First-row CKM unitarity
Experimental status and perspectives on kaon and light-flavor physics

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First-row CKM unitarity

Standard-model coupling of quarks and leptons to W:

$$\frac{g}{\sqrt{2}} W_\alpha^+ \left(\overline{\mathbf{U}}_L \mathbf{V}_{\mathrm{CKM}} \gamma^\alpha \mathbf{D}_L + \overline{e}_L \gamma^\alpha \nu_{e\,L} + \overline{\mu}_L \gamma^\alpha \nu_{\mu\,L} + \overline{\tau}_L \gamma^\alpha \nu_{\tau\,L}\right) \quad + \quad \mathrm{h.c.}$$

Single gauge coupling

Unitary matrix

$$\Delta_{\mathrm{CKM}} \equiv |V_{ud}^2| + |V_{us}^2| + \cancel{|V_{ub}^2|} - 1$$

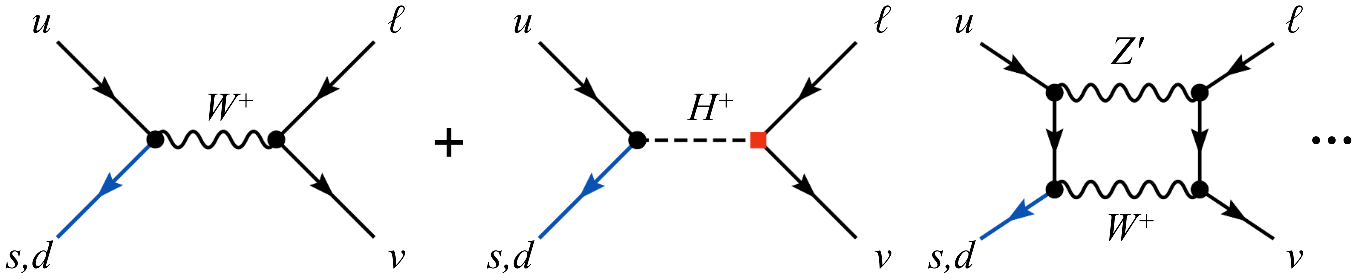
≈ 2×10^{-5}

Most precise test of CKM unitarity!

Universality: Is G_F from μ decay equal to G_F from π, K, nuclear β decay?

$$G_\mu^2 = (g_\mu g_e)^2 / M_W^4 \quad \stackrel{?}{=} \quad G_{\mathrm{CKM}}^2 = (g_q g_\ell)^2 \left(|V_{ud}|^2 + |V_{us}|^2\right) / M_W^4$$

Physics beyond the Standard Model can break gauge universality:



First-row CKM unitarity

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Single gauge coupling

Unitary matrix

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$\approx 2\times10^{-5}$

Most precise test of CKM unitarity!

$$G_{\mathrm{CKM},ij} \sim G_\mu V_{ij} \sim (g^2/M_W^2)\, V_{ij}$$

$$\delta G_{\mathrm{CKM}} \sim 1/\Lambda^2$$ ← energy scale for NP

BSM effects scale as $(M_W^2/g^2)/\Lambda^2$

For measurement of Δ_{CKM} with total uncertainty σ:

- Scale probed is $\Lambda \sim (M_W/g)/\sqrt{\sigma}$
- For $\sigma \sim 10^{-4}$ → probe **Λ ~ 20 TeV**

V_{us} from kaon decays

Determination of V_{us} from $K_{\ell 3}$ data

$$\Gamma(K_{\ell 3(\gamma)}) = \frac{C_K^2 G_F^2 m_K^5}{192\pi^3} S_{\mathrm{EW}} |V_{us}|^2 |f_+^{K^0\pi^-}(0)|^2 \times I_{K\ell}(\lambda_{K\ell}) \left(1 + 2\Delta_K^{SU(2)} + 2\Delta_{K\ell}^{\mathrm{EM}}\right)$$

with $K \in \{K^+, K^0\}$; $\ell \in \{e, \mu\}$, and:

C_K^2	1/2 for K^+, 1 for K^0
S_{EW}	Universal SD EW correction (1.0232)

Inputs from experiment:

$\Gamma(K_{\ell 3(\gamma)})$ Rates with well-determined treatment of radiative decays:

- Branching ratios: K_S, K_L, $K^\pm$
- Kaon lifetimes

$I_{K\ell}(\{\lambda\}_{K\ell})$ Integral of form factor over phase space: λs parameterize evolution in t

- K_{e3}: Only λ_+ (or λ_+', λ_+'')
- $K_{\mu 3}$: Need λ_+ and λ_0

Inputs from theory:

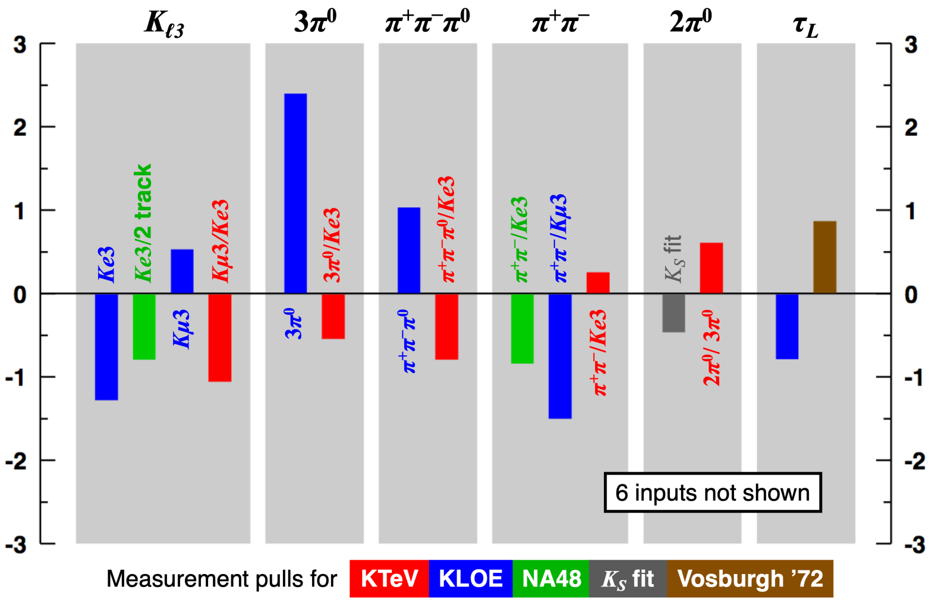
$f_+^{K^0\pi^-}(0)$ Hadronic matrix element (form factor) at zero momentum transfer ($t = 0$)

$\Delta_K^{SU(2)}$ Form-factor correction for $SU(2)$ breaking

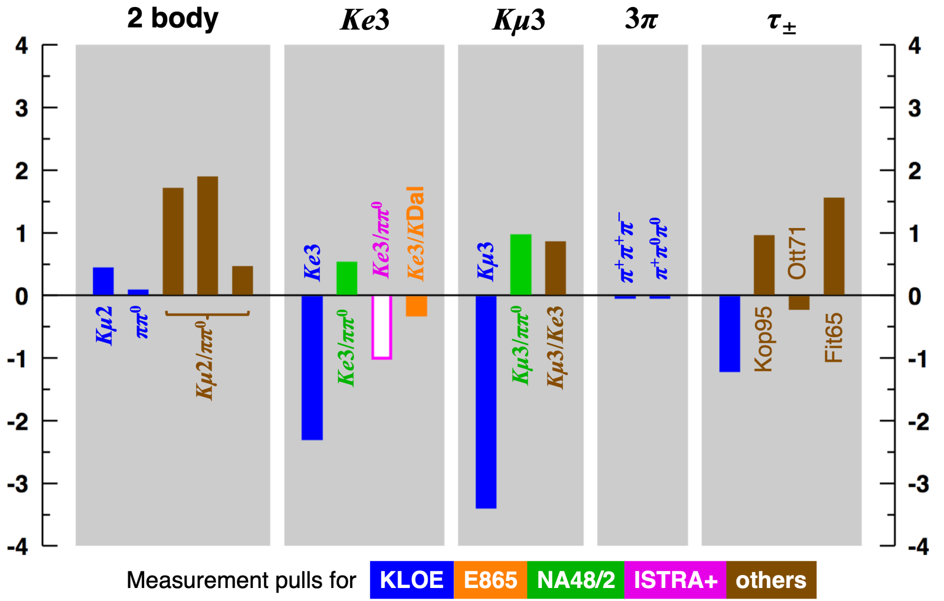
$\Delta_{K\ell}^{\mathrm{EM}}$ Form-factor correction for long-distance EM effects

BR(K^+, $K_L \to \pi\ell\nu$) from world data

Fit to K_L rate data (2010)



Fit to K^+ rate data (2014)

