A REVIEW ON THE THEORY OF EXOTIC HADRONS

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OUTLINE

I will give an overview of the current understanding of the nature of exotic hadrons

- What's up with exotic hadrons?
- EFT formulation of the problem
- Possible hints from "crowded" QCD
- Models and frameworks for the internal structure

Disclaimer: this will be a critical review, not free of personal opinions and likely incomplete

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QCD is the theory of strong interactions

$$\mathscr{L} = \sum_q \bar{q}_i \left(i\gamma \cdot D_{ij} - m_q \delta_{ij} \right) q_j - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu,a}$$

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Only color singlets as asymptotic states

A priori, not only mesons and baryons

$$\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{1} \oplus \dots \quad \text{meson}$$

$$\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = \mathbf{1} \oplus \dots \quad \text{baryon}$$

$$\mathbf{3} \otimes \mathbf{3} \otimes \bar{\mathbf{3}} \otimes \bar{\mathbf{3}} = \mathbf{1} \oplus \dots \quad \text{tetraquark}$$

$$\mathbf{8} \otimes \mathbf{8} \otimes \dots \otimes \mathbf{8} = \mathbf{1} \oplus \dots \quad \text{glueball}$$

And many more…

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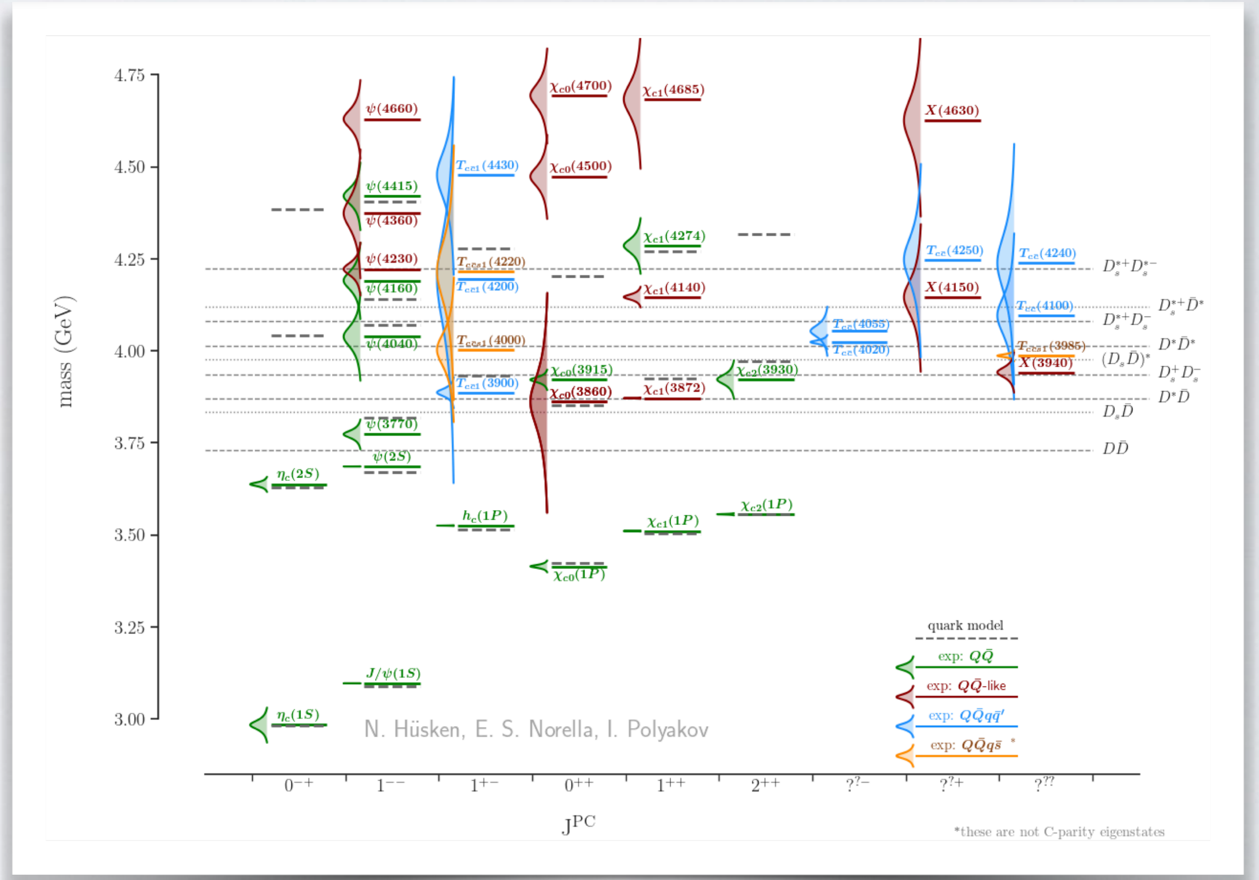
From 2003 on, several new states in the heavy $Q\bar{Q}$ sector



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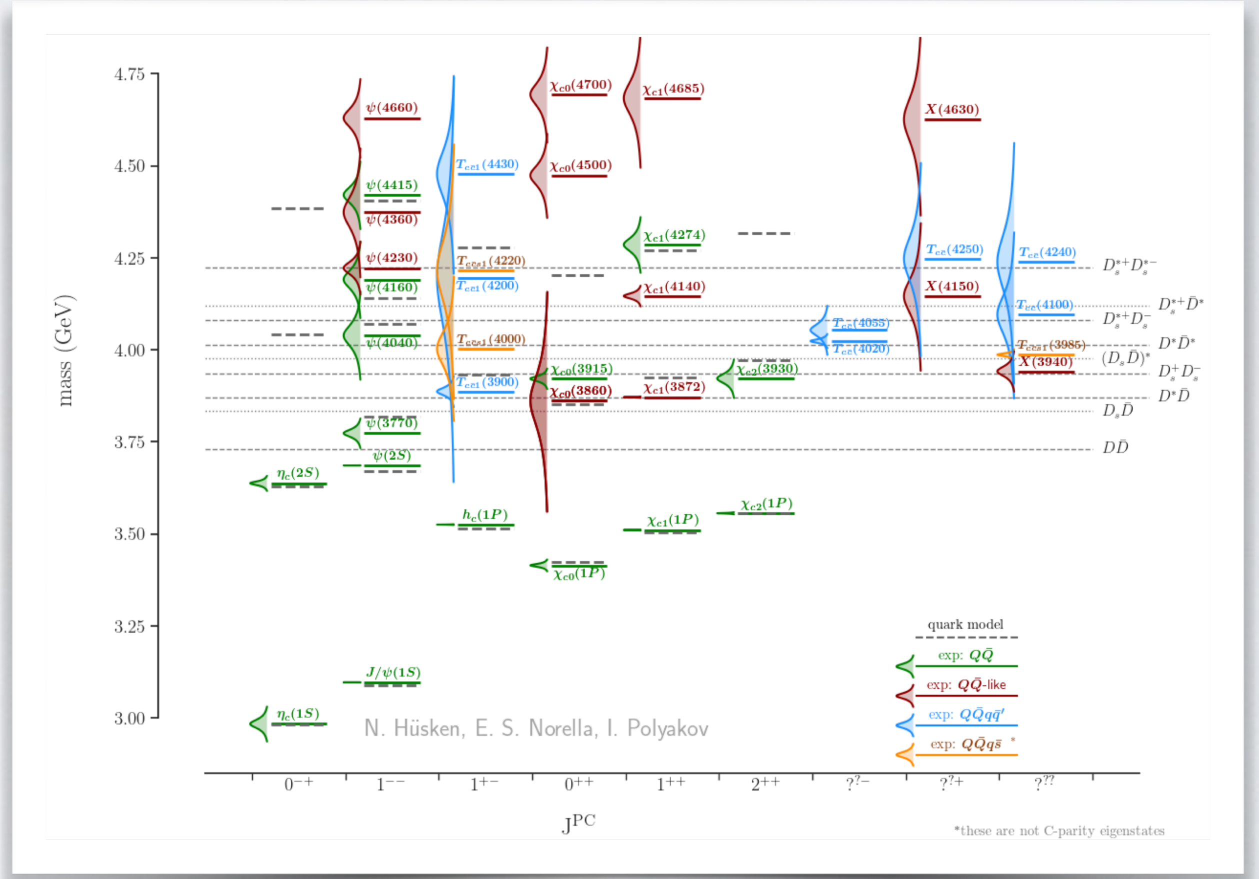
- Unconventional masses and J^{PC}
- Unexpectedly narrow
- Isospin violation
- *Very* close to threshold



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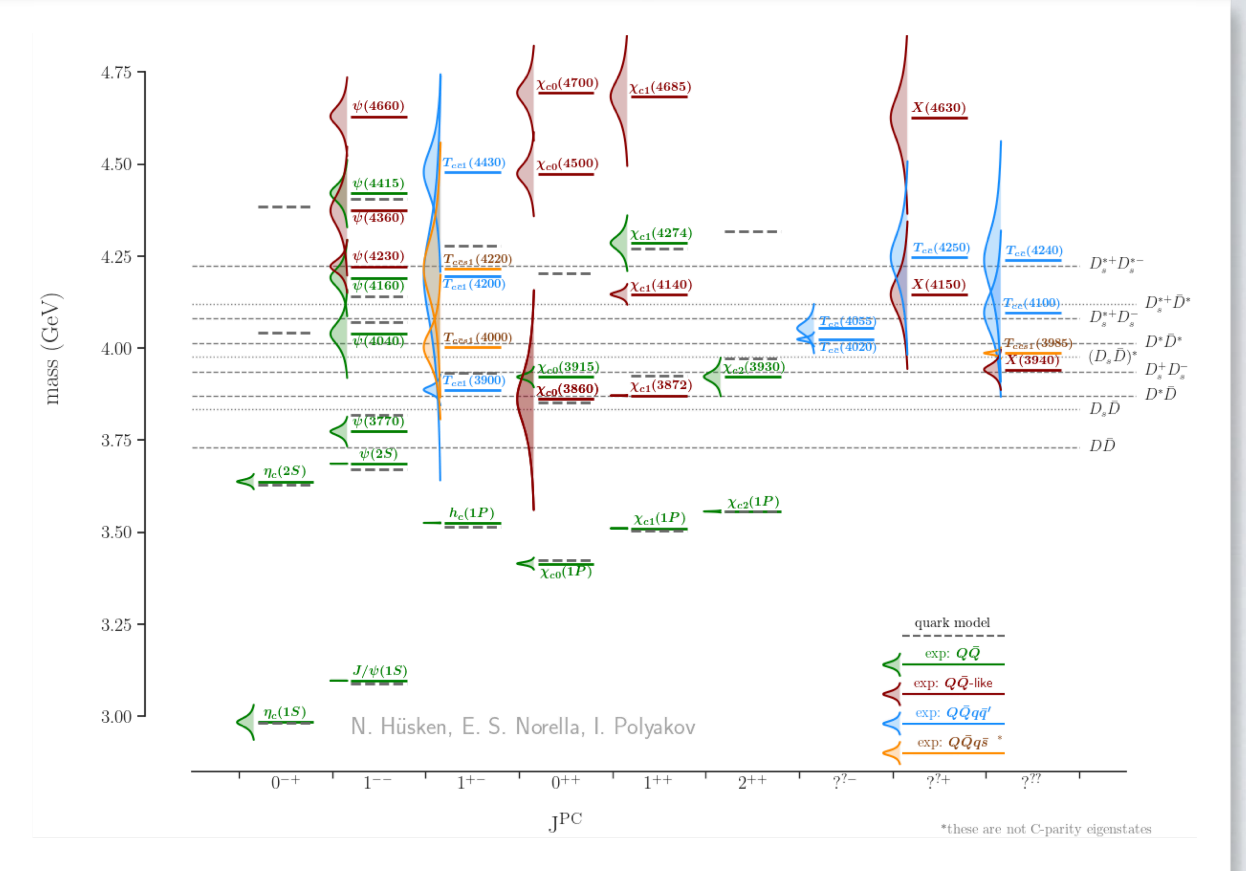

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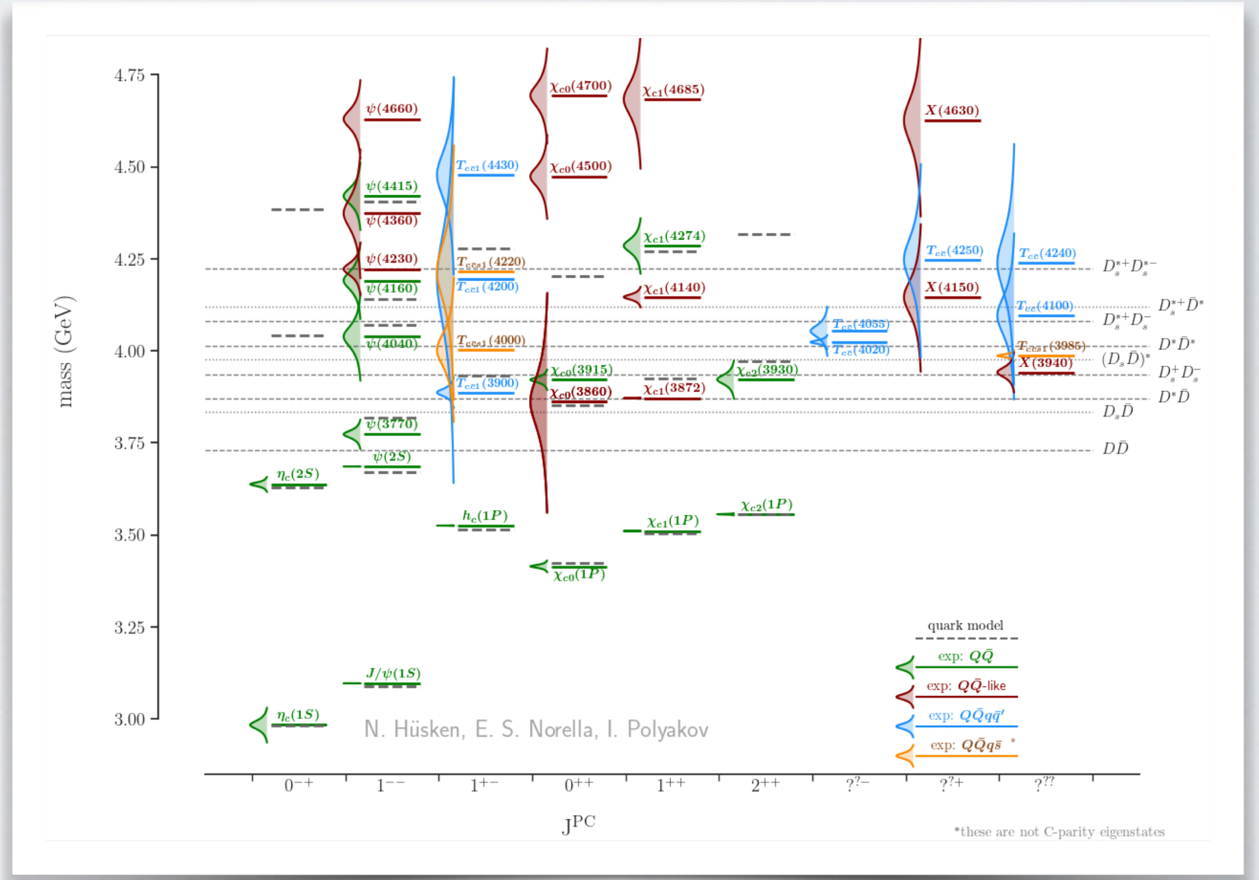

