Born-Oppenheimer EFT

– an unified description of ordinary and exotic quarkonia –

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Based on:

(1) N. Brambilla, A. Mohapatra and A. Vairo
Unravelling pentaquarks with Born-Oppenheimer effective theory
arXiv:2508.13050

(2) N. Brambilla, A. Mohapatra, T. Scirpa and A. Vairo
The nature of $\chi_{c1}(3872)$ and $T_{cc}^{+}(3875)$
Phys. Rev. Lett. 135 (2025) 13 arXiv:2411.14306

(3) M. Berwein, N. Brambilla, A. Mohapatra and A. Vairo
Hybrids, tetraquarks, pentaquarks, doubly heavy baryons, and quarkonia in Born-Oppenheimer effective theory
Phys. Rev. **D 110** (2024) 094040 arXiv:2408.04719

(4) M. Berwein, N. Brambilla, J. Tarrus and A. Vairo
Quarkonium hybrids with nonrelativistic effective field theories
Phys. Rev. **D 92** (2015) 114019 arXiv:1510.04299

XYZ

Quarkonia and exotic states

QCD allows for more color singlet bound states than conventional hadrons i.e. mesons and baryons. This can be seen already at the level of the constituent hadron model.

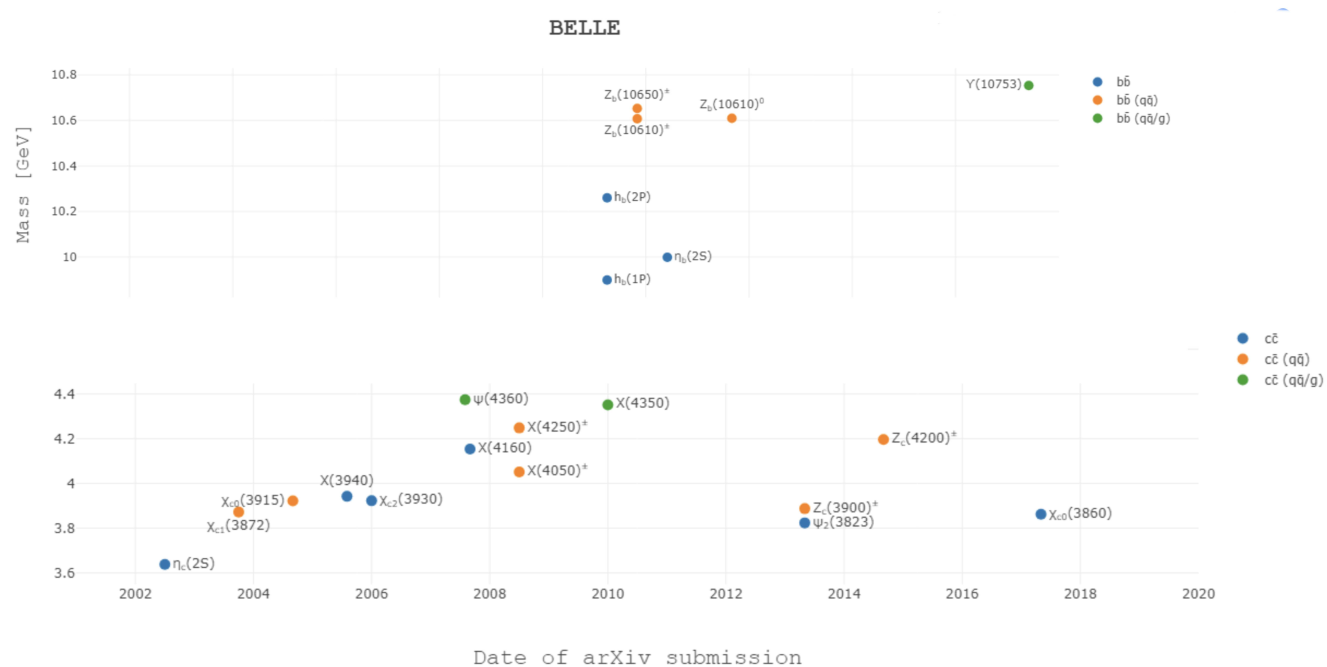
Constituents	Combinations	Naming convention (quark model)
$\mathbf{3} \otimes \bar{\mathbf{3}}$	$\mathbf{1} \oplus \mathbf{8}$	Meson
$\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3}$	$\mathbf{1} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{10}$	Baryon
$\mathbf{8} \otimes \mathbf{8}$	$\mathbf{1} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{10} \oplus \mathbf{10} \oplus \mathbf{27}$	Glueball
$\bar{\mathbf{3}} \otimes \mathbf{8} \otimes \mathbf{3}$	$\mathbf{1} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{10} \oplus \mathbf{10} \oplus \mathbf{27}$	Hybrid
$\bar{\mathbf{3}} \otimes \bar{\mathbf{3}} \otimes \mathbf{3} \otimes \mathbf{3}$	$\mathbf{1} \oplus \mathbf{1} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{10} \oplus \mathbf{10} \oplus \mathbf{27}$	Tetraquark/molecule
$\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} \otimes \bar{\mathbf{3}}$	$\mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{10} \oplus \mathbf{10} \oplus \mathbf{27} \oplus \mathbf{35} + \cdots$	Pentaquark
………..	………..	?

A constituent model of hadrons

- Gell-Mann Phys. Lett. 8 (1964) 214

22 years of XYZ

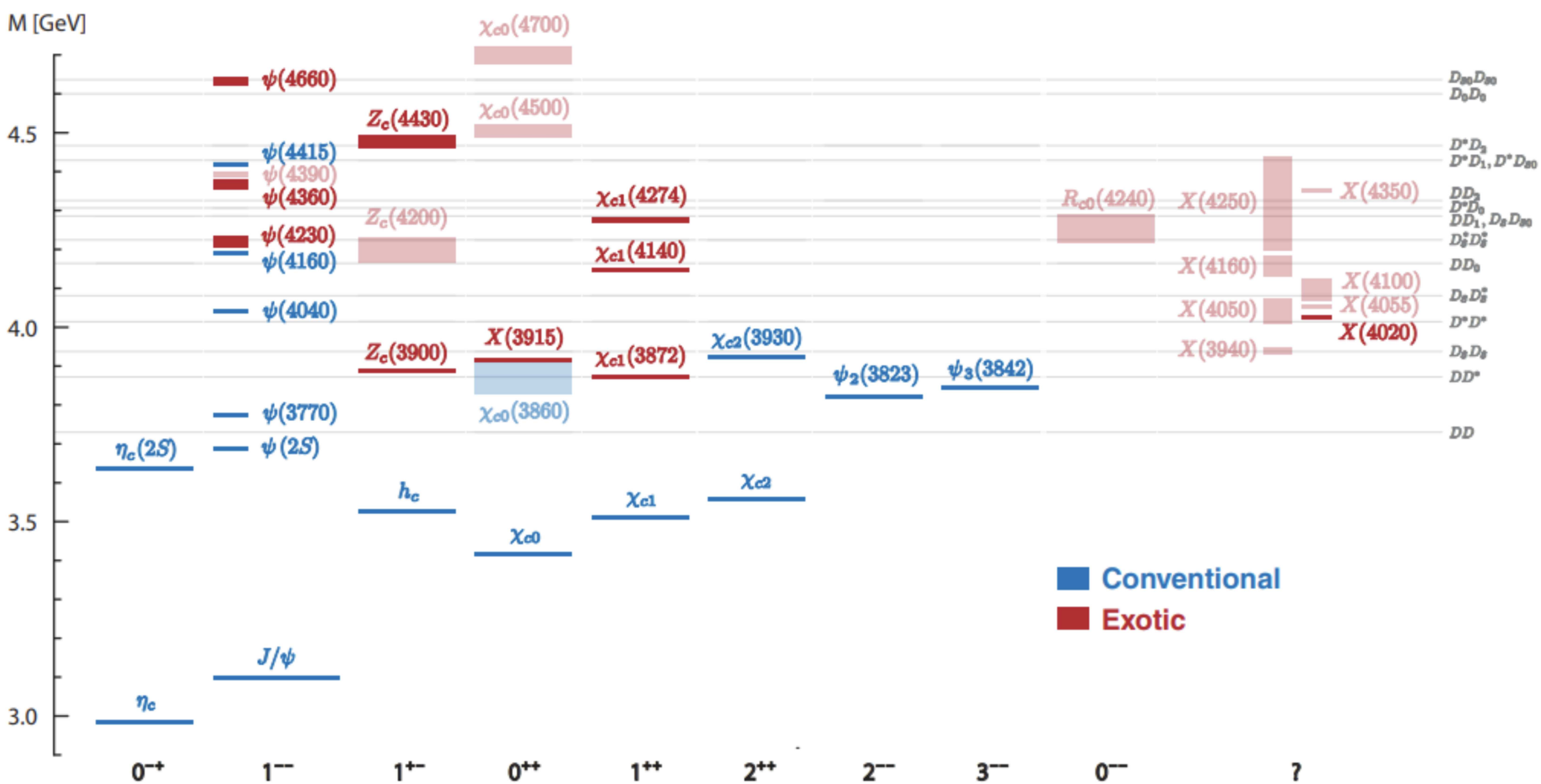
Since the discovery of the $\chi_{c1}(3872)$ in 2003 by Belle, 45 possibly non-conventional hadrons have been discovered in the charmonium and bottomonium sector.



Some of them are clearly non-conventional because they contain hidden charm or bottom and are charged (e.g. $T^+_{c\bar{c}}(4050)$, ..., $T^+_{b\bar{b}1}(10650)$, ...) or of other properties.

- Brambilla et al Phys. Rep. 873 (2020) 1
  qwg.ph.nat.tum.de/exoticshub/

Charmonia and XYZ in the charm sector



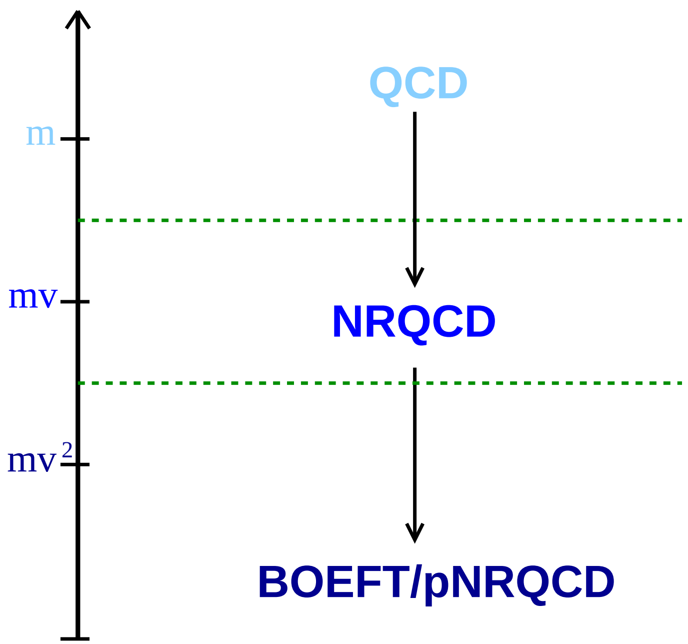
Born–Oppenheimer EFT

Energy scales

Quarkonia and quarkonium exotica are bound states made of a pair of heavy quarks.
The quarks being heavy guarantees the hierarchy of energy scales

$$m_Q \gg p \sim 1/r \sim m_Q v \gg E \sim m_Q v^2$$

where m_Q is the heavy quark mass and v the heavy quark relative velocity.
The hierarchy of energy scales calls for a hierarchy of effective field theories (EFTs).



BOEFT

The ultimate EFT is the Born–Oppenheimer EFT (BOEFT) that reduces to pNRQCD for heavy quarkonia. At first order, BOEFT reproduces the Born–Oppenheimer approximation: the heavy quarks move adiabatically in the presence of the light d.o.f., whose effect is encoded in a suitable set of potentials that depend on the distance $\boldsymbol{r}$ of the heavy quarks.