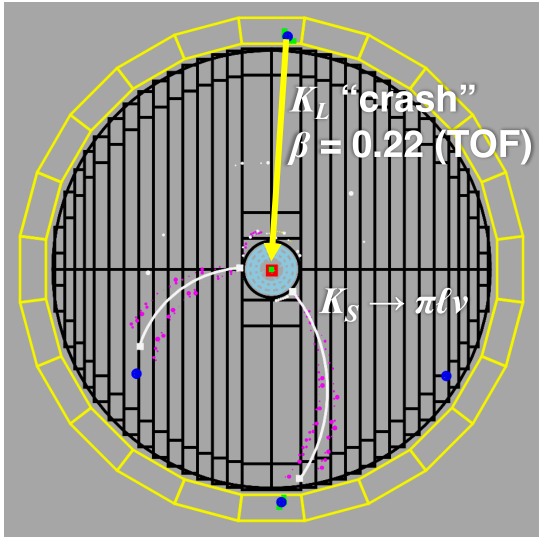


21 input measurements
χ^2/ndf = 19.8/12 (Prob = 7.0%)

17 input measurements
χ^2/ndf = 25.5/11 (Prob = 0.78%)

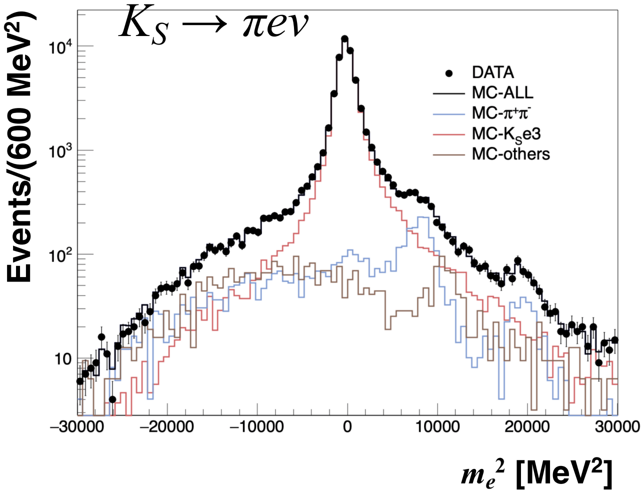
BR($K_S \to \pi\ell\nu$) from KLOE/KLOE-2

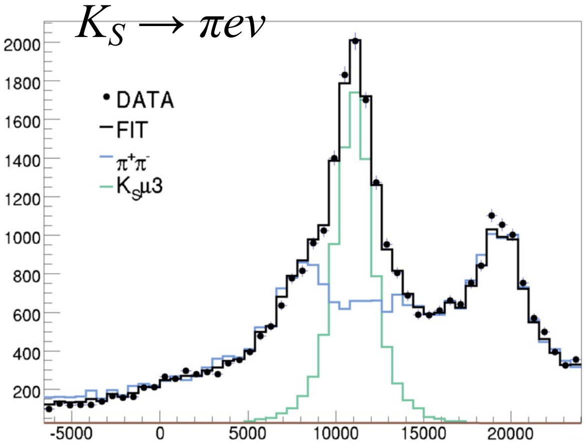
K_S from $\phi \to K_L K_S$ tagged by K_L interaction in calorimeter barrel



Preselection with kinematic BDT and time-of-flight $\pi\ell$ assignment

Fit to $m_\ell^2 = (E_{KS} - E_\pi - p_{\text{miss}})^2 - \mathbf{p}_\ell^2$





KLOE-2
JHEP 23 02 098

BR($K_S \to \pi e\nu$) = (7.153 ± 0.037 ± 0.043) × 10^{-4}
0.4 + 1.6 fb^{-1}: 0.8% uncertainty

KLOE-2
PLB 804 (2020)

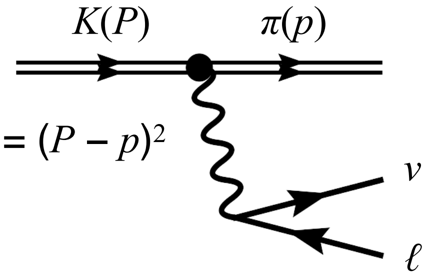
BR($K_S \to \pi\mu\nu$) = (4.56 ± 0.20) × 10^{-4}
First measurement of this BR

Normalizaton to $K_S \to \pi^+\pi^-$ for both channels

BR($K_S \to \pi^+\pi^-/\pi^0\pi^0$) also measured by KLOE

$K_{\ell 3}$ form factors

Hadronic matrix element:



$$\langle \pi | J_\alpha | K \rangle = f(0) \times \left[\tilde{f}_+(t)(P+p)_\alpha + \tilde{f}_-(t)(P-p)_\alpha \right]$$

K_{e3} decays: Only **vector form factor:** $\tilde{f}_+(t)$

$K_{\mu 3}$ decays: Also need **scalar form factor:** $\tilde{f}_0(t) = \tilde{f}_+ + \tilde{f}_- \dfrac{t}{m_K^2 - m_\pi^2}$

For V_{us}, need integral over phase space of squared matrix element:

Parameterize form factors and fit distributions in t (or related variables)

Form factor parameterization from dispersion relations

Bernard et al., PRD 80 (2009)

$$\tilde{f}_+(t) = \exp\left[\frac{t}{m_\pi^2} (\Lambda_+ - H(t)) \right]$$

$$\tilde{f}_0(t) = \exp\left[\frac{t}{m_K^2 - m_\pi^2} (\ln C - G(t)) \right]$$

- Allows tests of ChPT & low-energy dynamics
- Polynomials for $H(t)$, $G(t)$ from $K\pi$ scattering data

Uncertainties from $H(t)$, $G(t)$ contribute to fit results for Λ_+, $\ln C$

– Small compared to other uncertainties for single measurements (so far)

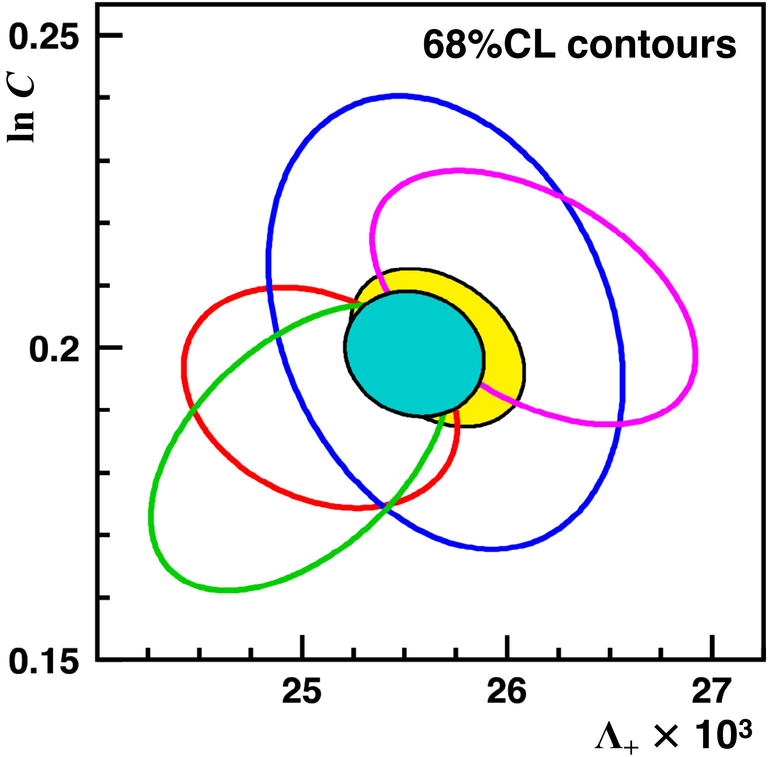
– Parameterization uncertainties beginning to dominate ***averages*** for Λ_+, $\ln C$

Dispersive parameters for $K_{\ell 3}$ form factors

$K_{\ell 3}$ avgs from KTeV | KLOE | ISTRA+ | NA48/2

NA48 K_{e3} data included in fits but not shown

2010 fit | Current



$\Lambda_+ \times 10^3$	=	25.55 ± 0.38
$\ln C$	=	0.1992(78)
$\rho(\Lambda_+, \ln C)$	=	−0.110
χ^2/ndf	=	7.5/7 (38%)

Integrals

Mode	Current value
$K^0{}_{e3}$	**0.15470(15)**
$K^+{}_{e3}$	**0.15915(15)**
$K^0{}_{\mu 3}$	**0.10247(15)**
$K^+{}_{\mu 3}$	**0.10553(16)**

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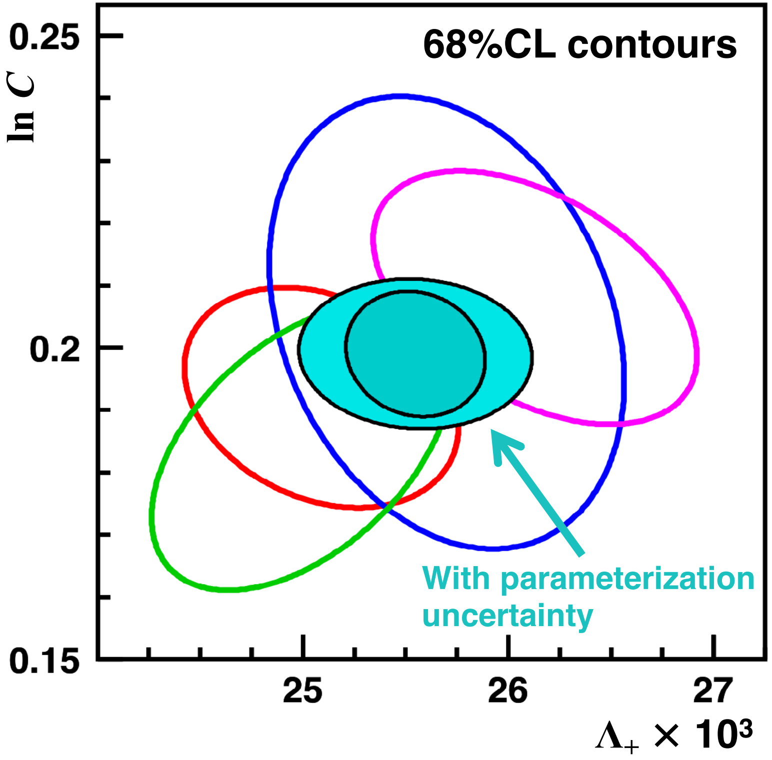

