

Theory Overview of Rare Decays within the Standard Model

Ludovico Vittorio (University of Rome Sapienza and INFN, Rome)

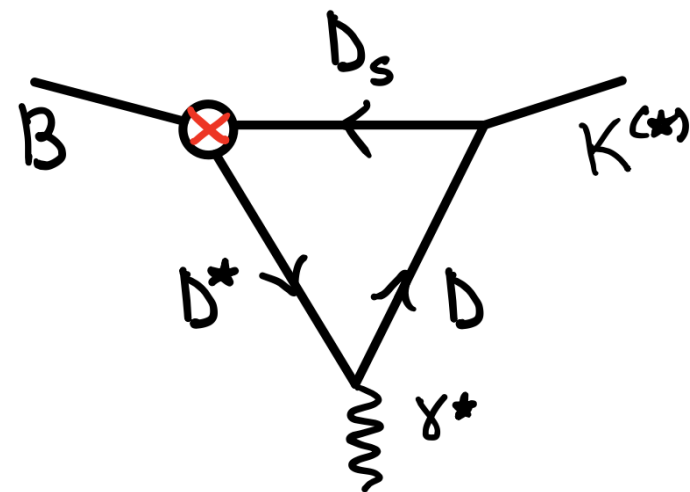
WIFAI 2025 Workshop, Bari – November the 13th, 2025



SAPIENZA
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(from PRD 107 (2023) 5, 055036)

***Many thanks to G. Martinelli, L. Silvestrini,
S. Simula and M. Valli for discussions and suggestions***

Introduction: why rare decays are so important

The most appealing places to look for possible NP effects are rare decays, since **rare decays are a manifestation of broken (accidental) symmetries**, e.g. of physics beyond the Standard Model

Some general examples:

1. Proton decay



*Test of baryon and lepton
number conservation*

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$$q_i \rightarrow q_j \ell^+ \ell^-,$$

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$$q_i \rightarrow q_j \gamma$$

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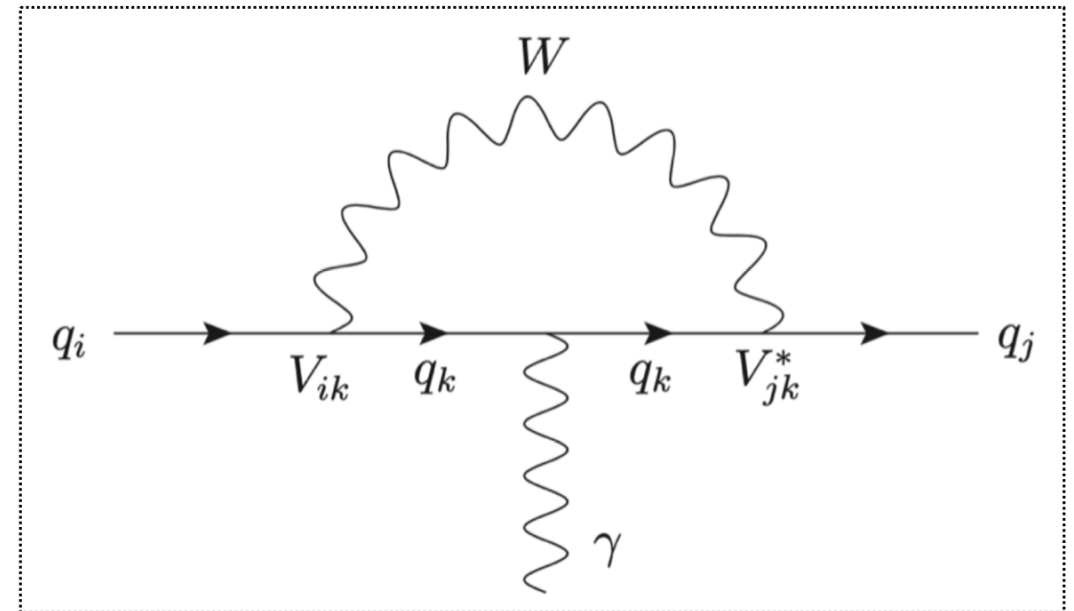


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$$\begin{aligned} q_i &\rightarrow q_j \ell^+ \ell^-, \\ q_i &\rightarrow q_j \nu \bar{\nu}, \\ q_i &\rightarrow q_j \gamma \end{aligned}$$



Plan of the talk

a) Semileptonic neutral-current $B_{(s)}$ -meson decays

b) Rare radiative-and-leptonic B_s -meson decay

c) Rare semileptonic decays with di-neutrino final states

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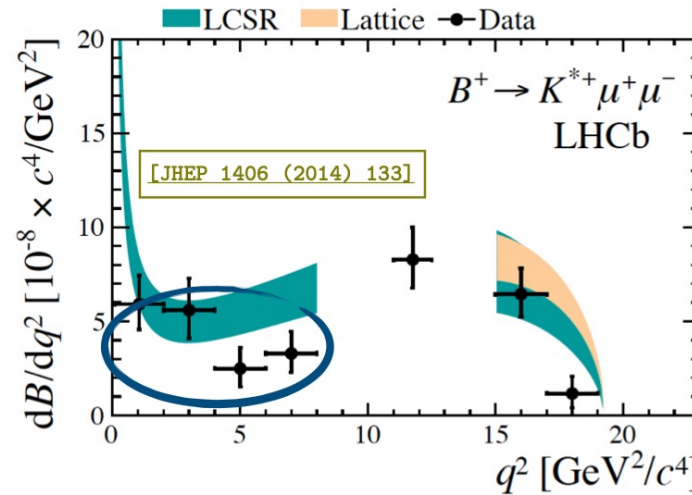
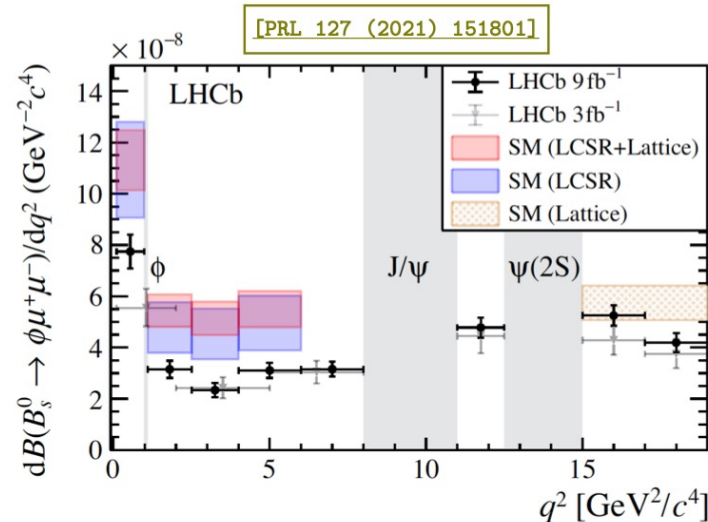
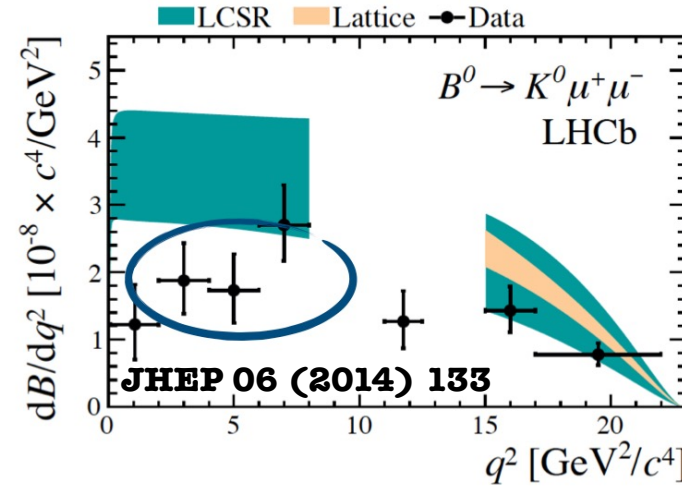
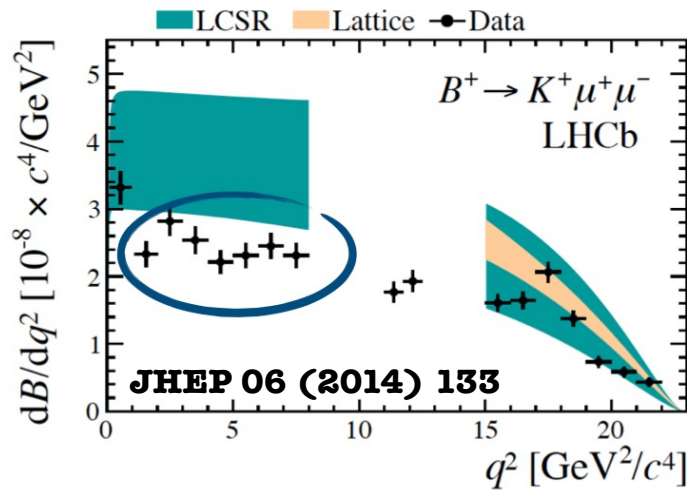
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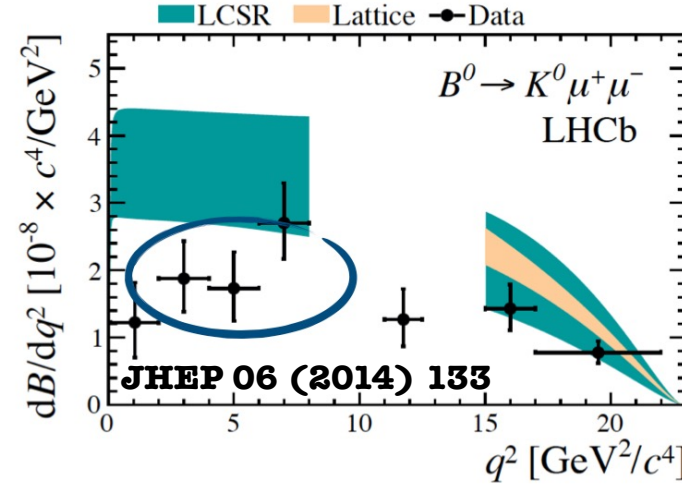
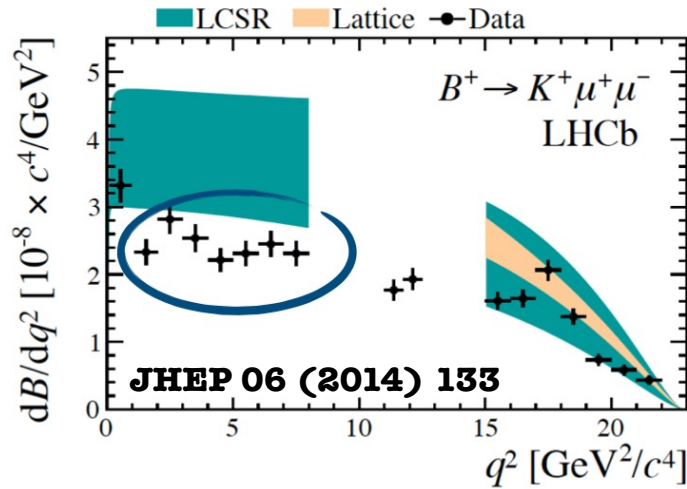
Tensions in BRs and angular observables (?)

Despite the disappearance of the $R(K^*)$ anomalies (**LHCb Coll., PRL 131 (2023) 5, 051803**), there are still **several discrepancies among theory and experiments in semileptonic neutral-current B decays data**:

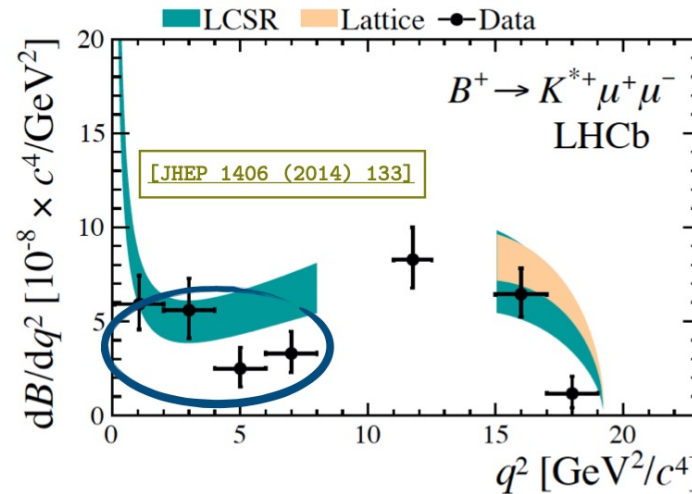
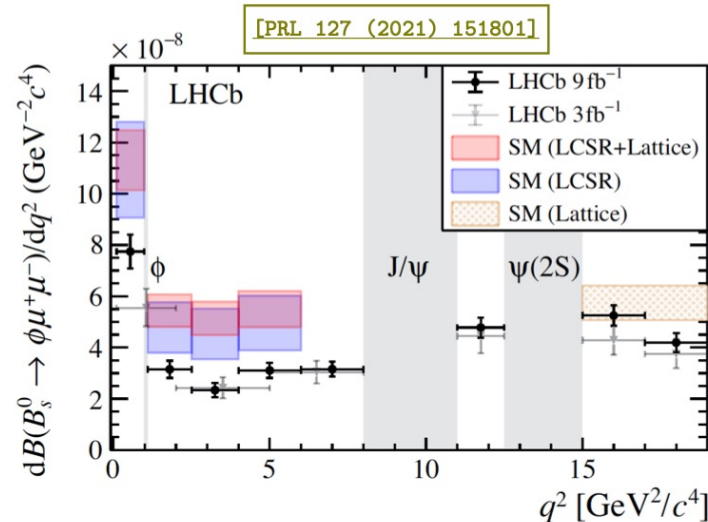


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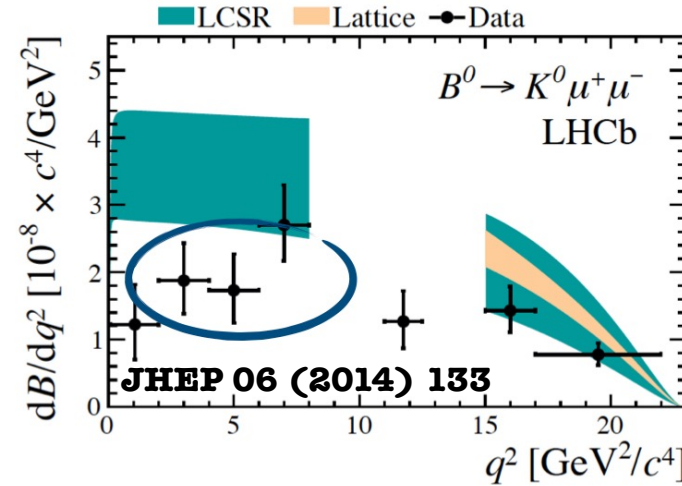
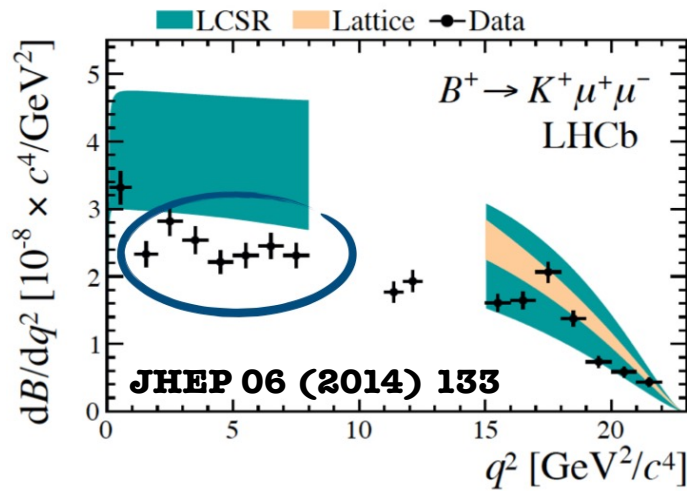


SM expectations seem to
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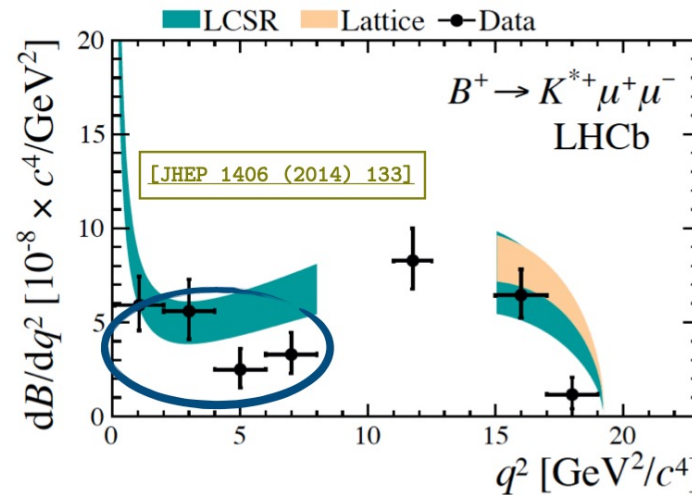
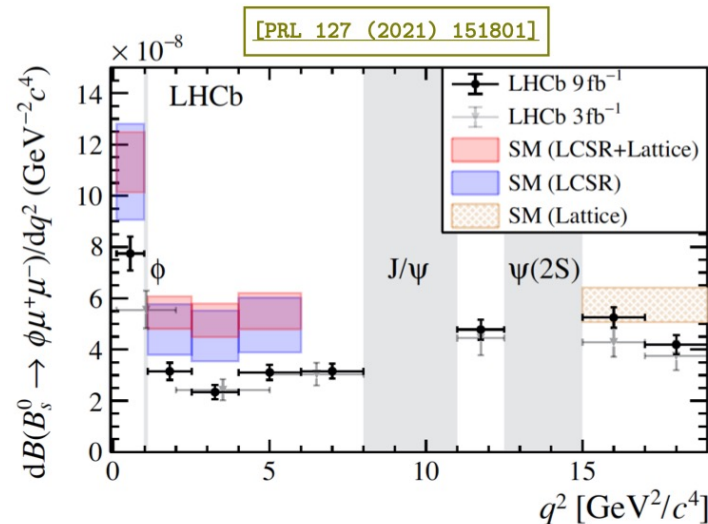


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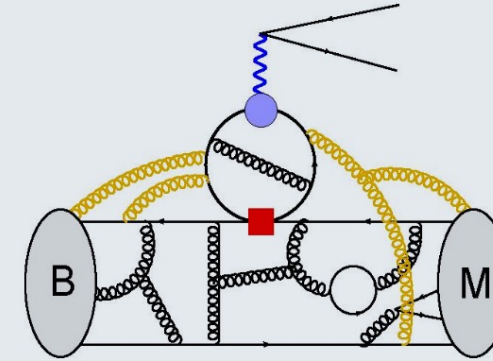
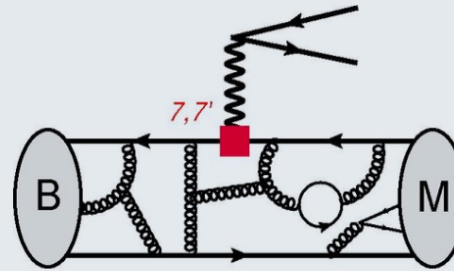
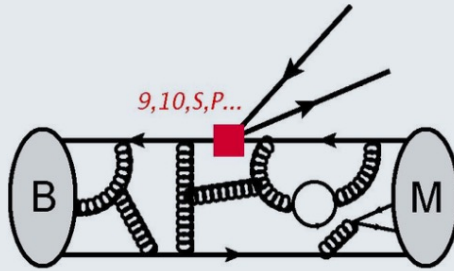


Important debate: is there really NP? Were the hadronic effects treated correctly ?

The central role of hadronic form factors

M. Reboud's talk @ LHC Implication Workshop 2022 @ CERN

$$\mathcal{H}(b \rightarrow s \ell \ell) = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i(\mu) \mathcal{O}_i(\mu)$$



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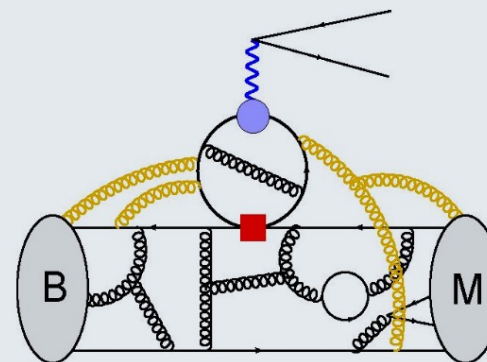
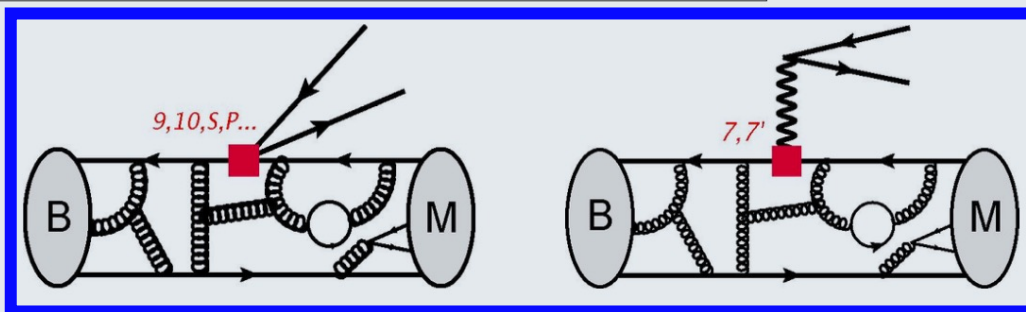
Local form-factors

Non-local form-factors

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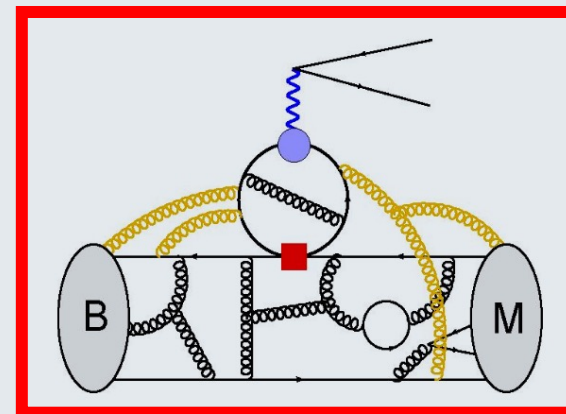
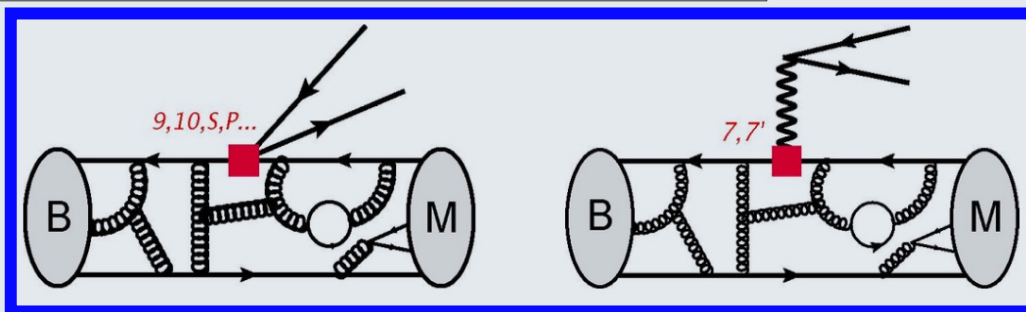
$$\langle P | J_{\text{had}}^\mu | B \rangle \text{ or } \langle V | J_{\text{had}}^\mu | B \rangle$$

$$J_{\text{had}}^\mu = \bar{s} \gamma^\mu (\gamma_5) b \text{ or } \bar{s} \sigma^{\mu\alpha} q_\alpha (\gamma_5) b$$

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$$i \int d^4 x e^{iq \cdot x} \langle P(V) | \mathcal{T} \{ j_{\mu}^{\text{em}}(x), (C_1 \mathcal{O}_1 + C_2 \mathcal{O}_2)(0) \} | B \rangle$$