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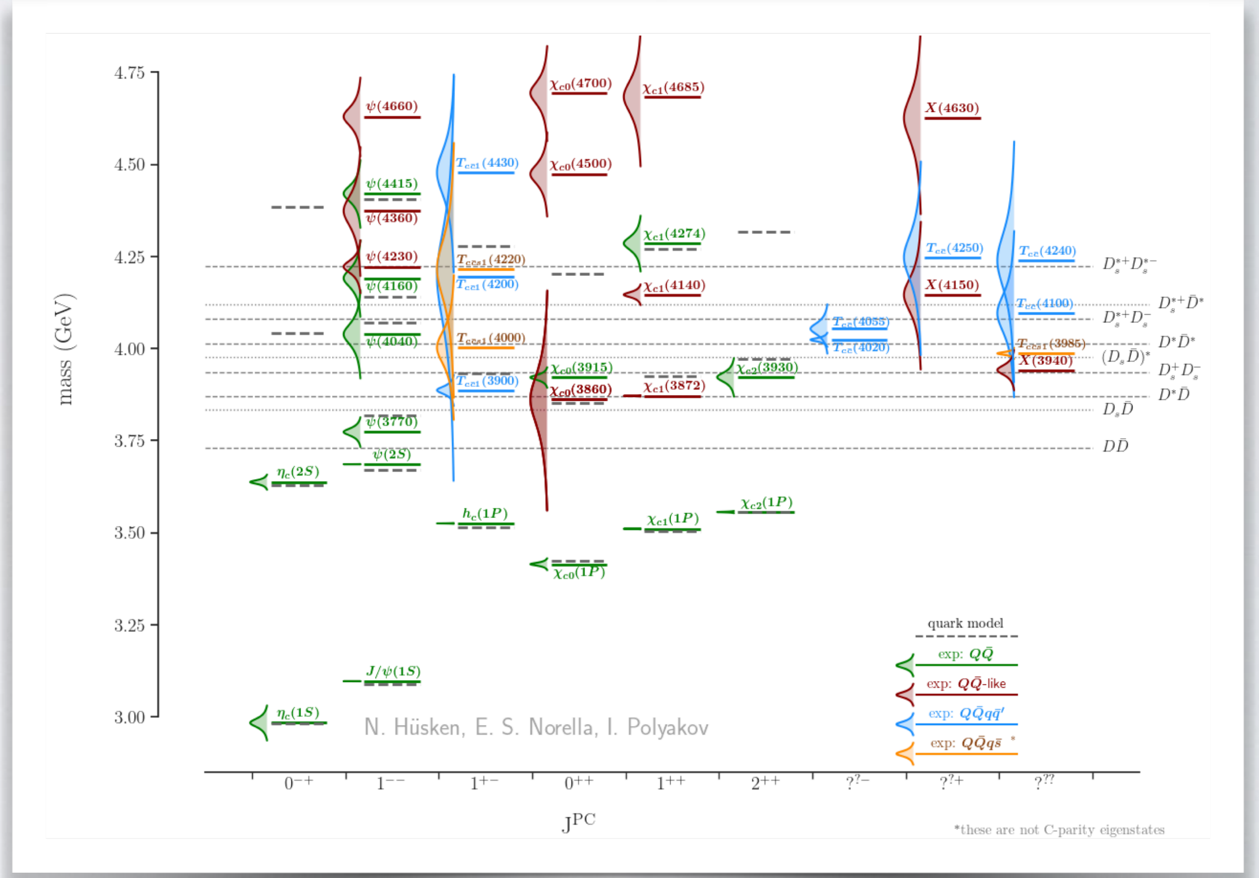

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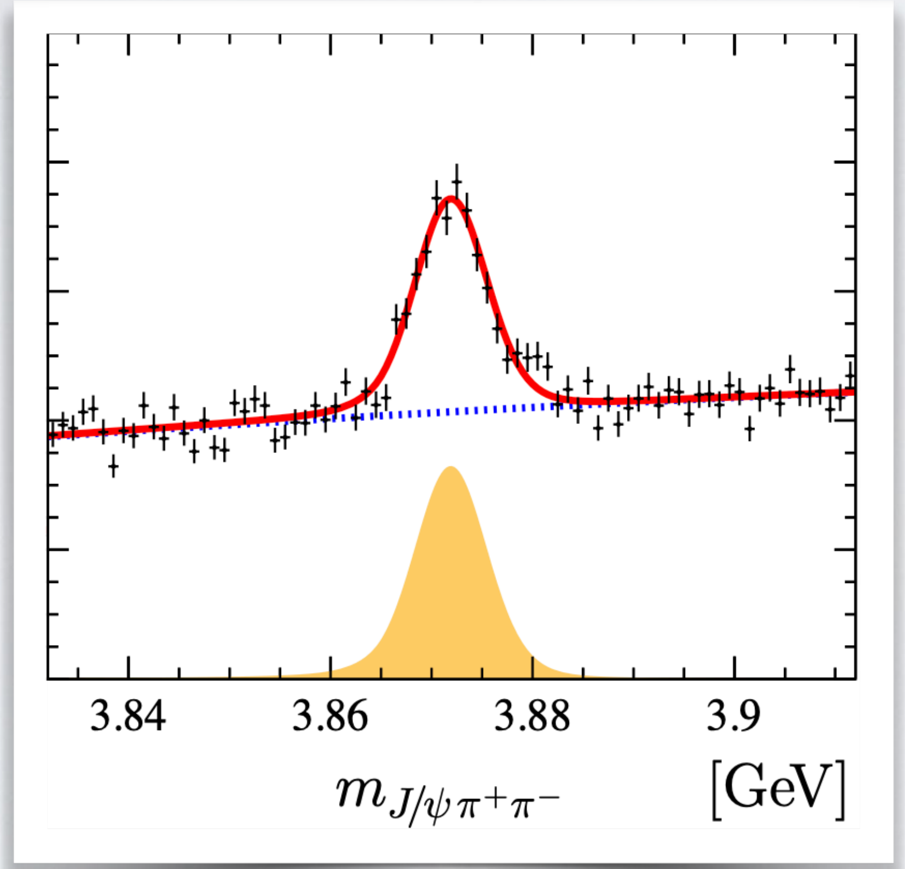

[LHCb — PRD 2020]

$$\left| m_X - m_{D^0} - m_{\bar{D}^{*0}} \right| \lesssim 0.1 \text{ MeV}$$

very close to threshold

$$\Gamma_X \simeq 0.2 - 1 \text{ MeV}$$

very narrow

$$\Gamma(X \to J/\psi\rho) \simeq \Gamma(X \to J/\psi\omega)$$

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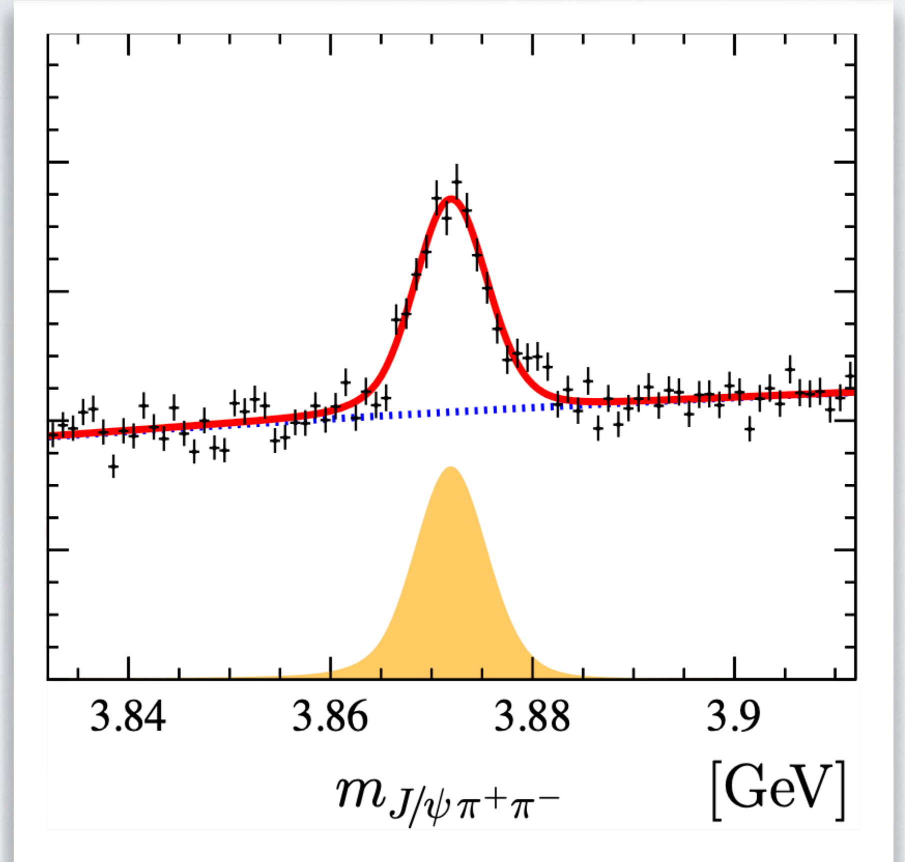

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Can be explained by a $Q\bar{Q}q\bar{q}$ structure with remarkable fine tunings

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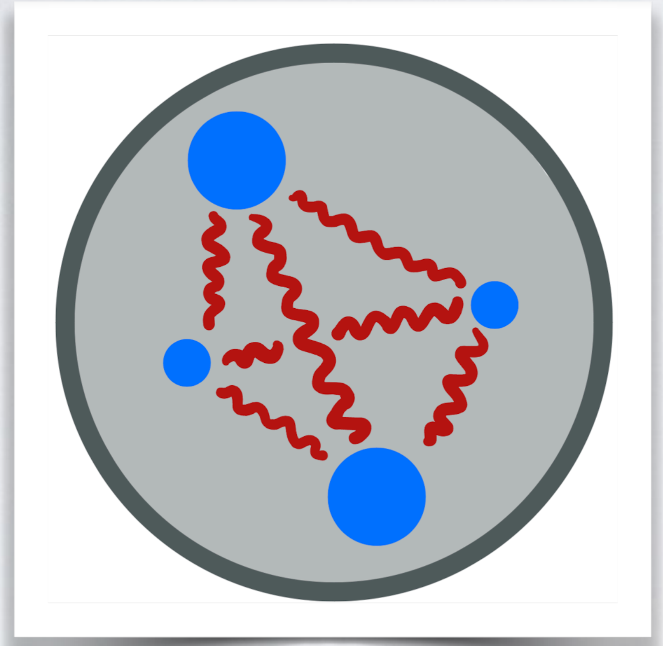

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Hadronic size ($r \lesssim 1$ fm)

Analogue to mesons/baryons

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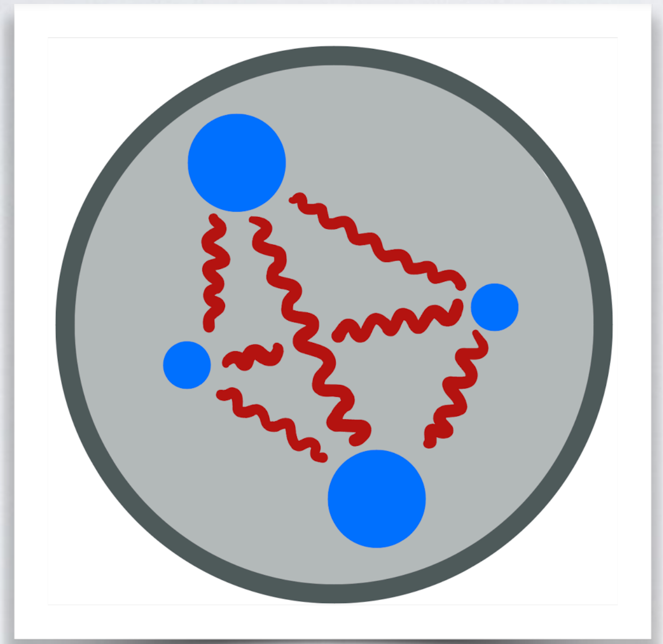

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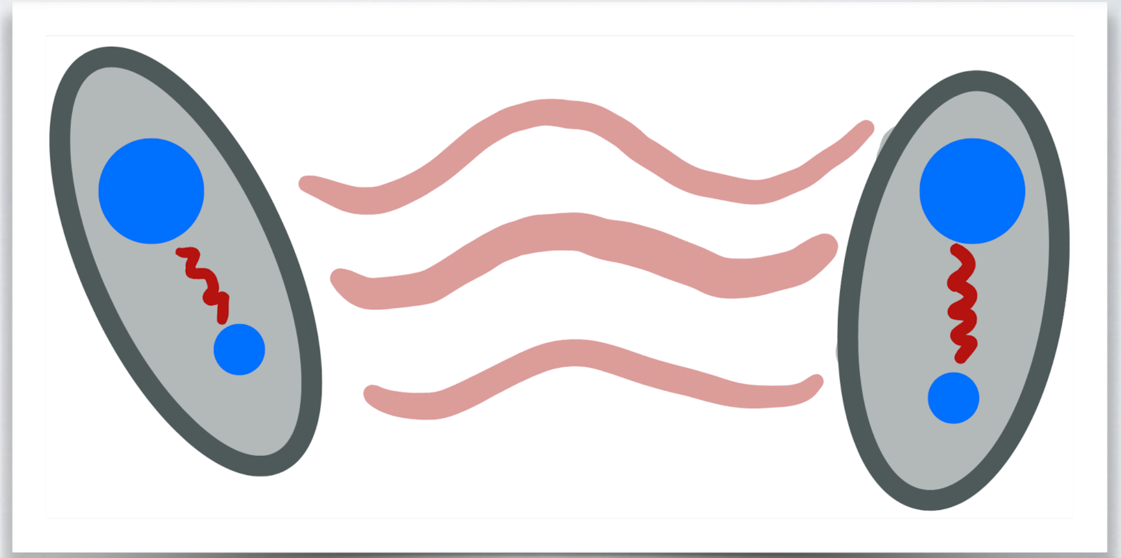

[e.g. Guo et al. — Rev.Mod.Phys. 2018]

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Short distance QCD

Hadronic size $(r \lesssim 1\ \mathrm{fm})$

Analogue to mesons/baryons

Hadron molecule

Long distance QCD

Very large $(r \gg 1\ \mathrm{fm})$

Analogue to deuteron

SHARP EFT FORMULATION

[Braaten, AE, Glioti, Guo, Hanhart, Hyodo, Kaplan, Kinugawa, Mikhasenko, Pilloni, Polosa, Rattazzi, and many more…]

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Input degrees of freedom: two standard hadrons

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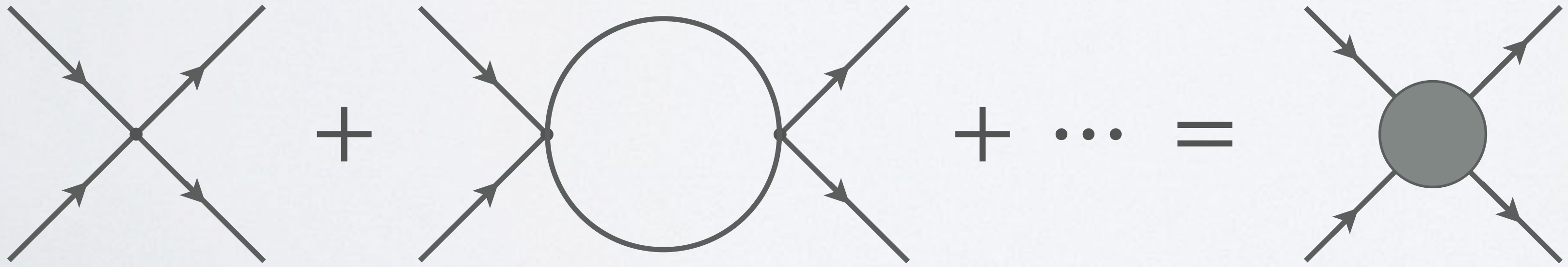

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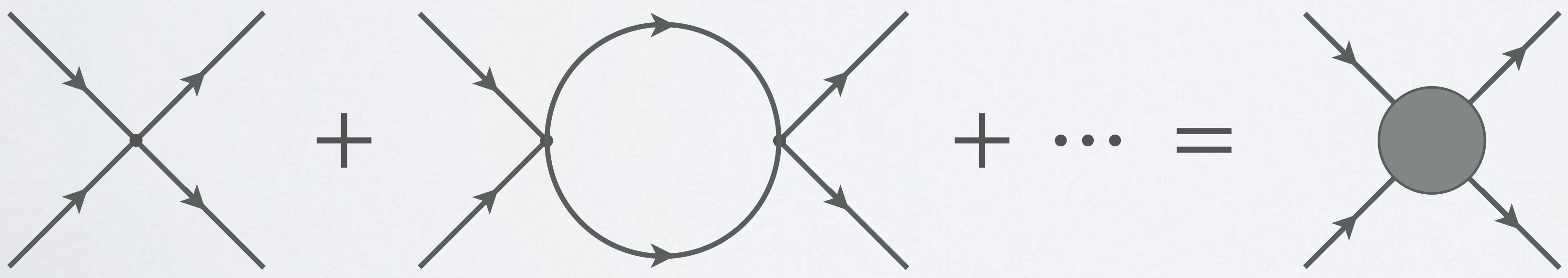

pole at

$$E_{\rm pole} = -\frac{2\pi^2}{\lambda_{\rm phys}^2 \, \mu^3}$$

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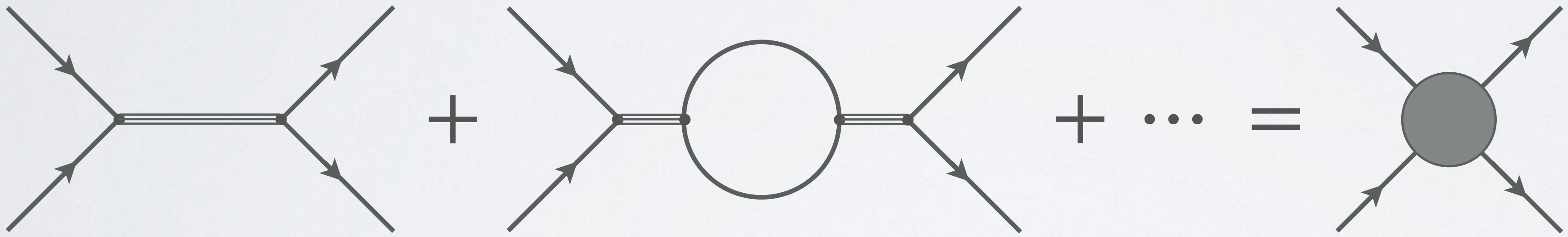
Option 2: tetraquark