- Berwein Brambilla Tarrus Vairo PRD 92 (2015) 114019
  Oncala Soto PRD 96 (2017) 014004
  Brambilla Krein Tarrus Vairo PRD 97 (2018) 016016
  Soto Tarrus PRD 102 (2020) 014012

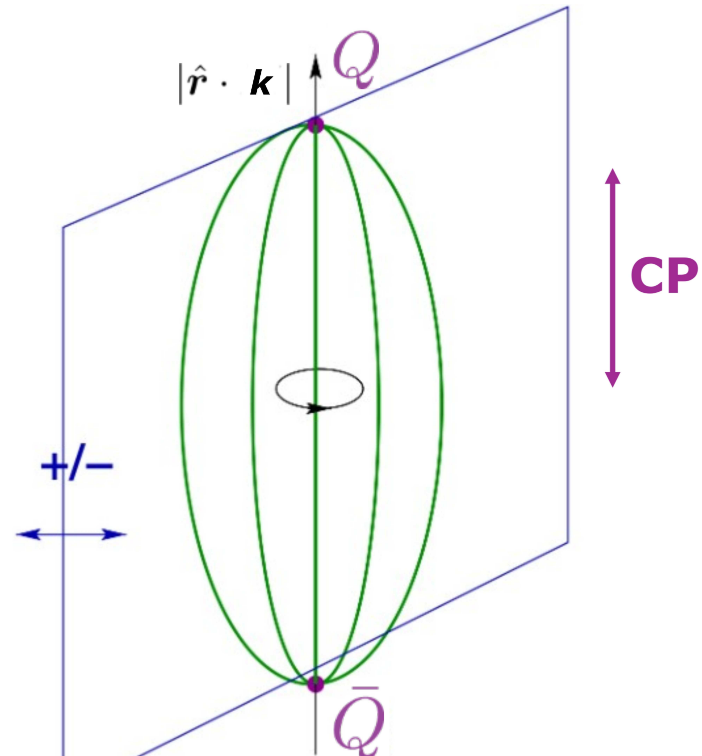
BOEFT: quantum numbers

Static $Q\bar{Q}(QQ)$ states are classified by the symmetry group $D_{\infty h}$
Representations are labeled Λ^σ_η
$\boldsymbol{k}$ is the angular momentum of the light d.o.f.
$\boldsymbol{r}\cdot\boldsymbol{k} = \Lambda = 0, 1, 2, \ldots \equiv \Sigma, \Pi, \Delta, \ldots$
η is the P(C) eigenvalue: $g \equiv 1$ and $u \equiv -1$
σ is the reflection eigenvalue (only for Σ)

Higher states for a given irreducible rep
are labeled by primes, e.g. Σ_g, Σ'_g, ...

For $r \to 0$, $D_{\infty h} \to O(3)(\times C)$
Hence several Λ^σ_η reps give one $k^{P(C)}$ rep

k^{PC}	Λ^σ_η	k^P	Λ^σ_η
0^{++}	Σ_g^+	0^+	Σ_g^+
0^{+-}	Σ_u^+	0^-	Σ_u^-
0^{-+}	Σ_u^-	1^+	$\{\Sigma_g^-, \Pi_g\}$
1^{+-}	$\{\Sigma_u^-, \Pi_u\}$	1^-	$\{\Sigma_u^+, \Pi_u\}$
1^{--}	$\{\Sigma_g^+, \Pi_g\}$	2^-	$\{\Sigma_u^-, \Pi_u, \Delta_u\}$
2^{--}	$\{\Sigma_g^-, \Pi_g, \Delta_g\}$	...	...
...	...		



CP $\to$ P for $\bar{Q} \to Q$

○ Foster Michael PRD 59 (1999) 094509

BOEFT: Lagrangian

$$L_{\mathrm{BOEFT}} = \int d^3\boldsymbol{R} \int d^3\boldsymbol{r} \sum_{\lambda\lambda'} \mathrm{Tr}\Big\{ \Psi^{\dagger}_{\kappa\lambda}(\boldsymbol{r},\, \boldsymbol{R},\, t) \Big[i\partial_t\, \delta_{\lambda\lambda'} - V_{\kappa,\lambda\lambda'}(r)$$
$$+ \sum_{\alpha} P^{\alpha\dagger}_{\kappa\lambda}(\theta,\varphi)\, \frac{\boldsymbol{\nabla}^2_r}{m_Q} P^{\alpha}_{\kappa\lambda'}(\theta,\varphi) \Big] \Psi_{\kappa\lambda'}(\boldsymbol{r},\, \boldsymbol{R},\, t) \Big\} + \dots$$

The fields $\Psi_{\kappa\lambda}$ identify the states, with $P^{\alpha}_{\kappa\lambda}$ suitable projectors.

- $\kappa = \{k^{P(C)}, \mathrm{flavor}\}$ are the light d.o.f. quantum numbers;
- $\lambda = \pm\Lambda$ is the BO quantum number.

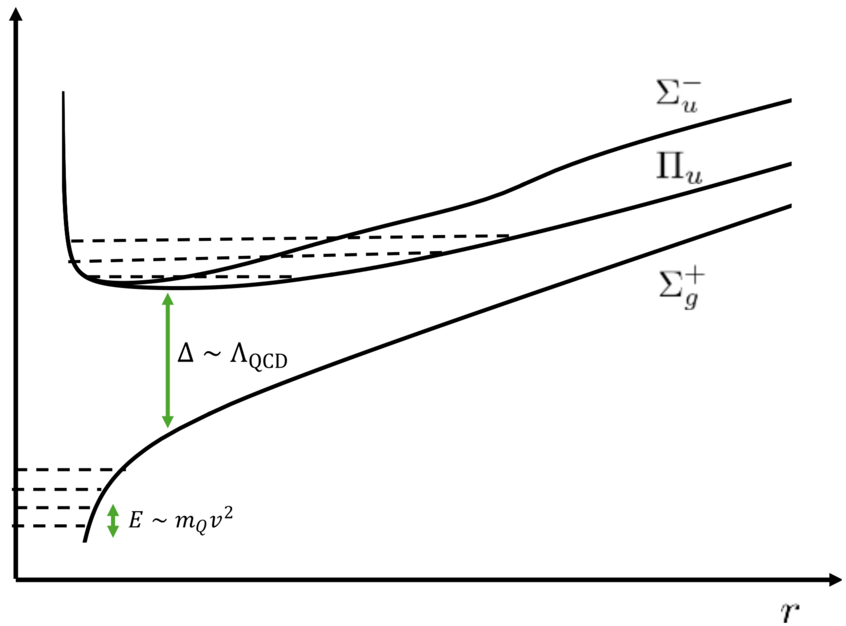
The potentials are expanded in $1/m_Q$:

$$V_{\kappa\lambda\lambda'}(r) = V^{(0)}_{\kappa\lambda\lambda'}(r) + \frac{V^{(1)}_{\kappa\lambda\lambda'}(r)}{m_Q} + \dots$$

with $V^{(0)}_{\kappa\lambda\lambda'}(r)$ the static potential and $V^{(1)}_{\kappa\lambda\lambda'}(r)$ includes the leading spin term.

Mixing at short distance: gluelumps and adjoint hadrons

We consider isospin $I = 0$ potentials between a heavy Q and $\bar{Q}$.



- Σ_g^+ is the quarkonium potential. The gap of order $\Lambda_{\rm QCD}$ guarantees that there is no mixing between quarkonium and excited states at short distance. A Cornell-like potential describes well quarkonium below threshold.
- At very short distance, excited potentials behave like the color octet $Q\bar{Q}$ potential. At short distance, they group in degenerate multiplets with the same gluelump mass (for hybrids), adjoint/triplet meson mass (for $Q\bar{Q}q\bar{q}/QQ\bar{q}\bar{q}$ tetraquarks), and adjoint baryon mass (for $Q\bar{Q}qqq$ pentaquarks) reflecting the restored $O(3)$ symmetry.

Mixing at short distance: coupled Schrödinger equations

Coupled (radial) equations of motion reflecting the short-distance Σ_u^--Π_u mixing for the lowest hybrids ($Q\bar{Q}g$) and tetraquarks ($Q\bar{Q}q\bar{q}$ or $QQ\bar{q}\bar{q}$) for $k = 1$ ($l(l+1)$ is the eigenvalue of $\boldsymbol{L}^2 = (\boldsymbol{k} + \boldsymbol{L}_Q)^2$) and parity σ_P:

$$\left[-\frac{1}{m_Q r^2}\,\partial_r r^2 \partial_r + \frac{1}{m_Q r^2}\begin{pmatrix} l(l+1)+2 & -2\sqrt{l(l+1)} \\ -2\sqrt{l(l+1)} & l(l+1) \end{pmatrix} + \begin{pmatrix} V_{\Sigma_u^-} & 0 \\ 0 & V_{\Pi_u} \end{pmatrix}\right]\begin{pmatrix} \psi^{(N)}_{\Sigma,\sigma_P} \\ \psi^{(N)}_{\Pi,\sigma_P} \end{pmatrix} = \mathcal{E}_N \begin{pmatrix} \psi^{(N)}_{\Sigma,\sigma_P} \\ \psi^{(N)}_{\Pi,\sigma_P} \end{pmatrix}$$

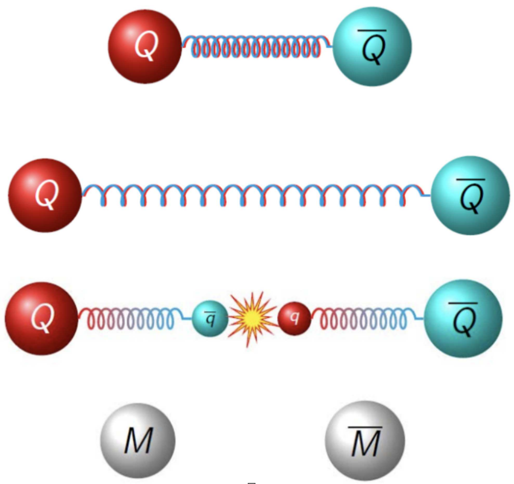
and opposite parity $-\sigma_P$:

$$\left[-\frac{1}{m_Q r^2}\,\partial_r\, r^2\, \partial_r + \frac{l(l+1)}{m_Q r^2} + V_{\Pi_u}\right]\psi^{(N)}_{\Pi,-\sigma_P} = \mathcal{E}_N\, \psi^{(N)}_{\Pi,-\sigma_P}$$

Note the parity doubling phenomenon.

Long distance and open flavor thresholds

Due to the string breaking mechanism,



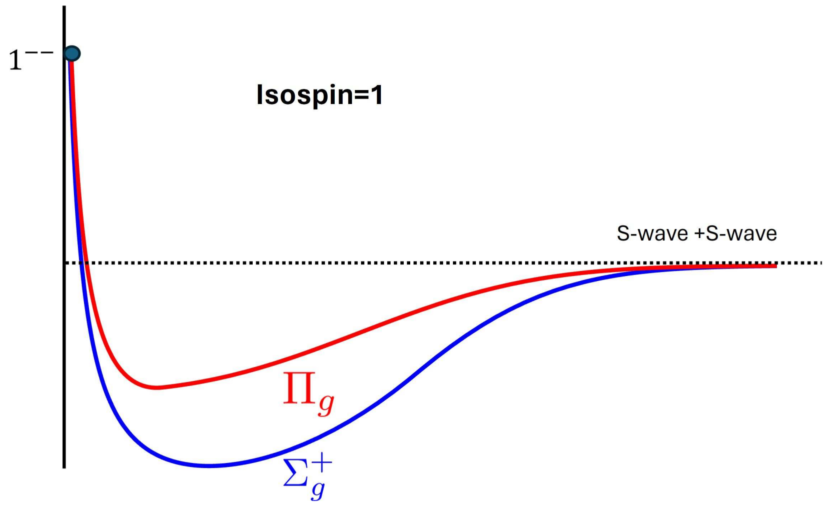
at long distance the potentials go into open flavor hadronic states whose light d.o.f. have the same BO quantum numbers, e.g.

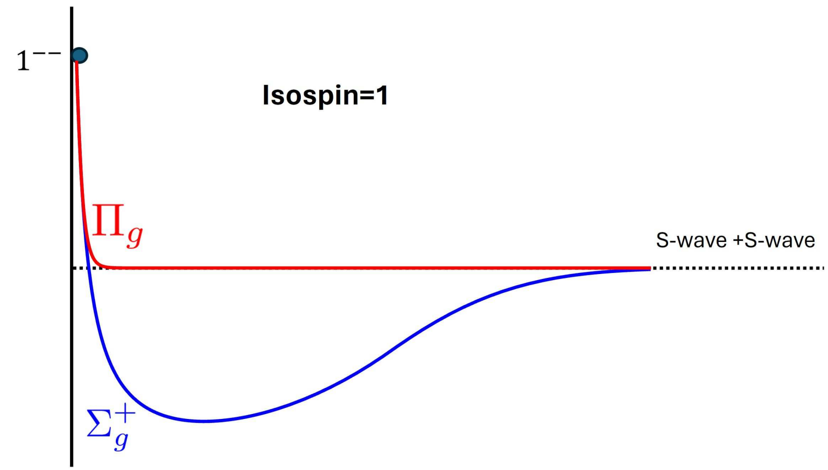
$k_{\bar{q}}^P \otimes k_q^P$	k^{PC}	BO quantum # Λ_η^σ
$(1/2)^- \otimes (1/2)^+$	0^{-+} 1^{--}	Σ_u^- Σ_g^+, Π_g

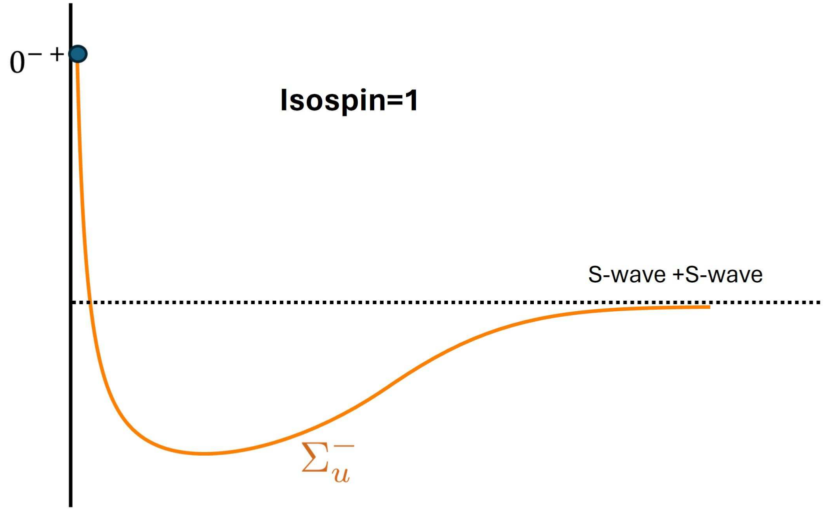
$k_{\bar{q}}^P \otimes k_{\bar{q}}^P$	k^P	BO quantum # Λ_η^σ
$(1/2)^- \otimes (1/2)^-$	0^+ 1^+	Σ_g^+ Σ_g^-, Π_g

where k_q ($k_{\bar{q}}$) is the angular momentum of the light (anti)quark.