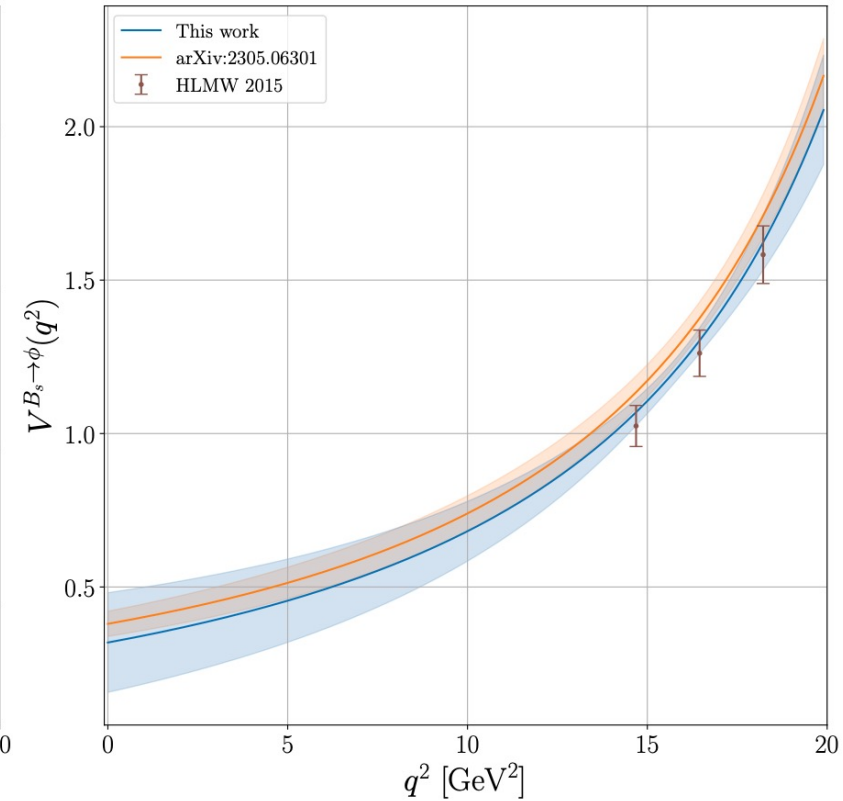
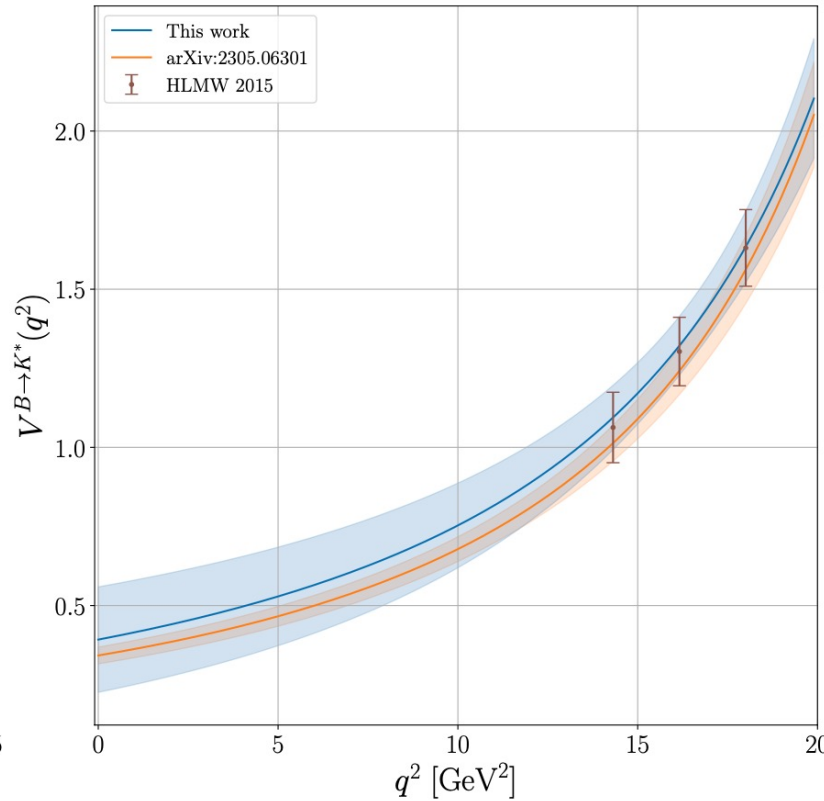
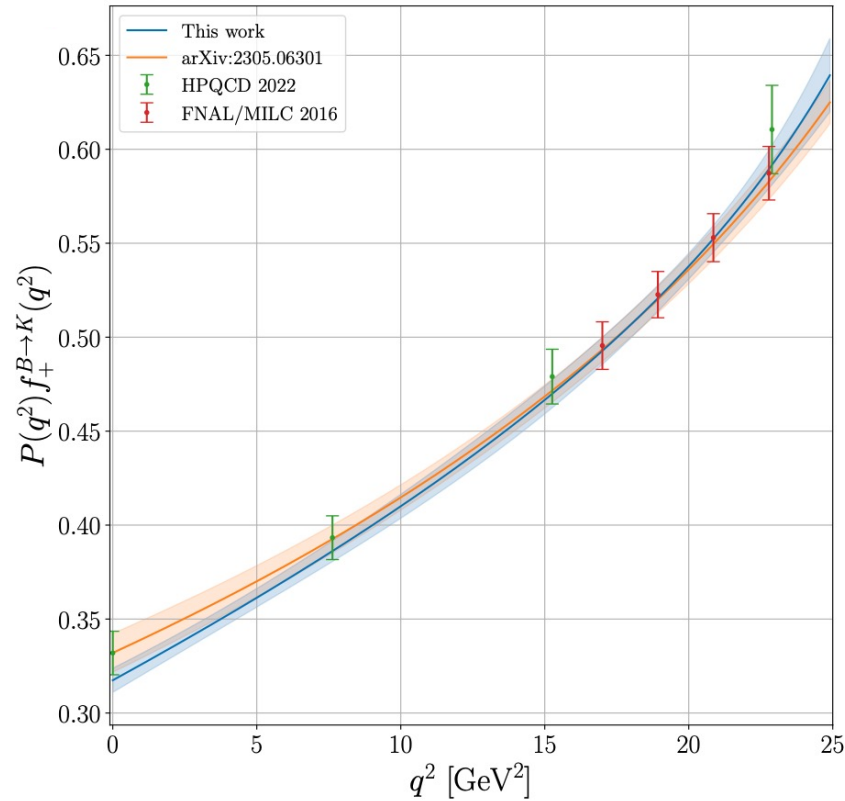
$$\mathcal{O}_1 = (\bar{s}_L \gamma_{\mu} T^a q_L) (\bar{q}_L \gamma^{\mu} T^a b_L),$$

$$\mathcal{O}_2 = (\bar{s}_L \gamma_{\mu} q_L) (\bar{q}_L \gamma^{\mu} b_L)$$



Local hadronic form factors

Study of local form factors through the Dispersive Matrix (DM) method:



In preparation with M. Fedele, L. Silvestrini, S. Simula, M. Valli

Available lattice datasets:

- **FNAL/MILC 2016 (1509.06235)**

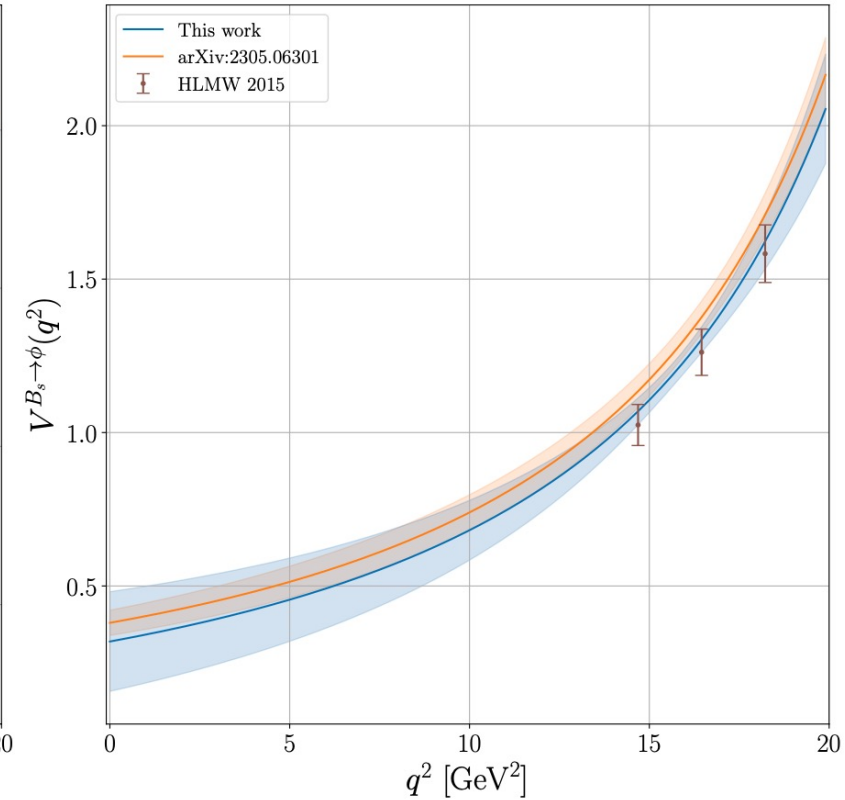
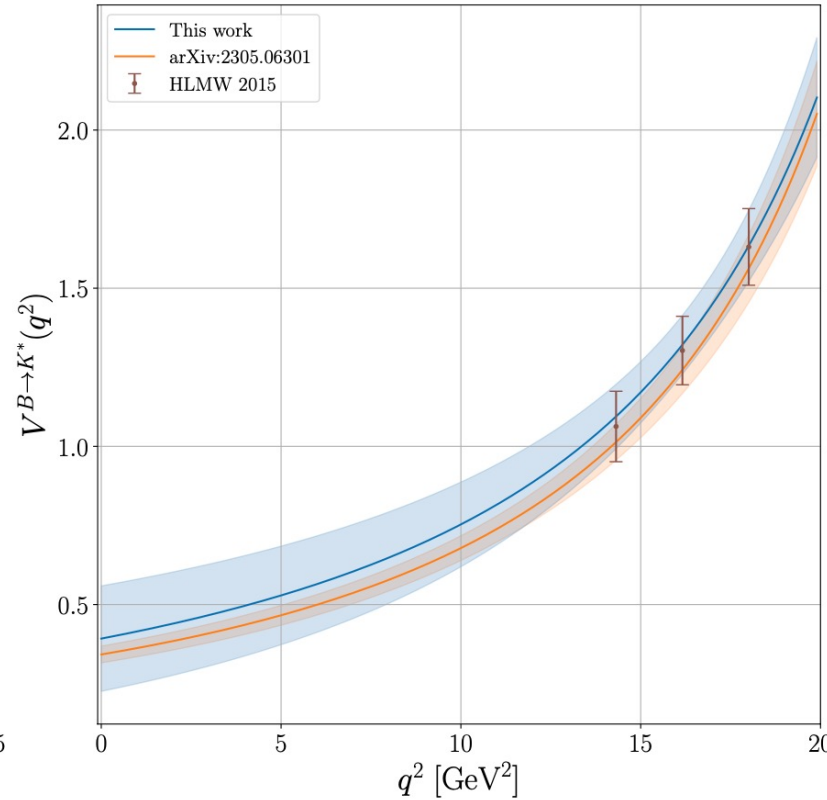
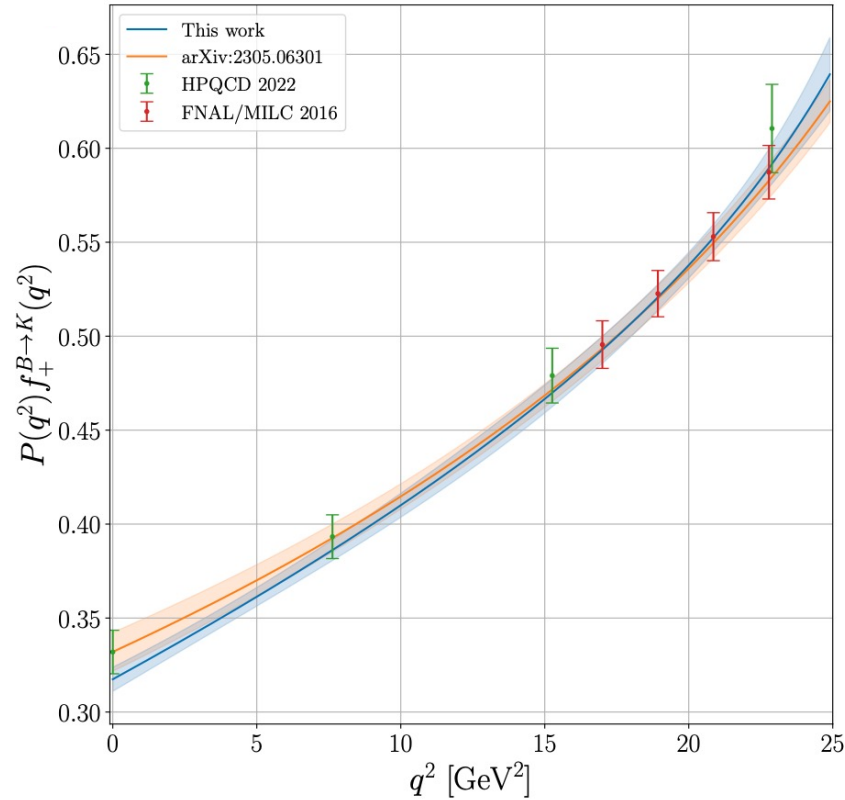
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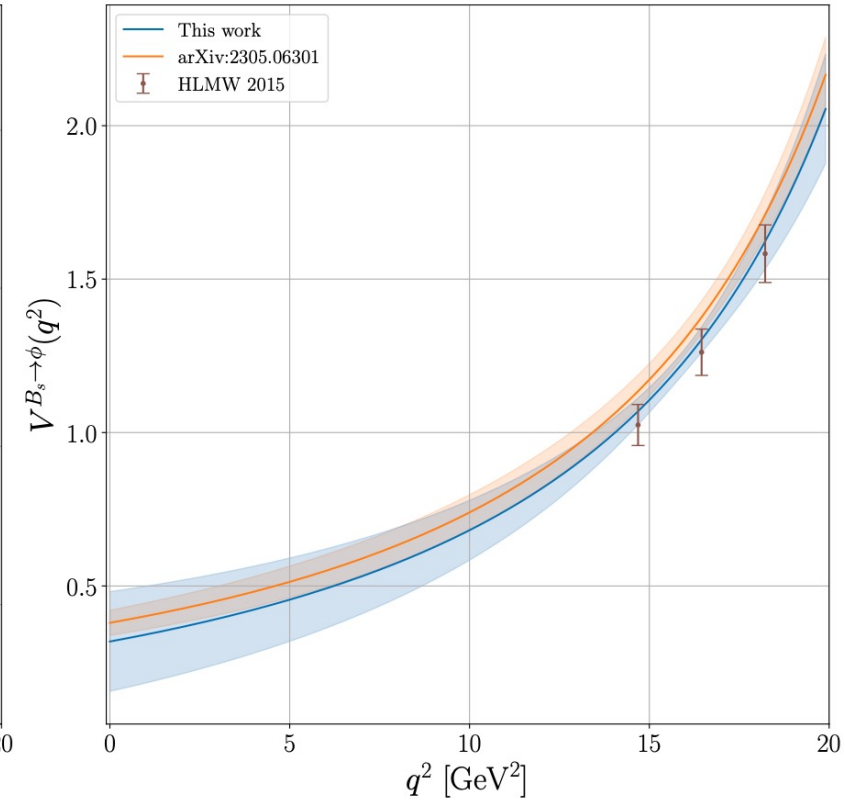
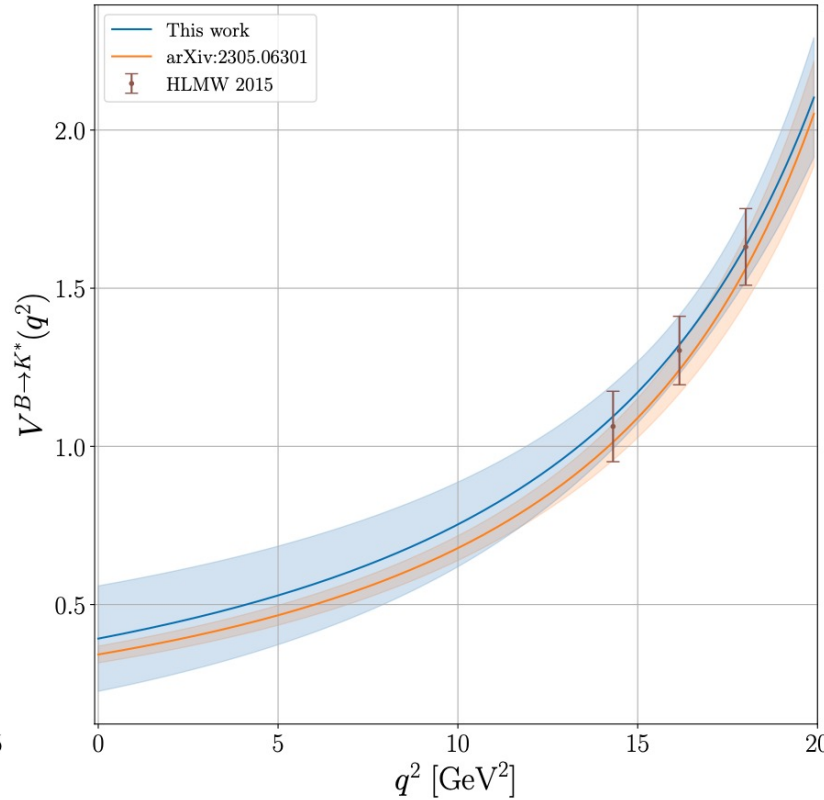
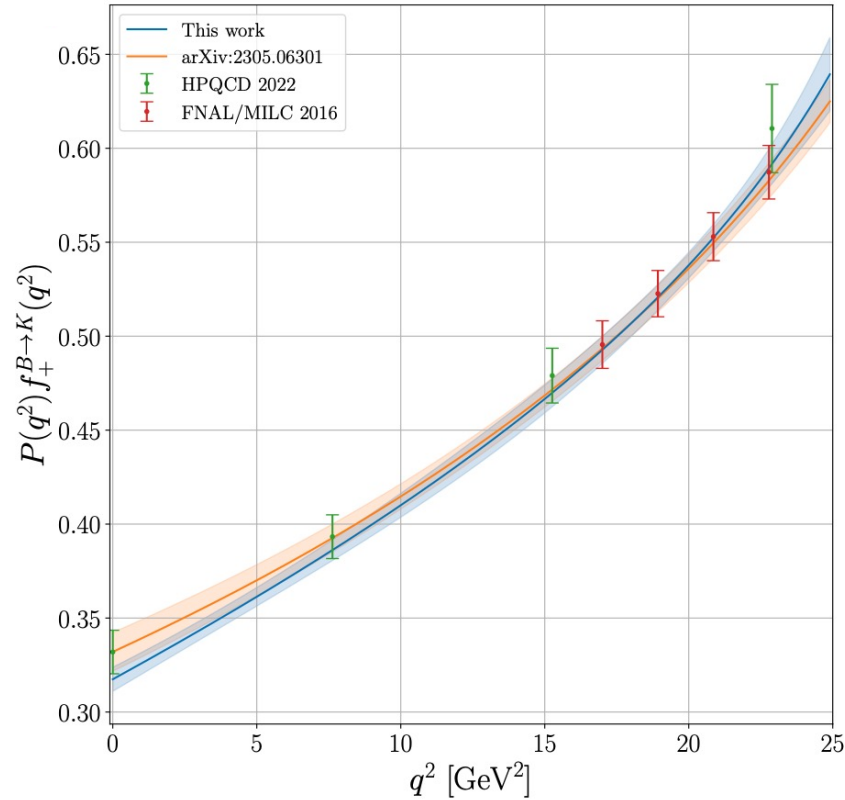
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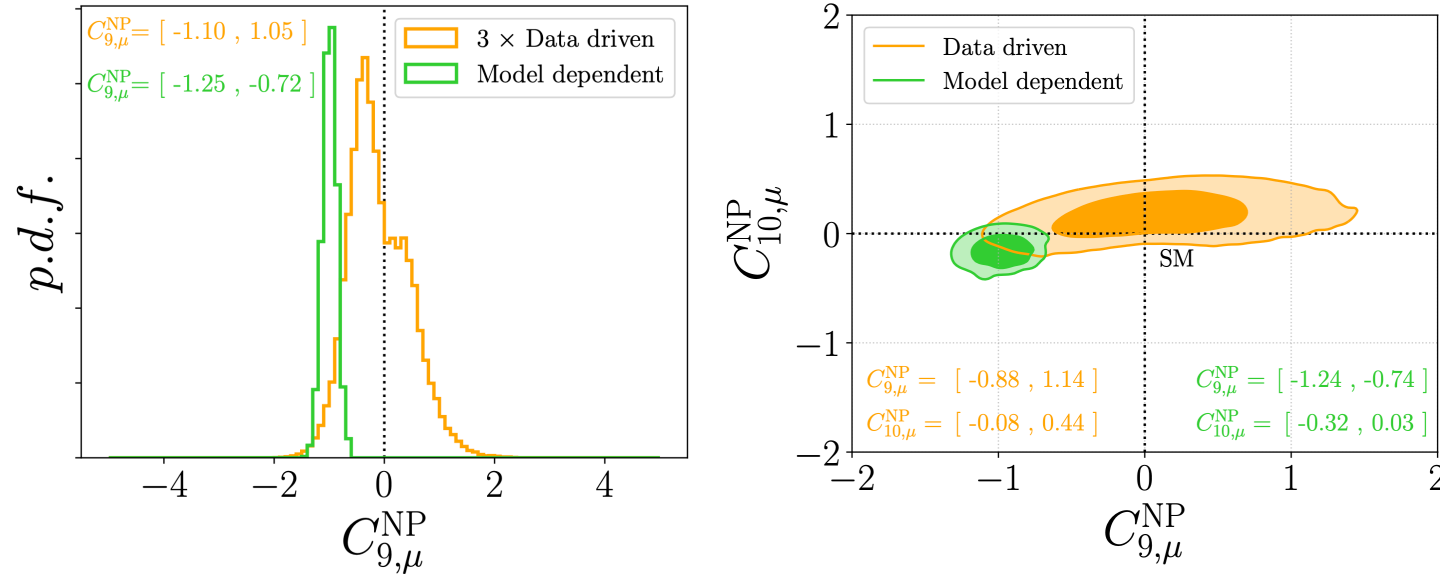
Nice consistency w/ the results of
Gubernari et al, JHEP '23 [2305.06301]

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Non-local hadronic form factors

Which is their contribution ? No general consensus on the answer to this question:

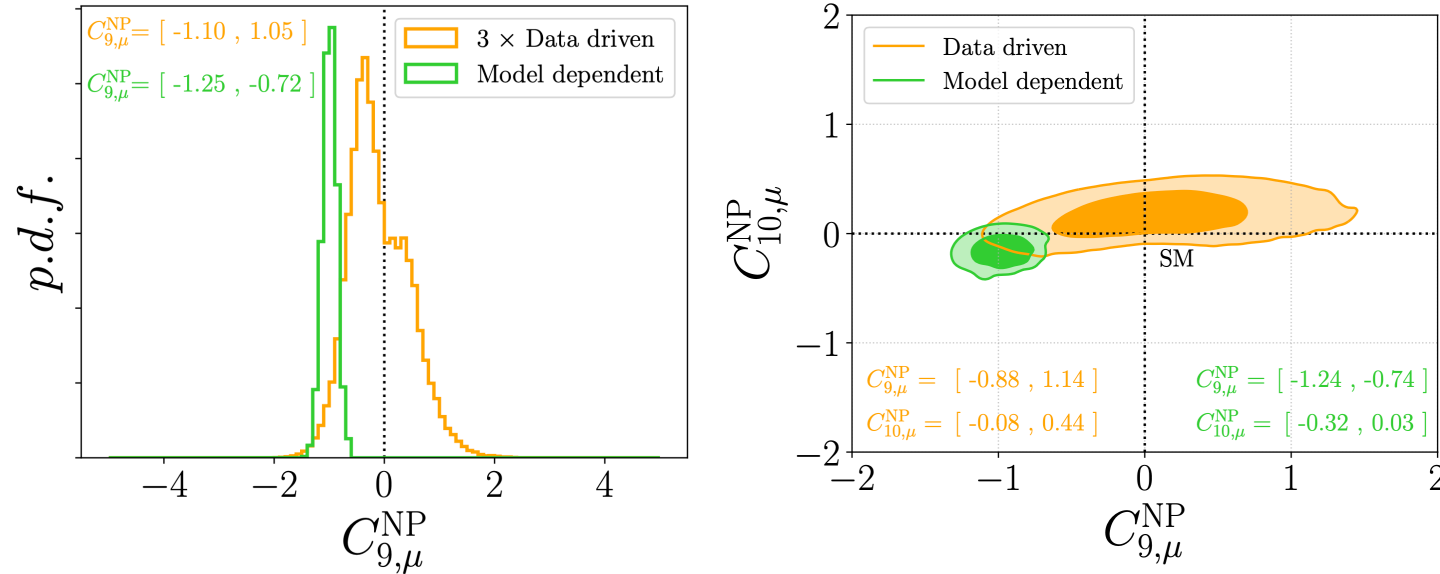
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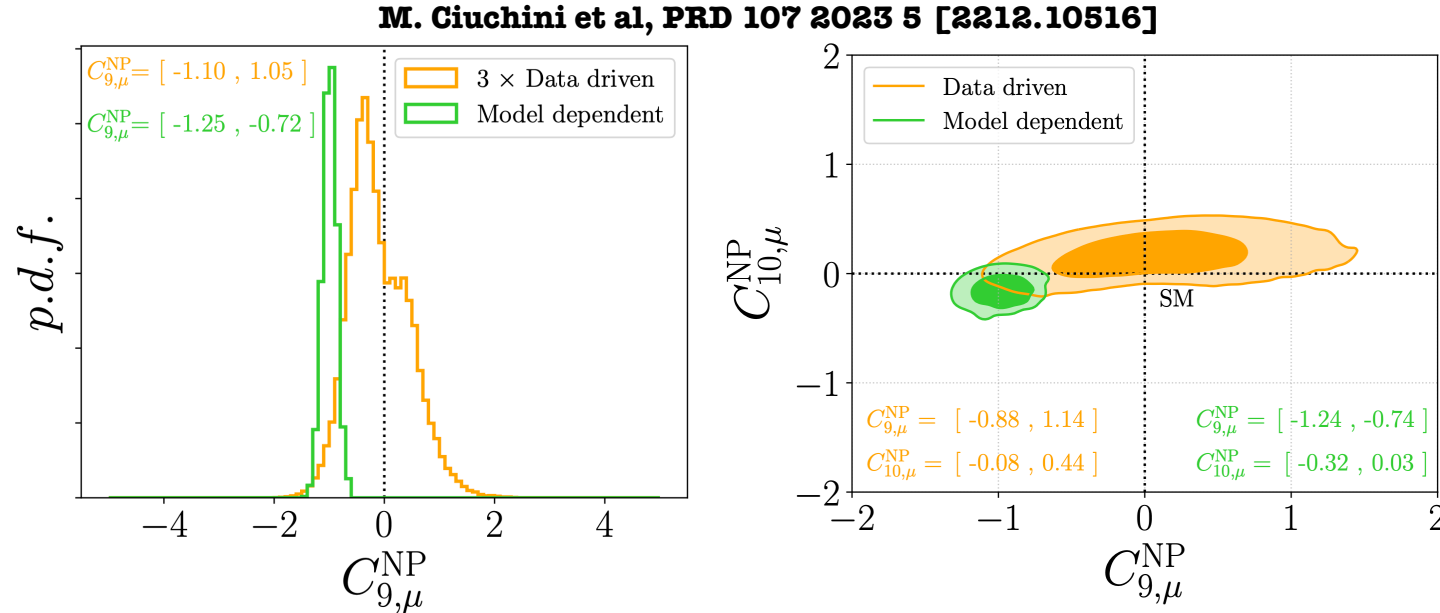
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Data-driven: naïve expansion in q^2

