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Correlating lepton flavor violating $b \to s$ and leptonic decay modes in a minimal abelian extension of the SM

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based on arXiv:2506.02552 (P. Colangelo, F. De Fazio, D. Milillo)

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Overview

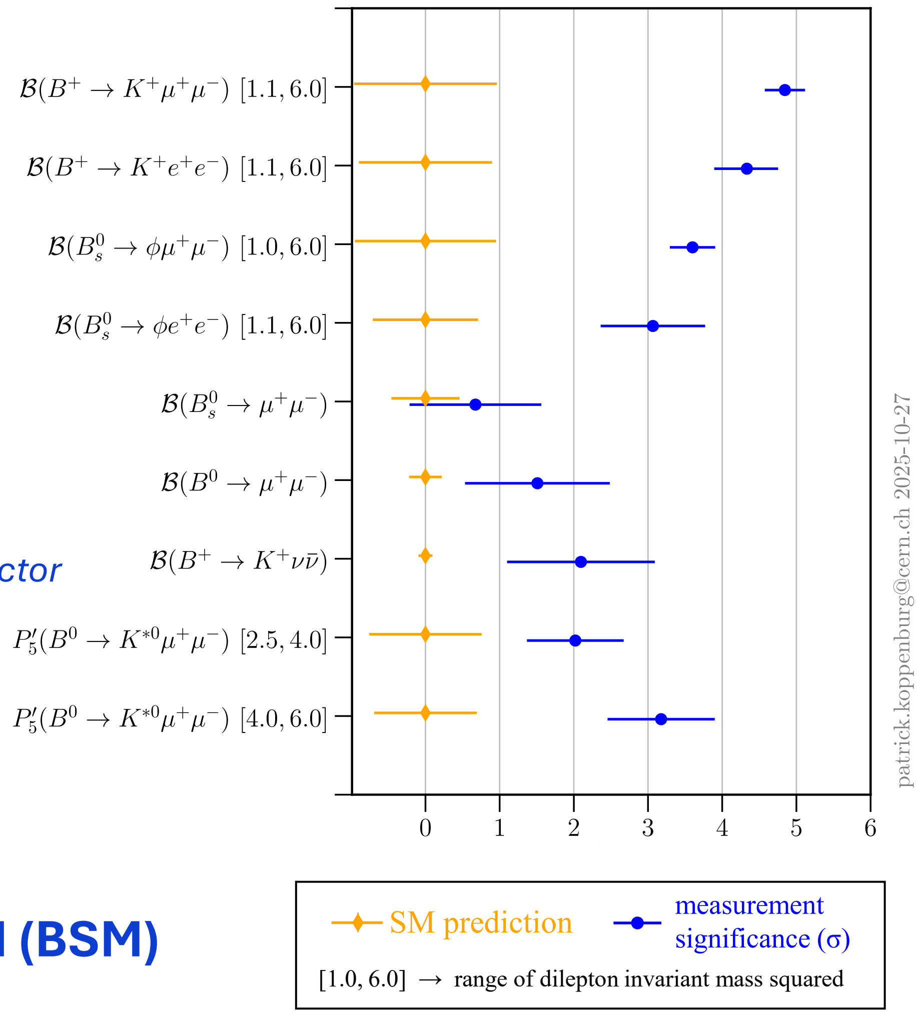
- Motivations to BSM
- Abelian extension of SM: ABCD model
- Lepton Flavor Violation in $b \to s$ & leptonic modes
- Conclusions

SM & beyond: open problems & tensions

- SM is the best tested theory of fundamental particles & interactions

- However, many tensions observed:
  - Predicted CP-violation too small to explain baryogenesis
  - CKM unitarity & determination of matrix elements
  - $b \to d, s$ observables (BRs, angular distributions, etc.)
  - $R_{D^{(*)}} = \dfrac{\mathcal{B}(B \to D^{(*)}\tau\bar{\nu}_\tau)}{\mathcal{B}(B \to D^{(*)}\ell\bar{\nu}_\ell)}$ exceed SM by > 3σ, possibly suggesting lepton flavor universality violation (LFUV)

flavor sector



- Flavor anomalies motivate the search for **physics beyond SM (BSM)**

ABCD model [1]

- Minimal abelian extension of the SM:

$$\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times \mathrm{U}(1)' \;\rightarrow\; \text{new massive neutral } \boldsymbol{Z'} \textbf{ boson}$$