Adiabatic potentials for isospin $\mathsf{I} = 1$



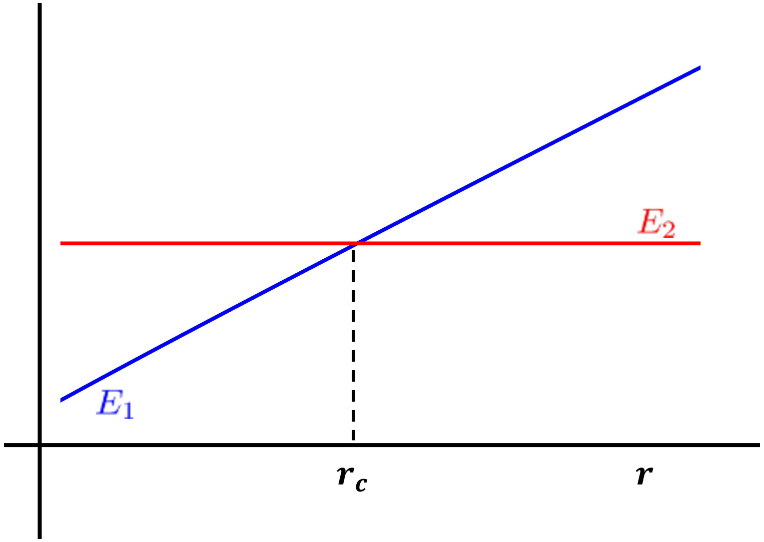




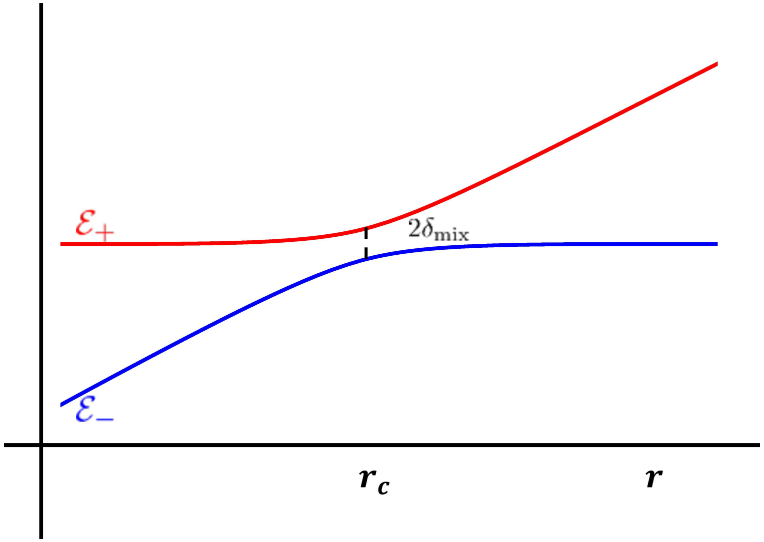
Mixing at intermediate distance: avoided level crossing

Potentials with the same BO quantum numbers mix, but mixing is a small effect if the potentials are separated by a large gap.

However, if potentials with the same BO quantum numbers in the $Q\bar{Q}$ and $M\bar{M}$ basis cross and mix, these are the diabatic potentials, then the potentials eigenstates of the static theory, the adiabatic potentials, show avoided level crossing.



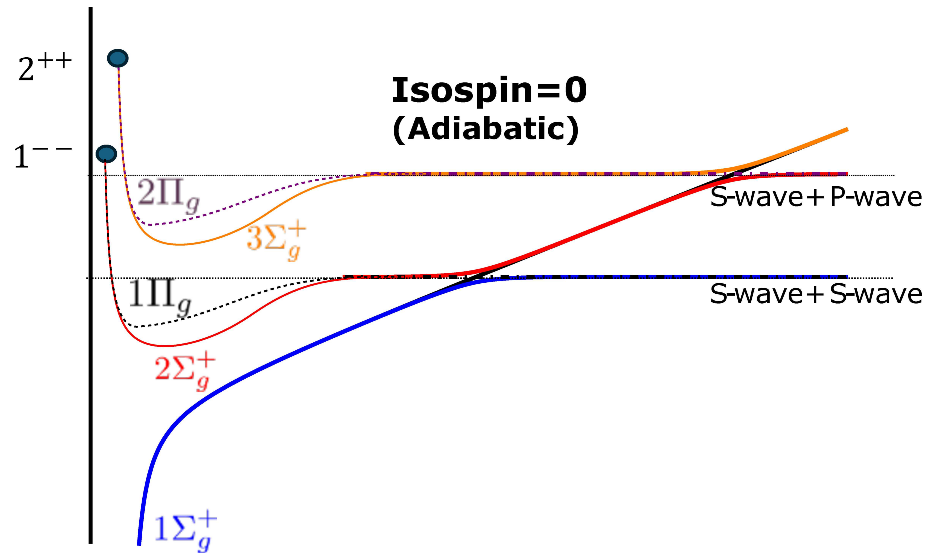
Diabatic potentials



Adiabatic potentials

This is particularly relevant in the isospin $I = 0$ sector when mixing with the quarkonium potential Σ_g^+ occurs.

Adiabatic potentials for isospin $\mathrm{I} = 0$



- At short distance the adiabatic potential $1\Sigma_g^+$ has attractive behavior, the adiabatic potentials $\left\{2\Sigma_g^+, 1\Pi_g\right\}$ and $\left\{3\Sigma_g^+, 2\Pi_g\right\}$ have repulsive behavior and form degenerate multiplets corresponding to 1^{--} and 2^{++} adjoint mesons.
- At about 1.2 fm, the $1\Sigma_g^+$ adiabatic potential and the $2\Sigma_g^+$ tetraquark potential undergo avoided level crossing. The $2\Sigma_g^+$ potential assumes the confining behavior of Σ_g^+ up to the next avoided level crossing. The $1\Sigma_g^+$ potential goes into the S-wave + S-wave static heavy-light meson-antimeson threshold.
- The avoided crossing is not affecting the BO potentials Π_g and the Π_g', which therefore coincide with the adiabatic potentials $1\Pi_g$ and $2\Pi_g$.

Coupled Schrödinger equations for Σ_g, Σ_g' and Π_g

$$\left[-\frac{1}{m_Q r^2}\,\partial_r r^2 \partial_r + \frac{1}{m_Q r^2}\begin{pmatrix} l\,(l+1) & 0 & 0 \\ 0 & l(l+1)+2 & -2\sqrt{l(l+1)} \\ 0 & -2\sqrt{l(l+1)} & l(l+1) \end{pmatrix}\right.$$
$$\left. + \begin{pmatrix} V_{\Sigma_g^+}(r) & V_{\Sigma_g^+ - \Sigma_g^{+\prime}}(r) & 0 \\ V_{\Sigma_g^+ - \Sigma_g^{+\prime}}(r) & V_{\Sigma_g^{+\prime}}(r) & 0 \\ 0 & 0 & V_{\Pi_g}(r) \end{pmatrix}\right]\begin{pmatrix} \psi^{(N)}_{\Sigma_g} \\ \psi^{(N)}_{\Sigma_g'} \\ \psi^{(N)}_{\Pi_g} \end{pmatrix} = \mathcal{E}_N \begin{pmatrix} \psi^{(N)}_{\Sigma_g} \\ \psi^{(N)}_{\Sigma_g'} \\ \psi^{(N)}_{\Pi_g} \end{pmatrix}$$

$V_{\Sigma_g^+ - \Sigma_g^{+\prime}}(r)$ is the mixing potential.
For $l = 1$ the equations describe $\chi_{c1}(3872)$ and the $\chi_c(1P)$ charmonium state.

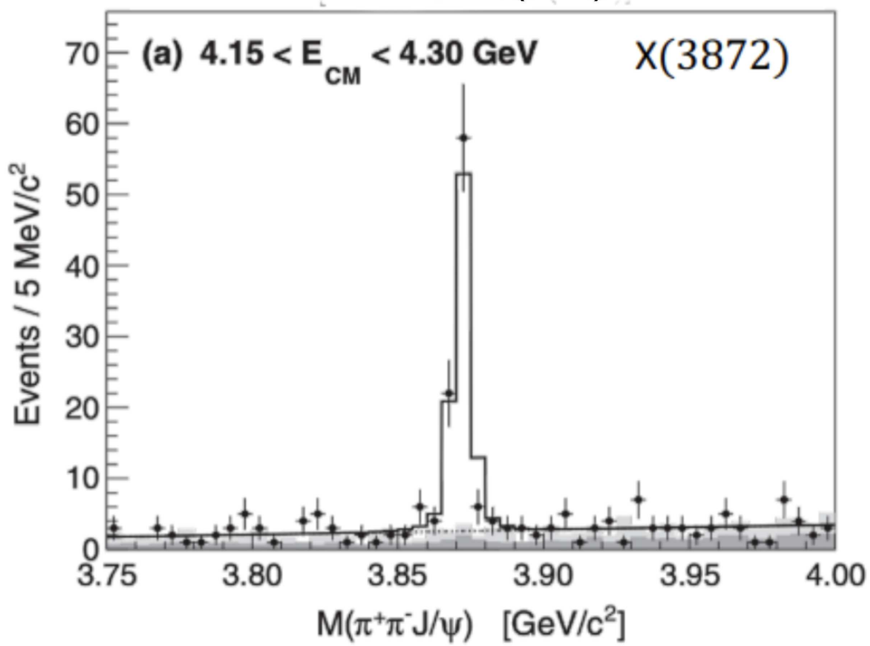
Tetraquarks: $\chi_{c1}(3872)$ and $T_{cc}^{+}(3875)$

$\chi_{c1}(3872)$

$\chi_{c1}(3872)$ has been the first observed XYZ.