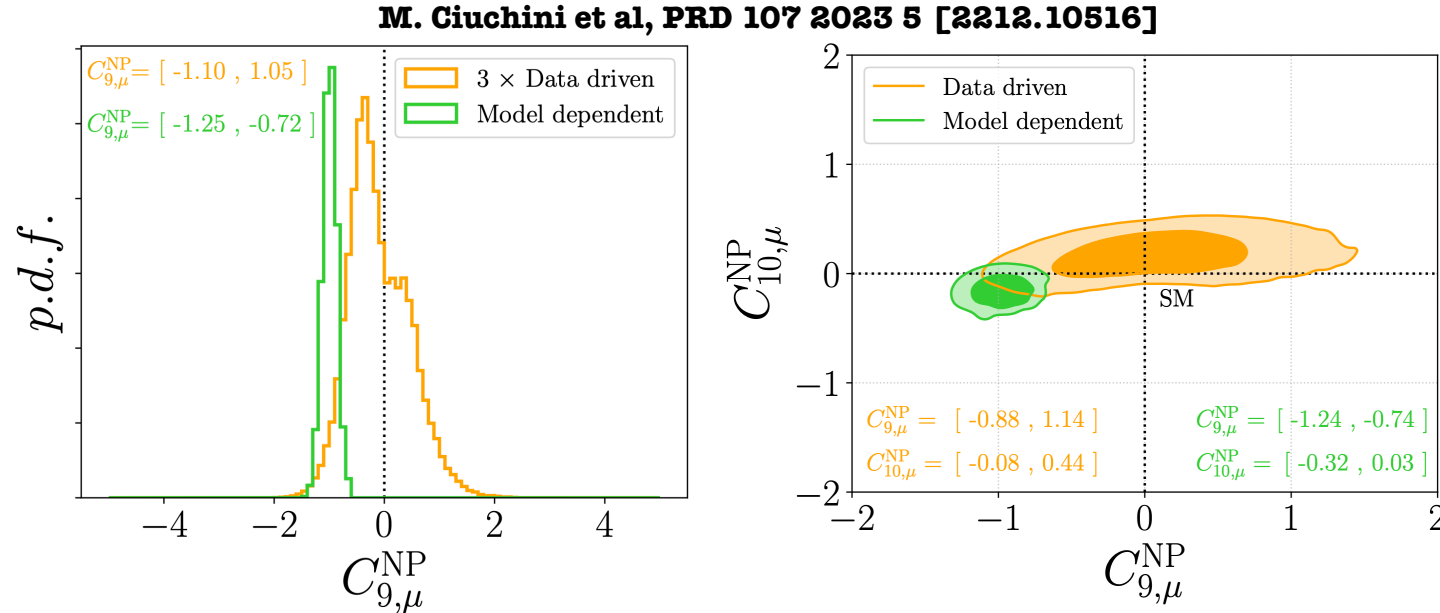
$$H_V^- \propto \frac{m_B^2}{q^2} \left[\frac{2m_b}{m_B} \left(C_7^{\text{SM}} + h_-^{(0)} \right) \tilde{T}_{L-} - 16\pi^2 h_-^{(2)} q^4 \right] + \left(C_9^{\text{SM}} + h_-^{(1)} \right) \tilde{V}_{L-},$$

Advantage: clear synergy between
hadronic contributions and New Physics!

M. Ciuchini et al, JHEP '16 [1512.07157], EPJC '17 [1704.05447],
EPJC '19 [1903.09632], PRD '21 [2011.01212],
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Model-dependent: the h -terms $[h^{(0)}, h^{(1)}, h^{(2)}]$ are considered *negligible*

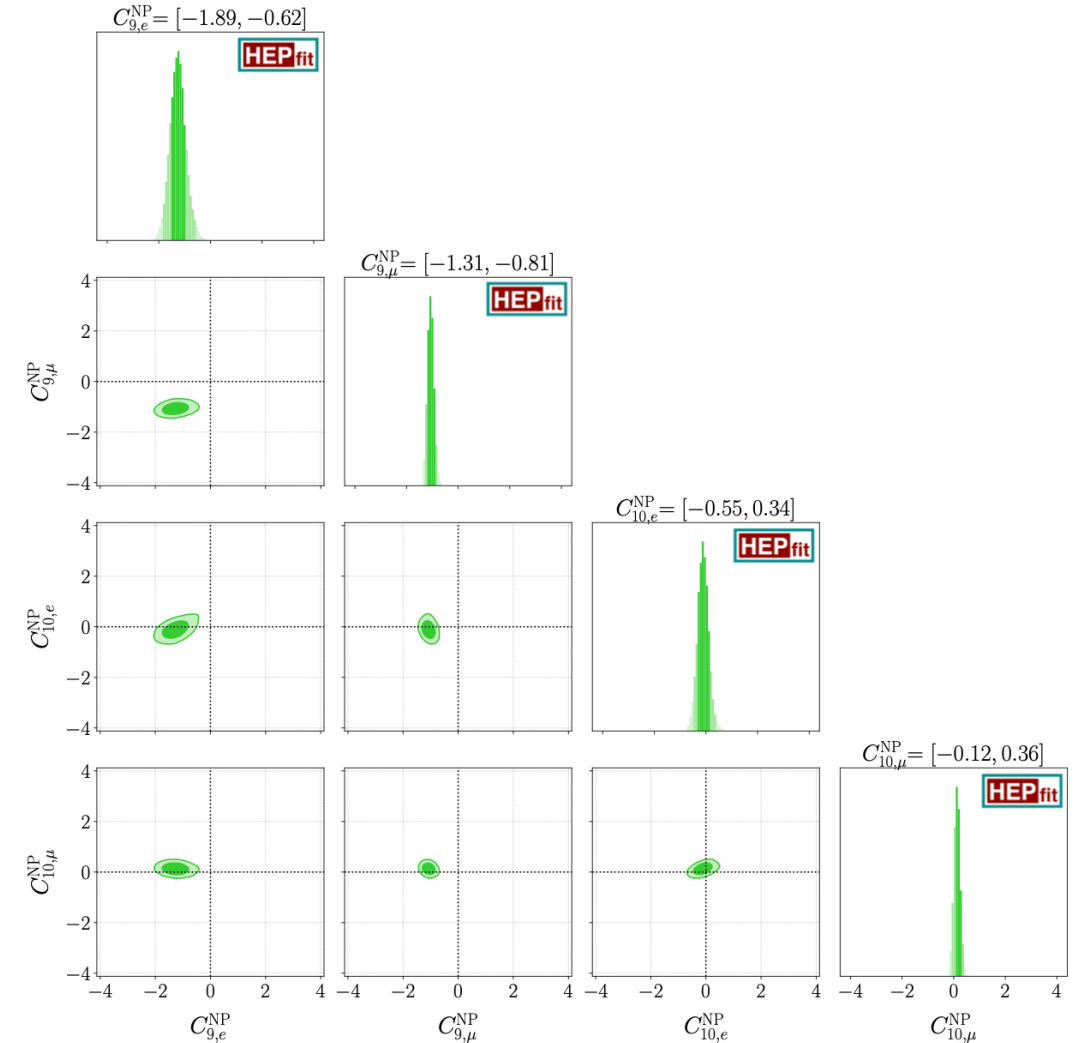
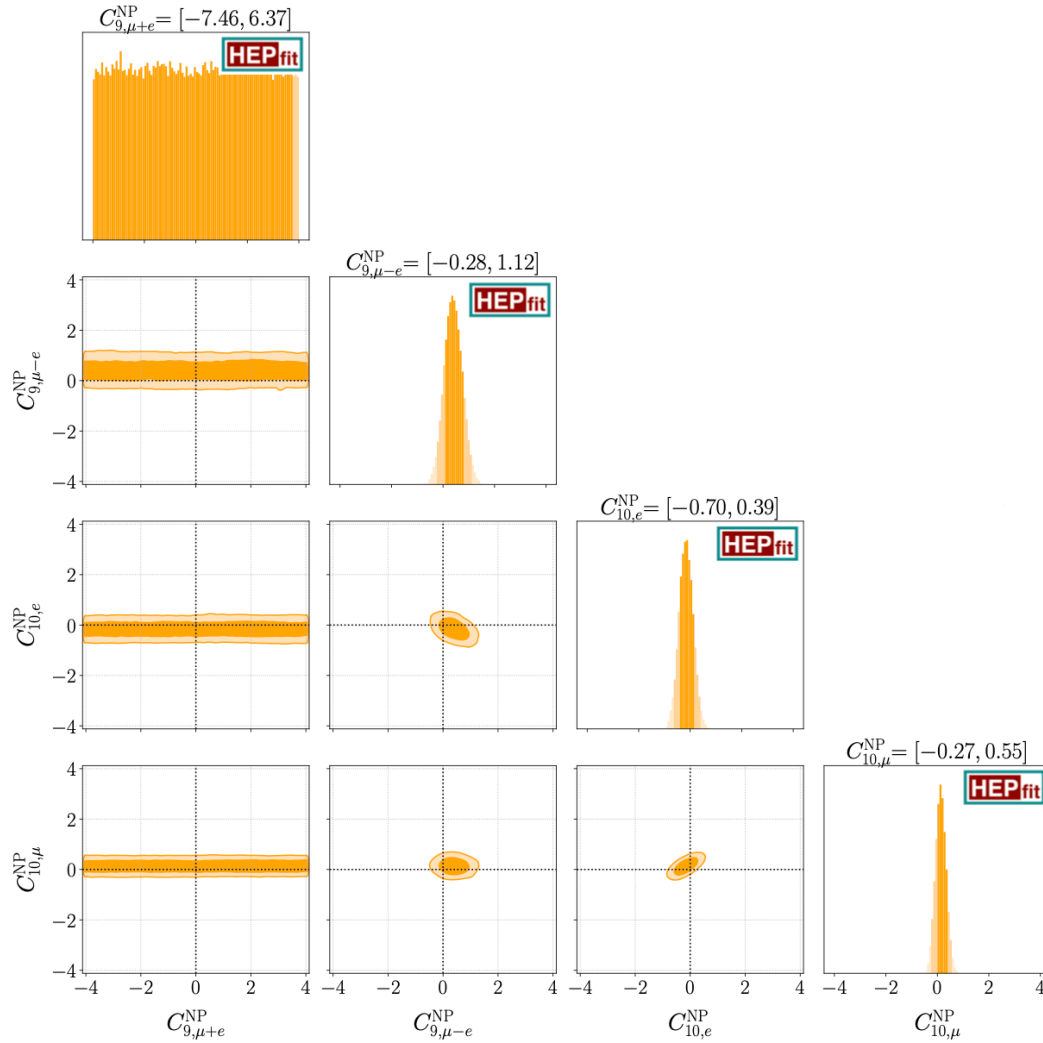
Rationale: this assumption is supported by the results of the application of dispersion relations, analyticity and unitarity (together with LCSR data used as inputs)!

C. Bobeth et al, EPJC '18 [1707.07305]
M. Chrzaszcz et al, JHEP '19 [1805.06378]
N. Gubernari et al, JHEP '21 [2011.09813], JHEP '22 [2206.03797], 2305.06301



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This conclusion is still true for the 2025 update of **Ciuchini et al., PRD 107 (2023) 5:**

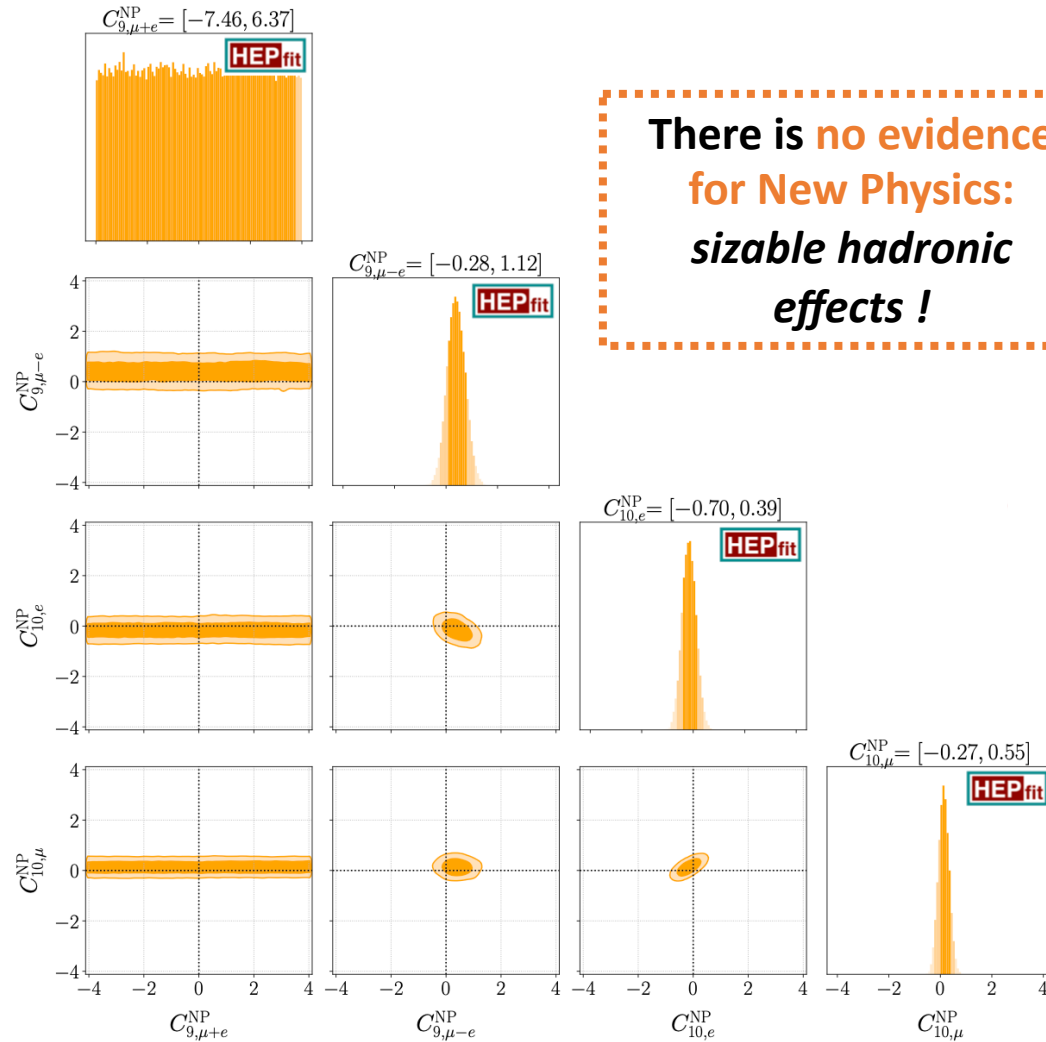


M. Valli's talk @ 21st Rencontres du Vietnam: Flavour Physics Conference 2025

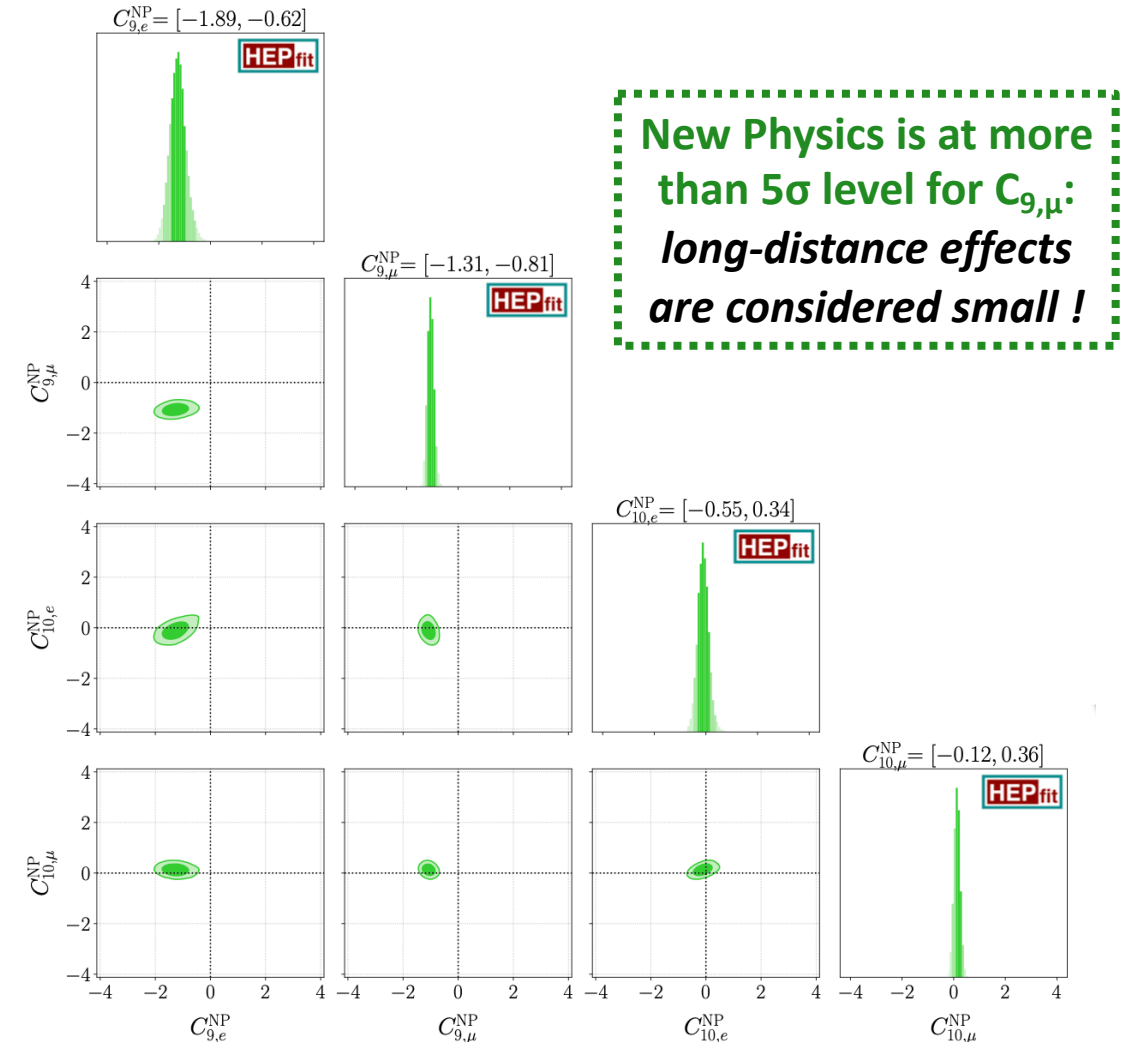


Non-local hadronic form factors

This conclusion is still true for the 2025 update of **Ciuchini et al., PRD 107 (2023) 5:**



There is **no evidence**
for New Physics:
sizable hadronic
effects !

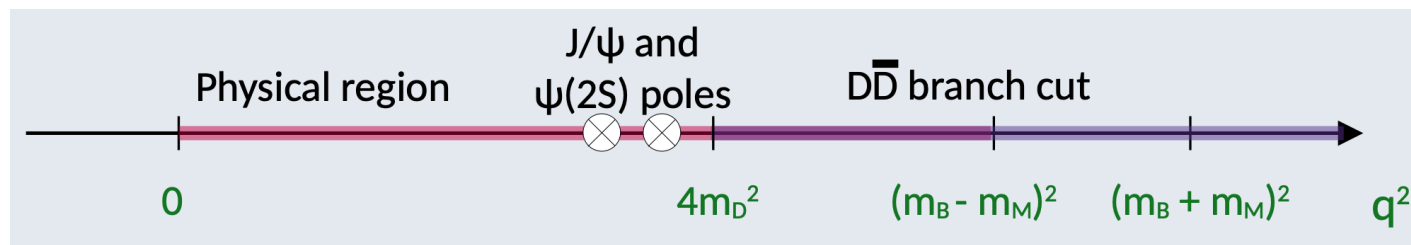


New Physics is at more
than 5σ level for $C_{9,\mu}$:
long-distance effects
are considered small !

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Possible directions for future work

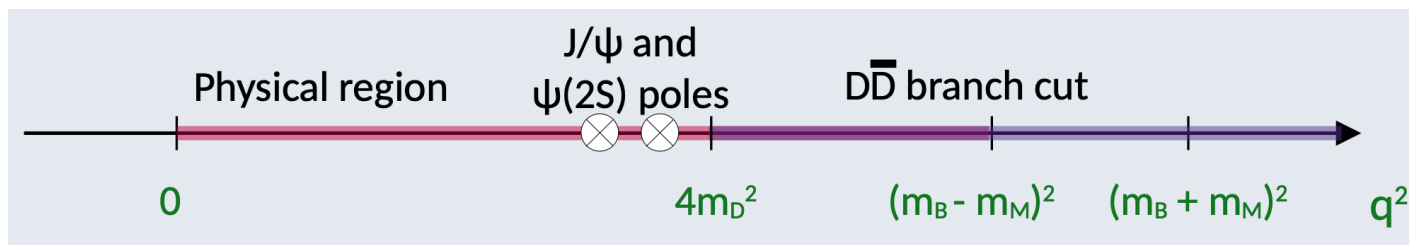
1. **Further investigation of sub-threshold effects for non-local FFs:** such FFs are indeed characterized by a branch-cut starting @ $q^2 = (2m_D)^2$



M. Reboud's talk @ Moriond EW 2025 Workshop

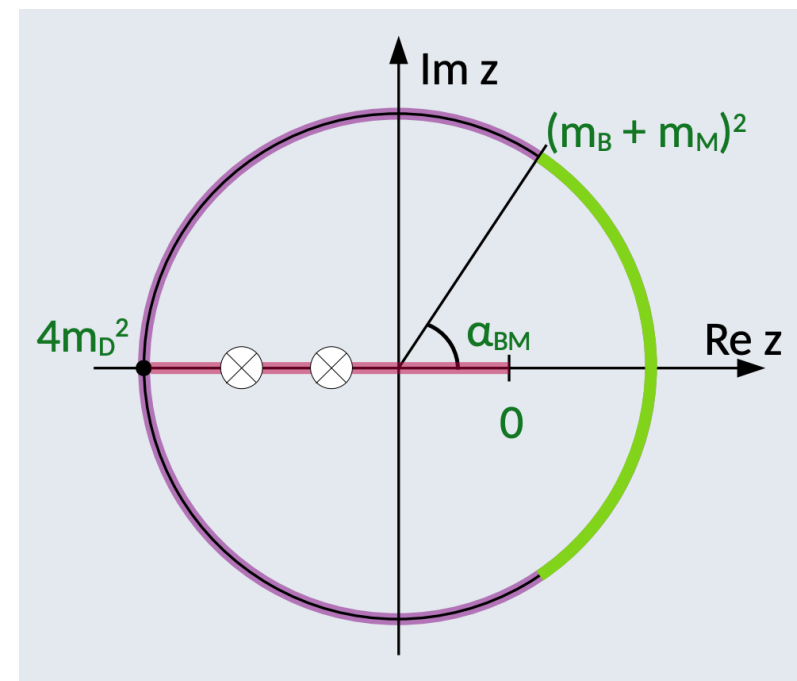
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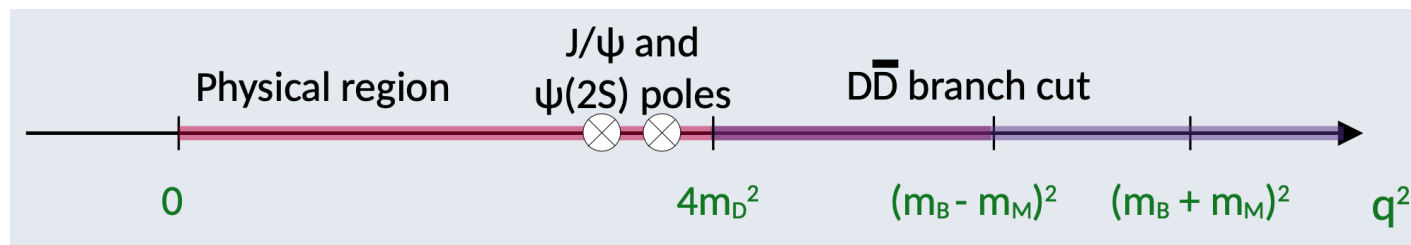
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through an appropriate
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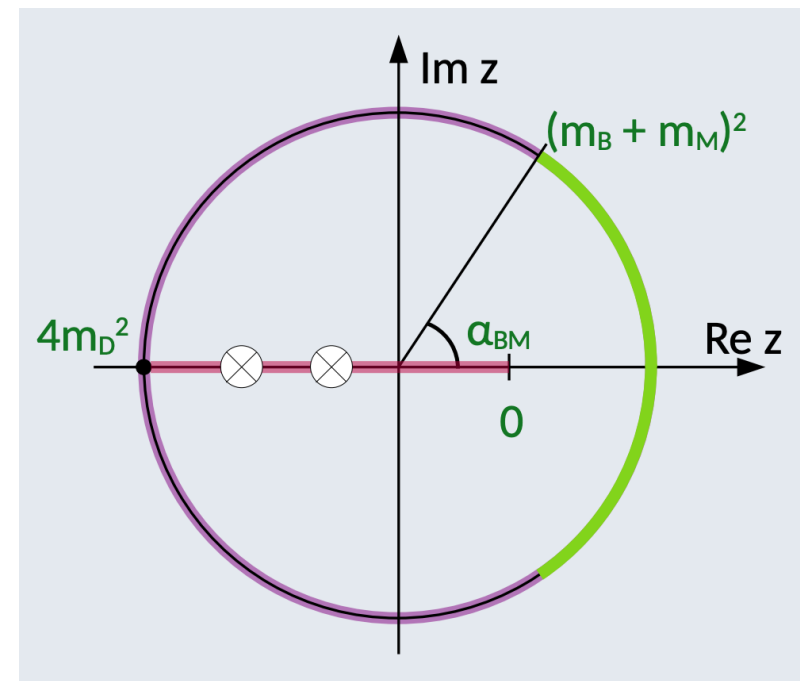
through an appropriate
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This conformal mapping allows to **expand the non-local FFs** on the green arc **using a set of orthonormal polynomials p_k** (see **Gubernari et al., 2011.09813, 2206.03797, 2305.06301**):

$$f(z) = \frac{\sqrt{\chi_+^U}}{\phi(z)B(z)} \sum_{k=0}^{\infty} c_k p_k(z; \alpha_{BM}), \quad \sum_{k=0}^{\infty} c_k^2 \leq 1$$

