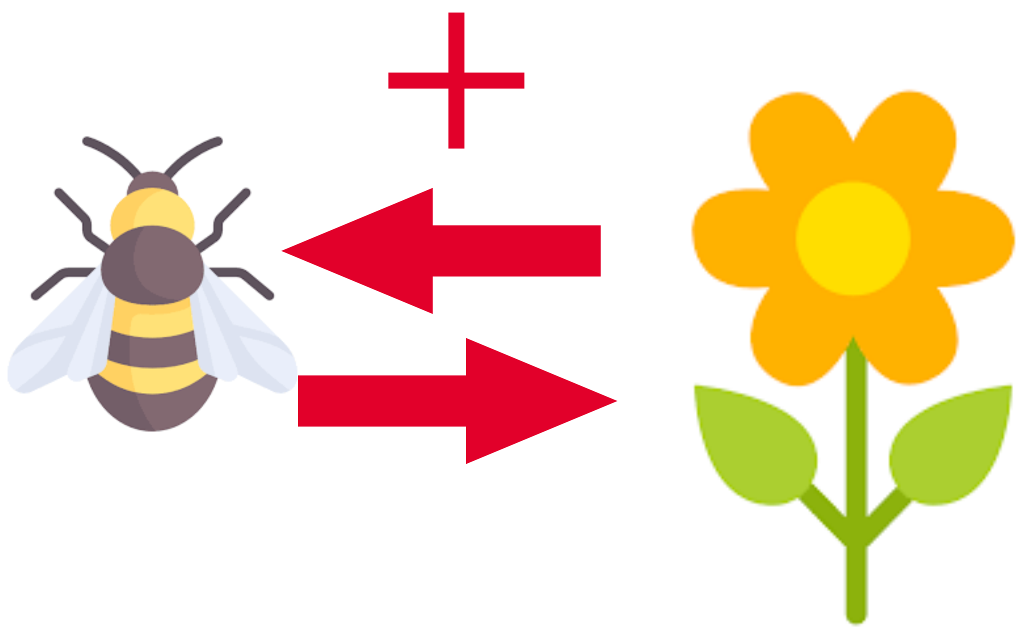
“Effect that a pair of organisms living together in a community have on each other.”

Ecological interactions are both diverse and ever-present

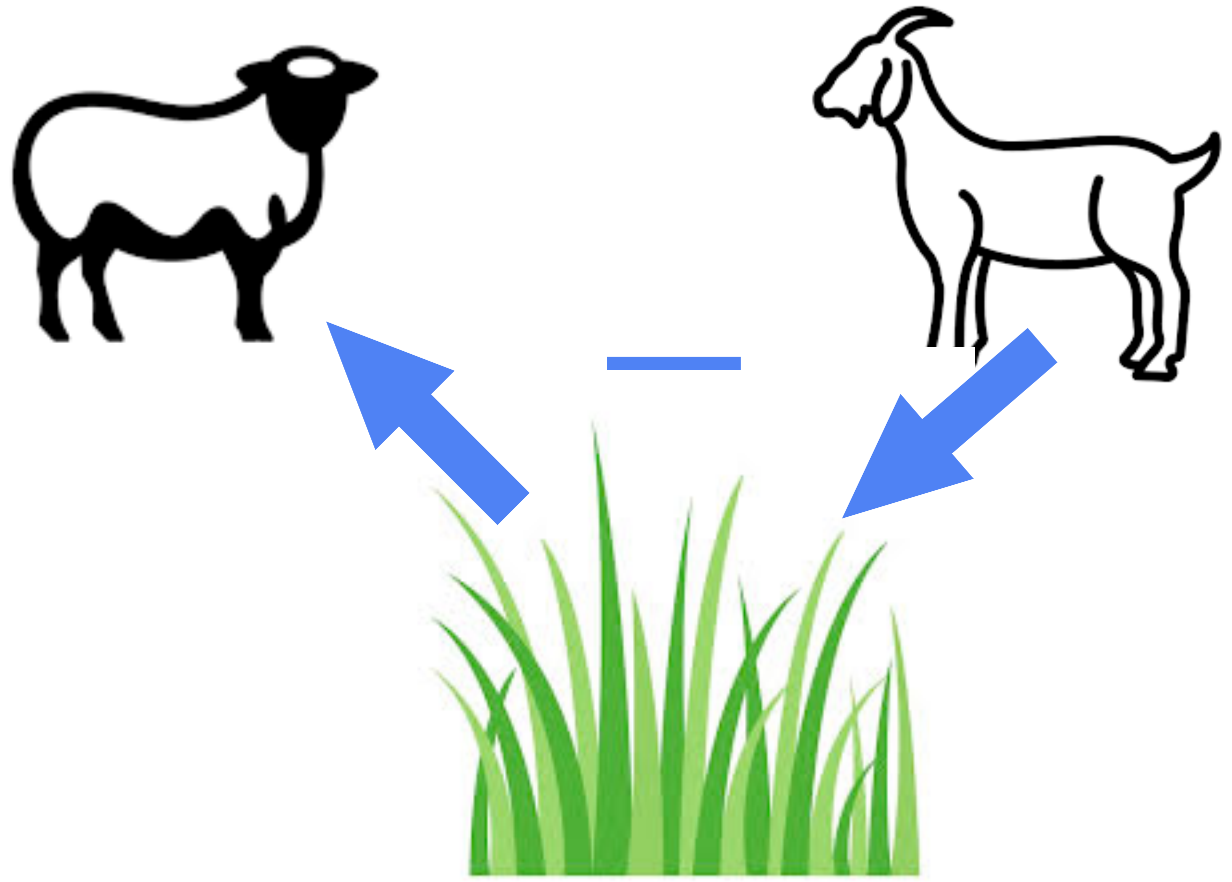
Predator - prey



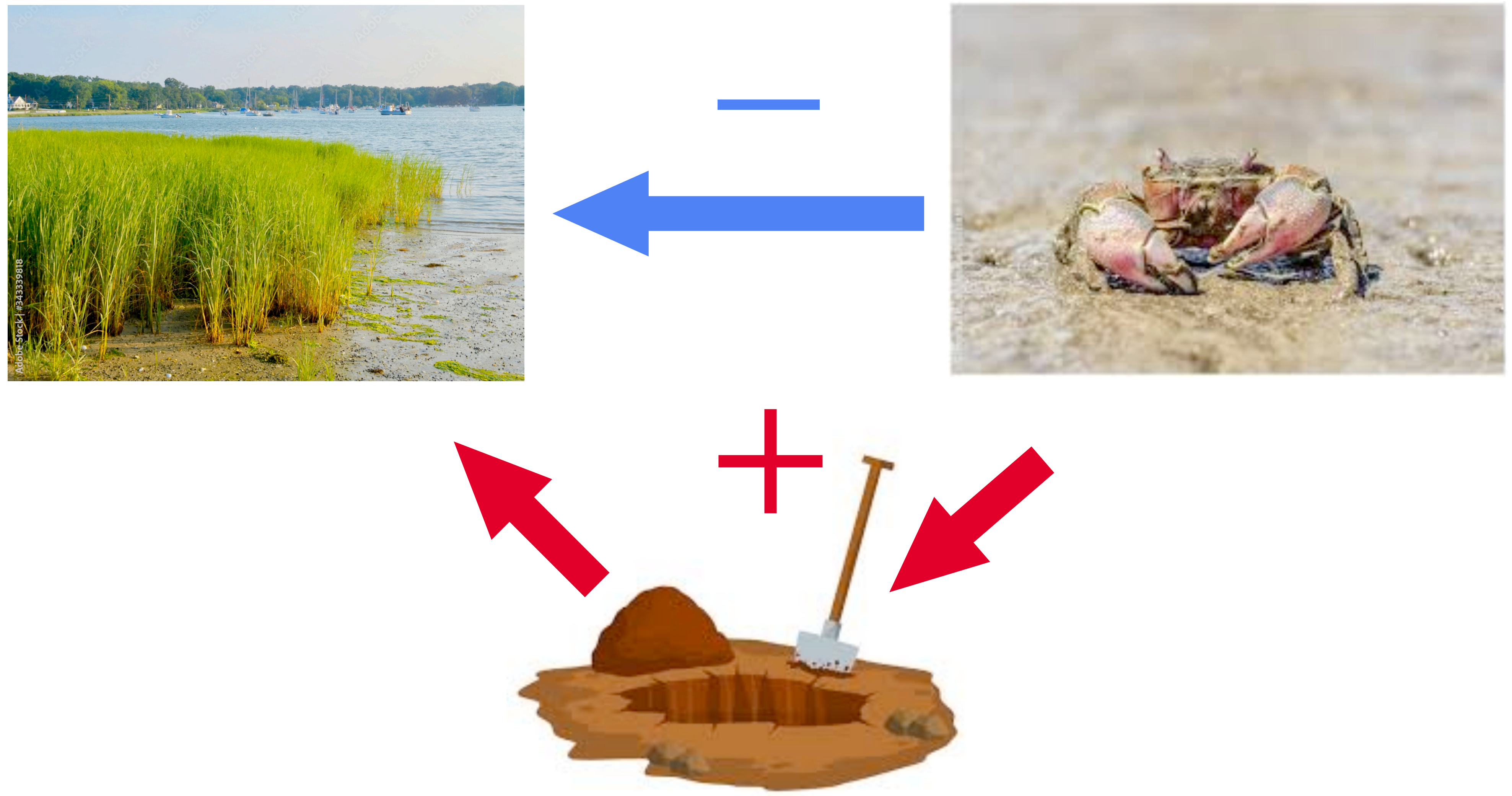
Mutualism

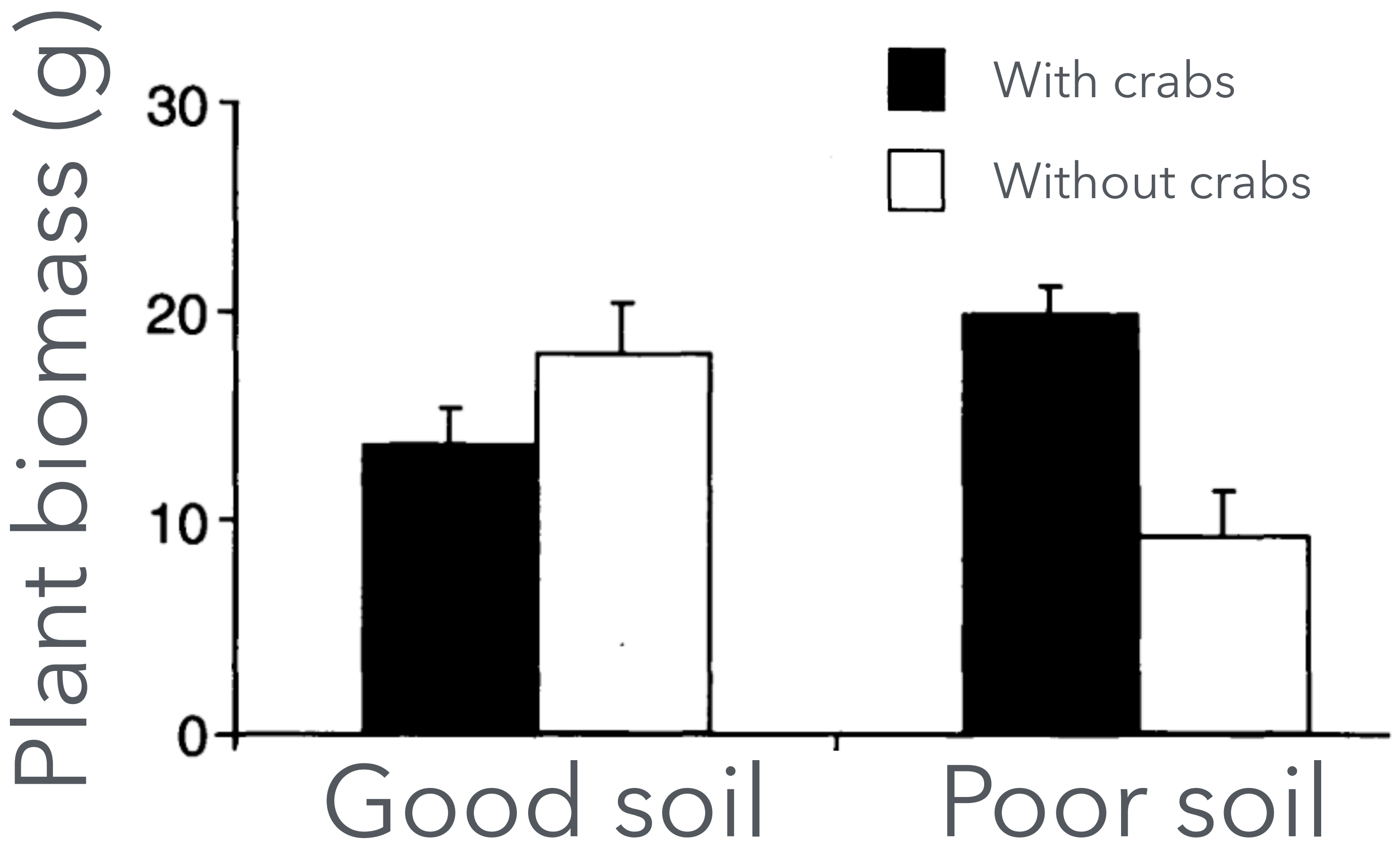


Mediated



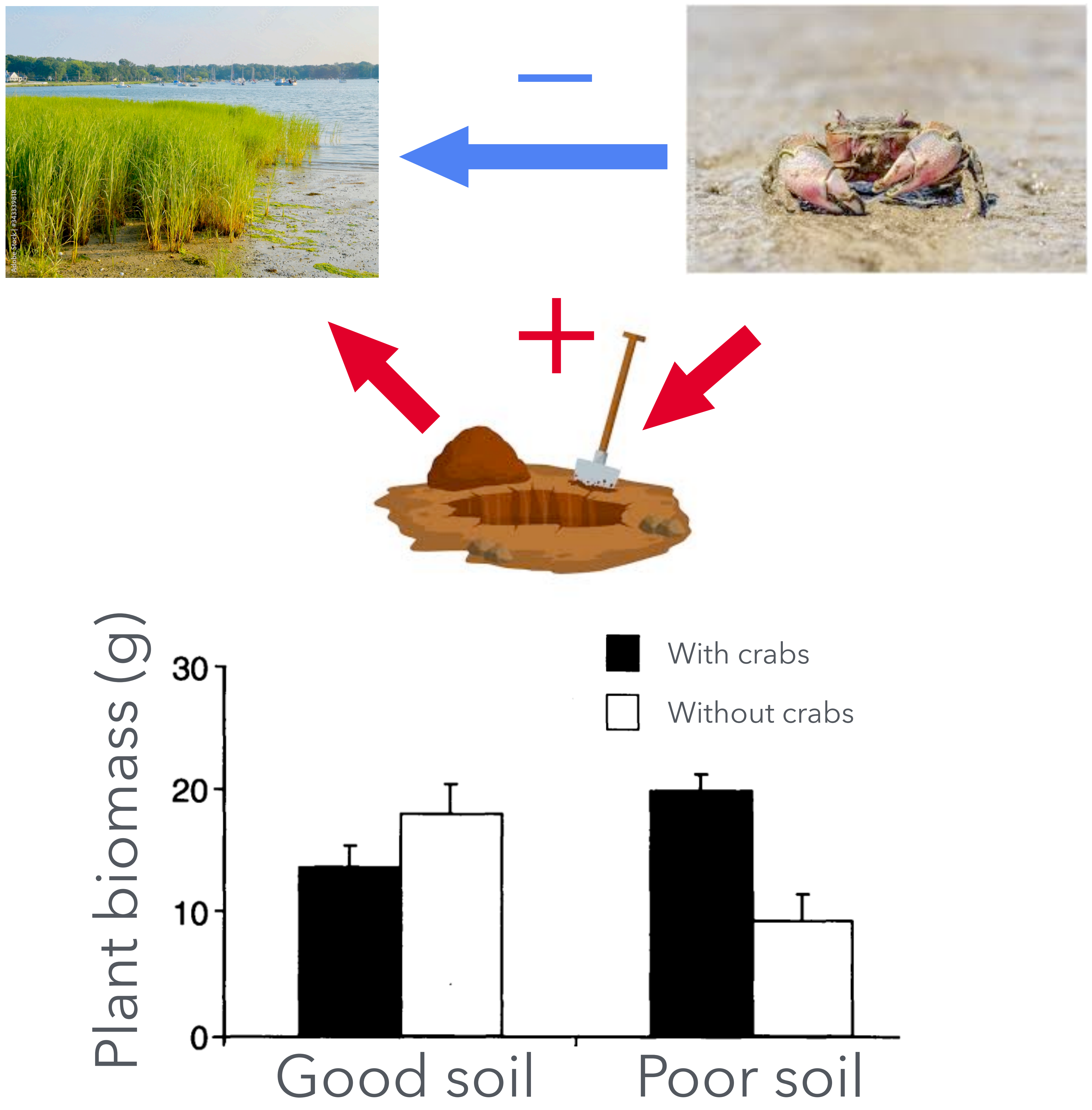
Interactions are contextual: Stress-gradient hypothesis

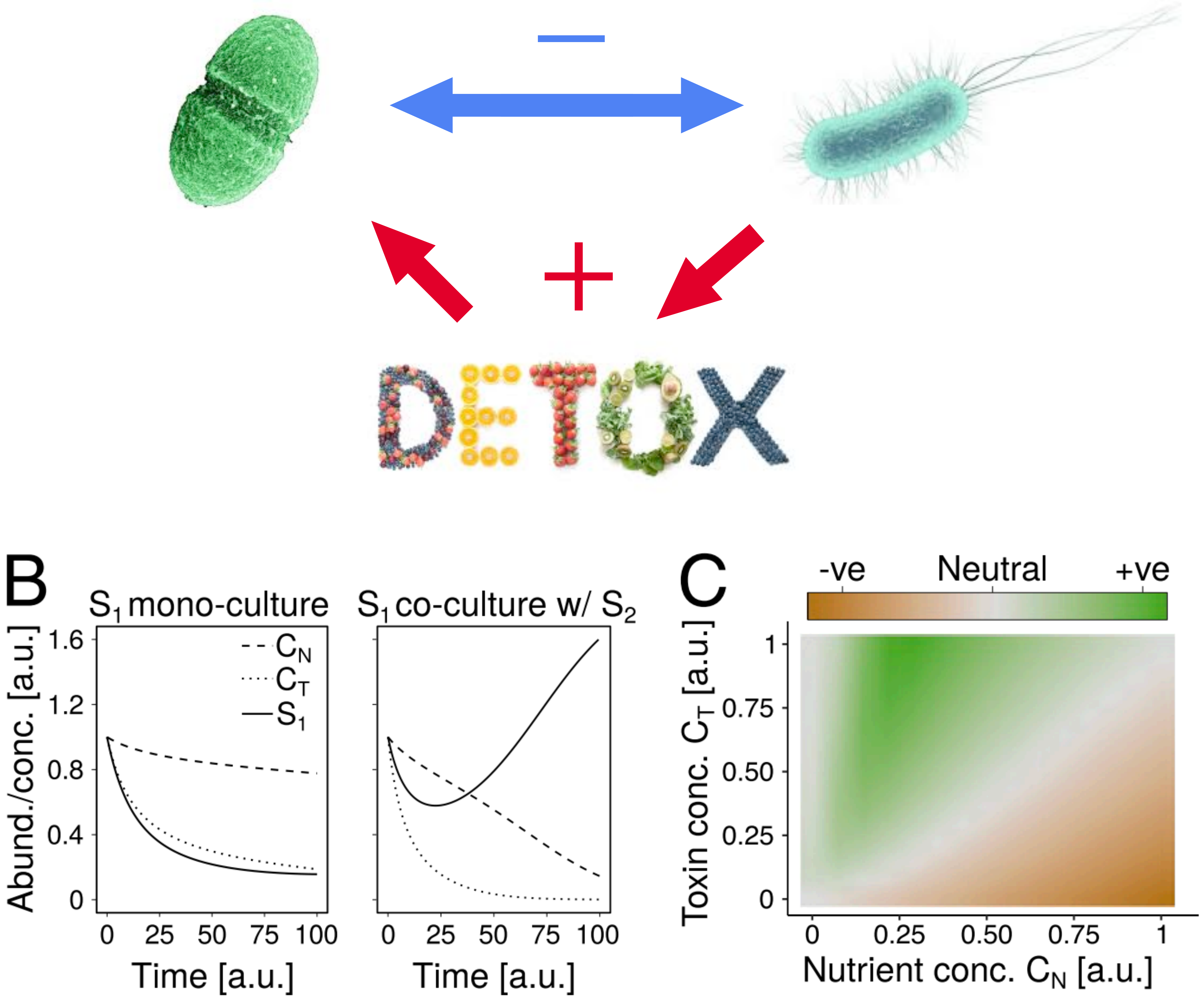




Daleo and Iribarne, 2009

Interactions are contextual: Stress-gradient hypothesis





Daleo and Iribarne, 2009

Piccardia, Vessmana, and Mitri, 2019

Measures of interactions: Perturbation experiments

Measures of interactions: Perturbation experiments

Measures of interactions: Perturbation experiments

$$\mathscr{M}^{(E)}(\text{🐐} \rightarrow \text{🐑}) = g^{(E)}\left[\text{Perturbed}\right] - g^{(E)}\left[\text{Baseline}\right]$$

