Kaon radiative leptonic decays from lattice QCD

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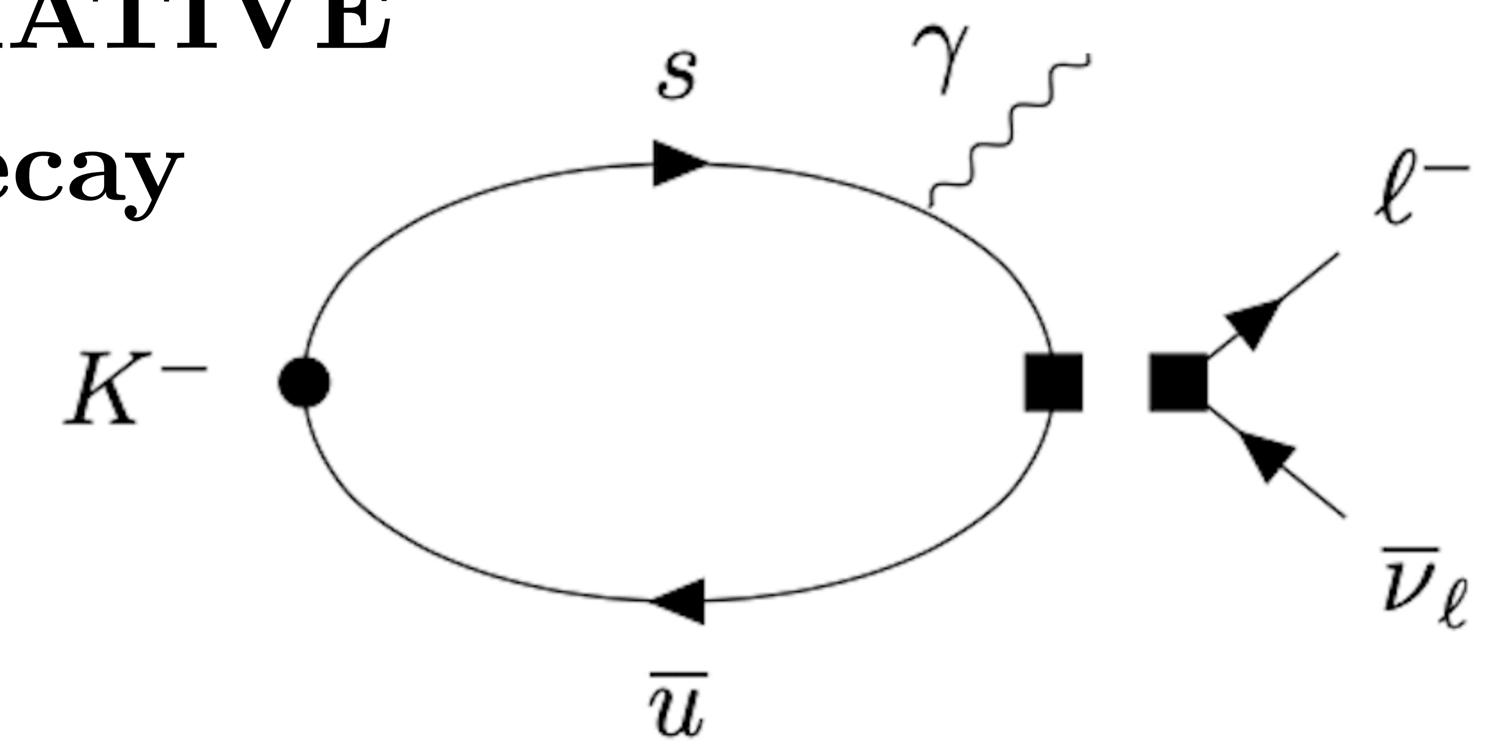
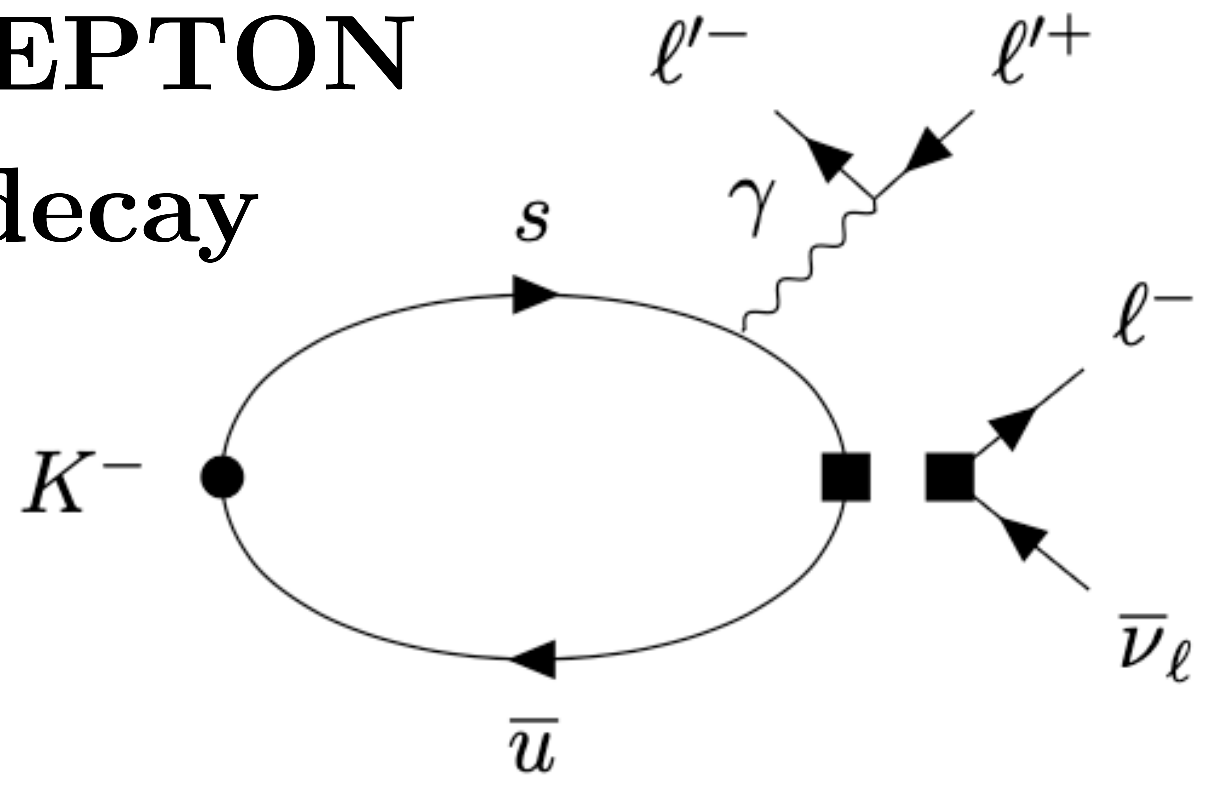
Kaon leptonic decays

RADIATIVE
decay



4-LEPTON
decay



- Same non-perturbative input for radiative and 4-Lepton decay rates

$$d\Gamma\left(\boldsymbol{k}_\gamma, \sqrt{k_\gamma^2}\right) = \left| f_K A^{\mathrm{pert}} + A^{\mathrm{non-pert}} \right|^2$$

- The non-perturbative part is encoded in the hadronic tensor $H_w^{\mu\nu}(E_\gamma, \boldsymbol{k}_\gamma)$

$$A^{\mathrm{non-pert}} = \boxed{H_w^{\mu\nu}(E_\gamma, \boldsymbol{k}_\gamma)}\, L_{\mu\nu}$$

Form factors

- The hadronic tensor is defined as

$$H_w^{\mu\nu}(E_\gamma, \boldsymbol{k}_\gamma) = \int dt\, e^{iE_\gamma t} \int d^3x\, e^{-i\boldsymbol{k}_\gamma\cdot\boldsymbol{x}}\, \langle 0 \,|\, \mathrm{T}[J^\mu_{\mathrm{em}}(t, \boldsymbol{x}) J^\nu_w(0)] \,|\, K^- \rangle$$