- Z' boson interaction with SM fermions:

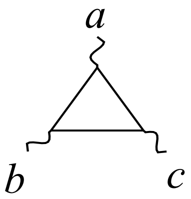
$$\mathscr{L}'_{\mathrm{int}} = -\underbrace{g_{Z'}}_{\text{gauge coupling constant}} \sum_{\psi} \sum_{i,j} \left[Z^{ij}_{\psi_L} \, \overline{\psi}_{iL} \gamma^\mu \psi_{jL} + Z^{ij}_{\psi_R} \, \overline{\psi}_{iR} \gamma^\mu \psi_{jR} \right] \underbrace{\mathbf{Z}'_\mu}_{Z' \text{ gauge field}}$$

- Couplings: $Z^{ij}_\psi = \underbrace{z_{\psi_i}}_{z\text{-hypercharges}} \delta^{ij}$ (diagonal in flavor basis)

[1] JHEP 02(2020)183 (Aebischer, Buras, Cerdà-Sevilla, De Fazio)

Gauge anomalies cancellation

- **Gauge anomalies** lead to profound inconsistencies (e.g. electric charge not conserved)
- Anomalous contribution comes from 1-loop correction to 3-gauge bosons vertex function (triangle diagrams)

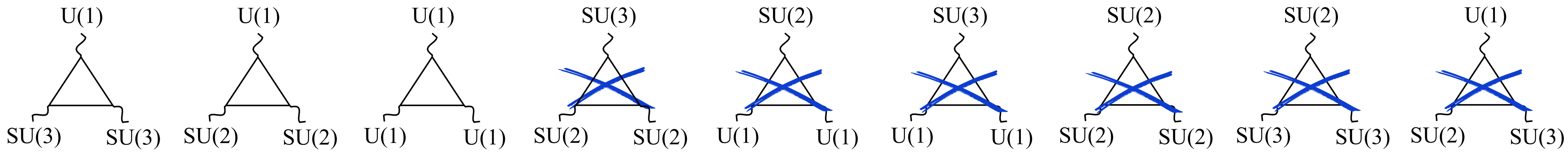


$$\propto \mathrm{Tr}\left[\mathbf{T}_a\{\mathbf{T}_b, \mathbf{T}_c\}\right]$$

group generators

- Gauge theories (e.g. the SM) must be *anomaly free* ➔ triangle diagrams must cancel

Gauge anomalies cancellation in SM

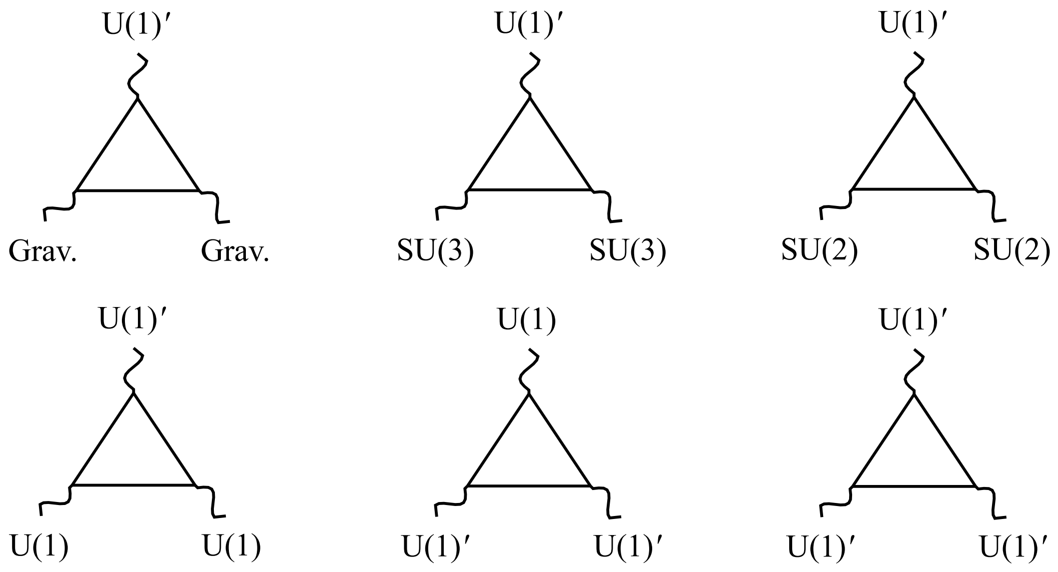


3 anomaly cancellation equations (ACEs)

verified independently by each fermion generation

$$\begin{cases} A_{331} = 2y_q - y_u - y_d = 0 \\ A_{221} = 3y_q + y_\ell = 0 \\ A_{111} = 3(2y_q^3 - y_u^3 - y_d^3) + 2y_\ell^3 - y_e^3 = 0 \end{cases}$$