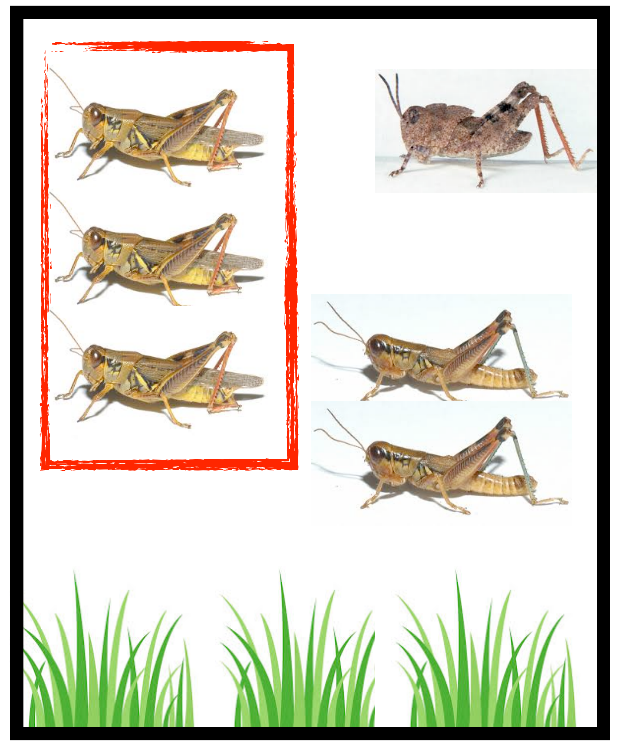
Measures of interactions: Perturbation experiments

M. femur-rubrum

S. collare

P. nebrascensis

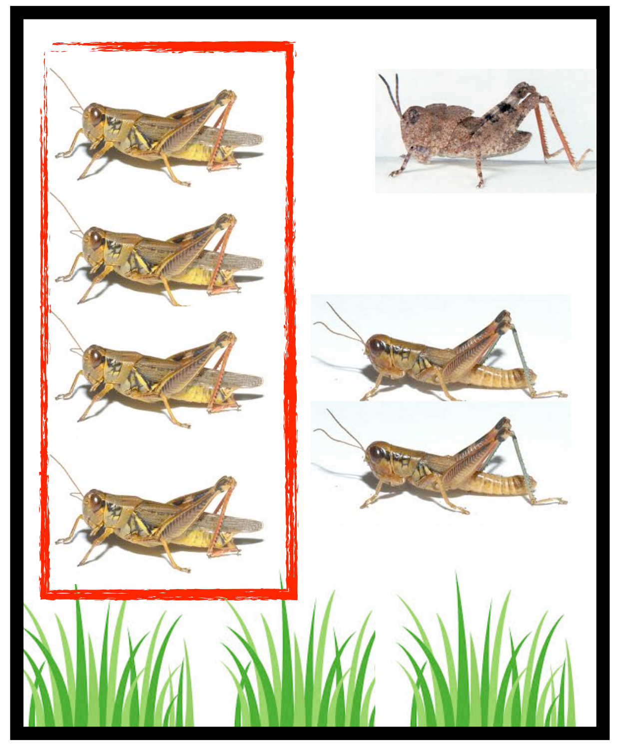
Initial
Condition 1



Experiment duration

$g^{(E)}\,[\text{Baseline}]$

Initial
Condition 2



Experiment duration

$g^{(E)}\,[\text{Perturbed}]$

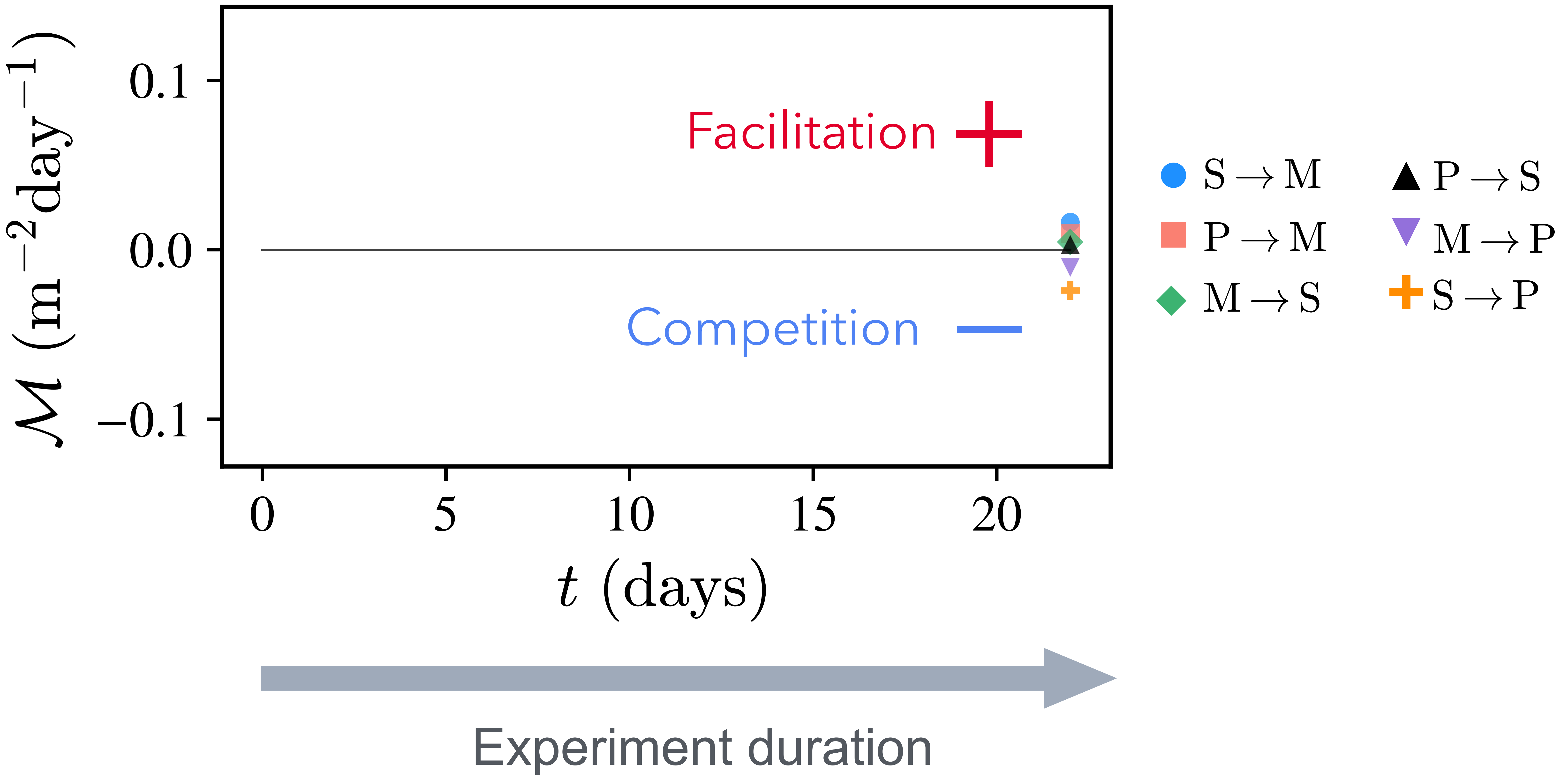
Ritchie and Tilman, 1993

Measures of interactions: Perturbation experiments

M. femur-rubrum

S. collare

P. nebrascensis



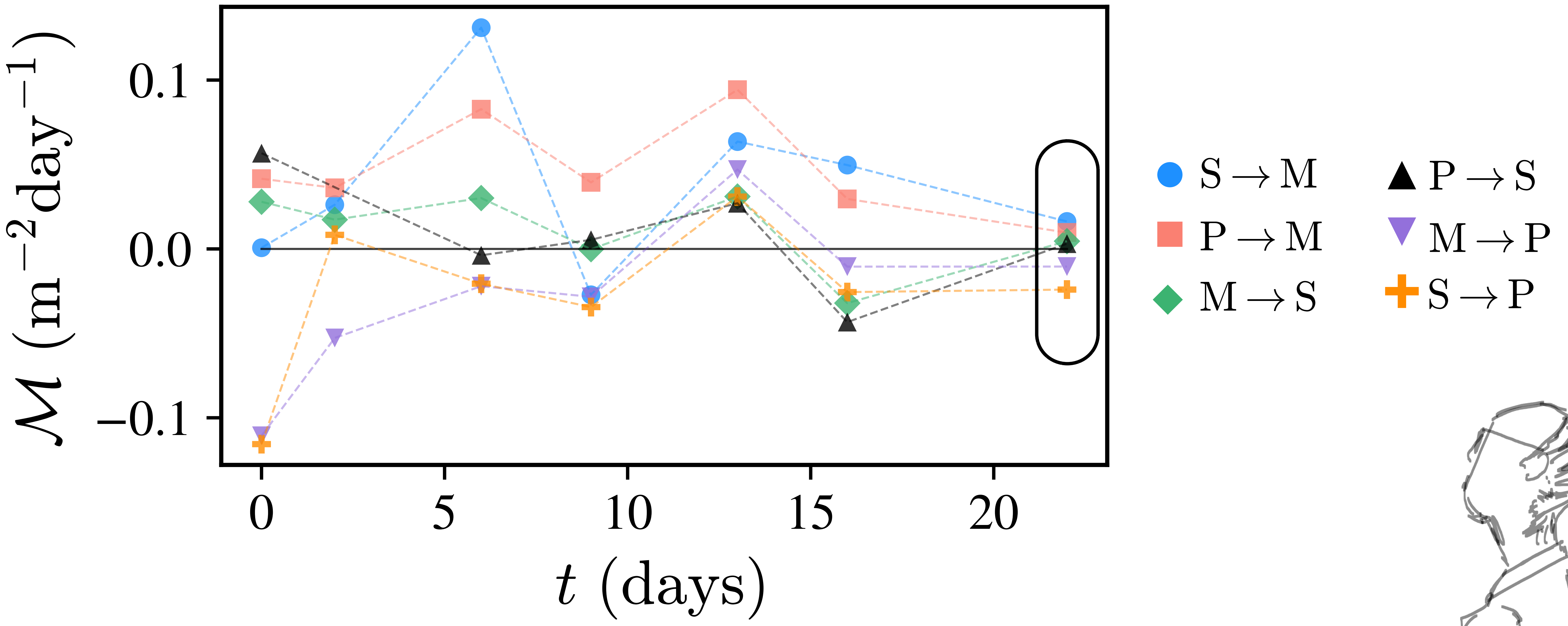
Ritchie and Tilman, 1993

Measures of interactions: Perturbation experiments

M. femur-rubrum

S. collare

P. nebrascensis



Ritchie and Tilman, 1993

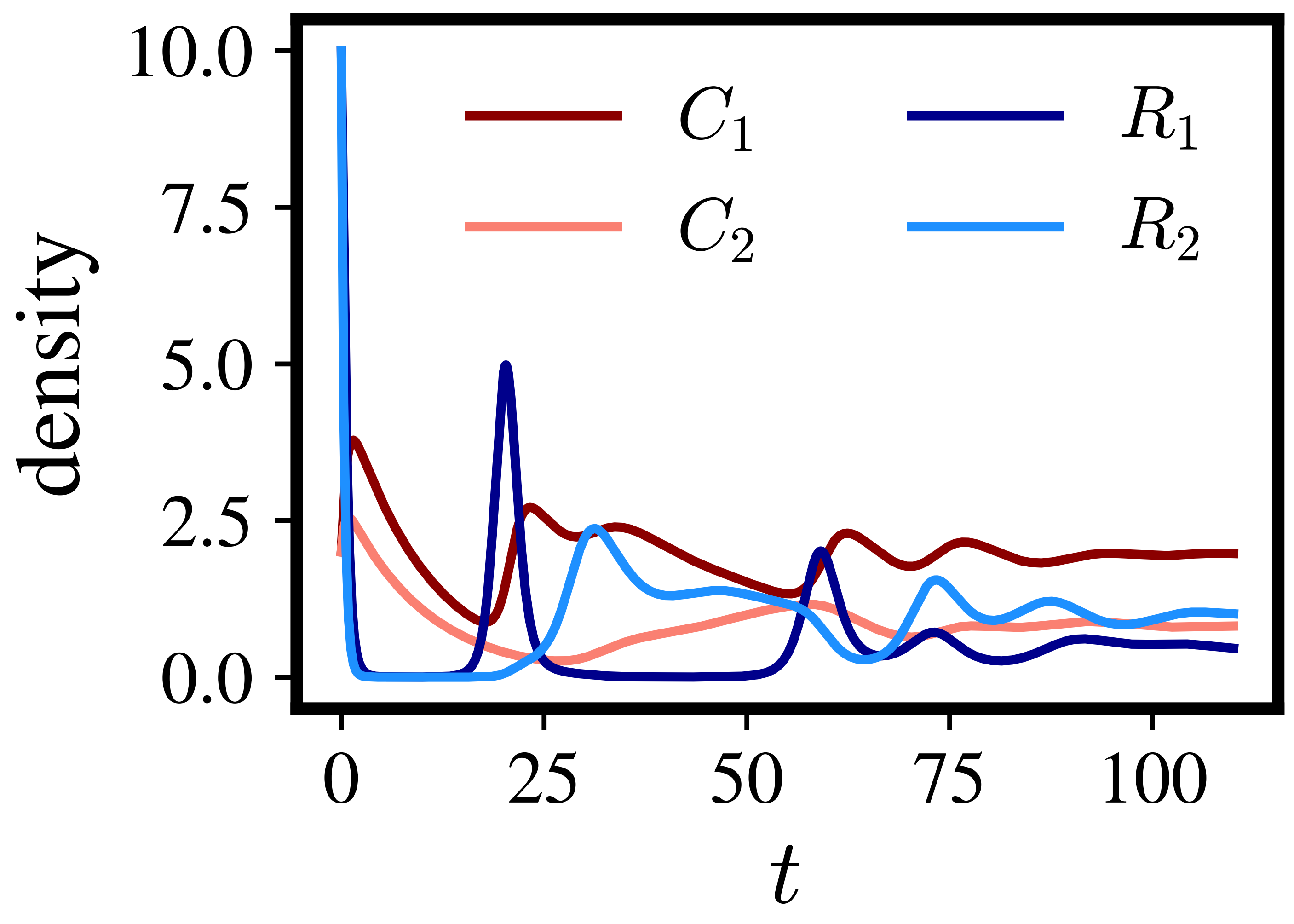
Questions:

How do changes in the sign of interactions emerge?

How can we differentiate between direct and indirect interactions?

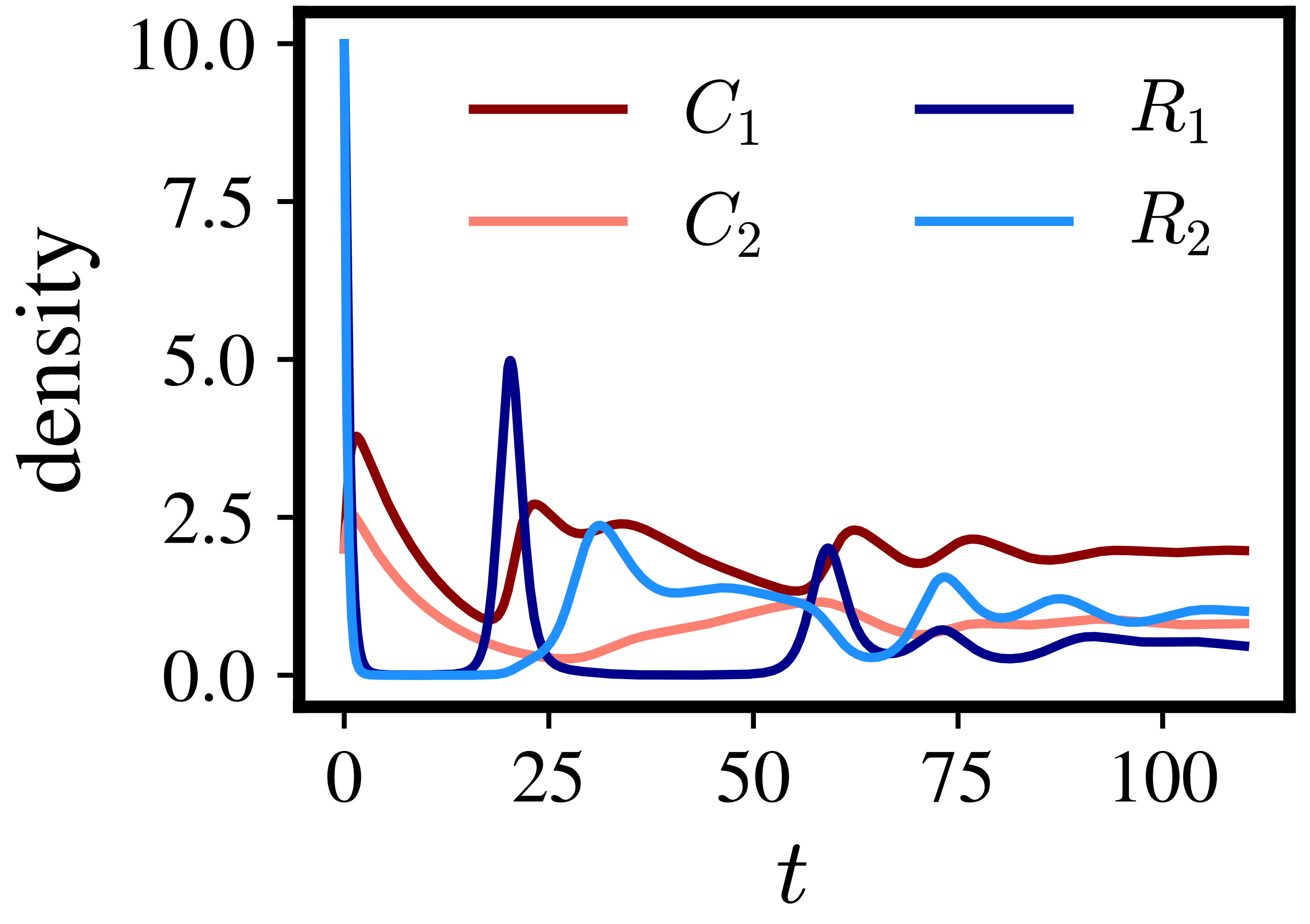
Synthetic datasets:

Generative model → $\dot{x}_i = x_i \, g_i\left(\vec{x}\right)$ ➡ <u>Test datasets</u>



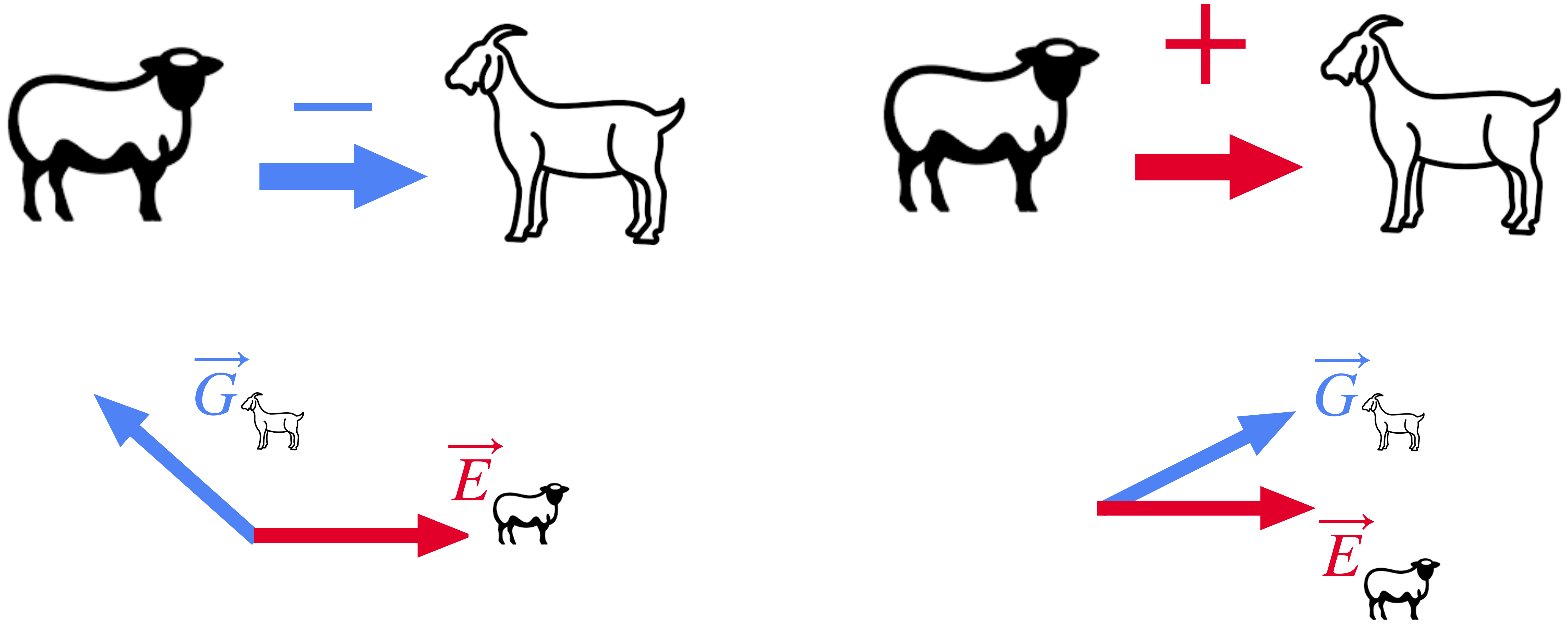
Synthetic datasets:

Generative model → $\dot{x}_i = x_i \, g_i\left(\vec{x}\right)$ → Test datasets



Analytical evaluation

$$\mathscr{M}_{i,j}^{(E)} \approx \vec{G}_i \cdot \vec{E}_j \, \Delta x$$

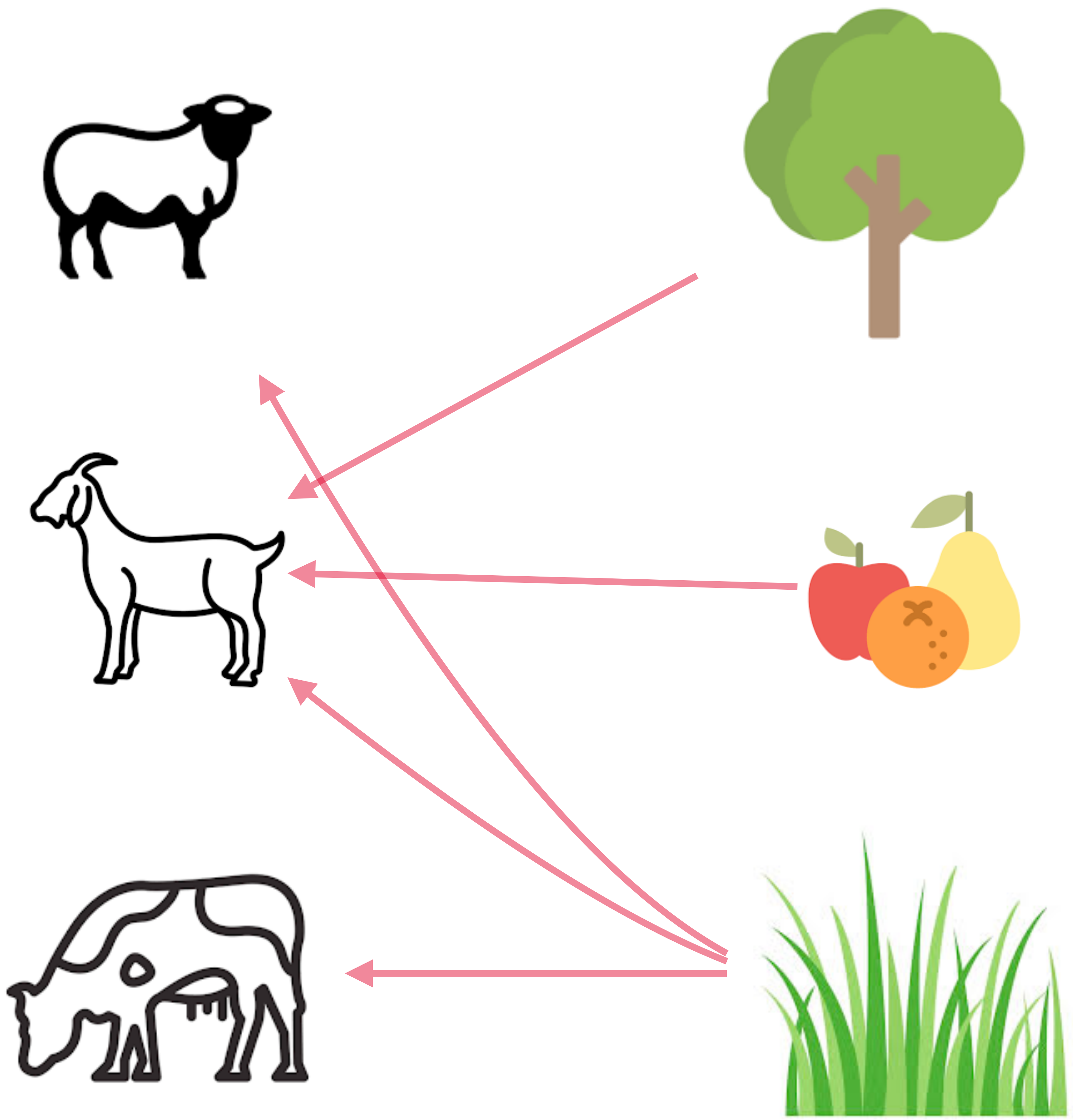


Gradient vector: $\vec{G}_i\,(\vec{x}) = \nabla g_i\,(\vec{x})$

Evolution vector: $\vec{E}_j = \partial_{x_j^{(0)}} \vec{x}\,(t \,|\, \vec{x}^{(0)})$

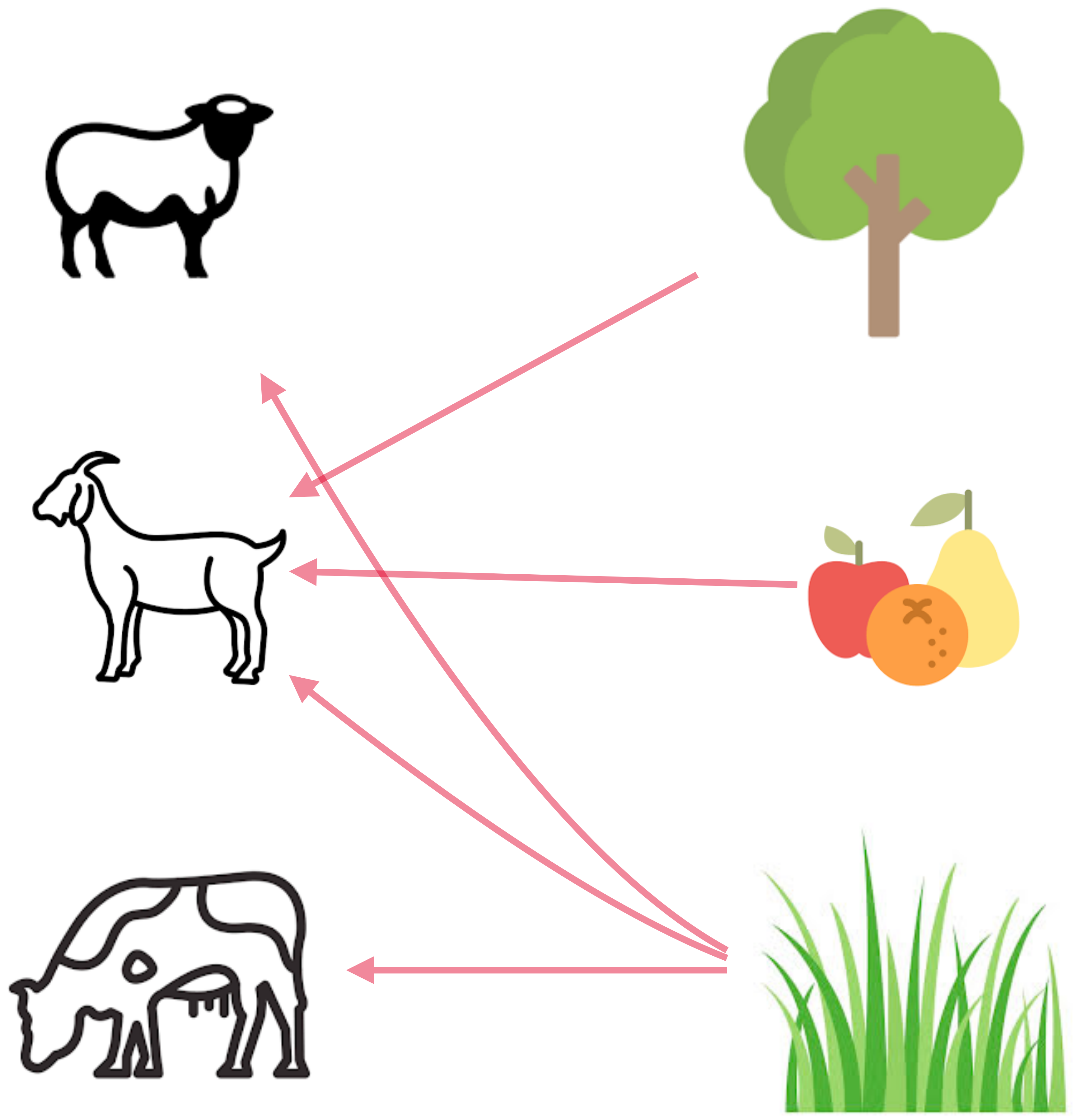
The consumer-resource model

Species within the same trophic level



The consumer-resource model

Species within the same trophic level



$$\begin{cases} \frac{d}{dt}C_i = & \varepsilon_i \sum_{\ell=1}^{n_R} \alpha_{i\ell}\, C_i\, R_\ell \; - d_i\, C_i, \\ \frac{d}{dt}R_\ell = & - \sum_{j=1}^{n_C} \alpha_{j\ell}\, C_j\, R_\ell + h_\ell\,. \end{cases}$$

No direct coupling between species

$\alpha_{ij} > 0$: Consumer-Resource coupling (uptake rate)

n_R : number of resources

n_C : number of "consumers"

$\varepsilon_i > 0$: uptake efficiency

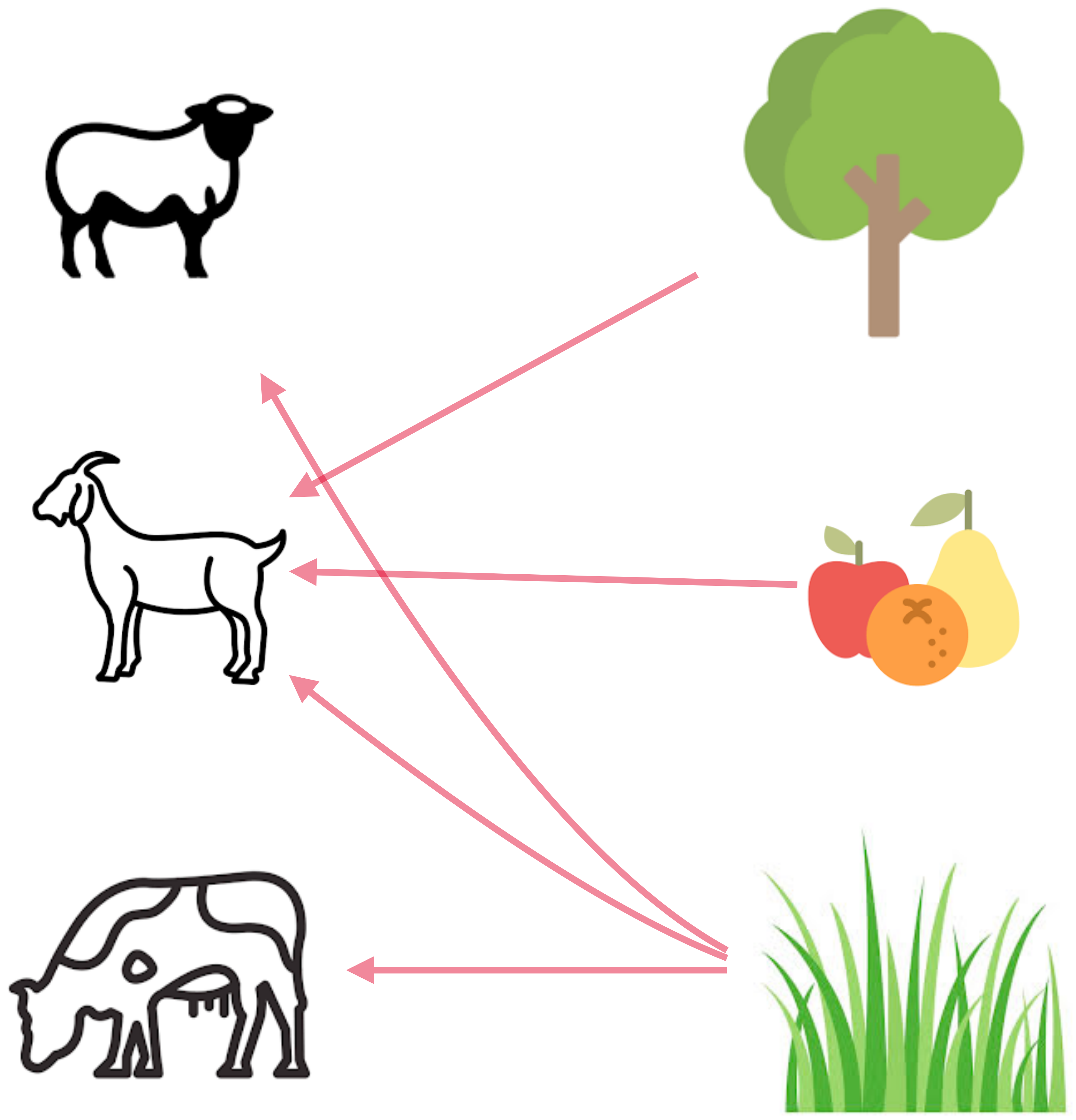
$d_i > 0$: death rate

$h_\ell > 0$: Renewal function

MacArthur 1970

The consumer-resource model

Species within the same trophic level



$$\begin{cases} \frac{d}{dt}C_i = & \varepsilon_i \sum_{\ell=1}^{n_R} \alpha_{i\ell}\, C_i\, R_\ell \; - d_i\, C_i, \\ \frac{d}{dt}R_\ell = & -\sum_{j=1}^{n_C} \alpha_{j\ell}\, C_j\, R_\ell + h_\ell\,. \end{cases}$$

No direct coupling between species

$\alpha_{ij} > 0$: Consumer-Resource coupling (uptake rate)

n_R : number of resources

n_C : number of "consumers"

$\varepsilon_i > 0$: uptake efficiency

$d_i > 0$: death rate

$h_\ell > 0$: Renewal function

$$\frac{d}{dt}R_\ell(t) \approx 0,$$

Effective Lotka-Volterra model

$$\frac{d}{dt}C_i(t) \approx \varepsilon_i\, C_i(t)\left(\tilde{K}_i + \sum_{\ell=1}^{m} \tilde{\alpha}_{i\ell} C_j(t)\right),$$

$\tilde{\alpha}_{ij} < 0$

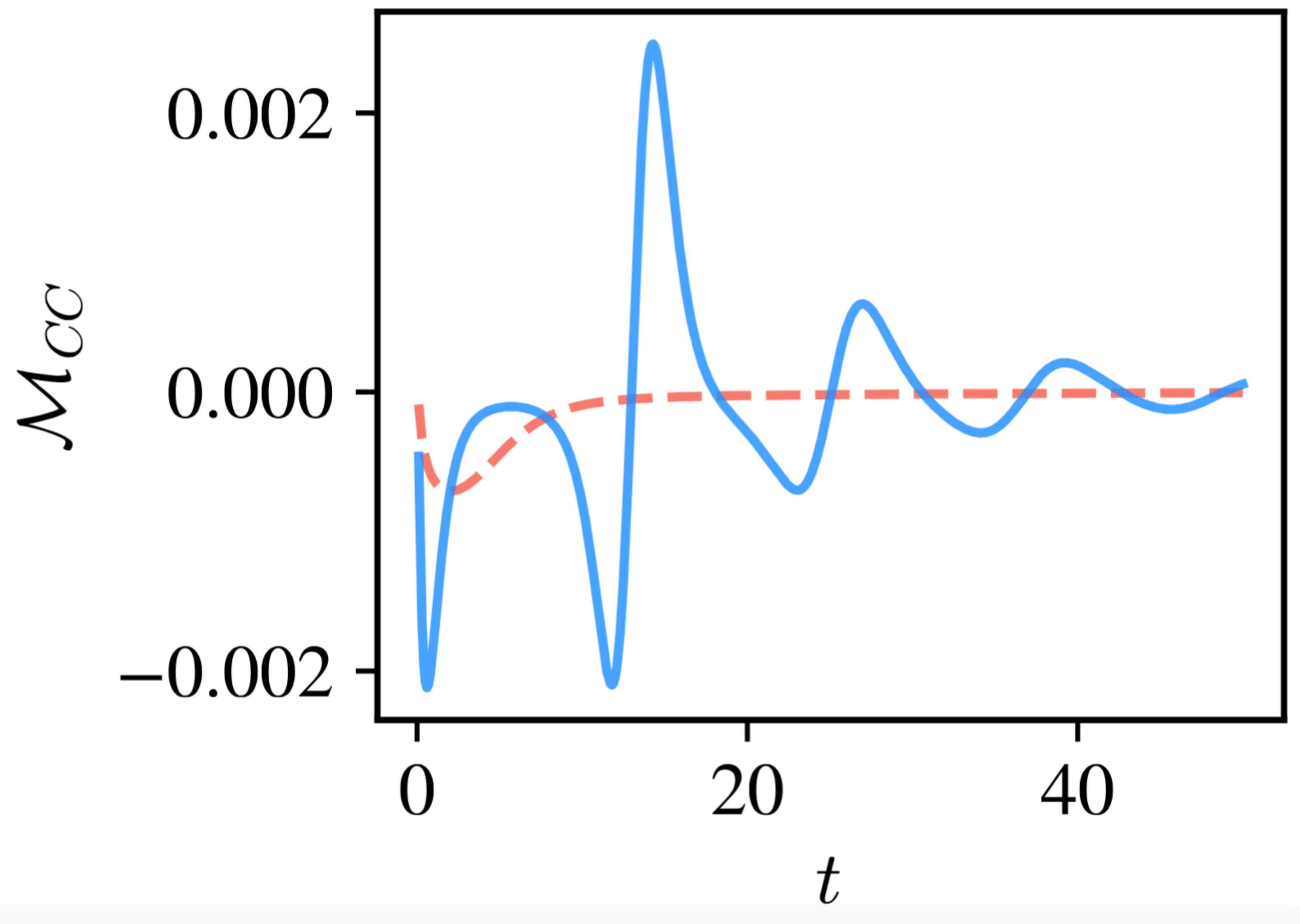
Competitive system

MacArthur 1970

Interactions in stationary regime

$$\mathscr{M}^{(E)}_{C_i,C_j} \approx \epsilon_i \sum_{\ell} \left(e^{\hat{J}t}\right)_{j,\ell} a_{i,\ell}\, \Delta x\,.$$

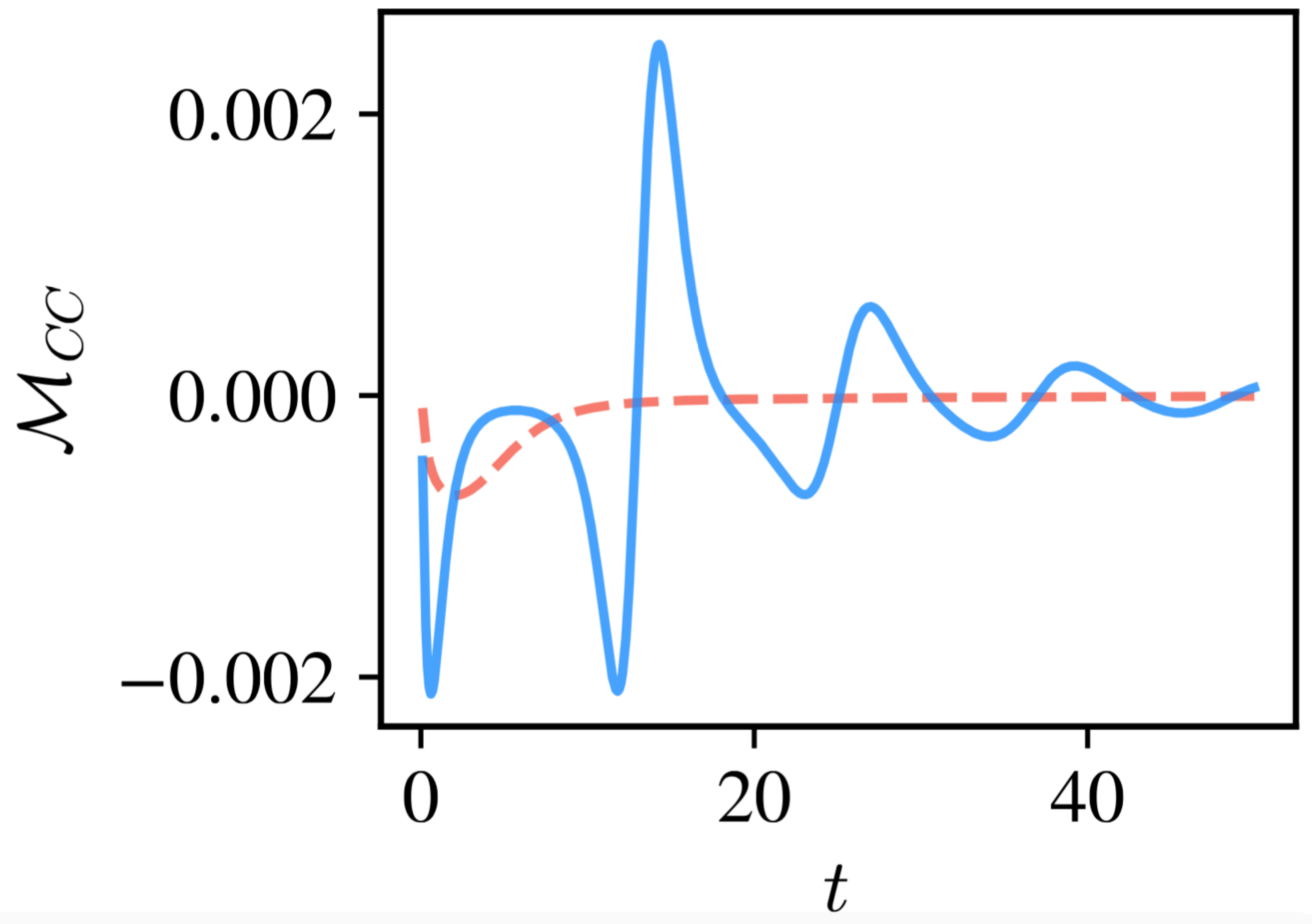
Changes of sign may arise in purely competitive systems!