- The hadronic tensor is described in terms of

Point-like term

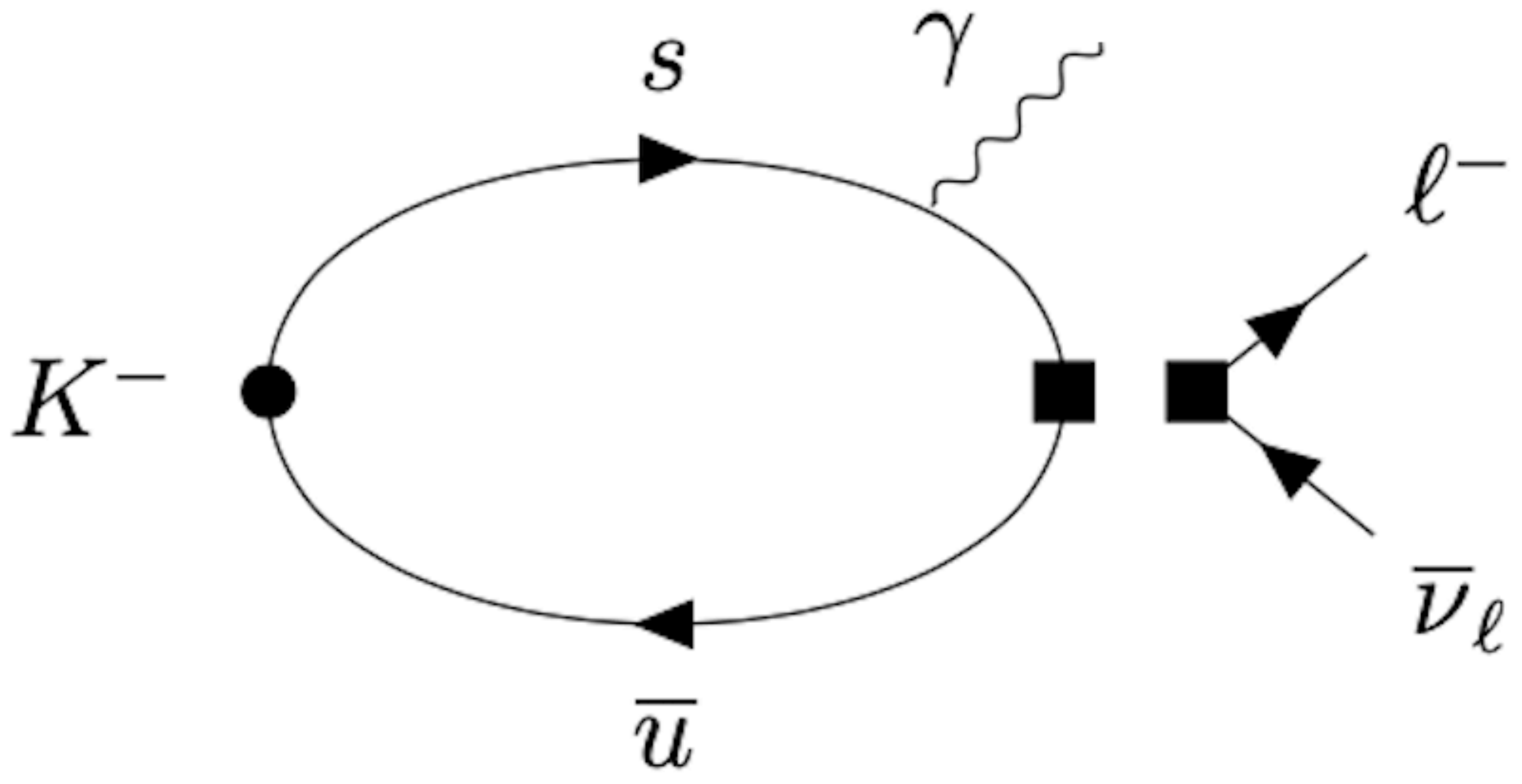
f_K

$+$

4 Structure-dependent form factors

F_V, F_A, H_1, H_2

Radiative decay



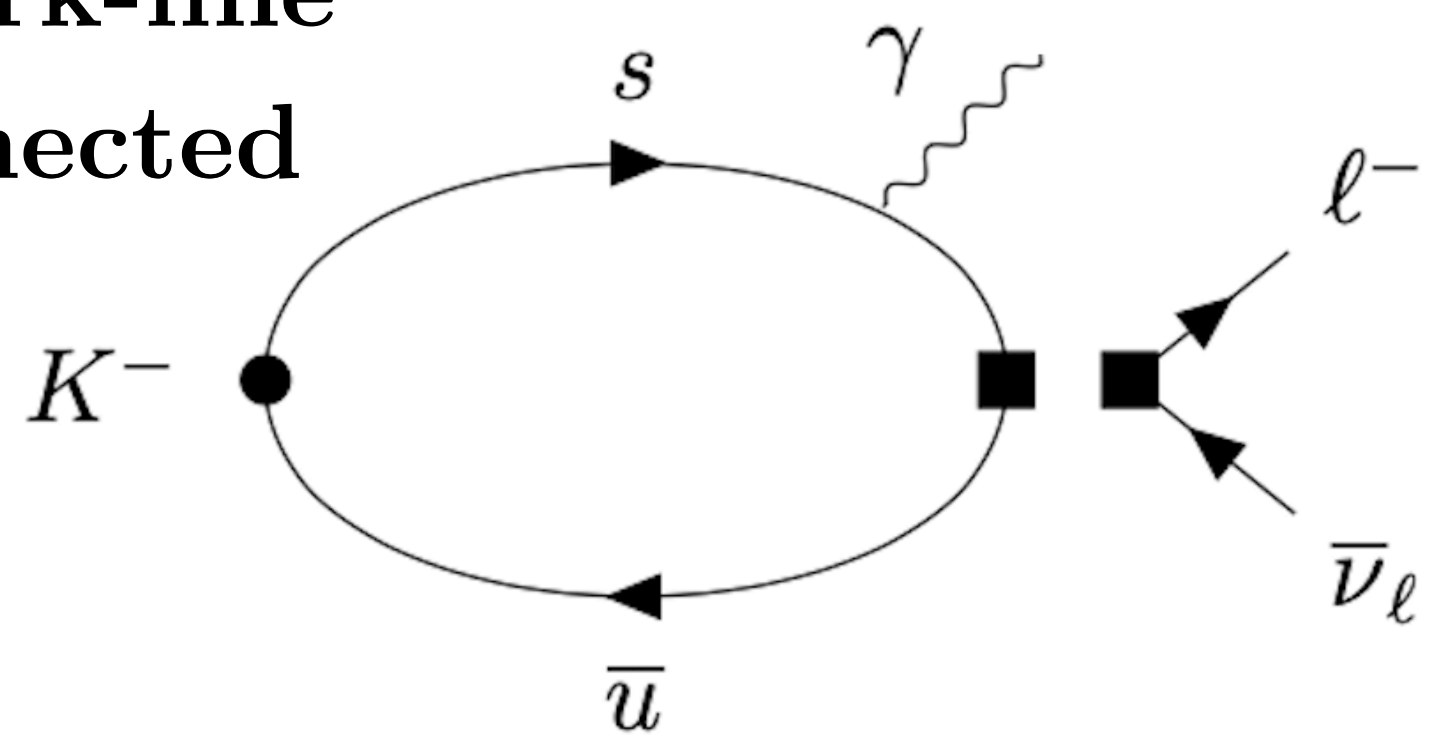
from *Phys.Rev.D* 111 (2025) 11, 114523

R. Di Palma, R. Frezzotti, G. Gagliardi,
V. Lubicz, G. Martinelli, C.T. Sachrajda,
F. Sanfilippo, S. Simula, N. Tantalo

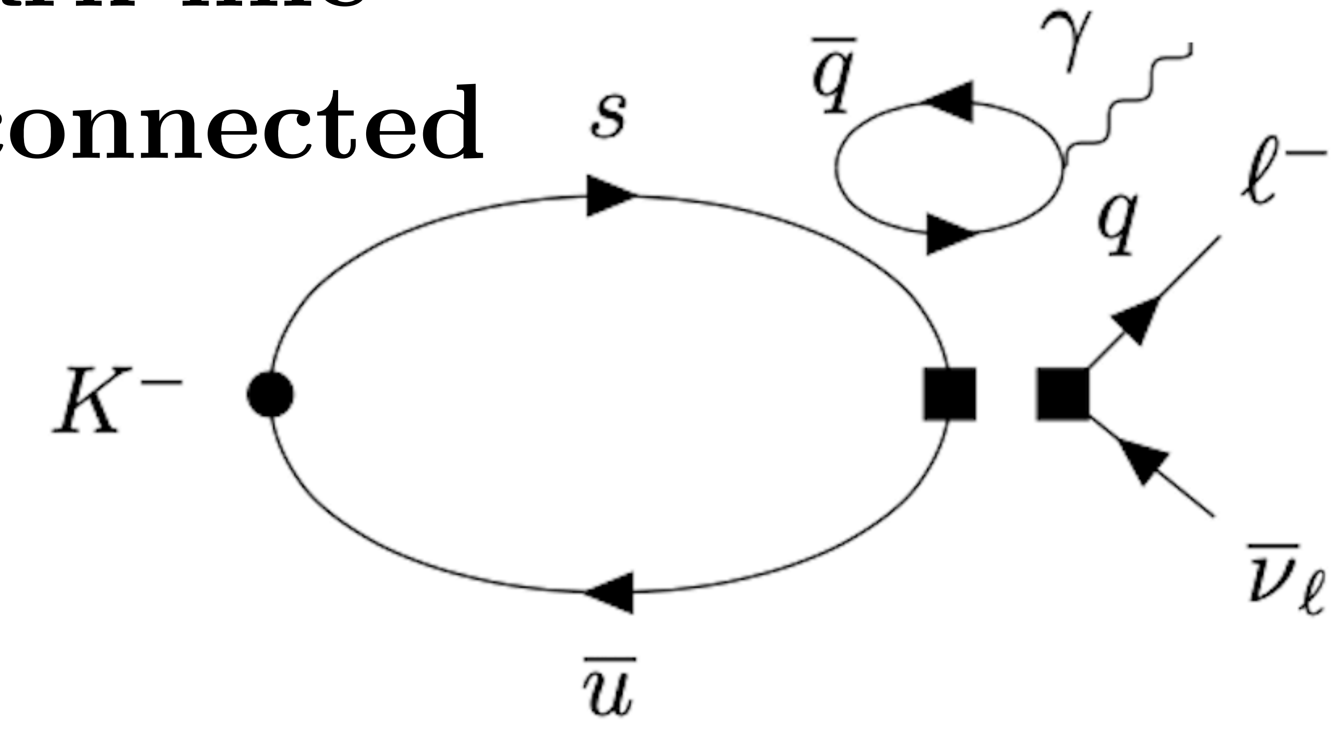
Why radiative decay?

- Sensitive to BSM currents different from the standard V−A
- Alternative determination of the Cabibbo angle $|V_{us}|$
- ChPT not enough and model-dependent at this level of precision ($\simeq 5/6\,\%$)
- $\boxed{\text{First time all the contributions at } O(\alpha_{\text{em}}) \text{ are computed}}$