2010 fit | Current

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Fit results include common uncertainty from $H(t)$, $G(t)$:

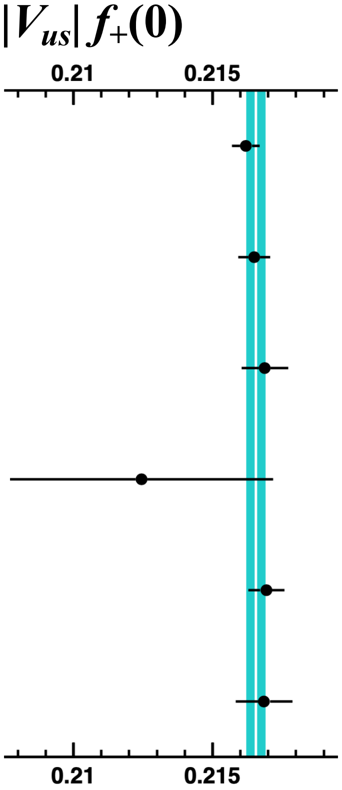
$\sigma_{\text{param}}(\Lambda_+) = 0.3 \times 10^{-3}$

$\sigma_{\text{param}}(\ln C) = 0.0040$

KTeV, Bernard et al. '09

Confidence ellipses shown **without** common uncertainty (except as indicated)

$|V_{us}| f_+(0)$ from world data (2022)

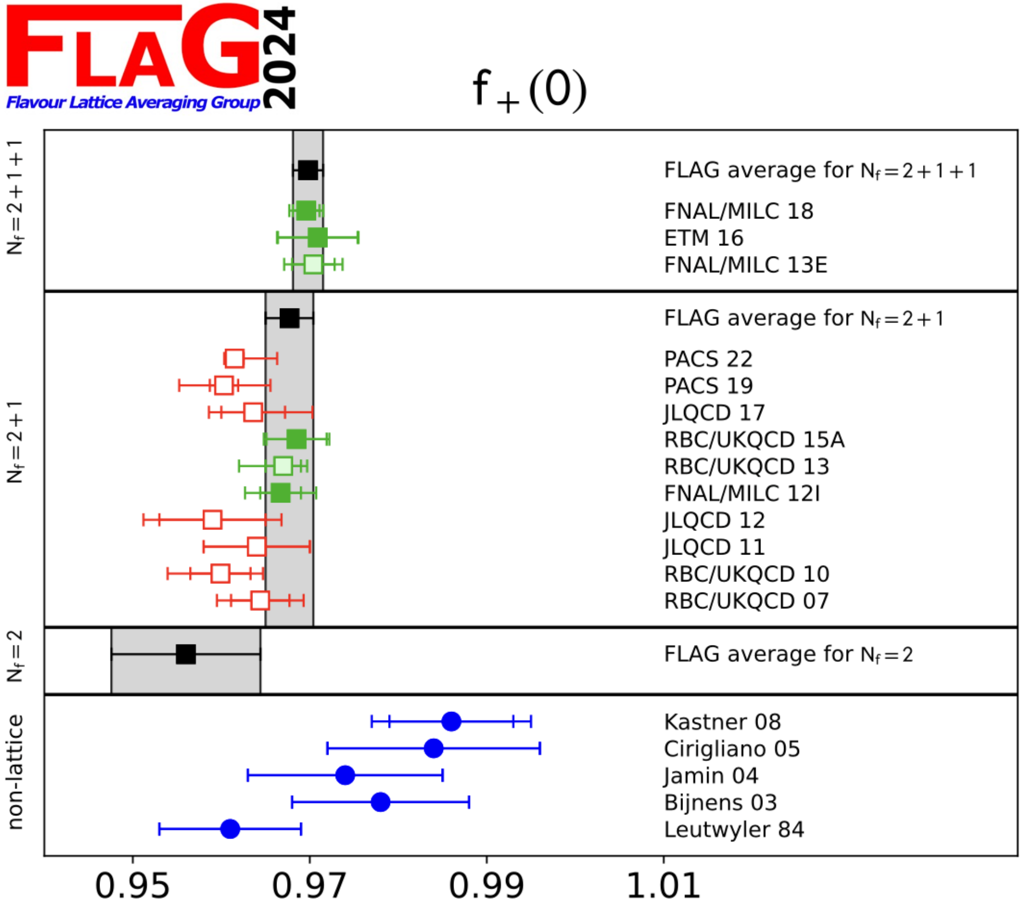


		% err	BR	τ	Δ	Int
$K_L e3$	**0.2162(5)**	**0.23**	0.09	0.20	0.02	0.05
$K_L \mu 3$	**0.2165(6)**	**0.26**	0.15	0.18	0.02	0.07
$K_S e3$	**0.2169(8)**	**0.39**	0.38	0.02	0.02	0.05
$K_S \mu 3$	**0.2125(47)**	**2.2**	2.2	0.02	0.02	0.08
$K^\pm e3$	**0.2169(6)**	**0.30**	0.27	0.06	0.11	0.05
$K^\pm \mu 3$	**0.2168(10)**	**0.47**	0.45	0.06	0.11	0.08

Approx. contrib. to % err from: BR, τ, Δ, Int

Average: $|V_{us}| f_+(0) = 0.21656(35)$ χ^2/ndf = 1.89/5 (86%)

Lattice results for $f_+(0)$



FLAG '24 averages:

N_f = 2+1+1 **$f_+(0)$ = 0.9698(17)**

Uncorrelated average of:

FNAL/MILC 18: HISQ, 5sp, $m_\pi \to$ 135 MeV, new ensembles added to FNAL/MILC 13E

ETM 16: TwMW, 3sp, $m_\pi \to$ 210 MeV, full q^2 dependence of f_+, f_0

N_f = 2+1 **$f_+(0)$ = 0.9677(27)**

Uncorrelated average of:

FNAL/MILC 12I: HISQ, $m_\pi \sim$ 300 MeV

RBC/UKQCD 15A: DWF, $m_\pi \to$ 139 MeV

JLQCD 17 not included because only single lattice spacing used

ChPT **$f_+(0)$ = 0.970(8)**

Ecker 15, Chiral Dynamics 15:
Calculation from Bijnens 03,
with new LECs from Bijnens, Ecker 14

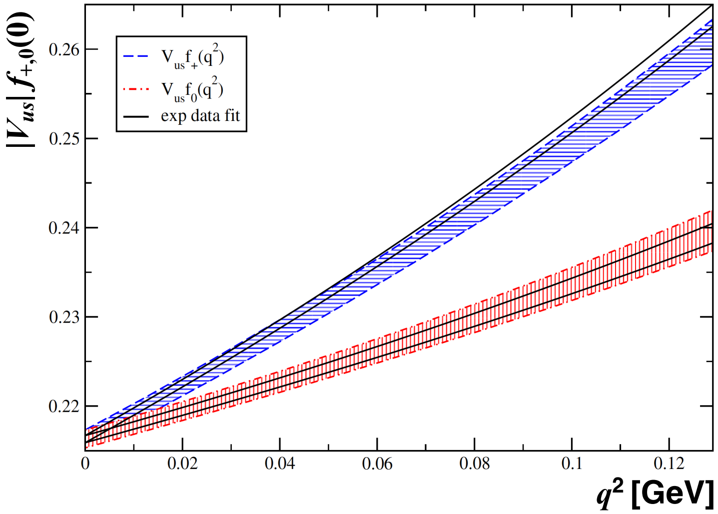
Lattice results for $f_+(0)$

ETM
PRD 93 (2016)

N_f **= 2+1+1**

$f_+(0) = 0.9709(44)_{st}(9)_{sy}(11)_{ext}$

Full q^2 dependence of f_+, f_0



Fit synthetic data points with dispersive parameterization

$\boldsymbol{\Lambda_+ = 24.22(1.16) \times 10^{-3}}$

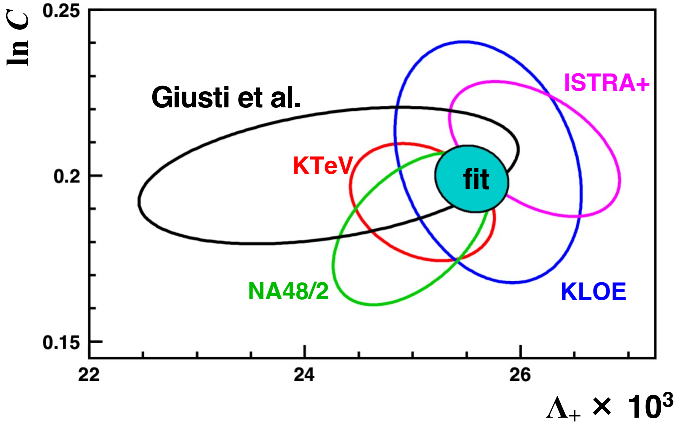
$\boldsymbol{\ln C = 0.1998(138)}$

$\rho(\Lambda_+, f_+(0)) = -0.228$

$\rho(\ln C, f_+(0)) = -0.719$

$\rho(\Lambda_+, \ln C) = +0.376$

See also:
PACS PRD 101 (2020)
ETM PRD 105 (2022)



- Basic agreement with experimental results
- Confirms basic correctness of lattice calculations for $f_+(0)$
- In the near future FF parameters will be obtained on lattice?