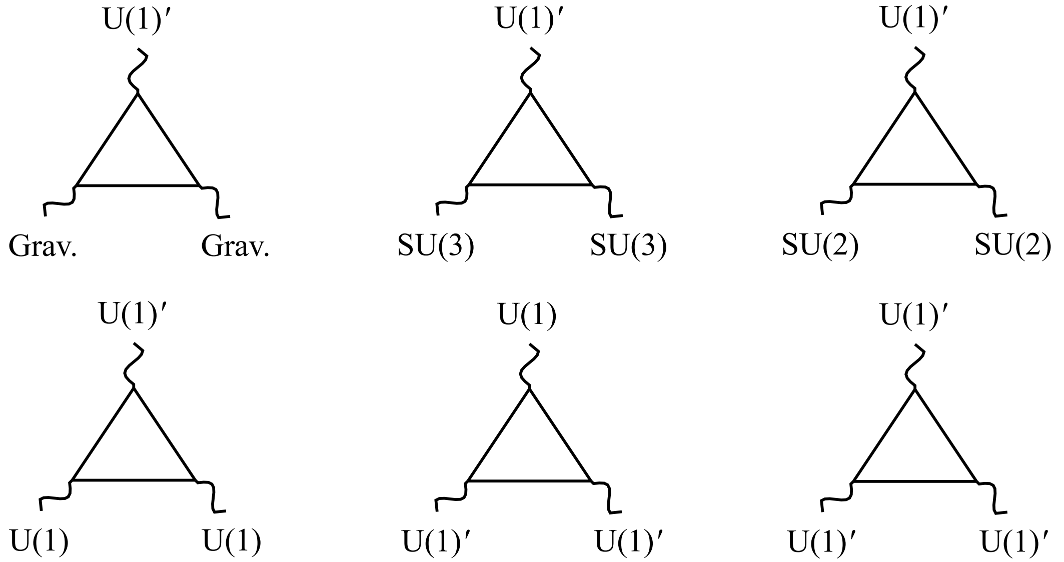
Gauge anomalies cancellation (ABCD model)



6 additional ACEs:

$$
\begin{cases}
A_{GG1'} = 2z_\ell - z_\nu - z_e = 0 \\
A_{331'} = 2z_q - z_u - z_d = 0 \\
A_{221'} = 3z_q + z_\ell = 0 \\
A_{111'} = \frac{1}{6}z_q - \frac{4}{3}z_u - \frac{1}{3}z_d + \frac{1}{2}z_\ell - z_e = 0 \\
A_{1'1'1} = z_q^2 - 2z_u^2 + z_d^2 - z_\ell^2 + z_e^2 = 0 \\
A_{1'1'1'} = 3(2z_q^3 - z_u^3 - z_d^3) + 2z_\ell^3 - z_\nu^3 - z_e^3 = 0
\end{cases}
$$

Gauge anomalies cancellation (ABCD model)



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ABCD assumption: generation-dependent *z*-hypercharges

$$z_{\psi_i} = y_\psi + \epsilon_i$$

(*i*=1,2,3 generation index)

SM weak hypercharges / rational numbers

i.e. same ϵ_i for all fermions (quark & leptons) of a given generation

→ ACEs satisfied if:

$$\epsilon_1 + \epsilon_2 + \epsilon_3 = 0$$

#free parameters reduced:

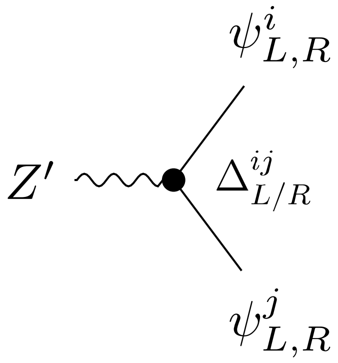
$$\epsilon_3 = -(\epsilon_1 + \epsilon_2)$$