Quark-line connected

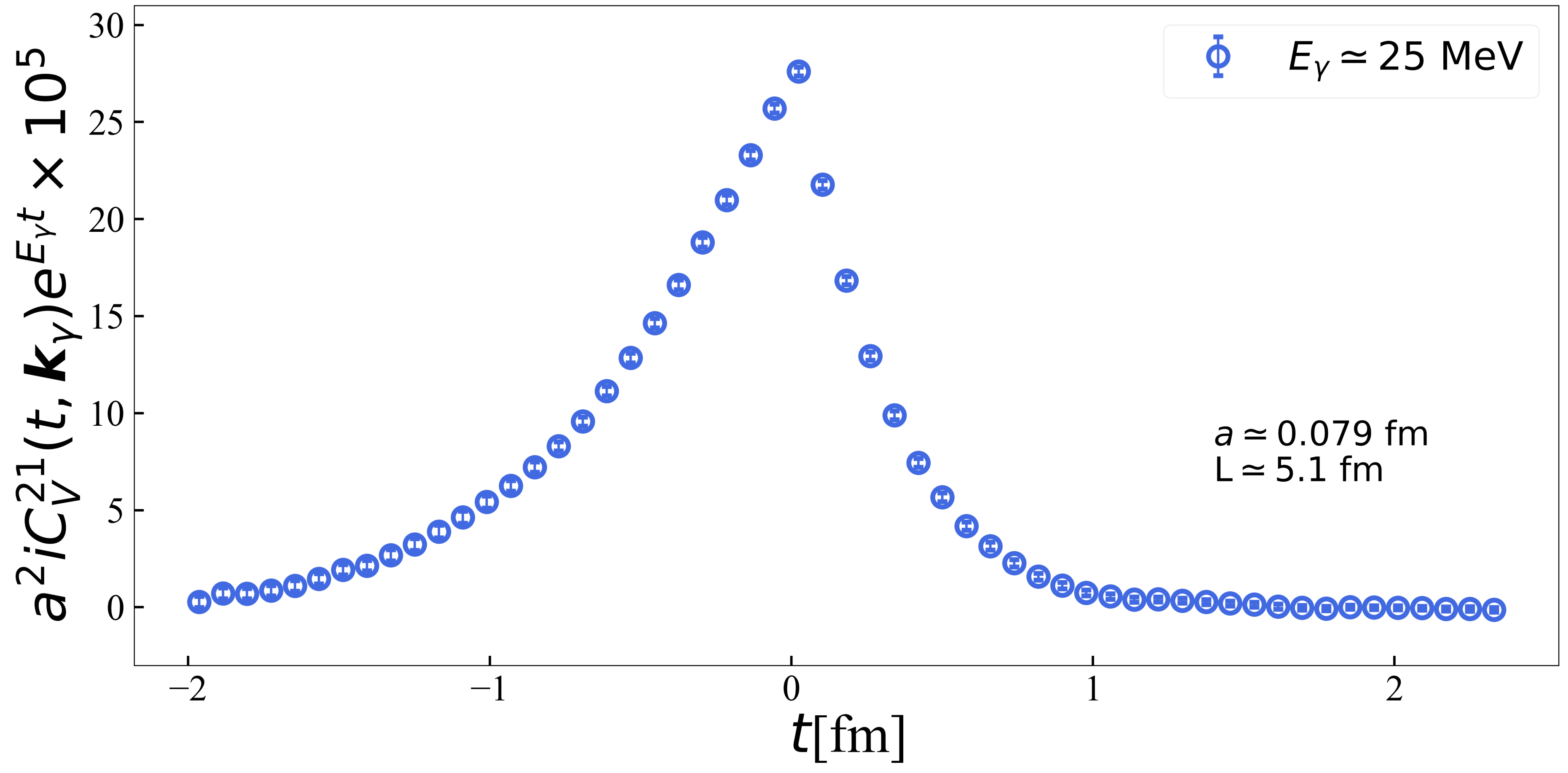


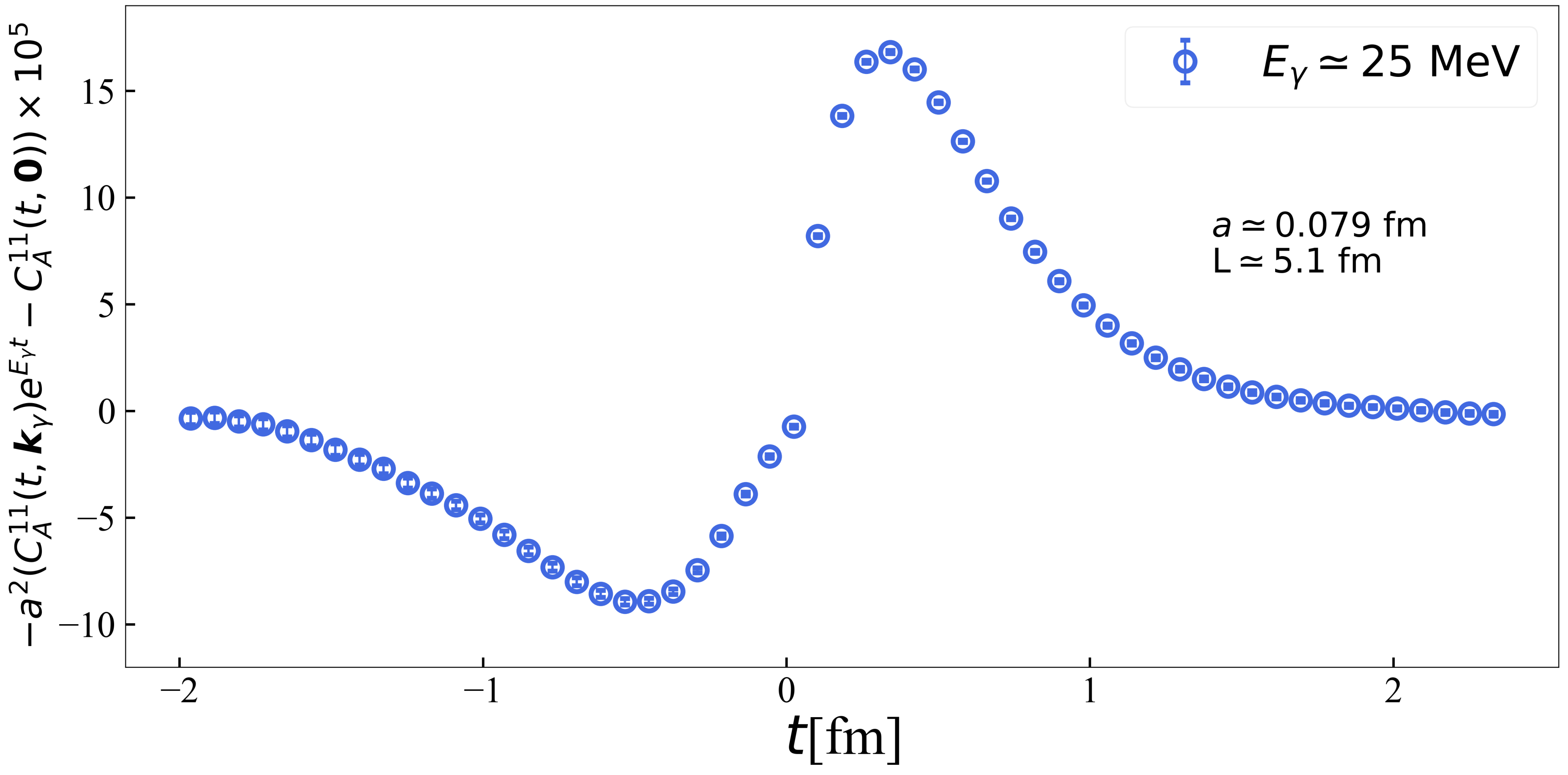
Quark-line disconnected



Lattice input

- Our lattice input are the so-called Euclidean correlation functions

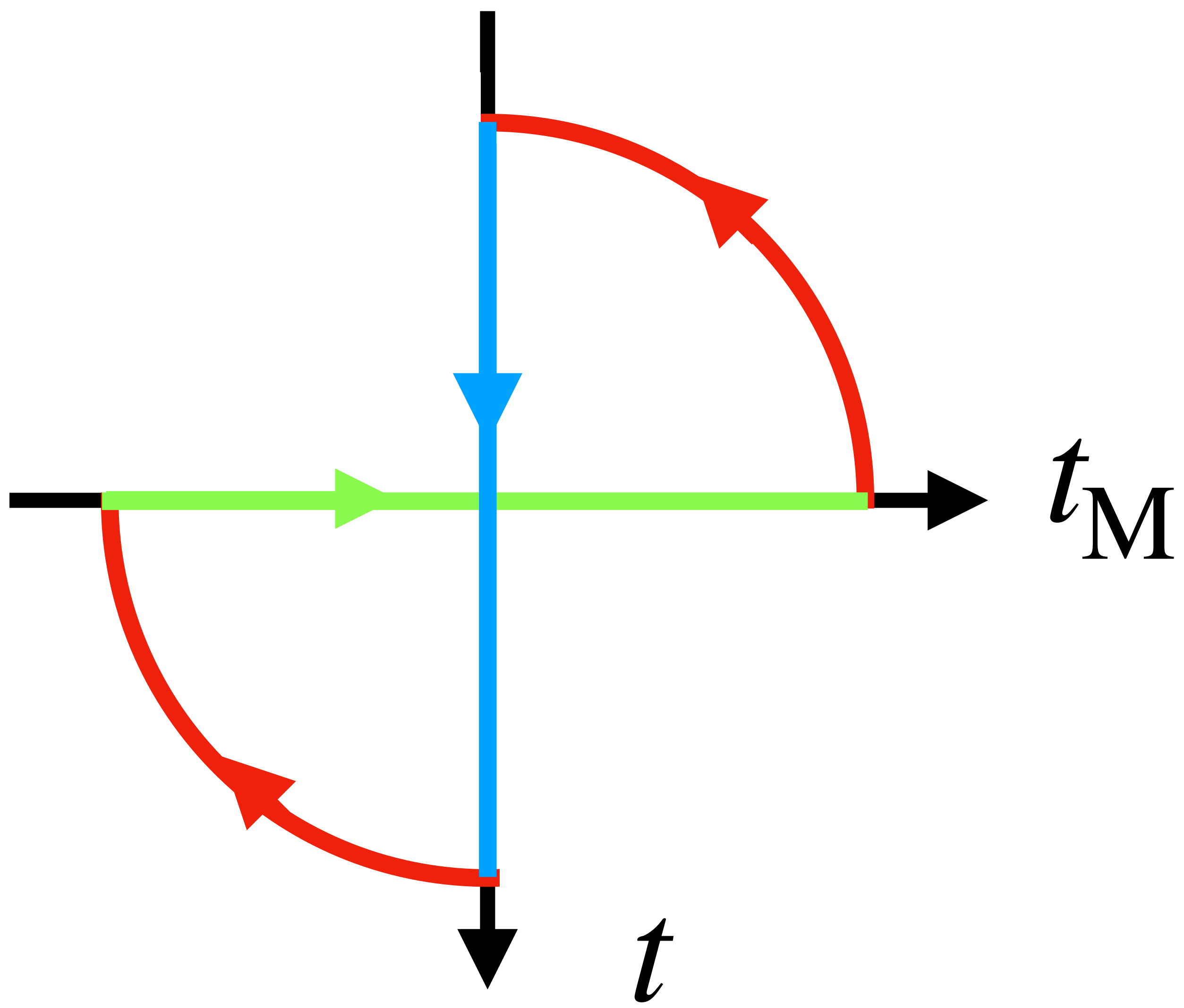




- How we compute the hadronic tensor

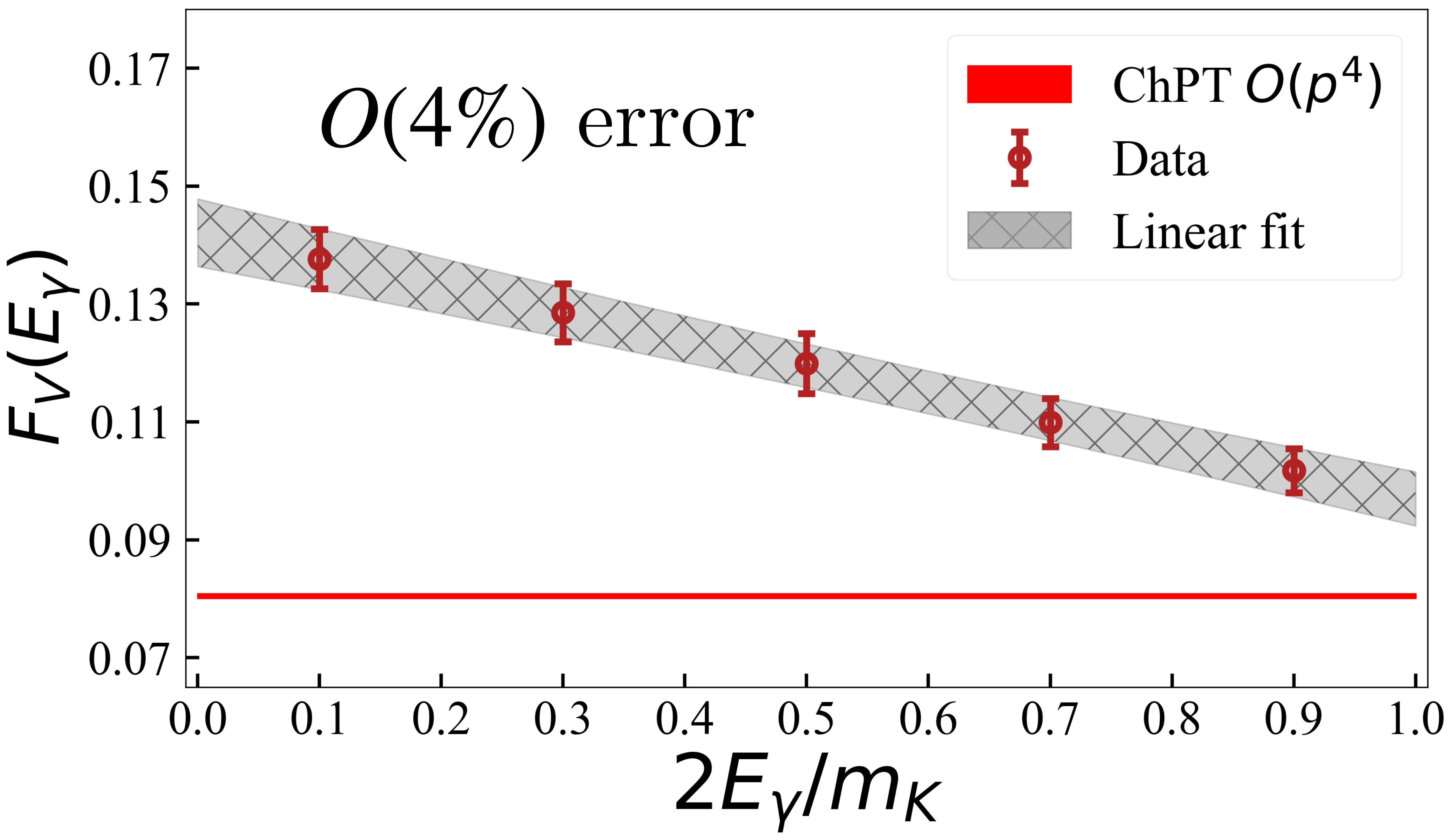
Target: $H_w^{\mu\nu}(E_\gamma, \boldsymbol{k}_\gamma) = \int_{-\infty}^{\infty} dt_{\mathrm{M}}\, e^{iE_\gamma t_{\mathrm{M}}}\, C_w^{\mu\nu}(t_{\mathrm{M}}, \boldsymbol{k}_\gamma)$

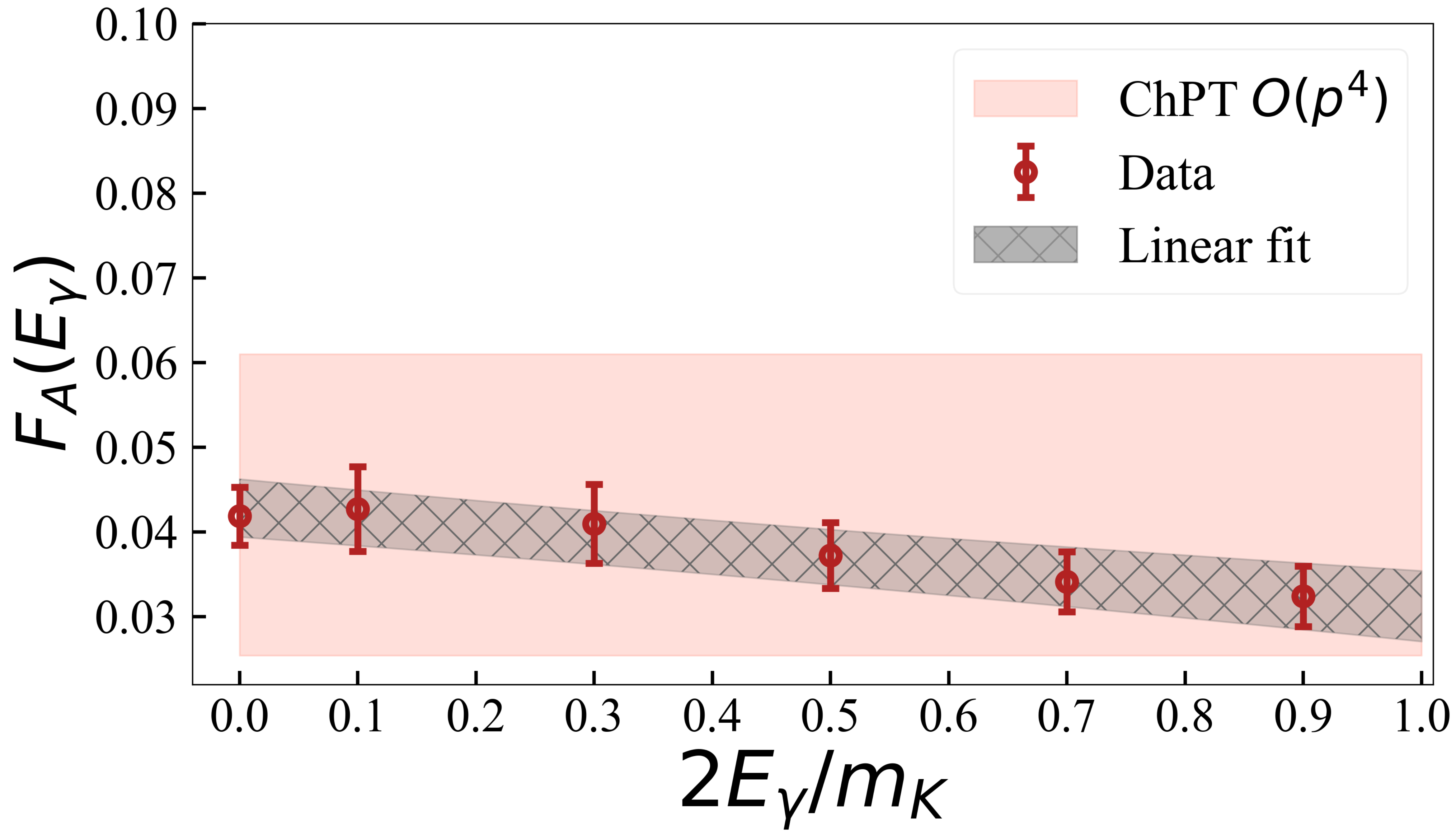
Lattice: $H_w^{\mu\nu}(E_\gamma, \boldsymbol{k}_\gamma) = \int_{-\infty}^{\infty} dt\, e^{E_\gamma t}\, C_w^{\mu\nu}(t, \boldsymbol{k}_\gamma)$



Form factors from the lattice

- Only 2 form factors needed for the radiative decay: $\boxed{F_V(E_\gamma),\ F_A(E_\gamma)}$
- Infinite-volume ($L \to \infty$) and continuum limits ($a \to 0$) performed





$K \to e\nu_e\gamma$: KLOE

- KLOE has performed a binned analysis of [EPJC 64 (2009) 627-636]

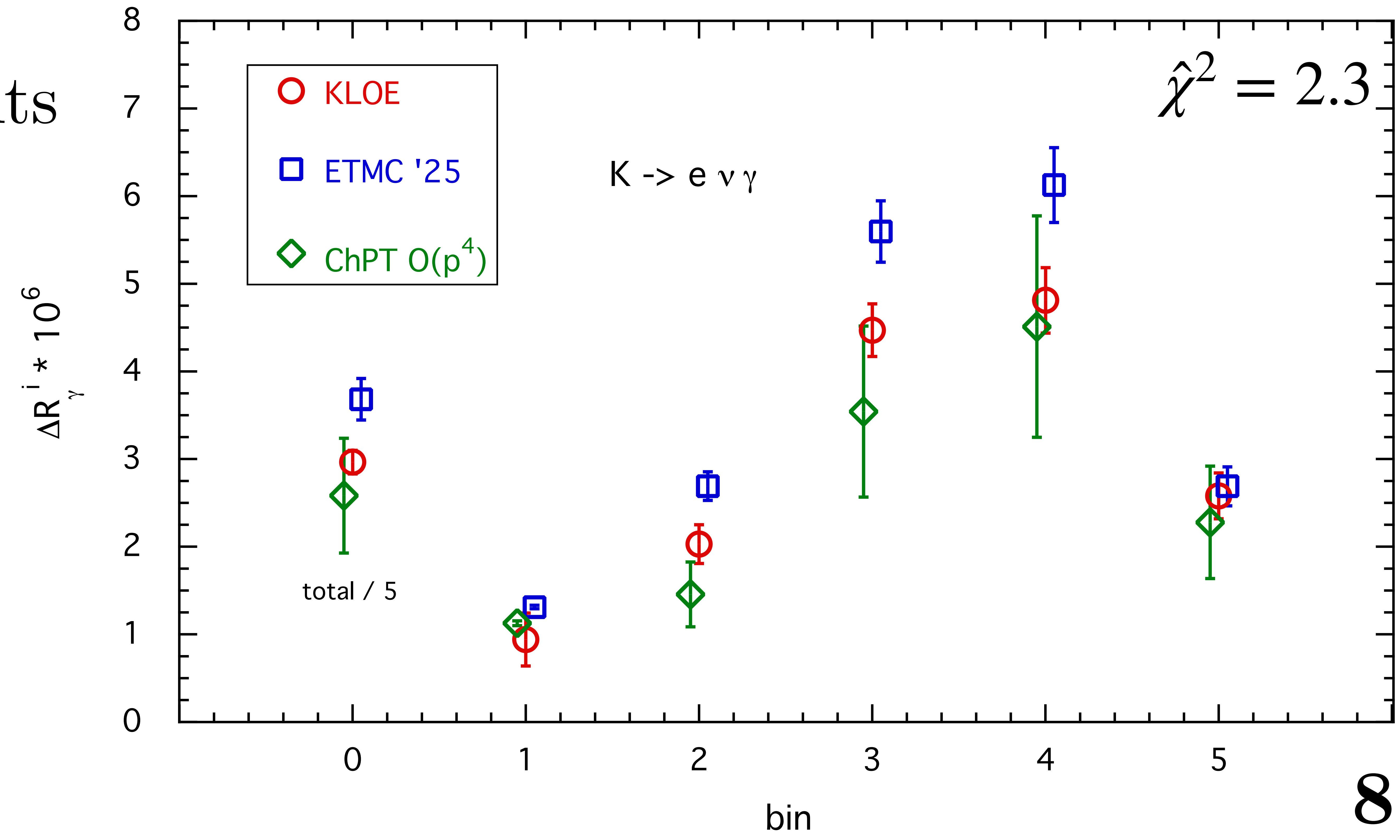
$$\Delta R^i_\gamma = \int_{E^i_\gamma}^{E^{i+1}_\gamma} dE_\gamma \frac{1}{\Gamma(K^- \to \mu^-\bar{\nu}_\mu(\gamma))} \frac{d\Gamma(K^- \to e^-\bar{\nu}_e\gamma)}{dE_\gamma}$$

- They impose kinematical constraints

$|p_e| > 200$ MeV,

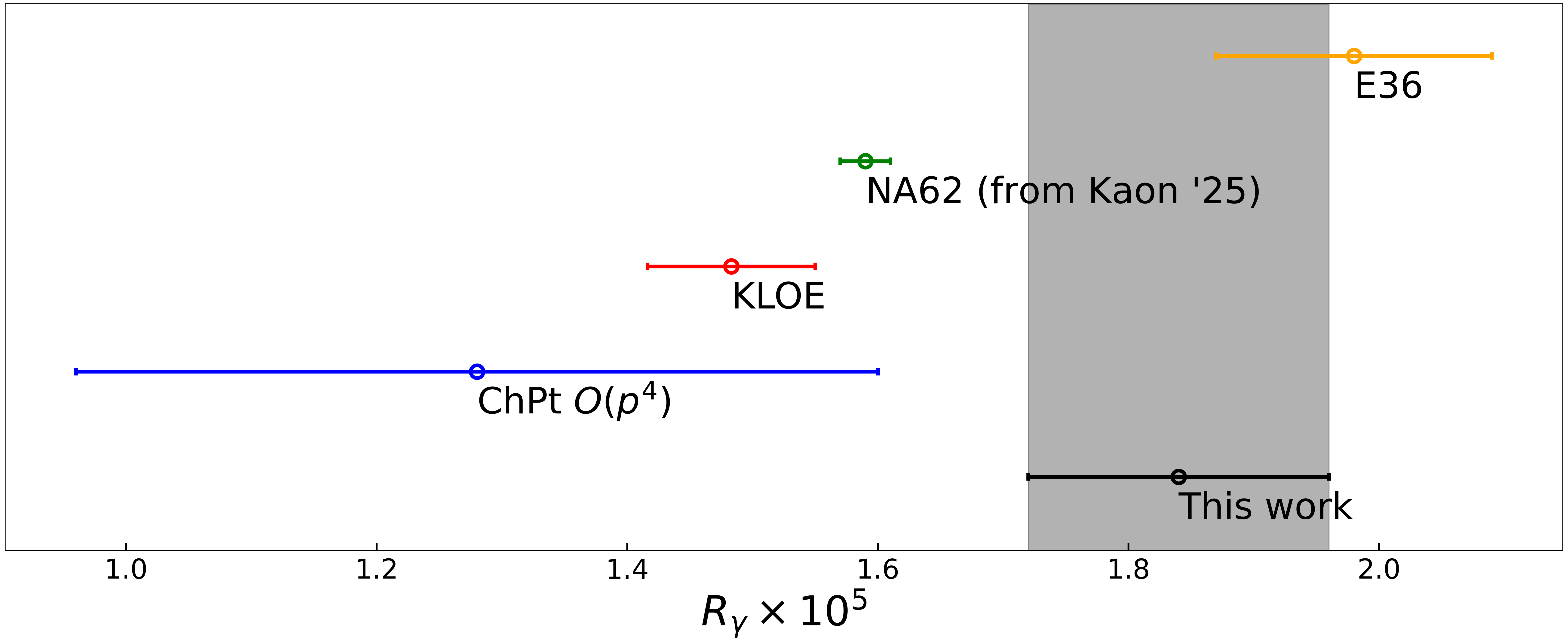
$E_\gamma \in [10{,}250]$ MeV

- Dominant contribution $\propto F^+ = F_V + F_A$



Energy integrated analysis: R_γ

- KLOE and the E36 experiment at J-PARC have provided an integrated analysis [*P.Lett.B 843 (2023) 138020*]
- Our estimate of R_γ include all the contributions at $O(\alpha_{em})$
- A new measurement presented by NA62 @ KAON 25!!