(Alternative expansion presented in **Flynn et al, PRD '23 [2303.11280]**)



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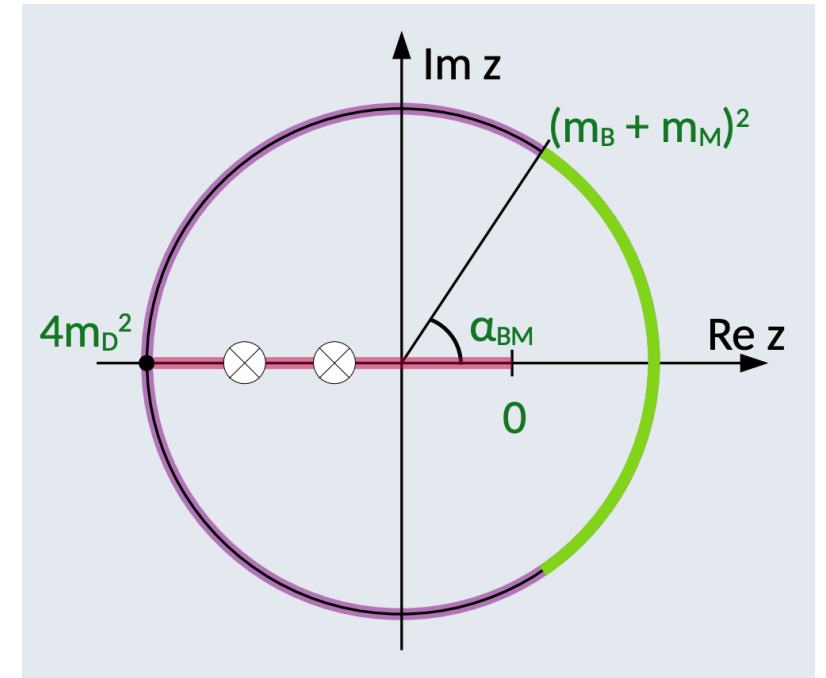
1. **Further investigation of sub-threshold effects for non-local FFs:** such FFs are indeed characterized by a branch-cut starting @ $q^2 = (2m_D)^2$

This parametrization has been also used in recent analyses by the **LHCb Collaboration**, see for instance the “Amplitude analysis of the $B \rightarrow K^* \mu^+ \mu^-$ decay” in ***PRL 132 (2024) 13 [2312.09115]***

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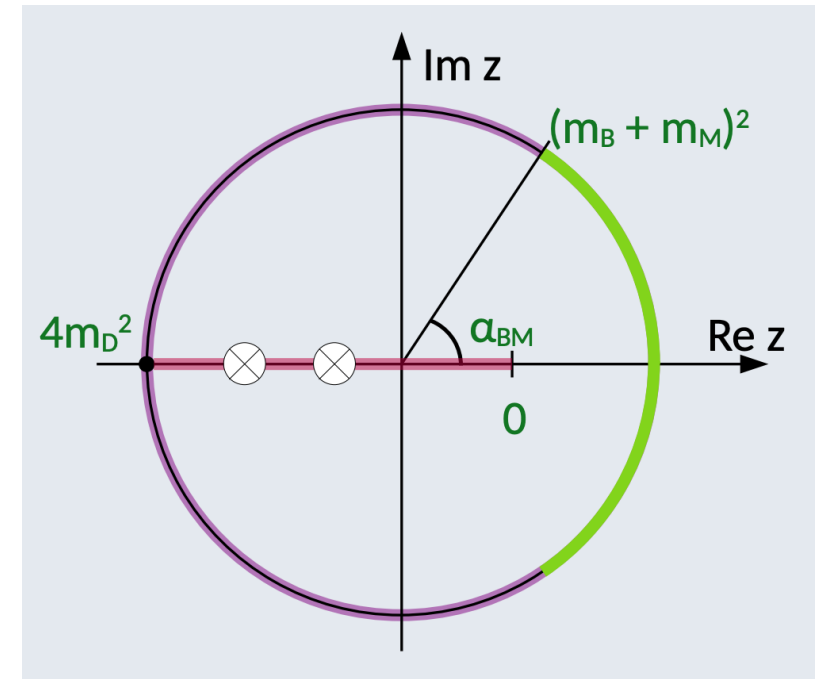
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HOWEVER: the non-local FFs have non-zero imaginary parts
not only **on the green arc**, but also **on the purple one!**



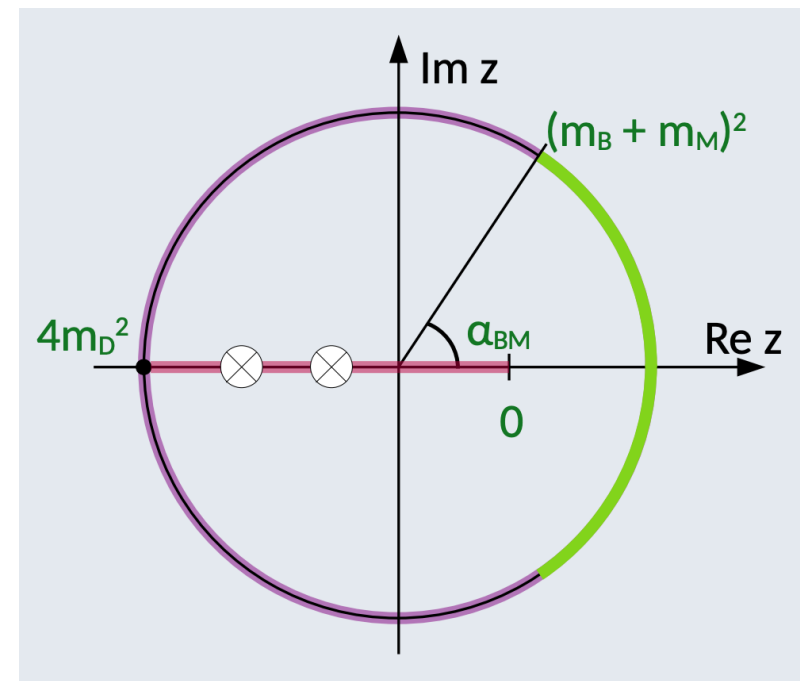
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not only **on the green arc**, but also **on the purple one!**

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$$\frac{1}{2\pi} \int_{-\alpha_{BM}}^{\alpha_{BM}} d\alpha \left| \phi(e^{i\alpha}) f(e^{i\alpha}) \right|^2 \leq \chi_+^U ,$$
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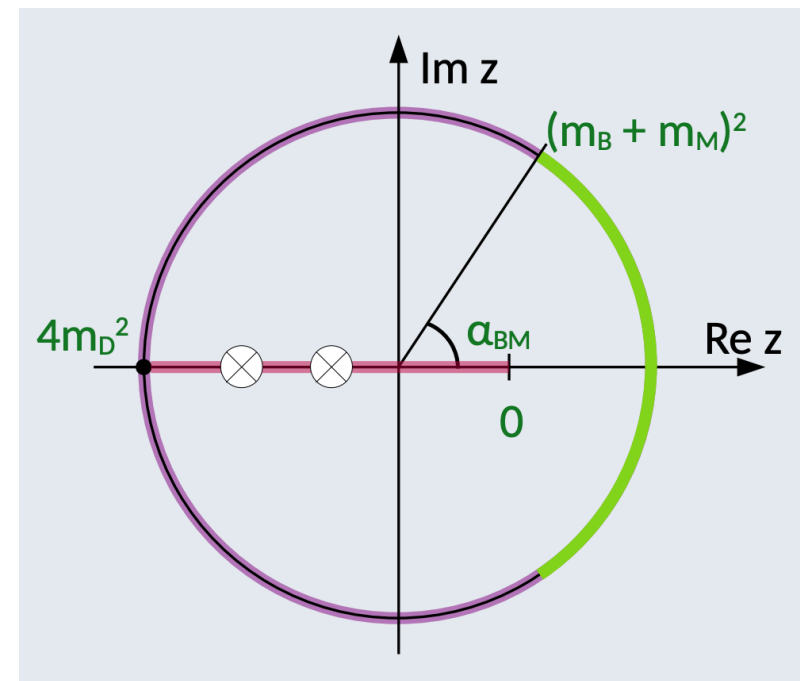
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Derivatives of suitable 2-point corr. functions

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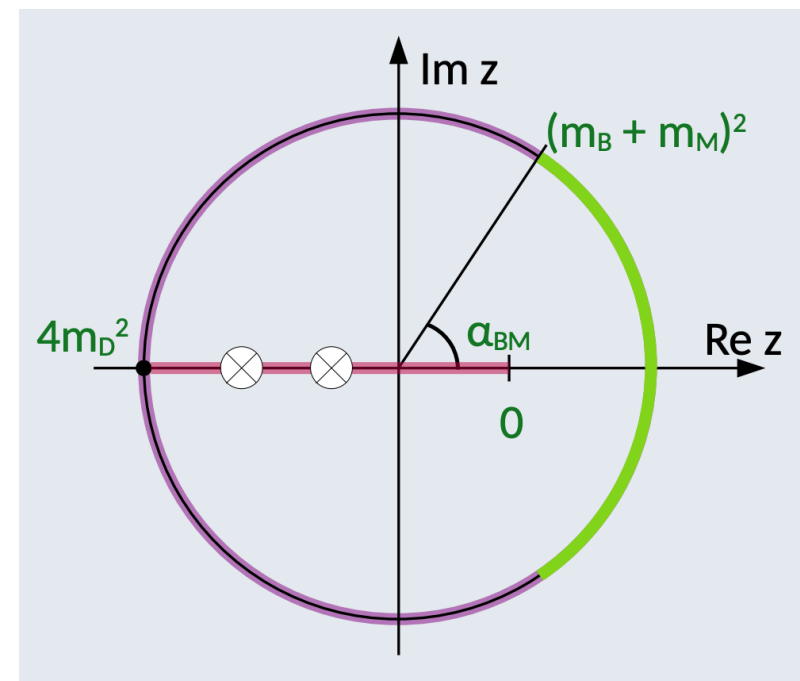
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Above-threshold poles



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This **double dispersive bound** may allow to corroborate previous findings concerning the application of unitarity and analyticity on non-local FFs !

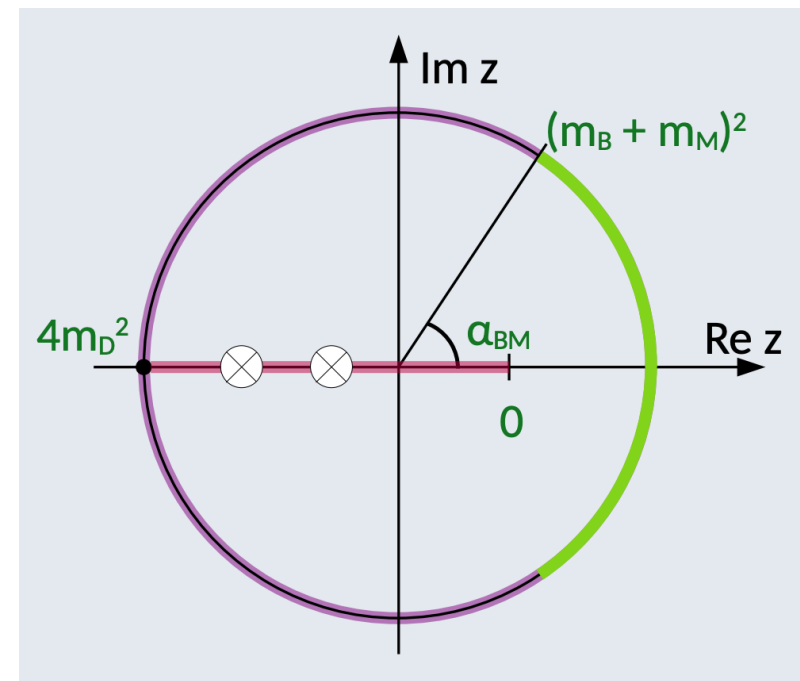
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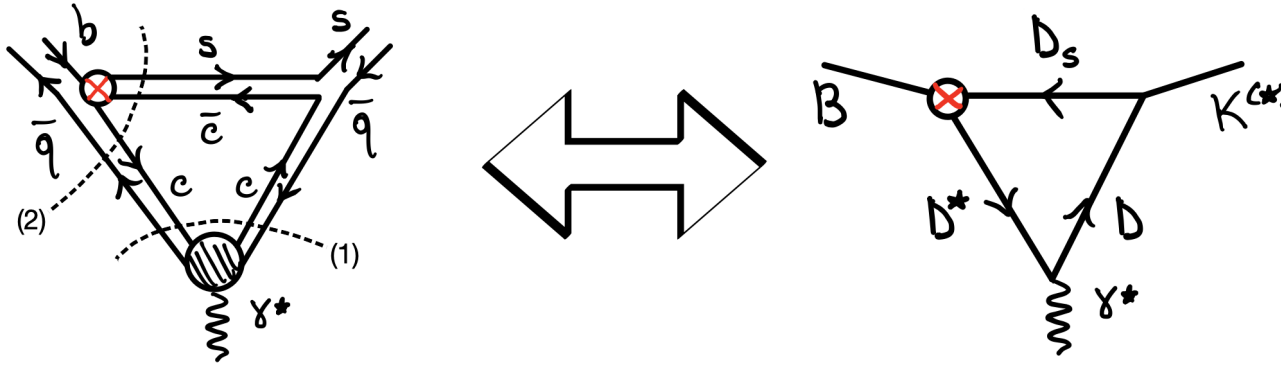
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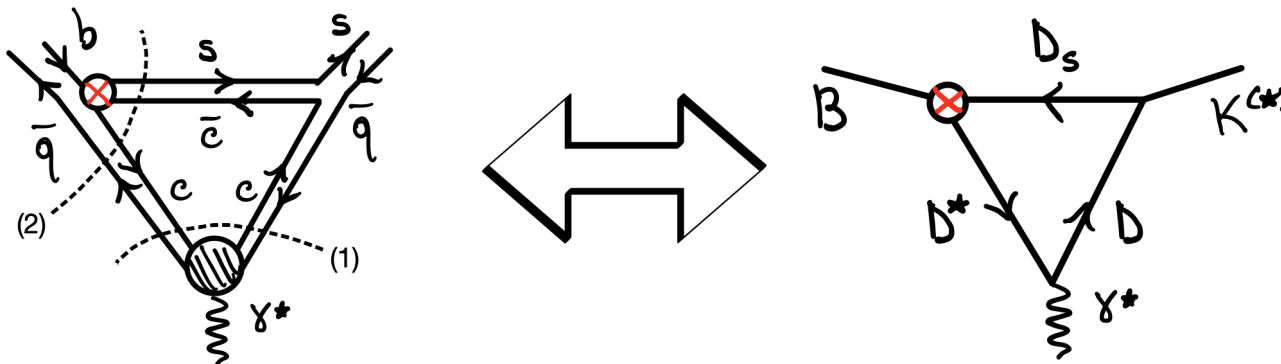
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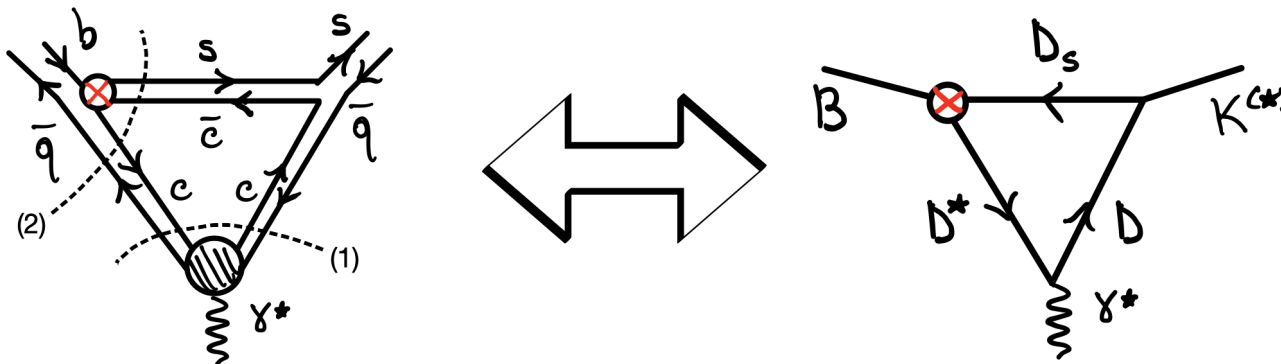


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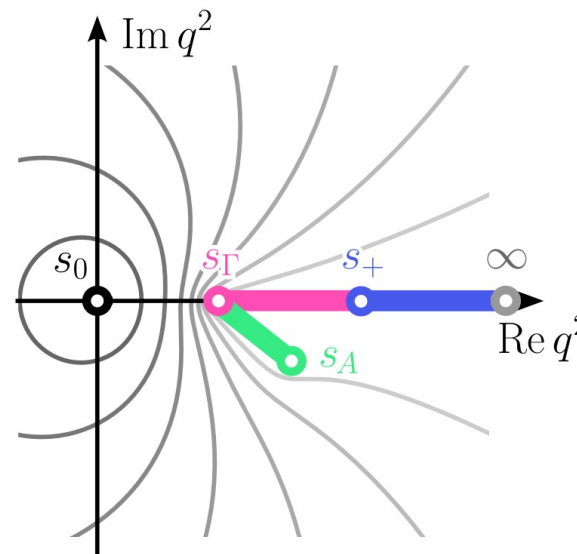
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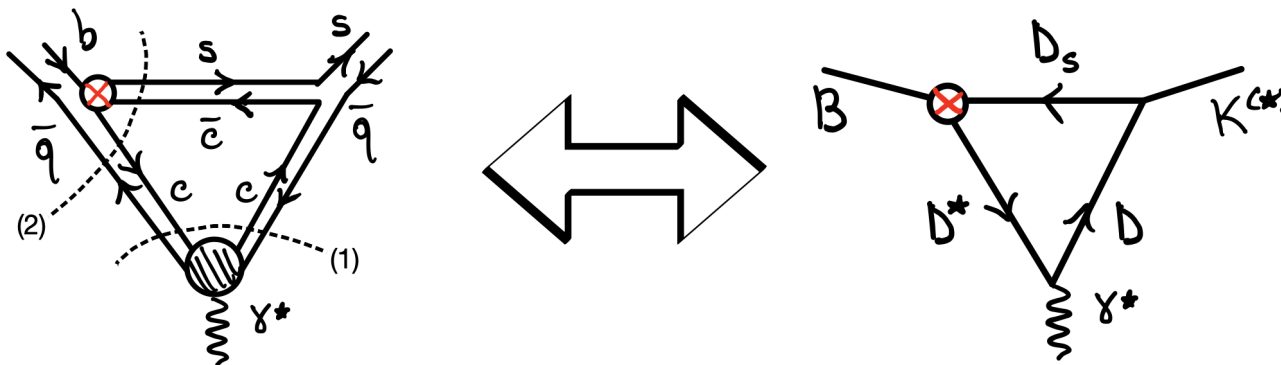
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taken from 2412.04388

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Pair-production threshold

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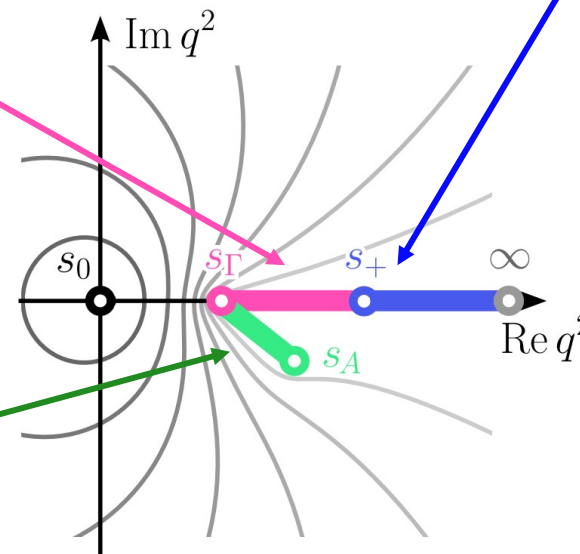
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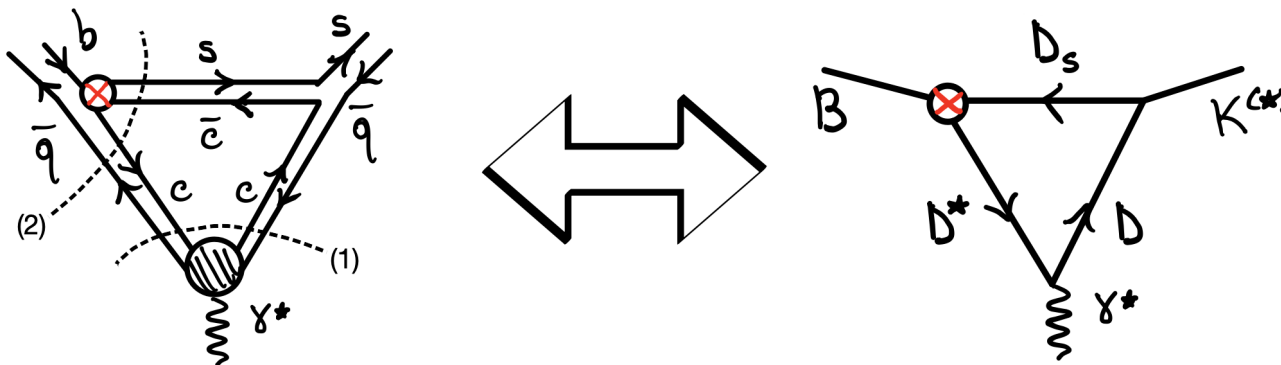
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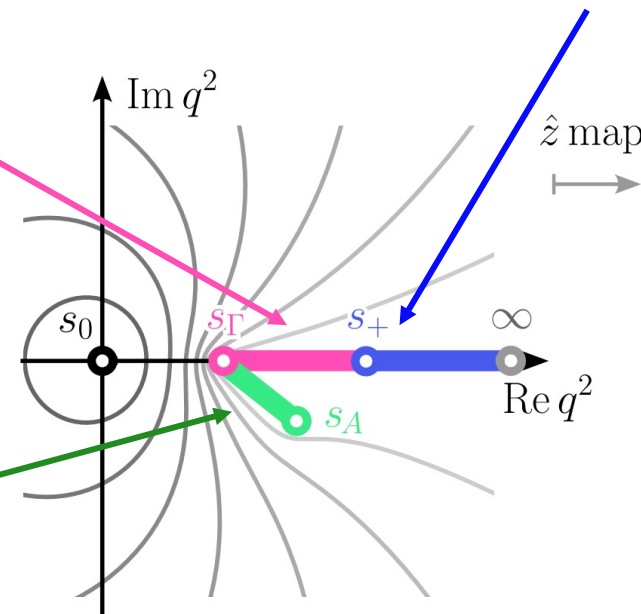
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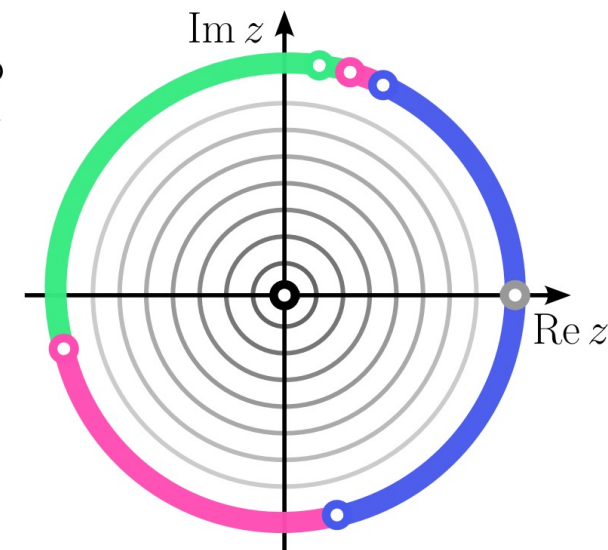
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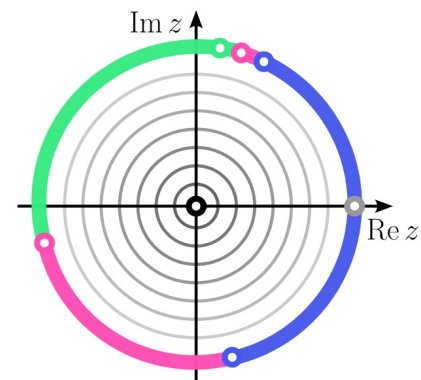