V_{us}/V_{ud} and $K_{\ell 2}$ decays

$$\frac{|V_{us}|}{|V_{ud}|}\frac{f_K}{f_\pi} = \left(\frac{\Gamma_{K_{\mu 2(\gamma)}} m_{\pi^\pm}}{\Gamma_{\pi_{\mu 2(\gamma)}} m_{K^\pm}}\right)^{1/2} \frac{1 - m_\mu^2/m_{\pi^\pm}^2}{1 - m_\mu^2/m_{K^\pm}^2} \left(1 - \frac{1}{2}\delta_{\mathrm{EM}} - \frac{1}{2}\delta_{SU(2)}\right)$$

Inputs from experiment:

From $K^\pm$ BR fit:

BR($K^\pm_{\mu 2(\gamma)}$) = 0.6358(11)

$\tau_{K\pm}$ = 12.384(15) ns

From PDG:

BR($\pi^\pm_{\mu 2(\gamma)}$) = 0.9999

$\tau_{\pi\pm}$ = 26.033(5) ns

Inputs from theory:

δ_{EM} Long-distance EM corrections

$\delta_{SU(2)}$ Strong isospin breaking

$f_K/f_\pi \rightarrow f_{K\pm}/f_{\pi\pm}$

f_K/f_π Ratio of decay constants

Cancellation of lattice-scale uncertainties from ratio

NB: Most lattice results already corrected for $SU(2)$-breaking: $f_{K\pm}/f_{\pi\pm}$

V_{us}/V_{ud} and $K_{\ell 2}$ decays

Giusti et al.
PRL 120 (2018)

First lattice calculation of EM corrections to P_{l2} decays

- Ensembles from ETM
- $N_f = 2+1+1$ Twisted-mass Wilson fermions

$$\delta_{SU(2)} + \delta_{\mathrm{EM}} = -0.0122(16)$$

- Uncertainty from quenched QED included (0.0006)

Compare to ChPT result from Cirigliano, Neufeld '11:

$$\delta_{SU(2)} + \delta_{\mathrm{EM}} = -0.0112(21)$$

Di Carlo et al.
PRD 100 (2019)

Update, extended description, and systematics of above

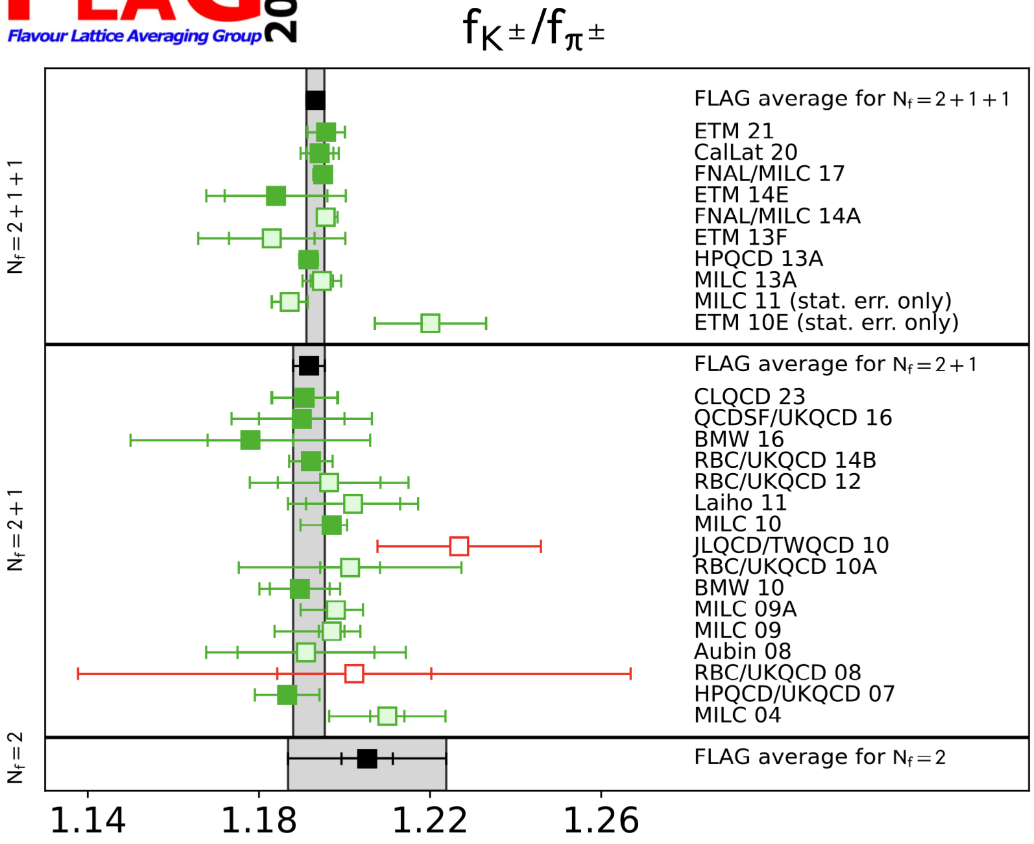
$$\delta_{SU(2)} + \delta_{\mathrm{EM}} = -0.0126(14)$$

See also:
RBC/UKQCD
JHEP 23 02 242

$$|V_{us}/V_{ud}| \times f_K/f_\pi = 0.27679(28)_{\mathrm{BR}}(20)_{\mathrm{corr}}$$

Lattice results for $f_{K\pm}/f_{\pi\pm}$

FLAG 2024
Flavour Lattice Averaging Group



$N_f = 2+1+1$ $f_{K\pm}/f_{\pi\pm} = 1.1934(19)$

ETM 21
TwM quarks, 3sp, $m_\pi \to$ physical
Updates ETM 14E

Miller 20
HISQ + DWF, 4sp, $m_\pi \to$ 130 MeV
Uses FNAL/MILC ensembles

FNAL/MILC 17
HISQ, 4sp, m_π phys
Updates MILC 13A, FNAL/MILC 14A

HPQCD 13A
HISQ, 3sp, m_π phys
Uses FNAL/MILC ensembles

$N_f = 2+1$ $f_{K\pm}/f_{\pi\pm} = 1.1916(34)$

CLQCD 23
Clover, 3sp, $m_\pi \to$ physical

QCDSF/UKQCD 16
Clover, 4sp, $m_\pi \to$ 220 MeV

BMW 16
Clover, 5sp, $m_\pi \to$ 139 MeV

V_{us} from kaon decays: Summary

$K_{\ell 3}$

$f_+(0) = 0.9698(17)$

$N_f = 2+1+1$

V_{us} **= 0.22330(35)$_{\text{exp}}$(39)$_{\text{lat}}$(8)$_{\text{IB}}$**

(53)$_{\text{tot}}$ = 0.24%

$K_{\mu 2}$

$f_K/f_\pi = 1.1978(22)$*

$N_f = 2+1+1$

V_{us}/V_{ud} **= 0.23108(23)$_{\text{exp}}$(42)$_{\text{lat}}$(16)$_{\text{IB}}$**

(51)$_{\text{tot}}$ = 0.22%

*Recalculated FLAG average for results without $SU(2)$-breaking as reported in original works

First hint of an anomaly: *Without information from β decays*

$$\Delta^{(3)}_{\text{CKM}} = |V_{us}^{K\ell 3}|^2 \left[\left(\frac{1}{|V_{us}/V_{ud}|^{K\mu 2}} \right)^2 + 1 \right] - 1$$

$\Delta^{(3)}_{\text{CKM}}$ = –0.0164(63)

–2.6σ

Need additional information to test consistency of $K_{\ell 3}$ and $K_{\mu 2}$

V_{us} from τ decays

Based mainly on work by HFLAV and talks by Alberto Lusiani

V_{us} from exclusive τ decays

$$\Gamma(\tau \to K\nu_\tau) = \frac{G_F^2}{16\pi\hbar} |V_{us}|^2 f_{K^\pm}^2 m_\tau^3 \left(1 - \frac{m_K^2}{m_\tau^2}\right)^2 S_{\mathrm{EW}} (1 + \delta R_K)$$

$$\frac{\Gamma(\tau \to K\nu_\tau)}{\Gamma(\tau \to \pi\nu_\tau)} = \frac{|V_{us}|^2}{|V_{ud}|^2} \frac{f_{K^\pm}^2}{f_{\pi^\pm}^2} \frac{(m_\tau^2 - m_K^2)^2}{(m_\tau^2 - m_\pi^2)^2} (1 + \delta R_{K/\pi})$$

Inputs from experiment:

HFLAV 2023 fit:

BR($K^-\nu_\tau$) = 0.00696(10)
BR($\pi^-\nu_\tau$) = 0.1081(5)
BR($K^-\nu_\tau/\pi^-\nu_\tau$) = 0.0644(9)

Radiative corrections:

Arroyo-Ureña et al., PRD104 (2021)
Large-N_C expansion

δR_K = (–0.15 ± 0.57)%
$\delta R_{K/\pi}$ = (0.10 ± 0.80)%

$\tau \to K\nu_\tau$

V_{us} = 0.2224(17)

HFLAV
2411.18639

$\tau \to K\nu_\tau/\tau \to \pi\nu_\tau$

V_{us}/V_{ud} = 0.2289(19)

N_f = 2+1+1, FLAG ’23:
$f_{K\pm}$ = 155.7 ± 0.3 MeV
$f_{K\pm}/f_{\pi\pm}$ = 1.1934(19)

Both about 2σ below unitarity

V_{us} from inclusive hadronic τ decays

$$R_\tau = \frac{\Gamma(\tau^- \to [\text{hadrons}]^- \nu_\tau(\gamma))}{\Gamma(\tau^- \to e^- \bar{\nu}_e \nu_\tau(\gamma))}$$

$$= R_{\tau\,\text{non-strange}} + R_{\tau\,\text{strange}}$$

vector + axial current

$SU(3)$ breaking:

$$\delta R_\tau^{\text{th}} = \frac{R_{\tau\,\text{non-strange}}}{|V_{ud}|^2} - \frac{R_{\tau\,\text{strange}}}{|V_{us}|^2}$$

Experimental inputs from HFLAV 2023:

$R_{\tau\ \text{non-strange}}/|V_{ud}|^2 \approx 3.470(8)$

$R_{\tau\ \text{strange}} = 0.1632(27)$

Theoretical inputs:

$\delta R_\tau^{\text{th}} = 0.239(32)$ for $m_s(m_\tau) = 93 \pm 7$ MeV

OPE with fixed-order or contour-improved perturbation theory for contributions up to $D = 2$

E. Gamiz et al., hep-ph/0612154v1

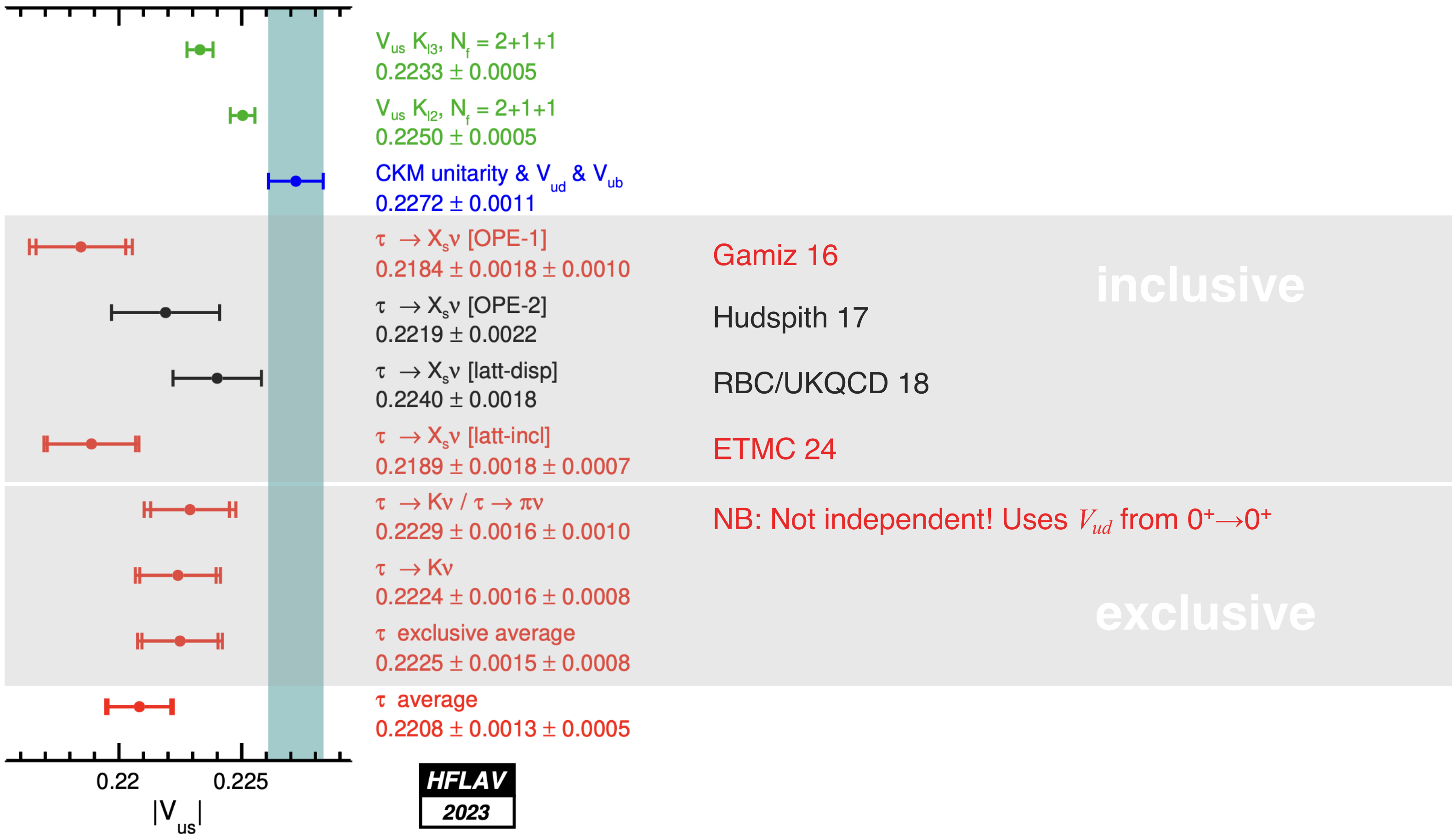
$\delta R_\tau^{\text{th}}$ from finite-energy sum rules (FESR):

$$R_\tau^w(s_0) \equiv \int_0^{s_0} ds \, \frac{w(s)}{w_\tau(s)} \frac{dR_\tau(s)}{ds}$$

$$\delta R_\tau^w(s_0) = \frac{R^w_{\tau\,\text{non-strange}}(s_0)}{|V_{ud}|^2} - \frac{R^w_{\tau\,\text{strange}}(s_0)}{|V_{us}|^2}$$

- $\delta R_\tau^w(s_0)$ has contributions up to $D = 8$
- $\delta R_\tau^w(s_0)$ has substantial dependence on s_0, w if contributions with $D > 4$ not negligible
  Hudspith et al., 2017
- Use lattice inputs for $D = 6$, 8 contributions
  Boyle et al. (RBC/UKQCD), 2018
- **New!** Direct lattice calculation of $R_{\tau\ \text{strange}}/|V_{us}|^2$
  Alexandrou et al. (ETMC), PRL 132 (2024)
  $R_{\tau\ \text{strange}}/|V_{us}|^2 = 3.407(22)$

V_{us} from exclusive τ decays

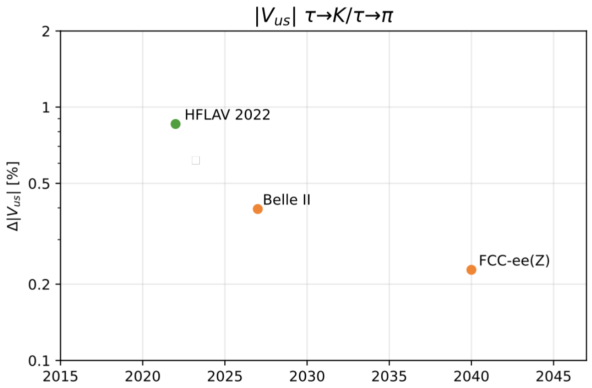


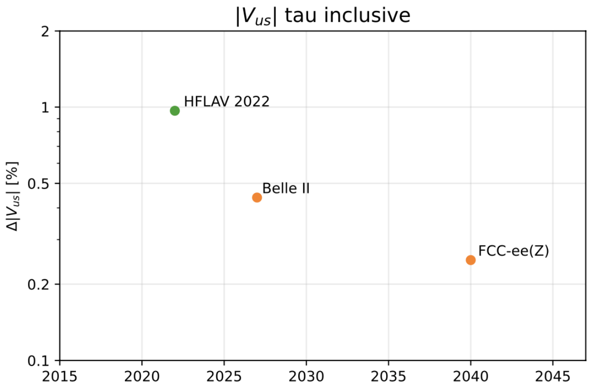
V_{us} from τ decays: Status and prospects

τ average	**V_{us} = 0.2208(14)**	0.63%

Currently uncertainty about 3x larger than for K decays
Significance of unitarity deficit is about the same (τ value for V_{us} a bit lower)