Input degrees of freedom: two standard hadrons and one tetraquark

$$\mathscr{L}_{\text{tetra}} \sim \mathscr{L}_0(h_1, h_2) + \phi^\dagger \left(i\partial_t + \frac{\nabla^2}{2M} \right) \phi - \varepsilon\, \phi^\dagger \phi + g\, \phi\, h_1^\dagger h_2^\dagger + \text{h.c.}$$



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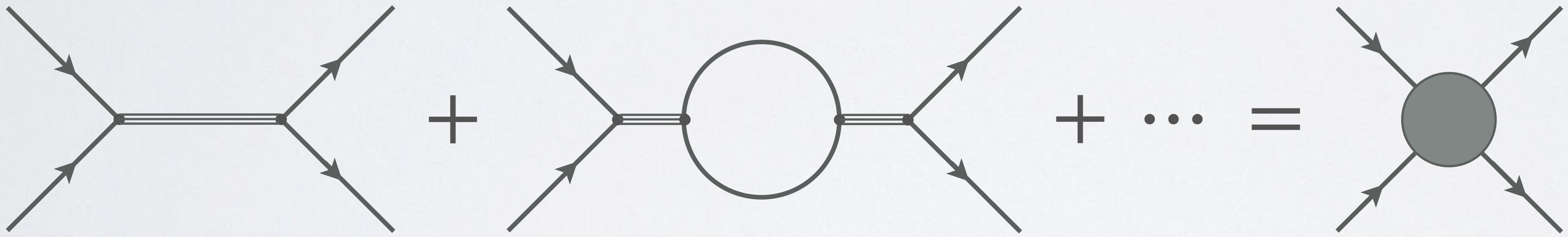

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Both options need fine tuning to have the state near threshold

$$\frac{1}{\lambda_{\text{phys}}} = \frac{1}{\lambda} + \mu\Lambda \ll \mu\Lambda \quad \text{or} \quad \varepsilon_{\text{phys}} = \varepsilon - g^2 \mu\Lambda \ll \frac{\Lambda^2}{\mu}$$

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$$\mathscr{A}_{\rm mol}(E) \sim \frac{\lambda_1\sqrt{-2\mu E - i\epsilon} + \lambda_2}{1 - i\lambda_3\left[\sqrt{2\mu E} + \sqrt{2\mu(E-\Delta)}\right] - \lambda_4\sqrt{2\mu E}\sqrt{2\mu(E-\Delta)}}$$

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$$\mathscr{A}_{\rm tetra}(E) \sim \frac{g^2}{E - \varepsilon + ig^2\left[\sqrt{2\mu E} + \sqrt{2\mu(E-\Delta)}\right]}$$

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- …and so on…

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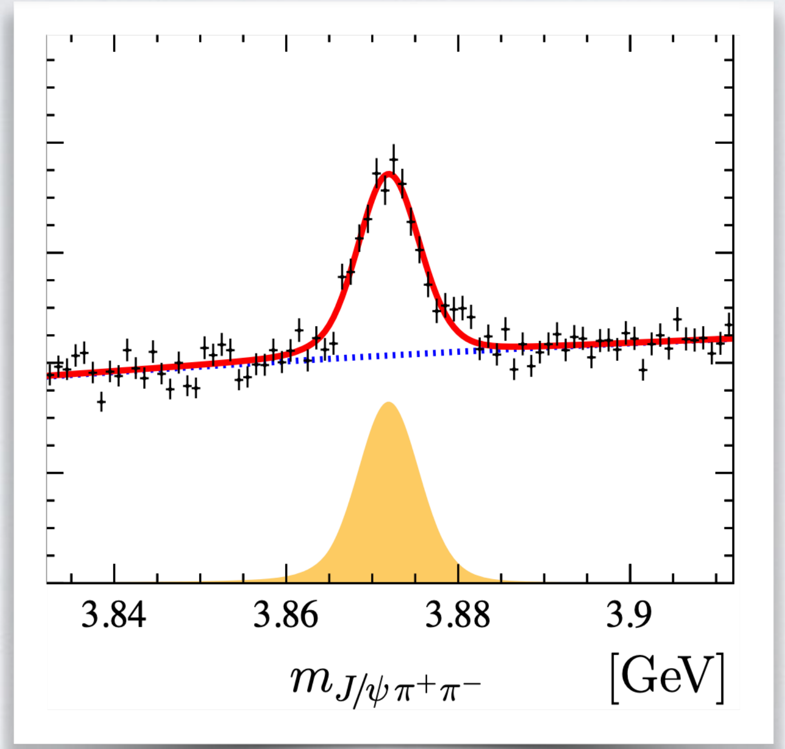
The lineshapes of the X and T_{cc}^{+} have been measured by LHCb



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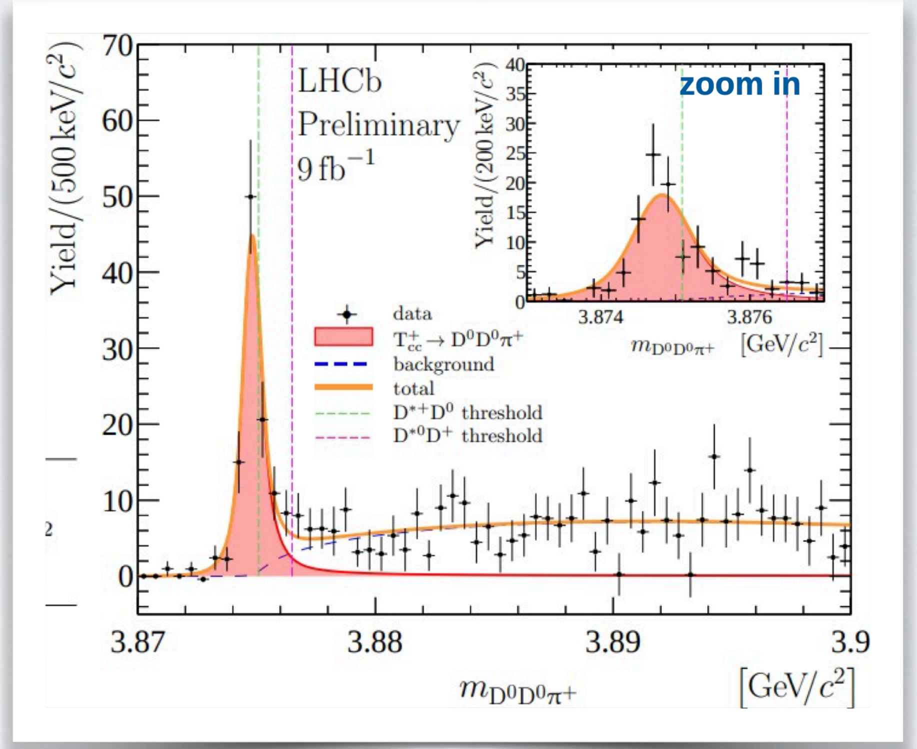

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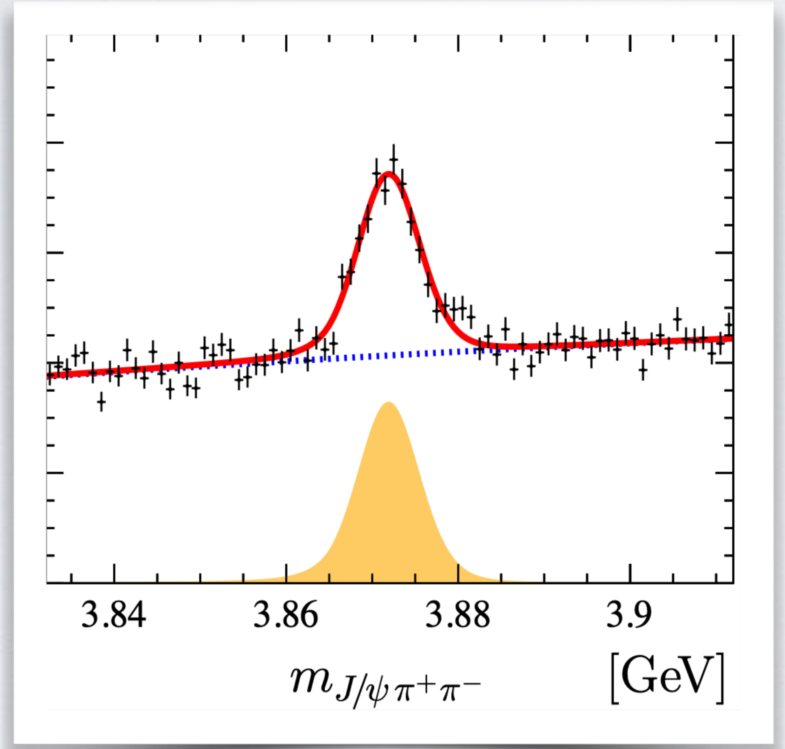

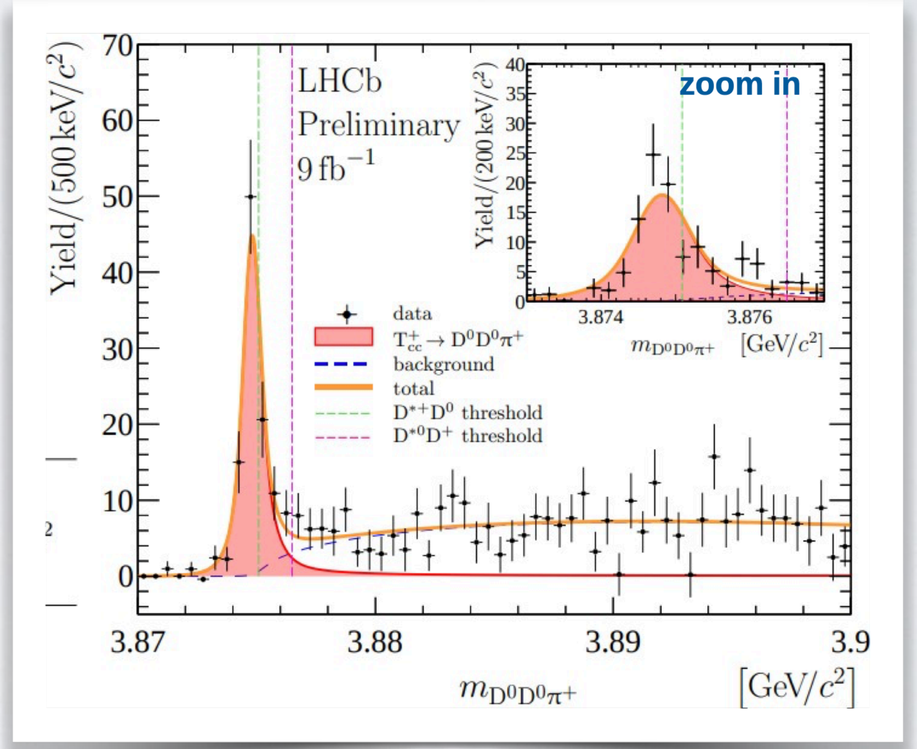

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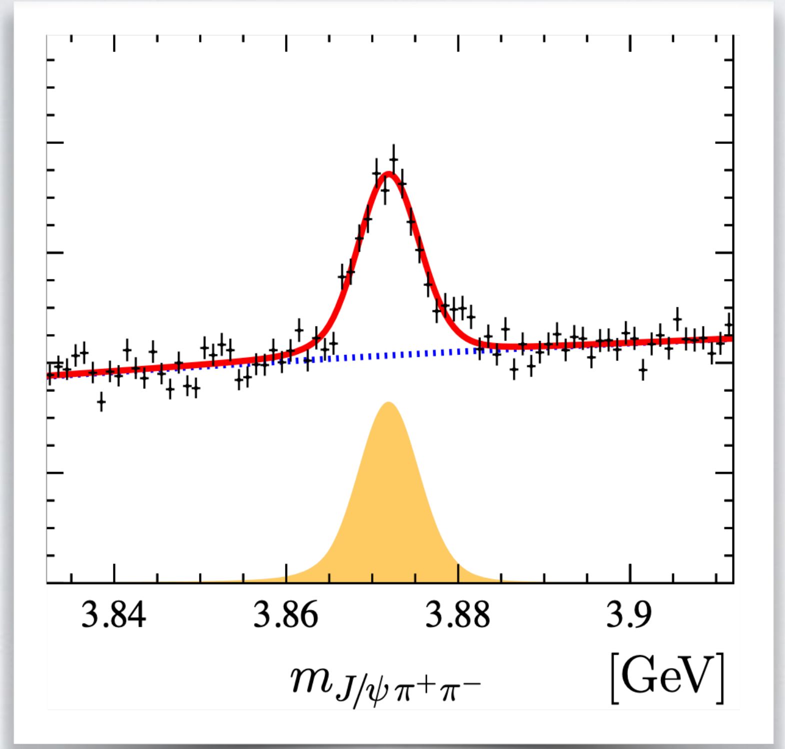

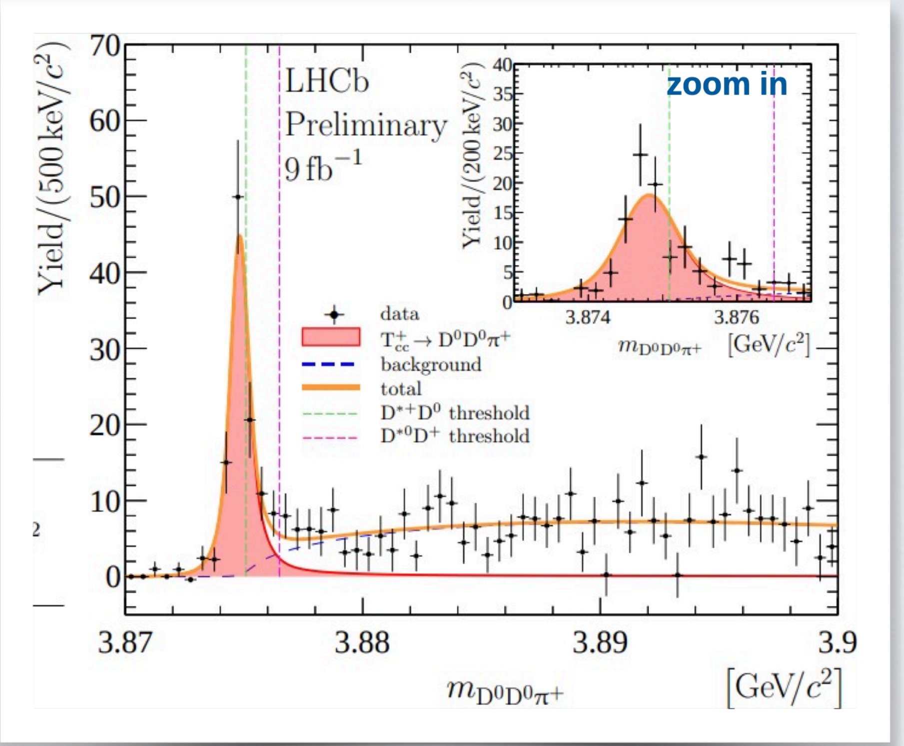

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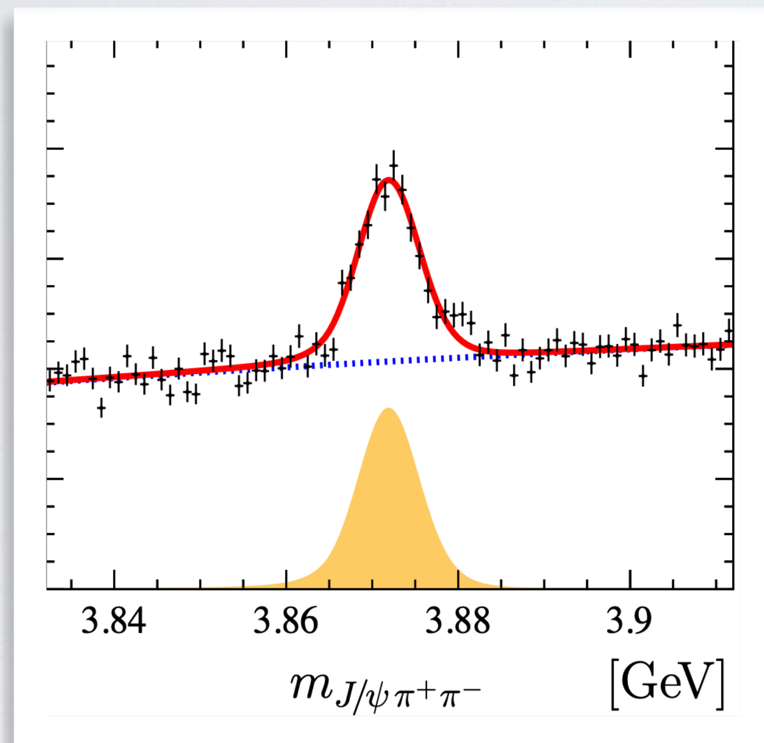