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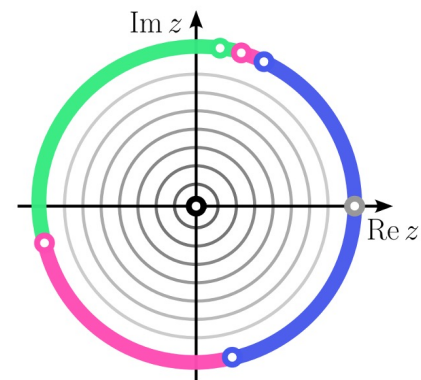
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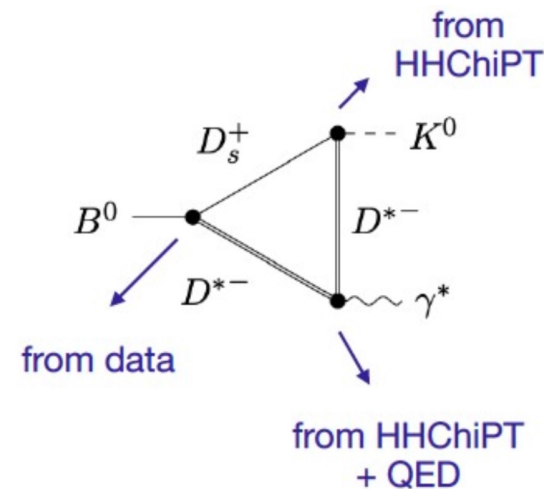
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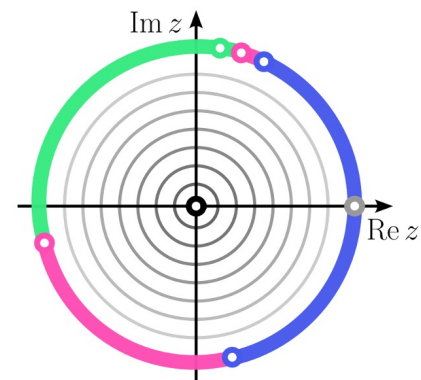
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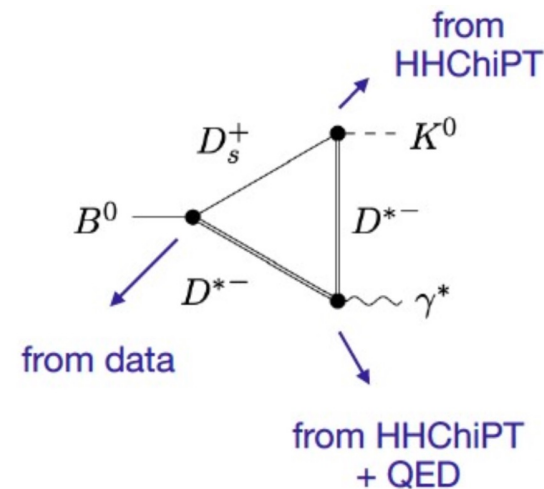
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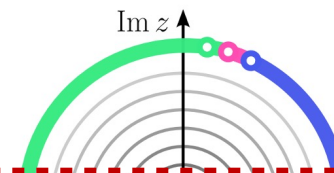
Take-home message:

a lot of work still to be done !! 😊

- Ph
com
[24]

This is, however, mandatory in order to
confirm or not the presence of New
Physics in semileptonic rare $B_{(s)}$ decays ...

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Plan of the talk

a) Semileptonic neutral-current $B_{(s)}$ -meson decays

b) Rare radiative-and-leptonic B_s -meson decay

c) Rare semileptonic decays with di-neutrino final states

Radiative-and-leptonic Bs decays

A novel possibility to analyze **$b \rightarrow s$ quark transitions** is the study of rare **radiative-and-leptonic Bs decays**. This is experimentally challenging, and the **first world limit** on this decay was **set by LHCb** (very close to the SM signal):

$$\mathcal{B}(B_s^0 \rightarrow \mu^+ \mu^- \gamma) < 2.0 \times 10^{-9} \quad \left\{ m_{\mu\mu} > 4.9 \text{ GeV}/c^2 \right\}$$

LHCb Collaboration, LHCb-PAPER-2021-007 & LHCb-PAPER-2021-008

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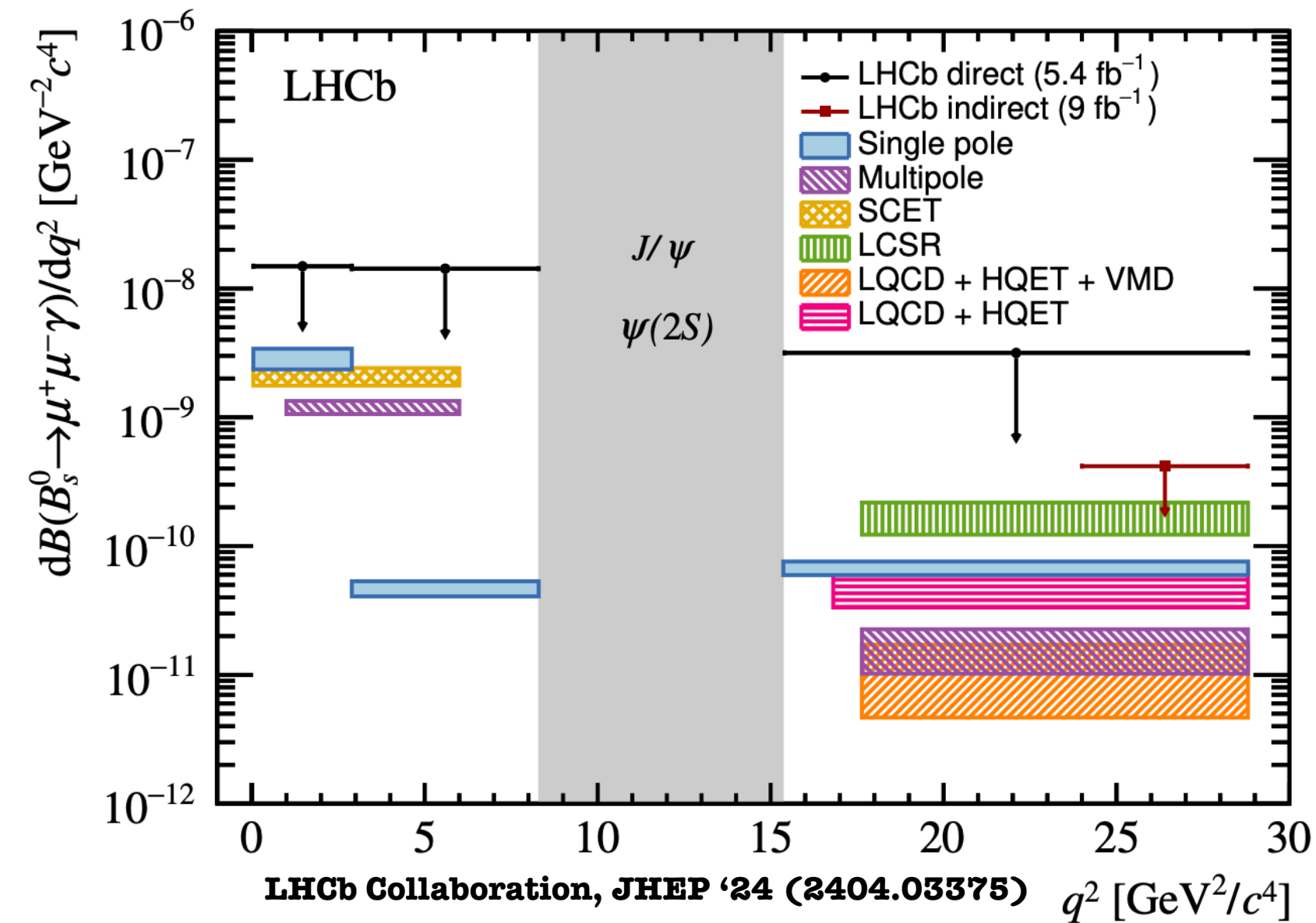
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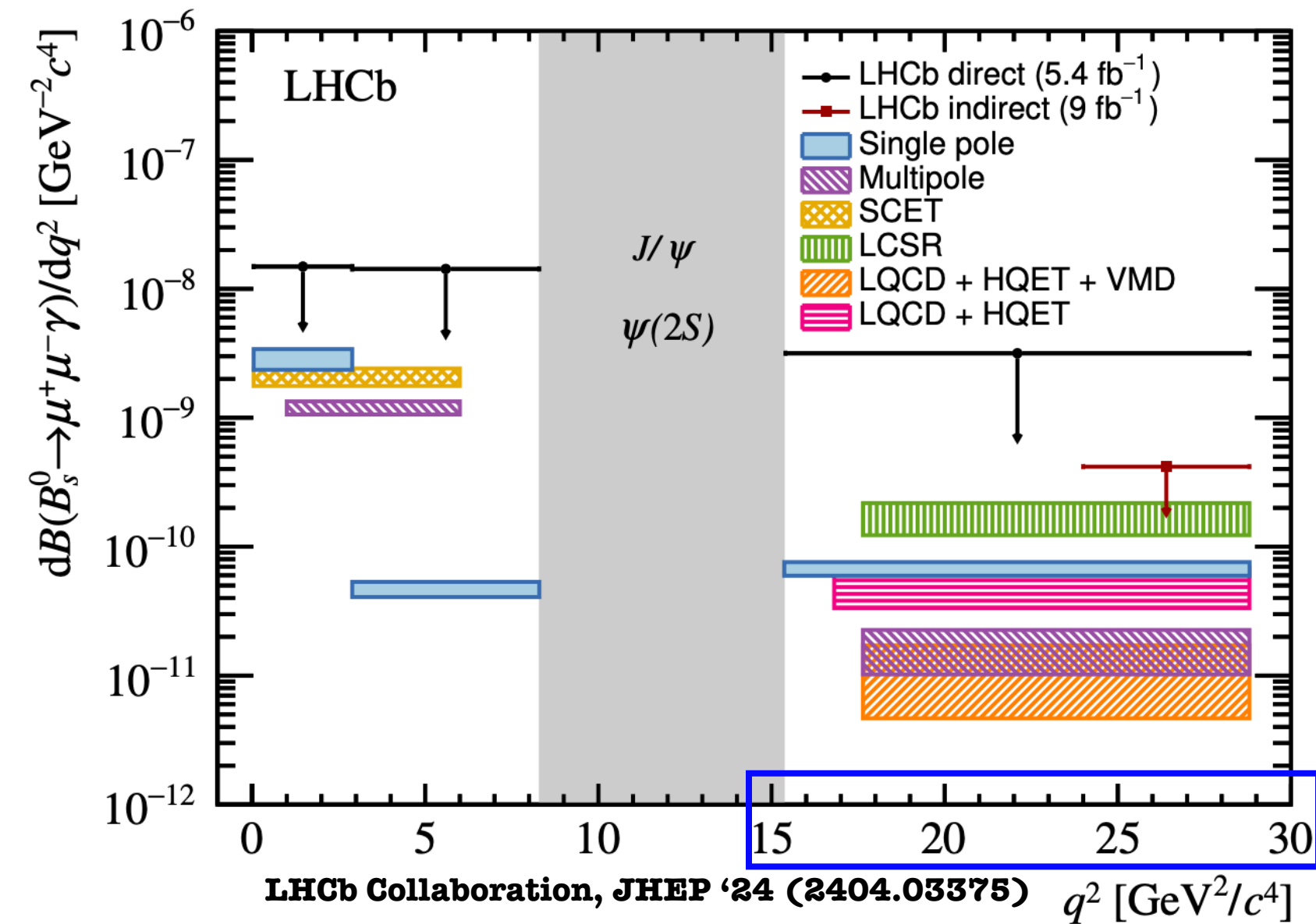
Several advantages from the phenomenological point of view:

- 1. No chirality suppression** (thanks to the additional photon): **enhancement w.r.t. the leptonic counterpart!**
- 2. Sensitivity to a larger set of WCs:** not only $O_{10}(')$, also $O_7(')$ and $O_9(')$
(reminder: $O_9(')$ and $O_{10}(')$ are particularly relevant @ high- q^2)
- 3. Two ways to detect it experimentally:**
 - directly (i.e. w/ photon reconstruction)** [LHCb Coll., JHEP '24 (2404.03375)]
 - indirectly (i.e. w/out photon reconstruction)** [Dettori et al, PLB '17 (1610.00629)]

Summary of all (th. and exp.) results within the SM

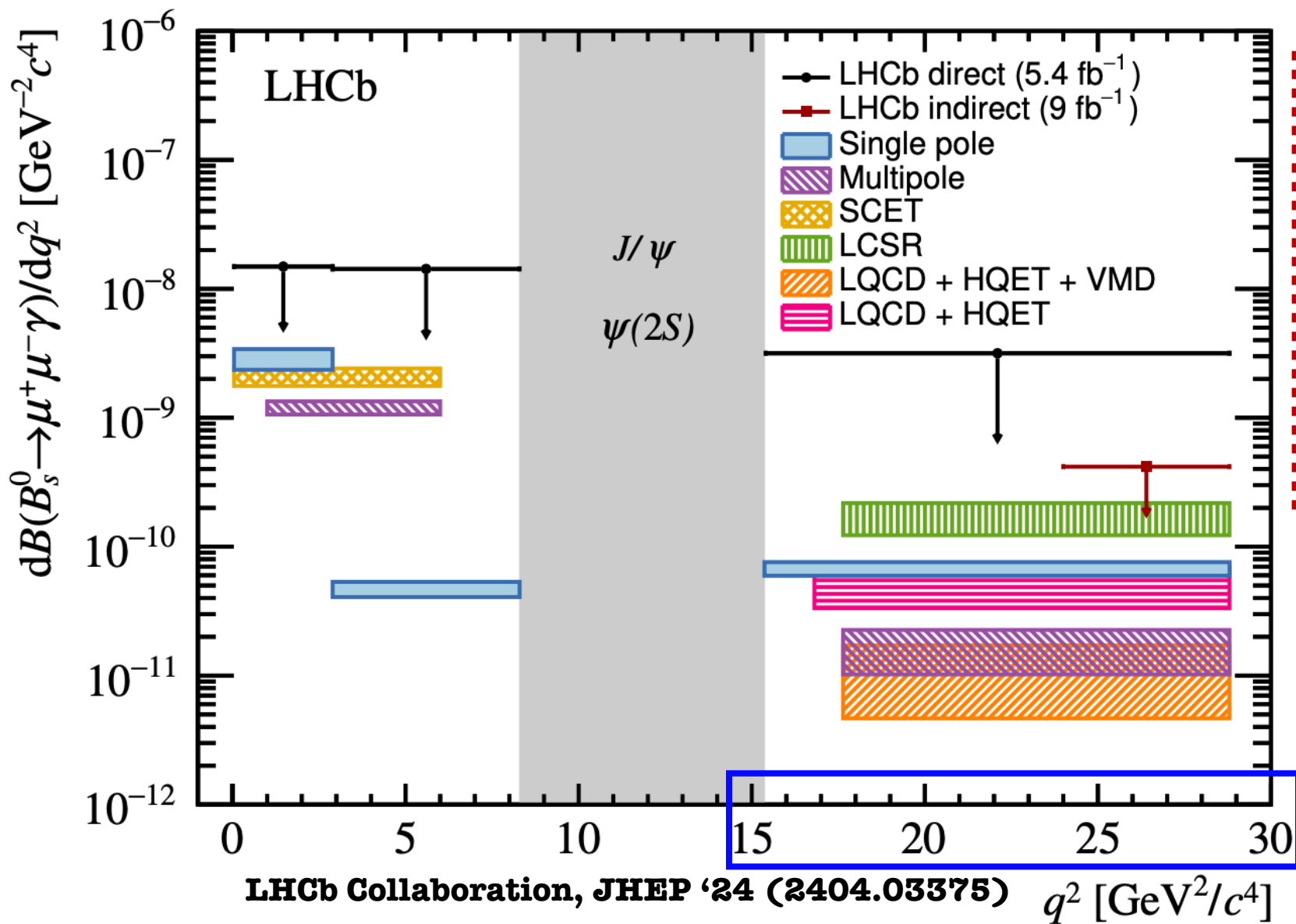


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*As we will see in the next slide, the **high- q^2** region can be particularly interesting for phenomenology*

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In the next years, it is well reasonable to expect that a measurement will be finally available !

As we will see in the next slide, the high- q^2 region can be particularly interesting for phenomenology

$B_s \rightarrow \mu\mu\gamma$ at high- q^2 in connection with semilept. B decays

KEY IDEA: $\text{BR}(B_s \rightarrow \mu\mu\gamma)$ @ high- q^2 is sensitive to the very same short-distance physics present in $B \rightarrow K(^*)$ decays, without being affected by the same long-distance effects !

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Hence: if there is really a NP contribution to C_9 , this effect should influence as well the $\text{BR}(B_s \rightarrow \mu\mu\gamma)$ @ high- q^2

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Hence: if there is really a NP contribution to C_9 , this effect should influence as well the $\text{BR}(B_s \rightarrow \mu\mu\gamma)$ @ high- q^2

However, we do not have a direct measurement of $\text{BR}(B_s \rightarrow \mu\mu\gamma)$ @ high- q^2 at present ...

Thus, the best that we can do at present is a sensitivity study !

$B_s \rightarrow \mu\mu\gamma$ at high- q^2 in connection with semilept. B decays

$$\boxed{\frac{d^2\Gamma^{(1)}}{d\hat{s} d\hat{t}}} = \frac{G_F^2 \alpha_{em}^3 M_1^5}{2^{10} \pi^4} |V_{tb} V_{tq}^*|^2 \left[x^2 B_0(\hat{s}, \hat{t}) + x \xi(\hat{s}, \hat{t}) \tilde{B}_1(\hat{s}, \hat{t}) + \xi^2(\hat{s}, \hat{t}) \tilde{B}_2(\hat{s}, \hat{t}) \right], \quad (6.1)$$

Emission of the
photon from
valence quarks or
FCNC vertex
(DE component)

$$B_0(\hat{s}, \hat{t}) = (\hat{s} + 4\hat{m}_l^2) (F_1(\hat{s}) + F_2(\hat{s})) - 8\hat{m}_l^2 |C_{10A}(\mu)|^2 (F_V^2(q^2) + F_A^2(q^2)),$$

$$\begin{aligned} \tilde{B}_1(\hat{s}, \hat{t}) = & 8 \left[\hat{s} F_V(q^2) F_A(q^2) \text{Re} \left(C_{9V}^{eff*}(\mu, q^2) C_{10A}(\mu) \right) \right. \\ & \left. + \hat{m}_b F_V(q^2) \text{Re} \left(C_{7\gamma}^*(\mu) \bar{F}_{TA}^*(q^2) C_{10A}(\mu) \right) + \hat{m}_b F_A(q^2) \text{Re} \left(C_{7\gamma}^*(\mu) \bar{F}_{TV}^*(q^2) C_{10A}(\mu) \right) \right], \end{aligned}$$

$$\tilde{B}_2(\hat{s}, \hat{t}) = \hat{s} (F_1(\hat{s}) + F_2(\hat{s})),$$

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$$\begin{aligned} F_2(\hat{s}) = & \left(|C_{9V}^{eff}(q^2, \mu)|^2 + |C_{10A}(\mu)|^2 \right) F_A^2(q^2) + \left(\frac{2\hat{m}_b}{\hat{s}} \right)^2 |C_{7\gamma}(\mu) \bar{F}_{TA}(q^2)|^2 \\ & + \frac{4\hat{m}_b}{\hat{s}} F_A(q^2) \text{Re} \left(C_{7\gamma}(\mu) \bar{F}_{TA}(q^2) C_{9V}^{eff*}(\mu, q^2) \right). \end{aligned}$$

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Bremsstrahlung

$$\boxed{\frac{d^2\Gamma^{(12)}}{d\hat{s} d\hat{t}}} = -\frac{G_F^2 \alpha_{em}^3 M_1^5}{2^{10} \pi^4} |V_{tb} V_{tq}^*|^2 \frac{16f_{B_q}}{M_B} \hat{m}_l^2 \frac{x^2}{(\hat{u} - \hat{m}_l^2)(\hat{t} - \hat{m}_{l34}^2)} \quad (6.3)$$

Interference

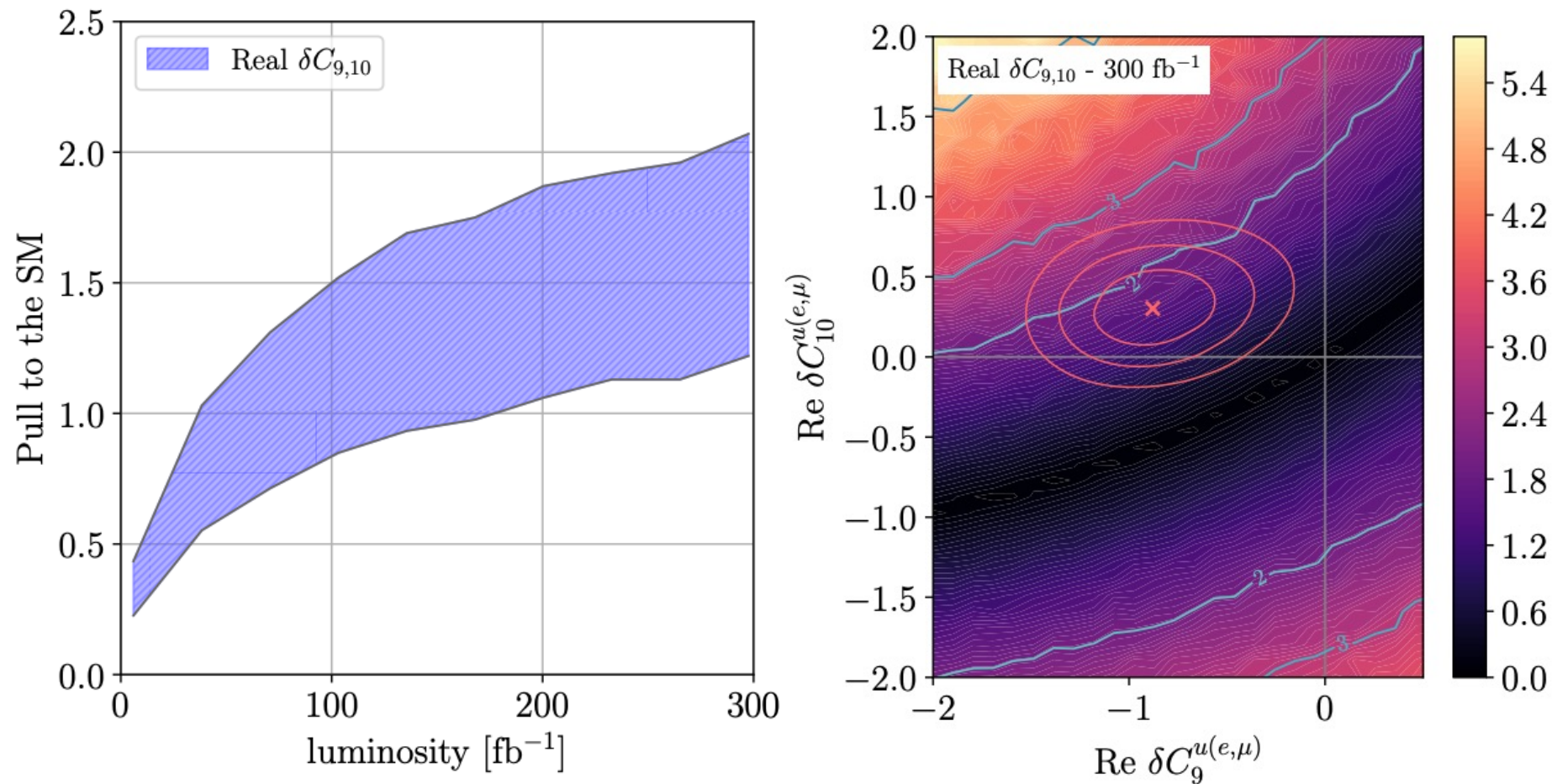
$$\times \left[\frac{2x\hat{m}_b}{\hat{s}} \text{Re} \left(C_{10A}^*(\mu) C_{7\gamma}(\mu) \bar{F}_{TV}(q^2, 0) \right) + x F_V(q^2) \text{Re} \left(C_{10A}^*(\mu) C_{9V}^{eff}(\mu, q^2) \right) + \xi(\hat{s}, \hat{t}) F_A(q^2) |C_{10A}(\mu)|^2 \right].$$