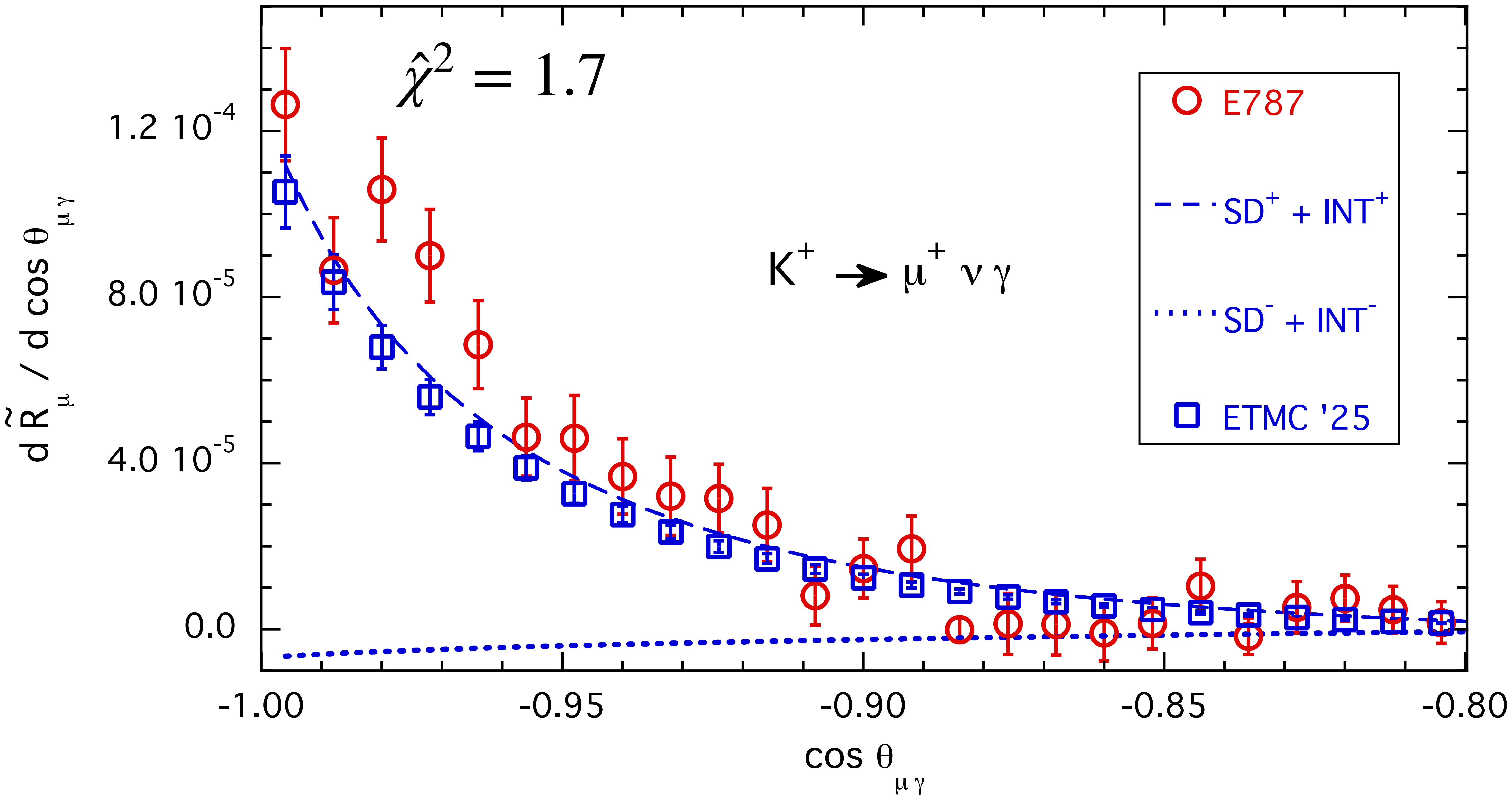


$K \to \mu\nu_\mu\gamma$: E787

- The E787 experiment has measured the muon channel

[Phys.Rev.Lett. 85 (2000) 2256-2259]

$$\frac{dR_\mu}{d\cos\theta_{\mu\gamma}} = \frac{1}{\Gamma(K^- \to \mu^-\bar{\nu}_\mu(\gamma))} \frac{d\Gamma(K^- \to \mu^-\bar{\nu}_\mu\gamma)}{d\cos\theta_{\mu\gamma}}$$



$K \to \mu\nu_\mu\gamma$: ISTRA+ and OKA

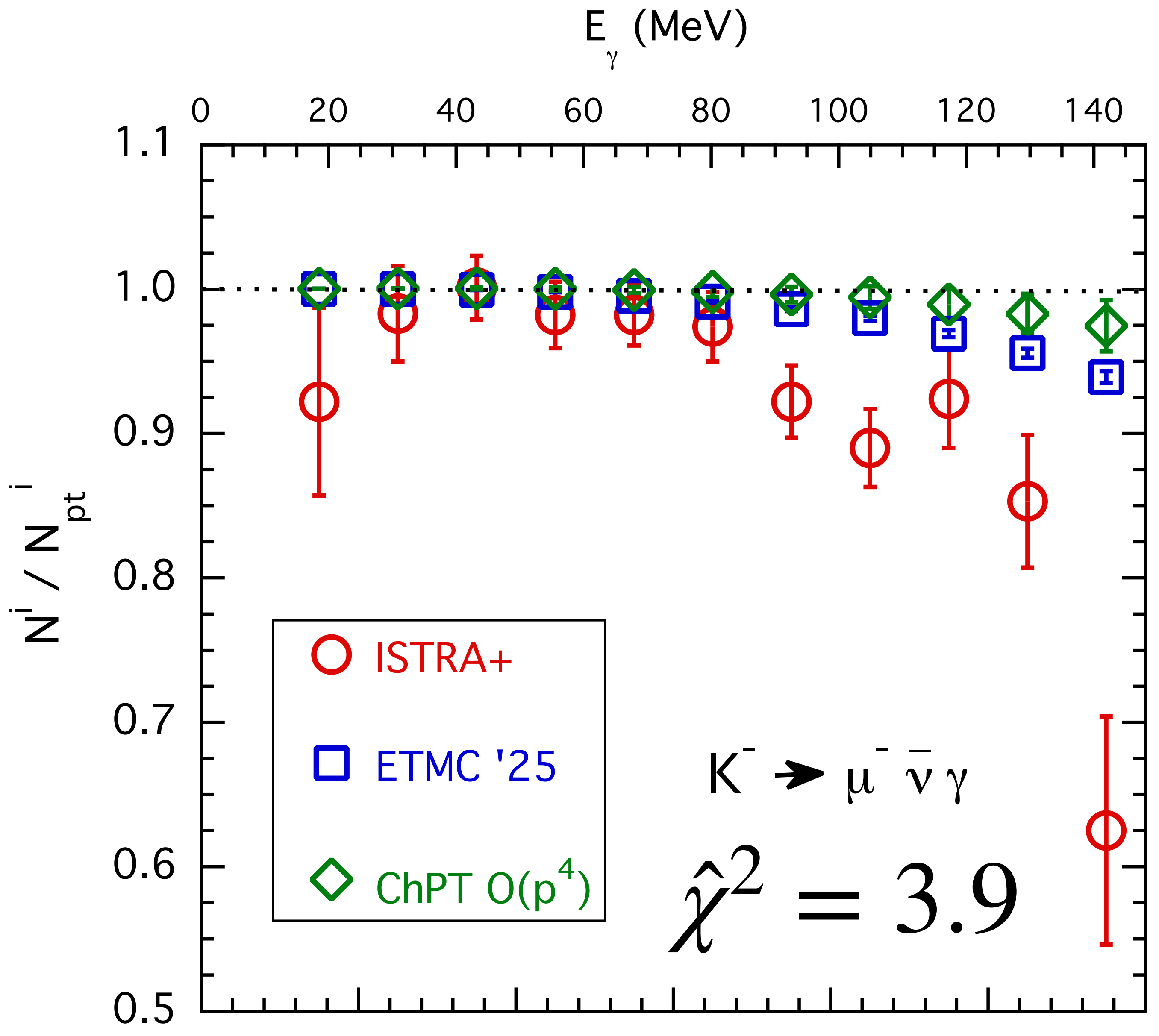
- ISTRA+ and OKA collaborations have measured the muon channel

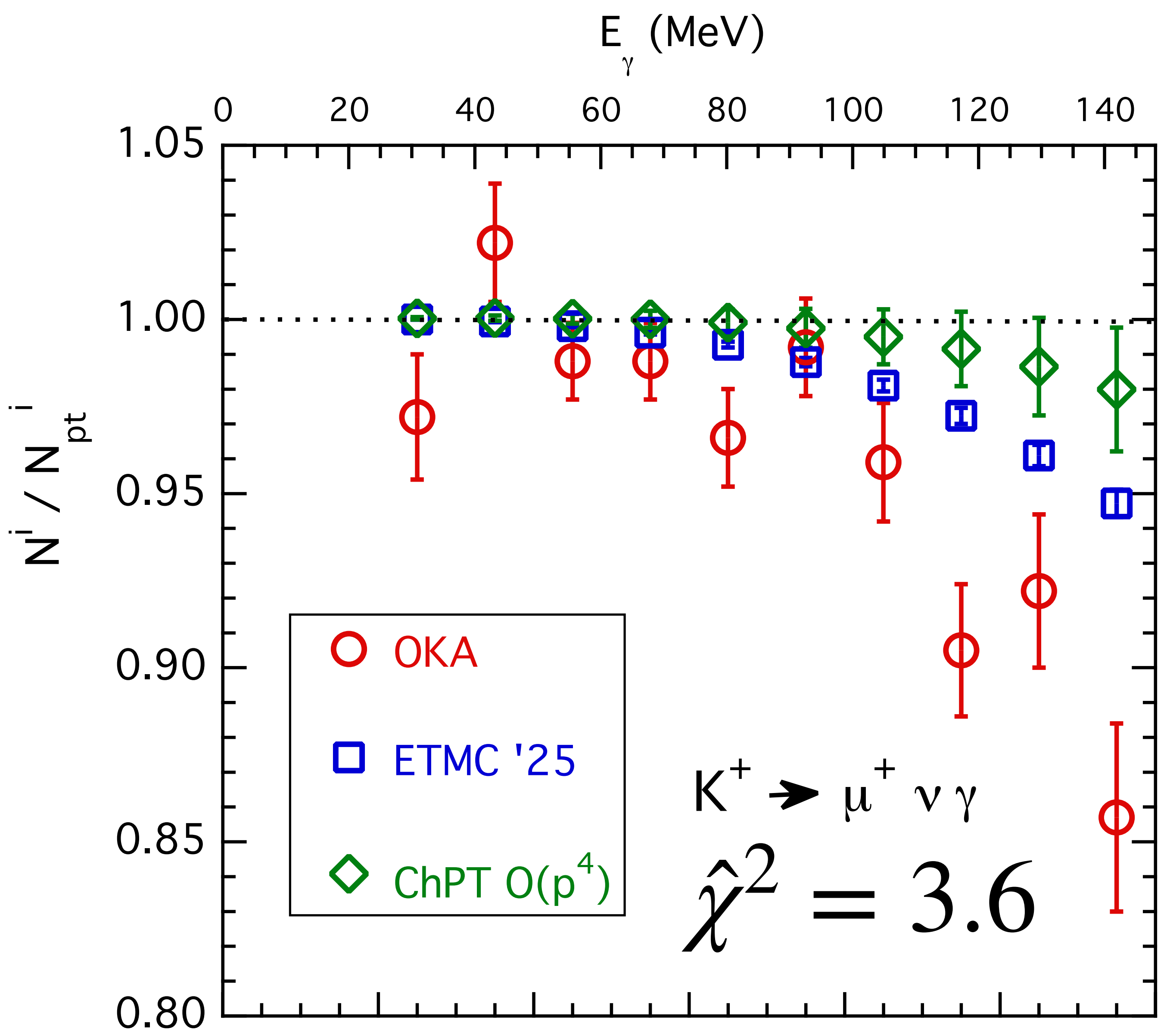
[*Phys.Lett.B 695 (2011) 59-66*]

[*Eur. Phys. J. C 79 (2019) 635*]

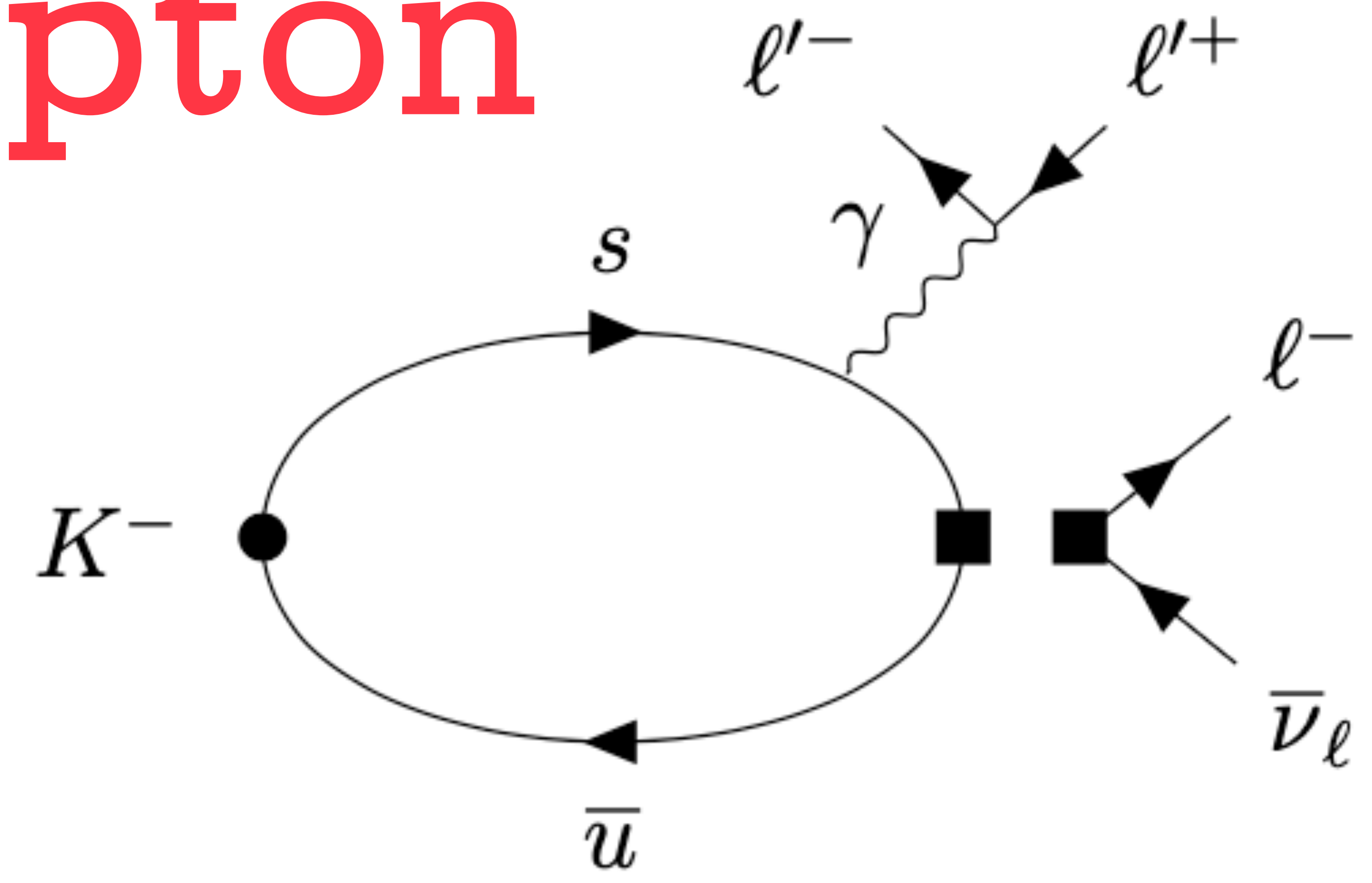
$$N^i = \int_{i-\text{strip}}^{i+1-\text{strip}} dE_\gamma\, dE_\mu\, \frac{d^2\Gamma(K^- \to \mu^-\bar{\nu}_\mu\gamma)}{dE_\gamma dE_\mu}$$

- These experiments are mostly sensitive to $F^- = F_A - F_V$





4-Lepton



soon on ArXiv

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Why 4-Lepton decays?

- Starting at $O(\alpha_{\rm em}^2)$ in the SM. Probe for NP
  (Heavy neutral leptons, non-universal gauge boson, ...)

