Measurements of Unitary Triangle sides: experimental status and perspectives

Tommy Martinov

INFN, Trieste

On behalf of the Belle, Belle II and LHCb collaborations

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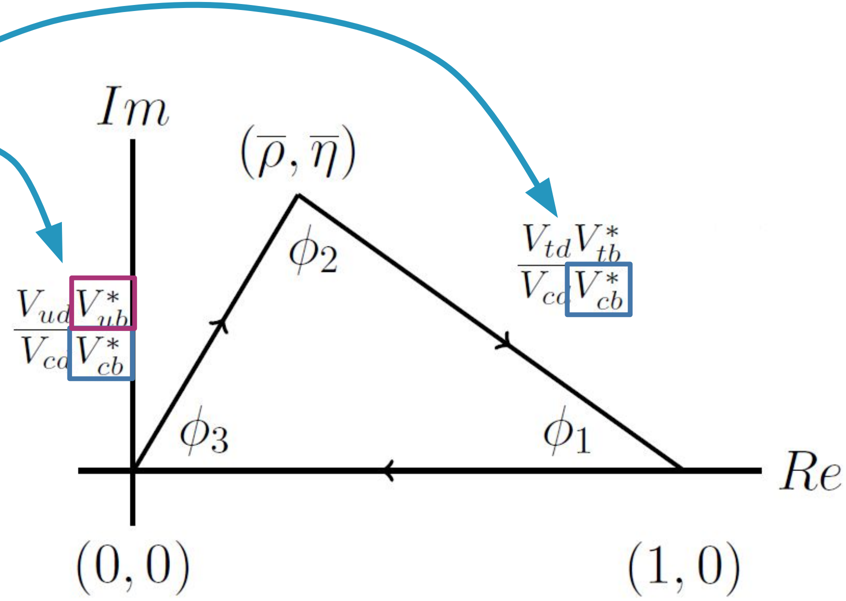
INTRODUCTION

Unitarity triangle sides

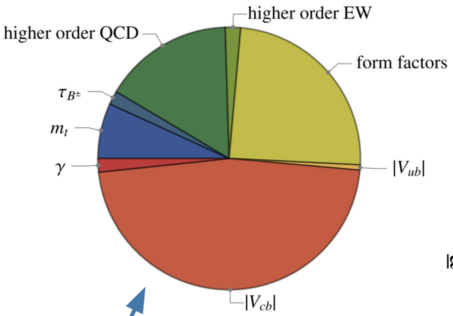
$$\bar{\rho} + i\bar{\eta} = -(V_{ud}V_{ub}^{*})/(V_{cd}V_{cb}^{*})$$

- Different parametrisations of the CKM matrix and unitarity triangle
- In the $\rho - \eta$ plane, the two sides are defined as a function of 6 of the CKM matrix elements
- The CKM matrix is unitary by construction and therefore any deviation observed in tests of the UT would be a clear sign of non-SM physics
- How are the UT sides measured?
  - V_{ud} → beta decays
  - V_{cd} → charm decays
  - V_{td} → rare penguin and box decays
  - V_{tb} → top quark decays
  - V_{ub} and V_{cb} → semileptonic *B* decays



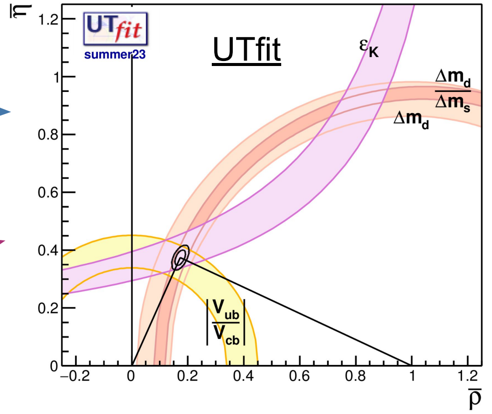
$|V_{ub}|$ and $|V_{cb}|$

- Measured in tree-level decays
  - → useful to anchor the UT and increase the sensitivity of unitarity tests to deviations in loop-dominated observables
- The precision of UT tests is limited by the uncertainty on $|V_{ub}|$ and $|V_{cb}|$ and inconsistencies between different measurements

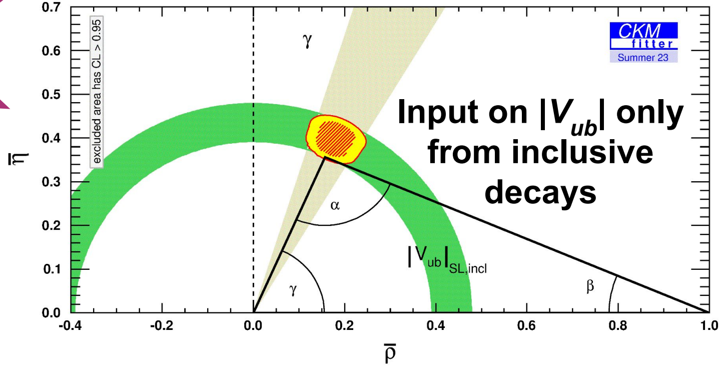


Wolfgang Altmannshofer
Belle II Physics Week

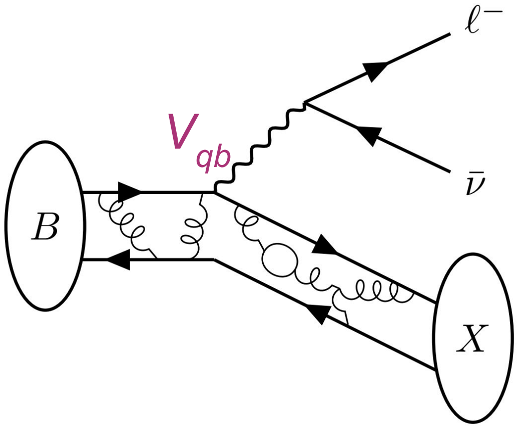
Error budget of BF($B \to K\,\nu\,\nu$) theory prediction



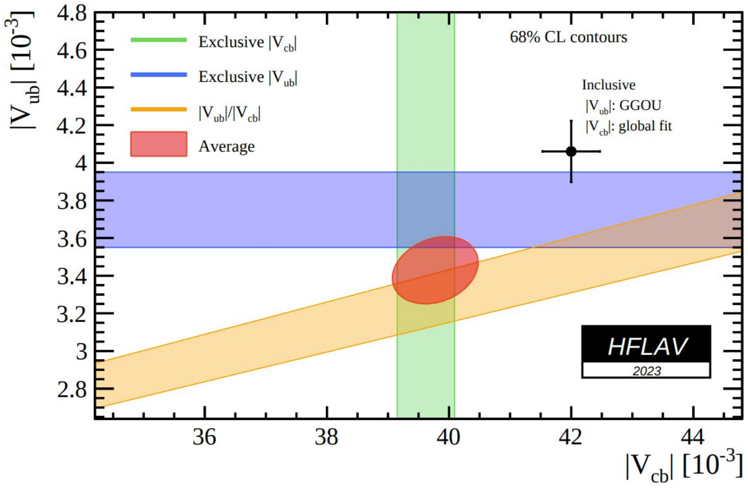
CKMfitter



$|V_{ub}|$ and $|V_{cb}|$ measurements



- Usually measured using semileptonic *B* decays
  - Via **exclusive decays**
    - $B \to \pi\, l\, \nu$, $B \to D\, l\, \nu$ …
  - Or via **inclusive decays** where no explicit requirements are applied on the hadronic system