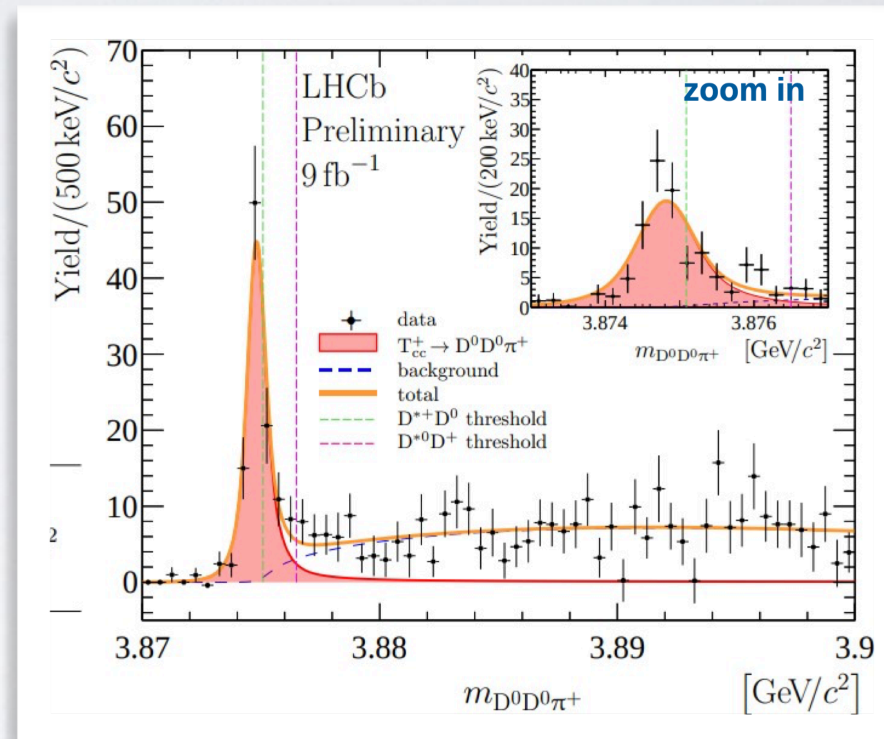

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…but the analysis is hard and the topic is still very controversial

[e.g., Baru et al. — PLB 2022]

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EFT amplitude → data → Lagrangian parameters

However, are current data sufficiently precise to even discriminate between different options?

CROWDED QCD

[Abreu, Albaladejo, Braaten, AE, Ferreiro, Ingles, Liao, Navarra, Nieves, Pilloni, Polosa, Zang, and many more…]

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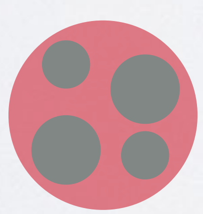

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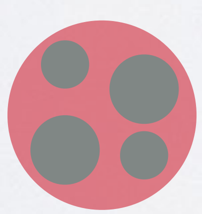
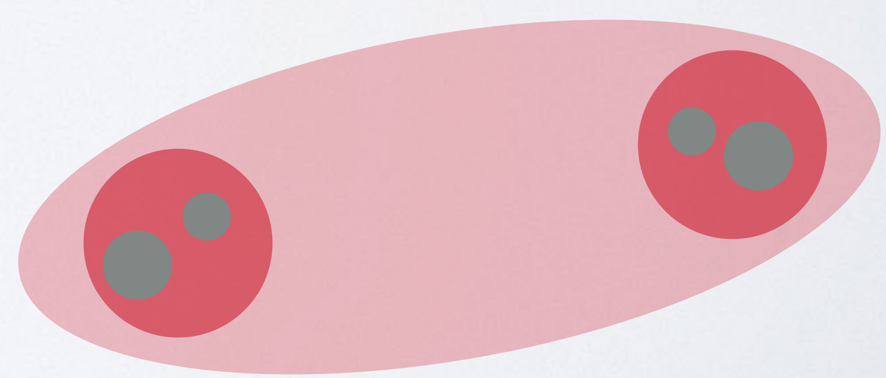


Molecule

Binding of two $Q\bar{q}$ in a volume $V \sim 1\ \mathrm{fm}^3$ + their coalescence



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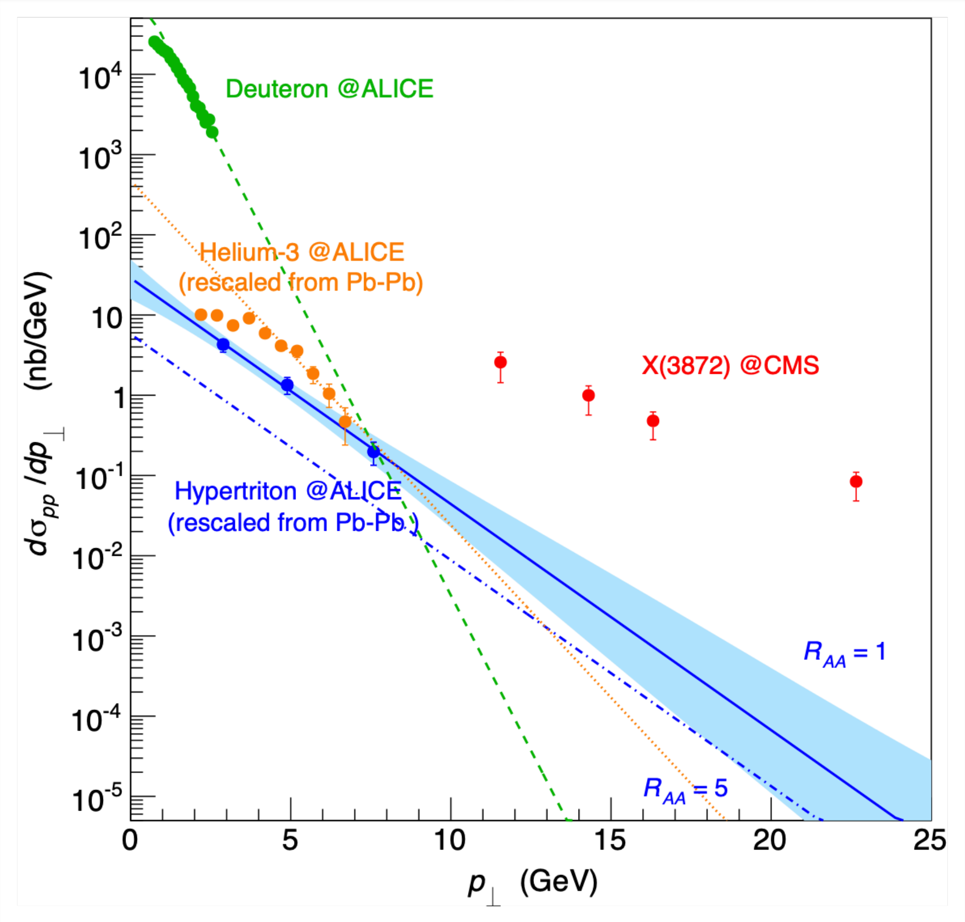
This seems to be confirmed by data on light nuclei [AE, Guerrieri, Maiani, Piccinini, Pilloni, Polosa, Riquer — PRD 2015]



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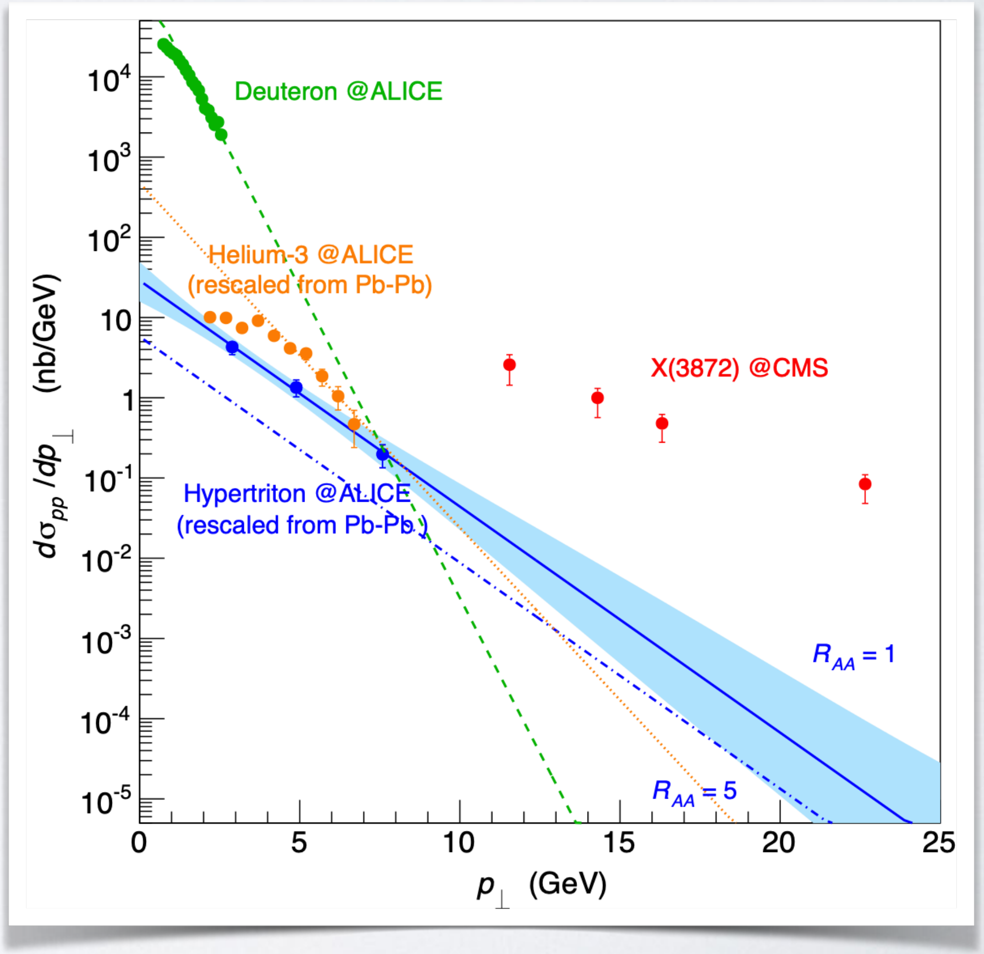
… although this intuition has been challenged [e.g., Artoisenet, Braaten — PRD 2010]



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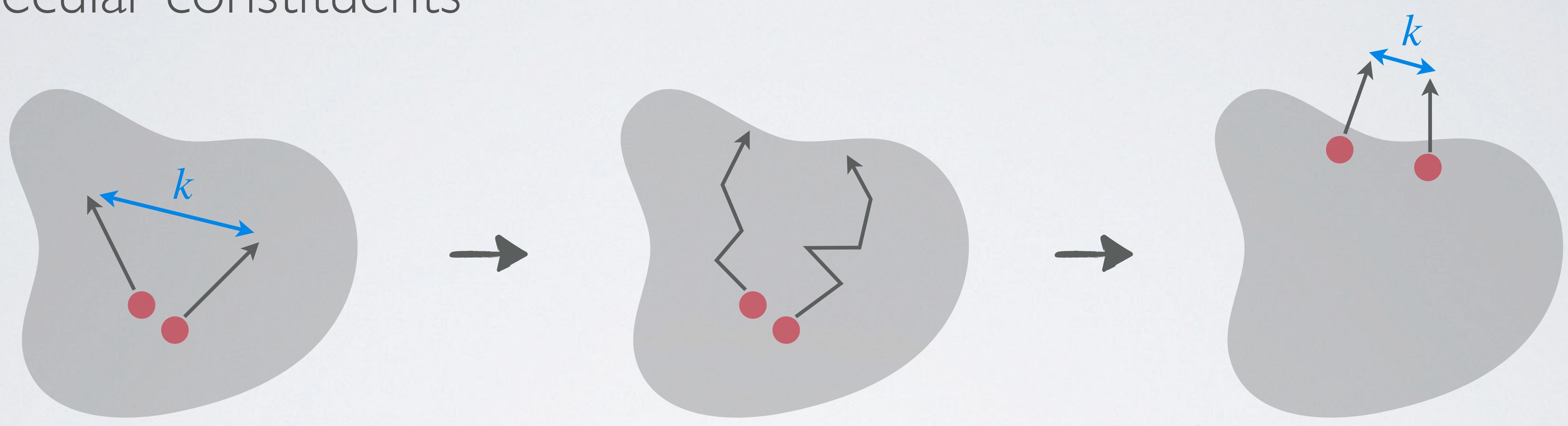
The presence of a "medium" should have the effect of softening the molecular constituents



large relative k softening due to medium smaller relative k

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On the contrary, tetraquarks should dissociate more due to higher number of collisions

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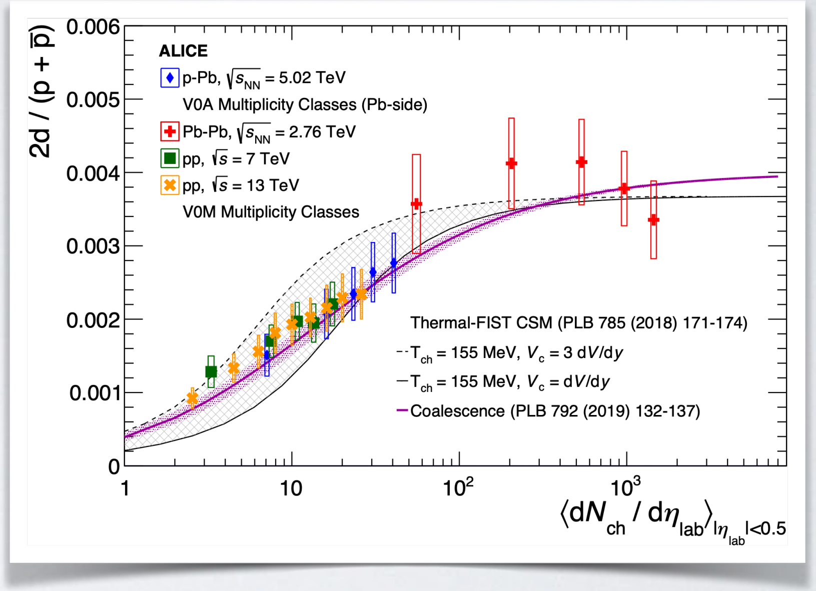
Confirmed by ALICE data on deuteron production in high-multiplicity pp-collisions



[ALICE — PLB 2019, EPJC 2020]