$e^+e^- \to \gamma X(3872);\quad X(3872) \to \pi^+\pi^- J/\psi$

BesIII coll. PRL 122 (2019) 202001



- Quantum numbers: $J^{PC} = 1^{++} \quad (I = 0)$
- Very close the $D^{*0}\bar{D}^0$ threshold: $M_{\chi_{c1}(3872)} - (M_{D^{*0}} + M_{\bar{D}^0}) = -0.07 \pm 0.12$ MeV
- Most likely quark content: $c\bar{c}q\bar{q}$

∘ Belle coll PRL 91 (2003) 262001; LHCb coll PRL 110 (2013) 222001, PRD 92 (2015) 011102, JHEP 08 (2020) 123

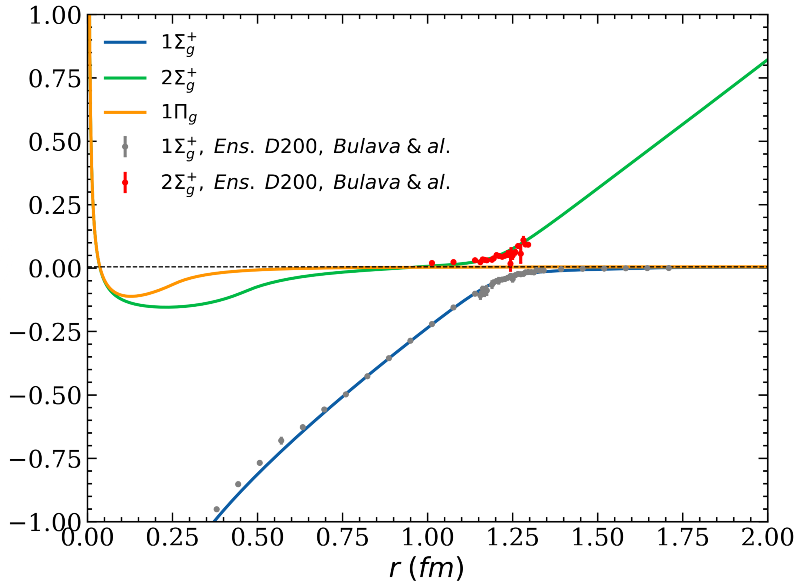
$Q\bar{Q}q\bar{q}$ multiplets

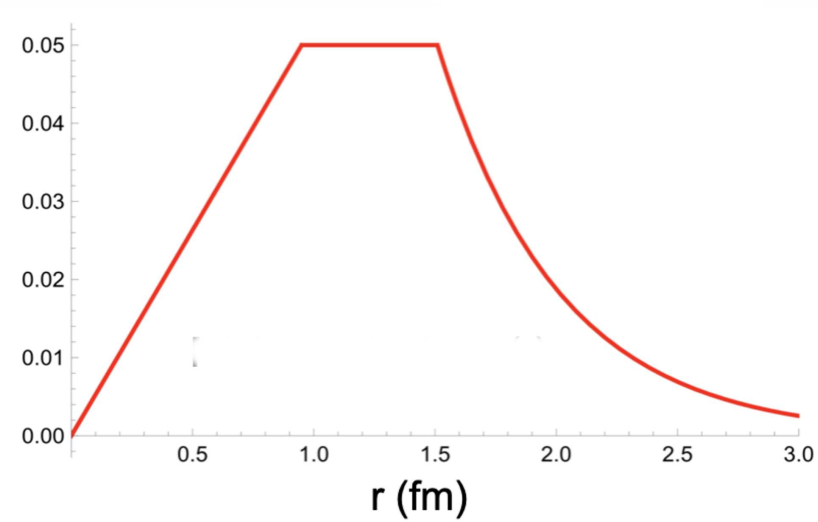
$Q\bar{Q}$ color state	Light spin k^{PC}	BO quantum # Λ_η^σ	l	J^{PC} $\{S_Q=0, S_Q=1\}$	Multiplets
Octet **8**	0^{-+}	Σ_u^-	0	$\{0^{++}, 1^{+-}\}$	T_1^0
			1	$\{1^{--}, (0,1,2)^{-+}\}$	T_2^0
			2	$\{2^{++}, (1,2,3)^{+-}\}$	T_3^0
	1^{--}	$\Sigma_g^{+\prime}, \Pi_g$	1	$\{1^{+-}, (0,1,2)^{++}\}$	T_1^1
		$\Sigma_g^{+\prime}$	0	$\{0^{-+}, 1^{--}\}$	T_2^1
		Π_g	1	$\{1^{-+}, (0,1,2)^{--}\}$	T_3^1
		$\Sigma_g^{+\prime}, \Pi_g$	2	$\{2^{-+}, (1,2,3)^{--}\}$	T_4^1

- States are ordered according to the $Q\bar{Q}$ orbital angular momentum.
- We identify $\chi_{c1}(3872)$ with the 1^{++} (isospin $I=0$) state of the lowest $k^{PC}=1^{--}$ multiplet, the one with $l=1$.
- 1^{+-} states are candidate for the isospin $I=1$ states $T_{c\bar{c}1}(3900)$ and $T_{c\bar{c}1}^{+}(4200)$ ($T_{b\bar{b}1}(10610)$ and $T_{b\bar{b}1}^{+}(10650)$ in the bottomonium sector).
  Note the mixing between $k^{PC}=0^{-+}$ and $k^{PC}=1^{--}$.
  - Voloshin PRD 93 (2016) 074011

Potentials

$\chi_{c1}(3872)$ is the solution of the coupled Schrödinger equations for Σ_g^+, $\Sigma_g^{+\prime}$ and Π_g. The adiabatic potentials $V_{1\Sigma_g^+}(r)$, $V_{2\Sigma_g^+}(r)$, $V_{1\Pi_g}(r)$ and the mixing potential $V_{\Sigma_g^+ - \Sigma_g^{+\prime}}(r)$ may be extracted from lattice QCD (+ symmetry constraints).



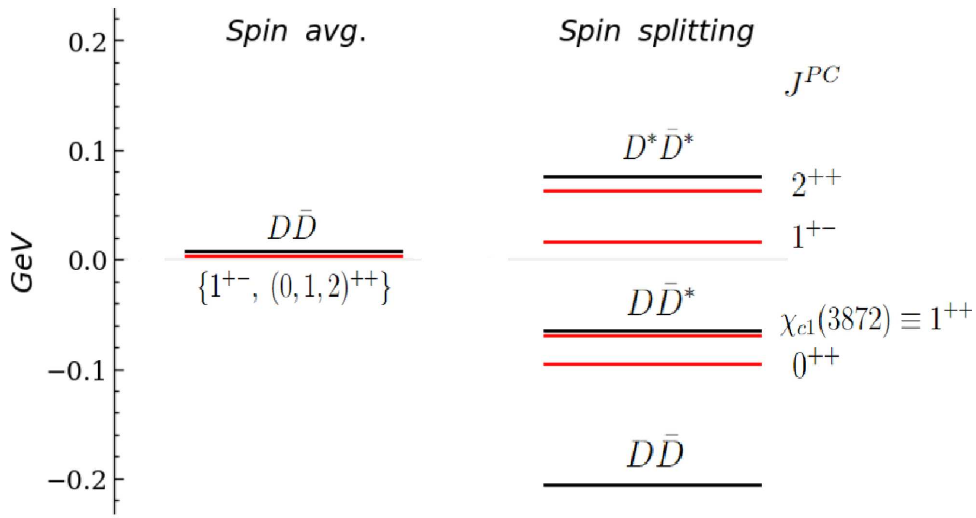


Adiabatic potentials

Mixing potential $V_{\Sigma_g^+ - \Sigma_g^{+\prime}}(r)$

- Lattice data from Bulava Knechtli Koch Morningstar Peardon PLB 854 (2024) 138754

$\chi_{c1}(3872)$ and fine structure



- After using all constraints on the potential, it remains only one free parameter: the overall normalization that we identify with the adjoint meson mass Λ^a_{1--}.
- Spin splittings are taken from lattice hybrid $c\bar{c}g$ spin splittings.
- For the critical value $\Lambda^a_{1--} \approx 914$ MeV, we could find a 1^{++} state with mass 3872 MeV that we identify with $\chi_{c1}(3872)$.
- Other members of the multiplet are a 1^{+-} state with mass $3957(11)$ MeV, a 0^{++} state with mass $3846(11)$ MeV, and a 2^{++} state with mass $4004(14)$ MeV. The 1^{+-} state could be a candidate for $X(3940)$ seen by Belle (not confirmed).
- In the spin-averaged case we find a deeper bound state with mass 3537 MeV that we identify with the spin-averaged $\chi_c\,(1P)$ state.

$\chi_{c1}(3872)$ compositness and radiative decays

The radius of the state is about 15 fm.

- The charmonium percentage $\sim |\psi_{\Sigma_g^+}|^2$ is about 8%.
- The $\Sigma_g^{+\prime}$ tetraquark percentage $\sim |\psi_{\Sigma_g^{+\prime}}|^2$ is about 38%.
- The Π_g tetraquark percentage $\sim |\psi_{\Pi_g}|^2$ is about 54%.

From this, it follows (assuming radiative decays via quarkonium component)

$$\frac{\Gamma_{\chi_{c1}(3872)\to\gamma\psi(2S)}}{\Gamma_{\chi_{c1}(3872)\to\gamma J/\psi}} = 2.99 \pm 2.36$$

to be compared with the LHCb measure 1.67 ± 0.25

- LHCb coll JHEP 11 (2024) 121

  Brambilla Mohapatra Scirpa Vairo PRL 135 (2025) 13

$\chi_{c1}(3872)$ inclusive production

Because the $c\bar{c}$ pair in the $\chi_{c1}(3872)$ at short distance is in an octet configuration, at leading order in v the $\chi_{c1}(3872)$ is formed in hadroproduction and in B decay from an octet $c\bar{c}$ pair. In the framework of NRQCD factorization, we have

$$\sigma_{\chi_{c1}(3872)} = \sigma_{Q\bar{Q}(^3S_1^{[8]})} \langle\Omega|\, \mathcal{O}^{\chi_{c1}(3872)}(^3S_1^{[8]})\,|\Omega\rangle$$

$$\mathrm{Br}(B \to X(3872) + X) = \mathrm{Br}(b \to c\bar{c}(^3S_1^{[8]}) + X)\ \langle\Omega|\, \mathcal{O}^{\chi_{c1}(3872)}(^3S_1^{[8]})\,|\Omega\rangle$$

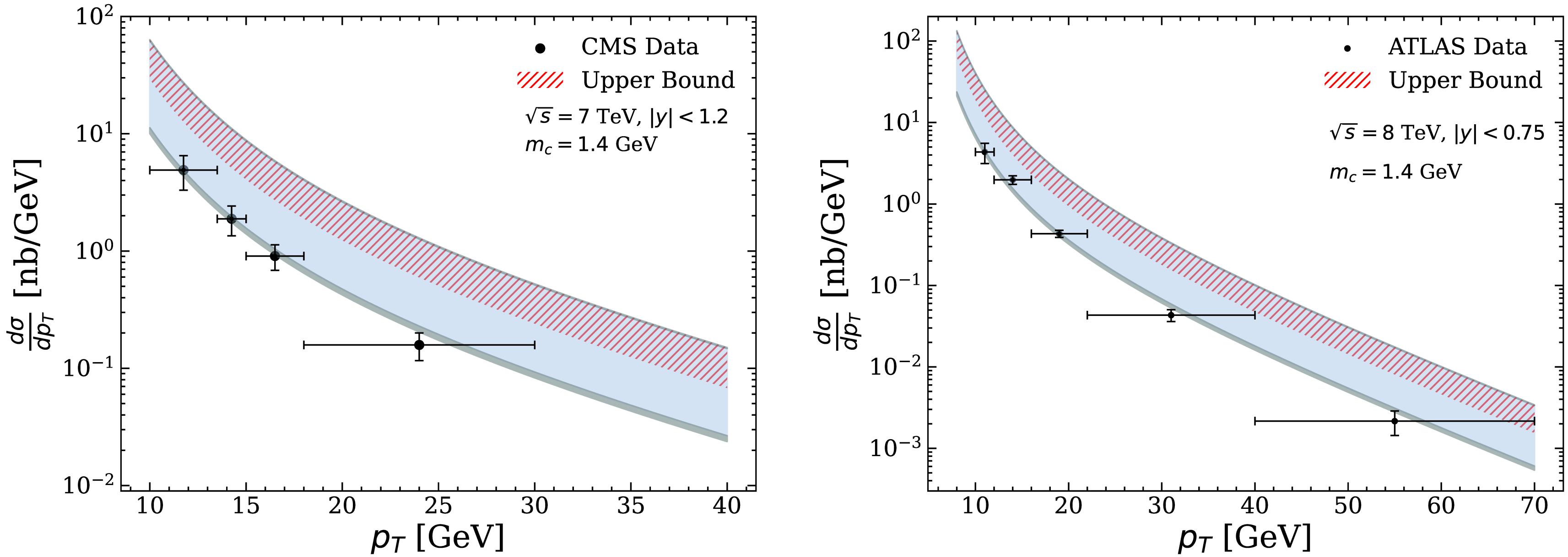
pNRQCD allows to factorize the octet matrix element in the BO wave function and an universal correlator $\mathcal{M}_S$, which has an upper bound,

$$\langle\Omega|\, \mathcal{O}^{\chi_{c1}(3872)}(^3S_1^{[8]})\,|\Omega\rangle = \frac{9}{4\pi}\,\left|\psi_{\Pi_g}(0)\right|^2 \mathcal{M}_S \lesssim \frac{3}{\pi}\,\left|\psi_{\Pi_g}(0)\right|^2$$

- Lai Chung PRD 112 (2025) 054005
  Brambilla Butenschoen Hibler Mohapatra Vairo Wang in preparation

$\chi_{c1}(3872)$ inclusive hadroproduction @ LHC

Fixing the octet matrix element on the $\chi_{c1}(3872)$ formation in B decay and using pNRQCD factorization leads to a prediction for $\chi_{c1}(3872)$ inclusive production at LHC.