- The two methods yield values which **differ by ~3σ** for both $|V_{ub}|$ and $|V_{cb}|$

$|V_{ub}|$ and $|V_{cb}|$ measurements

Semileptonic decays

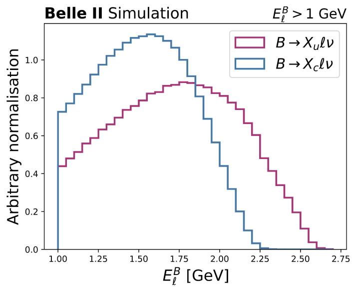
$$\lambda(q^2) = (m_B^2 + m_\pi^2 - q^2)^2 - 4m_B^2 m_\pi^2$$

$B \to \pi\, l\, \nu$

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{192\pi^3 m_B^3} \lambda^{3/2}(q^2) \boxed{|f_+(q^2)|}^2$$

- **3 main kinematical variables**
  - $E_l^{(B)}$: lepton energy (in B meson rest-frame)
  - M_X : mass of hadronic system
  - q^2 : momentum transfer
    - $q = p_l + p_\nu$
- Resonant decays are described via form factors
  - Functions which encapsulate the non-perturbative dynamics of the transition
- Inclusive decays
  - Described by Heavy Quark Expansion (HQE) → expansion in powers of α_s and $1/m_b$
  - $B \to X_c\, l\, \nu$ → use HQE to extract non-perturbative parameters and $|V_{cb}|$
  - $B \to X_u\, l\, \nu$ → apply kinematic selections to suppress large $B \to X_c\, l\, \nu$ background, HQE breaks down and decay rate prediction relies on specific calculations, combine resonant + non-resonant contribution

$X_c \sim 50 \times X_u$

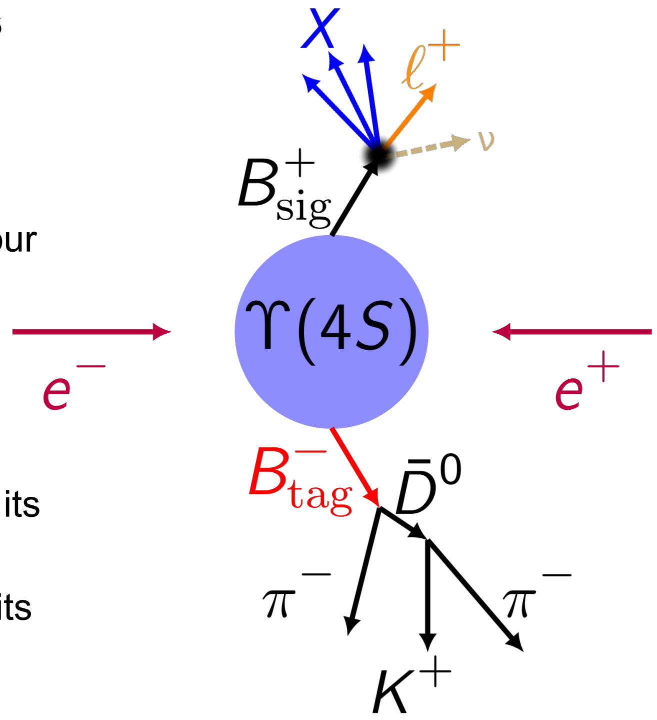


$$|V_{ub}| = \sqrt{\frac{\Delta\mathcal{B}(B \to X_u \ell \nu)}{\tau_B \Delta\tilde{\Gamma}(B \to X_u \ell \nu)}}$$

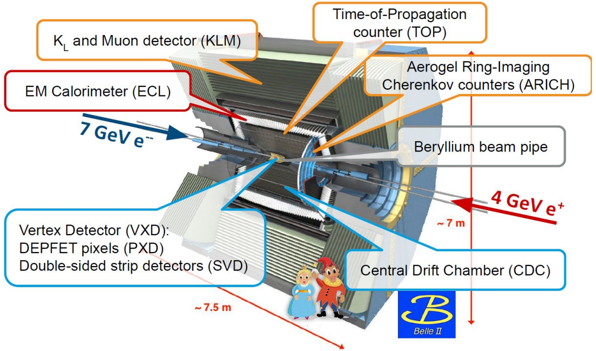
$|V_{ub}|$ and $|V_{cb}|$ measurements

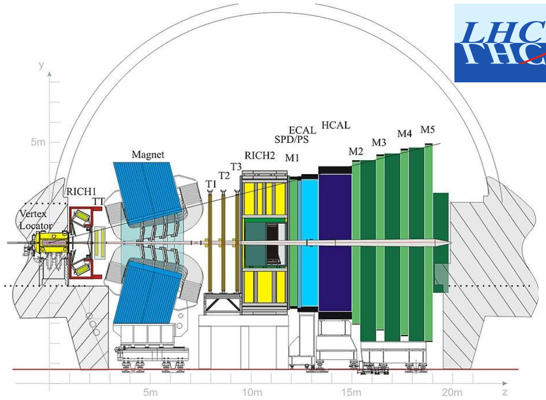
Challenges

- **We measure branching fractions of semileptonic decays but a theory input is needed to obtain $|V_{ub}|$ and $|V_{cb}|$**
  - Exclusive decays: form factor parameters
  - Inclusive decays: decay rate prediction via HQE
  - → theory and experiment must work hand-in-hand to improve our understanding
- **Experimental challenges of semileptonic decays**
  - Presence of neutrino ⇒ missing momentum
  - Inclusive hadronic system → need to reconstruct kinematic properties of a hadronic object without explicit requirements on its composition
  - *B*-factories: possibility to “tag” (*i.e.* reconstruct) non-signal *B* in its hadronic or semileptonic decay chains to constrain the event kinematics and derive properties of signal side particles



Experimental apparatus





SuperKEKB: asymmetric-energy $e^+e^- \rightarrow Y(4S) \rightarrow B\bar{B}$

~4π spatial coverage

Well known initial state

Tag non-signal *B*

⇒ Measurements with missing energy and inclusive systems

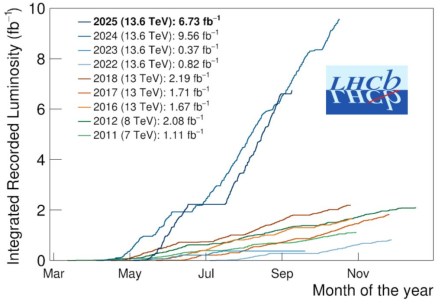
Run 1 luminosity: 365.37 ± 1.70 fb^{-1}

LHC: *pp* collider at various centre-of-mass energies

Access to a wide variety of *b* hadrons → unique opportunity to study baryon decays