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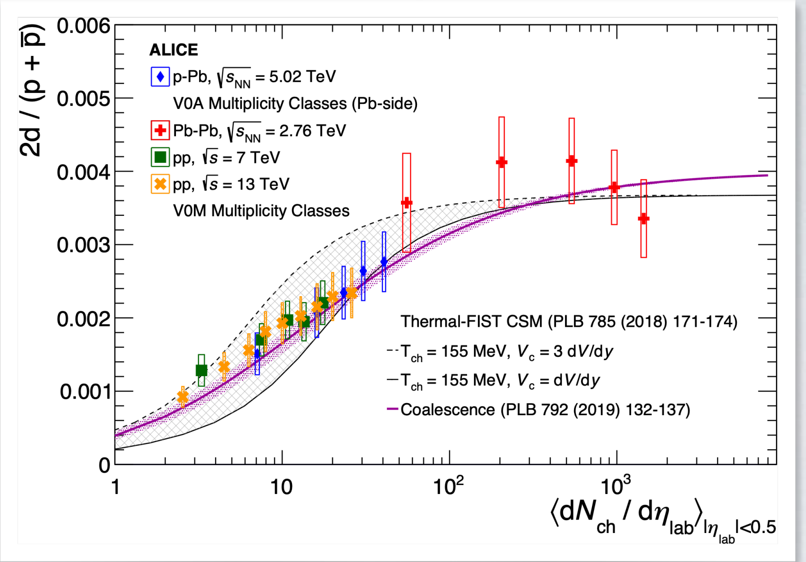

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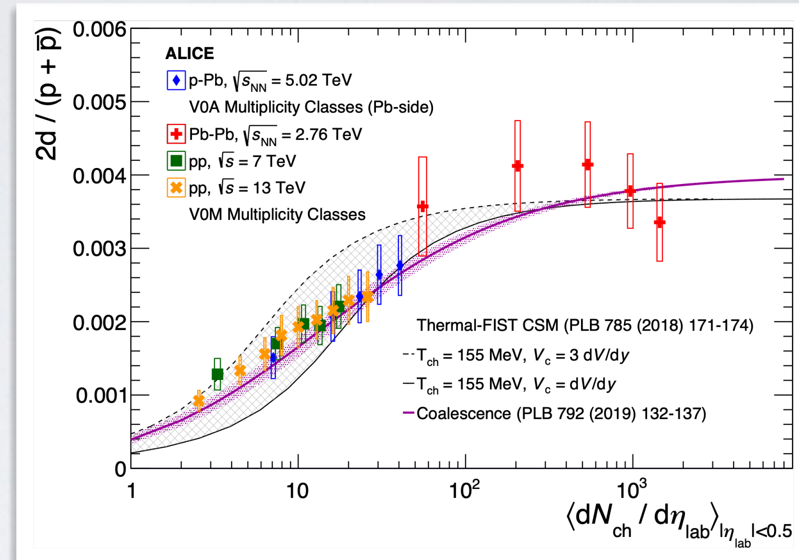

[ALICE – PLB 2019, EPJC 2020]

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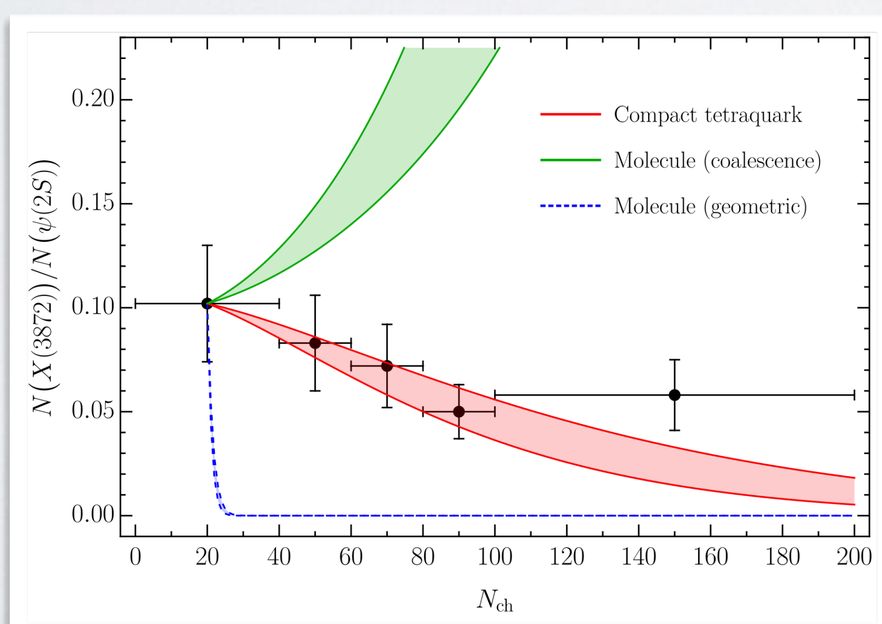
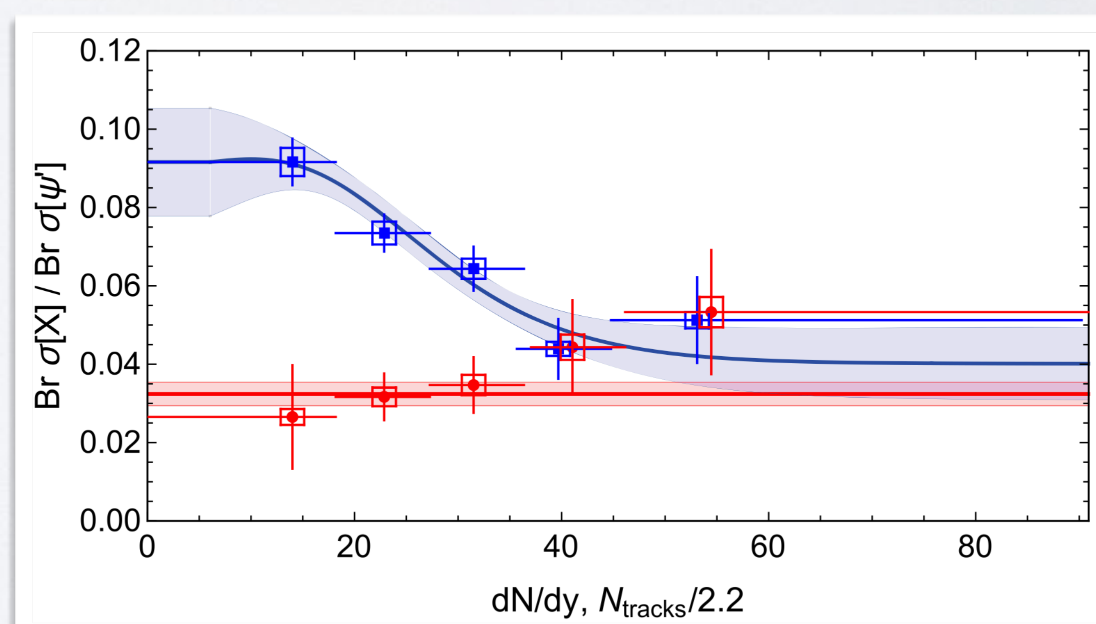
This points to a tetraquark behavior… but adding one more fitting parameter molecules can also be accommodated

[AE, Ferreiro, Pilloni, Polosa, Salgado – EPJC 2021]





[Braaten, He, Ingles, Jiang – PRD 2021]

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A similar trend (more molecules, less tetraquarks) is also expected in heavy ion collisions

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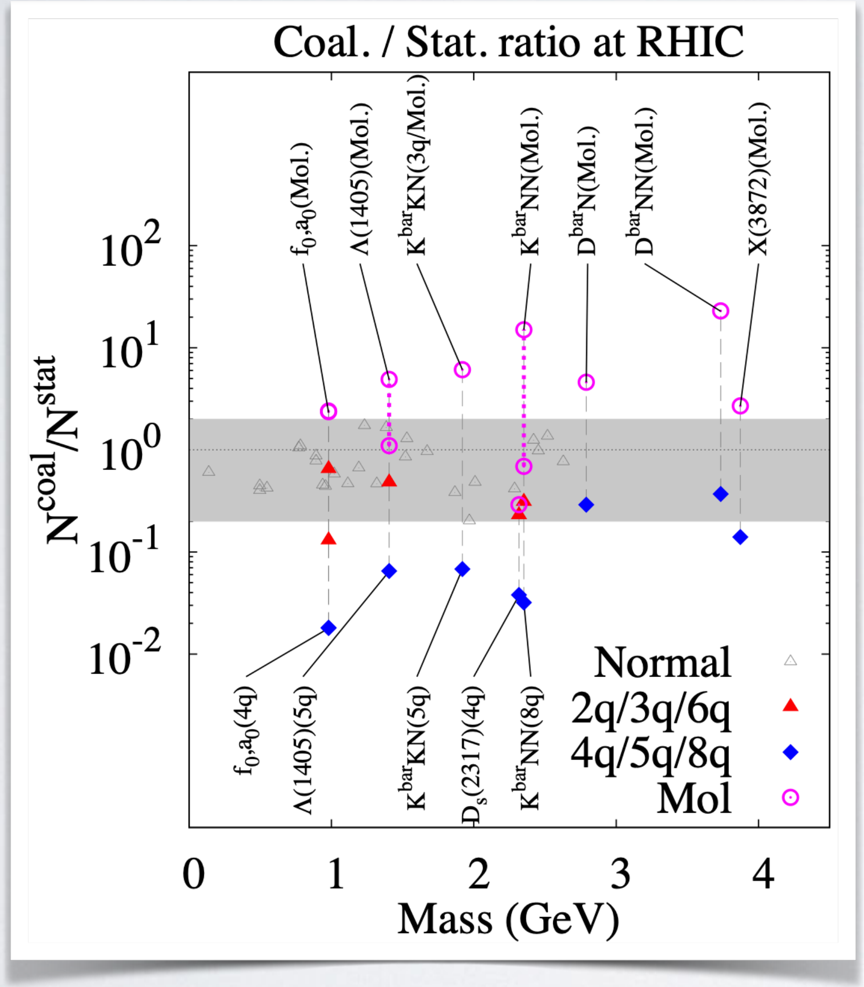

[ExHIC coll. — PRL 2011]

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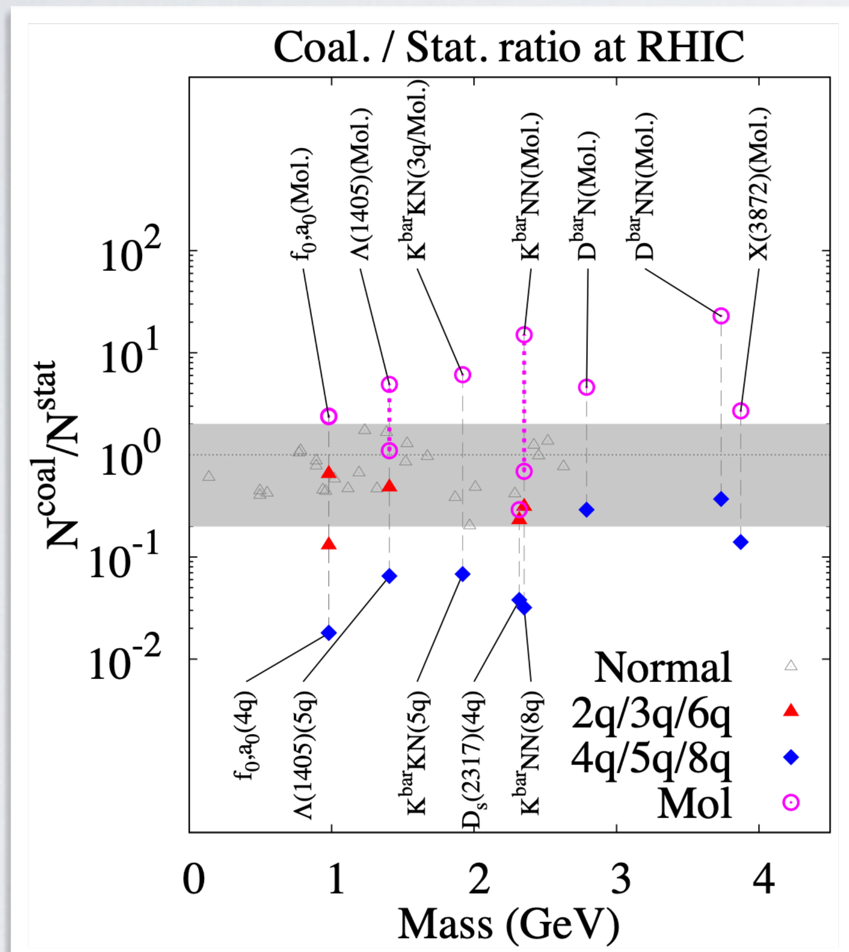

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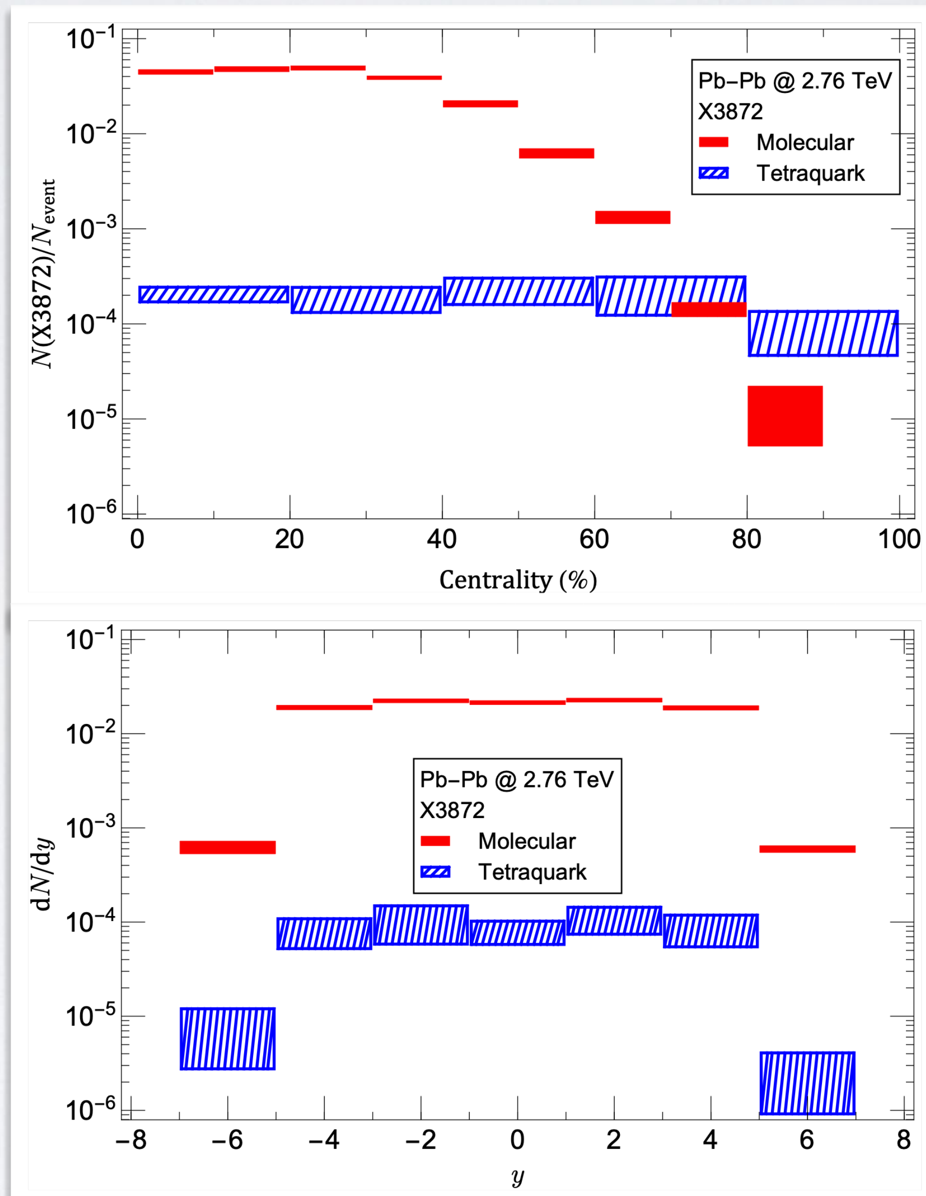

[Zhang et al. — PRL 2021]