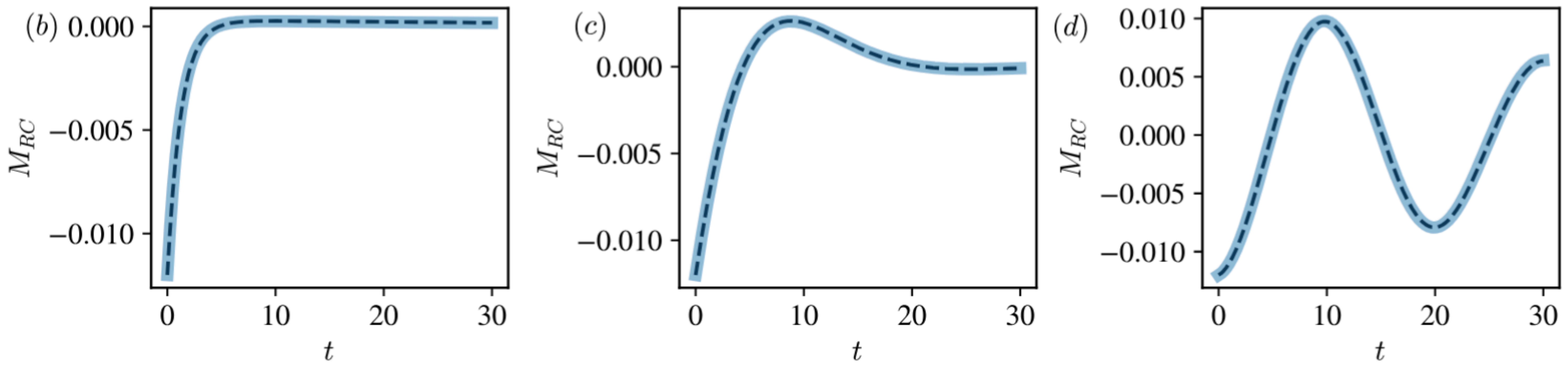
Interactions in stationary regime

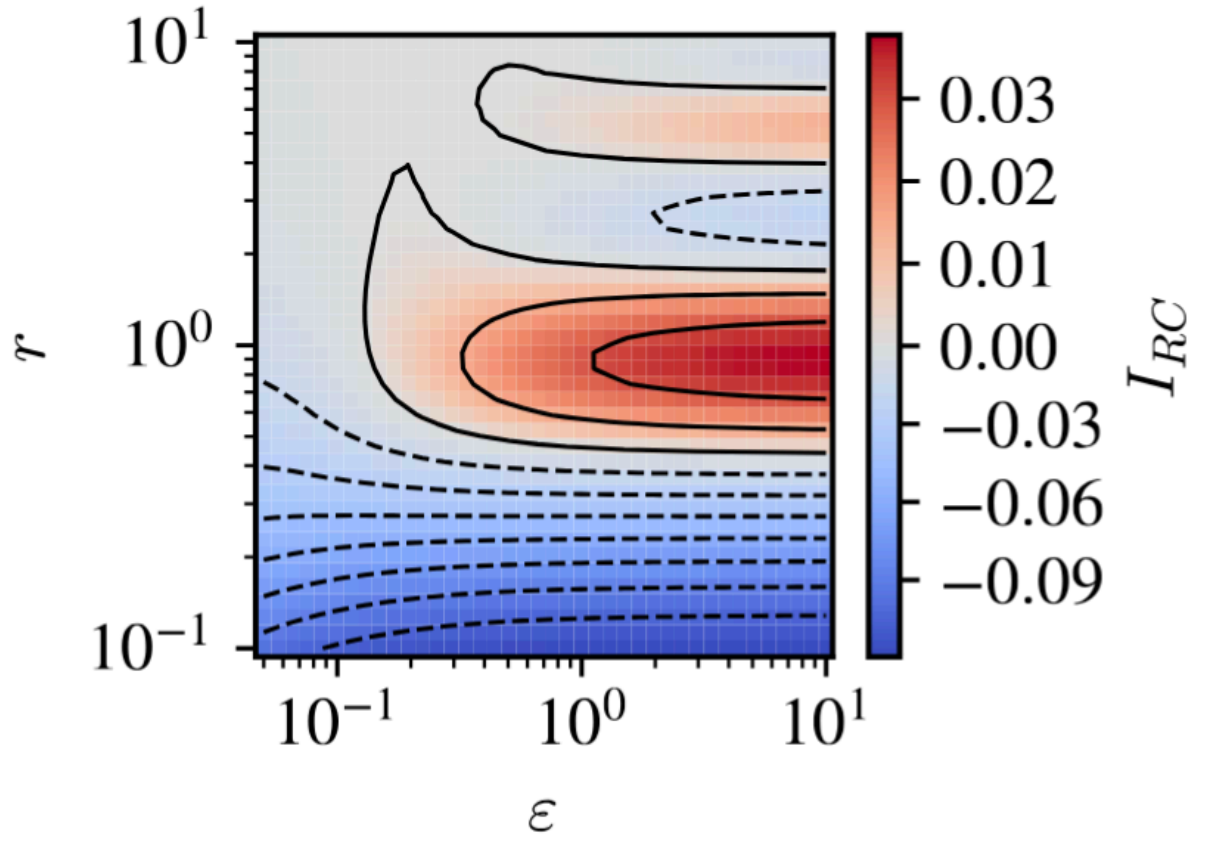
$$\mathscr{M}^{(E)}_{C_i,C_j} \approx \epsilon_i \sum_{\ell} \left(e^{\hat{J}t}\right)_{j,\ell} a_{i,\ell}\,\Delta x\,.$$

Changes of sign may arise in purely competitive systems!



Measures of interactions have its own non-trivial dynamics!

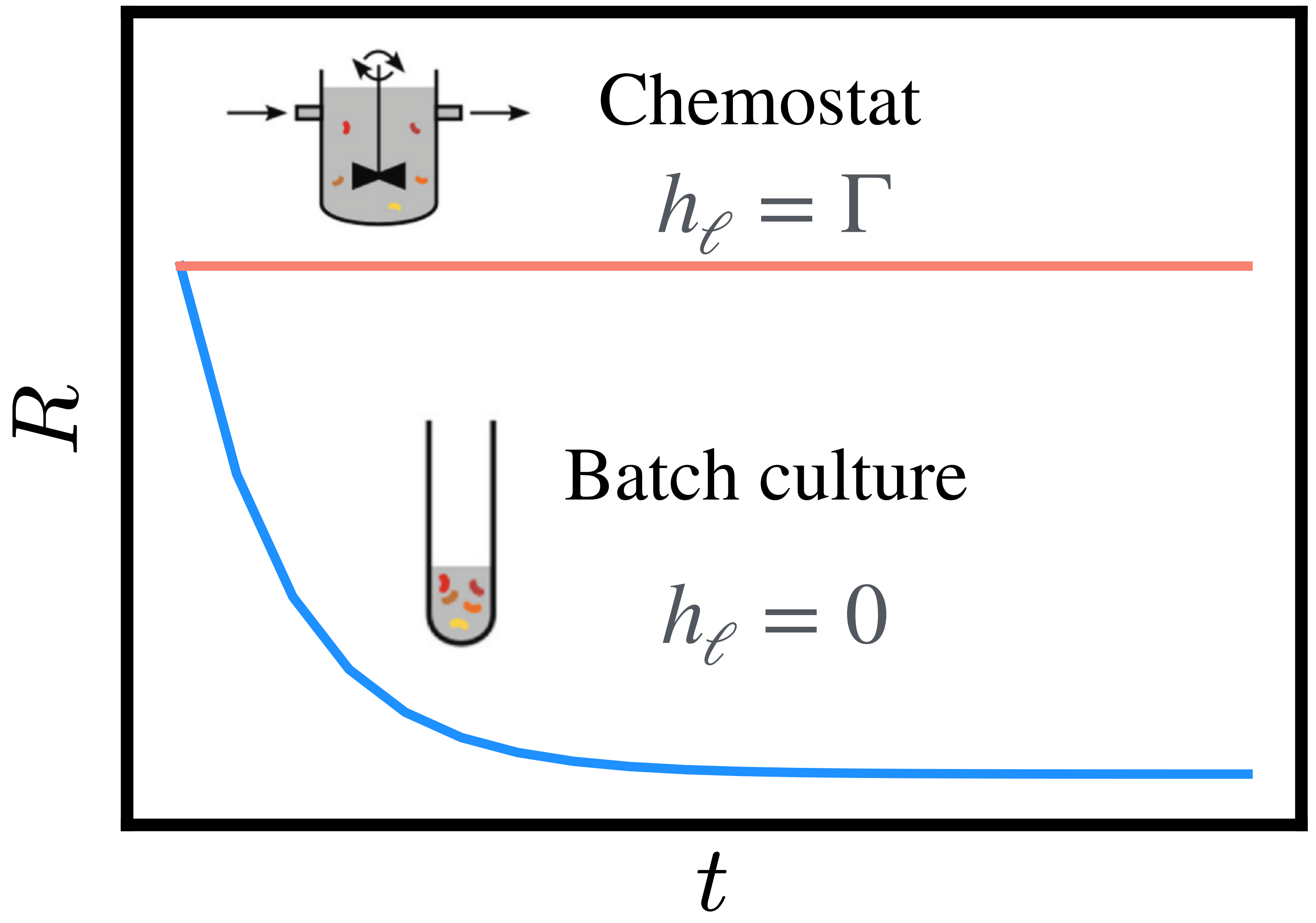




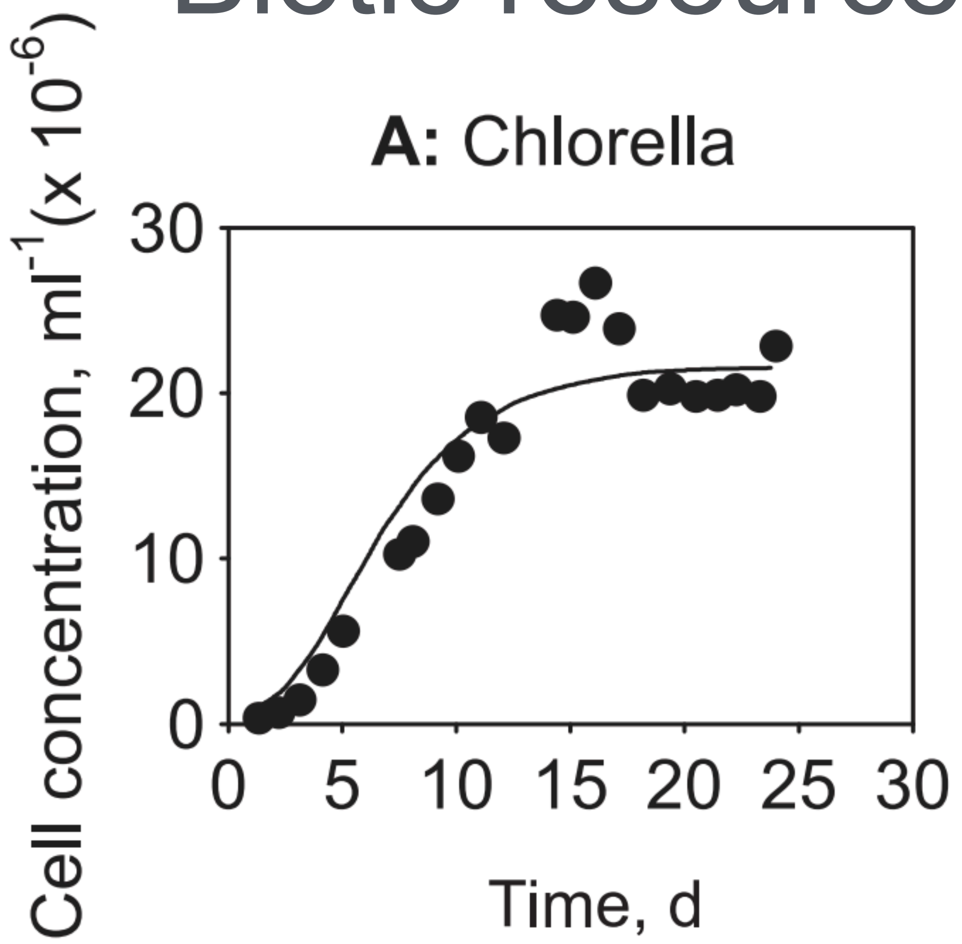
Interactions depend on the experimental protocol

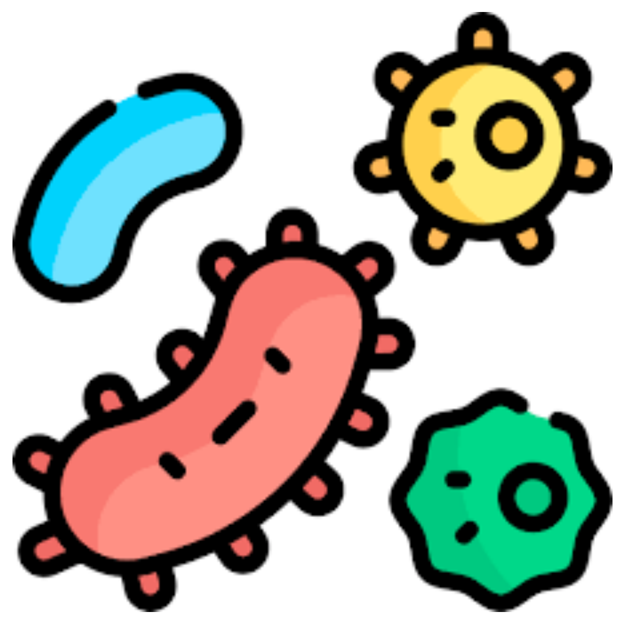
$$\frac{d}{dt}R_\ell = -\sum_{j=1}^{n_C} \alpha_{j\ell}\, C_j\, R_\ell + h_\ell\,.$$

Abiotic resources



Biotic resources $h_\ell = r\,R_\ell\,(1 - \frac{R_\ell}{K})$





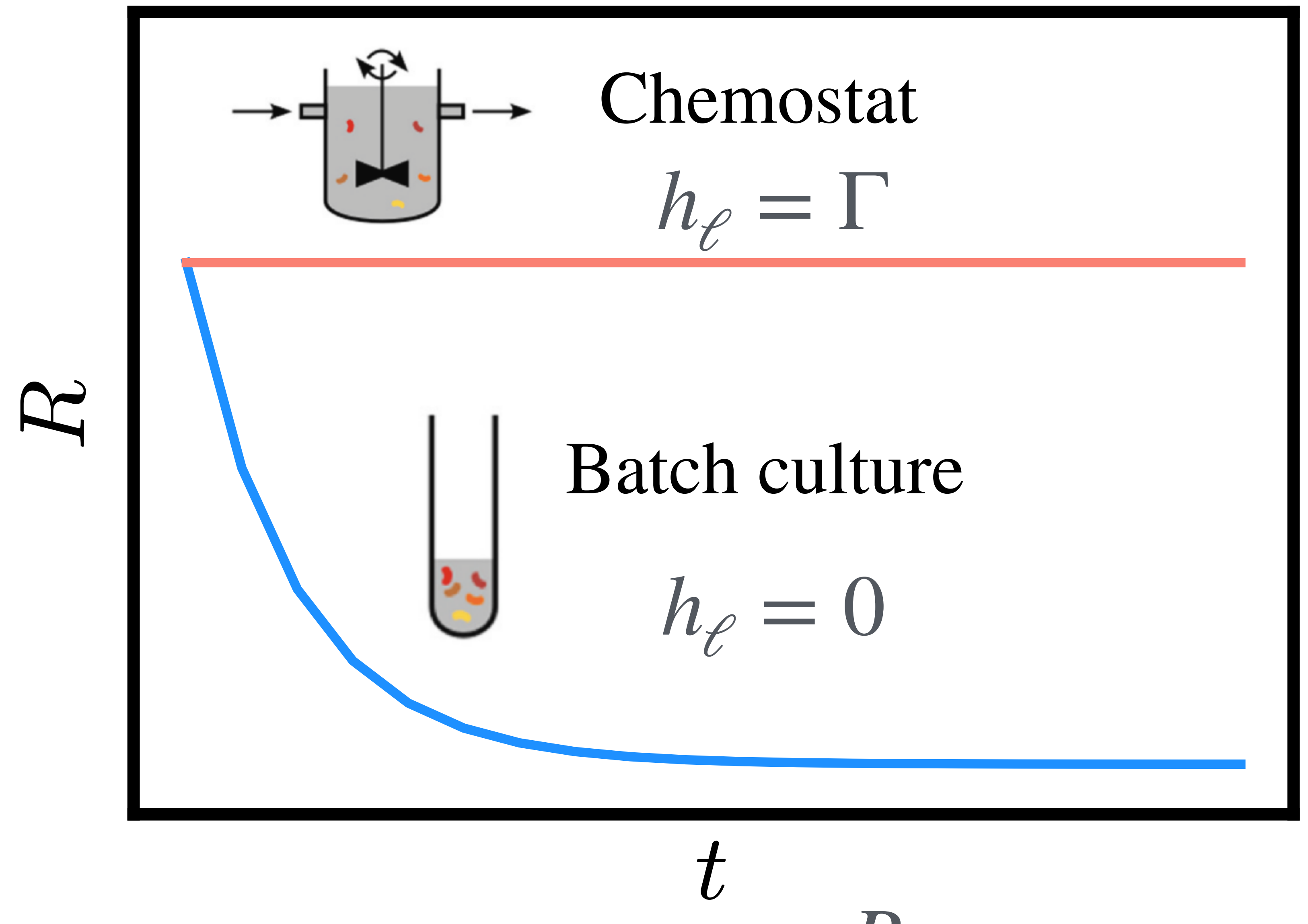
Kiørboe, 2008

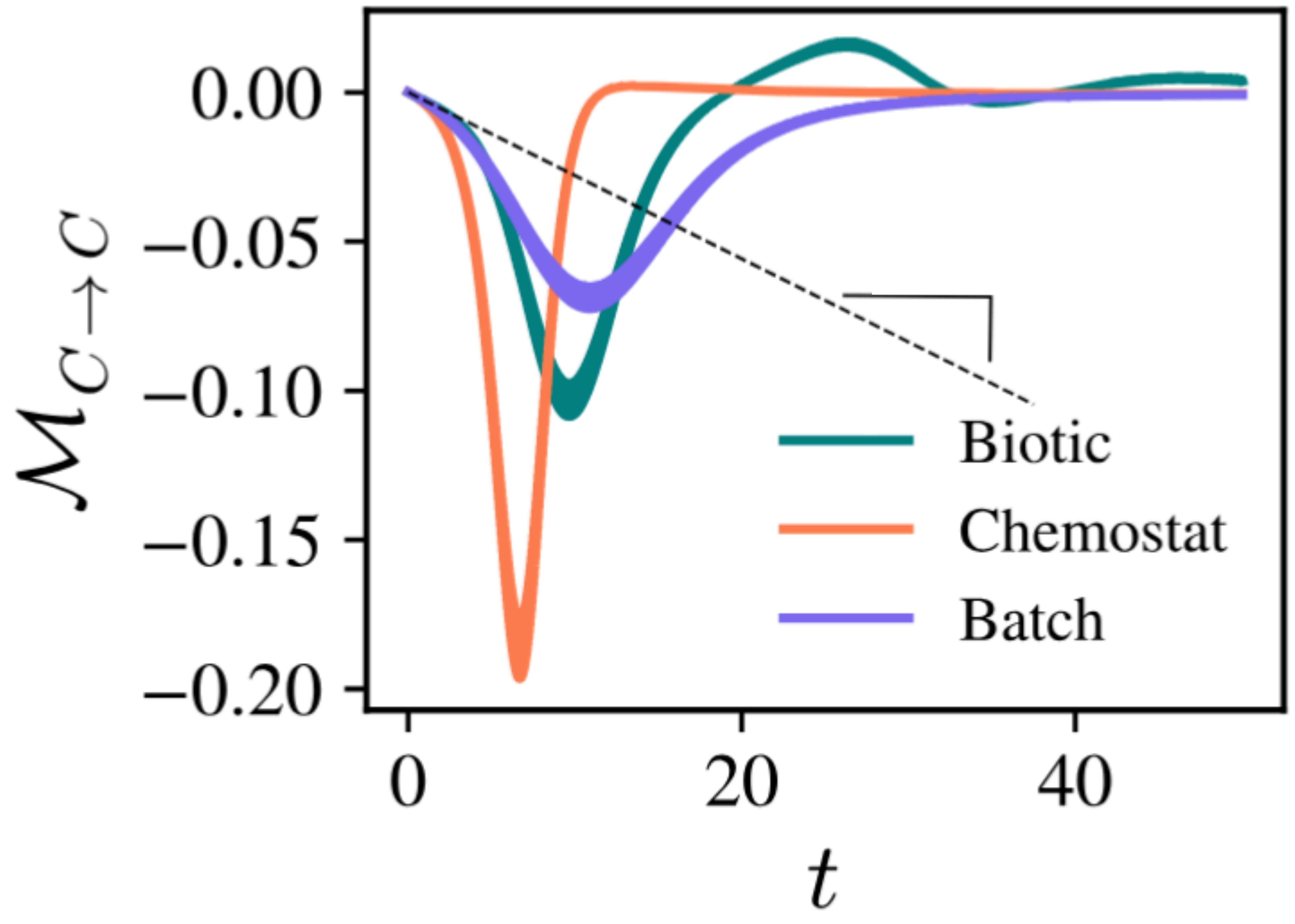
Picot, Shibasaki, Meacock, Mitri 2023

Interactions depend on the experimental protocol

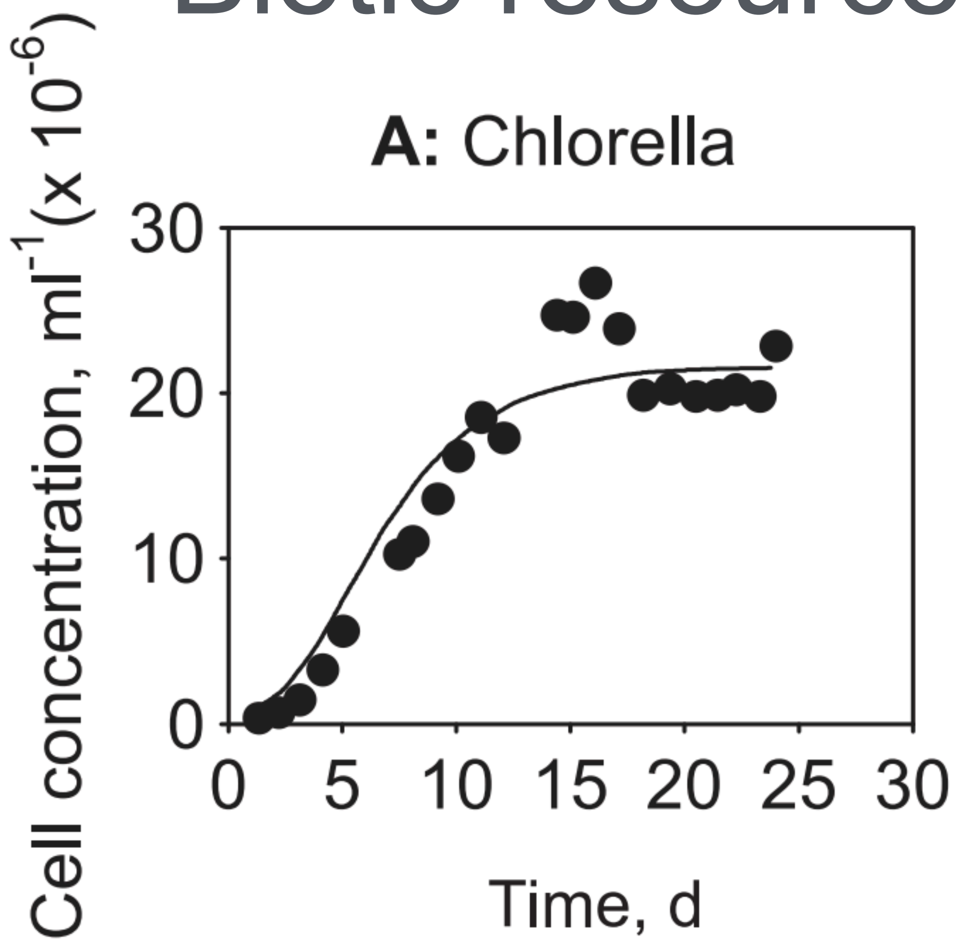
$$\frac{d}{dt}R_\ell = -\sum_{j=1}^{n_C} \alpha_{j\ell}\, C_j\, R_\ell + h_\ell\,.$$