Very good performance of muon reconstruction

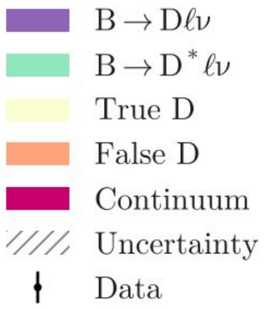
Luminosity as of now



EXCLUSIVE MEASUREMENTS

$B \to D\, l\, \nu$ measurement at Belle II

Exclusive $|V_{cb}|$ measurement

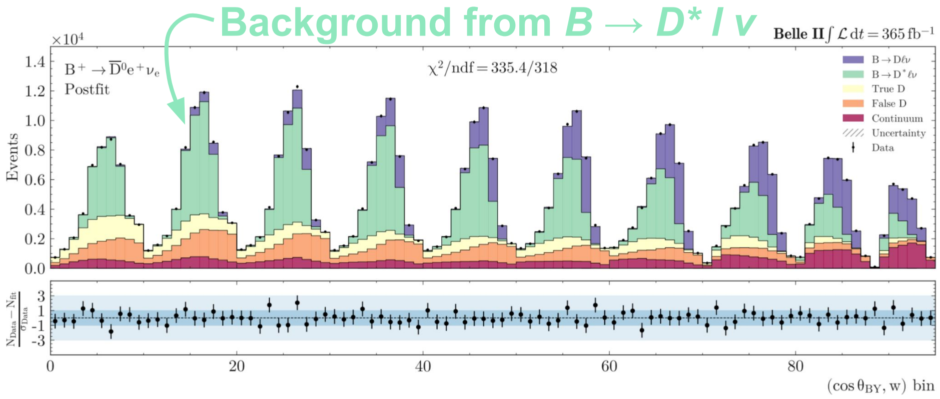


- Most commonly extracted from $B \to D^* l\, \nu$ decays because of the larger branching fraction
- Studying $B \to D\, l\, \nu$ has several advantages
  - Avoid reconstruction of low momentum pion from $D^* \to D\, \pi$ decays which represents the leading systematic uncertainty in $B \to D^* l\, \nu$ measurements
  - More precise lattice QCD predictions of form factor parameters
- Belle II 2025: arXiv:2506.15256 (accepted by PRD)
- Measurement performed using B^0 and B^+ decays **without explicitly reconstructing the partner *B* meson** from the $Y(4S) \to B\bar{B}$ decay
- The signal is **extracted from a 2D fit of** $\boldsymbol{\cos\theta_{BY}}$ and $\boldsymbol{w}$

***Y* = *Dl* system**

$$\cos\theta_{BY} = \frac{2\,E_B^* E_Y^* - M_B^2 - M_Y^2}{2|p_B^*||p_Y^*|}$$

$$w = \frac{M_B^2 + M_D^2 - q^2}{2 M_B M_D}$$

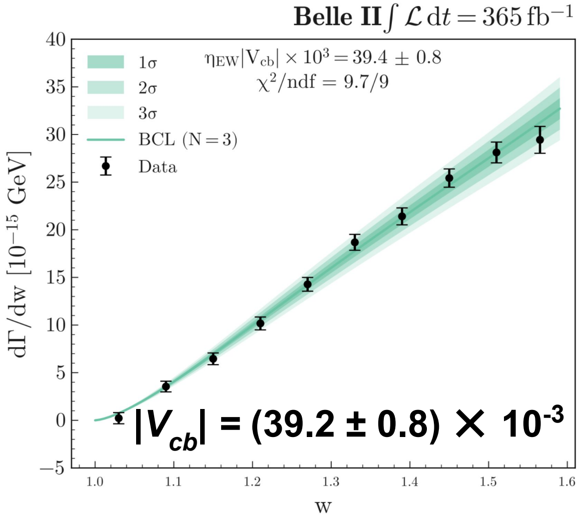


10 $\cos\theta_{BY}$ distributions in 10 w bins

$B \to D\, l\, \nu$ measurement at Belle II

Results

- The differential decay rate ***ΔΓ/Δw*** in 10 *w* bins is obtained from the **same fit**
- The obtained values of ***ΔΓ/Δw*** **are fitted** to the differential rate expressed using the Bourrely, Caprini, Lellouch (BCL) form factor parametrisation with a χ^2 fit with lattice QCD constraints → **extraction of $|V_{cb}|$ and BCL form factor parameters**



$|V_{cb}|$ = (39.2 ± 0.8) × 10^{-3}

$$|V_{cb}|_{\text{excl.}} = (39.77 \pm 0.46) \times 10^{-3}$$

$$|V_{cb}|_{\text{incl.}} = (41.97 \pm 0.48) \times 10^{-3}$$

HFLAV 2023 averages

arXiv:2506.15256

Measured values   Predicted values

$$\chi^2 = \sum_{i,j}^{10} \left(\boxed{\frac{\Delta\Gamma_i}{\Delta w}} - \boxed{\frac{\Delta\Gamma_{i,\mathrm{BCL}}}{\Delta w}} \right) \mathbf{C}^{-1}_{ij} \left(\frac{\Delta\Gamma_j}{\Delta w} - \frac{\Delta\Gamma_{j,\mathrm{BCL}}}{\Delta w} \right)$$

$$\boxed{+ \sum_{k,l}^{5} (c_k - c_{k,\mathrm{FLAG}}) \mathbf{D}^{-1}_{kl} (c_l - c_{l,\mathrm{FLAG}})}$$

LQCD constraints (FNAL/MILC + HPQCD)

2.1% relative uncertainty → Significant improvement of $|V_{cb}|$ precision from $B \to D\, l\, \nu$ decays and most precise fully exclusive measurement ever

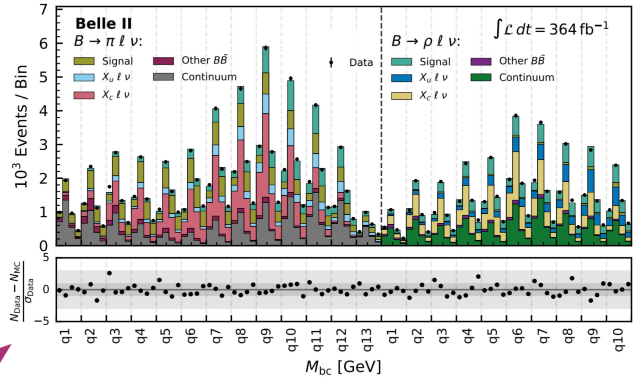
$B \to \pi/\rho\ l\ \nu$ measurement at Belle II