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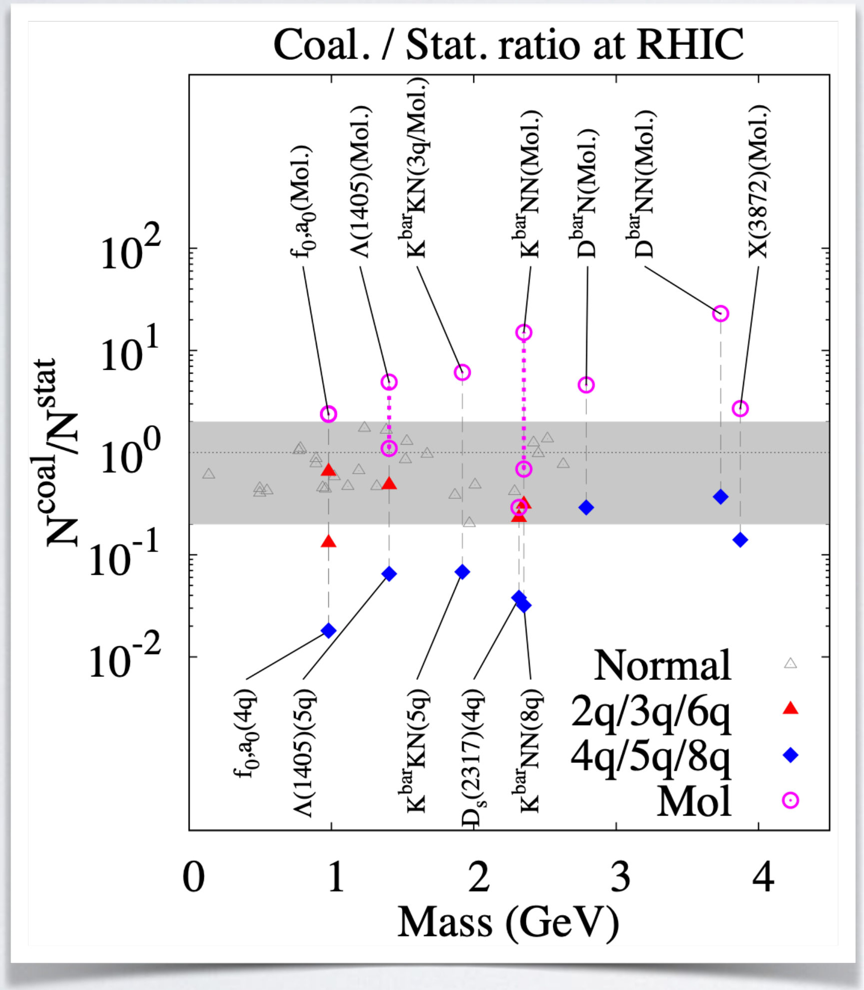

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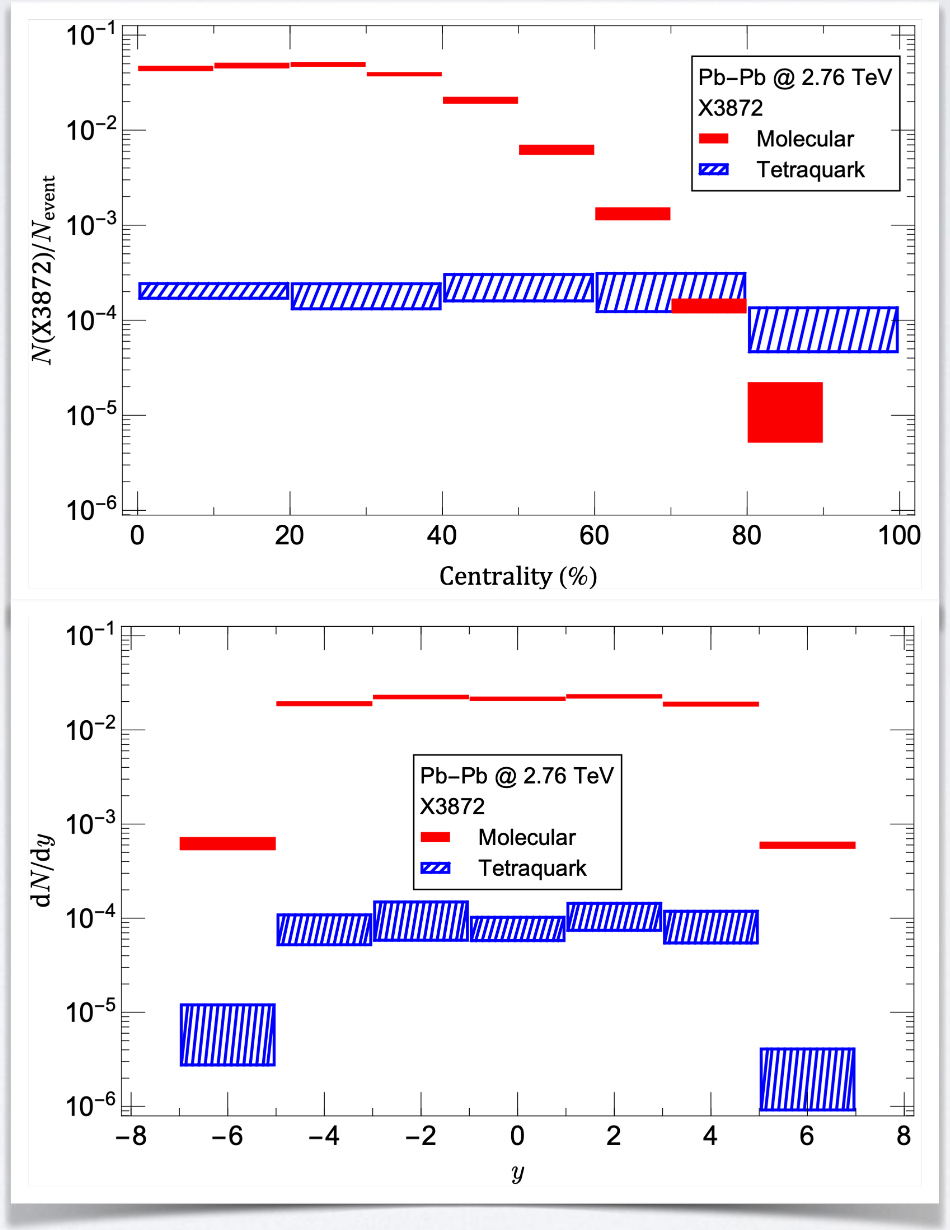

[Zhang et al. — PRL 2021]

Although, not always…

[Chen et al. — PRC 2022]

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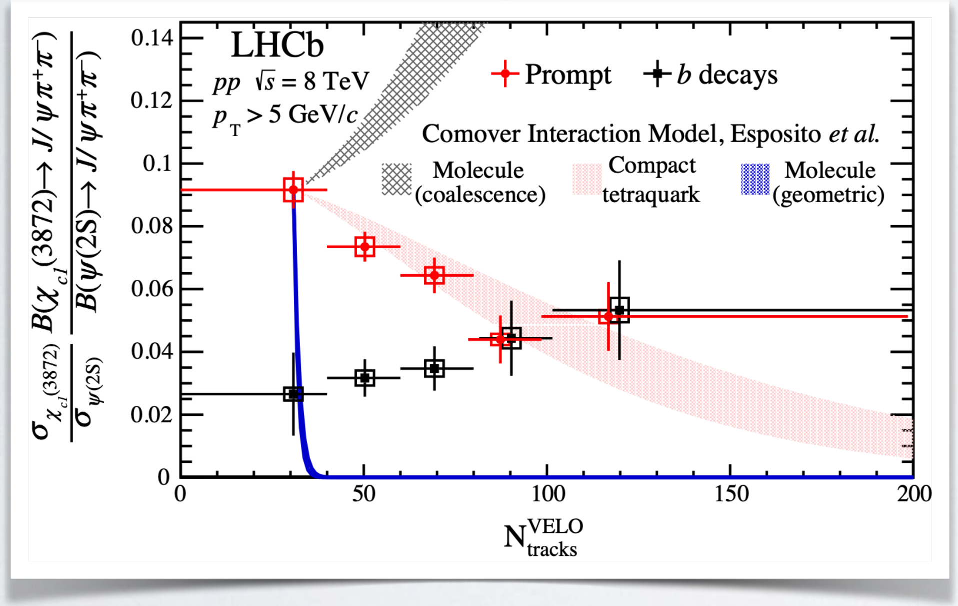

[LHCb — PRL 2021]

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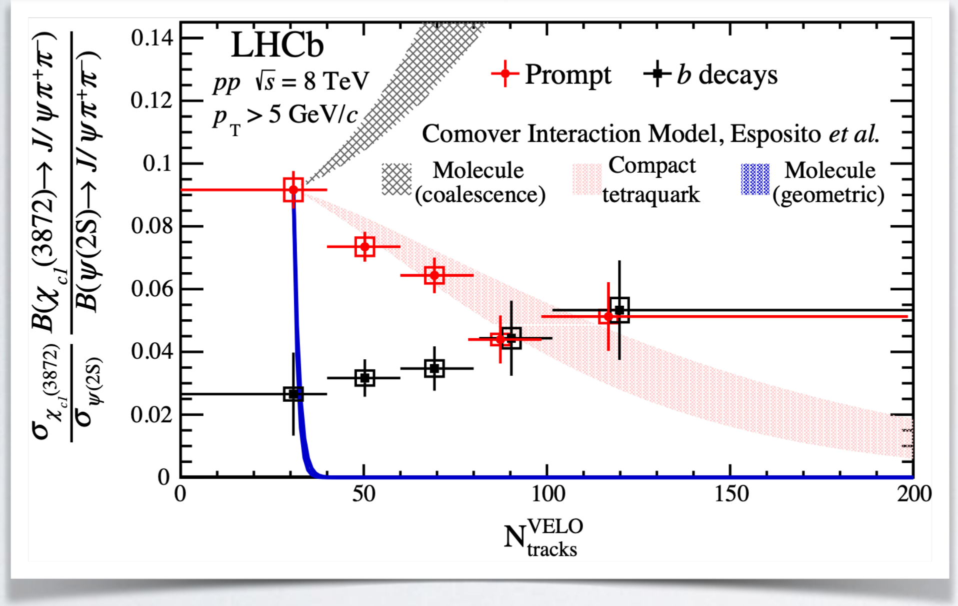

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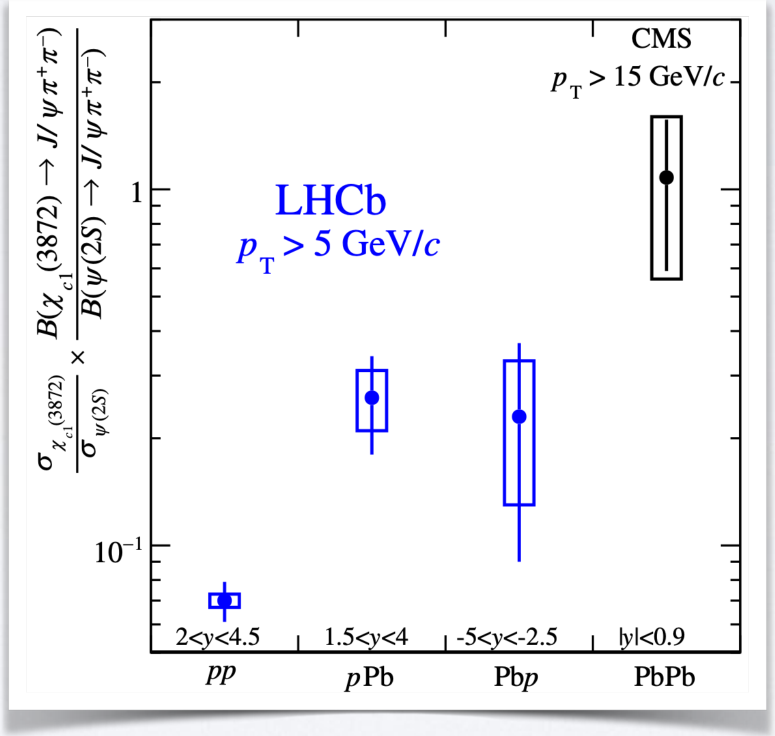

[LHCb — PRL 2024]

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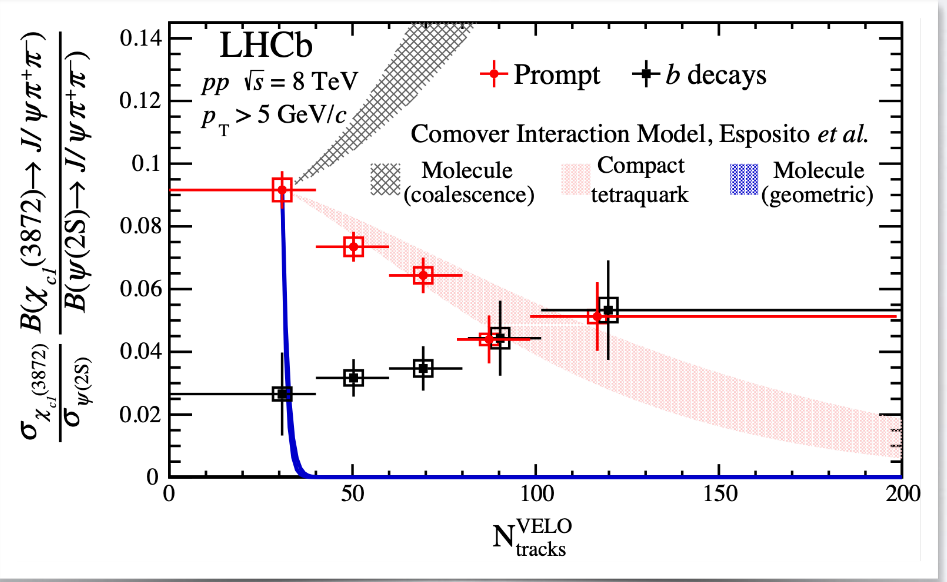

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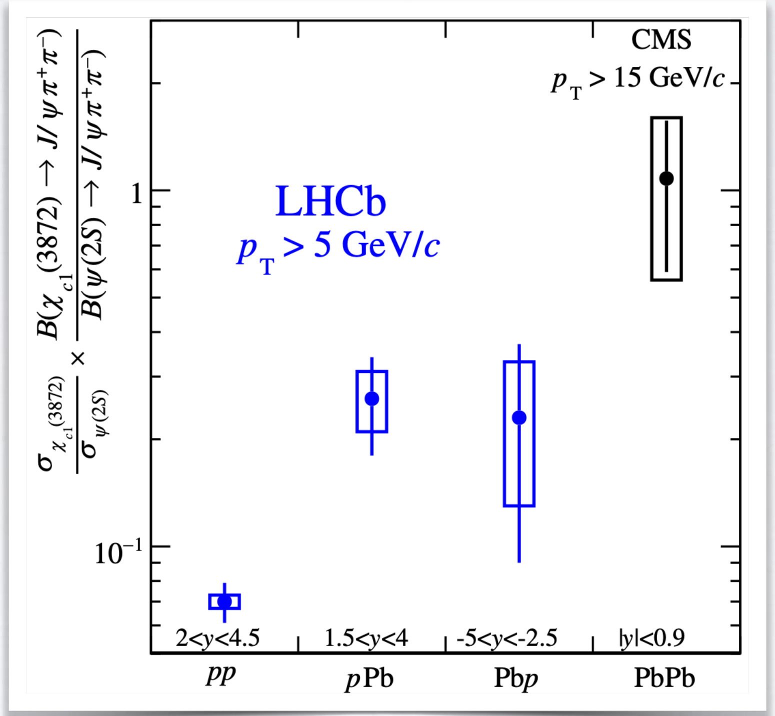
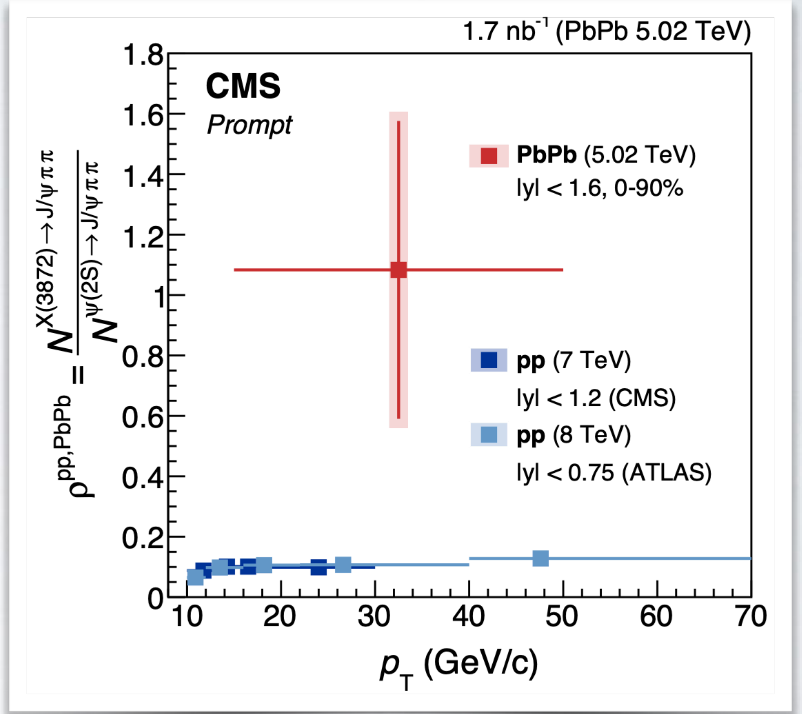