- Using the same lattice inputs of the radiative decay: $C_w^{\mu\nu}(t, \boldsymbol{k}_\gamma)$

- No complete calculation yet:
  - No continuum and infinite-volume limit
  - $\boxed{\text{Employed heavier pions } m_\pi \simeq 300 \text{ MeV}}$

ETMC - [Phys.Rev.D 105 (2022) 11, 114507]

Tuo et al. - [Phys.Rev.D 105 (2022) 5, 054518]

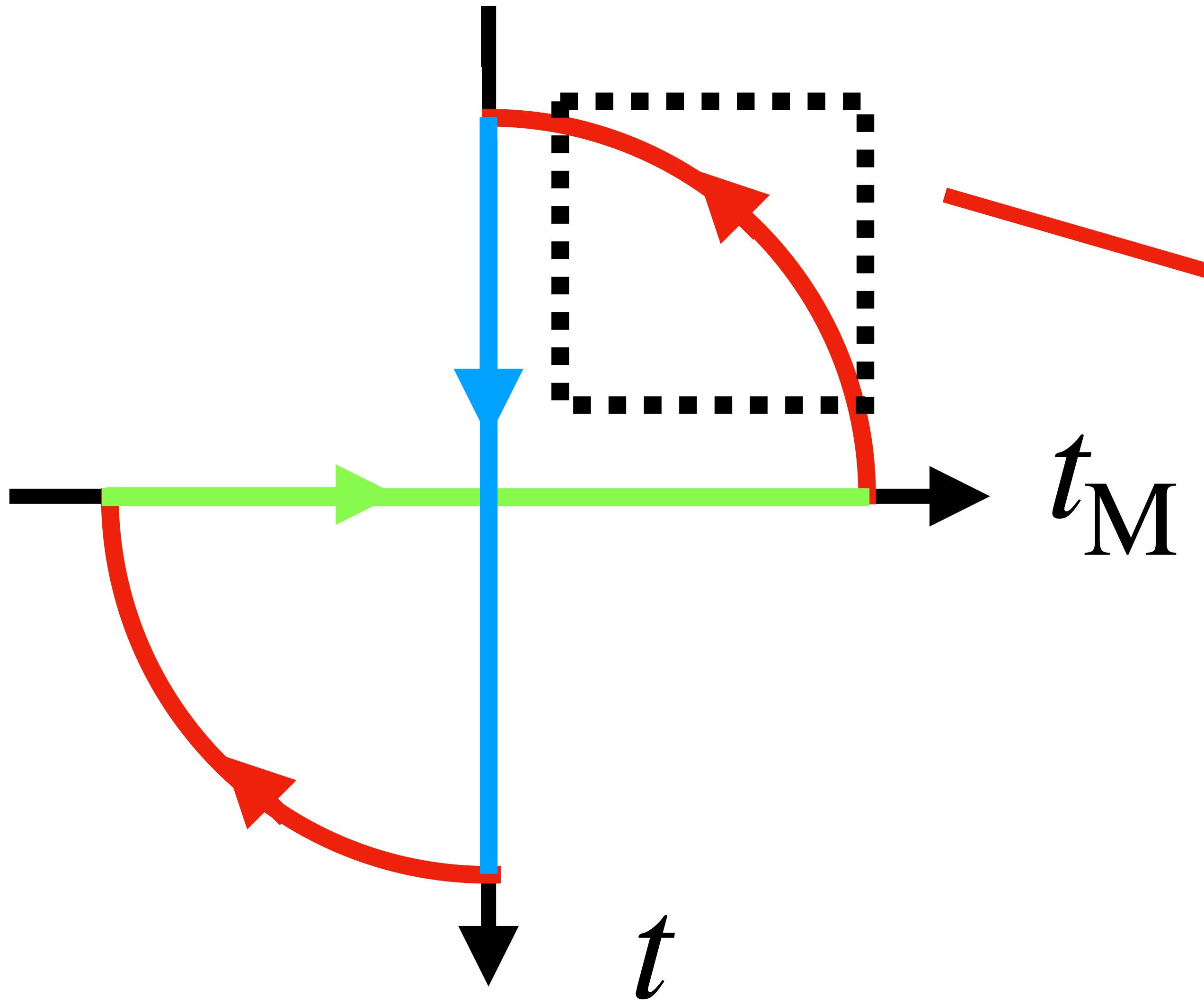
Wick rotation is obstructed!

- Consider the hadronic tensor in the region $t > 0$

$$H_w^{\mu\nu}(E_\gamma, \boldsymbol{k}_\gamma) = \int dt\; e^{iE_\gamma t} \int d^3x\; e^{-i\boldsymbol{k}_\gamma\cdot\boldsymbol{x}}\; \langle 0 \,|\, J^\mu_{\text{em}}(t,\boldsymbol{x}) J^\nu_w(0) \,|\, K^- \rangle$$

- Inserting a complete set of intermediate states

$$H_w^{\mu\nu}(E_\gamma, \boldsymbol{k}_\gamma) = \sum_{n(\boldsymbol{k}_\gamma)} \langle 0 \,|\, J^\mu_{\text{em}}(0) \,|\, n(\boldsymbol{k}_\gamma)\rangle \langle n(\boldsymbol{k}_\gamma) \,|\, J^\nu_w(0) \,|\, K^- \rangle \int_0^\infty dt\; e^{i(E_\gamma - E_n)t}$$



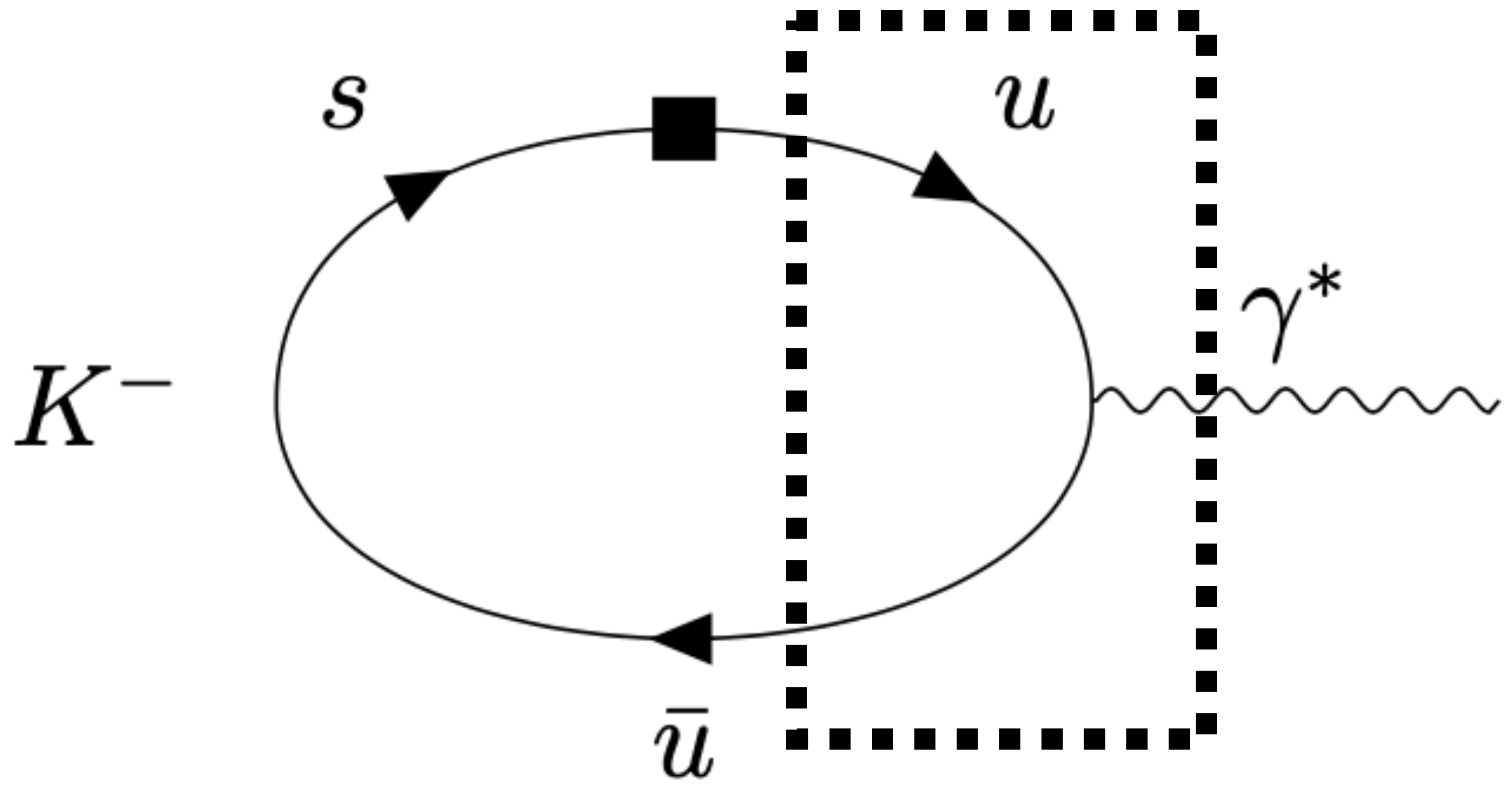
$$\sim \int_0^{\pi/2} d\theta \int_0^\infty dR\;\; \exp[iR\cos\theta]\;\; \exp[-R\sin\theta(E_n - E_\gamma)]$$

$$\boxed{E_n > E_\gamma?}$$

Intermediate states

- In this time-ordering, when the photon is emitted by the up-type quark, among the intermediate states contributing, there are also $\pi\pi(\boldsymbol{k})$

$$n = \cdots, \pi\pi(\boldsymbol{k}), \cdots$$



- Wick rotation is obstructed for

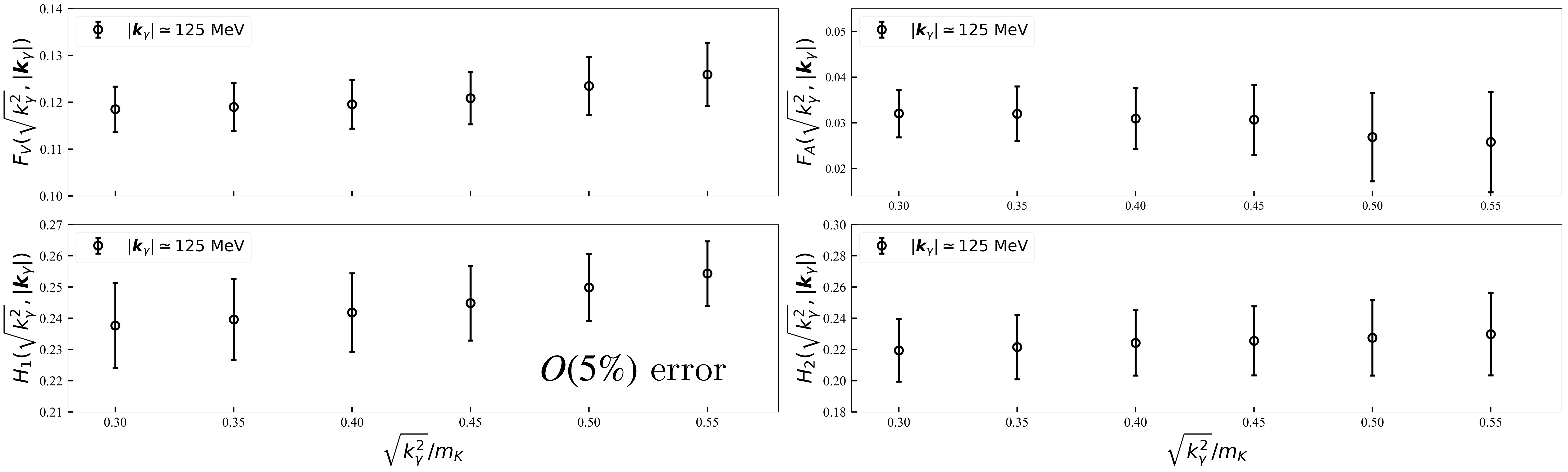
$$\boxed{(2m_\pi)^2 < k_\gamma^2 < m_K^2}$$

- Older works did not have this problem, since $2m_\pi > m_K$!

Below the 2π threshold

- We can use the same strategy adopted for radiative decays

$$H_w^{\mu\nu}(E_\gamma, \boldsymbol{k}_\gamma) = \int_{-\infty}^{\infty} dt\ e^{E_\gamma t} C_w^{\mu\nu}(t, \boldsymbol{k}_\gamma)$$



Above 2π: the SFR method

- The first ingredient of the SFR method is the spectral representation of the hadronic tensor

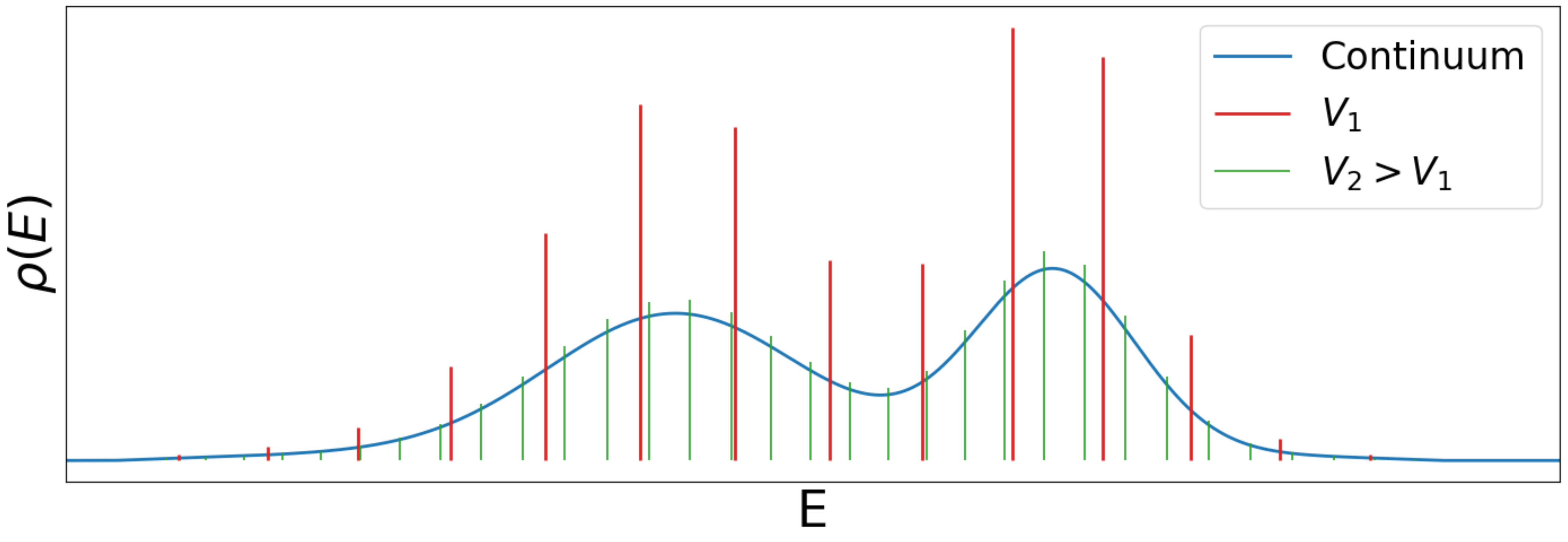
[*Phys.Rev.D 108 (2023) 7, 074510*]