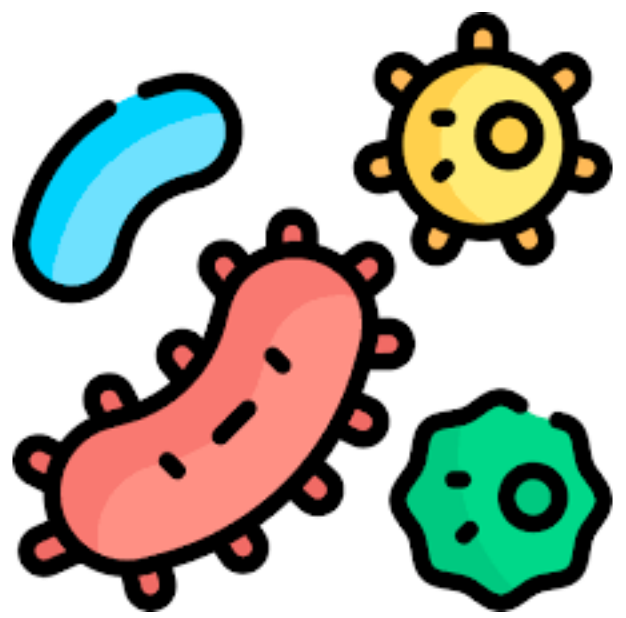
Abiotic resources





Biotic resources $h_\ell = r\,R_\ell\,(1 - \frac{R_\ell}{K})$





Kiørboe, 2008

Picot, Shibasaki, Meacock, Mitri 2023

Interactions close to the perturbation

$$\mathscr{M}_{i,j}(\vec{x}, t) = \mathscr{M}_{i,j}^{(0)}(\vec{x}) + \mathscr{M}_{i,j}^{(1)}(\vec{x})\, t + \mathscr{M}_{i,j}^{(2)}(\vec{x})\, t^2 + \dots$$

Interactions close to the perturbation

$$\mathscr{M}_{i,j}(\vec{x}, t) = \mathscr{M}^{(0)}_{i,j}(\vec{x}) + \mathscr{M}^{(1)}_{i,j}(\vec{x})\, t + \mathscr{M}^{(2)}_{i,j}(\vec{x})\, t^2 + \dots$$

$$\mathscr{M}^{(0)}_{i,j}(\vec{x}) = \frac{\partial}{\partial x_j} g_i(\vec{x})\, \Delta x \quad \Rightarrow \quad \text{Direct interactions}$$

Interactions close to the perturbation

$$\mathscr{M}_{i,j}(\vec{x},t) = \mathscr{M}^{(0)}_{i,j}(\vec{x}) + \mathscr{M}^{(1)}_{i,j}(\vec{x})\,t + \mathscr{M}^{(2)}_{i,j}(\vec{x})\,t^2 + \dots$$

$$\mathscr{M}^{(0)}_{i,j}(\vec{x}) = \frac{\partial}{\partial x_j} g_i(\vec{x})\,\Delta x \quad \Longrightarrow \quad \text{Direct interactions}$$

$$\mathscr{M}^{(1)}_{i,j}(\vec{x}) = \sum_k x_k\,\partial_{x_k} g_i(\vec{x})\,\partial_{x_j} g_k(\vec{x}) \quad \Longrightarrow \quad \text{Indirect interactions}$$

$$\vdots$$

As the experiment duration goes on, more indirect effects come into play

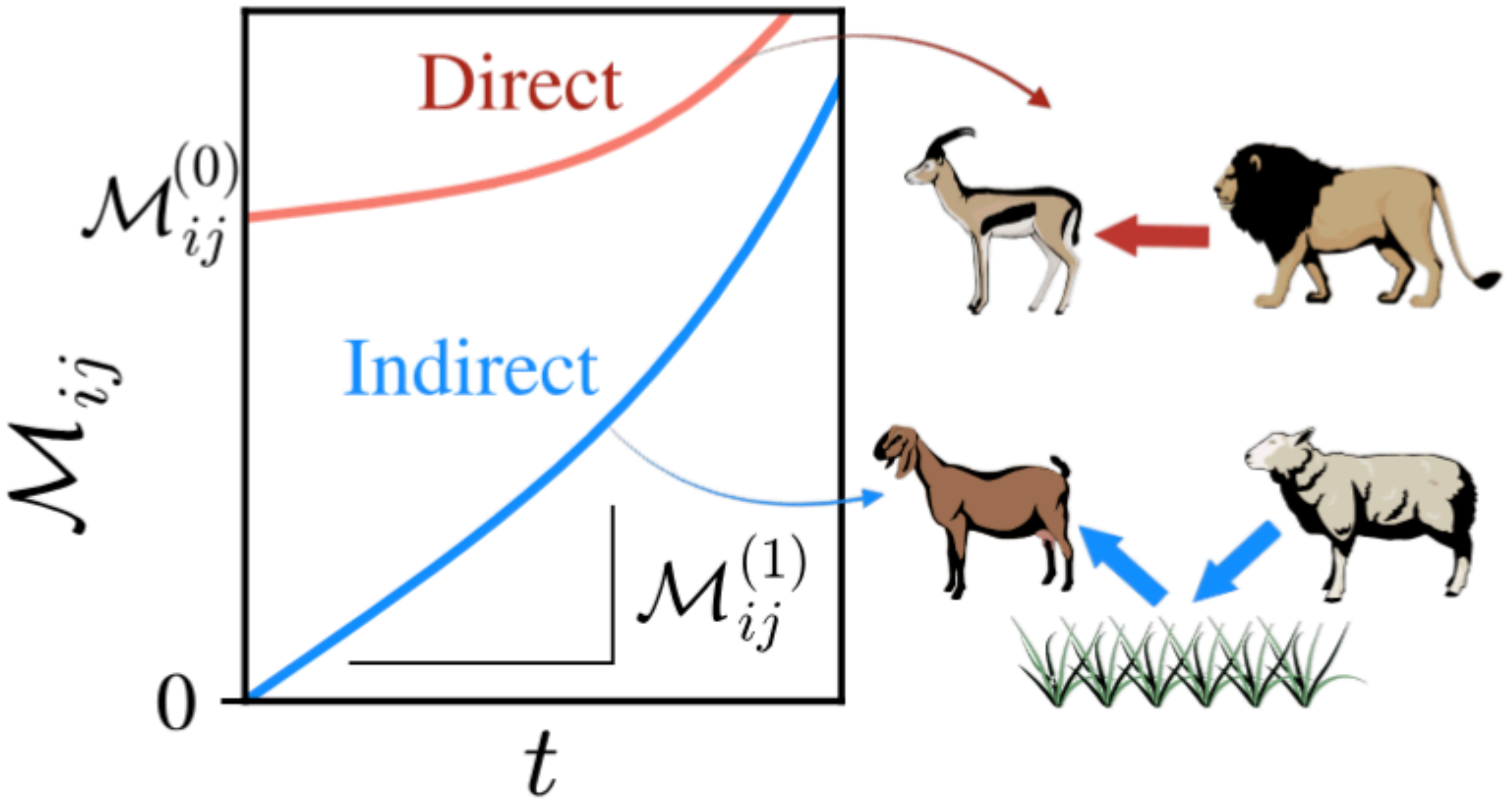
Consumer-resource model:

$$\mathscr{M}^{(0)}_{C_i,C_j}(\vec{x}) = 0 \qquad \mathscr{M}^{(1)}_{C_i,C_j} = -\,\varepsilon_i \sum_{\ell=1}^{n_R} R_\ell\,\alpha_{i,\ell}\,\alpha_{j,\ell} < 0$$

$$\mathscr{M}^{(2)}_{C_i,C_j}(\vec{x}) = F_2[\{h_\ell\}]$$

Interactions close to the perturbation

$$\mathscr{M}_{i,j}(\vec{x},t) = \mathscr{M}^{(0)}_{i,j}(\vec{x}) + \mathscr{M}^{(1)}_{i,j}(\vec{x})\,t + \mathscr{M}^{(2)}_{i,j}(\vec{x})\,t^2 + \dots$$



$$\mathscr{M}^{(0)}_{i,j}(\vec{x}) = \frac{\partial}{\partial x_j} g_i(\vec{x})\,\Delta x$$ ➡ Direct interactions

$$\mathscr{M}^{(1)}_{i,j}(\vec{x}) = \sum_k x_k\,\partial_{x_k} g_i(\vec{x})\,\partial_{x_j} g_k(\vec{x})$$ ➡ Indirect interactions

$\vdots$

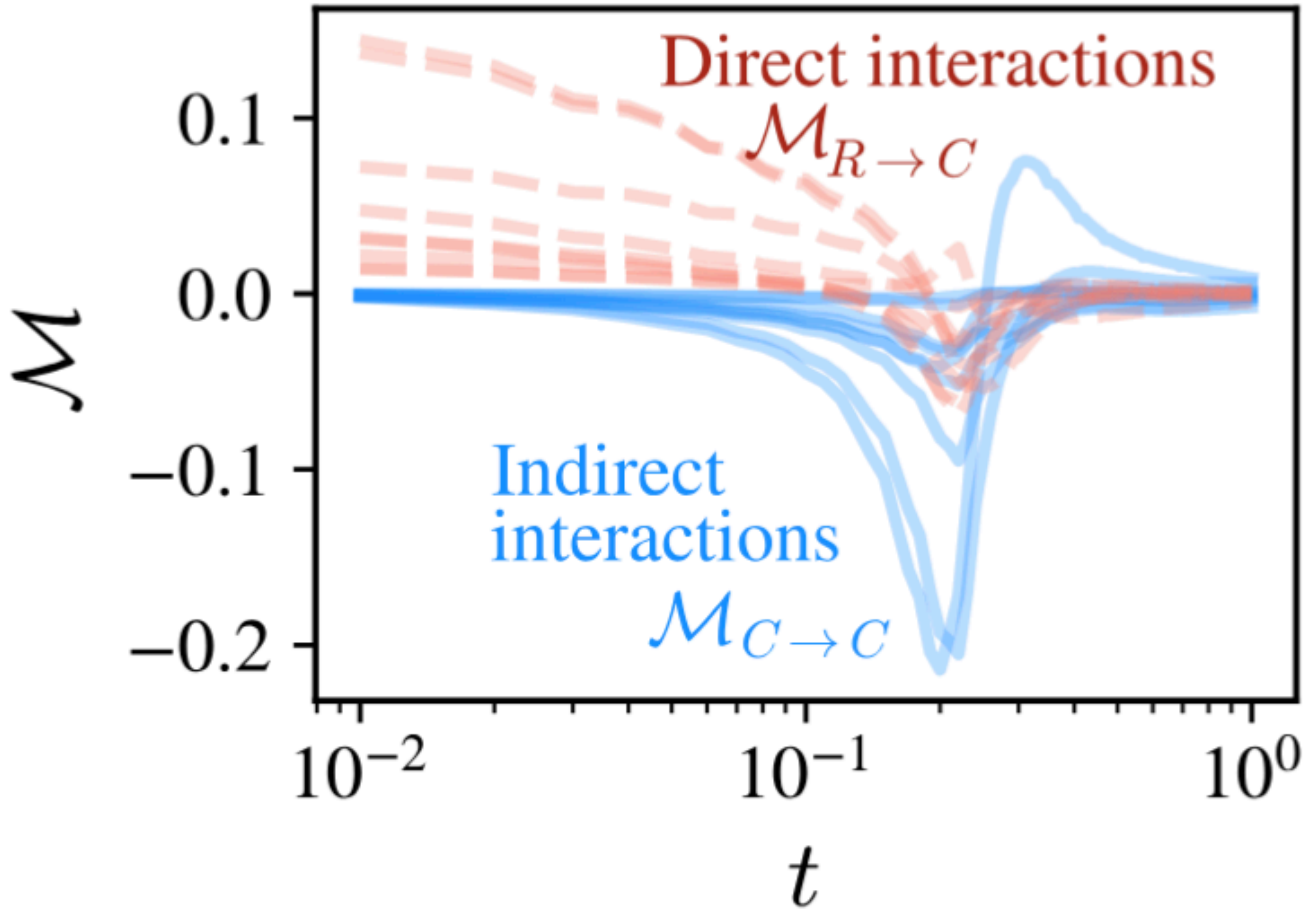
As the experiment duration goes on, more indirect effects come into play

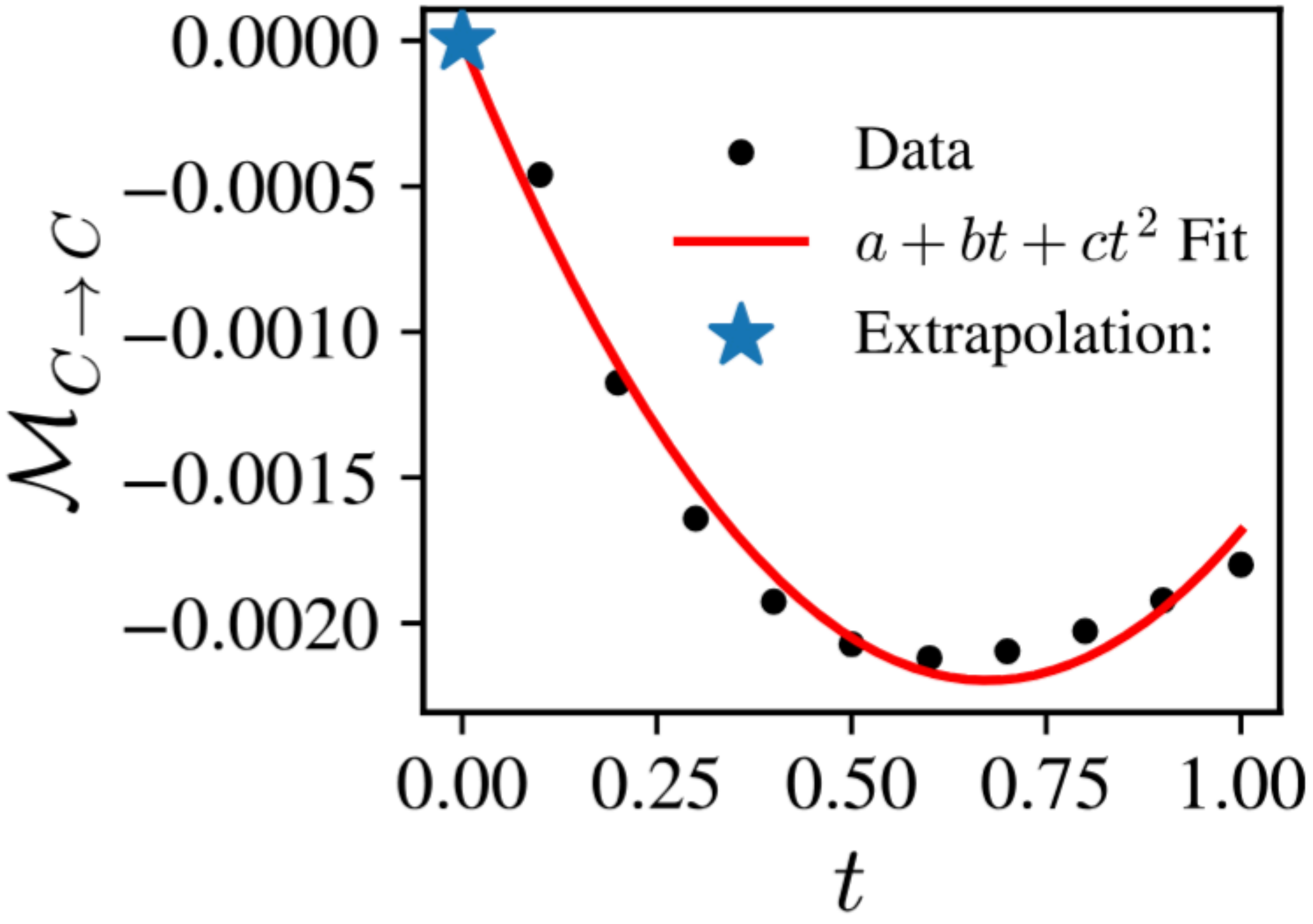
Consumer-resource model:

$$\mathscr{M}^{(0)}_{C_i,C_j}(\vec{x}) = 0 \qquad \mathscr{M}^{(1)}_{C_i,C_j} = -\,\varepsilon_i \sum_{\ell=1}^{n_R} R_\ell\,\alpha_{i,\ell}\,\alpha_{j,\ell} < 0$$

$$\mathscr{M}^{(2)}_{C_i,C_j}(\vec{x}) = F_2[\{h_\ell\}]$$

Disentangle direct and indirect interactions





Inferring model from interactions

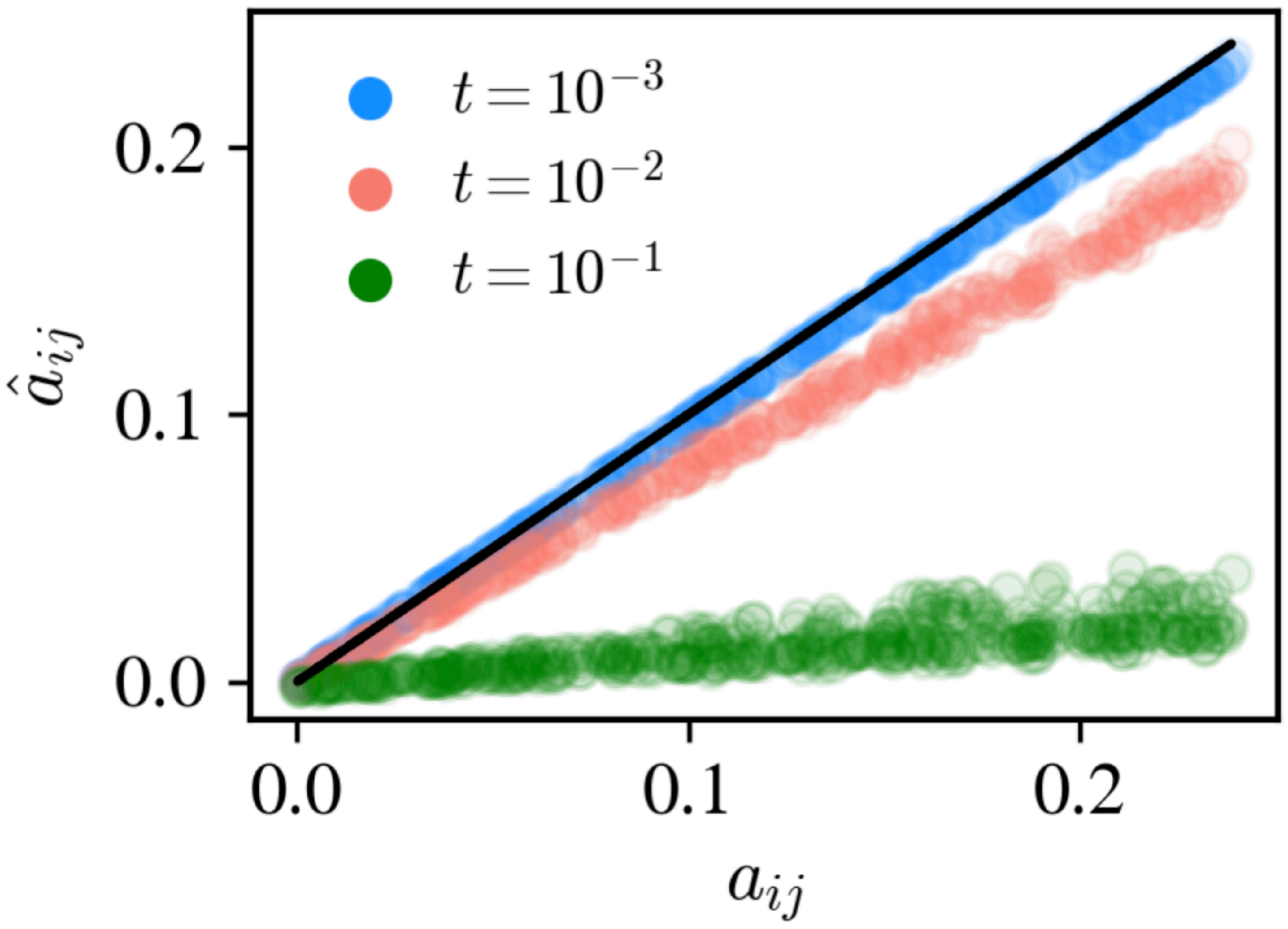
Measures: $\hat{a}_{ij} = \frac{1}{\Delta x} \mathscr{M}_{i,j}(\vec{x}, t, \Delta x, \Delta t)$