[LHCb — PRL 2024]



[CMS — PRL 2022]

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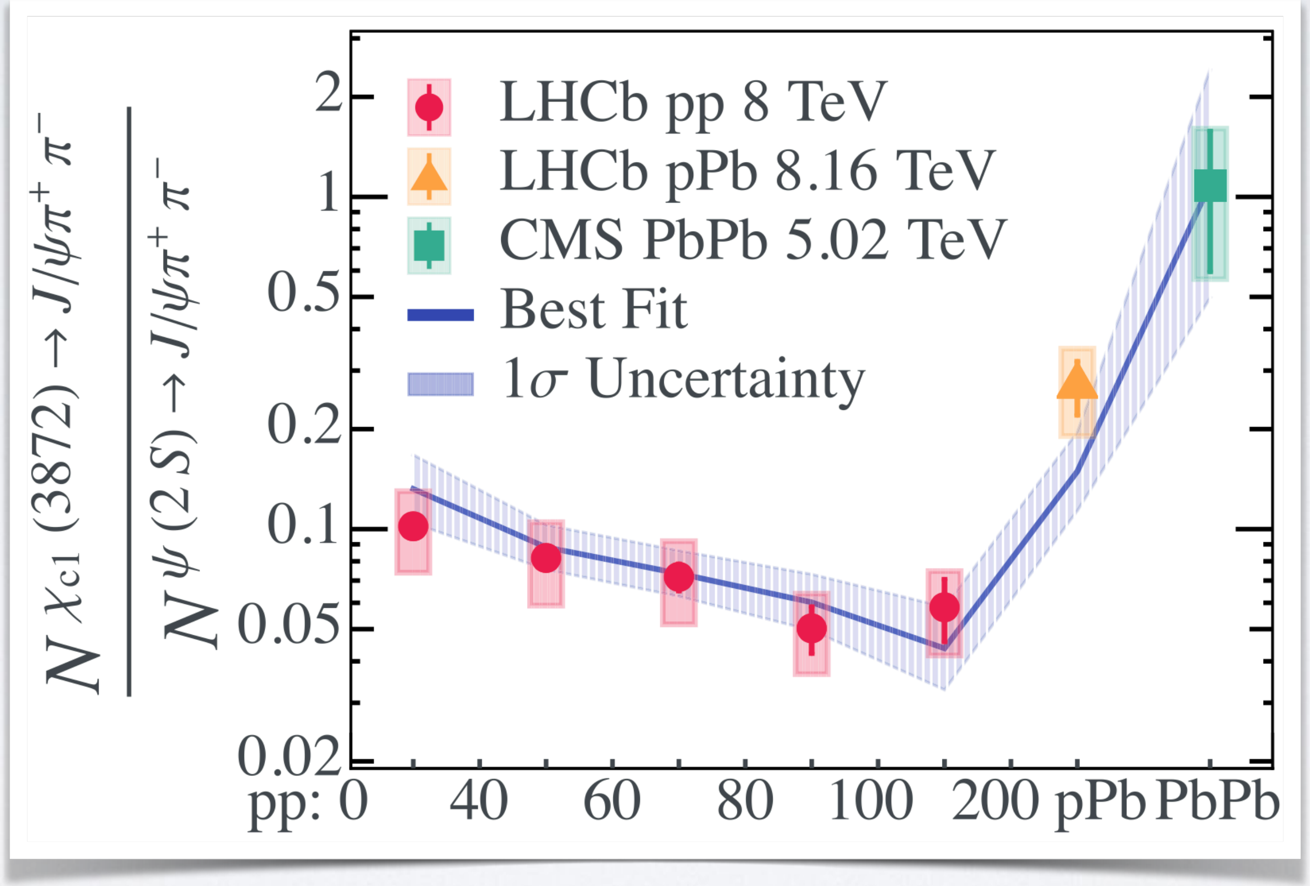

[Guo, Guo, Liao, Wang, Xing – PRC 2024]

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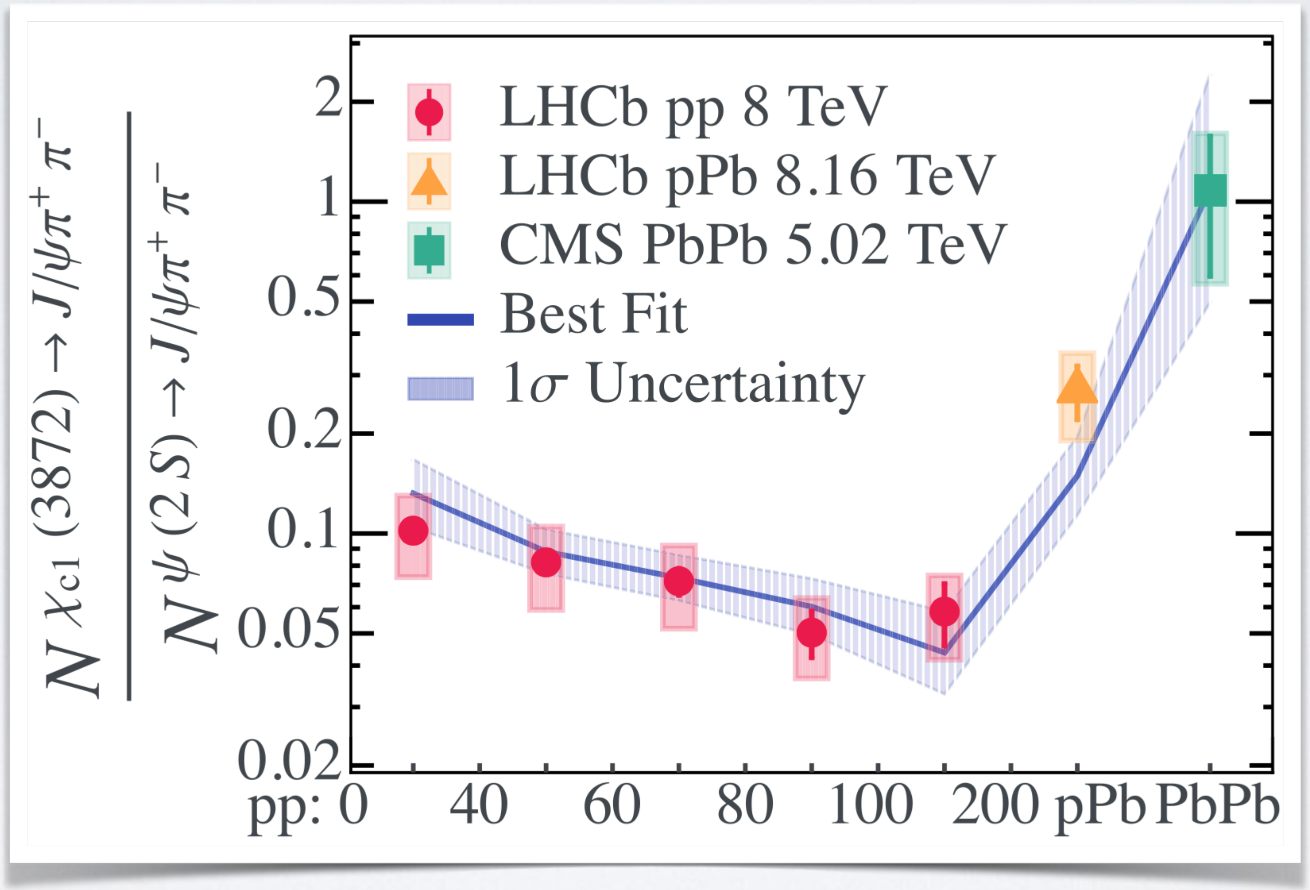

[Guo, Guo, Liao, Wang, Xing — PRC 2024]

Could be due to new creation mechanisms kicking in for larger systems

OUTLOOK

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- …and so on…

However: are we sure we can learn anything about puzzling particles from systems we do not fully understand yet?

FRAMEWORKS FOR THE INTERNAL STRUCTURE

[Brambilla, Brodsky, Germani, Grinstein, Guo, Hanart, Lebed, Maiani, Piccini, Polosa, Vairo, and many more…]

ZOOMING IN

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Very first attempt:

$$H = \sum_i m_i + \sum_{i,j} \kappa_{i,j} \mathbf{S}_i \cdot \mathbf{S}_j$$

[Maiani, Piccini, Polosa, Riquer — PRD 2005;
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$$V_\pi(r) \sim \frac{e^{-\mu r}}{m_\pi r}$$

[Swanson — PLB 2004; Tornqvist — PLB 2004]

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In both instances, models, parameters and mechanism have quickly proliferated

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$$
\begin{aligned}
V_{E0} &= +\frac{g_V^2}{q^2+m_V^2}\,,\\
V_{M1} &= -\frac{f_V^2}{4M^2}\,\frac{(\hat{S}_{L1}\times\vec{q})\cdot(\hat{S}_{L2}\times\vec{q})}{q^2+m_V^2}\\
&= -\frac{2}{3}\,\frac{f_V^2}{4M^2}\,\hat{S}_{L1}\cdot\hat{S}_{L2}\,\frac{q^2}{q^2+m_V^2}+\dots\,,\\
V_{E2} &= +\frac{h_V^2}{4M^4}\,\frac{(\hat{Q}_{L1,ij}\,q_iq_j)(\hat{Q}_{L2,lm}\,q_lq_m)}{q^2+m_V^2}\\
&= +\frac{h_V^2}{36M^4}\,(\hat{Q}_{L1,ij}\hat{Q}_{L2,ij})\,\frac{q^4}{q^2+m_V^2}+\dots\,,
\end{aligned}
$$

Molecule	J^P	V	Attractive?
$D\bar{D}$	0^{++}	C_0	Yes
$D^*\bar{D}$	1^{++}	$C_0 + C_1$	Most
$D^*\bar{D}$	1^{+-}	$C_0 - C_1$	Likely
$D^*\bar{D}^*$	0^{++}	$C_0 - 2C_1$	Likely
$D^*\bar{D}^*$	1^{+-}	$C_0 - C_1$	Likely
$D^*\bar{D}^*$	2^{++}	$C_0 + C_1$	Most

Molecule	J^P	V	Attractive?
$\bar{D}\Sigma_c$	$\frac{1}{2}^-$	D_0	Yes
$\bar{D}\Sigma_c^*$	$\frac{3}{2}^-$	D_0	Yes
$\bar{D}^*\Sigma_c$	$\frac{1}{2}^-$	$D_0 - \frac{4}{3}D_1$	Likely
$\bar{D}^*\Sigma_c$	$\frac{3}{2}^-$	$D_0 + \frac{2}{3}D_1$	Most
$\bar{D}^*\Sigma_c^*$	$\frac{1}{2}^-$	$D_0 - \frac{5}{3}D_1$	Likely
$\bar{D}^*\Sigma_c^*$	$\frac{3}{2}^-$	$D_0 - \frac{2}{3}D_1$	Likely
$\bar{D}^*\Sigma_c^*$	$\frac{5}{2}^-$	$D_0 + D_1$	Most

Molecule	J^P	V	Attractive?
$\Sigma_c\bar{\Sigma}_c$	0^{-+}	$E_0 - \frac{4}{3}E_1$	Likely
$\Sigma_c\bar{\Sigma}_c$	1^{--}	$E_0 + \frac{4}{9}E_1$	Yes
$\Sigma_c^*\bar{\Sigma}_c$	1^{-+}	$E_0 - E_1 - \frac{15}{2}E_2$	Likely
$\Sigma_c^*\bar{\Sigma}_c$	1^{--}	$E_0 - \frac{11}{9}E_1 + \frac{15}{2}E_2$	Likely
$\Sigma_c^*\bar{\Sigma}_c$	2^{-+}	$E_0 + \frac{1}{3}E_1 - \frac{3}{2}E_2$	Likely
$\Sigma_c^*\bar{\Sigma}_c$	2^{--}	$E_0 + E_1 + \frac{3}{2}E_2$	Most
$\Sigma_c^*\bar{\Sigma}_c^*$	0^{-+}	$E_0 - \frac{15}{9}E_1 + \frac{15}{2}E_2$	Likely
$\Sigma_c^*\bar{\Sigma}_c^*$	1^{--}	$E_0 - \frac{11}{9}E_1 + \frac{3}{2}E_2$	Likely
$\Sigma_c^*\bar{\Sigma}_c^*$	2^{-+}	$E_0 - \frac{1}{3}E_1 - \frac{9}{2}E_2$	Likely
$\Sigma_c^*\bar{\Sigma}_c^*$	3^{--}	$E_0 + E_1 + \frac{3}{2}E_2$	Most

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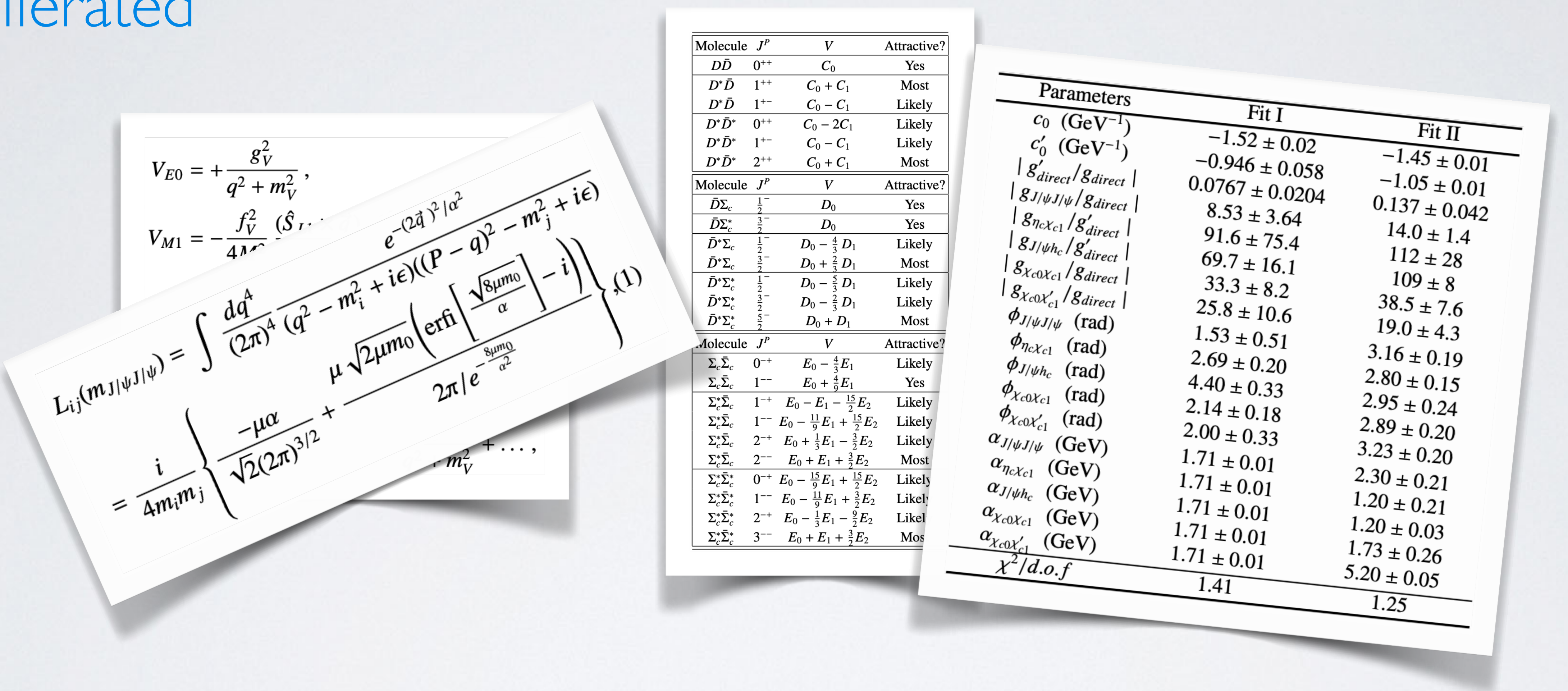

$$V_{E0} = +\frac{g_V^2}{q^2+m_V^2},$$

$$L_{ij}(m_{J/\psi J/\psi}) = \int \frac{dq^4}{(2\pi)^4} \frac{e^{-(2\vec{q})^2/\alpha^2}}{(q^2-m_i^2+i\epsilon)((P-q)^2-m_j^2+i\epsilon)} = \frac{i}{4m_i m_j}\left\{\frac{-\mu\alpha}{\sqrt{2}(2\pi)^{3/2}} + \frac{\mu\sqrt{2\mu m_0}\left(\mathrm{erfi}\left[\frac{\sqrt{8\mu m_0}}{\alpha}\right]-i\right)}{2\pi/e^{-\frac{8\mu m_0}{\alpha^2}}}\right\}, (1)$$