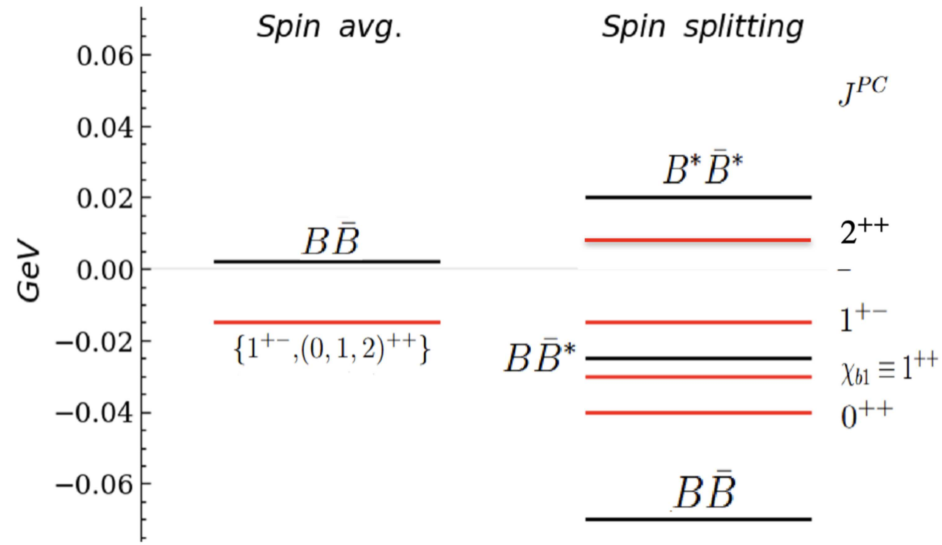


Preliminary

- Lai Chung PRD 112 (2025) 054005

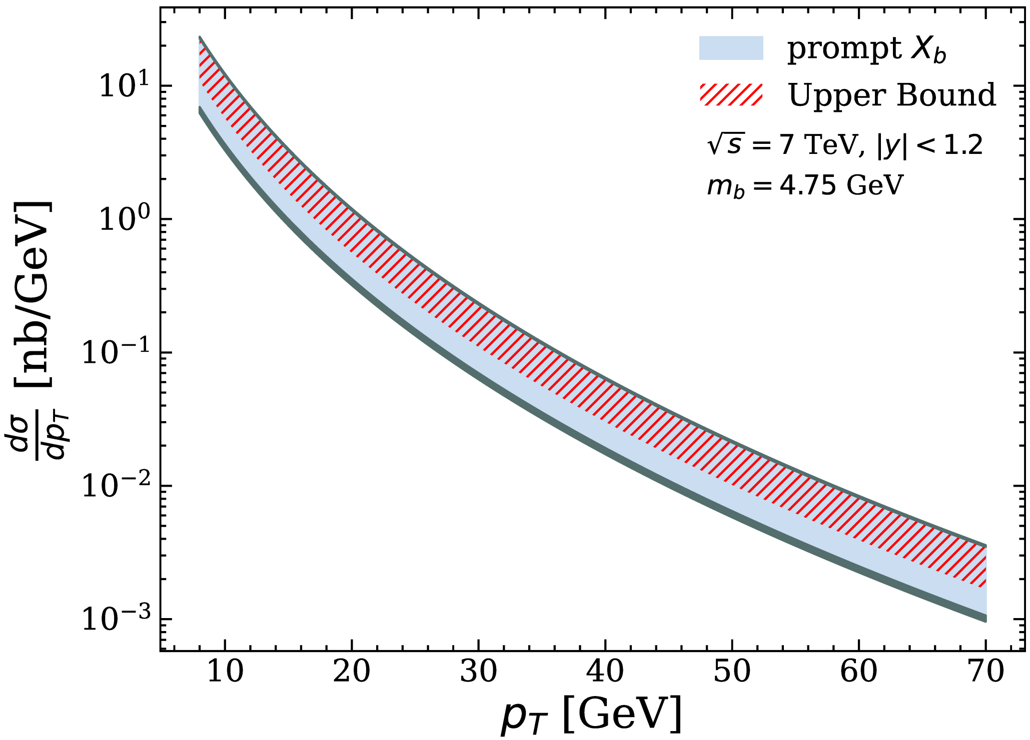
  Brambilla Butenschoen Hibler Mohapatra Vairo Wang in preparation

X_b and fine structure



- With the same input fixed on the charm case we can look at X_b, the equivalent of the $\chi_{c1}(3872)$ in the bottom sector. This state has not been observed yet.
- The lowest spin multiplet states 1^{++}, 1^{+-}, 0^{++} and 2^{++} get masses about 10595 MeV, 10612 MeV, 10586 MeV and 10627 MeV, respectively.
- The bottomonium percentage $\sim |\psi_{\Sigma_g^+}|^2$ is about 1.5%.
- The $\Sigma_g^{+\prime}$ tetraquark percentage $\sim |\psi_{\Sigma_g^{+\prime}}|^2$ is about 44.9%.
- The Π_g tetraquark percentage $\sim |\psi_{\Pi_g}|^2$ is about 53.6%.
- Note that X_b, differently from $\chi_{c1}(3872)$, is not on a meson-antimeson threshold.

X_b inclusive hadroproduction @ ATLAS



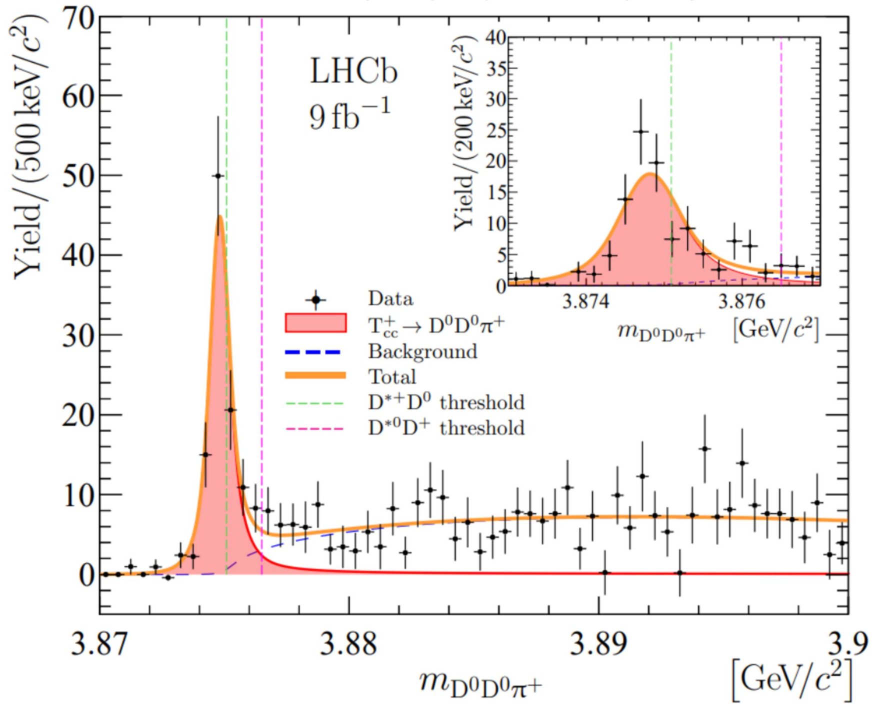
Preliminary

◦ Brambilla Butenschoen Hibler Mohapatra Vairo Wang in preparation

$T_{cc}^{+}(3875)$

$T_{cc}^{+}(3875)$ has been the first observed doubly charmed tetraquark at LHCb.

LHCb coll Nat. P 18 (2022)751, Nat. C 13 (2022) 3351



- Quantum numbers: $J^P = 1^+ \quad (I = 0)$
- Longest living exotic particle: $\Gamma \approx 50$ keV
- Below the $D^{*+}D^0$ threshold: $M_{T_{cc}^{+}(3875)} - (M_{D^{*+}} + M_{D^0}) = -0.27 \pm 0.06$ MeV
- Most likely quark content: $cc\bar{u}\bar{d}$

$QQ\bar{q}\bar{q}$ multiplets

QQ color state	$\bar{q}\bar{q}$ spin k^P	BO quantum # Λ^σ_η	Isospin I	l	J^P $S_Q = 0$	J^P $S_Q = 1$
Antitriplet $\bar{\mathbf{3}}$	0^+	Σ_g^+	0	0	—	1^+
				1	1^-	—
	1^+	Σ_g^-, Π_g	1	0	0^-	—
				1	1^-	$(0,1,2)^+$

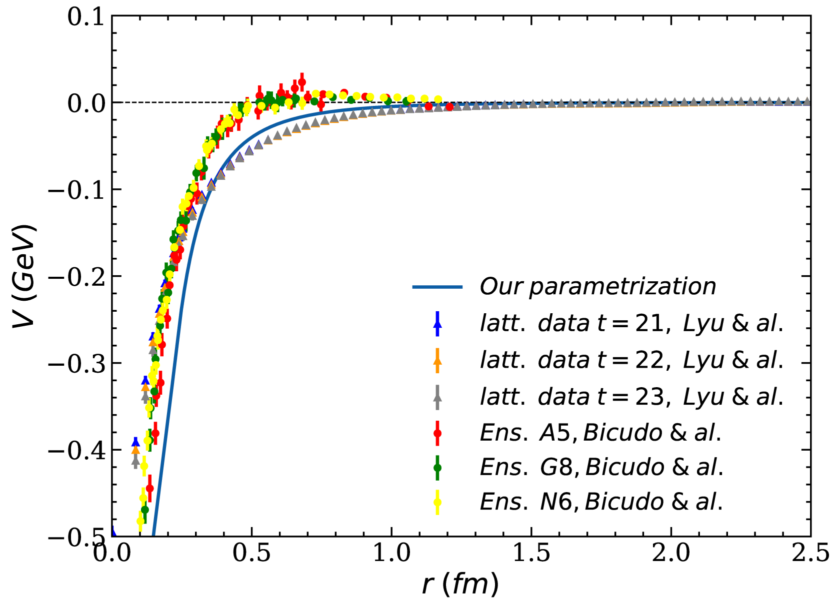
- Note that $3 \times 3 = \bar{3} + 6$; only the antitriplet channel is attractive ($\Rightarrow$ lower states).
- We identify $T_{cc}^+(3875)$ with the 1^+ (isospin $I = 0$) state of the lowest $k^P = 0^+$ multiplet, the one with $l = 0$ and $S_Q = 1$.
- The 1^+ state of Σ_g^+ is the lowest state if $V_{\Sigma_g^+}(r)$ lies below $V_{\Sigma_g^-}(r)$ and $V_{\Pi_g}(r)$.

Schrödinger equation and potential

$T_{cc}^{+}(3875)$ is the solution of the single Schrödinger equation for the BO state Σ_g^+ approaching a 0^+ triplet meson at short and a meson-meson $I=0$ pair at large distance:

$$\left[-\frac{1}{m_Q r^2}\,\partial_r\, r^2\,\partial_r + \frac{l(l+1)}{m_Q r^2} + V_{\Sigma_g^+}(r)\right]\psi^{(N)}_{\Sigma_g^+} = \mathcal{E}_N\,\psi^{(N)}_{\Sigma_g^+}$$

The potential $V_{\Sigma_g^+}(r)$ may be extracted from lattice QCD.

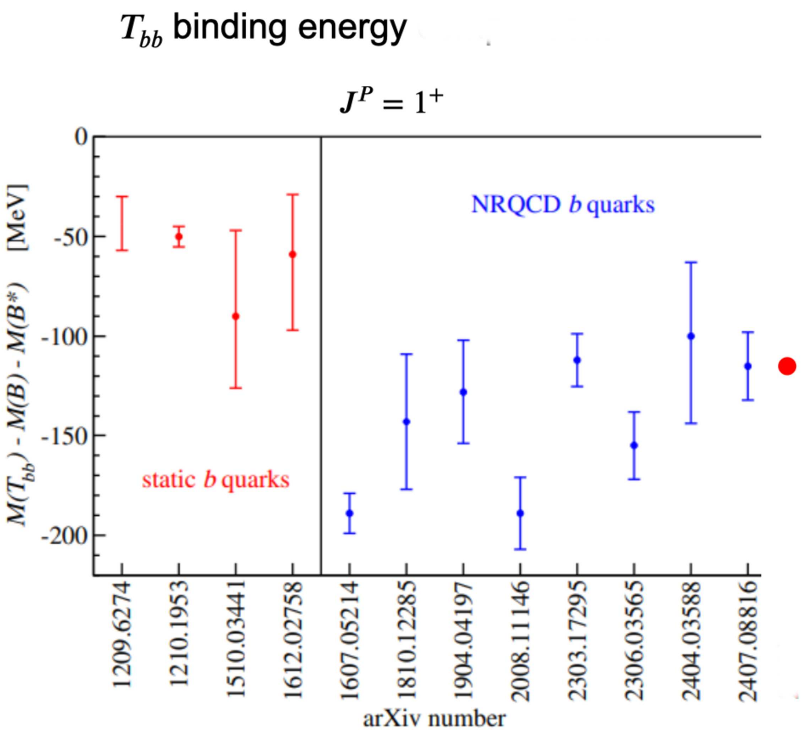


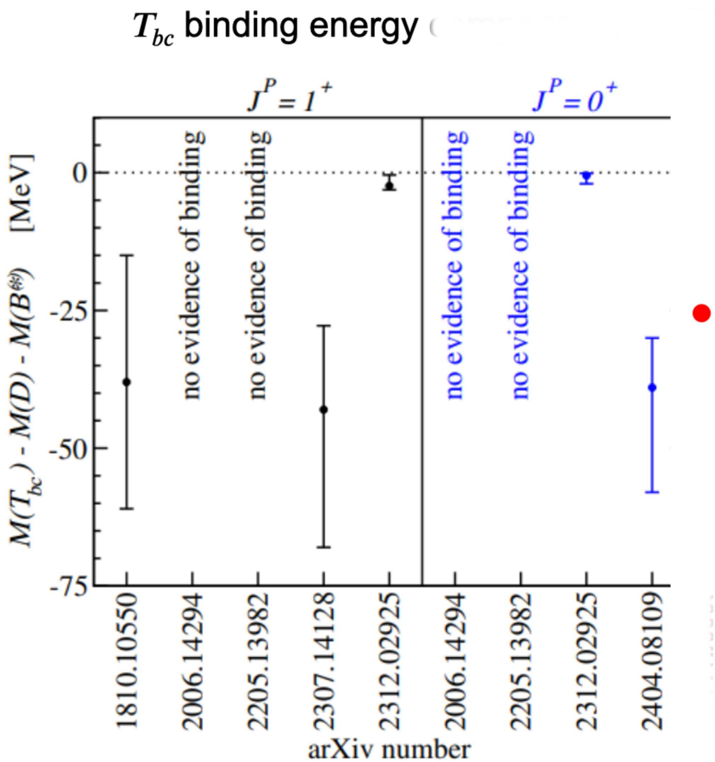
- Lattice data from
  Lyu et al PRL 131 (2023) 161901; Bicudo et al PoS LATTICE2024 124