Inferring model from interactions

Measures: $\hat{a}_{ij} = \frac{1}{\Delta x} \mathscr{M}_{i,j}(\vec{x}, t, \Delta x, \Delta t)$

Inference

$$\dot{x}_i = x_i \sum_j \int_0^{x_j} \hat{a}_{ij} \left(x_1, \ldots, x_{j-1}, s, x_{j+1}, \ldots \right) ds_j + x_i c_i \,.$$



Inferring model from interactions

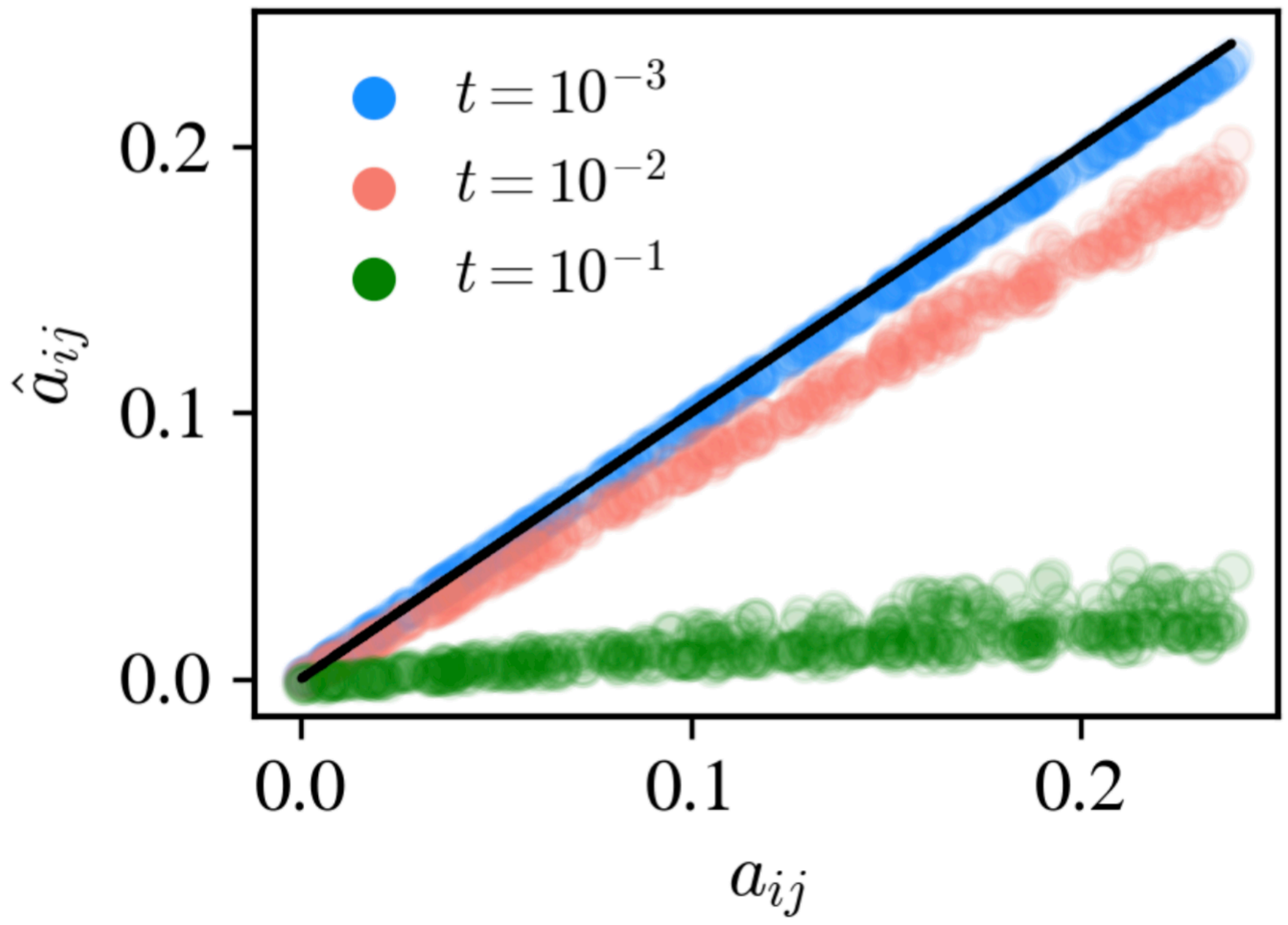
Measures: $\hat{a}_{ij} = \frac{1}{\Delta x}\mathscr{M}_{i,j}(\vec{x}, t, \Delta x, \Delta t)$

Inference

$$\dot{x}_i = x_i \sum_j \int_0^{x_j} \hat{a}_{ij}\left(x_1, \ldots, x_{j-1}, s, x_{j+1}, \ldots\right) ds_j + x_i c_i \, .$$

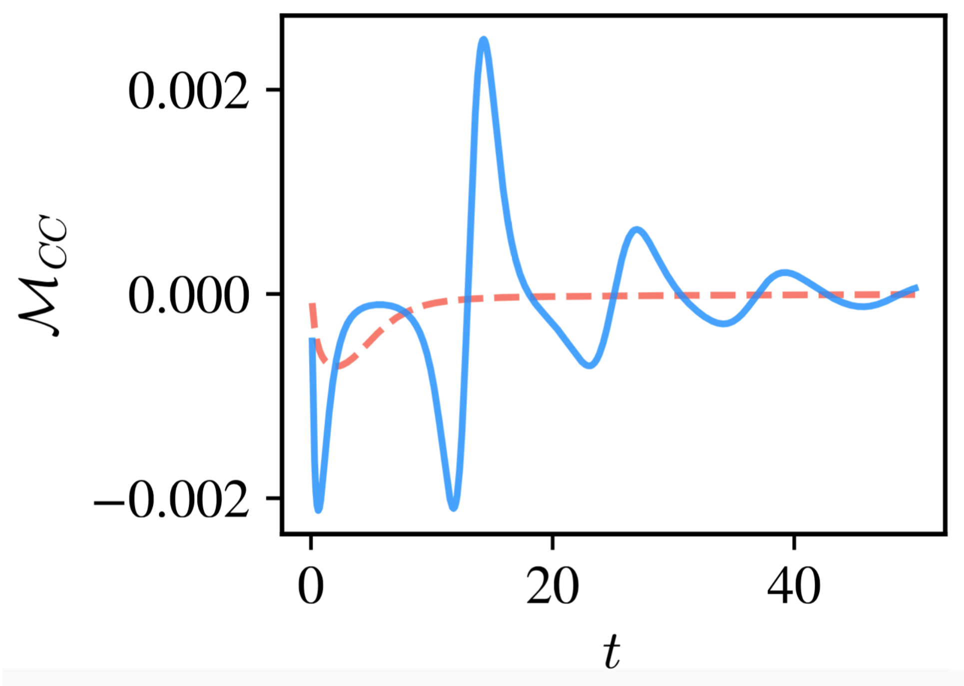
Approximation
$\hat{a}_{ij}\left(\vec{x}\right) \approx \hat{a}_{ij}$ constant

$$\dot{x}_i = x_i \sum_j \hat{a}_{ij} x_j + x_i c_i \, .$$

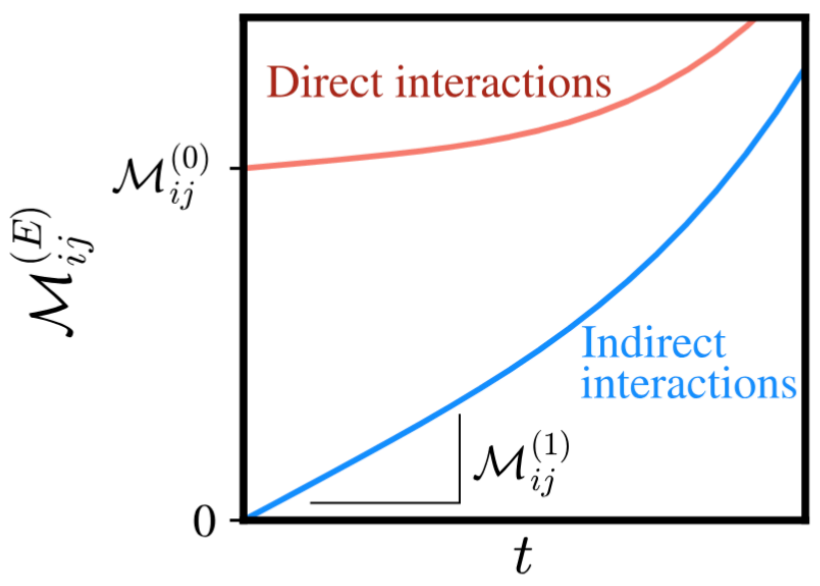


Lotka-Volterra as a first-order approximation of any inference!

Conclusions

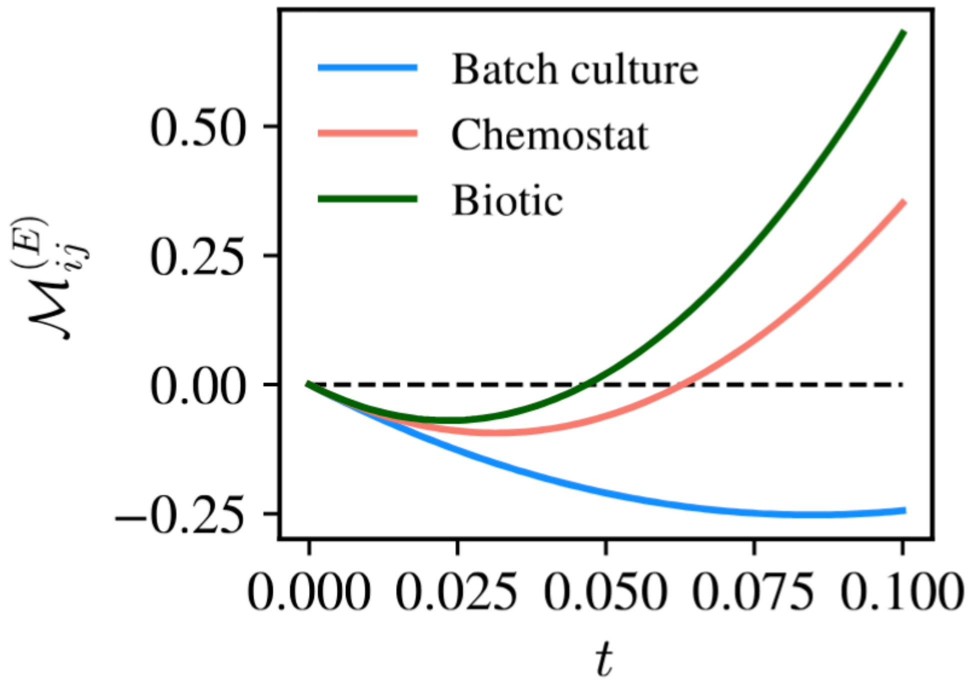


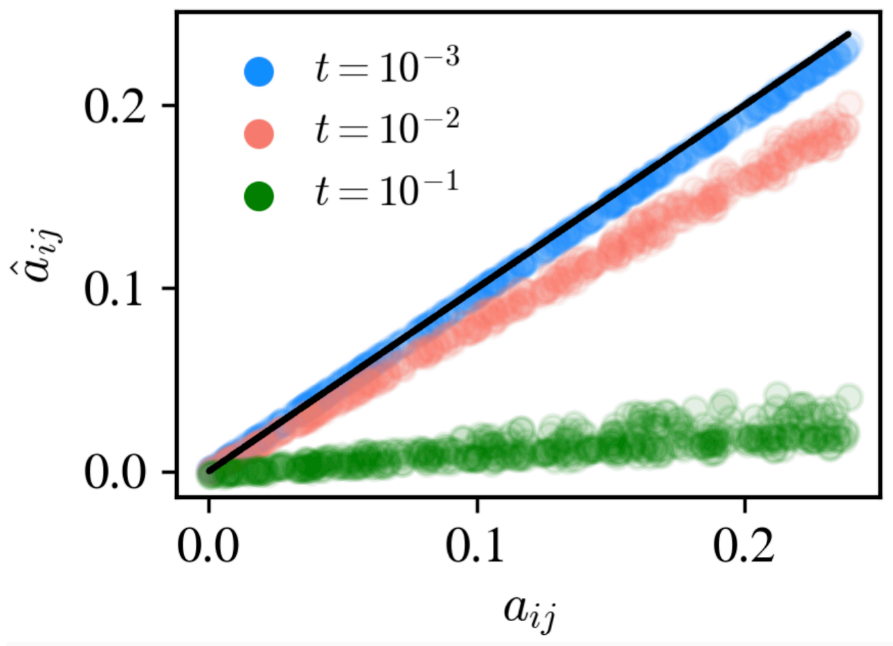
Sign of interactions depends on the experiment duration



Scaling with experiment duration determine nature of interactions

Measured interactions depend on the experimental setup





Interaction measures provides a method to infer models

Thank you for your attention!

bioRxiv, 2025.08. 29.673018

bioRxiv
THE PREPRINT SERVER FOR BIOLOGY

Amos Maritan

Sandro Azaele

Samir Suweis

Finanziato dall'Unione europea
NextGenerationEU

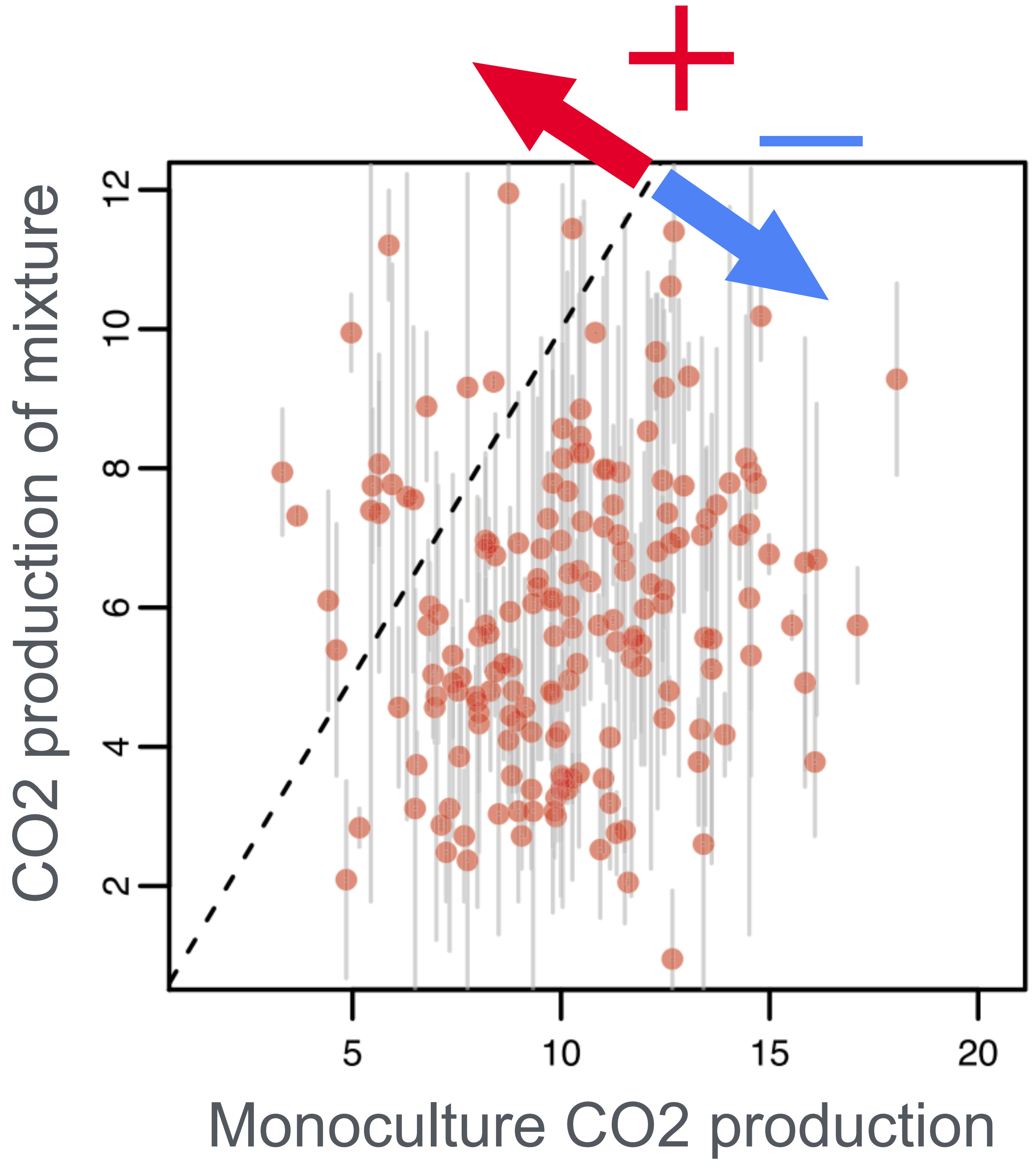
Ministero dell'Università e della Ricerca

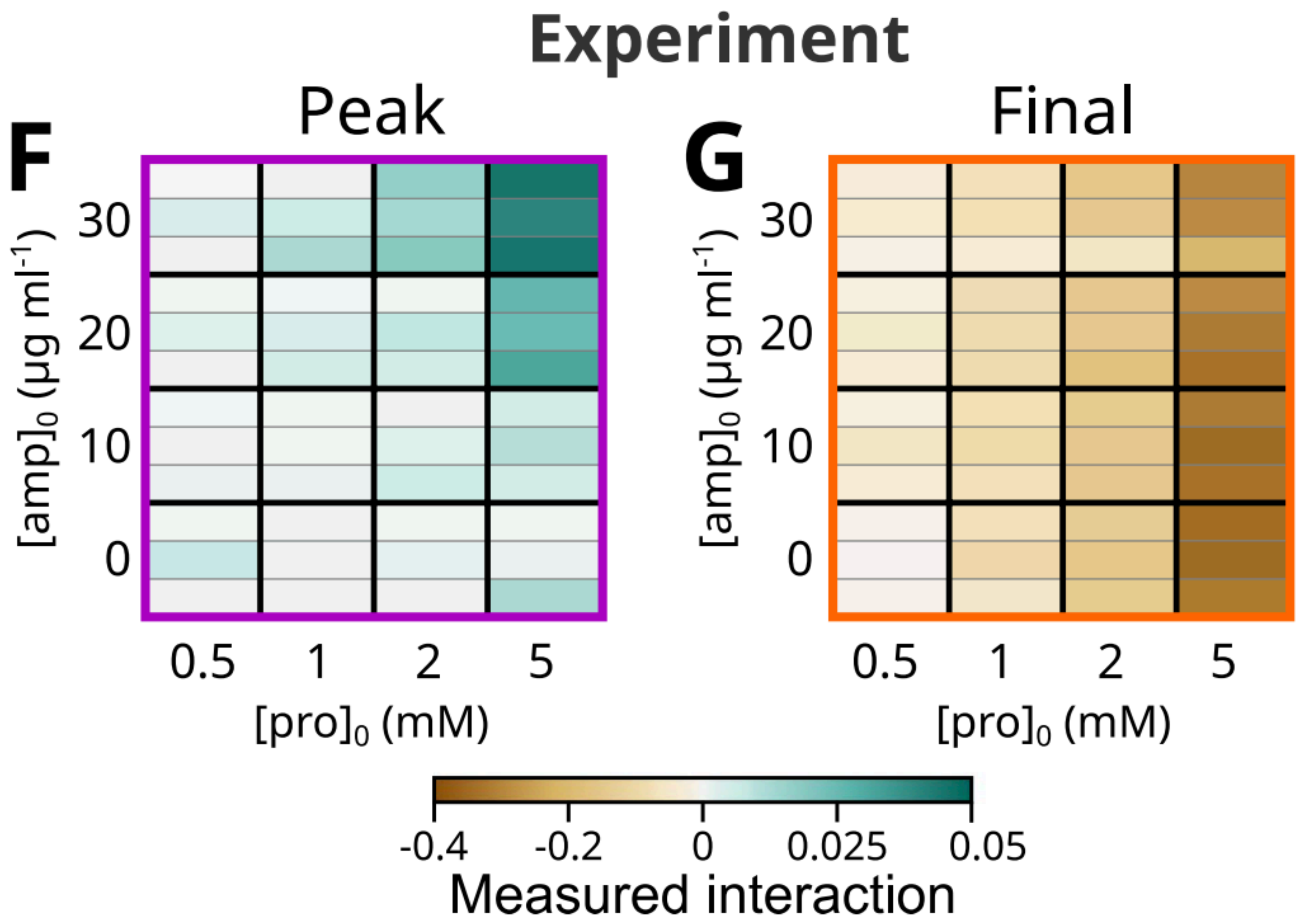
Italiadomani
PIANO NAZIONALE DI RIPRESA E RESILIENZA