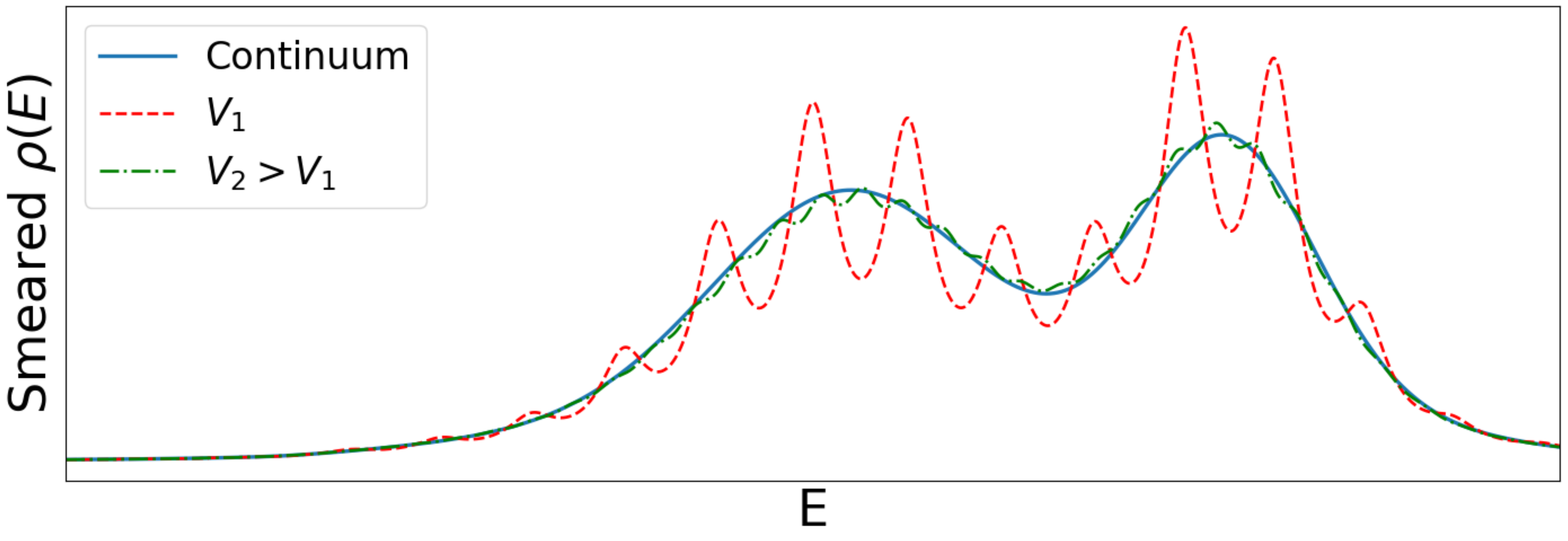
$$H_w^{\mu\nu}(\varepsilon, E_\gamma, \boldsymbol{k}_\gamma) = \int_{E_\gamma^*}^{\infty} \frac{dE}{2\pi} \, \frac{\rho_w^{\mu\nu}(E, \boldsymbol{k}_\gamma)}{E - E_\gamma - i\varepsilon}, \qquad \lim_{\varepsilon \to 0^+} H_w^{\mu\nu}(\varepsilon, E_\gamma, \boldsymbol{k}_\gamma)$$

- We can use $\varepsilon \neq 0^+$ as a regulator to compute the smeared amplitude $H_w^{\mu\nu}(\varepsilon, E_\gamma, \boldsymbol{k}_\gamma)$ (so-called Hansen-Lupo-Tantalo (HLT) method [*Phys.Rev.D* 99 (2019) 9, 094508])

- The infinite-volume limit MUST be performed at this point

$$\lim_{L \to \infty} H_w^{\mu\nu}(L, \varepsilon, E_\gamma, \boldsymbol{k}_\gamma)$$



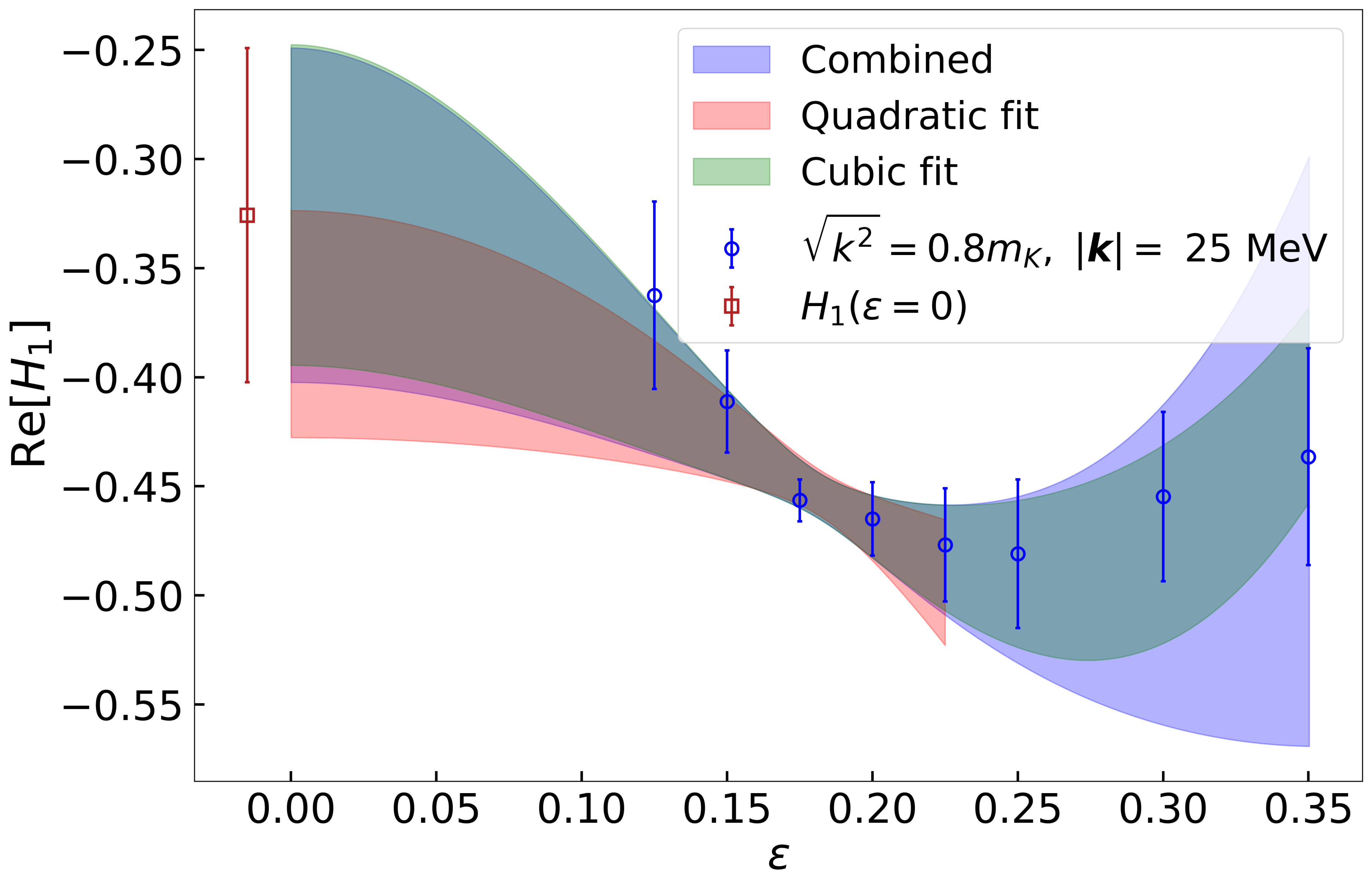


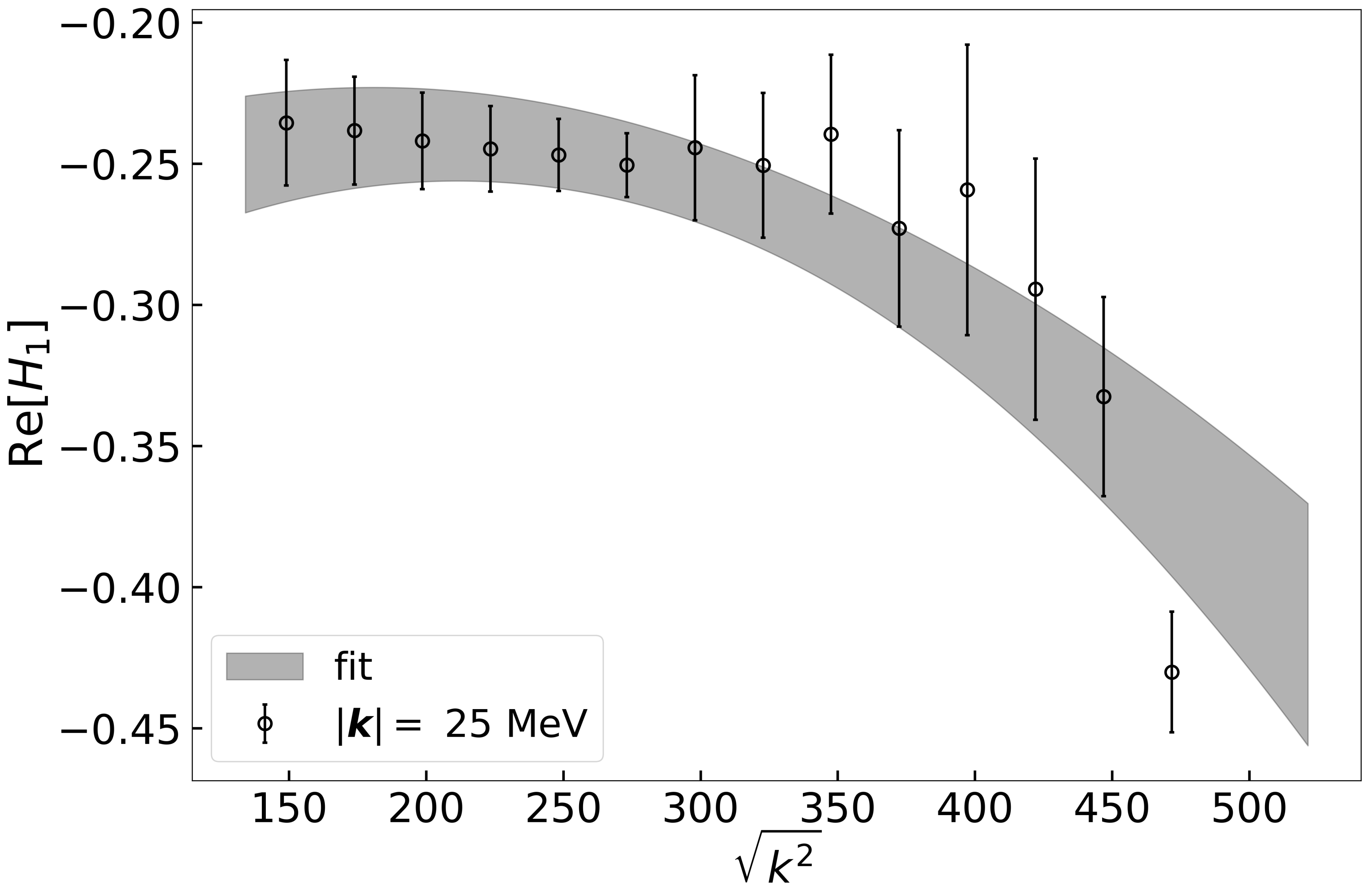
SFR results

- Finally, we extrapolate to $\varepsilon \to 0$ using a polynomial ansatz

$$H_w^{\mu\nu}(\varepsilon, E_\gamma, \boldsymbol{k}_\gamma) = \ H_w^{\mu\nu}(E_\gamma, \boldsymbol{k}_\gamma) + P(\varepsilon, n-1) + O(\varepsilon^n)$$

- Dominant form factor H_1

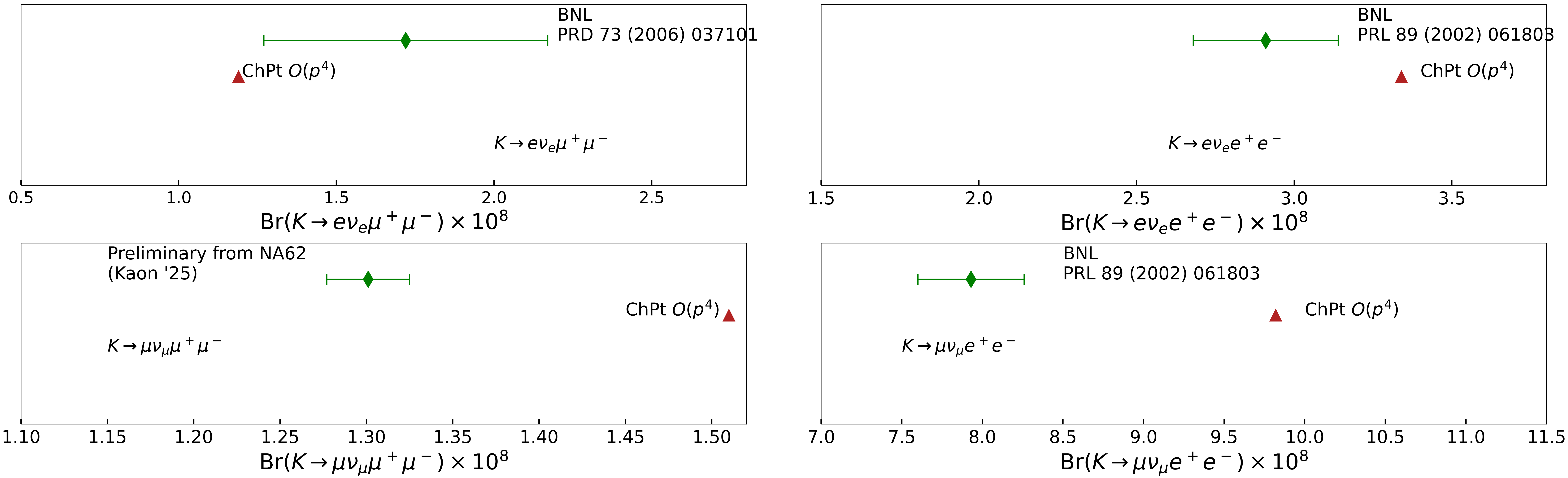




Next steps: 4-Lepton

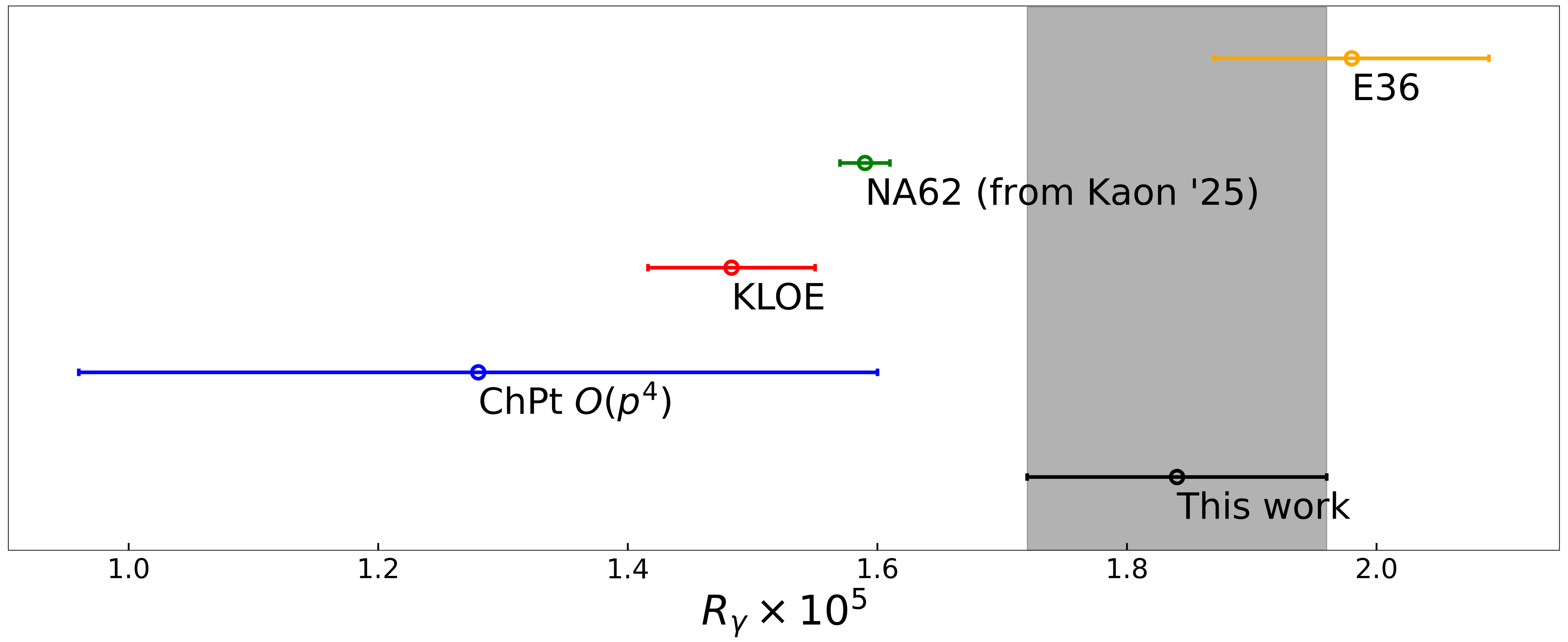
- Soon completing the prediction on $K \to l\bar{\nu}_l l'^+ l'^-$. Since H_1 is the dominant form factor, we expect a precision of $O(10\%)$ on $\Gamma(K \to e\bar{\nu}_e l'^+ l'^-)$