Results for $T_{cc}^{+}(3875)$

- We get a T_{cc} state 323 keV below the DD^* threshold that we identify with $T_{cc}^{+}(3875)$ by fixing the 0^+ triplet meson mass $\Lambda_{0^+}^{t}$ to be 664 MeV.
- Such a state is seen also in lattice studies:
  `Padmanath Prelovsek PRL 129 (2022) 032002` get a state $9.9^{+3.6}_{-7.1}$ MeV below threshold ($M_\pi = 280$ MeV),
  `Lyu et al PRL 131 (2023) 161901` get a state $59^{+53}_{-99}{}^{+2}_{-67}$ keV below threshold ($M_\pi = 146$ MeV).
- The radius of the state is about 8 fm.

T_{bb} and T_{bc}



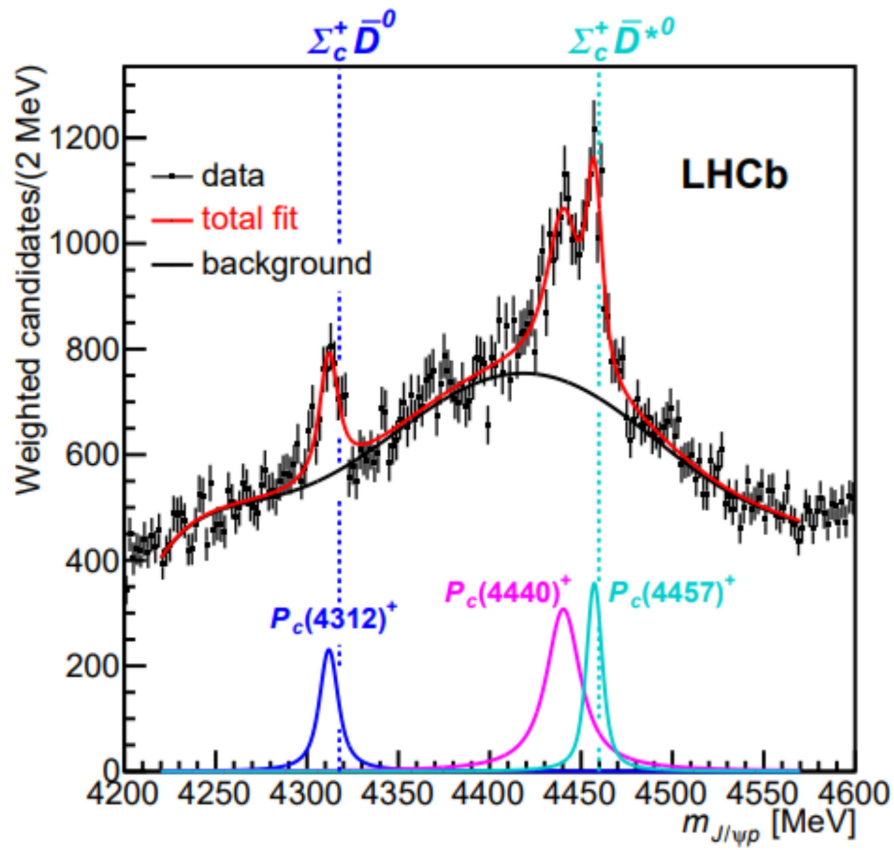


The dot ● is from

`Brambilla Mohapatra Scirpa Vairo PRL 135 (2025) 13`

Pentaquarks

Pentaquarks: observations



4 states with isospin $1/2$: $P_{c\bar{c}}(4312)^+$, $P_{c\bar{c}}(4380)^+$, $P_{c\bar{c}}(4440)^+$, $P_{c\bar{c}}(4457)^+$ with unknown J^P. Also 2 states with isospin 0 were found: $P_{c\bar{c}s}(4338)^0$, $P_{c\bar{c}s}(4459)^0$.

- LHCb coll PRL 122 (2019) 222001

Pentaquarks: thresholds

There are two thresholds relevant for the lowest lying pentaquarks:

- $\Lambda_c \bar{D}$, where the light quark pair in the baryon has spin 0.

  The $\Lambda_c \bar{D}$ threshold may be the long range tail of a $(1/2)_g$ Born–Oppenheimer potential joining at short range with an adjoint baryon $(1/2)^+$ mass.

- $\Sigma_c \bar{D}$, where the light quark pair in the baryon has spin 1.

  The $\Sigma_c \bar{D}$ threshold may be the long range tail of a $(1/2)_g$ Born–Oppenheimer potential joining at short range with an adjoint baryon $(1/2)^+$ mass or the long range tail of the two $\left\{(1/2)'_g\,,(3/2)_g\right\}$ Born–Oppenheimer potentials becoming degenerate at short range with an adjoint baryon $(3/2)^+$ mass.

In order to describe the lowest lying pentaquarks in the BOEFT framework, we need information about the above four Born–Oppenheimer potentials.
None of these potentials has been computed in lattice QCD yet.

Scenario I

If all four potentials support bound states, i.e. connect to the thresholds from below (at short distance all the potentials behave as repulsive octet potentials), then there could be ten low-lying pentaquark states. Ten low-lying pentaquark states are predicted in some compact pentaquark models.

- `Ali Parkhomenko PLB 793 (2019) 365`

Scenario II

The fact that there is no experimental evidence for states near the $\Lambda_c \bar{D}$ threshold could be explained by the Born–Oppenheimer potential connecting to that threshold falling off monotonically from above and therefore not supporting bound states. In such a scenario, bound states are supported only by the three Born–Oppenheimer potentials connecting to the $\Sigma_c \bar{D}$ threshold. These are the seven states grouped in the following spin multiplets:

$Q\bar{Q}$ color state	Light spin k^P	BO quantum # $D_{\infty h}$	l	J^P $\{S_Q = 0, S_Q = 1\}$
Octet	$(1/2)^+$	$(1/2)_g$	$1/2$	$\{1/2^-, (1/2, 3/2)^-\}$
8	$(3/2)^+$	$\{(1/2)'_g, (3/2)_g\}$	$3/2$	$\{3/2^-, (1/2, 3/2, 5/2)^-\}$

In this scenario, three out of seven expected low-lying states still need to be detected.

- Brambilla Mohapatra Vairo arXiv:2508.13050

Scenario II: Schrödinger equations

For $k^P = (1/2)^+$ states:

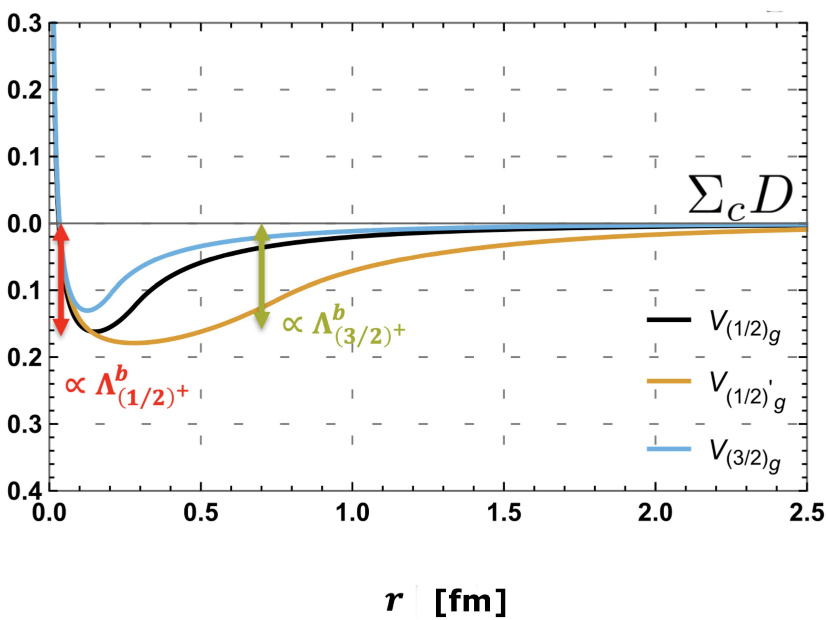
$$\left[-\frac{1}{m_Q r^2}\,\partial_r\, r^2\,\partial_r + \frac{(l-1/2)(l+1)}{m_Q r^2} + V_{(1/2)_g}\right]\psi^{(N)}_{(1/2)^+} = \mathcal{E}_N\,\psi^{(N)}_{(1/2)^+}$$

For $k^P = (3/2)^+$ states:

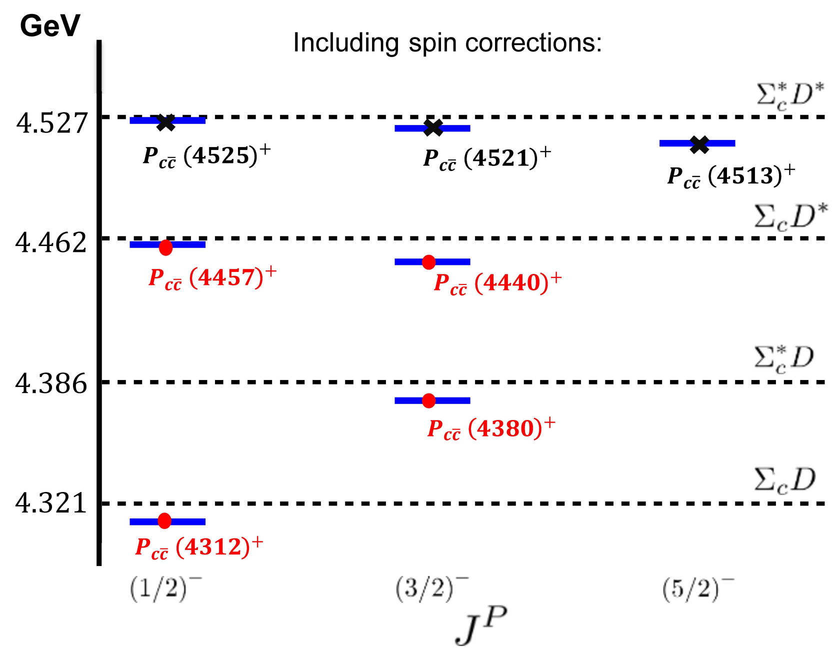
$$\left[-\frac{1}{m_Q r^2}\,\partial_r r^2 \partial_r + \frac{1}{m_Q r^2}\begin{pmatrix} l(l-1)-9/4 & -\sqrt{3l(l+1)-9/4} \\ -\sqrt{3l(l+1)-9/4} & l(l+1)-3/4 \end{pmatrix} + \begin{pmatrix} V_{(1/2)'_g} & 0 \\ 0 & V_{(3/2)_g} \end{pmatrix}\right]\begin{pmatrix} \psi^{(N)}_{(1/2)^{+\prime}} \\ \psi^{(N)}_{(3/2)^+} \end{pmatrix} = \mathcal{E}_N \begin{pmatrix} \psi^{(N)}_{(1/2)^{+\prime}} \\ \psi^{(N)}_{(3/2)^+} \end{pmatrix}$$

There are no lattice determinations of these potentials and of their overall normalization (the adjoint baryon masses $\Lambda^b_{(1/2)^+}$ and $\Lambda^b_{(3/2)^+}$). Their parameterization may be taken from hybrid potentials with similar BO quantum numbers.