Z' couplings to fermions

- Rotate fermion fields to mass basis by unitary matrices $\hat{V}_{L/R}$ (PMNS, CKM, CKM-like):

$$\mathscr{L}'_{\text{int}} = -\sum_{\psi}\sum_{i,j}\left[\overline{\psi}_{iL}\gamma^{\mu}\Delta^{ij}_{\psi_L}\psi_{jL} + \overline{\psi}_{iR}\gamma^{\mu}\Delta^{ij}_{\psi_R}\psi_{jR}\right]\mathbf{Z}'_{\mu}$$

- Couplings: $\Delta^{ij}_{L/R} = g_{Z'}\sum_{k}\epsilon_k(\hat{V}_{L/R})^{*}_{ki}(\hat{V}_{L/R})_{kj}$ *(generally non-diagonal)*



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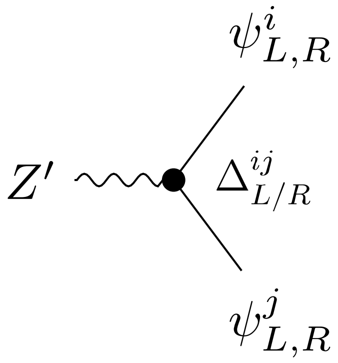


the set $\{\epsilon_1, \epsilon_2, \epsilon_3\}$ is the same for both quark & lepton sectors

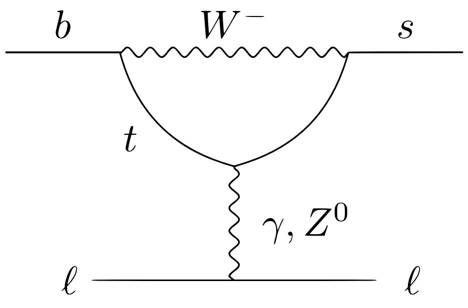
- quark & lepton sectors act together to avoid large deviations from SM
- correlations between hadron & lepton decays can be established

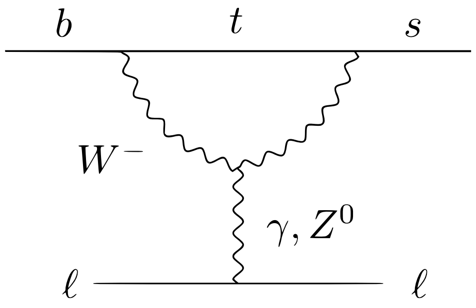
Most distinctive feature of ABCD model!

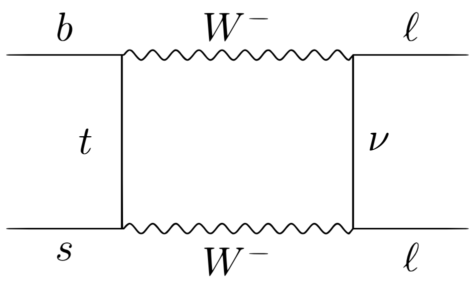
❖ Promising processes to be investigated: rare & SM-forbidden hadron & lepton decays

Effective Hamiltonian for $b \to s\, \ell^- \ell^+$ (SM)

- FCNCs (e.g. $b \to s\ell^-\ell^+$) forbidden at tree-level due to unitarity of CKM matrix & universality of weak interactions
- However, they can occur at 1-loop level through penguin & box diagrams







- At typical hadron energies (m_b ~ 4.2 GeV) heavy fields can be *integrated out* ➜ effective point-like interaction

$$\mathcal{H}_{\text{eff}} = -4\frac{G_F}{\sqrt{2}} \overbrace{V_{tb}V_{ts}^*}^{\text{CKM factor}} \sum_k C_k \overbrace{\mathcal{O}_k}^{\text{local 4-fermion operators}}$$

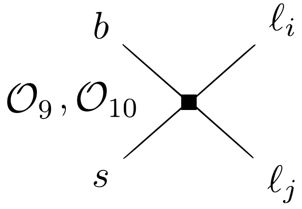
Wilson coefficients: short-distance effects at scales $\mu > m_b$ (*effective coupling constants*)