Soon Standard Model predictions



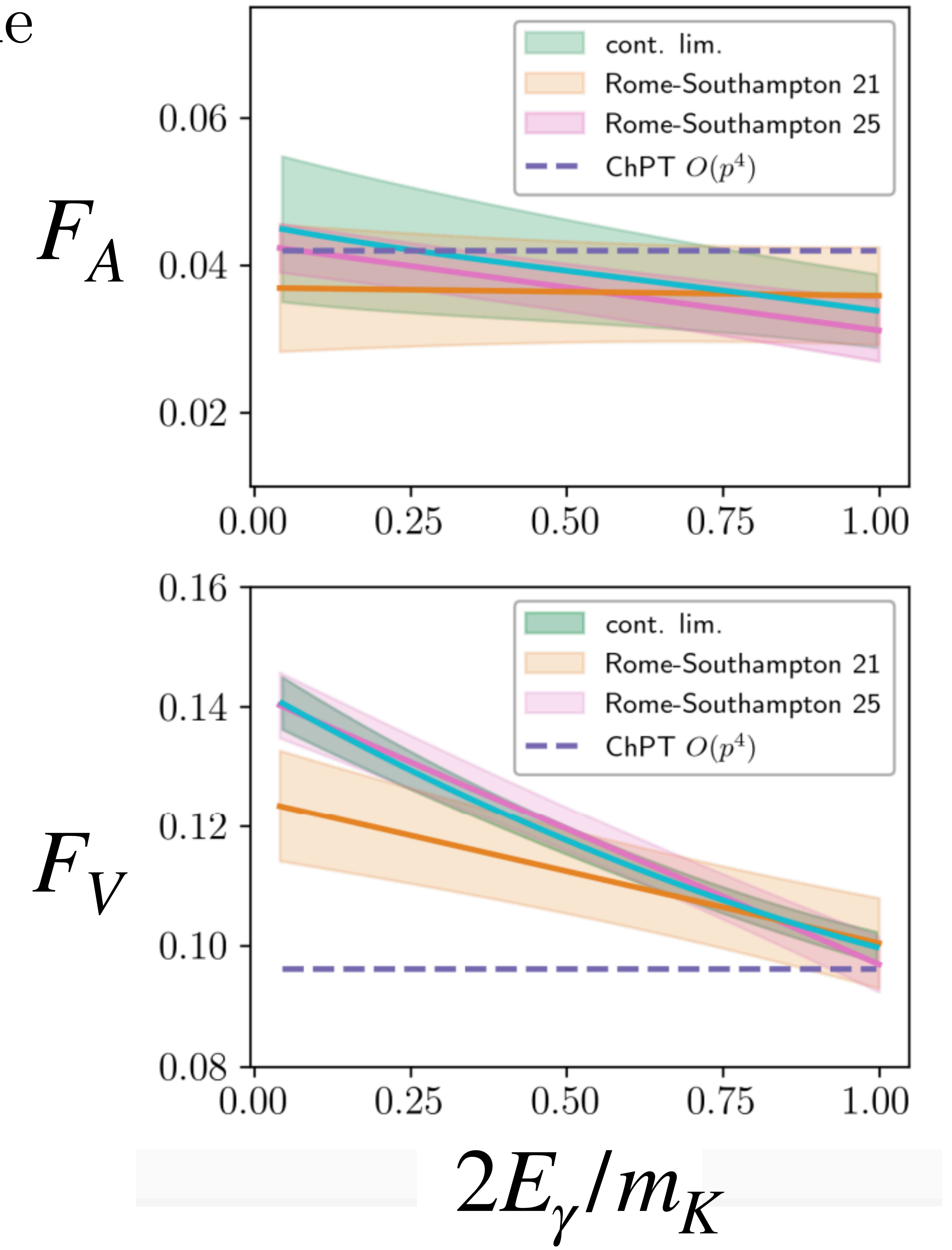
Next steps: R_γ

- New lattice results for the Form Factors describing the radiative decay from RBC/UKQCD [*Arxiv:2510.26993*] **(LATTICE EXTREMELY COMPATIBLE!!!)**



- Suggested issues in the treatment of collinear photons effects $O(\alpha_{em}^2 \ \ln(m_e^2/m_K^2))$

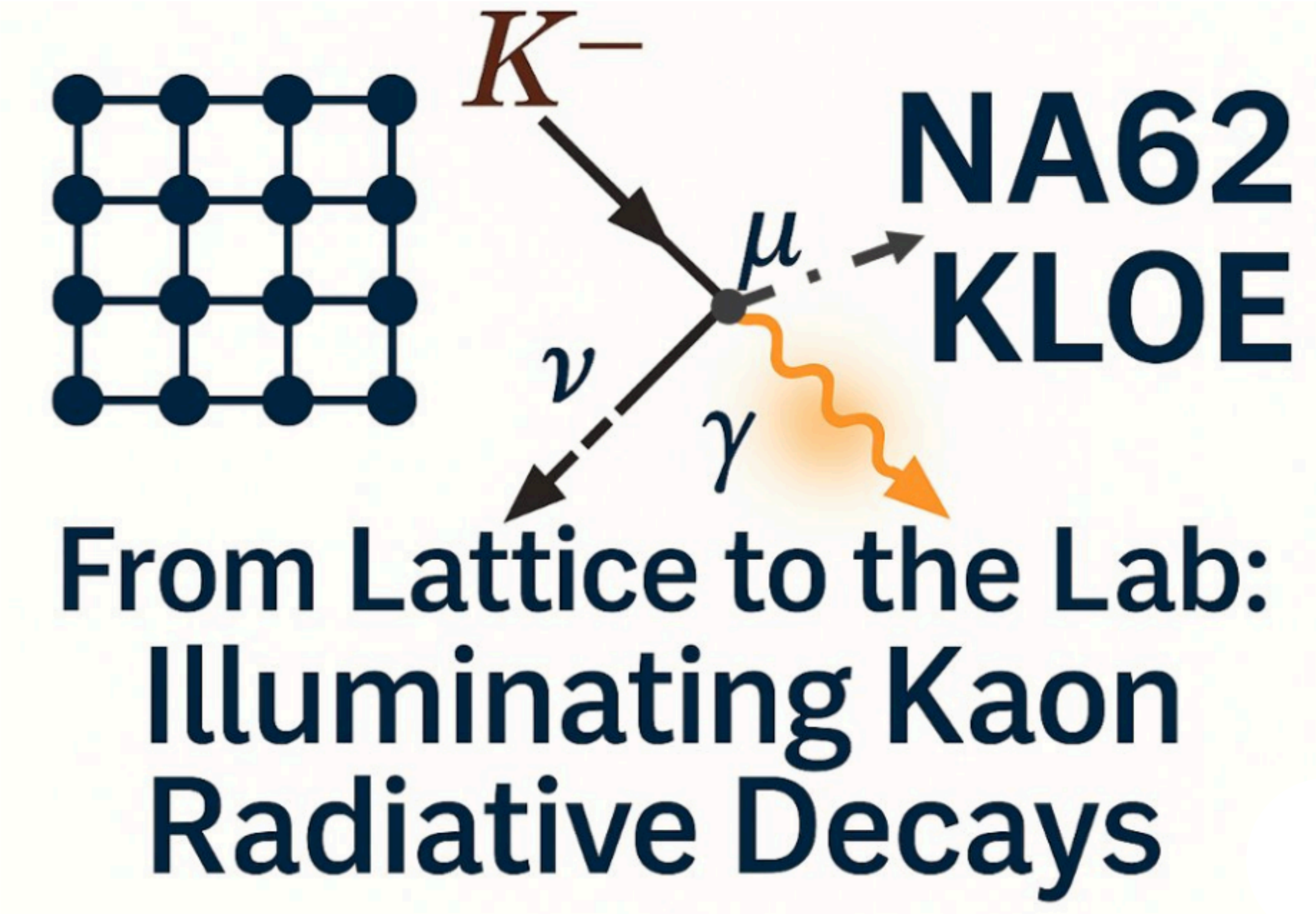
From Arxiv:2510.26993



Adv

Theorists and experimentalists together to talk and discuss Kaon decays

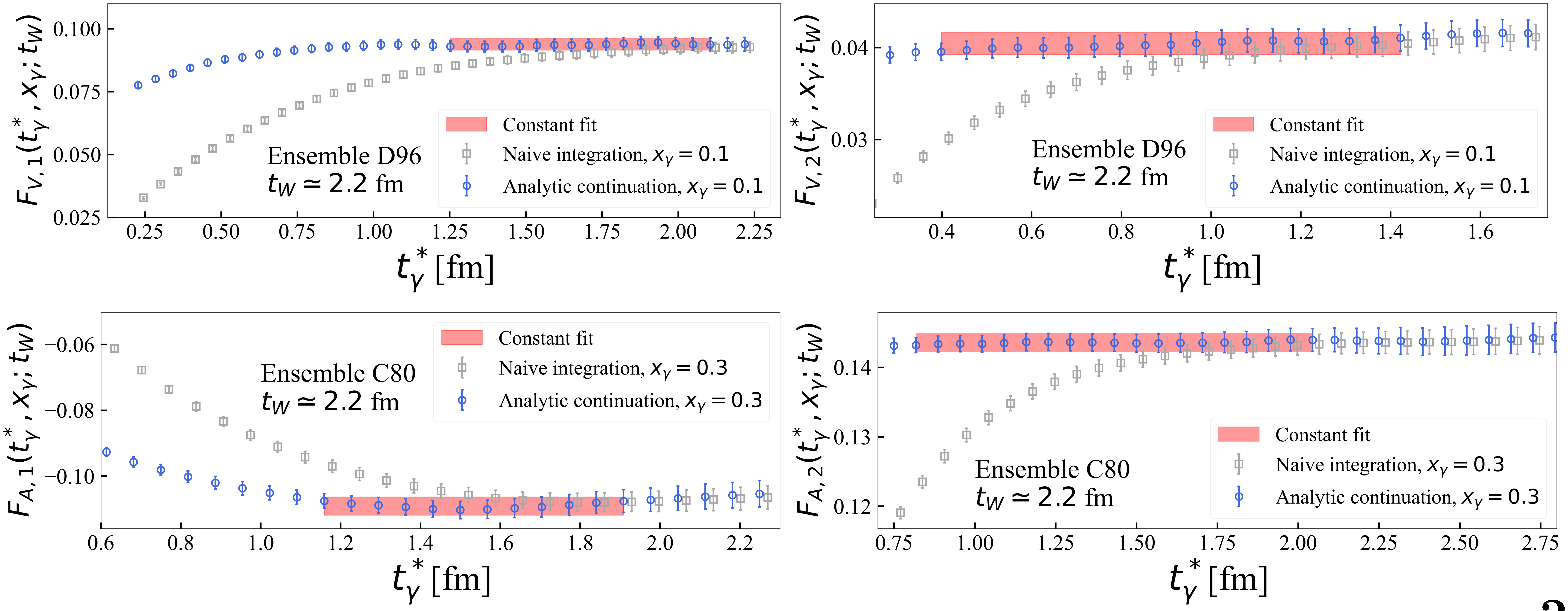
Frascati, Laboratori Nazionali (LNF), 24-25 March 2026



Backup slides

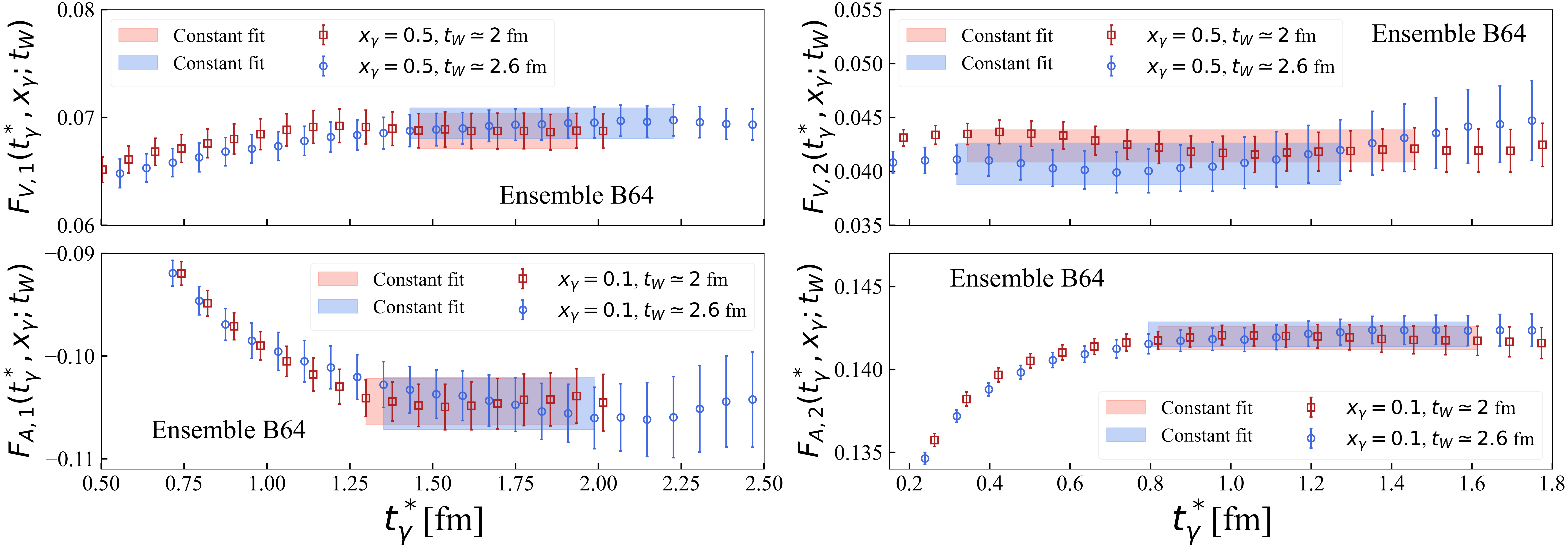
Finite-T effects

- We employ an asymptotic approximation to evaluate finite-T effects



Finite-t_w effects

- We check the effects due to Kaon isolation simulating two different values of t_w