Melikhov and Nikitin, Phys. Rev. D 70 (2004) 114028

Kozachuk, Melikhov and Nikitin, Phys. Rev. D 97 (2018) 053007

Guadagnoli, Melikhov and Reboud, Phys.Lett.B 760 (2016) 442-447

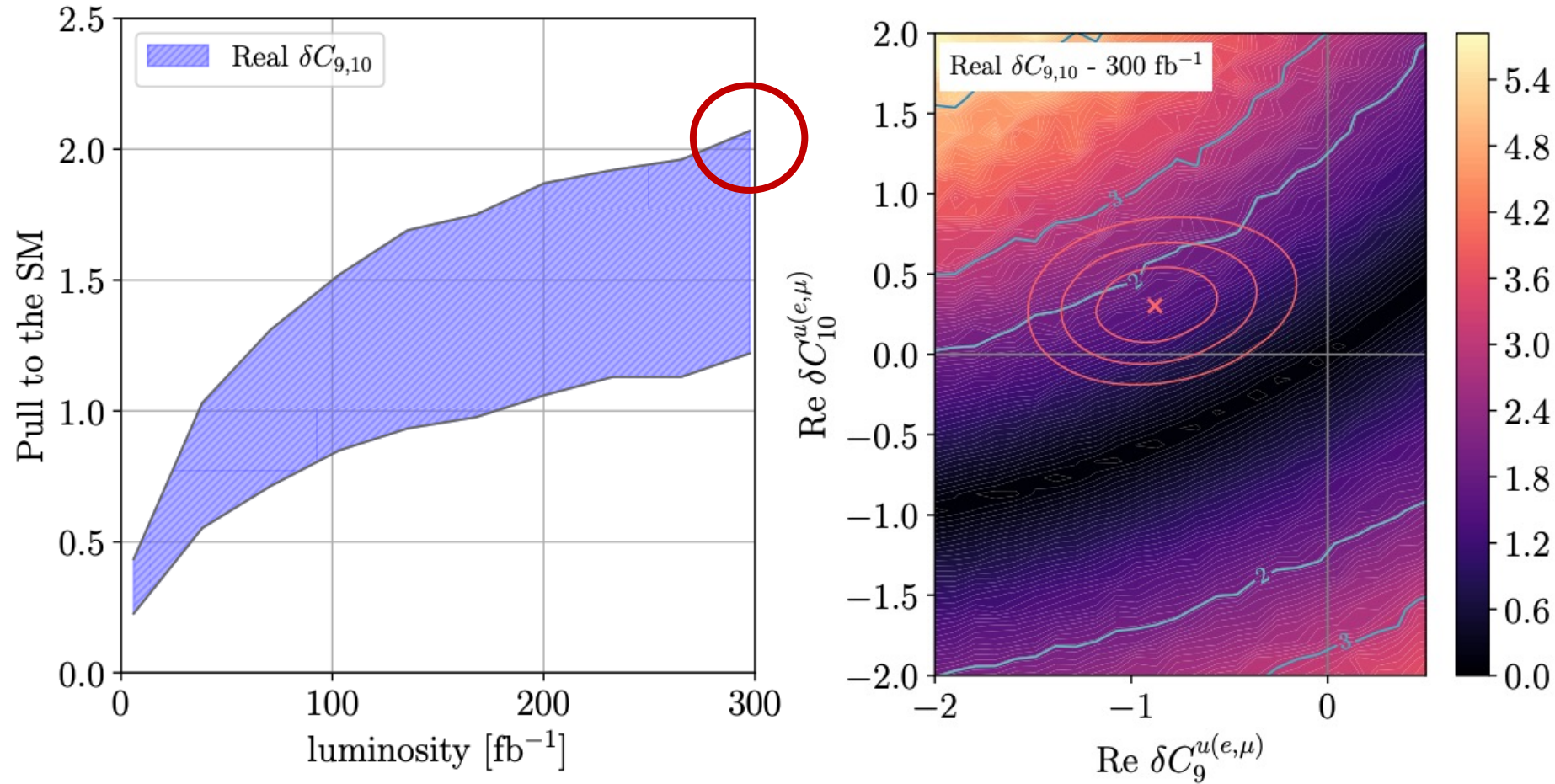
$B_s \rightarrow \mu\mu\gamma$ as a golden channel to study NP



Pull to the SM of the BR **assuming NP in both C_9 , C_{10}**

Guadagnoli et al., JHEP '23 [2308.00034]

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**It reaches the 2σ level
at the border of the
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Plan of the talk

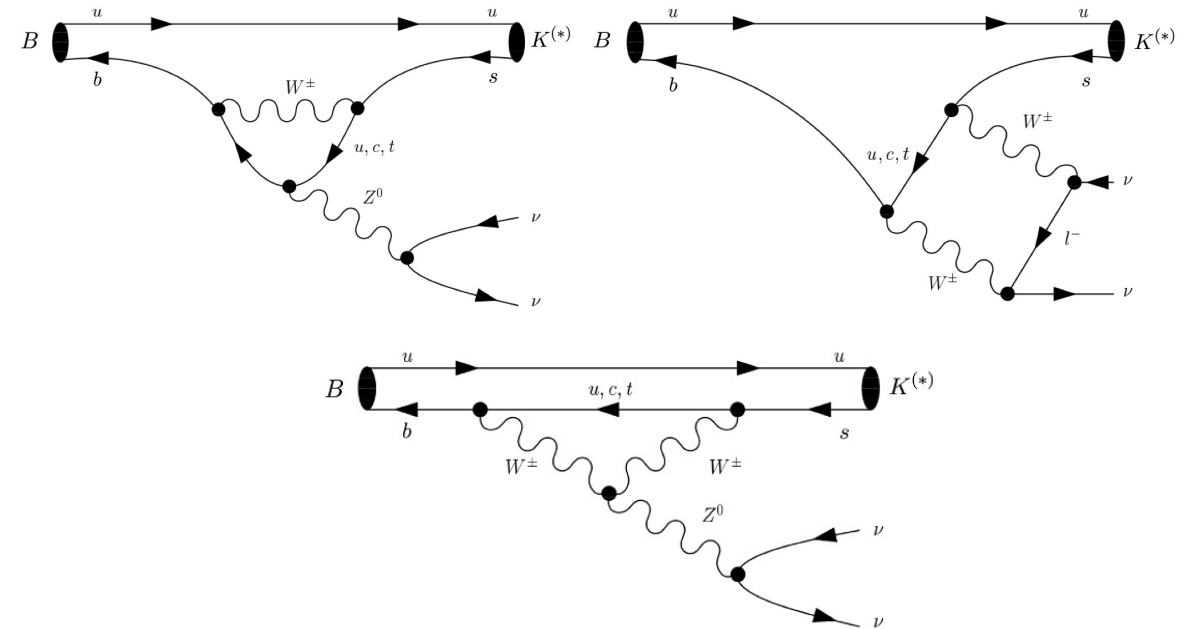
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Long-distance effects here are **much smaller than in $B \rightarrow K(^*)\ell^+\ell^-$**

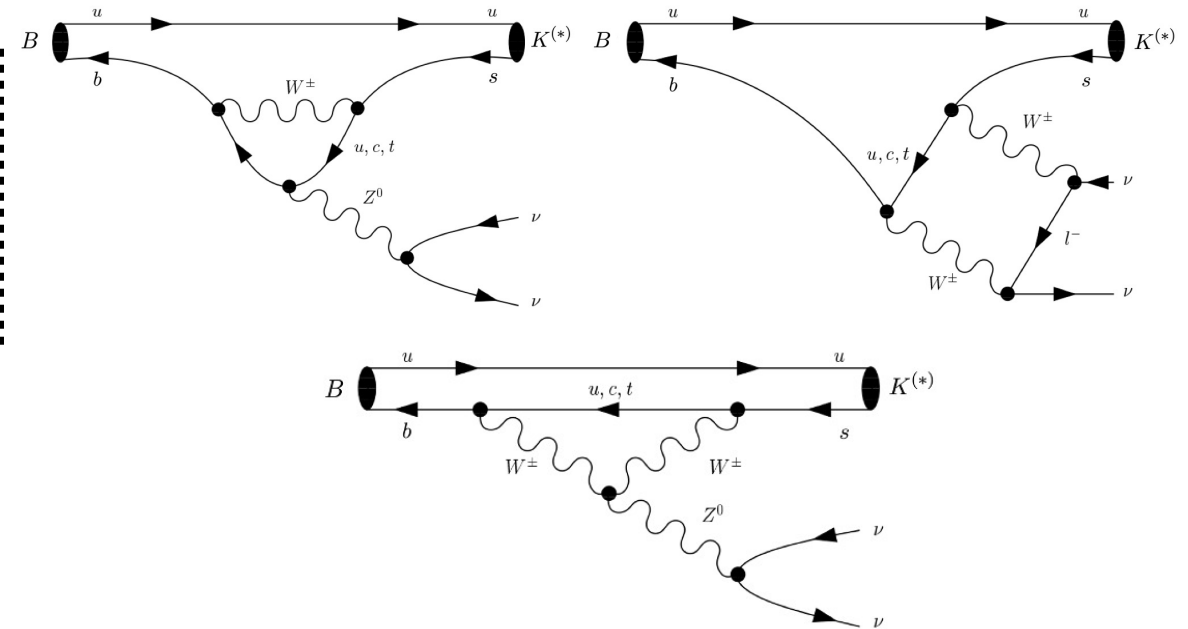


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Major sources of uncertainty:

1. $|V_{cb}|$ value (*exclusive vs. inclusive tension*)
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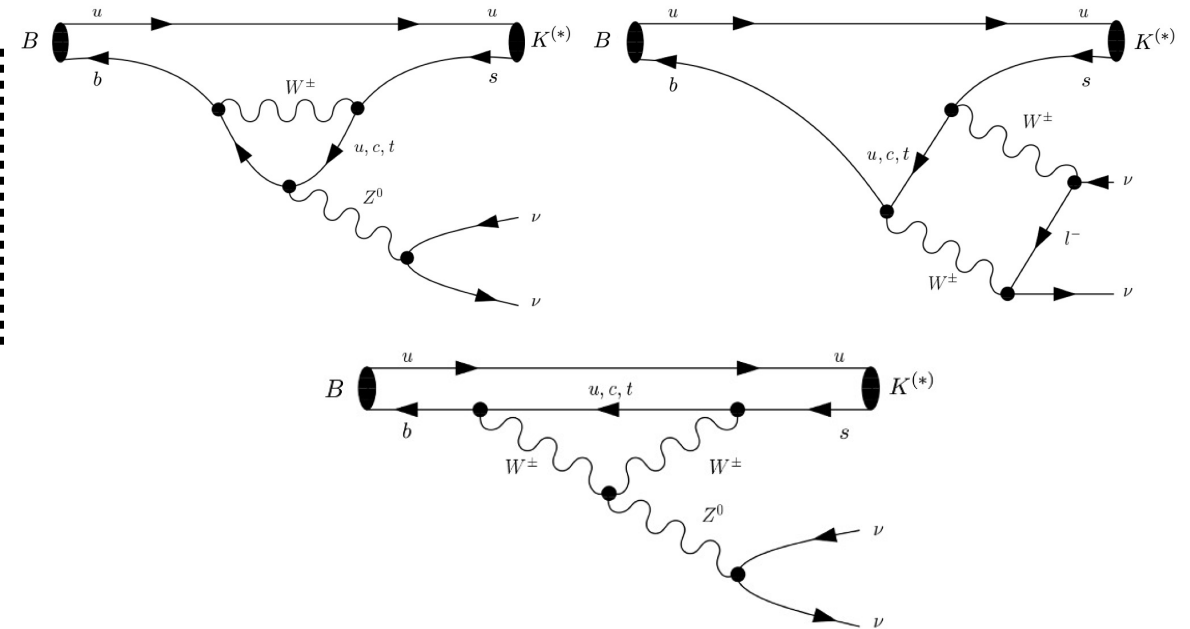
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D. Becirevic, G. Piazza & O. Sumensari, EPJC '23 [arXiv:2301.06990]



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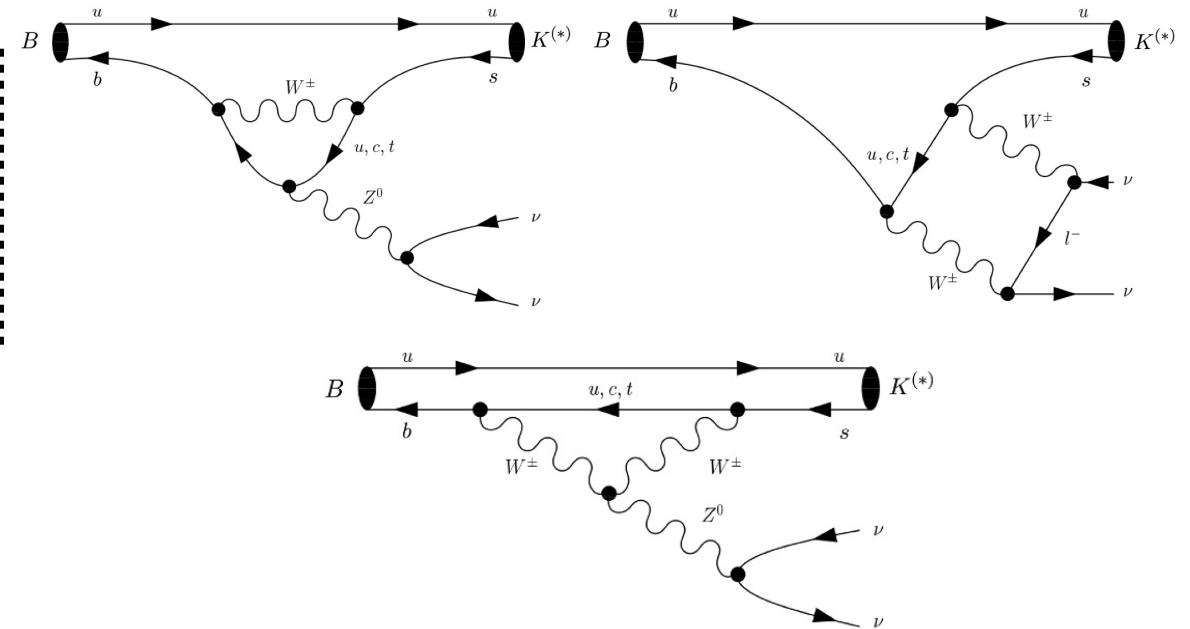
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to be compared with

$$\mathcal{B}(B^+ \rightarrow K^+ \nu \bar{\nu}) \Big|_{\text{Belle-II}} = (2.4 \pm 0.7) \times 10^{-5}$$

Belle-II Collaboration, PRD '24 [2311.14647]



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Final prediction SM :

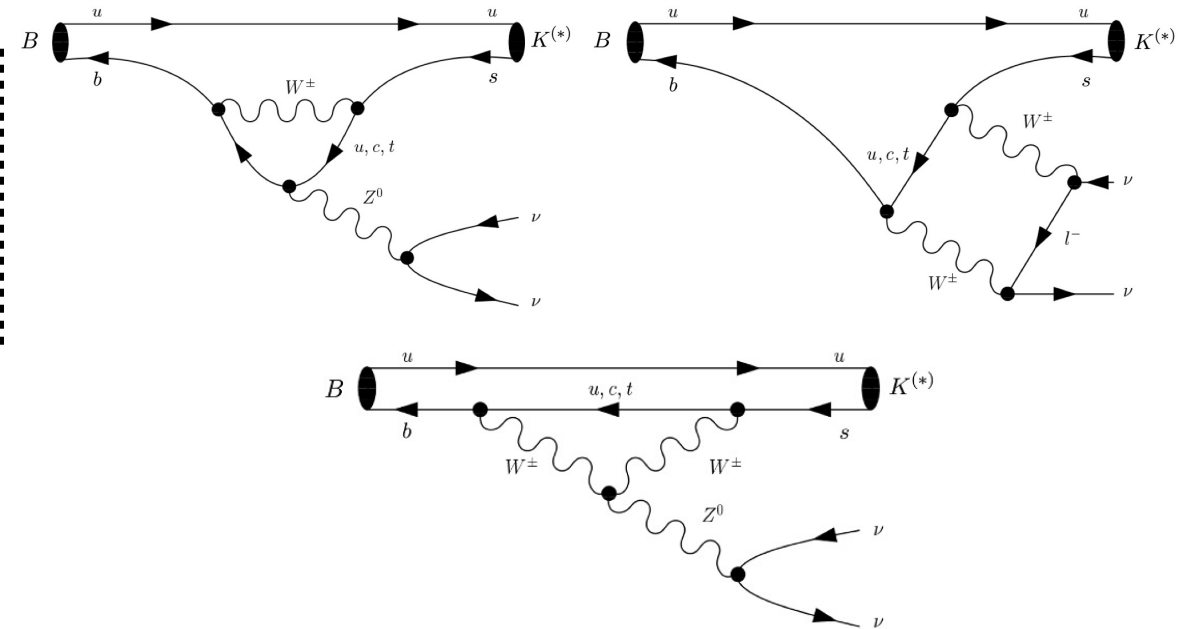
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D. Becirevic, G. Piazza & O. Sumensari, EPJC '23 [arXiv:2301.06990]

to be compared with

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Belle-II Collaboration, PRD '24 [2311.14647]



Tension in $B \rightarrow K\nu\nu$:
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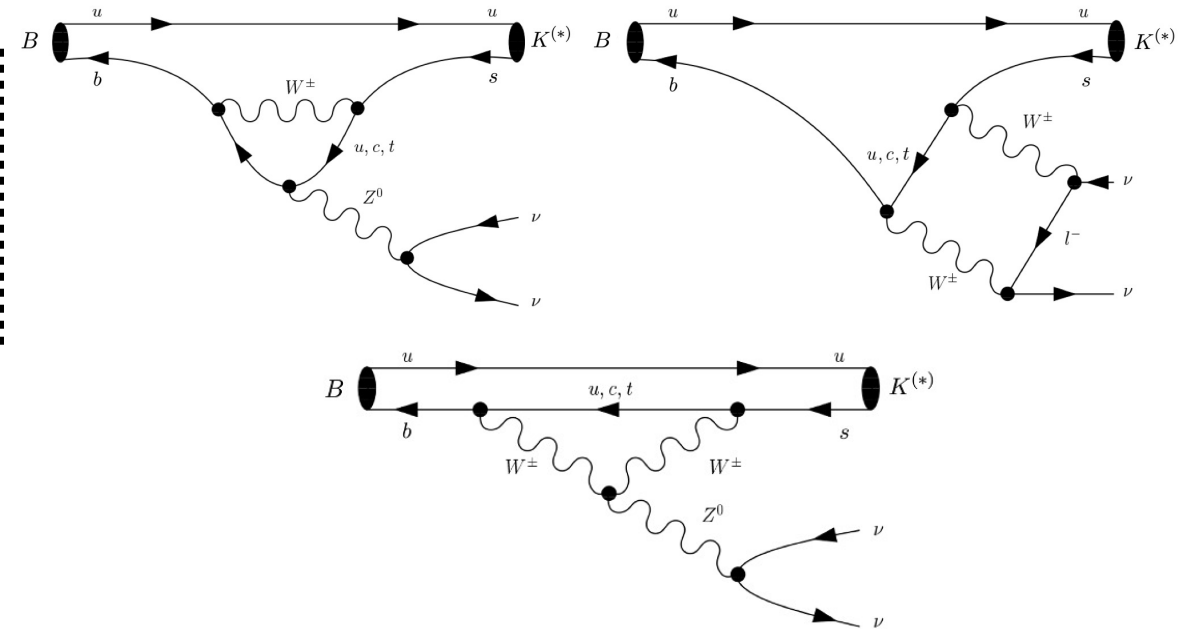
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$$\left\{ \mathcal{B}(B \rightarrow K^* \nu \bar{\nu}) < 2.7 \times 10^{-5} \quad \text{Belle Coll., PRD '17 [1702.03224]} \right\}$$

$K \rightarrow \pi \nu \bar{\nu}$ decays

Experiments are giving (or will give) insights on K^+ , K_L (and K_S ?) physics:

NA62 (K^+): $\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = (13.0^{+3.3}_{-3.0}) \times 10^{-11}$ **NA62 Collaboration,**
JHEP '25 [2412.12015]

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For what concerns the SM predictions :

- $$10^{11} \times \mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = 8.38 \pm 0.14_{X_t^{\text{QCD}}} \pm 0.01_{X_t^{\text{EW}}} \pm 0.11_{P_c} \pm 0.25_{\delta P_{cu}} \\ \pm 0.04_{\kappa_+} \pm 0.14_{\lambda} \pm 0.31_A \pm 0.12_{\bar{\rho}} \pm 0.03_{\bar{\eta}} \pm 0.05_{m_t} \pm 0.15_{m_c} \pm 0.06_{\alpha_s}$$
Anzivino et al., EPJC '24 [2311.02923] [Kaons@CERN23]

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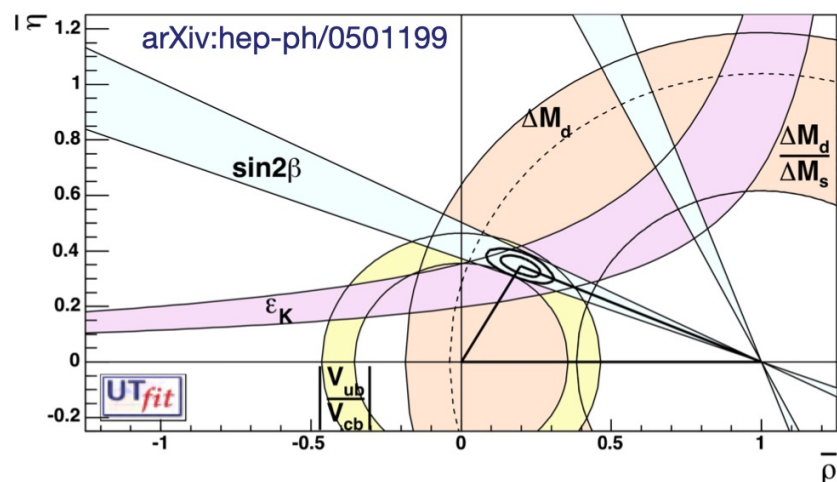
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 $\pm 0.15_{\bar{\eta}} \pm 0.15_A \pm 0.07_{\lambda} \pm 0.03_{m_t}$
Anzivino et al., EPJC '24 [2311.02923] [Kaons@CERN23]

Improving precision on the Unitarity Triangle



Marcella Bona¹ Marco Ciuchini² Denis Derkach³ Fabio Ferrari^{4,5} Vittorio Lubicz^{2,7} Guido Martinelli^{6,8}
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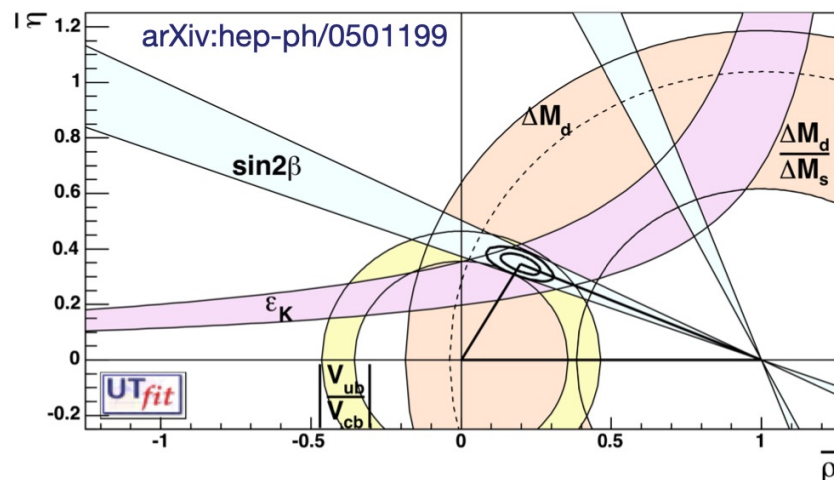


$$\bar{\rho} = 0.196 \pm 0.045 \sim 23\%$$
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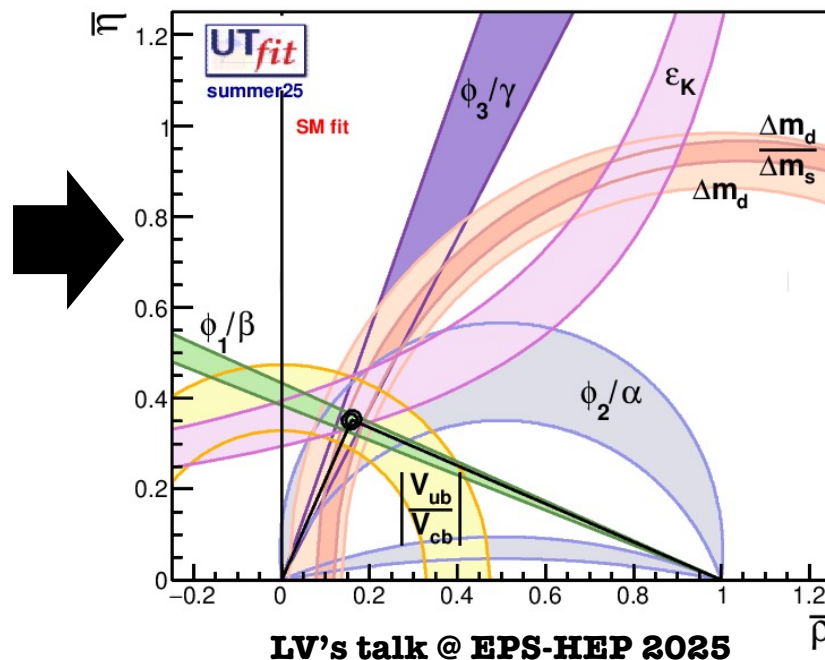