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$\bar{D}^*\Sigma_c^*$	$\frac{5}{2}^-$	D_0+D_1	Most
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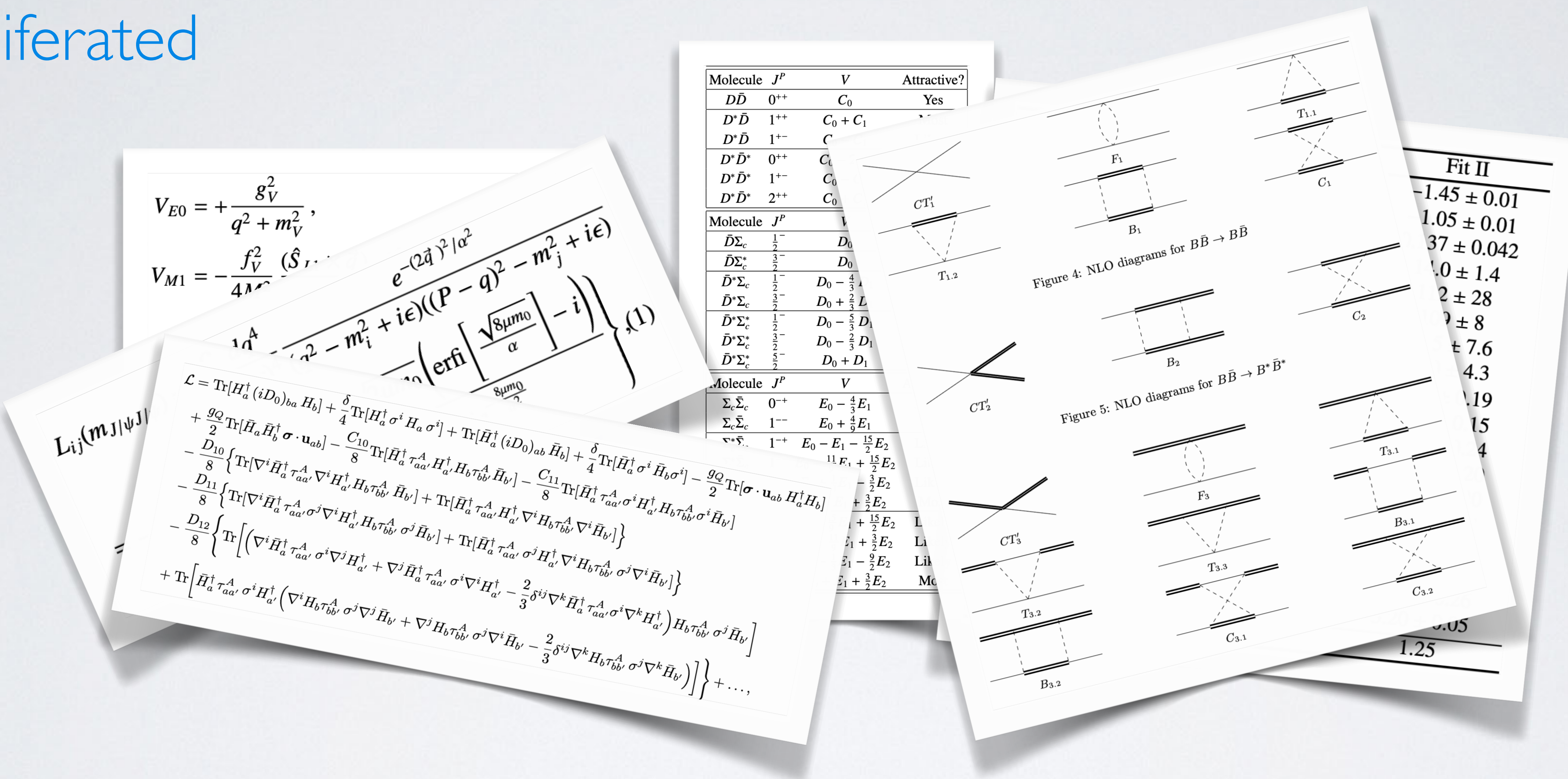
$\Sigma_c^*\bar{\Sigma}_c^*$	3^{--}	$E_0+E_1+\frac{3}{2}E_2$	Mos

Parameters	Fit I	Fit II
c_0 (GeV^{-1})	-1.52 ± 0.02	-1.45 ± 0.01
c_0' (GeV^{-1})	-0.946 ± 0.058	-1.05 ± 0.01
$\lvert g'_{direct}/g_{direct}\rvert$	0.0767 ± 0.0204	0.137 ± 0.042
$\lvert g_{J/\psi J/\psi}/g_{direct}\rvert$	8.53 ± 3.64	14.0 ± 1.4
$\lvert g_{\eta_c\chi_{c1}}/g'_{direct}\rvert$	91.6 ± 75.4	112 ± 28
$\lvert g_{J/\psi h_c}/g'_{direct}\rvert$	69.7 ± 16.1	109 ± 8
$\lvert g_{\chi_{c0}\chi_{c1}}/g_{direct}\rvert$	33.3 ± 8.2	38.5 ± 7.6
$\lvert g_{\chi_{c0}\chi'_{c1}}/g_{direct}\rvert$	25.8 ± 10.6	19.0 ± 4.3
$\phi_{J/\psi J/\psi}$ (rad)	1.53 ± 0.51	3.16 ± 0.19
$\phi_{\eta_c\chi_{c1}}$ (rad)	2.69 ± 0.20	2.80 ± 0.15
$\phi_{J/\psi h_c}$ (rad)	4.40 ± 0.33	2.95 ± 0.24
$\phi_{\chi_{c0}\chi_{c1}}$ (rad)	2.14 ± 0.18	2.89 ± 0.20
$\phi_{\chi_{c0}\chi'_{c1}}$ (rad)	2.00 ± 0.33	3.23 ± 0.20
$\alpha_{J/\psi J/\psi}$ (GeV)	1.71 ± 0.01	2.30 ± 0.21
$\alpha_{\eta_c\chi_{c1}}$ (GeV)	1.71 ± 0.01	1.20 ± 0.21
$\alpha_{J/\psi h_c}$ (GeV)	1.71 ± 0.01	1.20 ± 0.03
$\alpha_{\chi_{c0}\chi_{c1}}$ (GeV)	1.71 ± 0.01	1.73 ± 0.26
$\alpha_{\chi_{c0}\chi'_{c1}}$ (GeV)	1.71 ± 0.01	5.20 ± 0.05
$\chi^2/d.o.f$	1.41	1.25

ZOOMING IN

In both instances, models, parameters and mechanism have quickly proliferated



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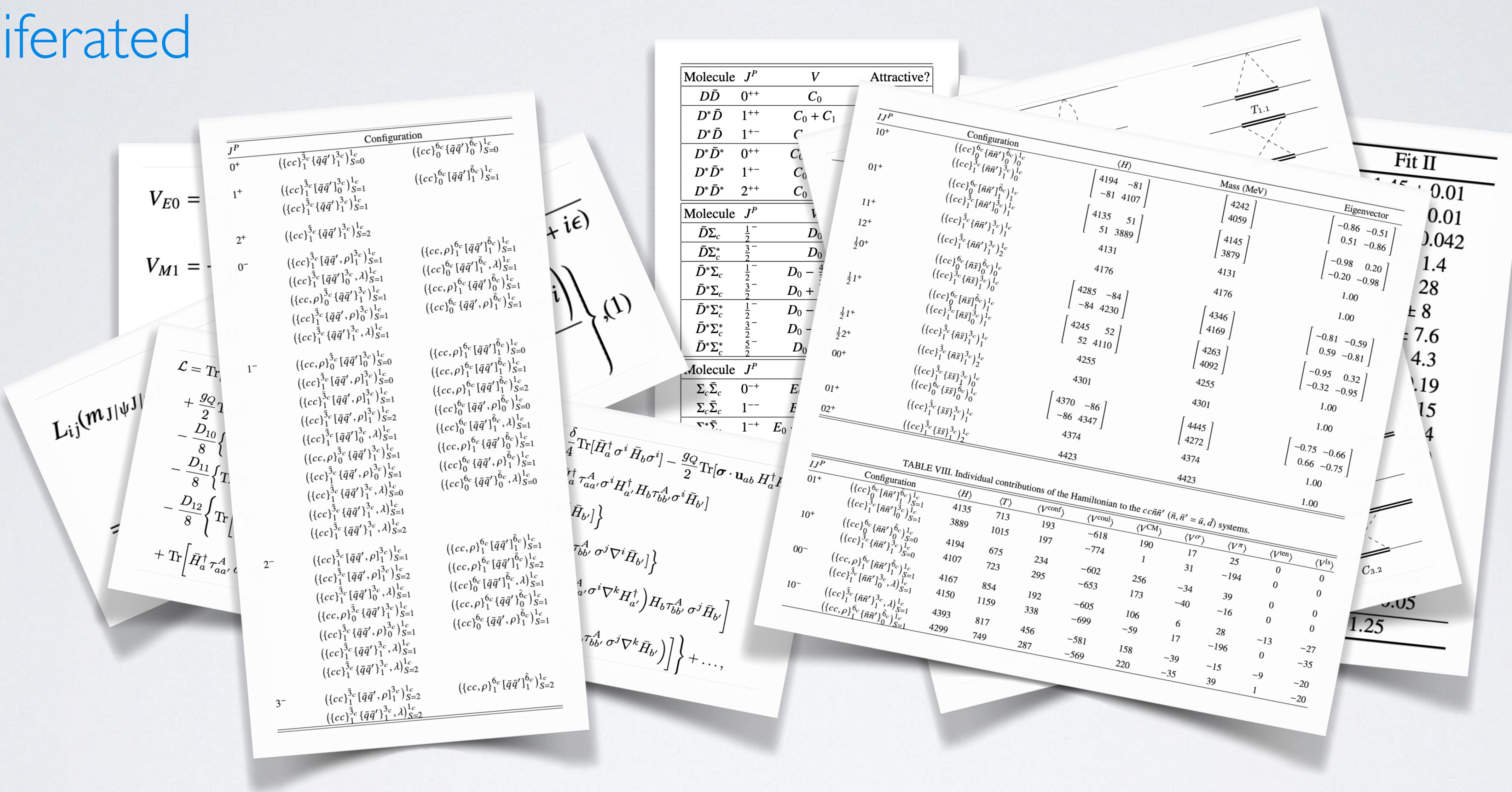

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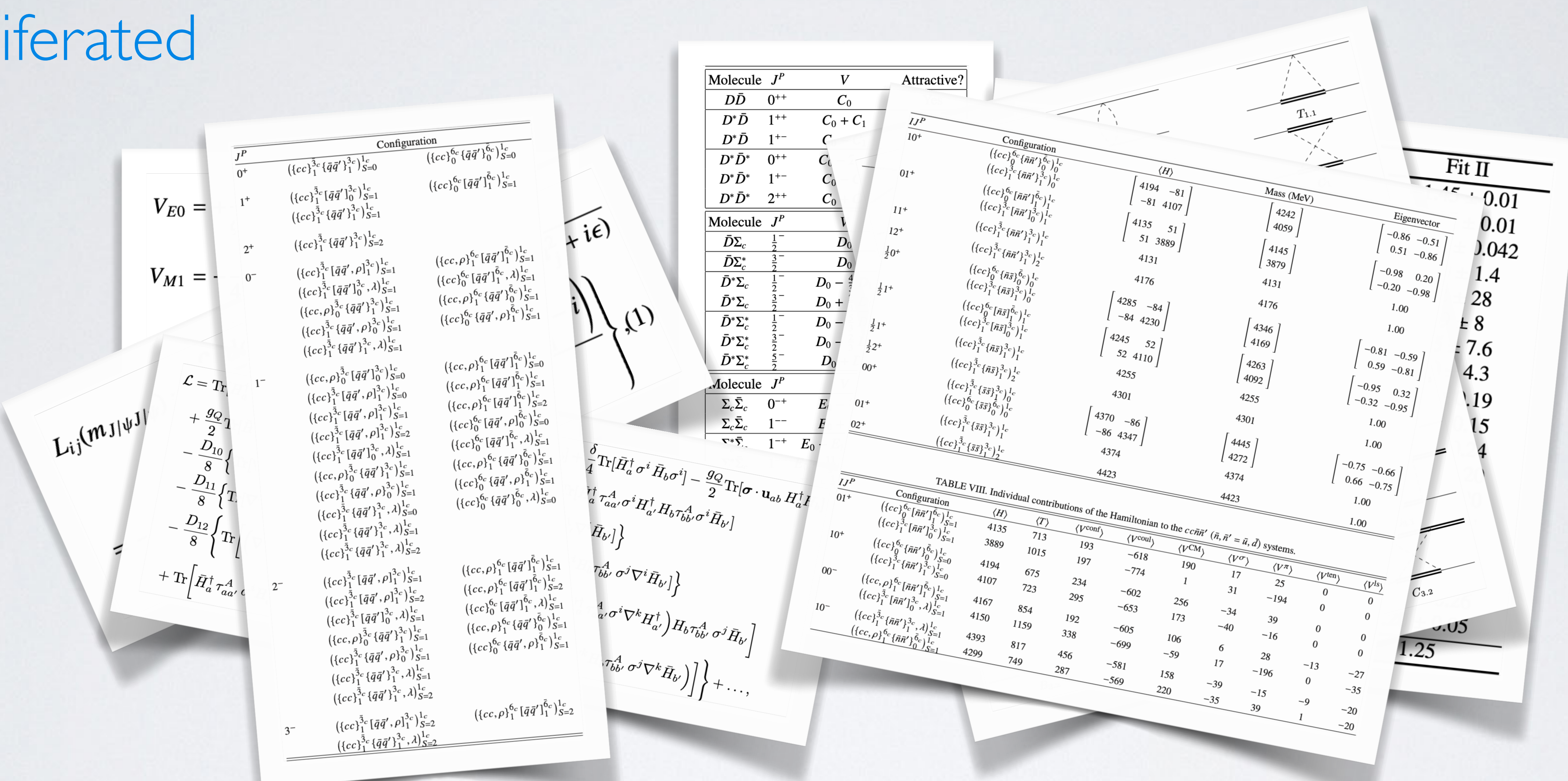

Beside a few exceptions, properties and spectrum are extremely model dependent → hardly the right way to shed light on the issue

$$m_Q \gg m_q \, , \Lambda_{\rm QCD}$$

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A more systematic framework leverages the existing hierarchies

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Since $m_Q \gg m_q\,, \Lambda_{\rm QCD}$ one develops a Born-Oppenheimer approximation for the 4-quark constituents

[e.g., Braaten, Langmack, Smith — PRD 2014; Berwein, Brambilla, Castellà, Vairo — PRD 2015; Maiani, Polosa, Riquer — PRD 2019; Allaman, Ekterachian, Nardi, Rattazzi, Stelz — JHEP 2024; Braaten, Bruschini — PLB 2025]

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Solve the light quark problem with static heavy quarks

↓

Light quark energy as effective potential for the heavy quarks

$m_Q \gg m_q\,, \Lambda_{\rm QCD}$

A more systematic framework leverages the existing hierarchies

Since $m_Q \gg m_q\,, \Lambda_{\rm QCD}$ one develops a Born-Oppenheimer approximation for the 4-quark constituents

[e.g., Braaten, Langmack, Smith — PRD 2014; Berwein, Brambilla, Castellà, Vairo — PRD 2015; Maiani, Polosa, Riquer — PRD 2019; Allaman, Ekterachian, Nardi, Rattazzi, Stelz — JHEP 2024; Braaten, Bruschini — PLB 2025]

Solve the light quark problem with static heavy quarks

↓

Light quark energy as effective potential for the heavy quarks

The separation of scales makes the problem clean and with corrections that can be estimated systematically

APPLICATIONS

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- Production in ultra-periferal heavy ion collisions [e.g., Moreira, Bertulani, Gonçalves, Navarra — PRD 2016; AE, Manzari, Pilloni, Polosa — PRD 2021; Gonçalves, Moreira — PLB 2021; Niu, Ding, Wang, Yang — CPL 2025]

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Thank you for the attention!