Università degli Studi di Padova

Laboratory of Interdisciplinary Physics

Interaction measures





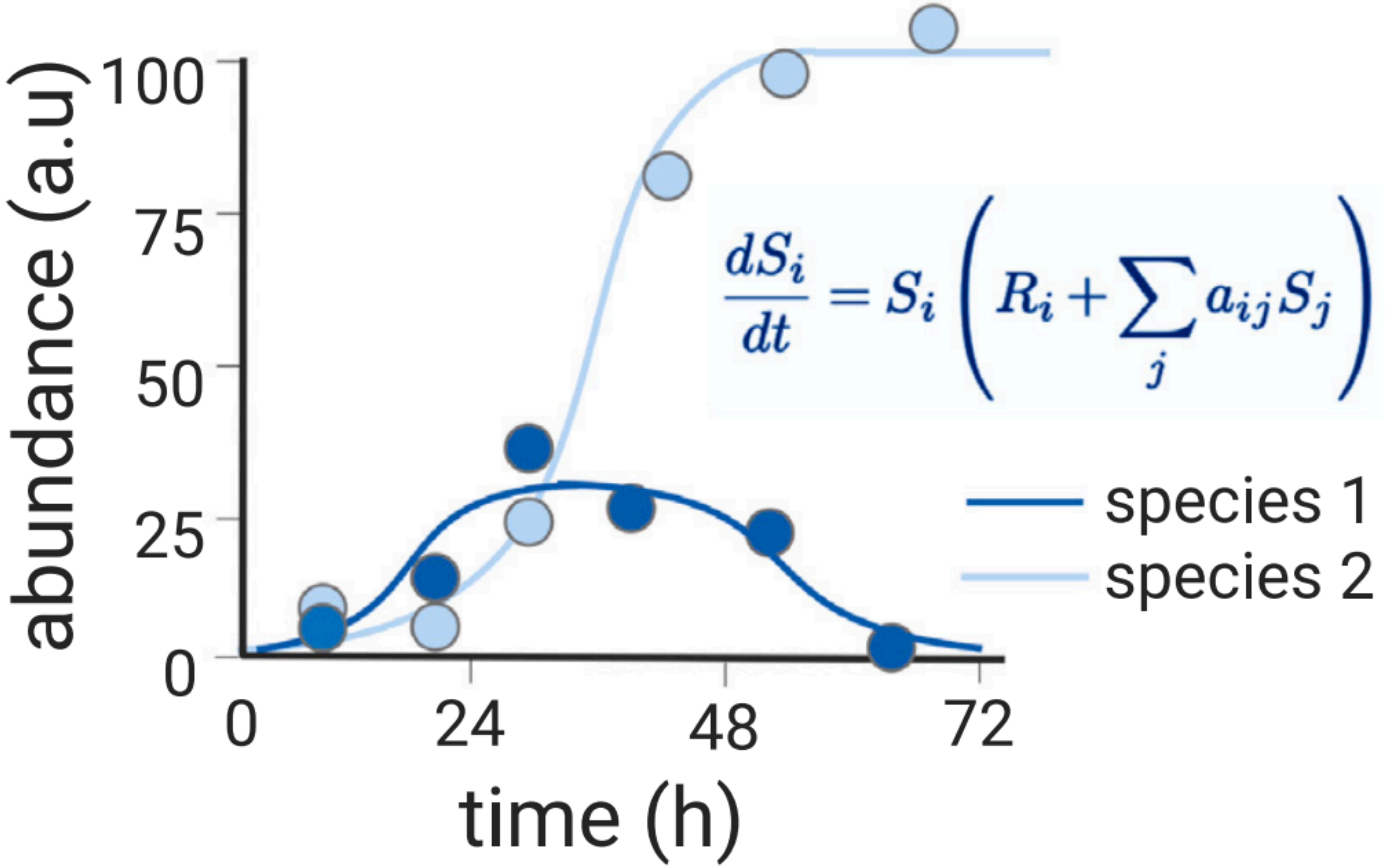
$$a'_{\alpha\beta}(\mathbf{r}) \equiv \nabla g_\alpha(\mathbf{r}) \cdot \mathbf{f}_\beta(\mathbf{r})$$

Foster, Bell 2012

Meacock, Mitri 2025

Measures of interactions: Parametric inference

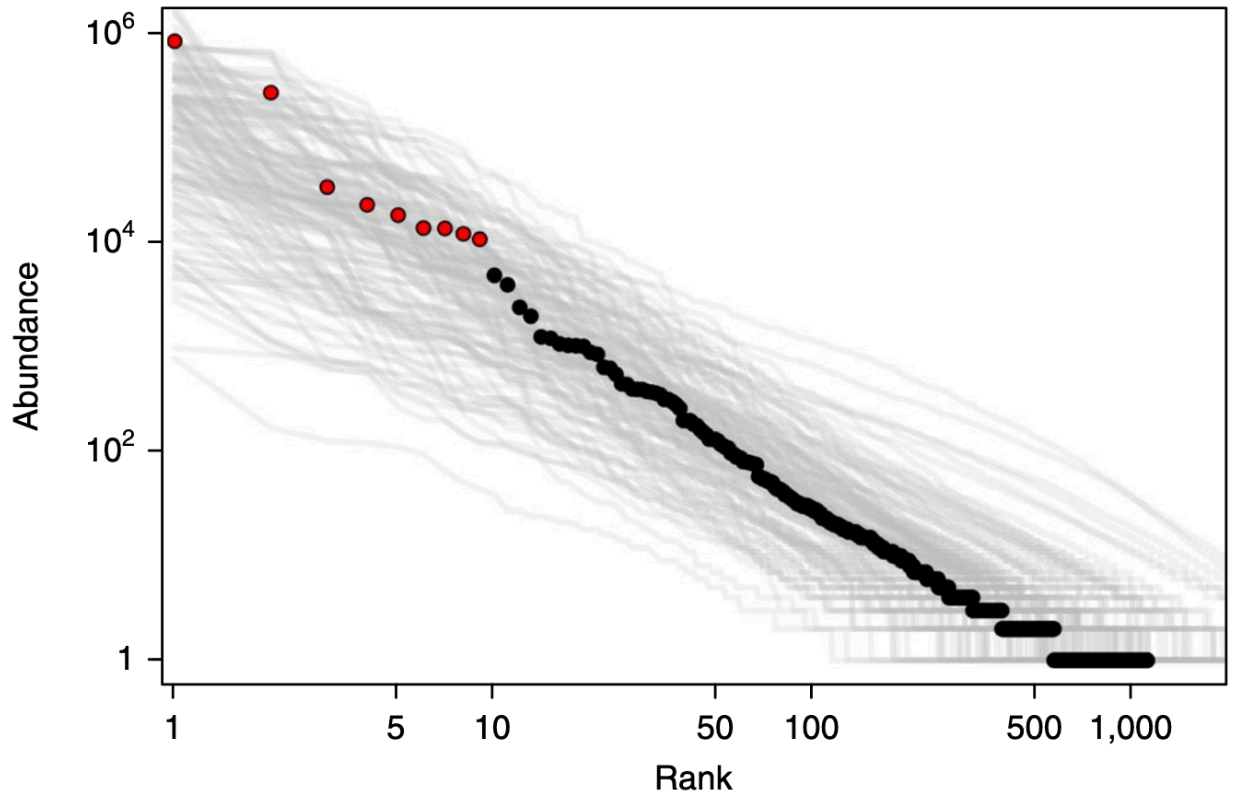
Generalized Lotka-Volterra model (gLV)

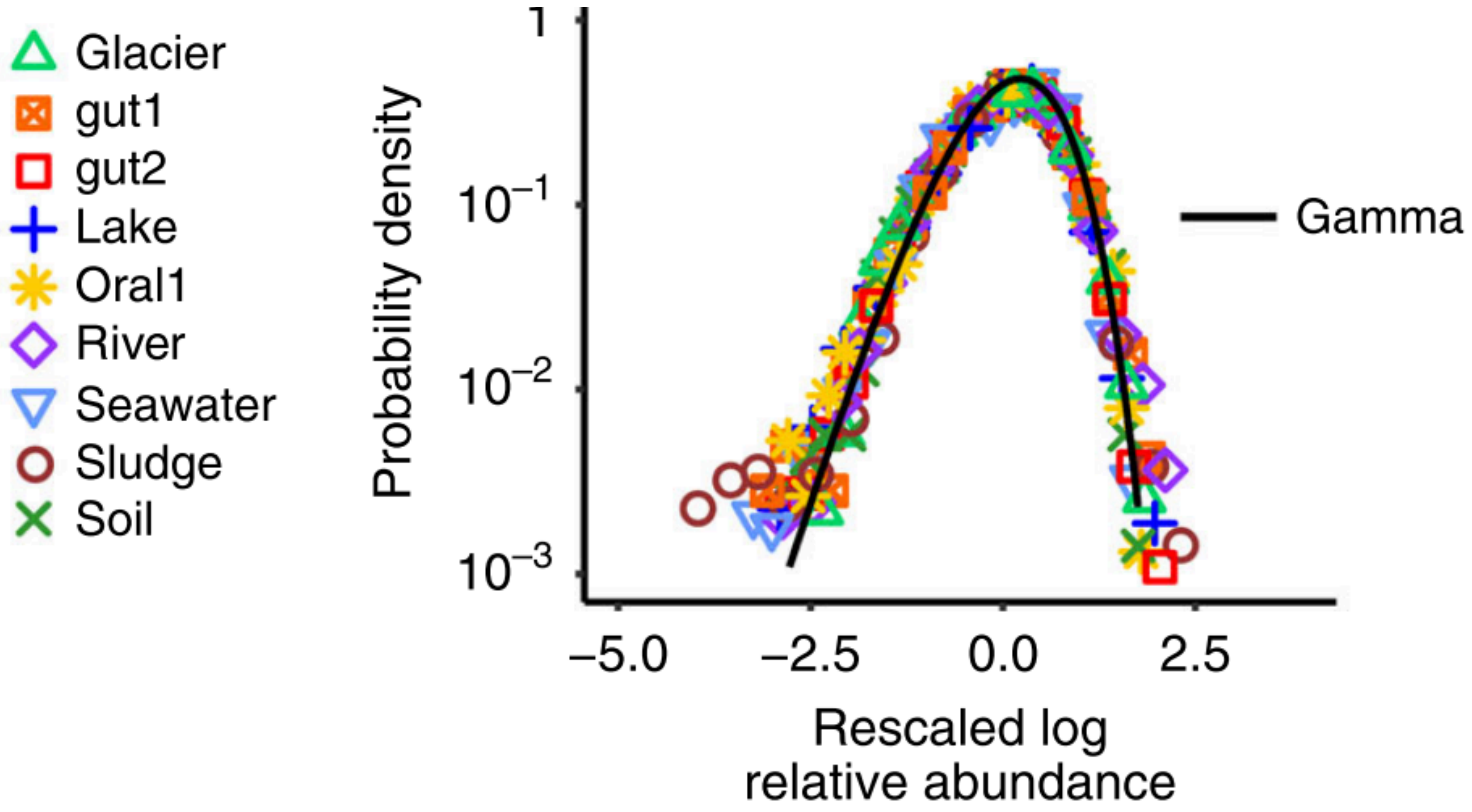


Picot, Shibasaki, Meacock, Mitri 2023

How to deal with complex interactions?

Approach I : Neglect them





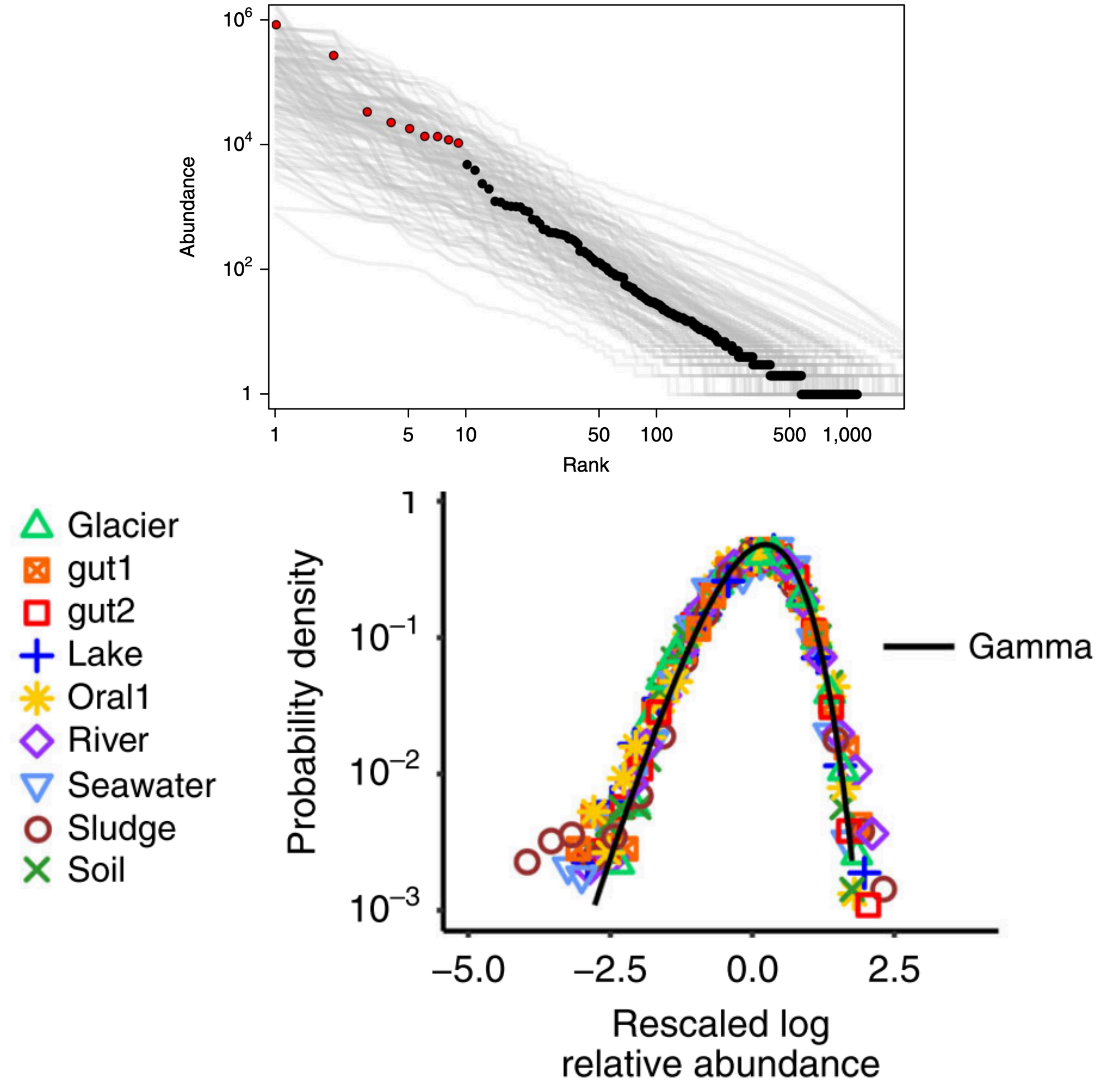
Ser-Giacomi et al. 2018

Grilli 2020

Interactions in a changing environment:

How to deal with complex interactions?