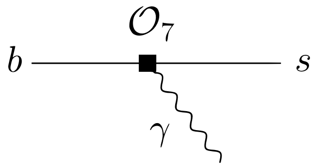
relevant operators for $b \to s\ell_i^-\ell_j^+$:

$$\mathcal{O}_9 = \frac{e^2}{16\pi^2}(\bar{\mathbf{s}}\gamma^\mu P_L \mathbf{b})(\bar{\boldsymbol{\ell}}_i\gamma_\mu \boldsymbol{\ell}_j)$$

$$\mathcal{O}_{10} = \frac{e^2}{16\pi^2}(\bar{\mathbf{s}}\gamma^\mu P_L \mathbf{b})(\bar{\boldsymbol{\ell}}_i\gamma_\mu\gamma_5 \boldsymbol{\ell}_j)$$

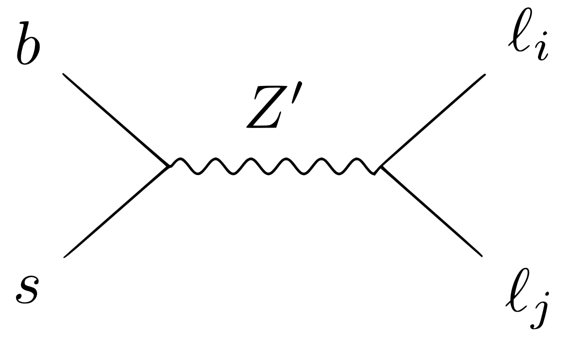
$$\mathcal{O}_7 = \frac{e}{16\pi^2}\left[\bar{\mathbf{s}}\sigma^{\mu\nu}(m_s P_L + m_b P_R)\mathbf{b}\right]\mathbf{F}_{\mu\nu}\delta_{ij}$$





Effective Hamiltonian for $b \to s\, \ell_i^- \ell_j^+$ (ABCD model)

In ABCD model, Z' boson can have flavor non-universal couplings



- lepton flavor conserving (**LFC**) FCNC transition $b \to s\ell^-\ell^+$ at tree level
- lepton flavor violating (**LFV**) transition $b \to s\ell_i^- \ell_j^+$ ($i \neq j$) allowed

NP effects → modification of Wilson coefficients: $(C_{9,10})_{ij} = C_{9,10}^{\rm SM}\delta_{ij} + (C_{9,10}^{\rm NP})_{ij}$

$C_7^{\rm SM}$	$C_9^{\rm SM}$	$C_{10}^{\rm SM}$
-0.450	4.273	-4.166

Bordone, Cornella, Davighi [arXiv:2503.22635]

and

$$(C_{9,10}^{\rm NP})_{ij} = -\frac{g_t^2}{M_{Z'}^2}\Delta_{d_L}^{bs}(\Delta_{e_R}^{ij} \pm \Delta_{e_L}^{ij})$$

with: $g_t^2 = \dfrac{4\pi^2}{8G_F^2 M_W^2 \sin^2\theta_W\, V_{tb}^* V_{ts}}$

Scenario A: no flavor violation for RH fermions ($\Delta_{\psi_R}^{ij} = 0$ if $i \neq j$)

Note: for increasing $M_{Z'}$ ABCD model approaches SM (NP contribution vanishes) → smaller deviations

(Semi)leptonic $B_{(s)}$ decay modes

$B_s \to \ell_i^- \ell_j^+$

$$\bar{\mathcal{B}}(B_s \to \ell_i^- \ell_j^+) = \frac{1}{(1-y_s)} \frac{\alpha^2 G_F^2 \tau_{B_s}}{64\pi^3 M_{B_s}^3} F_{B_s}^2 |V_{tb} V_{ts}^*|^2 \, \lambda^{1/2}(M_{B_s}^2, m_{\ell_i}^2, m_{\ell_j}^2)$$
$$\times \Big\{ [M_{B_s}^2 - (m_{\ell_i} + m_{\ell_j})^2] \, \big|(m_{\ell_i} - m_{\ell_j}) C_9\big|^2 + [M_{B_s}^2 - (m_{\ell_i} - m_{\ell_j})^2] \, \big|(m_{\ell_i} + m_{\ell_j}) C_{10}\big|^2 \Big\}$$