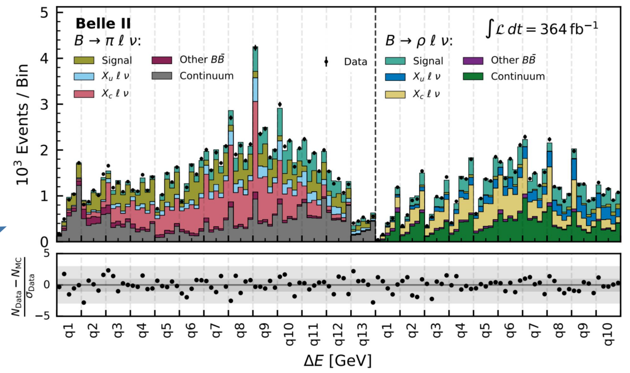
Exclusive $|V_{ub}|$ measurement

- Most precise determination of $|V_{ub}|$ using exclusive decays is obtained from $B \to \pi\ l\ \nu$ decays
- Simultaneous study of $B \to \pi\ l\ \nu$ and $B \to \rho\ l\ \nu$ performed at Belle II with untagged strategy [arXiv:2407.17403, accepted by PRD]
- Signal extracted from 2D fit of M_{bc} and ΔE
- Signal extracted in bins of q^2

$$M_{bc} = \sqrt{E_{\text{beam}}^{*2} - |\vec{p}_B^{\,*}|^2}$$

$$\Delta E = E_B^* - E_{\text{beam}}^*$$



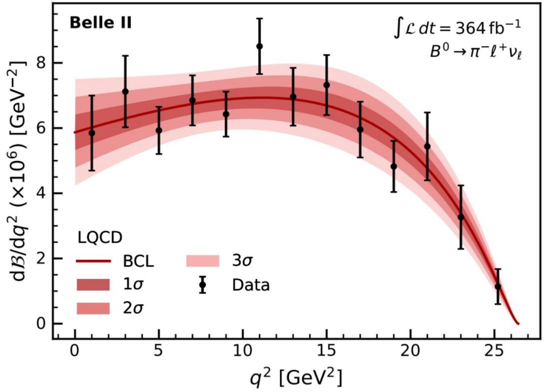


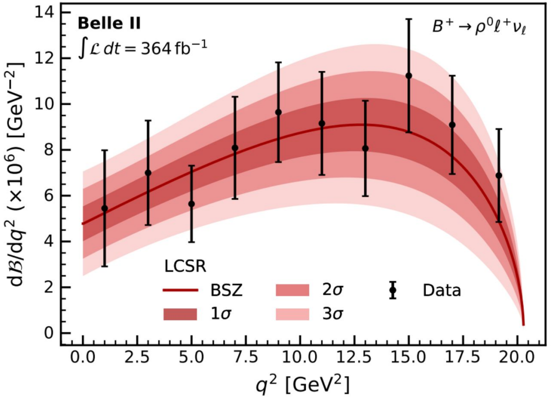
$B \to \pi/\rho\ l\ \nu$ measurement at Belle II

arXiv:2407.17403

Results

- Extract form factor parameters and $|V_{ub}|$ using χ^2 fit
- $B \to \pi\ l\ \nu \to$ BCL parametrisation, LQCD or LQCD + LCSR constraints
  - $|V_{ub}|_{LQCD} = (3.93 \pm \overset{stat}{0.09} \pm \overset{syst}{0.13} \pm \overset{theo}{0.19}) \times 10^{-3}$
  - $|V_{ub}|_{LQCD+LCSR} = (3.73 \pm 0.07 \pm 0.07 \pm 0.16) \times 10^{-3}$
- $B \to \rho\ l\ \nu \to$ Bharucha, Straub, Zwicky parametrisation and LCSR constraints
  - $|V_{ub}|_{LCSR} = (3.19 \pm 0.12 \pm 0.17 \pm 0.26)\times10^{-3}$
- Theoretical uncertainty dominates
- Light cone sum rules pull the central value down significantly





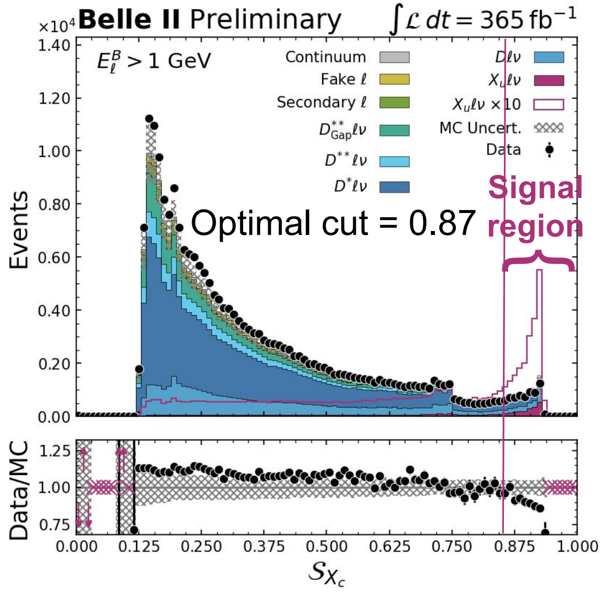
INCLUSIVE MEASUREMENTS

Preliminary

$B \to X_u\, l\, \nu$ measurement at Belle II

Inclusive $|V_{ub}|$ measurement

- Partner B meson is fully reconstructed in its hadronic decay channels → hadronic system is reconstructed inclusively
- Main challenge → large background from $e^+e^- \to q\bar{q}$ (q = u, d, s, c) and $B \to X_c\, l\, \nu$
  - Two separate neural networks to suppress background
  - Kaon veto to suppress decays to charm mesons
  - Poor modelling of $B \to X_c\, l\, \nu$ requires a complex fitting procedure to correct mismodelling without biasing the signal
- Signal extraction strategy
  - 2D fit in E_l^B and q^2
  - → Simultaneously fit the signal region and a control region to extract the shape of the $B \to X_c\, l\, \nu$ background from data



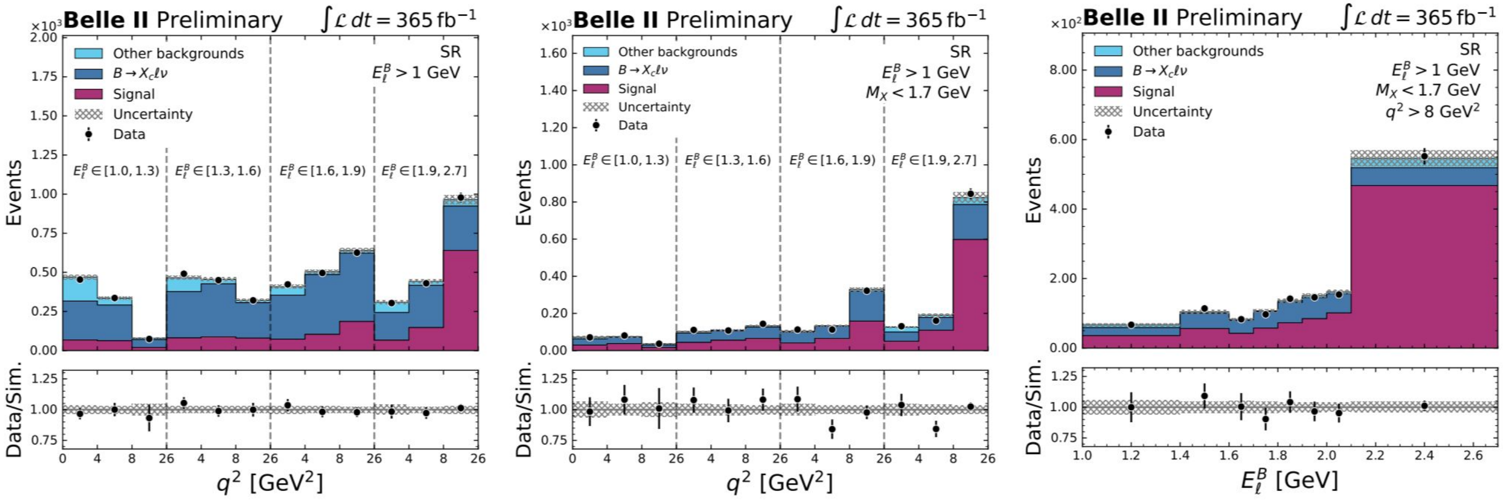
Preliminary

$B \to X_u\, l\, \nu$ measurement at Belle II

Extract $|V_{ub}|$ from 3 different regions of phase-space

- $E_l^B > 1.0$ GeV
- $E_l^B > 1.0$ GeV, $M_X < 1.7$ GeV
- $E_l^B > 1.0$ GeV, $M_X < 1.7$ GeV, $q^2 > 8$ GeV^2