Prospects for improvement: A. Lusiani, Electro 2022





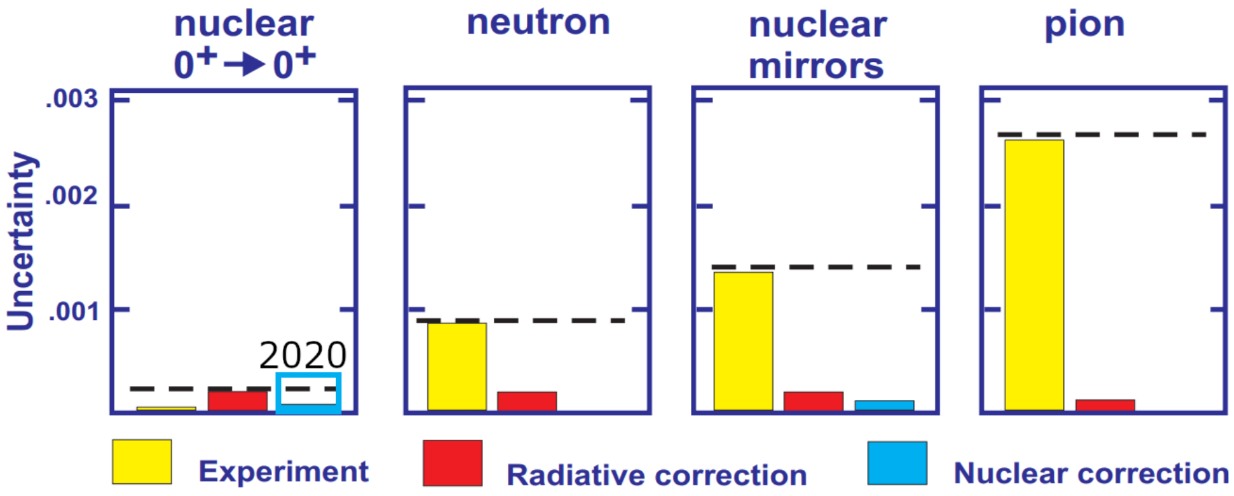
Enormous increases in statistics expected from Belle II (50-100x *BABAR*, Belle)
To be competitive with K decays, need statistics from FCC-ee (1000x ALEPH)

$$V_{ud}$$

Experimental determination of V_{ud}

1. **$0^+ \rightarrow 0^+$ nuclear β decays** (superallowed Fermi transitions)
2. **Neutron β decay**
3. **T = 1/2 nuclear mirror decays**
4. **Pion β decay**

J. Hardy, Amherst 2019



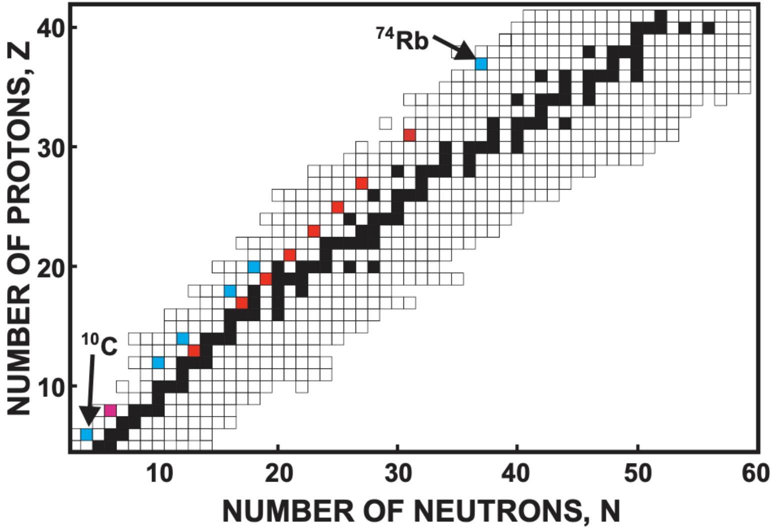
$|V_{ud}|$ from $0^+ \to 0^+$: World data

$$ft = \frac{K}{G_V^2 \langle\tau\rangle^2}$$

$f(Z, Q_{EC})$	statistical rate function
$t = t_{1/2}/\mathrm{BR}$	partial half life
$G_V = G_F V_{ud}$	vector coupling constant
$\langle\tau\rangle$	Fermi matrix element

Experimentally, need to measure:

- **BR branching ratios**
- $t_{1/2}$ **parent half-life**
- Q_{EC} **transition energy**



Hardy & Towner
PRC 102 (2020)

Comprehensive survey of ft measurements

- 9 cases with precision < 0.05%
- 6 cases with precision 0.05-0.23%
- About 220 individual measurements with compatible precison

$|V_{ud}|$ from $0^+ \to 0^+$: Corrections

$$\mathcal{F}t \equiv ft(1+\delta'_R)(1+\delta_{NS}-\delta_C) = \frac{K}{2G_V^2\left(1+\Delta_R^V\right)}$$

Δ_R Universal radiative correction

High-energy γW box + ZW box amplitudes

δ_R' Long-distance radiative correction

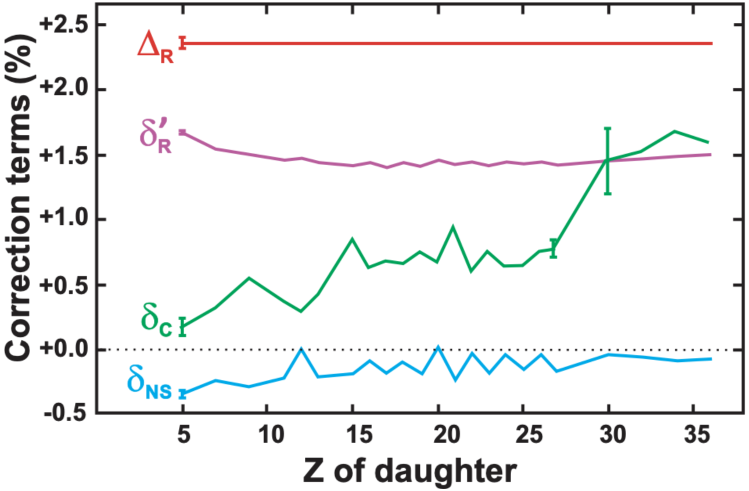
One-photon bremsstrahlung + low-energy γW box

δ_C Coulomb correction

Charge-dependent mismatch between parent and daughter analog states (members of same isospin triplet)

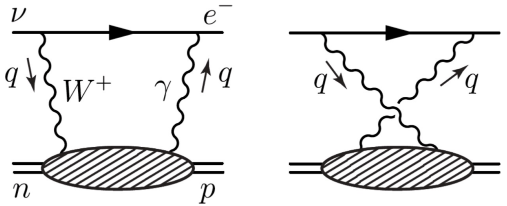
δ_{NS} Nuclear structure

O(α) axial photonic contributions



Consistency check: CVC demands equivalence of $\mathcal{F}t$ values after corrections

V_{ud} and inner radiative correction Δ_R^V



Box diagrams contributing at order α/π to neutron β decay at the hadronic scale

Hardy & Towner
1807.01146

$|V_{ud}|$ = 0.97420(21) Δ_R^V = 2.361(38)%
Δ_R^V from Marciano & Sirlin '06
First-row CKM unitarity respected

Seng et al.
PRD 100 (2019)

$|V_{ud}|$ = 0.97370(14) Δ_R^V = 2.467(22)%
New calculation of γW-box contribution to Δ_R^V using dispersion relations and DIS structure functions
3σ shift in Δ_R^V and V_{ud}: the birth of the anomaly!
Also identified need for new calculations of $\delta_{\rm NS}$

Hardy & Towner
PRC 102 (2020)

$|V_{ud}|$ = 0.97373(31) Δ_R^V = 2.454(19)%
23 new publications, some older measurements eliminated
Use weighted average of above values for Δ_R^V
Larger uncertainty for $\delta_{\rm NS}$

$|V_{ud}|$ from $0^+ \to 0^+$: Recent theoretical work

$\boldsymbol{\Delta_R}$ Universal RC	**Cirgliano et al. PRD108 (2023)** – EFT **Ma et al. PRL132 (2024)** – lattice estimate using RBC/UKQCD ensembles
$\boldsymbol{\delta_{\mathrm{NS}}}$ Nuclear structure RC **Currently dominates uncertainty in V_{ud}**	**Cirigliano et al. PRL133 (2024)** – EFT **Gennari et al. PRL143 (2025)** – ab initio calculation with shell model and chiral EFT, with $^{10}C \to {}^{10}B$ as an example
$\boldsymbol{\delta_R'}$ Long-distance RC	**Cao et al. 2511.05446** – analytical, relativistic limit **Crosas, Mereghetti 2511.05481** – EFT
$\boldsymbol{\delta_C}$ Coulomb IB correction	**Seng, Gorchtein PLB838 (2023), PRC109 (2024)** – ab initio calculation cross-checked with EW charge radii
$\boldsymbol{ft}$ Electroweak form factor	**Seng PRL 130 (2023)** – model independent technique from EW charge radius **Seng, Gorchtein PRC109 (2024)** – new ft values for 15 different decays
Alternative schemes for evaluation of V_{ud}	**Cirigliano et al. PRC110 (2024)** – ab initio calculation of V_{ud} in EFT, with $^{14}O \to {}^{14}N$ as an example

$|V_{ud}|$ from neutron β decays

$$|V_{ud}|^2 = \frac{5024.7\mathrm{s}}{\tau_n\,(1+3\lambda^2)\,(1+\Delta_R)}$$

τ_n Free neutron lifetime

$\lambda = g_A/g_V$ Ratio of axial to vector couplings

Δ_R Radiative correction (universal + outer)