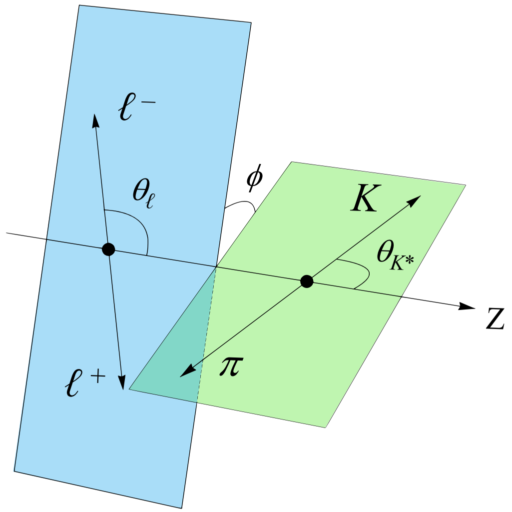
- Hadronic uncertainties only due to F_{B_s}
- In LFC case ($i = j$) only C_{10} contributes

$\bar{B} \to \bar{K}^* \ell_i^- \ell_j^+$

Fully differential decay distribution for $\bar{B} \to \bar{K}^*(\to K\pi)\ell_i^- \ell_j^+$:

$$\frac{d^4\Gamma}{dq^2 \, d\cos\theta_\ell \, d\cos\theta_{K^*} \, d\phi} = \frac{9}{32\pi} \Big\{ I_1^s \sin^2\theta_{K^*} + I_1^c \cos^2\theta_{K^*} + [I_2^s \sin^2\theta_{K^*} + I_2^c \cos^2\theta_{K^*}] \cos 2\theta_\ell$$
$$+ I_3 \sin^2\theta_{K^*} \sin^2\theta_\ell \cos 2\phi + I_4 \sin 2\theta_{K^*} \sin 2\theta_\ell \cos\phi$$
$$+ I_5 \sin 2\theta_{K^*} \sin\theta_\ell \cos\phi + [I_6^s \sin^2\theta_{K^*} + I_6^c \cos^2\theta_{K^*}] \cos\theta_\ell$$
$$+ I_7 \sin 2\theta_{K^*} \sin\theta_\ell \sin\phi + I_8 \sin 2\theta_{K^*} \sin 2\theta_\ell \sin\phi + I_9 \sin^2\theta_{K^*} \sin^2\theta_\ell \sin 2\phi \Big\}$$

angular coefficients



Parameter space

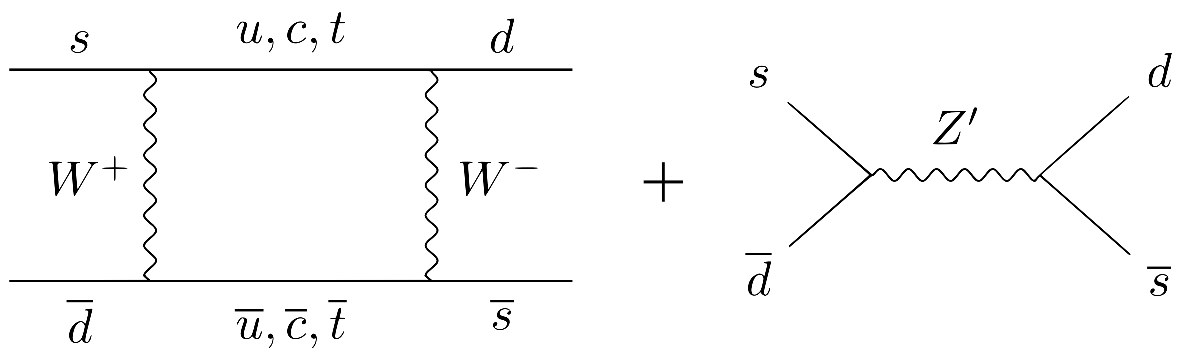
Free-parameters:

$$\{g_{Z'},\ M_{Z'},\ \epsilon_1,\ \epsilon_2,\ |V_{ub}|,\ |V_{cb}|\}$$

$$\begin{cases} 0.01 \leq g_{Z'} \leq 1 \\ M_{Z'} = 1\,\text{TeV and } 3\,\text{TeV} \end{cases} \qquad \begin{array}{l} |V_{ub}|_{\text{exc}} \leq |V_{ub}| \leq |V_{ub}|_{\text{inc}} \\ |V_{cb}|_{\text{exc}} \leq |V_{cb}| \leq |V_{cb}|_{\text{inc}} \end{array}$$