Tighter selection increases signal purity but also model dependence



$B \to X_u\, l\, \nu$ measurement at Belle II

Preliminary

Results

- Theoretical predictions are most reliable over broader regions of phase-space
  - $\Delta\mathcal{B}(B \to X_u\, l\, \nu) = (1.54 \pm 0.08_{\text{stat}} \pm 0.12_{\text{syst}}) \times 10^{-3}$ with $E_l^B > 1.0$ GeV

Preliminary

$B \rightarrow X_u\, l\, \nu$ measurement at Belle II

Results

$$|V_{ub}| = \sqrt{\frac{\Delta\mathcal{B}(B \rightarrow X_u \ell \nu)}{\tau_B \Delta\tilde{\Gamma}(B \rightarrow X_u \ell \nu)}}$$

B meson lifetime

- Theoretical predictions are most reliable over broader regions of phase-space
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- Using the Gambino, Giordano, Ossola, Uraltsev framework

$B \to X_u\, l\, \nu$ measurement at Belle II

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- Using the Gambino, Giordano, Ossola, Uraltsev framework
  - $|V_{ub}| = (4.01 \pm 0.19_{\text{exp}}\, {}^{+0.07}_{-0.08}\,{}_{\text{theo}}) \times 10^{-3}$

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Preliminary

$B \to X_u\, l\, \nu$ measurement at Belle II

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$$|V_{ub}| = \sqrt{\frac{\Delta\mathcal{B}(B \to X_u \ell \nu)}{\tau_B \Delta\tilde{\Gamma}(B \to X_u \ell \nu)}}$$

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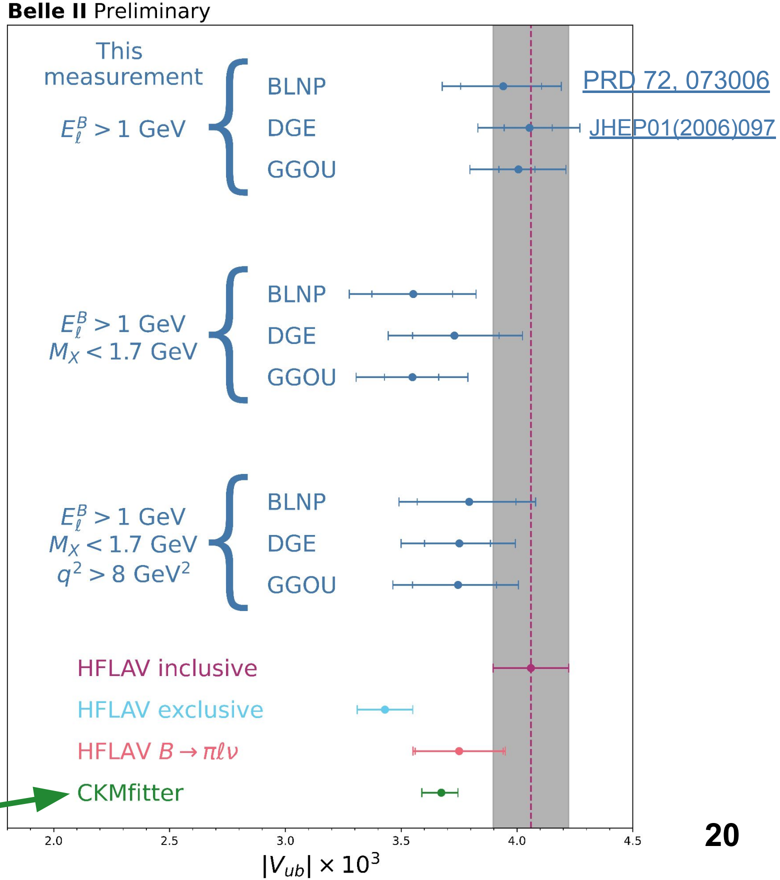


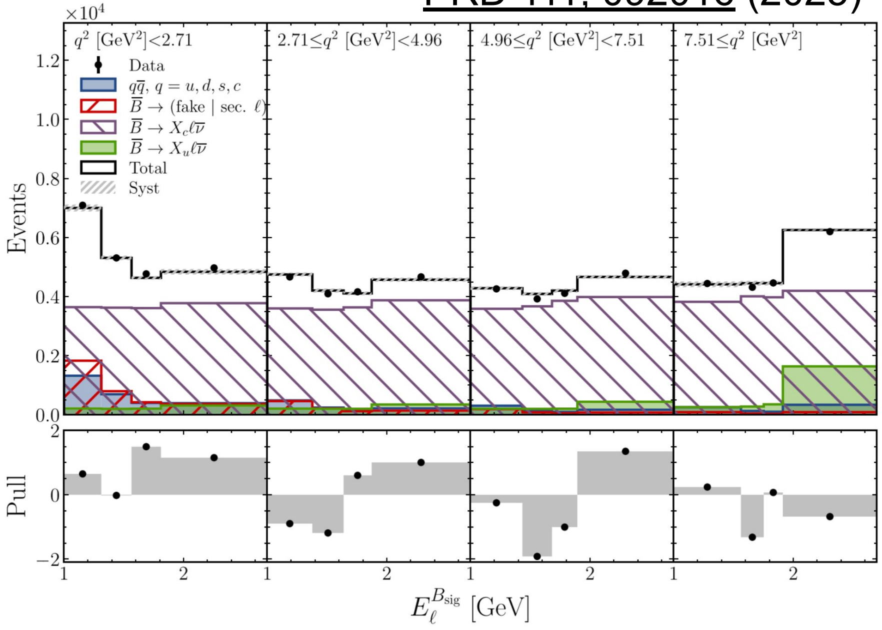
CKMfitter output without using $|V_{ub}|$ inputs

Inclusive $|V_{ub}|$ / $|V_{cb}|$ measurement at Belle

Inclusive $|V_{ub}|$ and $|V_{cb}|$ measurement

PRD 111, 092016 (2025)