- Δ_R under control to same extent as in $0^+\to0^+$
- To match precision from $0^+\to0^+$ require $\sigma_\tau \sim 0.3$ s and $\sigma_\lambda/\lambda \sim 3\times10^{-4}$
- World data set for τ and λ riddled by inconsistencies → large scale factors
  → Use recent high-precision measurements instead of averages

UCNτ PRL 127 (2021)	$\boldsymbol{\tau_n = 877.75(28)_{\mathrm{stat}}(^{+22}_{-16})_{\mathrm{sys}}}$ **s** Ultra-cold neutron trap Improves on precision of previous results by > 2x
PERKEO III PRL 122 (2019)	$\boldsymbol{\lambda = -1.27641(45)_{\mathrm{stat}}(33)_{\mathrm{sys}}}$ β decay asymmetry 5x improvement on precision of world average

Combined result for $|V_{ud}|$

Cirigliano et al.
PLB 838 (2023)

Evaluation of Δ_R^V and Δ_R:

- Hadronic scheme for resummation of infrared logs
- Non-correlated average of contributions to γW box

V_{ud} from neutron decays uses current best measurements (not averages) for τ_n and $\lambda = g_A/g_V$

$0^+ \to 0^+$ with $\Delta_R^V = 2.467(27)\%$

$\boldsymbol{V_{ud}^{0+\to 0+} = 0.97367(11)_{\text{exp}}(13)_{\Delta RV}(27)_{\text{NS}}}$ $[32]_{\text{tot}}$, 0.033%

n decays with $\Delta_R = 3.983(27)\%$

$\boldsymbol{V_{ud}^{n,\text{ best}} = 0.97413(13)_{\Delta R}(35)_{\lambda}(20)_{\tau_n}}$ $[43]_{\text{tot}}$, 0.044%

0.9σ agreement ▶

Average
$\boldsymbol{|V_{ud}| = 0.97384(26)}$

$|V_{ud}|$ from pion β decays

$$\Gamma_{\pi\beta} = \frac{G_\mu^2 |V_{ud}|^2}{30\pi^3}\left(1 - \frac{\Delta}{2M_+}\right)^3 \Delta^5 f(\epsilon, \Delta)(1+\delta)$$

$\Delta = m_{\pi+} - m_{\pi-}$
$\epsilon = (m_e/\Delta)^2$
$f(\epsilon, \Delta)$ = Fermi function

- Experimentally, need to measure BR($\pi^+ \to \pi^0 e\nu$) and lifetime $\tau_{\pi+}$
- Radiative correction $\delta \sim 3.3\%$, very well controlled (lattice, Feng et al., 2020)

PIBETA
PRL 93 (2004)

BR($\pi^+ \to \pi^0 e\nu$) = $1.036(4)_{\text{stat}}(4)_{\text{sys}}(3)_{\pi e2} \times 10^{-8}$
Decays at rest

$\boxed{V_{ud}^{\pi\beta} = 0.97386(281)_{BR}(9)_\tau(14)_\delta(28)_f}$

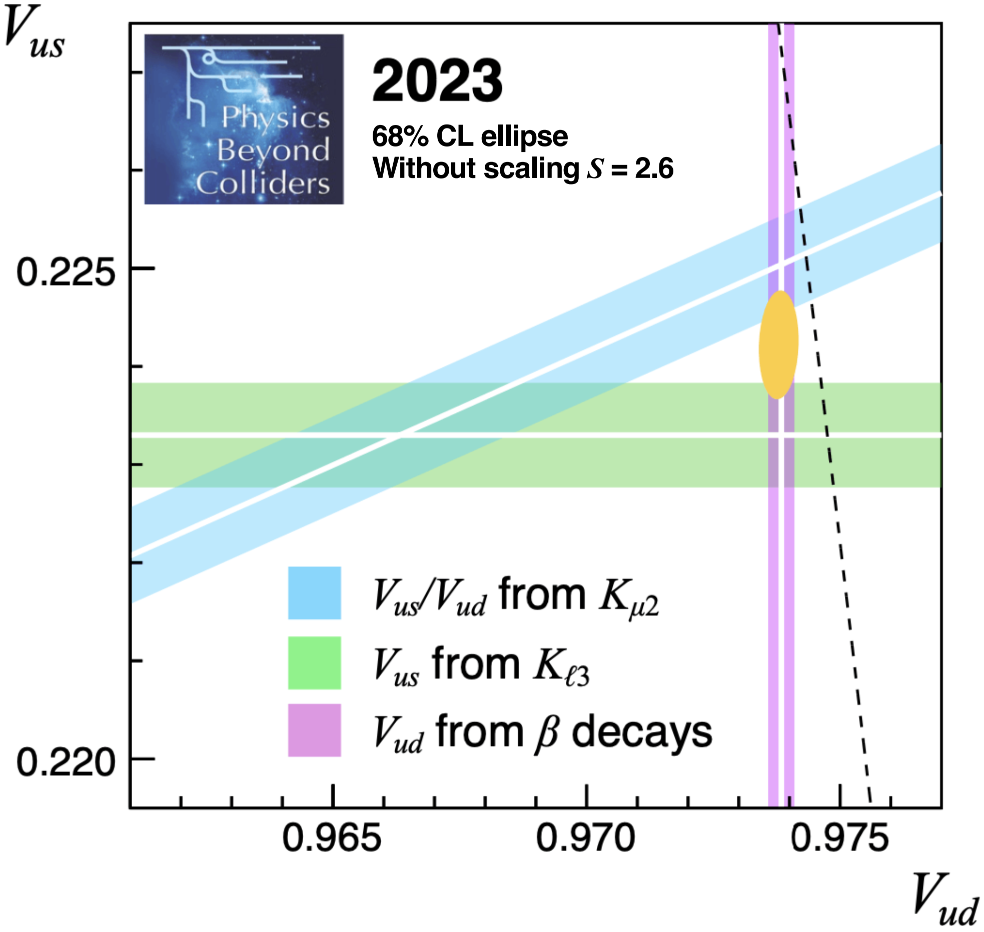
Cirigliano et al.
PLB838 (2023)

$\tau_{\pi+}$ from
PDG 2022

- Phase-2 goal of recently proposed PIONEER experiment:
  Reduce uncertainty on BR: 0.6% → **0.02%** (competitive with $0^+ \to 0^+$)
- Completely independent of $0^+ \to 0^+$: No nuclear-structure corrections and different radiative corrections

First-row CKM unitarity: Status and prospects

Status of first-row unitarity



Fit results, no constraint

V_{ud} = **0.97378(26)**

V_{us} = **0.22422(36)**

χ^2/ndf = 6.4/2 (4.1%)

Δ_{CKM} = **–0.0018(6)**

–2.8σ

With scale factor S = **2.6**

V_{ud} = **0.9737(8)**

V_{us} = **0.2242(10)**

Status of first-row unitarity

3 observables: $|V_{us}|^{K\ell3}$, $|V_{us}/V_{ud}|^{K\mu2}$, V_{ud}
2 quantities to determine: V_{us}, V_{ud}
▶ **3 ways to test unitarity**

$$\Delta^{(1)}_{\mathrm{CKM}} = |V_{ud}|^2 + |V_{us}^{K_{\ell3}}|^2 - 1$$ **= –0.00176(56) –3.1σ**

$$\Delta^{(2)}_{\mathrm{CKM}} = |V_{ud}|^2 \left[1 + \left(\left|\frac{V_{us}}{V_{ud}}\right|^{K_{\mu2}}\right)^2\right] - 1$$ **= –0.00098(58) –1.7σ**

$K_{\mu2}$ result shows better agreement with unitarity than $K_{\ell3}$ result when $|V_{ud}|$ obtained from beta decays:

$$\Delta V_{us}(K_{\ell3} - K_{\mu2}) = V_{us}^{K_{\ell3}} - V_{ud}\left(\frac{V_{us}}{V_{ud}}\right)^{K_{\mu2}}$$ **= –0.0174(73) –2.4σ**

$\Delta^{(3)}_{\mathrm{CKM}}$ uses no information from β decays:

$$\Delta^{(3)}_{\mathrm{CKM}} = |V_{us}^{K_{\ell3}}|^2 \left[\left(\frac{1}{|V_{us}/V_{ud}|^{K_{\mu2}}}\right)^2 + 1\right] - 1$$ **= –0.0164(63) –2.6σ**

Constraints on right-handed currents

Cirigliano et al.
PLB 838 (2023)

In SM, W couples only to LH chiral fermion states

New physics with couplings to RH currents could explain both unitarity deficit and $K_{\ell 3}$-$K_{\mu 2}$ difference

Define

ϵ_R = admixture of RH currents in non-strange sector

$\epsilon_R + \Delta\epsilon_R$ = admixture of RH currents in strange sector

$$\Delta^{(1)}_{\mathrm{CKM}} = 2\epsilon_R + 2\Delta\epsilon_R V_{us}^2$$
$$\Delta^{(2)}_{\mathrm{CKM}} = 2\epsilon_R - 2\Delta\epsilon_R V_{us}^2$$
$$\Delta^{(3)}_{\mathrm{CKM}} = 2\epsilon_R + 2\Delta\epsilon_R (2 - V_{us}^2)$$