Scenario II: potentials and spectrum of lowest lying $c\bar{c}qqq$



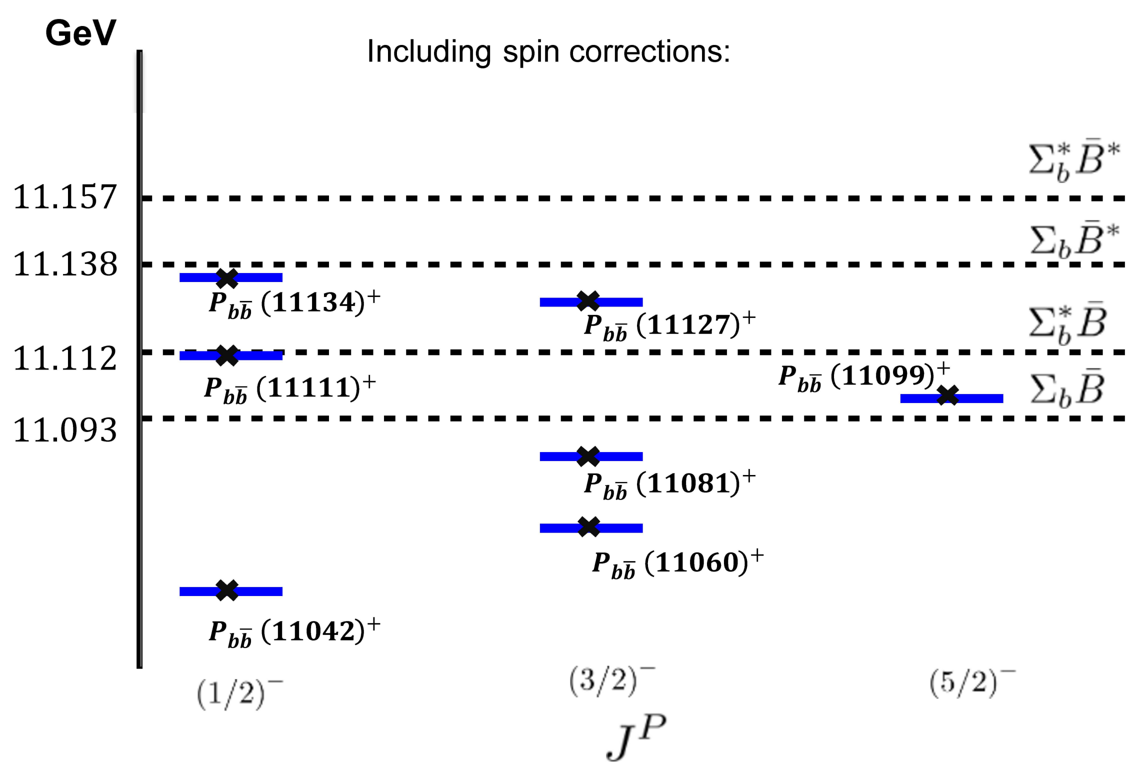
$$\Lambda^b_{(1/2)^+} \approx 1125 \text{ MeV}$$
$$\Lambda^b_{(3/2)^+} \approx 1152 \text{ MeV}$$



Seven low lying pentaquarks are also predicted by molecular models.

- Brambilla Mohapatra Vairo arXiv:2508.13050

Scenario II: spectrum of lowest lying $b\bar{b}qqq$



- Brambilla Mohapatra Vairo arXiv:2508.13050

Scenario III

A final scenario consists in assuming that only the potentials with BO quantum numbers $\{(1/2)'_g, (3/2)_g\}$ cross the $\Sigma_c \bar{D}$ threshold and then approach it from below, supporting bound states, while the potential with BO quantum number $(1/2)_g$ joins the $\Sigma_c \bar{D}$ threshold from above. This scenario implies that the lowest pentaquark states are only four in agreement with current observations.

- Alasiri Braaten Bruschini arXiv:2507.06991

Hybrids

Hybrid multiplets

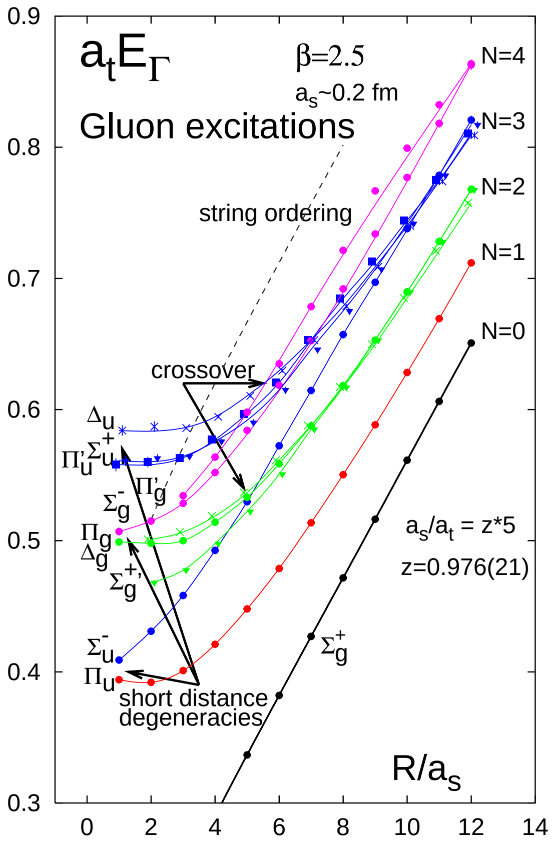
The lowest $Q\bar{Q}g$ hybrid multiplets and the corresponding quantum numbers:

BO quantum #	l	$J^{PC}\{S_Q = 0, S_Q = 1\}$	Multiplets
Σ_u^-, Π_u	1	$\{1^{--}, (0,1,2)^{-+}\}$	H_1
Π_u	1	$\{1^{++}, (0,1,2)^{+-}\}$	H_2
Σ_u^-	0	$\{0^{++}, 1^{+-}\}$	H_3
Σ_u^-, Π_u	2	$\{2^{++}, (1,2,3)^{+-}\}$	H_4
Π_u	2	$\{2^{--}, (1,2,3)^{-+}\}$	H_5

The Σ_u^- and Π_u potentials become degenerate with the $k^{PC} = 1^{+-}$ gluelump mass in the short distance limit where the $O(3) \times C$ symmetry is restored.

Hybrid potentials

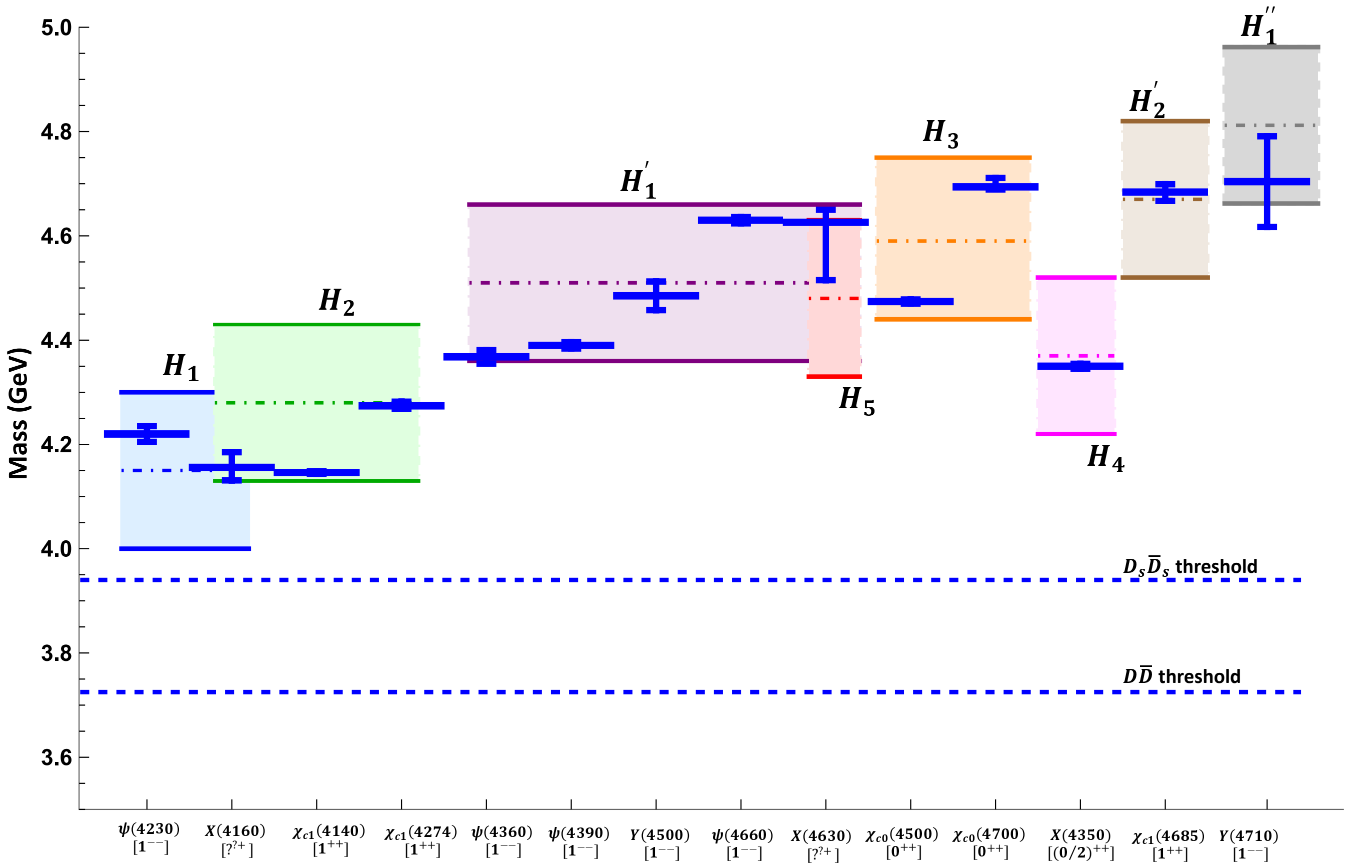
Hybrid potentials are well known from lattice QCD (mostly in pure SU(3) gauge).



- Juge Kuti Morningstar PRL 90 (2003) 161601
  see also: Bali Pineda PRD 69 (2004) 094001
  Capitani et al PRD 99 (2019) 034502
  Höllwieser et al PoS LATTICE2024 102 [in this case $N_f = 3 + 1$!]

Spectrum of the lowest lying $c\bar{c}g$

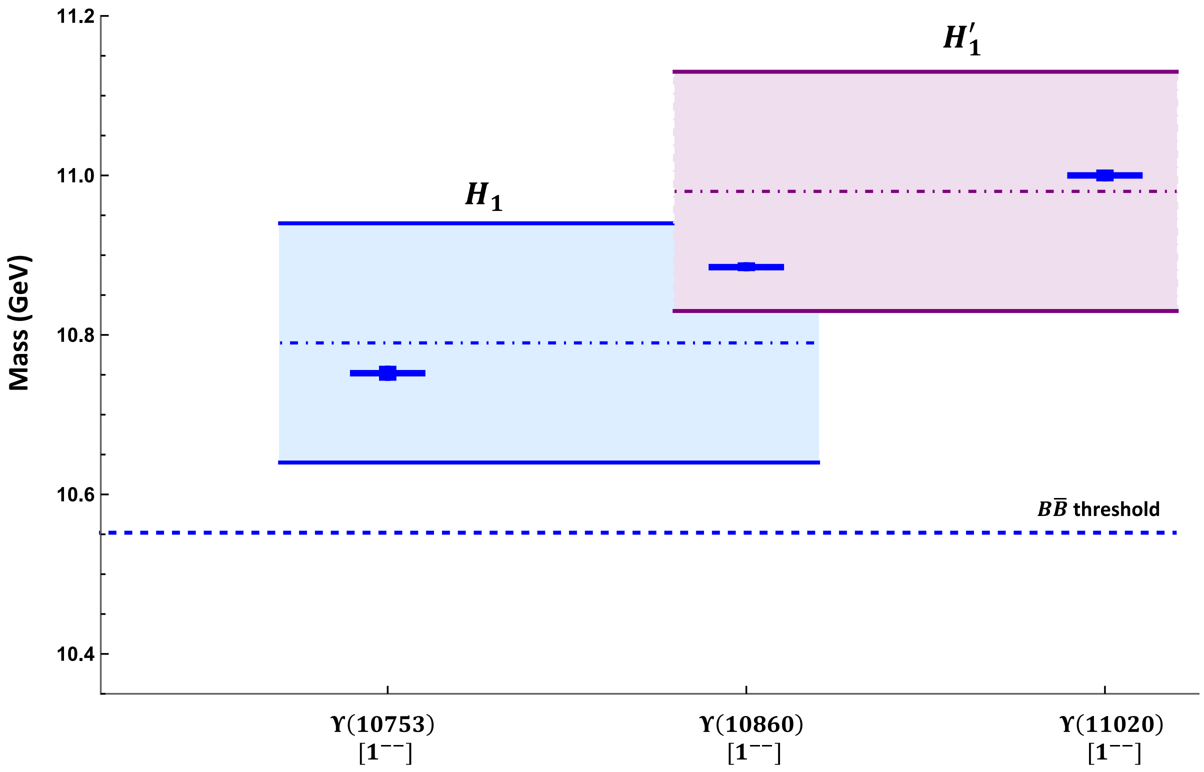
Lowest multiplets of the Σ_u^-, Π_u charmonium hybrid spectrum:



- Brambilla Mohapatra Vairo PRD 107 (2023) 054034

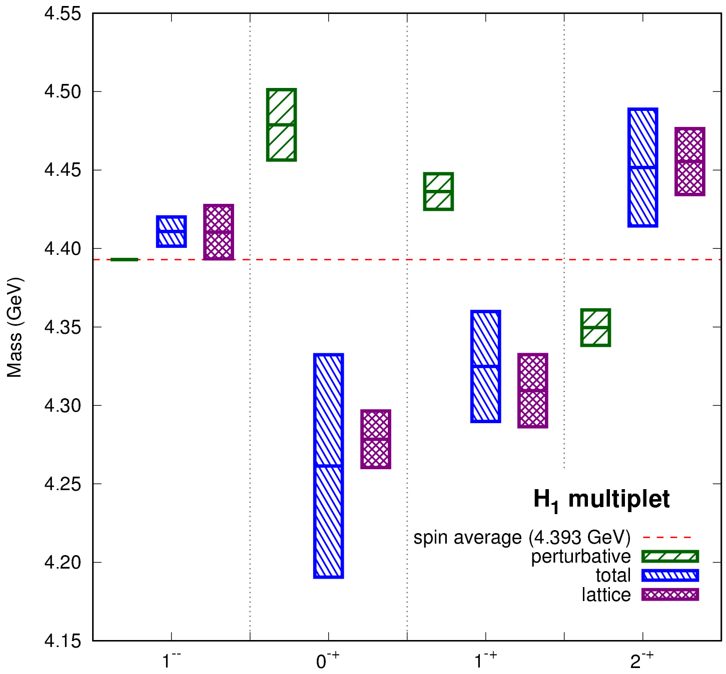
Spectrum of the lowest lying $b\bar{b}g$

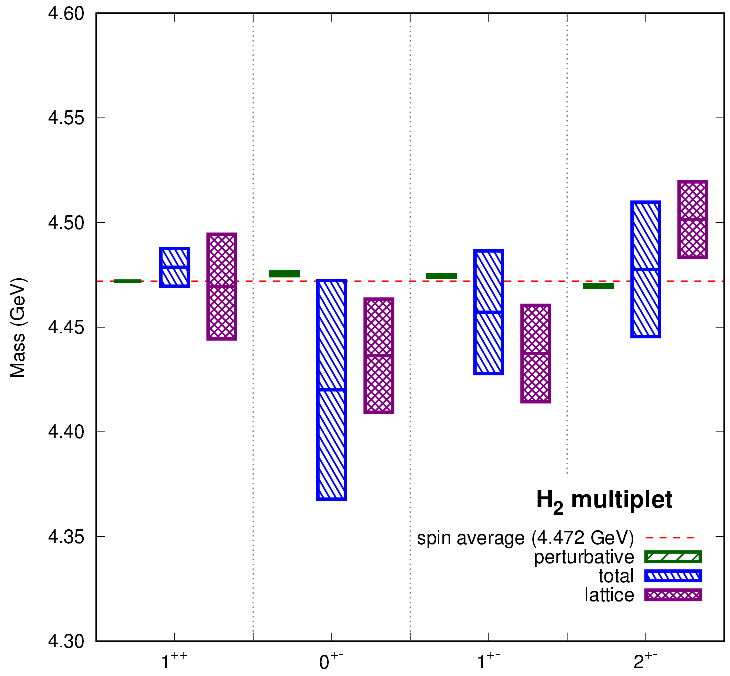
Lowest multiplets of the Σ_u^-, Π_u bottomonium hybrid spectrum:



- Brambilla Mohapatra Vairo PRD 107 (2023) 054034

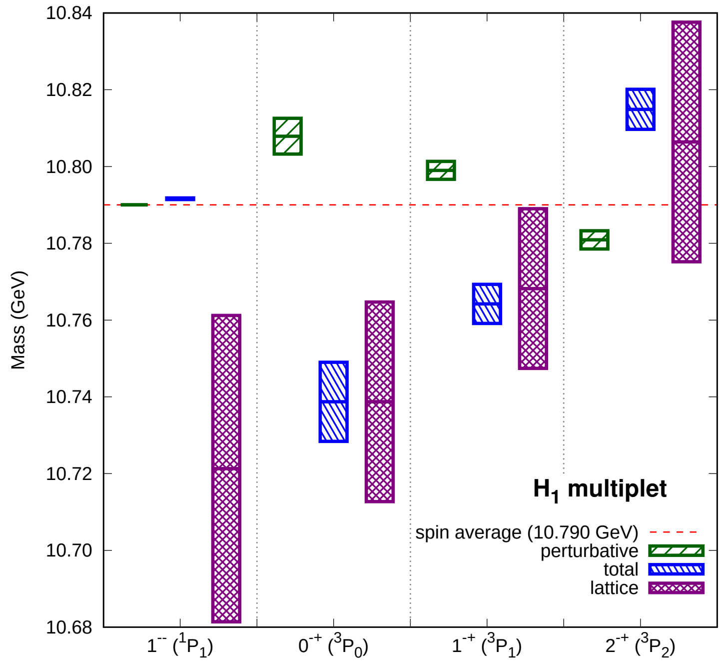
Charmonium hybrid fine structure





- Brambilla Lai Segovia Tarrus Vairo PRD 99 (2019) 014017
  lattice data from Liu et al JHEP 1612 (2016) 089
  [2+1 flavors, $m_\pi = 240$ MeV]
  similar results from Ryan Wilson JHEP 02 (2021) 214
  see also Soto Valls PRD 108 (2023) 014025

Bottomonium hybrid fine structure



- Brambilla Lai Segovia Tarrus Vairo PRD 99 (2019) 014017
  lattice data based on Ryan Wilson JHEP 02 (2021) 214
  see also Soto Valls PRD 108 (2023) 014025

Hybrid decays

- Hybrid $\to$ quarkonium transitions are a crucial element for the identification of hybrids among the XYZ.
  - Brambilla Mohapatra Vairo PRD 107 (2023) 054034

- Hybrid decay to a heavy-light meson pair requires coupling the hybrid to the heavy-light meson-meson threshold. It turns out that the decay of quarkonium hybrids to two S-wave heavy-light mesons is not suppressed (as stated in the literature for long time)!
  - Bruschini PRD 109 (2024) L031501; Tarrus JHEP 06 (2024) 107 Braaten Bruschini PRD 109 (2024) 094051

Coupled Schrödinger equations with heavy-light mixing

In the BO language, an S-wave + S-wave meson-antimeson pair can have $k^{PC} = 0^{-+}$, in which case this threshold is embedded in the tetraquark potential $V_{\Sigma_u^-}(r)$ (here $V_{\Sigma_u^-{}'}(r)$ to distinguish it from the hybrid potential).
The coupled Schrödinger equations read

$$\left[-\frac{1}{m_Q r^2}\,\partial_r r^2 \partial_r + \frac{1}{m_Q r^2}\begin{pmatrix} l(l+1)+2 & -2\sqrt{l(l+1)} & 0 \\ -2\sqrt{l(l+1)} & l(l+1) & 0 \\ 0 & 0 & l\,(l+1) \end{pmatrix} + \begin{pmatrix} V_{\Sigma_u^-}(r) & 0 & V_{\Sigma_u^- - \Sigma_u^-{}'}(r) \\ 0 & V_{\Pi_u}(r) & 0 \\ V_{\Sigma_u^- - \Sigma_u^-{}'}(r) & 0 & V_{\Sigma_u^-{}'}(r) \end{pmatrix} \right] \begin{pmatrix} \psi^{(N)}_{\Sigma_u} \\ \psi^{(N)}_{\Pi_u} \\ \psi^{(N)}_{\Sigma_u'} \end{pmatrix} = \mathcal{E}_N \begin{pmatrix} \psi^{(N)}_{\Sigma_u} \\ \psi^{(N)}_{\Pi_u} \\ \psi^{(N)}_{\Sigma_u'} \end{pmatrix}$$

The potentials $V_{\Sigma_u^-}(r)$ and $V_{\Pi_u}(r)$ are the lowest hybrid potentials, while the mixing potential between the hybrid Σ_u^- and the tetraquark $\Sigma_u^-{}'$, $V_{\Sigma_u^- - \Sigma_u^-{}'}(r)$, is unknown.

Because hybrids of the multiplets H_1, H_3, H_4, ... are in part or completely excitations of the Σ_u^- potential, they couple to the tetraquark $\Sigma_u^-{}'$ and therefore are allowed to decay into S-wave meson-meson pairs.

Quarkonium

Coupled Schrödinger equations

Thresholds are embedded in tetraquark potentials with the same BO quantum numbers as quarkonium, i.e. Σ_g^+. Hence, threshold effects are described in the BO language by three coupled Schrödinger equations involving the diabatic quarkonium potential with quantum numbers Σ_g^+ and the tetraquark ones with quantum numbers $\Sigma_g^{+\prime}$ and Π_g that correspond at short distance to the 1^{--} adjoint meson and at large distance to the isospin $(I=0)$ $k^{PC}=1^{--}$ S-wave plus S-wave meson-antimeson threshold:

$$\left[-\frac{1}{m_Q r^2}\,\partial_r r^2 \partial_r + \frac{1}{m_Q r^2}\begin{pmatrix} l\,(l+1) & 0 & 0 \\ 0 & l(l+1)+2 & -2\sqrt{l(l+1)} \\ 0 & -2\sqrt{l(l+1)} & l(l+1) \end{pmatrix}\right.$$
$$\left. + \begin{pmatrix} V_{\Sigma_g^+}(r) & V_{\Sigma_g^+-\Sigma_g^{+\prime}}(r) & 0 \\ V_{\Sigma_g^+-\Sigma_g^{+\prime}}(r) & V_{\Sigma_g^{+\prime}}(r) & 0 \\ 0 & 0 & V_{\Pi_g}(r) \end{pmatrix}\right]\begin{pmatrix} \psi^{(N)}_{\Sigma_g^+} \\ \psi^{(N)}_{\Sigma_g^{+\prime}} \\ \psi^{(N)}_{\Pi_g} \end{pmatrix} = \mathcal{E}_N \begin{pmatrix} \psi^{(N)}_{\Sigma_g^+} \\ \psi^{(N)}_{\Sigma_g^{+\prime}} \\ \psi^{(N)}_{\Pi_g} \end{pmatrix}$$

Bottomonium spectrum and threshold effects

nl $(b\bar{b})$	% Σ_g^+	% $\Sigma_g^{+'}$	% Π_g	ΔE_{nl} (MeV)
$1S$	100	≈ 0		-0.1
$2S$	99.9	0.01		-1.0
$3S$	99.1	0.9		-3.5
$4S$	78.8	21.2		-16.0
$1P$	99.9	0.1	≈ 0	-0.5
$2P$	99.5	0.5	≈ 0	-2.3
$3P$	95.8	4.1	0.1	-7.6
X_b	1.5	44.9	53.6	
$1D$	99.8	0.2	≈ 0	-1.2
$2D$	98.7	1.3	≈ 0	-4.2

Note that the tetraquark component is negligible for states well below threshold, but amounts to 20% for $\Upsilon(4S)$.

- Brambilla Mohapatra Scirpa Vairo in preparation

Conclusions

- BOEFT is an EFT of QCD able to describe quarkonia, hybrids, doubly heavy baryons, tetraquarks and pentaquarks in a consistent manner.
  It does not rely on any a priori assumption on the nature of the bound state (compact tetraquark, molecule, hadroquarkonium, ...). It is ultimately the solution of the coupled Schrödinger equations that fixes the content of the states, their binding energies and radii.

- Most of the present limitations come from the incomplete knowledge of the potentials. These should be provided ideally by lattice QCD. The situation is best for hybrids, but relevant tetraquark potentials are currently being computed by several collaborations.

  The knowledge of the potentials is fundamental. It may help to discriminate between different scenarios and explain on a dynamical basis why some bound states exist and some possible others do not. E.g. it could be that only the lowest lying BO potentials are able to support (few) bound states.

QWG 2025

17th International Workshop on Heavy Quarkonium Physics

CERN, November 17 – 21, 2025

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Geoffrey Bodwin
Nora Brambilla
Roberto Mussa
Vaia Papadimitriou
Antonio Vairo

Program topics
Quarkonium Production, Spectroscopy, Decays
Quarkonium in Media
SM measurements and BSM searches
Automated calculations & interface to Monte Carlo
New opportunities

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