- Simultaneous constraint on $|V_{ub}|$ and $|V_{cb}|$ with reduced systematic uncertainty
- Partner B reconstructed in hadronic decays
- $B \to X_u \, l \, \nu$ yields are extracted from a region enriched in such decays using a 2D E_l^B:q^2 fit
- $B \to X_c \, l \, \nu$ yields are obtained from a region enriched in such decays by subtracting the scaled $B \to X_u \, l \, \nu$ yields
- Ratio of partial branching fractions is obtained from ratio of X_u and X_c yields
- $|V_{ub}|$ / $|V_{cb}|$ = (10.06 ± 0.66) × 10^{-2}



$$\frac{|V_{ub}|}{|V_{cb}|} = \sqrt{\frac{\Delta\mathcal{B}(\bar{B} \to X_u \ell \bar{\nu})}{\Delta\mathcal{B}(\bar{B} \to X_c \ell \bar{\nu})} \frac{\Delta\Gamma(\bar{B} \to X_c \ell \bar{\nu})}{\Delta\Gamma(\bar{B} \to X_u \ell \bar{\nu})}}$$

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OTHER MEASUREMENTS

Other measurements

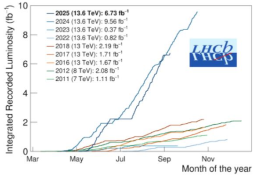
- Inclusive $|V_{cb}|$ → latest result from Belle II
  - Measurement of lepton mass squared moments in $B \to X_c\, l\, \nu$ decays with the Belle II experiment [PRD **107, 072002** (2023)]
- LHCb
  - Two measurements of $|V_{ub}|$ / $|V_{cb}|$ ratio
    - $\Lambda_{b/c}$ semileptonic decays [Nature Physics 11, 743–747 (2015)]
    - B_s semileptonic decays [PRL 126, 081804 (2021)]
  - One measurement of $|V_{cb}|$ using $B_s \to D^{(*)}_s\, \mu\, \nu$ [PRD 101, 072004 (2020)]

Large number of incoming results from LHCb

Upcoming measurements at LHCb

CKM 2025

1. **Extracting $|V_{ub}|$ from $B_s^0 \to K^-\mu^+\nu_\mu$** and $B_s^0 \to K^-$ FF parameters with Run 2 data
   - Measure shape of differential decay rate in eight bins of q^2
2. **Extracting $|V_{ub}|$ from $B^+ \to \rho^0\mu^+\nu_\mu$** and $B^+ \to \rho^0$ FF parameters with Run 2 data
   - Measure shape of differential decay rate in 10 bins of q^2
3. **Extracting $|V_{ub}|/|V_{cb}|$ from $B_c^+ \to D^{(*)0}\mu^+\nu_\mu$** with Run 2 data
   - Normalize to $B_c^+ \to J/\psi\mu^+\nu_\mu$
   - Profit from LQCD $B_c^+ \to D^0$ FF across full q^2 [Phys. Rev. D 105, 014503 (2022)]
4. **Extracting $|V_{cb}|$ form $\Lambda_b^0 \to \Lambda_c^+\mu^-\overline{\nu}_\mu$** with Run 2 data
   - Measure shape of differential decay rate
5. **Extracting $B^0 \to D^{*-}$ FF from $B^0 \to D^{*-}\mu^+\nu_\mu$** using Run 2 data



⇒ **Many exciting Run 2 results upcoming**

⇒ Already collected **more data in Run 3** than in Run 1+2 combined

Fabian Glaser | CKM 2025 | September 16th 2025 | 22 / 23

Leptonic *B* decays

$B^+ \to \tau^+/\mu^+ \nu$

$$\mathcal{B}(B^+ \to \ell^+\nu_\ell) = \frac{G_F^2 m_B}{8\pi} m_\ell^2 \left(1 - \frac{m_\ell^2}{m_B^2}\right)^2 f_B^2 |V_{ub}|^2 \tau_B$$

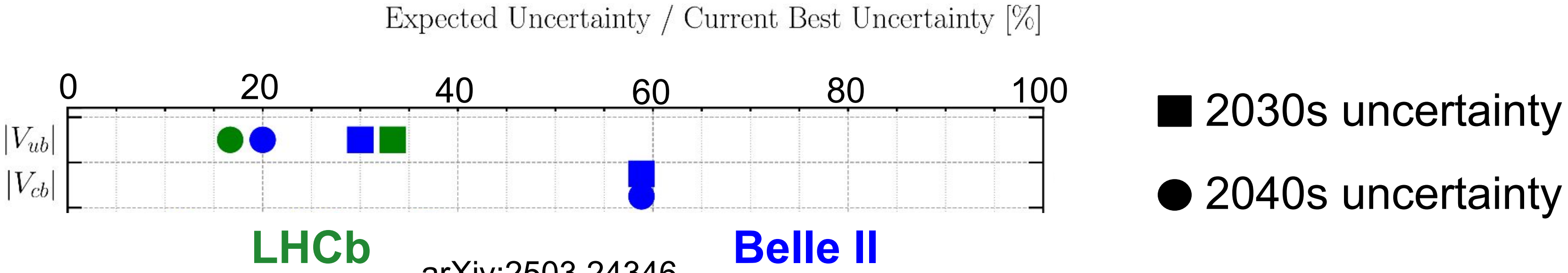
f_B = 190.0 ± 1.3 MeV
[FLAG]

- Cleanest channels to measure $|V_{ub}|$
- Challenging measurements because of helicity suppression
  - $B^+ \to \tau^+ \nu$ branching fraction is expected to be of the same order as that of $B \to \pi l \nu$ but it suffers from large backgrounds
- Belle II has performed two measurements
  - Hadronic tag $B^+ \to \tau^+ \nu \to |V_{ub}| = (4.41^{+0.74}_{-0.89}) \times 10^{-3}$ [PRD 112, 072002 (2025)]
  - Untagged $B^+ \to \mu^+ \nu \to |V_{ub}| = (3.90^{+0.88}_{-1.08}) \times 10^{-3}$ (preliminary)

PERSPECTIVES

$|V_{cb}|$ in the next 5 years

- In addition to statistical uncertainty, several systematic uncertainties in Belle II measurements can be reduced with larger dataset (number of *Y(4S)* events, particle identification…)
- But the bottleneck comes from theoretical predictions for form factors
- Cause of the exclusive/inclusive discrepancy hasn't been identified but investigations are ongoing
  - Understand better $B \to D^{**} l \nu$ and non-resonant decays (see next slides)
  - LHCb: potential study of inclusive B_s decays via sum of exclusive [JHEP06(2024)158]



arXiv:2503.24346
atas.web.cern.ch

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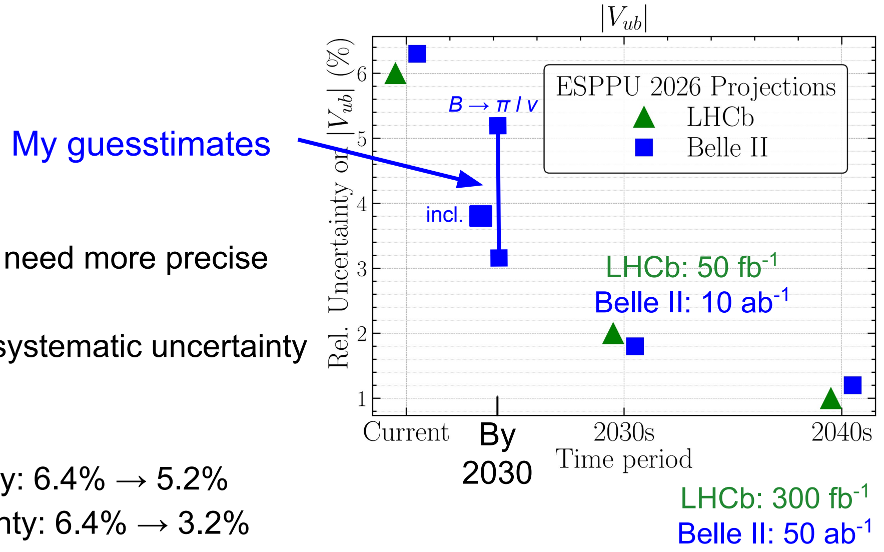
$|V_{ub}|$ in the next 5 years