Constrain $\epsilon_1,\ \epsilon_2$ by requiring $\Delta F = 2$ mixing observables ($\mathcal{F}$) to lie within experimental range:

$$\mathcal{F} = \mathcal{F}_{\text{SM}} + \mathcal{F}_{\text{NP}} \in \left[\mathcal{F}_{\text{exp}} - \delta\mathcal{F}_{\text{exp}},\ \mathcal{F}_{\text{exp}} + \delta\mathcal{F}_{\text{exp}}\right]$$



Example: $K^0 - \bar{K}^0$ mixing (SM+NP)

	$\mathcal{F}$	$\mathcal{F}_{\text{exp}} \pm \delta\mathcal{F}_{\text{exp}}$
$B_d - \bar{B}_d$ mixing	ΔM_d	$(0.5069 \pm 0.0019)\ \text{ps}^{-1}$
	$S_{\psi K_S}$	0.709 ± 0.011
$B_s - \bar{B}_s$ mixing	ΔM_s	$(17.765 \pm 0.004)\ \text{ps}^{-1}$
	$S_{\psi\phi}$	0.051 ± 0.046
$K^0 - \bar{K}^0$ mixing	ΔM_K	$(0.0059 \pm 0.0015)\ \text{ps}^{-1}$
	ε_K	$(2.25 \pm 0.25) \times 10^{-3}$

Parameter space

Free-parameters:

$$\{g_{Z'},\ M_{Z'},\ \epsilon_1,\ \epsilon_2,\ |V_{ub}|,\ |V_{cb}|\} \qquad \begin{cases} 0.01 \le g_{Z'} \le 1 \\ M_{Z'} = 1\,\text{TeV and } 3\,\text{TeV} \end{cases} \qquad \begin{array}{l} |V_{ub}|_{\text{exc}} \le |V_{ub}| \le |V_{ub}|_{\text{inc}} \\ |V_{cb}|_{\text{exc}} \le |V_{cb}| \le |V_{cb}|_{\text{inc}} \end{array}$$

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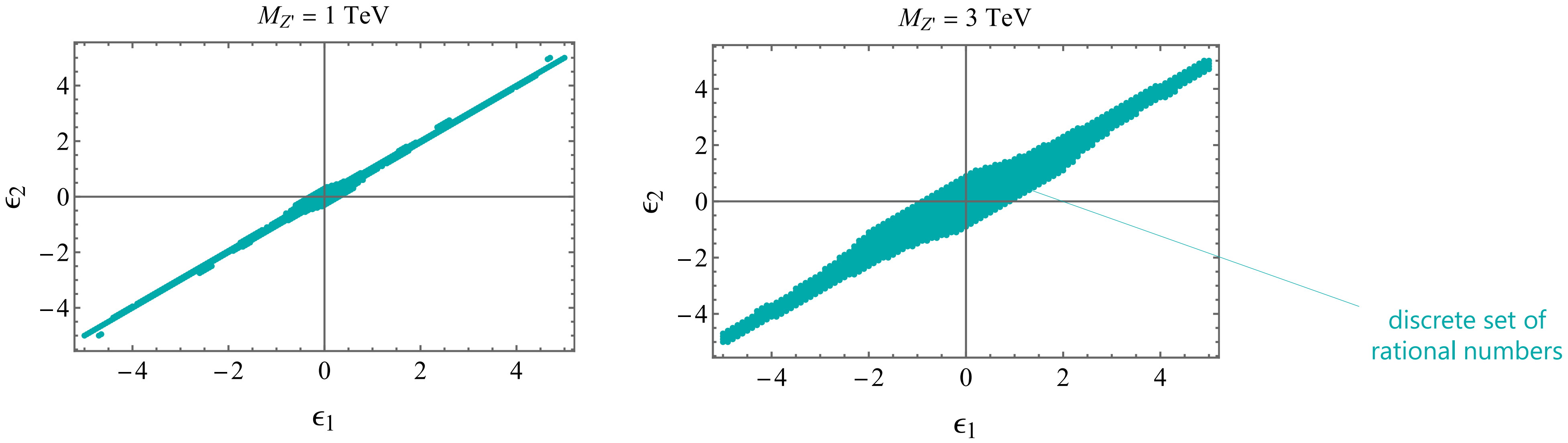