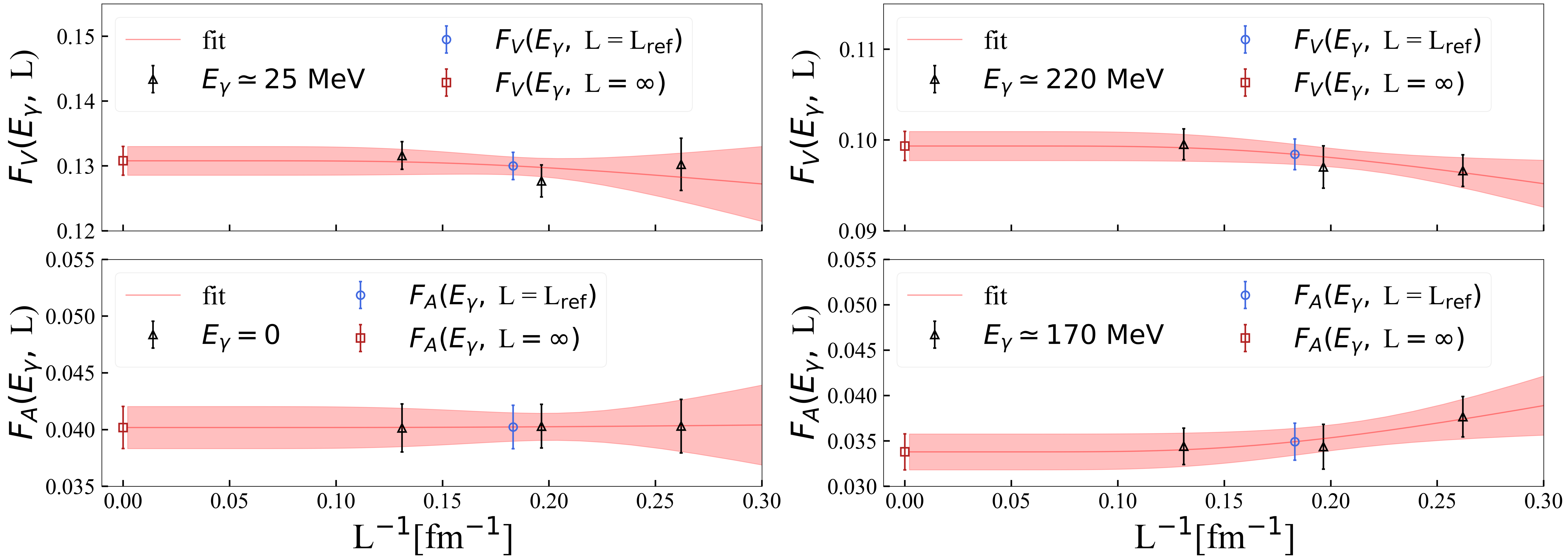


Finite-size effects

- Finite-size effects are parametrized as

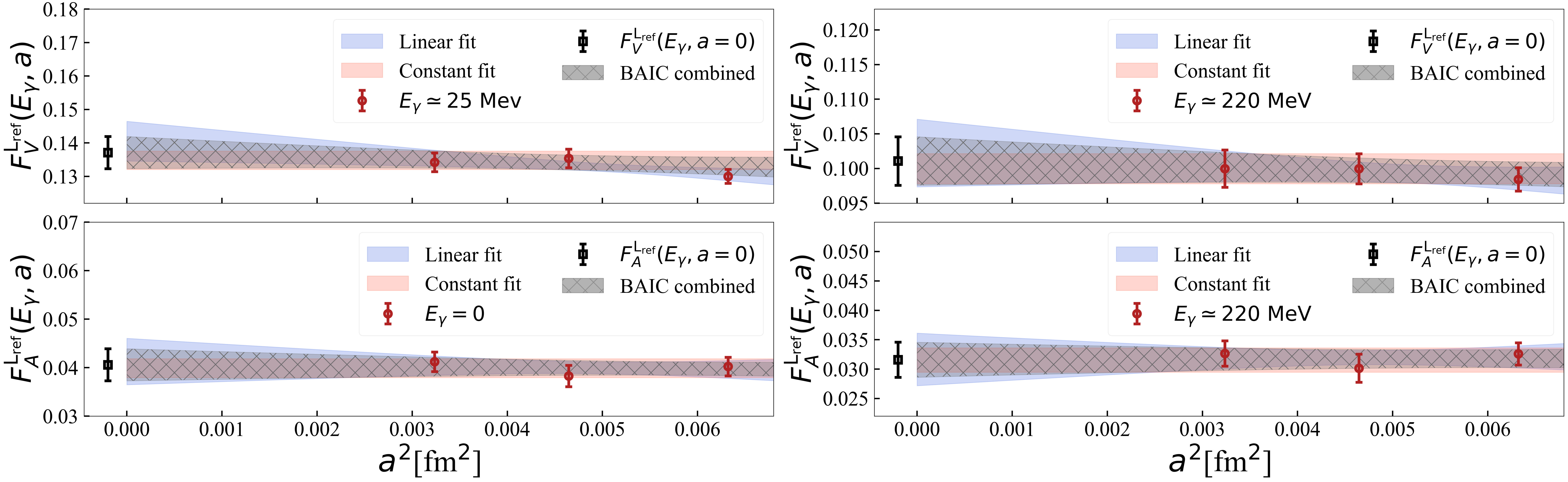
$$F_{A,V}(E_\gamma, \mathrm{L}) = F_{A,V}(E_\gamma, \mathrm{L} = \infty) + \Delta_{A,V}(E_\gamma)\exp[-m_\pi \mathrm{L}]$$

and fitted from data collected at $\mathrm{L} \simeq \{3.8,\ 5.1,\ 7.6\}$ fm and fixed $a \simeq 0.079$ fm



Continuum limit

- Continuum limit with three lattice spacings $a \simeq \{0.079, 0.068, 0.056\}$ fm, at fixed $\mathrm{L_{ref}} \simeq 5.5$ fm

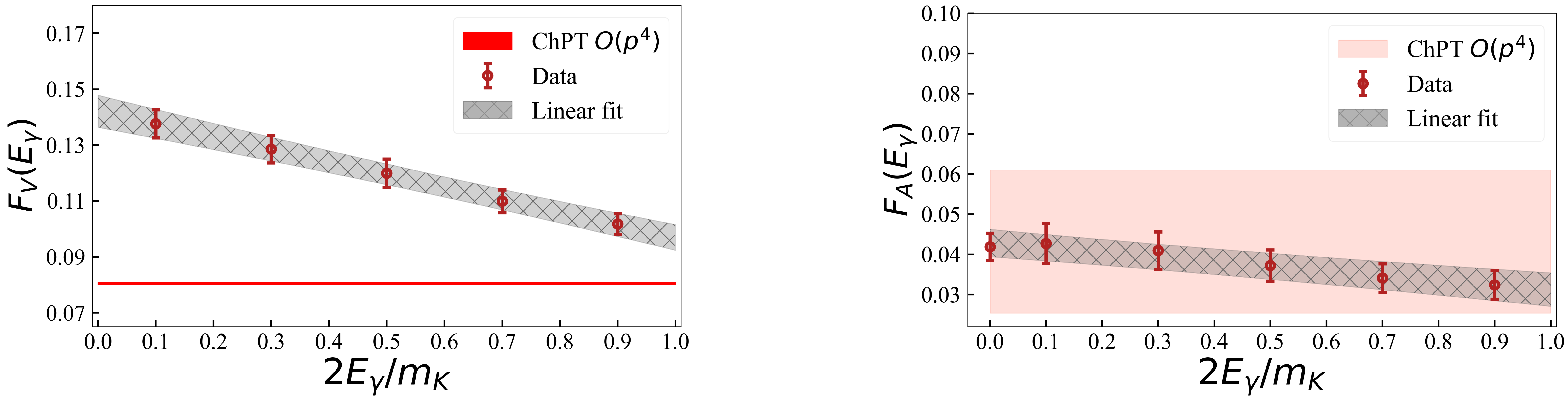


- Going to the infinite volume by adding the finite-size corrections

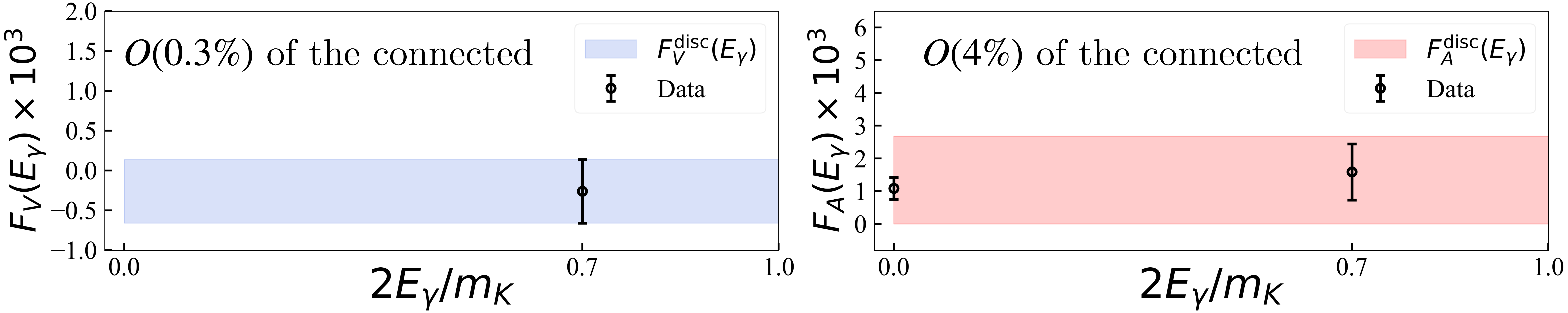
$$-\Delta_{A,V}(E_\gamma)\exp[-m_\pi \mathrm{L_{ref}}]$$

Form factors results

- We fit the form factors using a linear ansatz

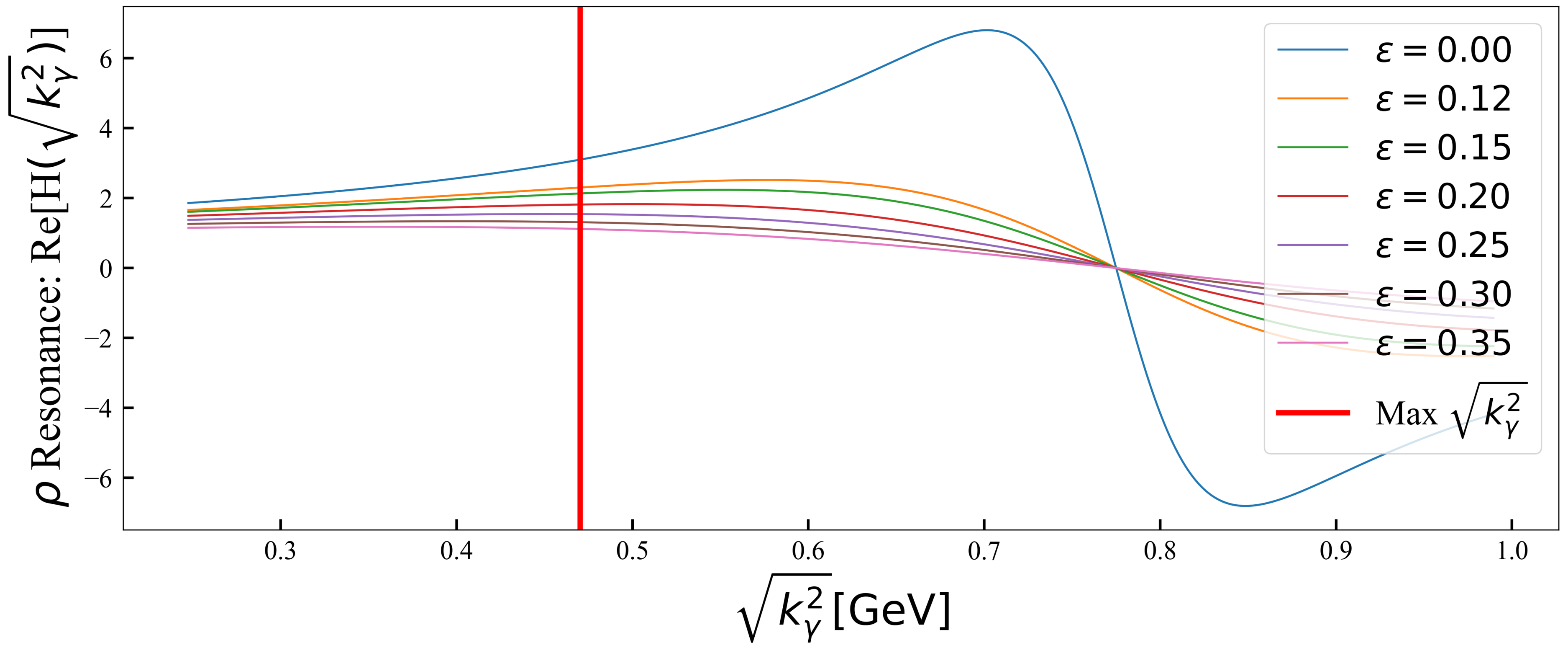


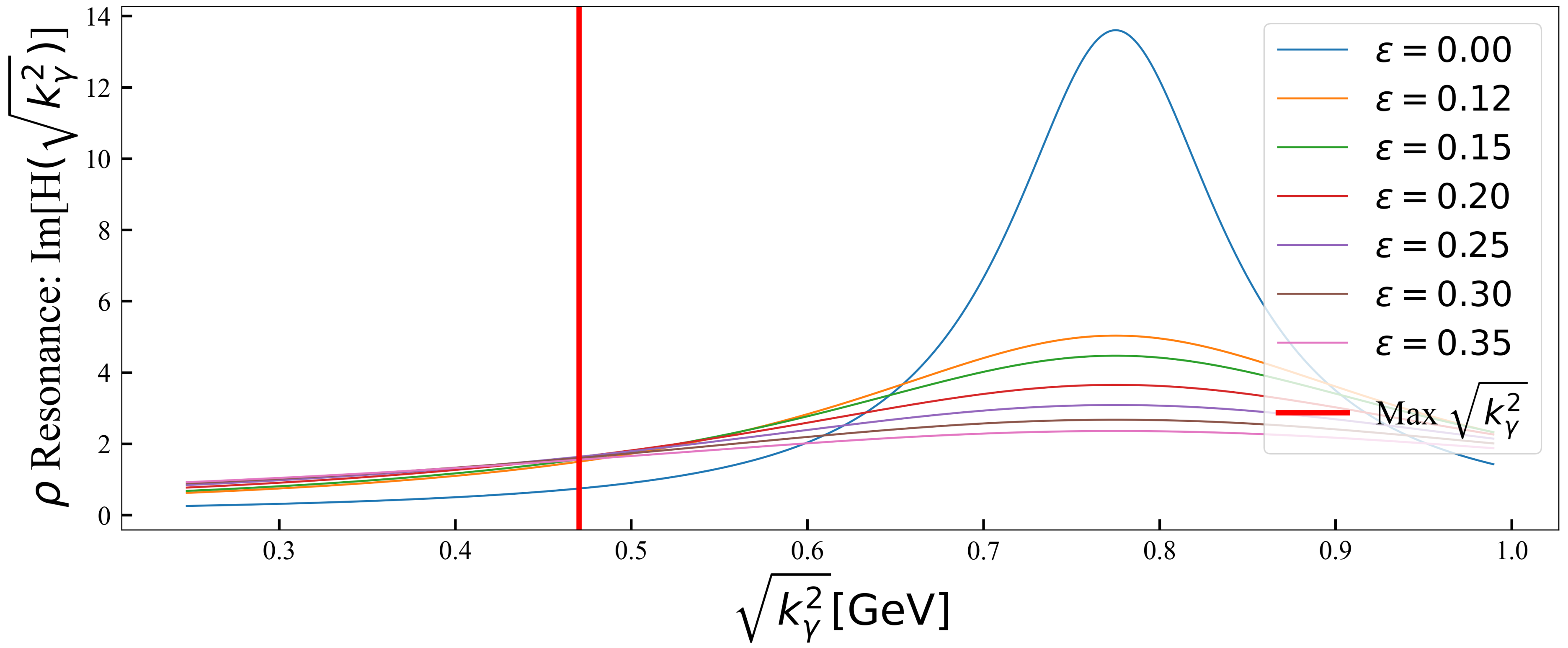
- The quark-line disconnected contribution is negligible at this level of precision



ρ Model

- Perfect case! Sitting on the left tail of the broad ρ resonance, we are in the polynomial regime already with large values of $\varepsilon < \Lambda \simeq 350$ MeV





The HLT method

- The smeared hadronic tensor can be computed as

$$H^{\mu\nu}(\varepsilon, E_\gamma, \boldsymbol{k}_\gamma) = \sum_t (g_R(t, E_\gamma, \varepsilon) + i g_I(t, E_\gamma, \varepsilon)) C_w^{\mu\nu}(t, \boldsymbol{k}_\gamma)$$

- The coefficients are determined by minimizing the functional

$$W[\mathbf{g}] = \frac{A[\mathbf{g}]}{A[\mathbf{0}]} + \lambda B[\mathbf{g}]$$

$$A[\mathbf{g}] = \int_{E^*}^{\infty} dE' \left| \sum_t g(t, E_\gamma, \varepsilon) e^{-E't} - \frac{1}{E' - E_\gamma - i\varepsilon} \right|^2 \qquad B[\mathbf{g}] \propto \sum_{t_1, t_2} g(t_1, E_\gamma, \varepsilon) g(t_2, E_\gamma, \varepsilon) \mathrm{Cov}(t_1, t_2)$$

4-Lepton: contributions