- **$|V_{ub}|$ from $B \to \pi l \nu$**
  - Measurements are limited by the theoretical uncertainty → need more precise lattice QCD results
  - Experimental uncertainty of Belle II result is dominated by systematic uncertainty but part of it can be reduced with higher statistics
  - Assume 5 ab^{-1} collected by Belle II
    - Same theoretical uncertainty → total $|V_{ub}|$ uncertainty: 6.4% → 5.2%
    - Theoretical uncertainty halved → total $|V_{ub}|$ uncertainty: 6.4% → 3.2%
- **Inclusive measurements of $|V_{ub}|$**
  - Latest Belle II result suffers from poor modelling of dominating charm background
  - Assuming 5 ab^{-1} of collected data and an improved background modelling
    - Total $|V_{ub}|$ uncertainty: 5.1% → 3.8%
- **Leptonic decays**
  - Unlikely to give competitive measurements in the next few years but with large datasets (Belle II, FCC-ee) they will yield the most precise determinations of $|V_{ub}|$
  - LHCb has the potential to measure $B \to \mu\nu\gamma$ (and $B \to \tau\nu$)



arXiv:2503.24346
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What about the $|V_{ub}|$ discrepancy?

- **Modelling of inclusive $B \to X_u\, l\, \nu$ decays ?**
  - Modelling is more reliable over broad regions of phase-space
  - But measurements are very sensitive to high signal purity region
  - Larger dataset and improved machine learning selection would help increase the signal purity over the entire phase-space → reduced model dependence
- **$B \to X_c\, l\, \nu$ background ?**
  - $B \to X_c\, l\, \nu$: gap between sum of exclusive BR and total inclusive BR is filled by *ad hoc* never-observed decays
  - Ongoing theoretical and experimental investigation of decay modes which could fill this gap
  - → **this could also explain the $|V_{cb}|$ discrepancy**

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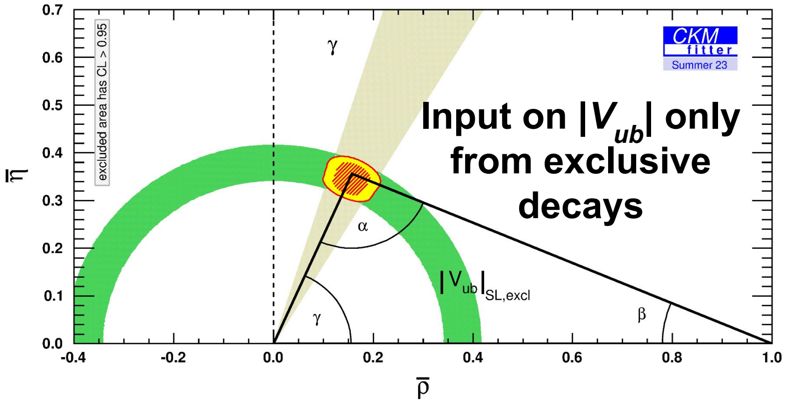

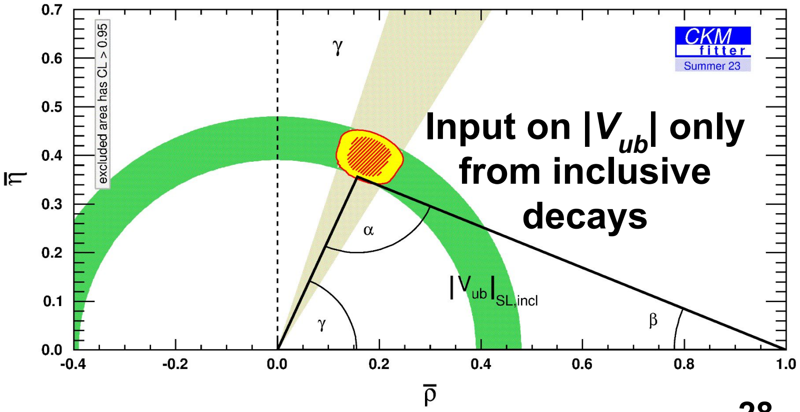

INFN

Summary

- **Already several measurements of $|V_{ub}|$ and $|V_{cb}|$ at Belle II**
  - Thanks to improved analysis techniques, these results are competitive with / better than BaBar and Belle measurements already with the current dataset
- **LHCb is on track to perform several measurements using Run 2 data**
  - The total collected dataset after Run 3 offers a chance to perform measurements with very low statistical uncertainty
- **Understanding the discrepancies** between exclusive and inclusive measurements of $|V_{ub}|$ and $|V_{cb}|$ is **crucial to constrain the unitarity triangle** sides and enhance the sensitivity of other CKM unitarity tests to possible non-SM effects
- **Exciting times for CKM matrix tests → stay tuned!**

THANKS FOR LISTENING!

BACKUP

$B^0 - \overline{B}^0$ mixing

SU(3) flavor-symmetry breaking factor

$$\left|\frac{V_{td}}{V_{ts}}\right| = \xi \sqrt{\frac{\Delta m_d}{\Delta m_s}\frac{m(B_s^0)}{m(B^0)}}$$

- Measured for the first time by UA1 and ARGUS in 1987
- Probability for B_q^0 or $\overline{B}_q^0$ meson to decay after time t in same flavour state (+1) or opposite (-1) is proportional to $1 \pm \cos(\Delta m_q t)$
- → oscillation frequency Δm_q is extracted from time-dependent studies of B^0 mixing
- ***B*-factories** (BaBar, Belle, Belle II)
  - Identify the flavour of the *signal* B using a high branching fraction decay
    - e.g. charge of lepton in $B \to D^* l \nu$ or pion in $B \to D^{*-} \pi^+$
  - Identify flavour of the other B using a flavour tagging algorithm
  - The decay time Δt is then measured as the time difference between the decays of both B
- **High-energy colliders** (LEP, Tevatron, LHC)
  - Infer the proper time of the B^0 from its decay length
  - Identify its flavour at time of production from information related this B or the other b hadron produced in the collision

CKM parameter combinations

- CKM matrix parameter values can also be inferred from global fits to measured values
  - CKMfitter (frequentist approach)
  - UTfit (Bayesian approach)
- Global fits incorporate unitarity constraints
- Precision of Unitarity Triangle fits is limited by the uncertainty on $|V_{ub}|/|V_{cb}|$