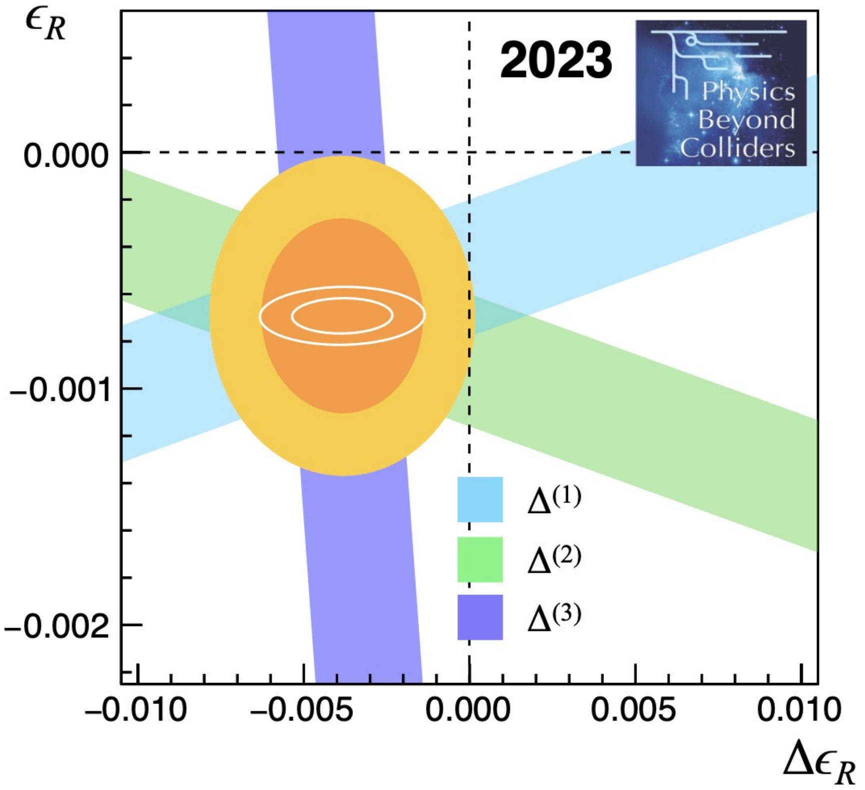
$$r \equiv \left(\frac{1 + \Delta^{(2)}_{\mathrm{CKM}}}{1 + \Delta^{(3)}_{\mathrm{CKM}}}\right)^{1/2} = \frac{\left.\frac{V_{us}}{V_{ud}}\right|_{K_{\ell 2}/\pi_{\ell 2}}}{\frac{V_{us}^{K_{\ell 3}}}{V_{ud}^{\beta}}} = 1 - 2\Delta\epsilon_R$$

From current fit:

$\epsilon_R = -0.69(27) \times 10^{-3}$ (2.5σ)

$\Delta\epsilon_R = -3.9(1.6) \times 10^{-3}$ (2.4σ)

$\epsilon_R = \Delta\epsilon_R = 0$ excluded at 3.1σ



What can NA62 contribute?

Dedicated low-intensity (~1% of nominal) data taking with minimum bias trigger:

One week each collected in 2024 and 2025!

- Sample size ensures minimal statistical uncertainties (0.1% for main K^+ BRs)
- Stable, low-intensity, min-bias conditions for minimal systematic uncertainties
- Suite of redundant measurements for good control of systematics
- Single analysis framework to maximize cancellation of systematics

Will allow key new measurements of the 6 main K^+ decays including:

- $K^+ \to \mu^+\nu$: BR from global fit currently dominated by a single result (KLOE)
- $K^+ \to \pi^0 e^+\nu$ and $K^+ \to \pi^0\mu^+\nu$: new inputs for BR($K_{\ell 3}$) $\propto |V_{us}|^2 f_+(0)^2$
- BR($K^+ \to \mu^+\nu$)/BR($\pi^+ \to \mu^+\nu$) $\propto |V_{us}|^2/|V_{ud}|^2\ (f_K/f_\pi)^2$ with $K^+ \to \mu^+$ selection:

  BR($\pi^+ \to \mu^+\nu$) from $K^+ \to \pi^+\pi^0$ with prompt $\pi^+ \to \mu^+\nu$ decay tagged by π^0

What can NA62 contribute?

NA62 can make a precision measurement of $K_{\mu 3}/K_{\mu 2}$, with many systematics cancelling
What can this measurement alone tell us?

r is proportional to $(K_{\mu 3}/K_{\mu 2})^{-1/2}$:

$$r \equiv \left(\frac{1+\Delta^{(2)}_{\mathrm{CKM}}}{1+\Delta^{(3)}_{\mathrm{CKM}}}\right)^{1/2} = \frac{\left.\frac{V_{us}}{V_{ud}}\right|_{K_{\ell 2}/\pi_{\ell 2}}}{\frac{V_{us}^{K_{\ell 3}}}{V_{ud}^{\beta}}} = 1 - 2\Delta\epsilon_R$$

- Uses input from β decays, but provides a qualified statement about consistency of data set
- Search for right-handed currents

NA62 hypothetical $K_{\mu 3}/K_{\mu 2}$ to 0.5%:

Result	$\Delta\epsilon_R$		Remarks
Same as fit	–4.0(1.9)×10^{-3}	2.1σ	Almost same precision as result from world average
+ 1.5σ	–0.4(1.9)×10^{-3}	0.2σ	$K_{\mu 2}$, $K_{\mu 3}$, V_{ud} consistent: current tensions have experimental origin?
– 1.5σ	–7.6(1.9)×10^{-3}	4.0σ	Evidence for contribution of right-handed currents to CKM non-unitarity

Longer term: V_{ud} from pion β decays with PIONEER

A next generation stopped-pion experiment at PSI

Proposal approved! (2203.01981)

Data taking to start in ~5 years

Phase 1: LFU tests

$R_{e/\mu}$ = BR($\pi_{e2(\gamma)}$)/BR($\pi_{\mu2(\gamma)}$) to 0.1% (2x improvement)

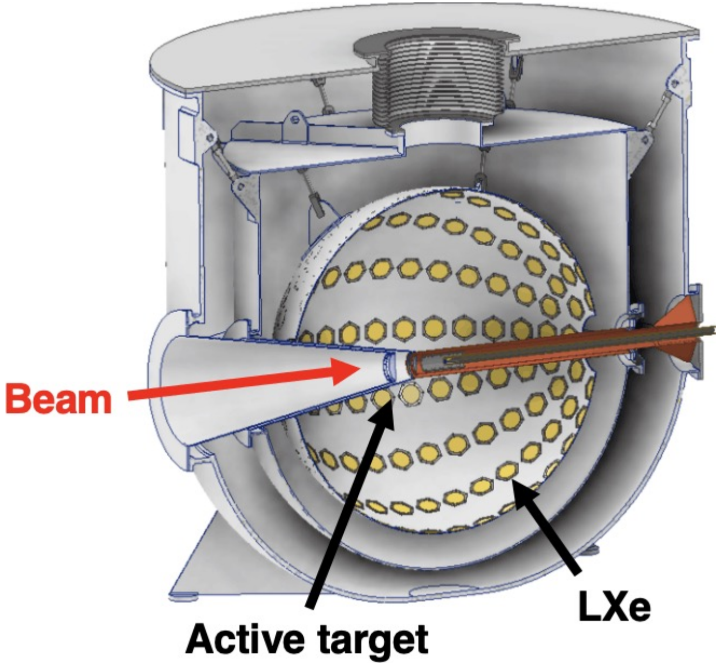
Distinguish e^+ from $\pi^+ \to e^+\nu$ vs. $\pi^+ \to \mu^+\nu$, $\mu^+ \to e^+\nu\nu$

Phases 2 & 3: BR($\pi^+ \to \pi^0 e^+ \nu$)

0.2% precision goal

Same concept as PIBETA:

- Detect kinematic signature of back-to-back $\gamma\gamma$ from π^0 decay at rest
- Normalize to $\pi^+ \to e^+\nu$



Goal: Reach precision of 0.02% on V_{ud} by end of phase 3

Competitive with $0^+ \to 0^+$ β decays with fewer theory uncertainties

Status of first-row unitarity

Experimental results from kaons

$|V_{us}|\, f_+(0) = 0.21656(35)$

$|V_{us}/V_{ud}| \times f_K/f_\pi = 0.27679(34)$

With $|V_{ud}|(\beta)$ and $N_f = 2{+}1{+}1$ lattice

$\Delta^{(1)}_{\rm CKM} = -0.00176(56) = -3.1\sigma$

$\Delta^{(2)}_{\rm CKM} = -0.00098(58) = -1.7\sigma$

Fit to both gives $\Delta_{\rm CKM} = -2.8\sigma$ and 3.1σ evidence for right-handed currents

$K_{\mu 2}$ result shows better agreement with unitarity than $K_{\ell 3}$ result when $|V_{ud}|$ obtained from beta decays

New measurement of $K_{\mu 3}/K_{\mu 2}$ (e.g. from NA62) could be very helpful in distinguishing if origin of discrepancy is experimental

- Other measurements of main K BRs also very important!

Precision in kaon sector has strongly motivated further progress on V_{ud}, especially in theoretical calculation of radiative corrections!