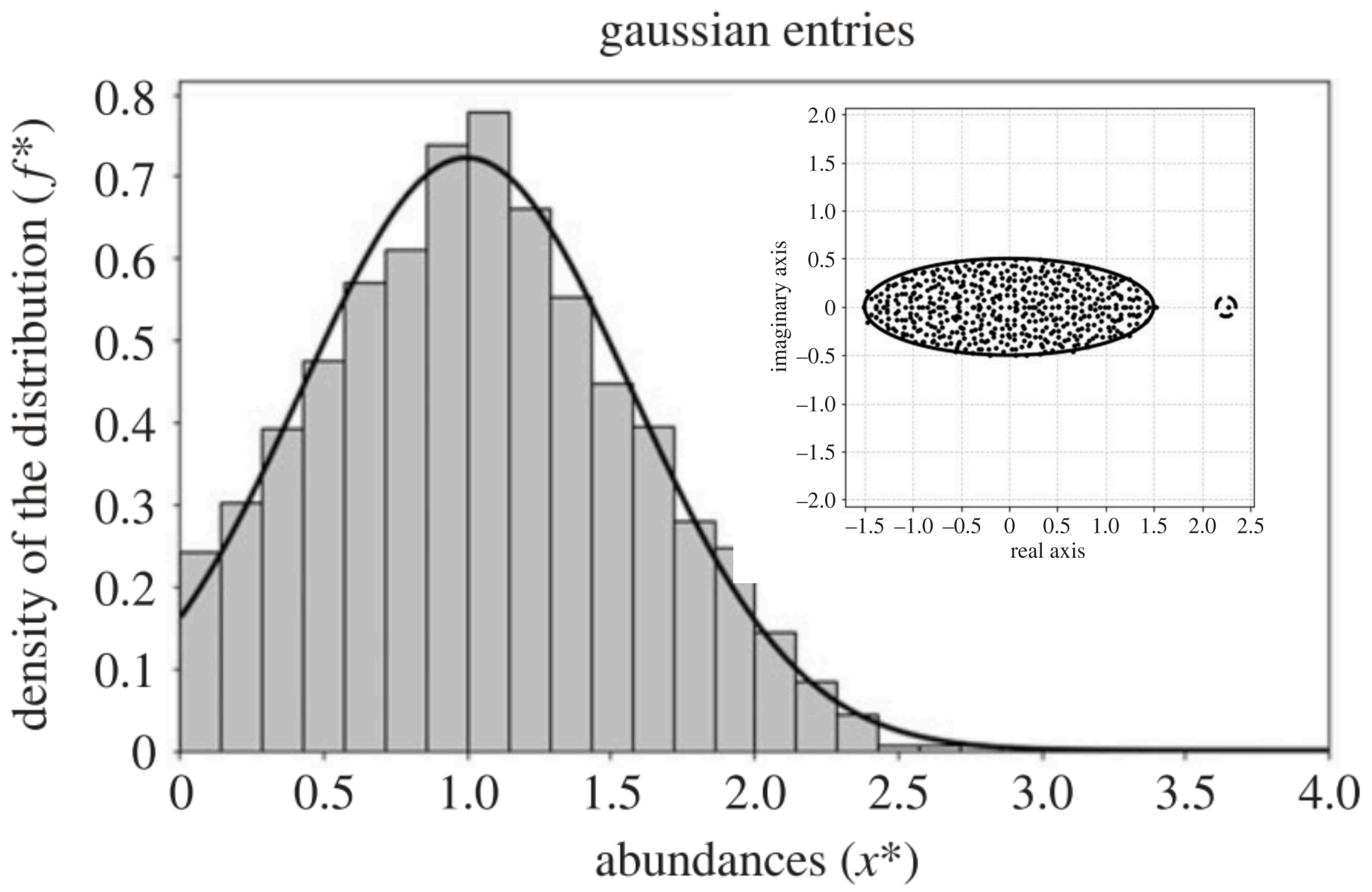
Approach I : Neglect them



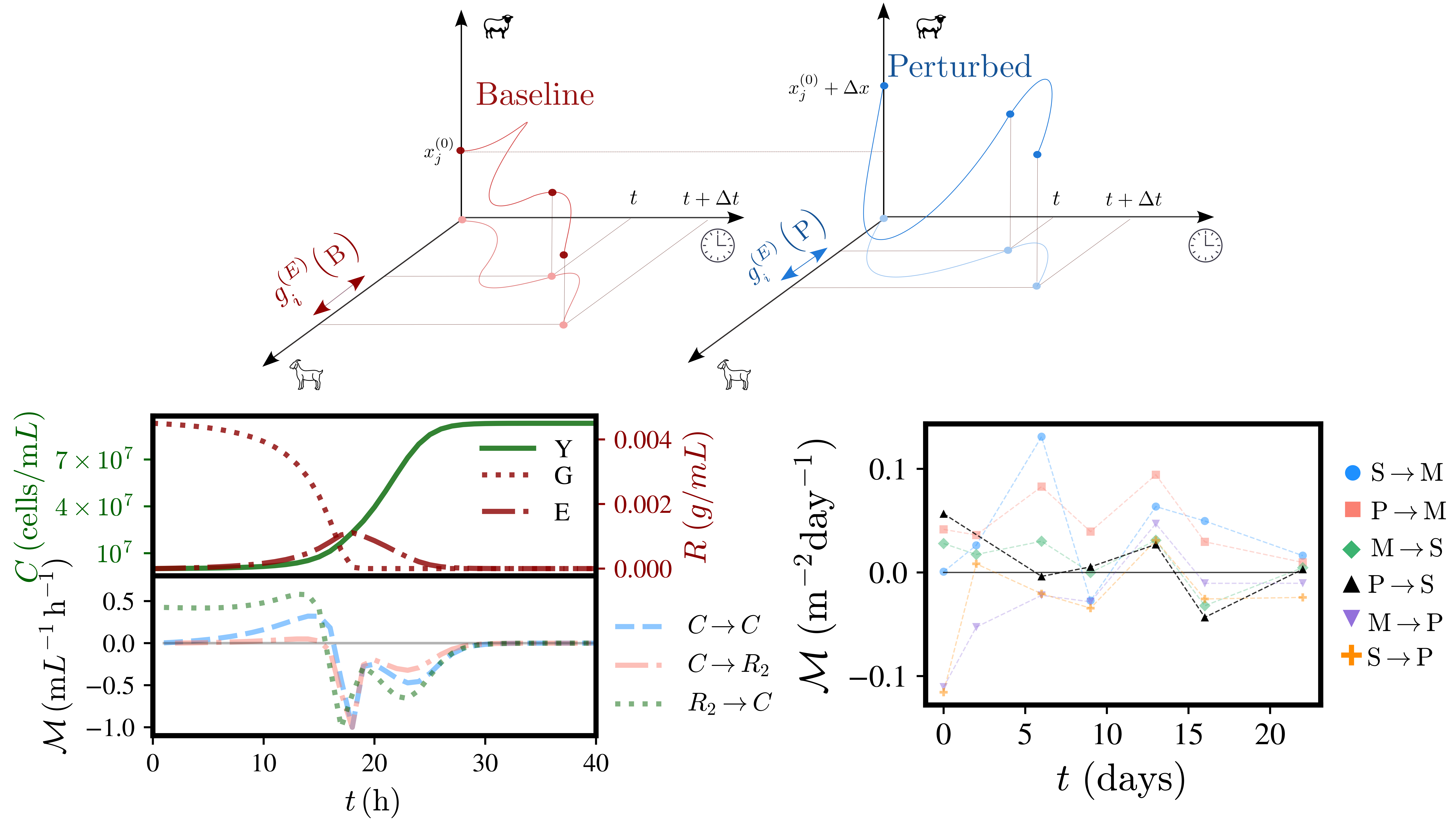
Ser-Giacomi et al. 2018

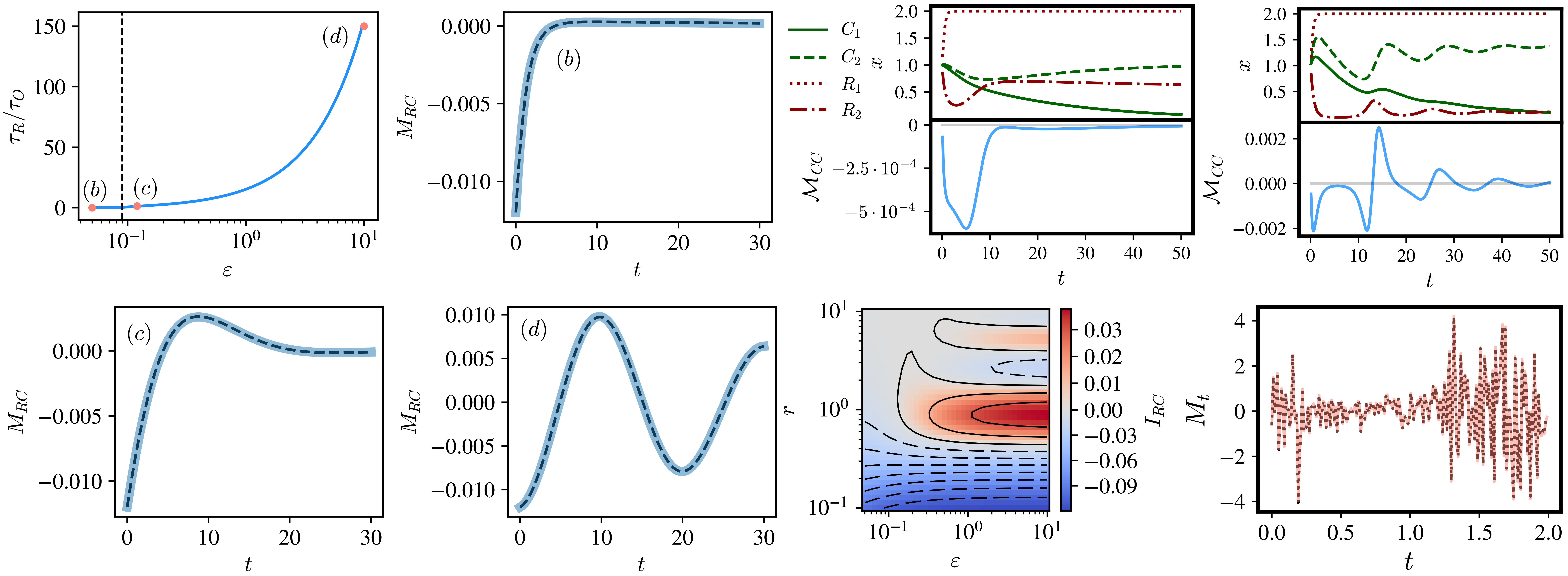
Grilli 2020

Approach II : Treat as random

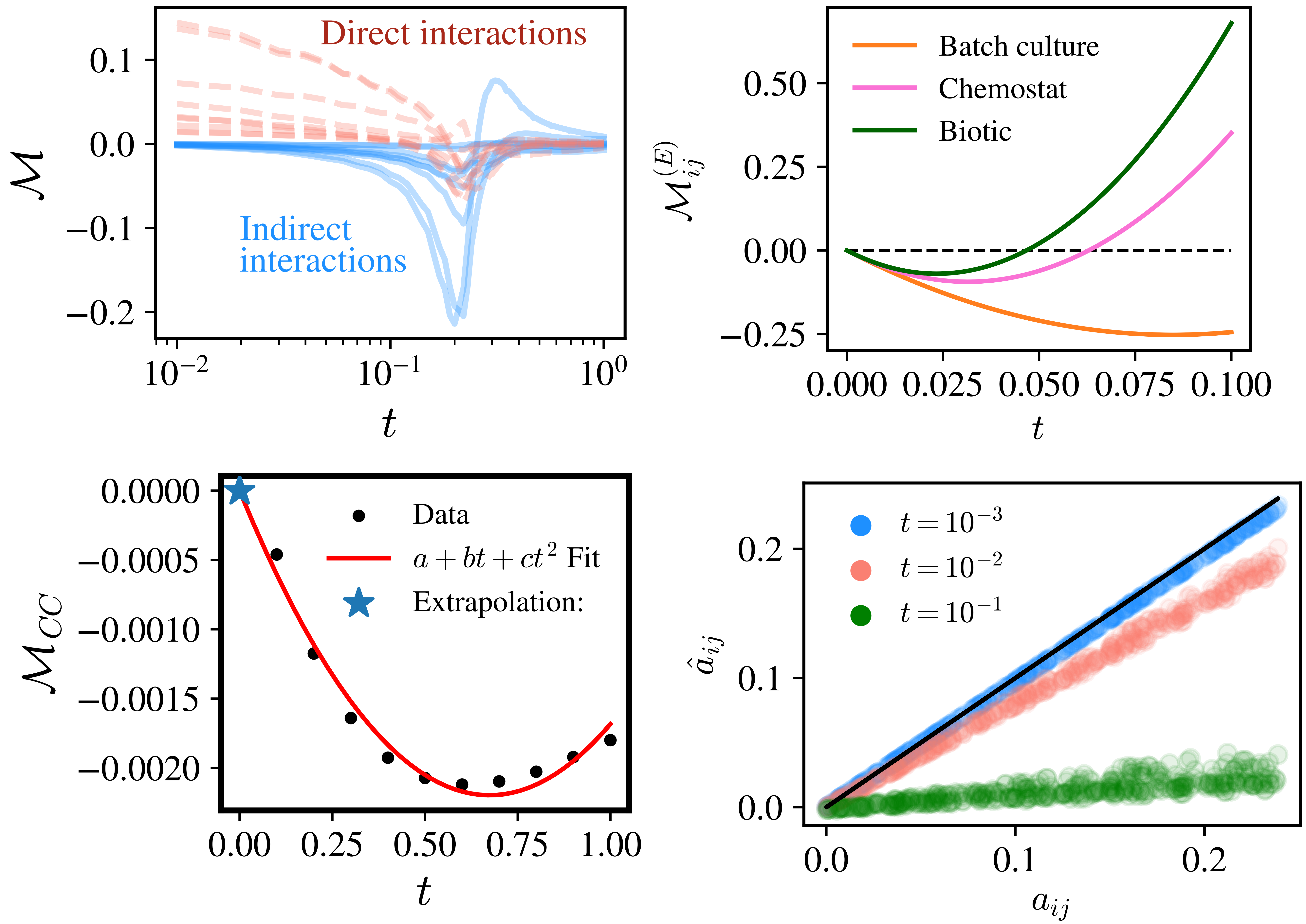


Akjouj et al. 2024


Baseline
Perturbed
$x_j^{(0)}$
$x_j^{(0)} + \Delta x$
t
$t + \Delta t$
$g_i^{(E)}$ (B)
$g_i^{(E)}$ (P)
C (cells/mL)
R (g/mL)
Y
G
E
7×10^7
4×10^7
10^7
0.004
0.002
0.000
$\mathcal{M}$ (mL^{-1} h^{-1})
0.5
0.0
−0.5
−1.0
$C \to C$
$C \to R_2$
$R_2 \to C$
0
10
20
30
40
t (h)
$\mathcal{M}$ (m^{-2}day^{-1})
0.1
0.0
−0.1
0
5
10
15
20
t (days)
S → M
P → M
M → S
P → S
M → P
S → P



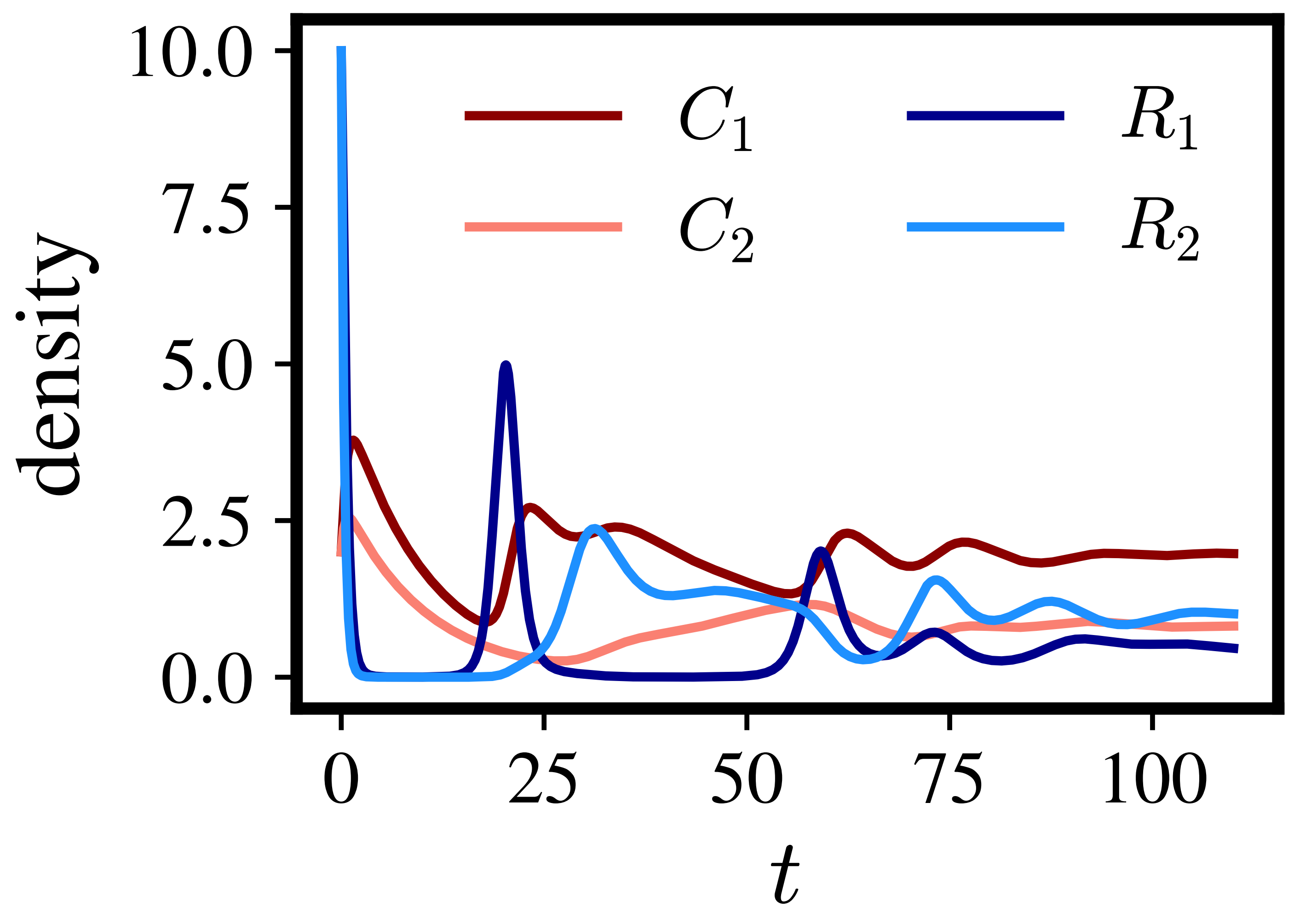
τ_R/τ_O
ε
(b)
(c)
(d)
M_{RC}
t
C_1
C_2
R_1
R_2
x
$\mathcal{M}_{CC}$
r
I_{RC}
M_t



Direct interactions
Indirect interactions
$\mathcal{M}$
t
Batch culture
Chemostat
Biotic
$\mathcal{M}_{ij}^{(E)}$
t
Data
$a+bt+ct^2$ Fit
Extrapolation:
$\mathcal{M}_{CC}$
t
$t=10^{-3}$
$t=10^{-2}$
$t=10^{-1}$
$\hat{a}_{ij}$
a_{ij}

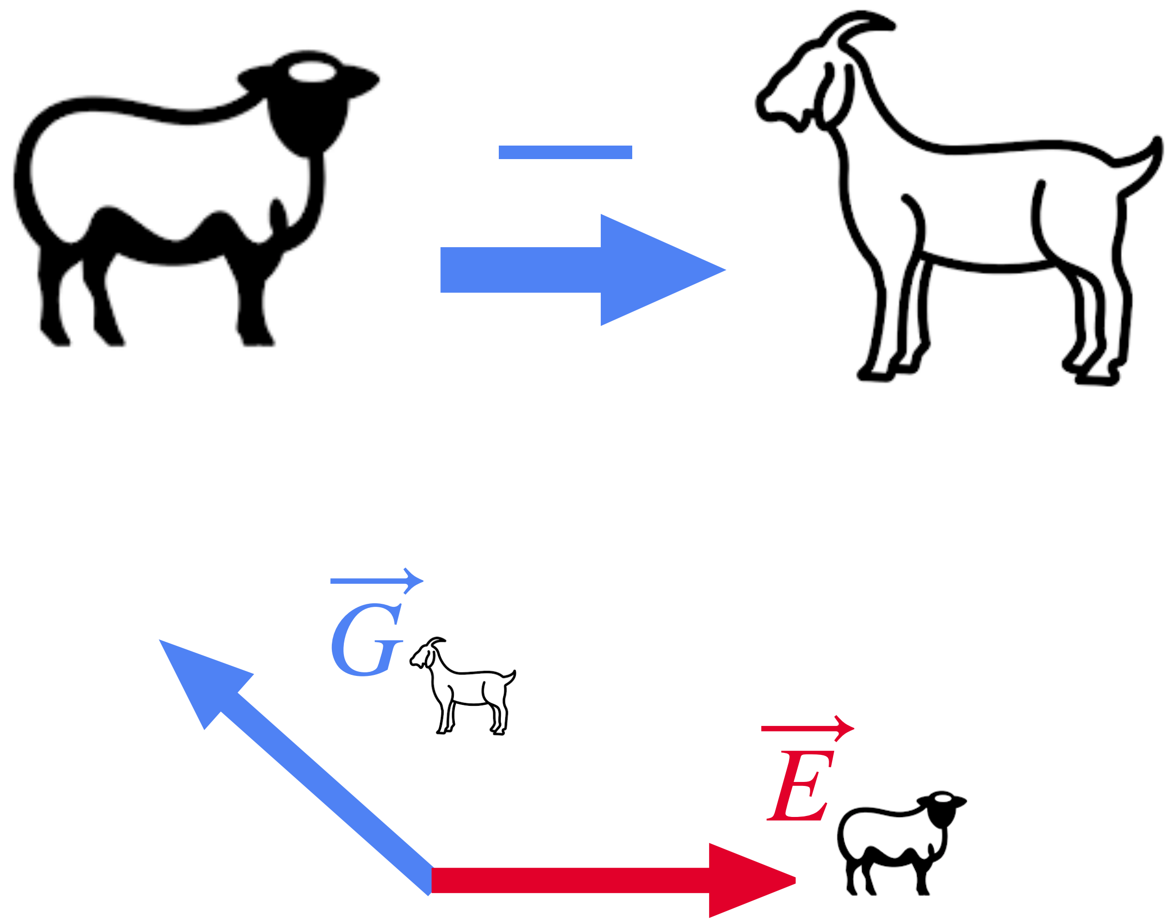
Synthetic datasets:

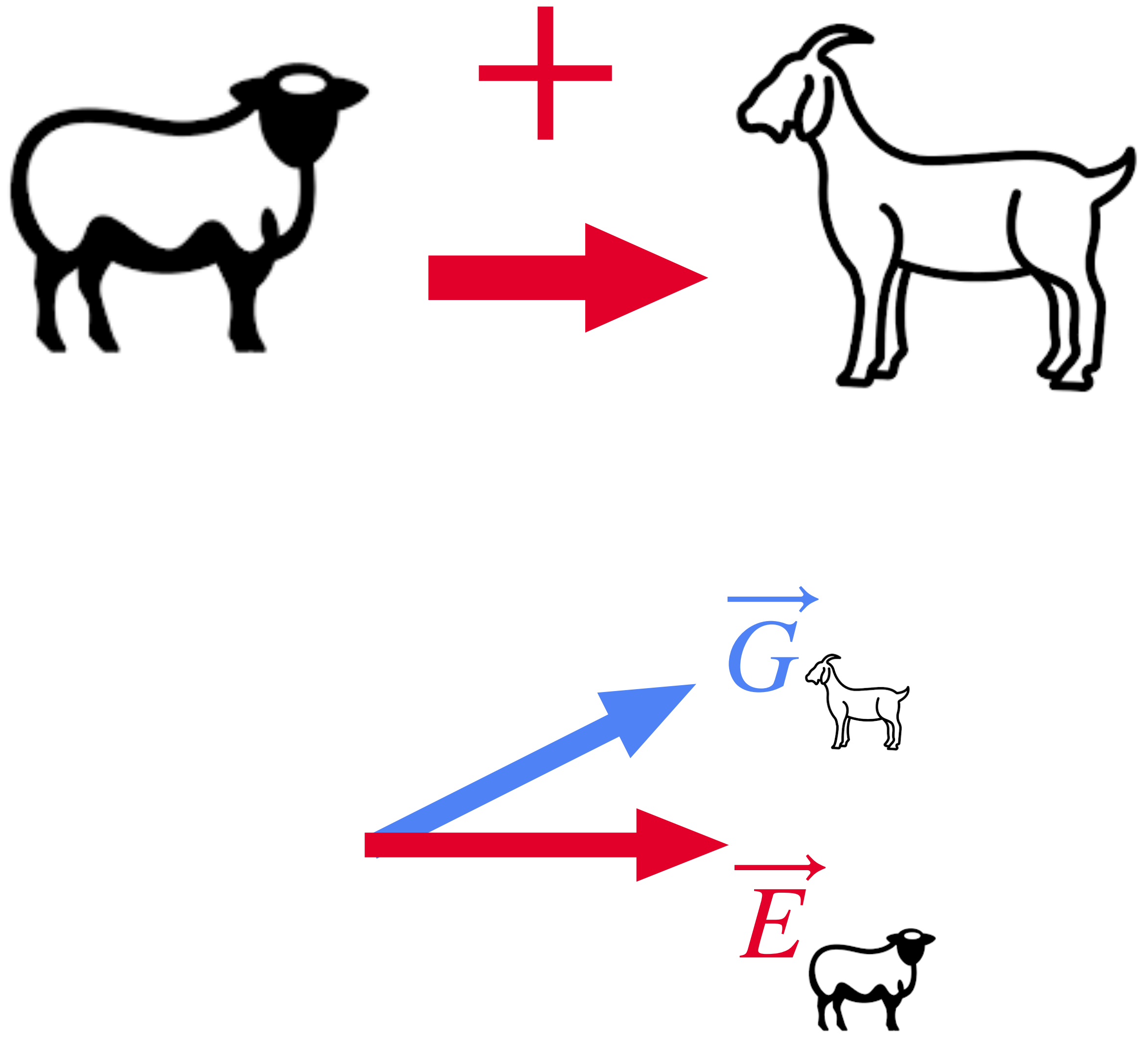
Generative model → $\dot{x}_i = x_i \, g_i\left(\vec{x}\right)$ → Test datasets



Analytical evaluation

$$\mathcal{M}^{(E)}_{i,j} \approx \vec{G}_i \cdot \vec{E}_j \, \Delta x$$





Gradient vector: $\vec{G}_i\left(\vec{x}\right) = \nabla g_i\left(\vec{x}\right)$

Evolution vector: $\vec{E}_j = \partial_{x_j^{(0)}} \vec{x}\left(t \,|\, \vec{x}^{(0)}\right)$