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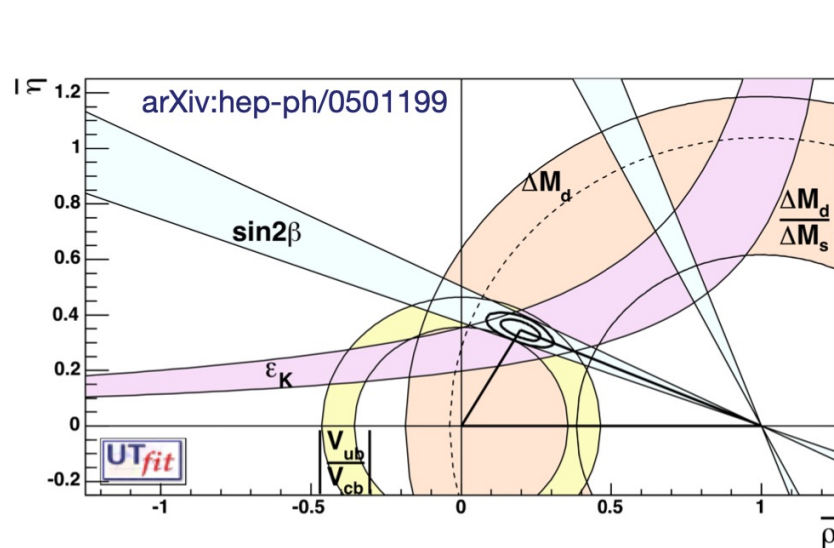
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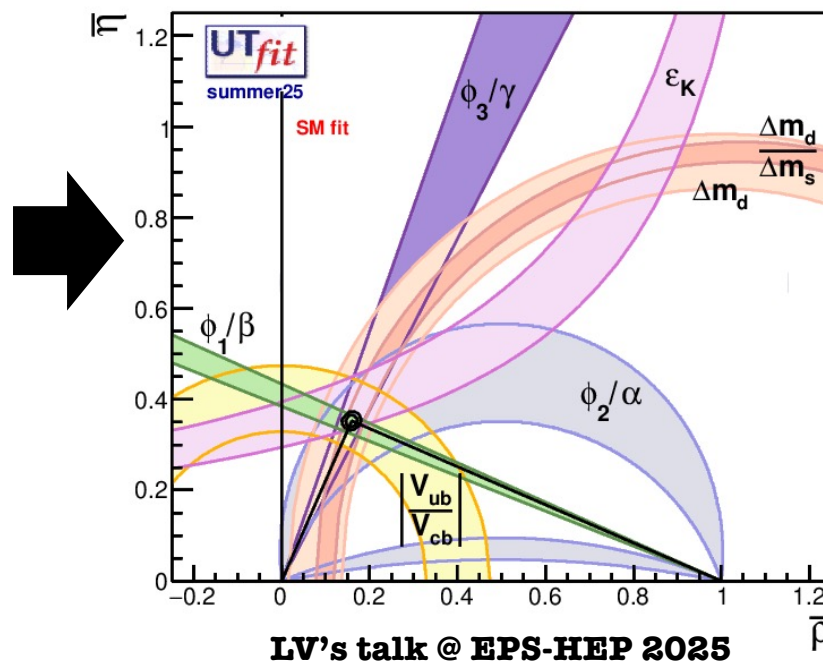


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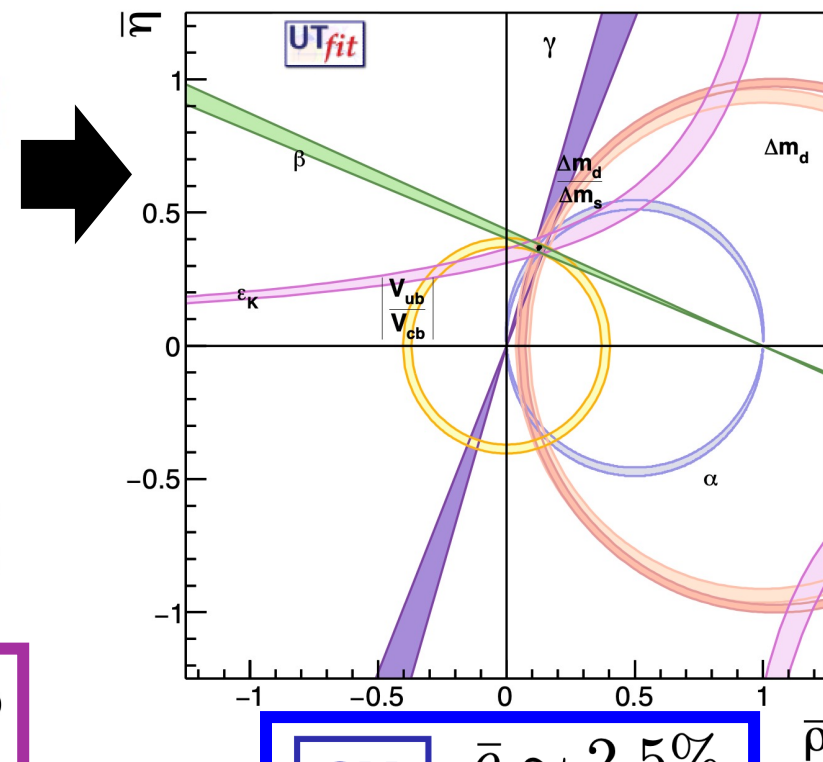
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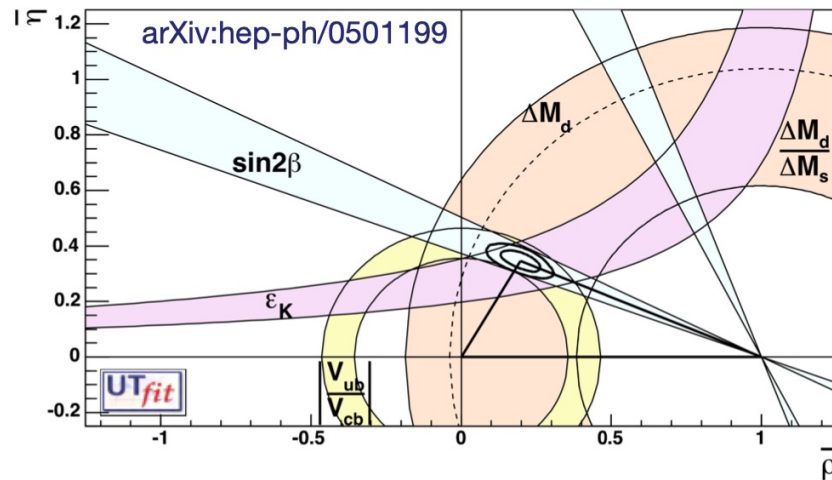
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Improving precision on the Unitarity Triangle



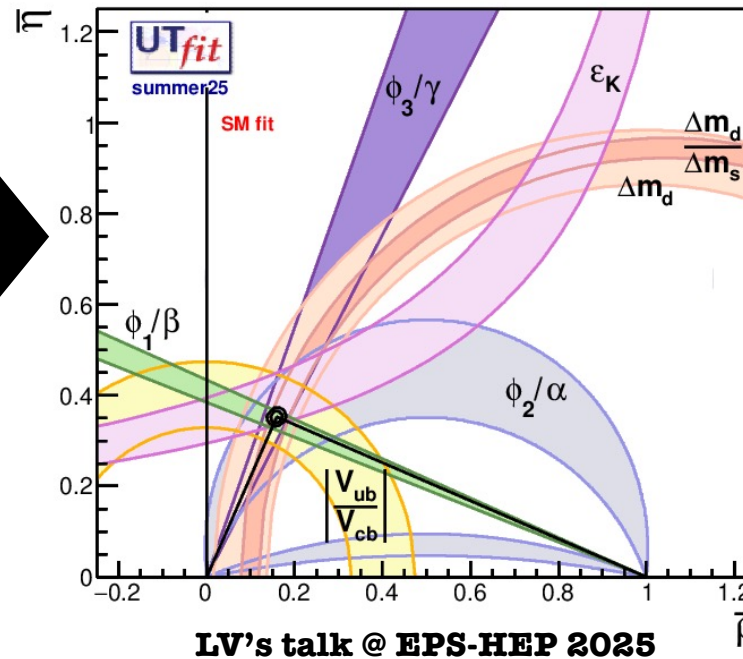
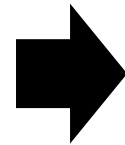
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**Assumption: 50 ab⁻¹ lum. @ Belle II
 (arXiv:1808.10567)**



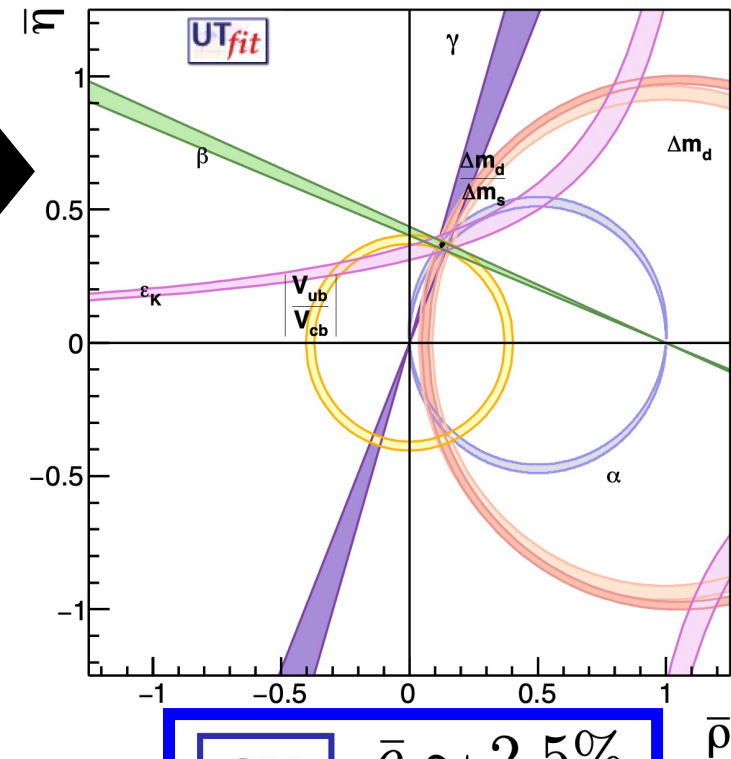
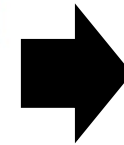
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SM fit

$$\bar{\rho} \sim 2.5\%$$

$$\bar{\eta} \sim 1\%$$

Conclusions

Many challenges have to be faced **from both the theoretical and the experimental points of view** in the next few years! To summarize:

- ***Semileptonic rare $B_{(s)}$ -meson decays***: lots of experimental data currently available, an important theoretical effort has to be done in order to clarify the precise magnitude of the contributions of non-local FFs

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**Flavour is the best framework to test indirectly very high energy scales
(much higher than the one presently investigated at colliders) 😊**

GRAZIE PER LA VOSTRA
ATTENZIONE !

BACK-UP SLIDES

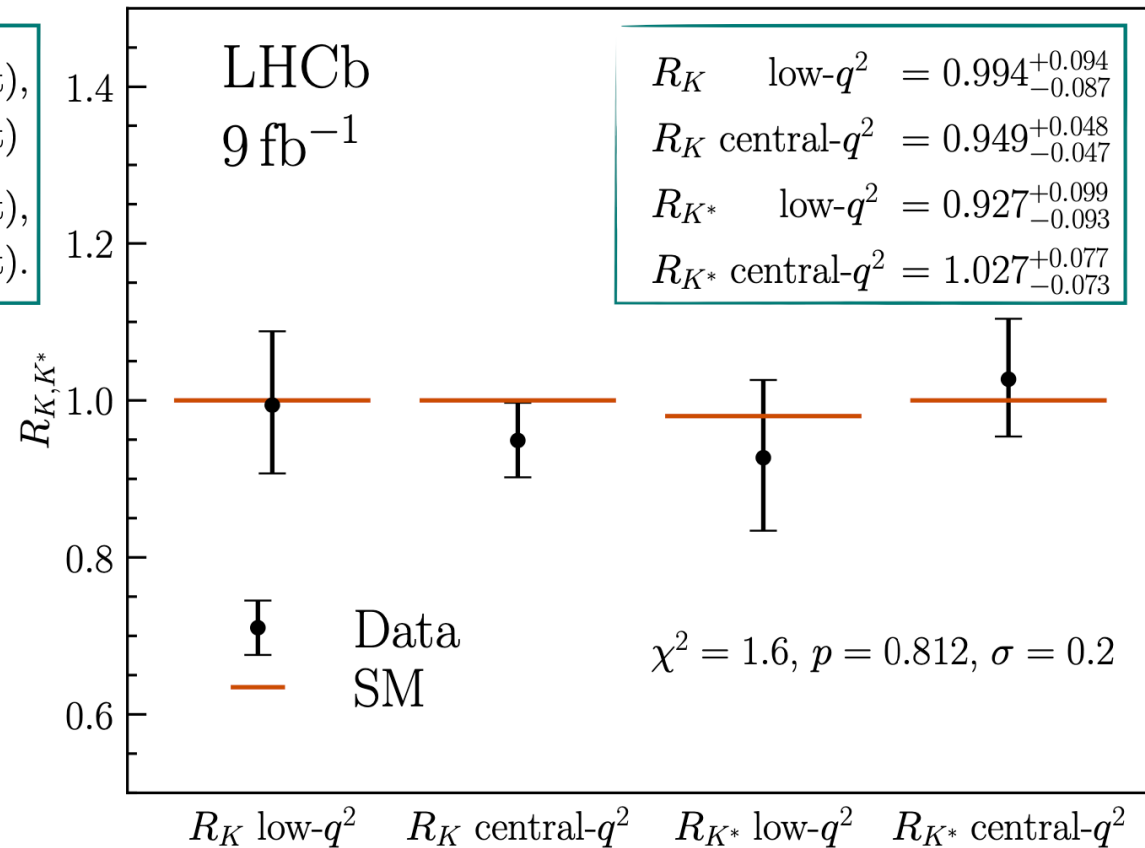
Disappearance of $R(K)$ and $R(K^*)$ anomalies

Analysis: results

Results

$$\begin{aligned} \text{low-}q^2 \begin{cases} R_K &= 0.994^{+0.090}_{-0.082} (\text{stat})^{+0.027}_{-0.029} (\text{syst}), \\ R_{K^*} &= 0.927^{+0.093}_{-0.087} (\text{stat})^{+0.034}_{-0.033} (\text{syst}) \end{cases} \\ \text{central-}q^2 \begin{cases} R_K &= 0.949^{+0.042}_{-0.041} (\text{stat})^{+0.023}_{-0.023} (\text{syst}), \\ R_{K^*} &= 1.027^{+0.072}_{-0.068} (\text{stat})^{+0.027}_{-0.027} (\text{syst}). \end{cases} \end{aligned}$$

- ◆ Most precise and accurate LFU test in $b \rightarrow s\ell\ell$ transition
- ◆ Compatible with SM with a simple χ^2 test on 4 measurement at 0.2σ



The Dispersive Matrix (DM) method

Our goal is to describe the FFs using a **novel, non-perturbative and model independent approach in the whole kinematical region!**

- Pioneering works from S. Okubo [PRD, 3 (1971); PRD, 4 (1971)], C. Bourrely et al [NPB, 189 (1981)] and L. Lellouch [NPB, 479 (1996)]
- New developments in M. di Carlo et al, PRD '21 (2105.02497)

Let us focus on a generic FF f : **we will determine $f(t)$ with $f(t_i)$ known at positions t_i ($i=1, \dots, N$)**

How? Through:

- An inner product
- An auxiliary function

$$\langle h_1 | h_2 \rangle = \int_{|z|=1} \frac{dz}{2\pi i z} \bar{h}_1(z) h_2(z)$$

$$g_t(z) \equiv \frac{1}{1 - \bar{z}(t)z}$$

$$\left(\begin{array}{l} z(t) = \frac{\sqrt{\frac{t_+ - t}{t_+ - t_-}} - 1}{\sqrt{\frac{t_+ - t}{t_+ - t_-}} + 1} \\ t_{\pm} \equiv (m_B \pm m_D)^2 \\ t: \text{momentum transfer} \end{array} \right)$$

We build up the matrix M of the scalar products of ϕf , g_t , g_{t_1} , ..., g_{t_N} :

$$M = \begin{pmatrix} \langle \phi f | \phi f \rangle & \langle \phi f | g_t \rangle & \langle \phi f | g_{t_1} \rangle & \cdots & \langle \phi f | g_{t_n} \rangle \\ \langle g_t | \phi f \rangle & \langle g_t | g_t \rangle & \langle g_t | g_{t_1} \rangle & \cdots & \langle g_t | g_{t_n} \rangle \\ \langle g_{t_1} | \phi f \rangle & \langle g_{t_1} | g_t \rangle & \langle g_{t_1} | g_{t_1} \rangle & \cdots & \langle g_{t_1} | g_{t_n} \rangle \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \langle g_{t_n} | \phi f \rangle & \langle g_{t_n} | g_t \rangle & \langle g_{t_n} | g_{t_1} \rangle & \cdots & \langle g_{t_n} | g_{t_n} \rangle \end{pmatrix}$$

The Dispersive Matrix (DM) method

CENTRAL ISSUE: since $\mathbf{M}$ contains only inner products, by construction its determinant is semipositive definite

$$\det \mathbf{M} \geq 0 \quad \Rightarrow \quad f_{\text{lo}}(z) \leq f(z) \leq f_{\text{up}}(z)$$

DISPERSION RELATIONS:

$$0 \leq \langle \phi f | \phi f \rangle \leq \chi(q^2)$$

$$\mathbf{M} = \begin{pmatrix} \langle \phi f | \phi f \rangle & \langle \phi f | g_t \rangle & \langle \phi f | g_{t_1} \rangle & \cdots & \langle \phi f | g_{t_n} \rangle \\ \langle g_t | \phi f \rangle & \langle g_t | g_t \rangle & \langle g_t | g_{t_1} \rangle & \cdots & \langle g_t | g_{t_n} \rangle \\ \langle g_{t_1} | \phi f \rangle & \langle g_{t_1} | g_t \rangle & \langle g_{t_1} | g_{t_1} \rangle & \cdots & \langle g_{t_1} | g_{t_n} \rangle \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \langle g_{t_n} | \phi f \rangle & \langle g_{t_n} | g_t \rangle & \langle g_{t_n} | g_{t_1} \rangle & \cdots & \langle g_{t_n} | g_{t_n} \rangle \end{pmatrix}$$

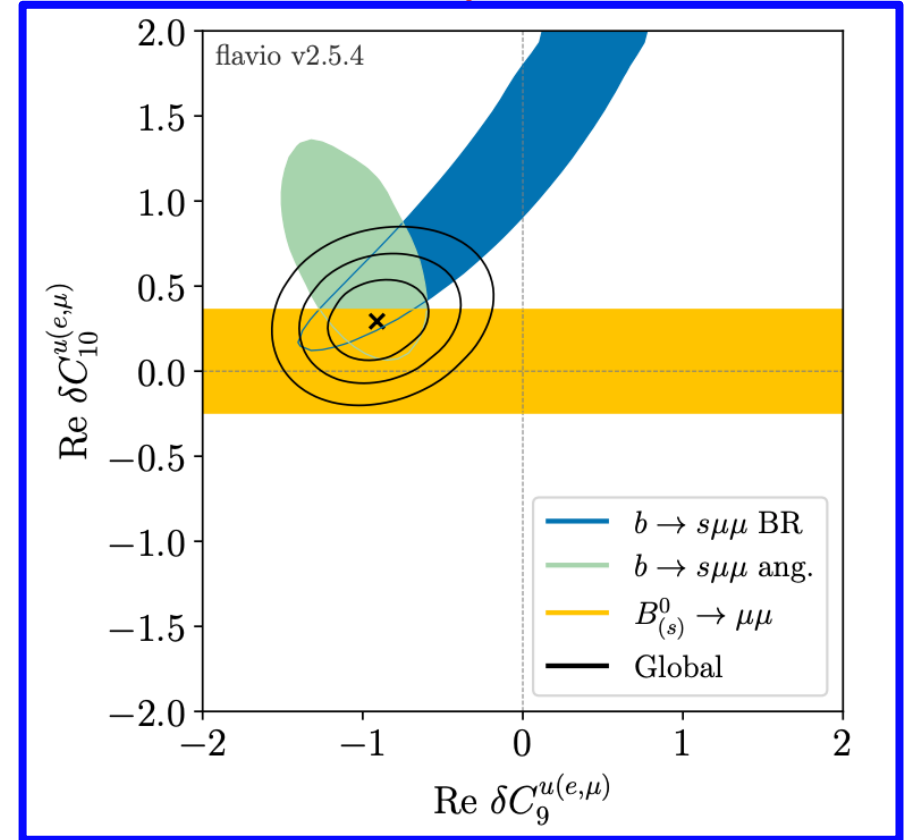


$B_s \rightarrow \mu\mu\gamma$ as a golden channel to study NP

Main ingredients of our sensitivity study:

1. **Identification of NP benchmarks**
from semileptonic neutral-current B decays:

	NP shift	ℓ -specific + ℓ -univ. parts
$(k = 9, 10)$	δC_k^{bsee}	$\equiv \delta C_k^{(e)} + \delta C_k^{u(e,\mu)}$
	$\delta C_k^{bs\mu\mu}$	$\equiv \delta C_k^{(\mu)} + \delta C_k^{u(e,\mu)}$
	$\left[\delta C_9^{(\ell)} = -\delta C_{10}^{(\ell)} \equiv \delta C_{LL}^{(\ell)}/2 \right]$	



E
X
A
M
P
L
E

REMARK: we are computing these NP shifts **assuming**,
as said before, that **SM long-distance effects are negligible !!**

(See also the global analyses in JHEP '23 [2212.10497], PRD '23 [2212.10516] ...)

Method I: measure $B_s \rightarrow \mu\mu\gamma$ with photon reconstruction

Pro: Sensitive to low- q^2 region, therefore, to larger set of Wilson coefficients

Con: Photon reconstruction worsen the resolution

Three q^2 regions:

Bin I: low- q^2

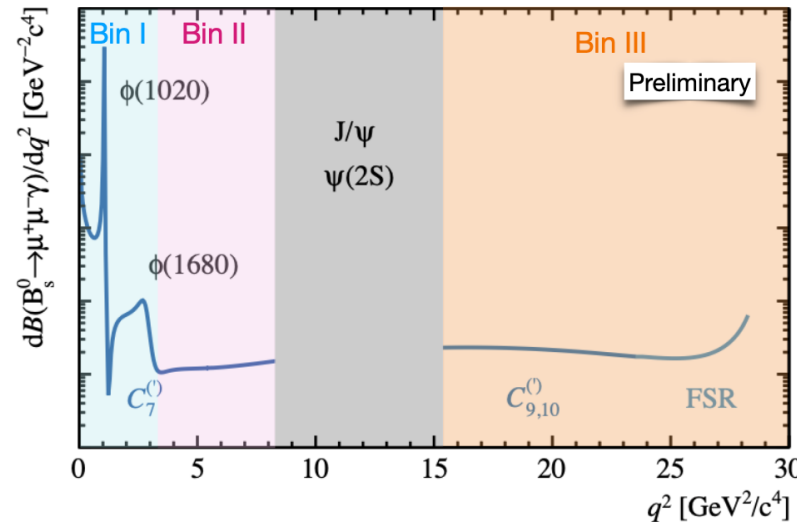
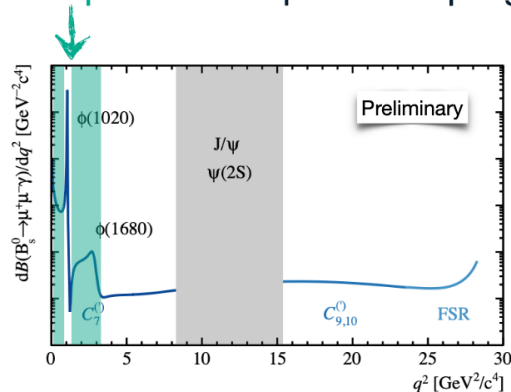
Bin II: middle- q^2

Bin III: high- q^2

Additionally, Bin I is also studied with a veto on the ϕ -resonance mass:

$$m(\mu^+\mu^-) = [989.6, 1073.4]\text{MeV}/c^2$$

Bin I ϕ -veto: low- q^2 without ϕ region



Phys. Rev. D70 (2004) 114028

CERN-THESIS-2020-303

$E_\gamma > 50\text{MeV}/c^2$

FSR = final state radiation

q^2 bin	I	II	III
q^2 [GeV^2/c^4]	$[4m_\mu^2, 2.89]$	$[2.89, 8.29]$	$[15.37, m_{B_s^0}^2]$
$m(\mu^+\mu^-)$ [GeV/c^2]	$[2m_\mu, 1.70]$	$[1.70, 2.88]$	$[3.92, m_{B_s^0}]$
$10^{10} \times \mathcal{B}(B_s^0 \rightarrow \mu^+\mu^-\gamma)$ [8]	82 ± 15	2.54 ± 0.34	9.1 ± 1.1
Fraction of $B_s^0 \rightarrow \mu^+\mu^-\gamma$	87%	2.7%	9.8%

Unfortunately, the measured BR is not statistically significant in any of the three regions...