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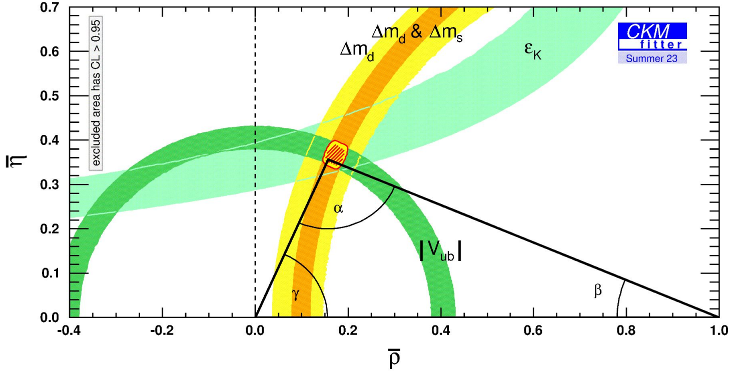

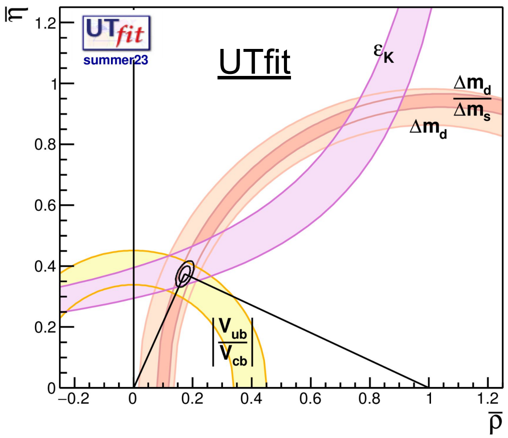

Inclusive or exclusive?

That is the question

CKMfitter average

Constraint from exclusive

Constraint from inclusive

Indirect determination from global fit

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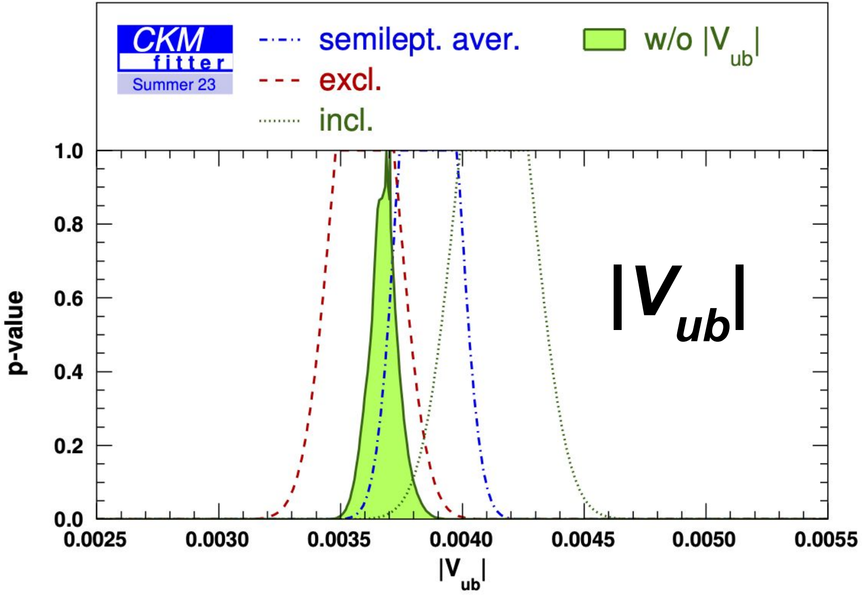
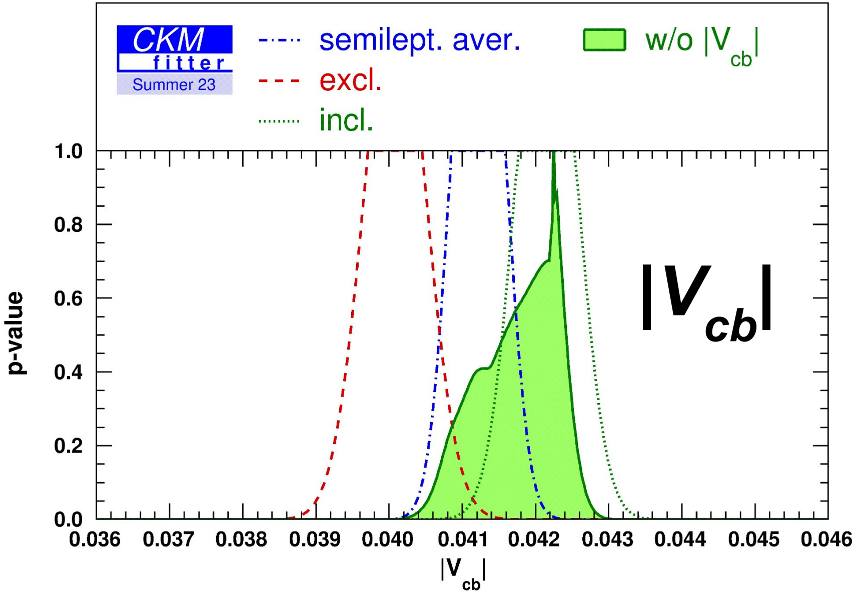




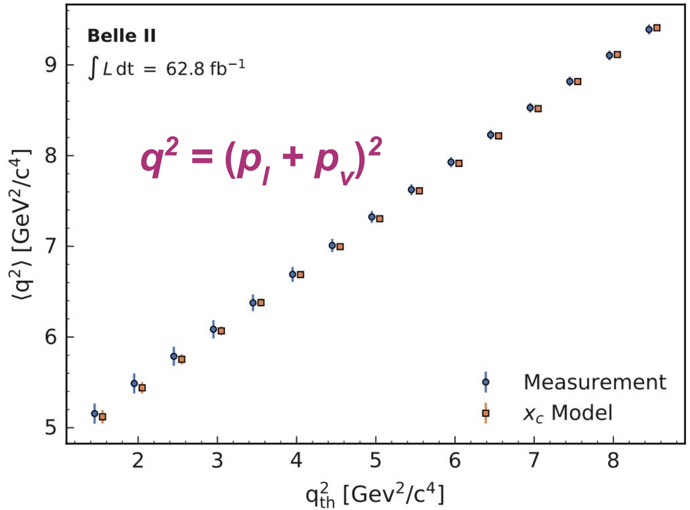
$|V_{cb}|$ measurements

Inclusive measurements

$$\langle Y^n \rangle = \int Y^n \frac{d\Gamma}{dY} dY$$

Measure moments of kinematic variables in inclusive $B \to X_c\, l\, \nu$ decays

- Y can be the momentum transfer q^2 for example
- Express moments using the *heavy quark expansion* (HQE) and simultaneously extract the HQE non-perturbative parameters and $|V_{cb}|$
- Moments measured most recently by Belle II
  - PRD **107, 072002** (2023)



$$|V_{cb}| = (41.69 \pm 0.63) \cdot 10^{-3}$$