Upper limits on BR in the regions I, II and III (see next slides)

The “indirect” method to detect radiative-and-leptonic decays

The basic idea is to reconstruct the radiative signal from the non-radiative counterpart, namely

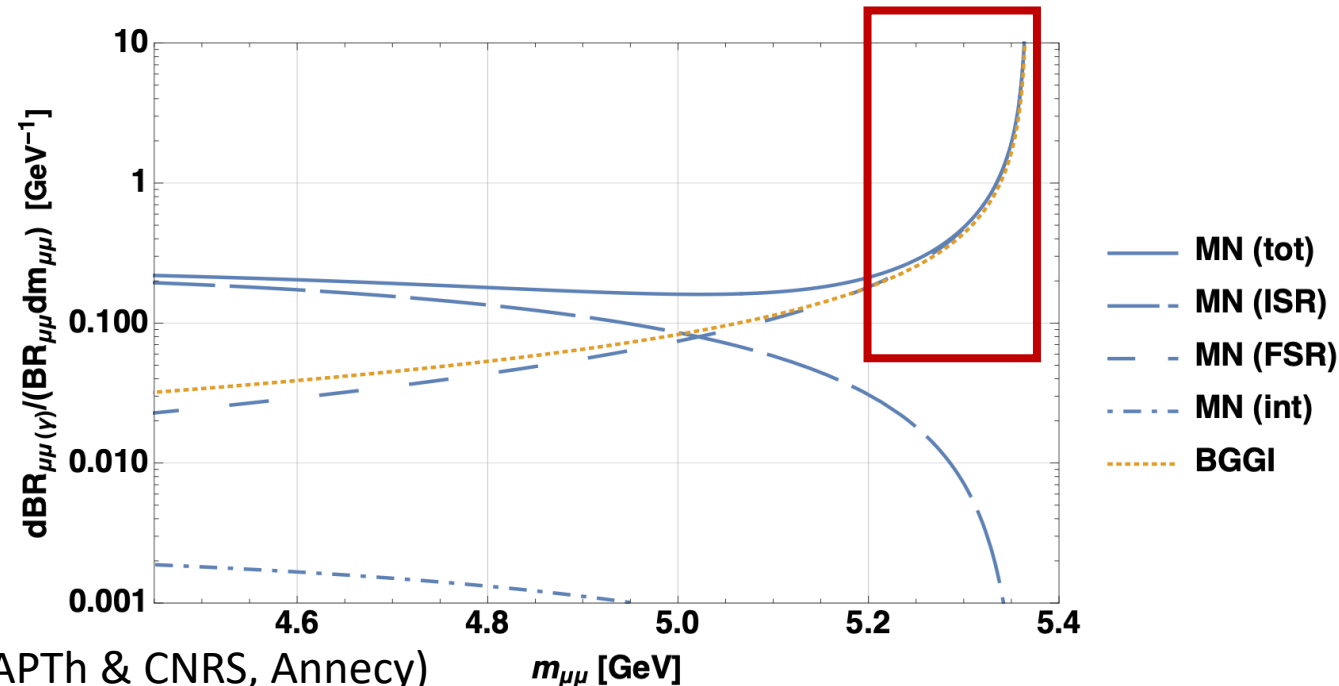
$$B_s^0 \rightarrow \mu^+ \mu^- \gamma \text{ from } B_s^0 \rightarrow \mu^+ \mu^-$$

Dettori, Guadagnoli, Reboud, Phys.Lett.B 768 (2017) 163-167

How? Enlarging the **dilepton invariant mass** below the B_s -peak (it works IF the bkg are well under control!)

The problem is in other words

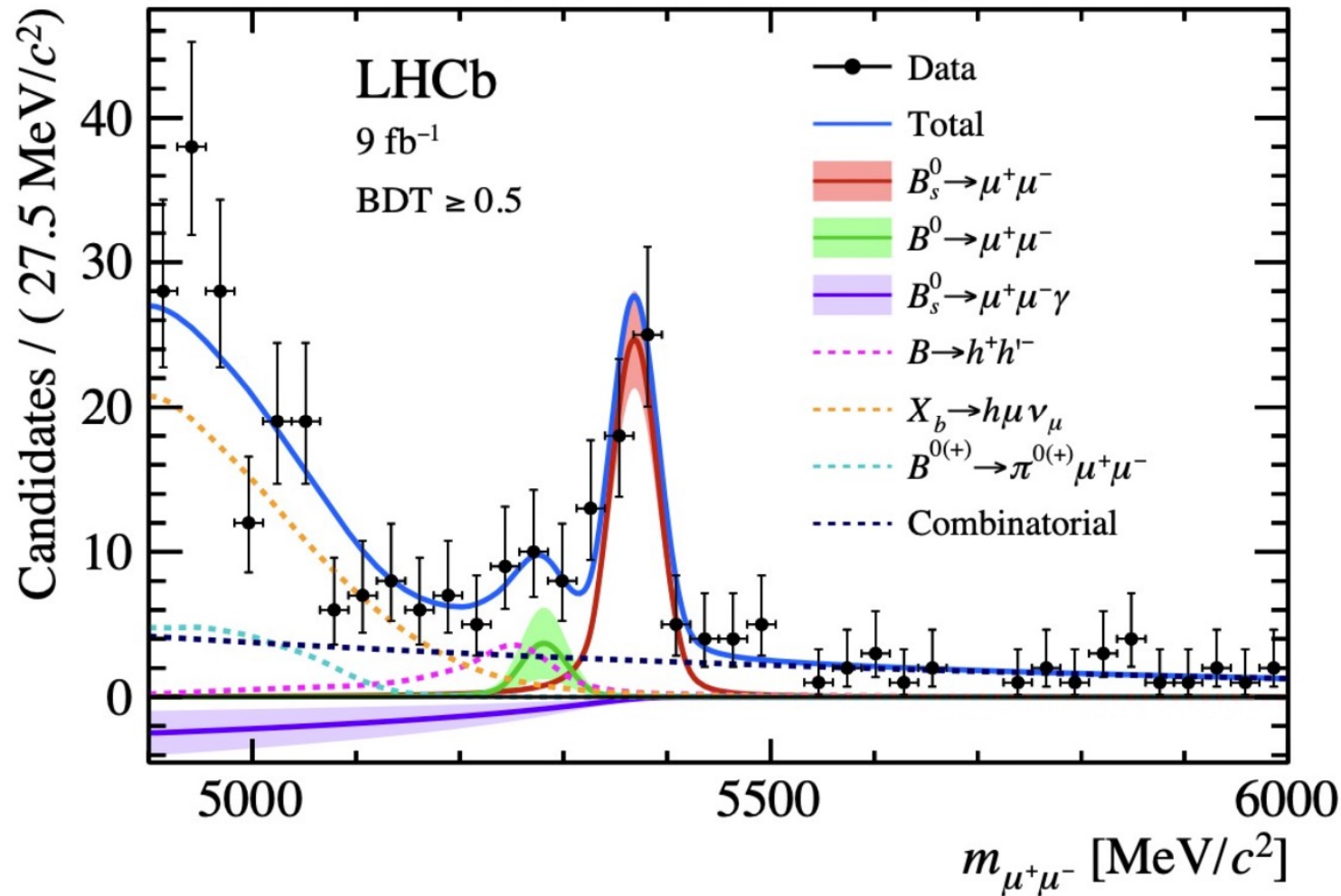
$$\mathcal{B}(B_s^0 \rightarrow \mu^+ \mu^- + n\gamma) \text{ VS } \mathcal{B}(B_s^0 \rightarrow \mu^+ \mu^- \gamma)$$



For very soft photons, the single-photon component of the former should be equal to the latter!

The “indirect” method to detect radiative-and-leptonic decays

$$\mathcal{B}(B_s^0 \rightarrow \mu^+ \mu^- \gamma)_{m_{\mu\mu} > 4.9 \text{ GeV}} = (-2.5 \pm 1.4 \pm 0.8) \times 10^{-9} < 2.0 \times 10^{-9}$$



(from LHCb-PAPER-2021-007, LHCb-PAPER-2021-008)

Some pros:

- i) No reconstruction of the photon, whose efficiency is inherently small
- ii) Measur. at high- q^2 , which is the best region for Lattice QCD and is also the region least affected by resonances
- iii) Sensitivity to C_9 , C_{10}

Some cons:

- i) Signal as a «shoulder», i.e. requires reliable estimation of all other «shoulders»
- ii) Difficult below $(4.2 \text{ GeV})^2$
- iii) Mass resolution crucial !!

$B_s \rightarrow \mu\mu\gamma$ as a golden channel to study NP

Main ingredients of our sensitivity study:

2. **Experimental uncertainties:** we will assume that all the backgrounds are under control, i.e. that their uncertainties will eventually fall safely below the signal yield (**“no-background” hypothesis**).

Thus, the $B_s \rightarrow \mu\mu\gamma$ -signal *uncertainty* will be *dominated by the sheer amount of data collected*

(Many effects to be taken into account: efficiencies, optimal choice of $(q_{min})^2$, ...)

3. **Theoretical uncertainties:**

- GNSV FFs with shrunk uncertainties, i.e. **$O(5\%)$ errors**, and KMN tensor FFs with $O(20\%)$ uncertainties
- assumption: broad charmonia play an entirely negligible role at high- q^2

Recall that we need to resolve

$$\delta C_9^{(\mu)} / C_9^{(\mu), \text{SM}} \simeq 15\% \quad !!$$

Motivated by precision on $D_s \rightarrow \gamma$ FFs as from the lattice study **PRD '23 (2306.05904)**, confirmed by the $B_s \rightarrow \gamma$ results in **PRD '24 (2402.03262)**

$B \rightarrow K\nu\nu$ as the fundamental link among $b \rightarrow c$ and $b \rightarrow s$

$b \rightarrow s\nu\nu$ is theoretically cleaner than $b \rightarrow s\mu\mu$: it is **not affected by charm-loop effects** !!

$$\mathcal{O}_L^{\nu_i\nu_j} = \frac{e^2}{(4\pi)^2} \left(\bar{s}_L \gamma_\mu b_L \right) \left(\bar{\nu}_i \gamma^\mu (1 - \gamma_5) \nu_j \right) \quad C_L^{\text{SM}} = -6.32(7) \quad \searrow \quad \mathcal{O}_R^{\nu_i\nu_j} = \frac{e^2}{(4\pi)^2} \left(\bar{s}_R \gamma_\mu b_R \right) \left(\bar{\nu}_i \gamma^\mu (1 - \gamma_5) \nu_j \right) \quad C_R^{\text{SM}} = 0$$

Buchalla & Buras, NPB '93

Misiak & Urban, arXiv:hep-ph/9901278

Buchalla & Buras, arXiv:hep-ph/9901288

Brod, Gorbahn & Stamou, arXiv:1009.0947

Major sources of uncertainty:

1. Value of $|V_{cb}|$ (due to CKM suppression)
2. Determination of hadronic FFs

Final prediction

$$\mathcal{B} \left(B^\pm \rightarrow K^\pm \nu \bar{\nu} \right) = (4.44 \pm 0.30) \times 10^{-6}$$

Final prediction

$$\mathcal{B} \left(B^\pm \rightarrow K^{\pm*} \nu \bar{\nu} \right) = (9.8 \pm 1.4) \times 10^{-6}$$

D. Becirevic, G. Piazza & O. Sumensari, EPJC '23 [arXiv:2301.06990]

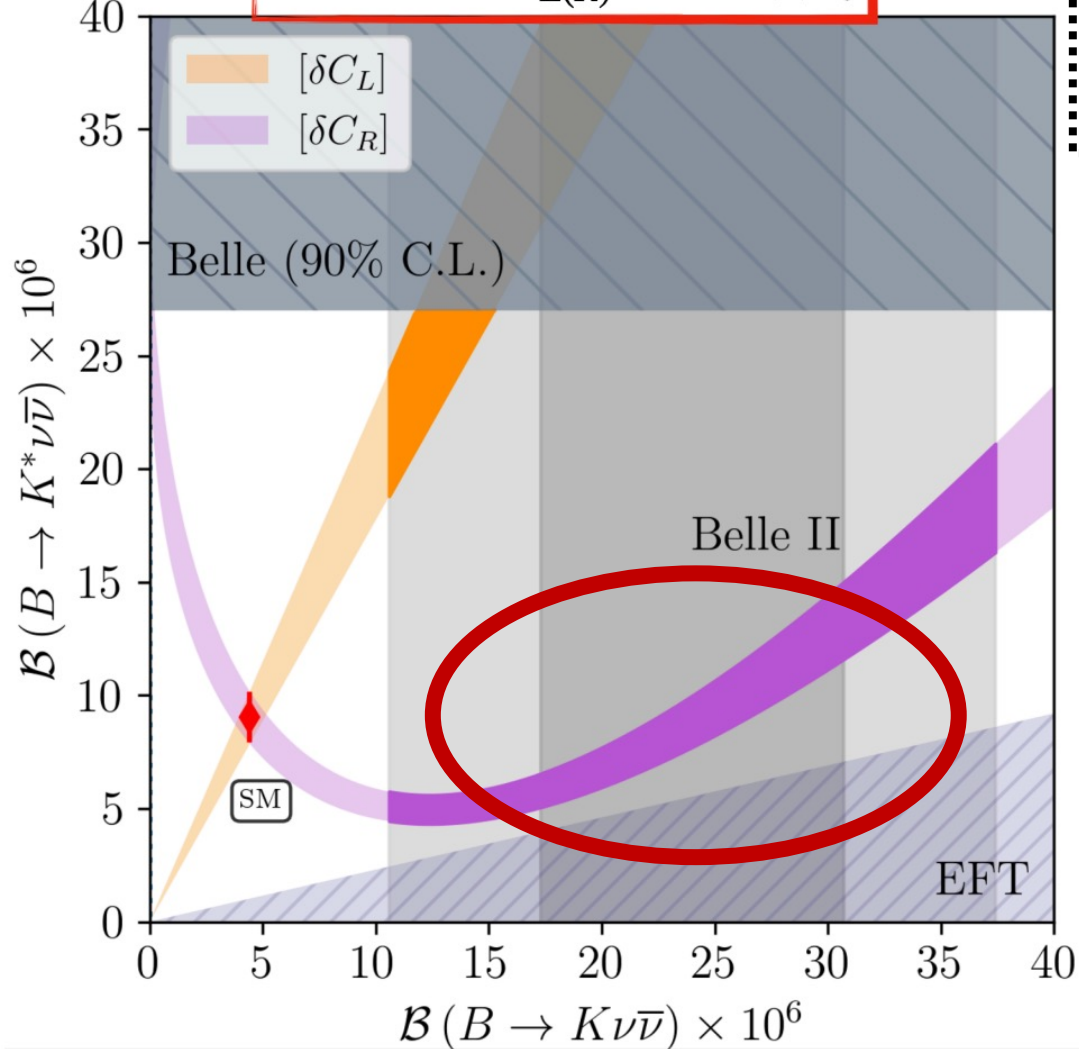
to be compared with

$$\mathcal{B} \left(B^+ \rightarrow K^+ \nu \bar{\nu} \right) \Big|_{\text{Belle-II}} = (2.4 \pm 0.7) \times 10^{-5}$$

A. Glazov, plenary talk EPS-HEP2023 Conference, Aug 20-25, 2023

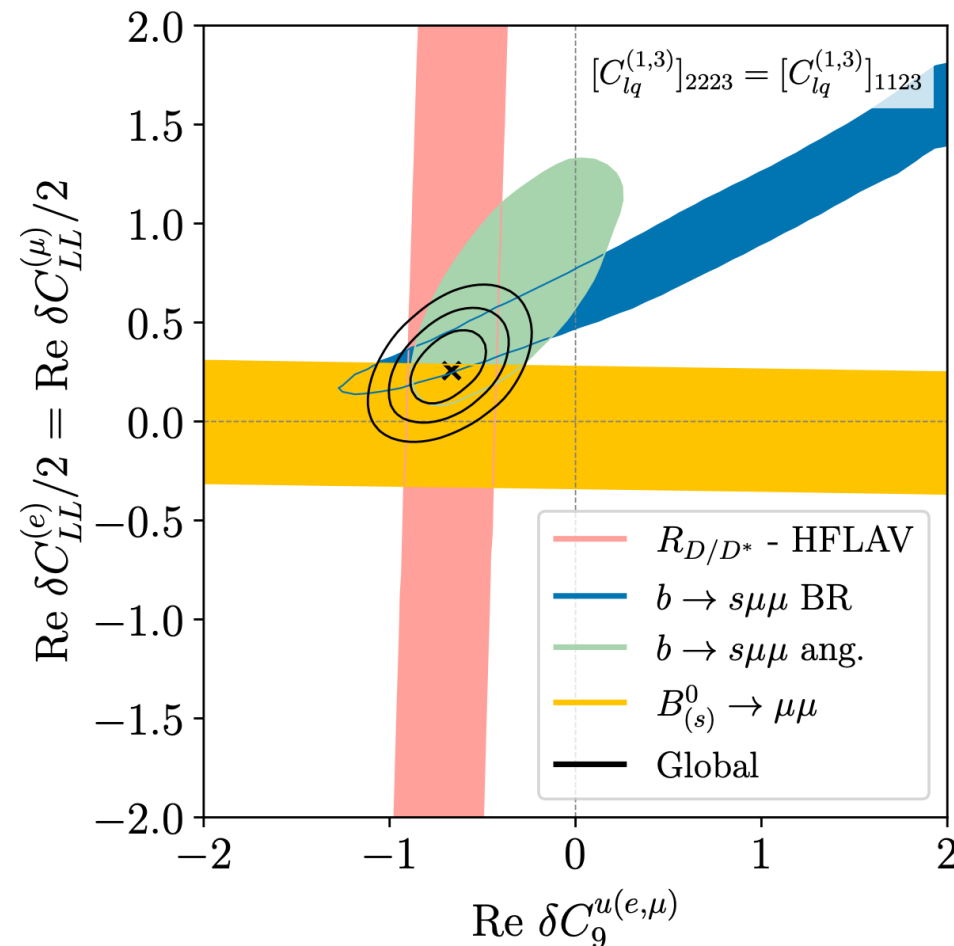
$B \rightarrow K\nu\nu$ as the fundamental link among $b \rightarrow c$ and $b \rightarrow s$

Assuming $\delta C_{L(R)}^{\nu_i \nu_j} = \delta C_{L(R)} \delta_{ij}$



L. Allwicher et al., arXiv:2309.02246

This kind of channels is important for SMEFT analyses, since $B \rightarrow K\nu\nu$ may influence $b \rightarrow s$ observables (with charged leptons) and $b \rightarrow c$ ones !



Light-lepton universal NP scenario [without $B \rightarrow K\nu\nu$ data]

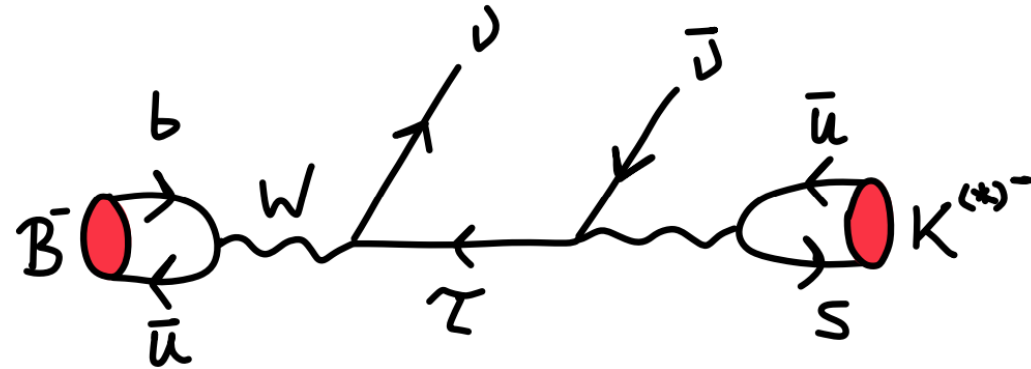
D. Guadagnoli et al, JHEP '23 [2308.00034]

Tree-level contribution

$$B^{\pm} \rightarrow K^{\pm(*)} \nu \bar{\nu}$$

J. F. Kamenik & C. Smith, arXiv:0908.1174

Charged meson decay modes have a **tree-level contribution** from the **annihilation to an intermediate τ**



$$m_B > m_{\tau} > m_{K^{(*)}}$$

Using the narrow width approximation

$$\mathcal{B}(B^+ \rightarrow K^{(*)+} \nu \bar{\nu}) \sim \mathcal{B}(B^+ \rightarrow \tau^+ \nu) \mathcal{B}(\tau^+ \rightarrow K^{(*)+} \bar{\nu})$$

$$\frac{\mathcal{B}(B \rightarrow K^{(*)} \nu \bar{\nu})_{\text{tree}}}{\mathcal{B}(B \rightarrow K^{(*)} \nu \bar{\nu})_{\text{loop}}} \simeq 14 \% (11 \%)$$

Non negligible contribution!

Belle-II can in principle disentangle these two contributions

Basics of the Unitarity Triangle analysis

The **Cabibbo-Kobayashi-Maskawa (CKM) matrix** describes the mixing among quarks (with different electric charges): it is a **unitary 3x3 matrix**

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \quad V_{CKM}^\dagger V_{CKM} = I$$

Wolfenstein parametrization (L. Wolfenstein, PRL 51 (1983) 1945-1947):

$$V_{CKM} = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\bar{\rho} - i\bar{\eta}) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \bar{\rho} - i\bar{\eta}) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

By exploiting the property of unitarity:

$$V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0$$



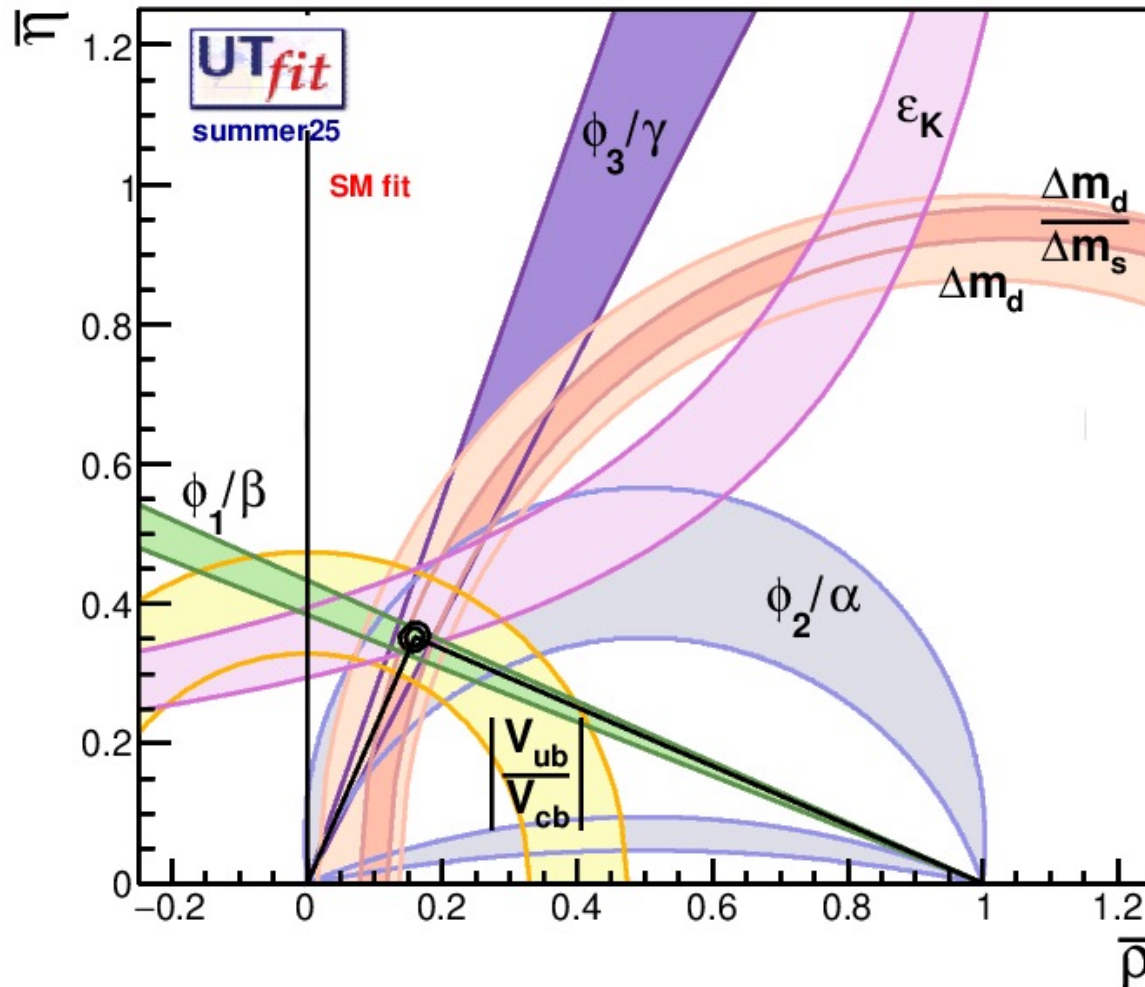
***Triangle in the
complex $(\bar{\rho}, \bar{\eta})$ plane***

UTA within the SM: results

Example of relations to understand the coloured bands:

$$\left| \frac{V_{ub}}{V_{cb}} \right| = \frac{\lambda}{1 - \frac{\lambda^2}{2}} \sqrt{\bar{\rho}^2 + \bar{\eta}^2}$$

$$\frac{\Delta m_d}{\Delta m_s} = \frac{m_{B_d} f_{B_d}^2 \hat{B}_{B_d}}{m_{B_s} f_{B_s}^2 \hat{B}_{B_s}} \left(\frac{\lambda}{1 - \frac{\lambda^2}{2}} \right)^2 [(1 - \bar{\rho})^2 + \bar{\eta}^2]$$



Fit results:

$$\bar{\rho} = 0.159 \pm 0.009$$

$$\bar{\eta} = 0.353 \pm 0.008$$

$$\lambda = 0.2250 \pm 0.0006$$

$$A = 0.827 \pm 0.008$$