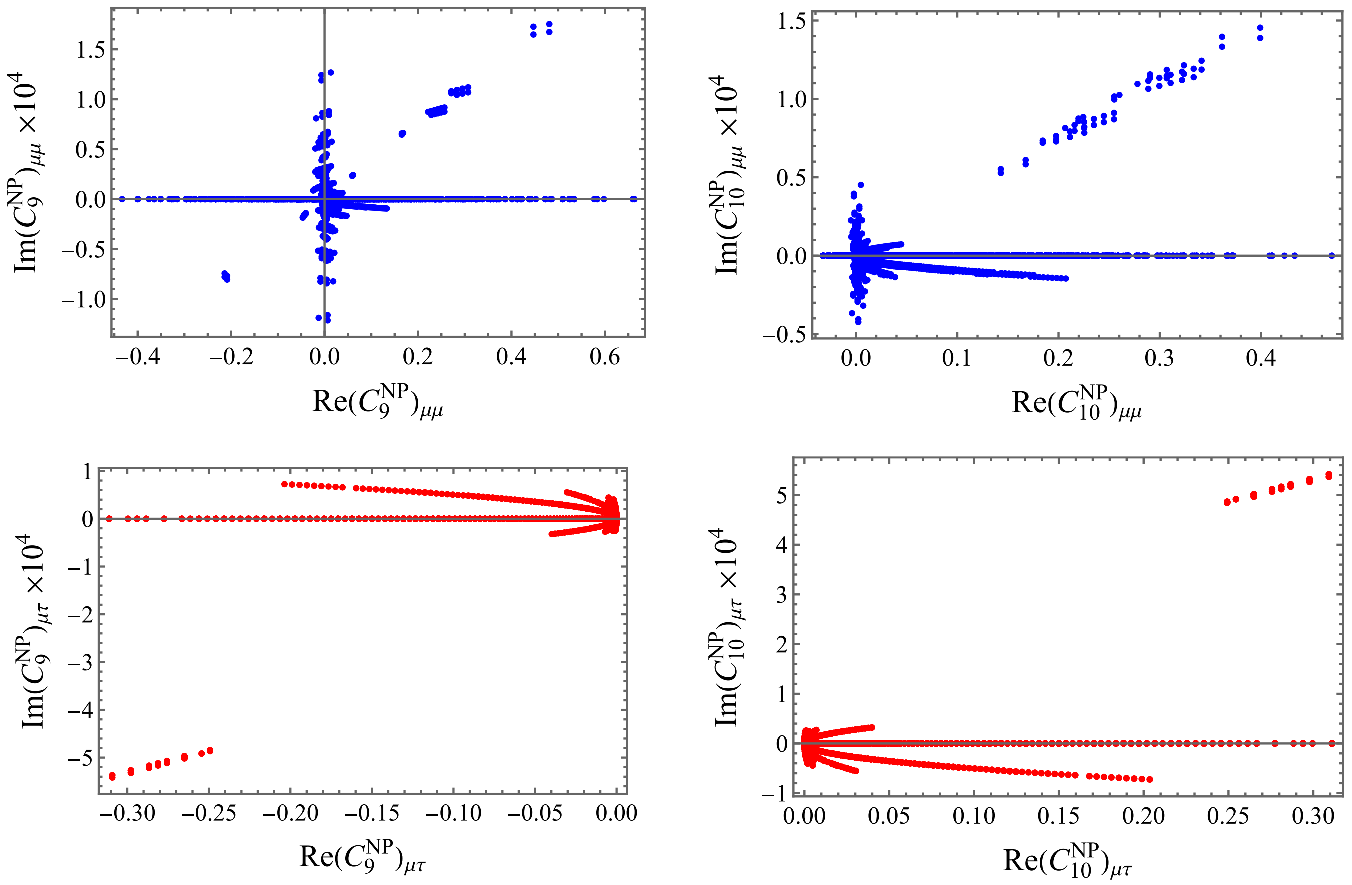
Hereafter only the case $M_{Z'} = 1$ TeV is shown, but similar results hold for $M_{Z'} = 3$ TeV

Wilson coefficients – $\mathrm{Re}(C_{9,10}^{\mathrm{NP}})$ vs $\mathrm{Im}(C_{9,10}^{\mathrm{NP}})$

NP contribution to relevant Wilson coefficients:

LFC ($b \to s\mu\mu$)

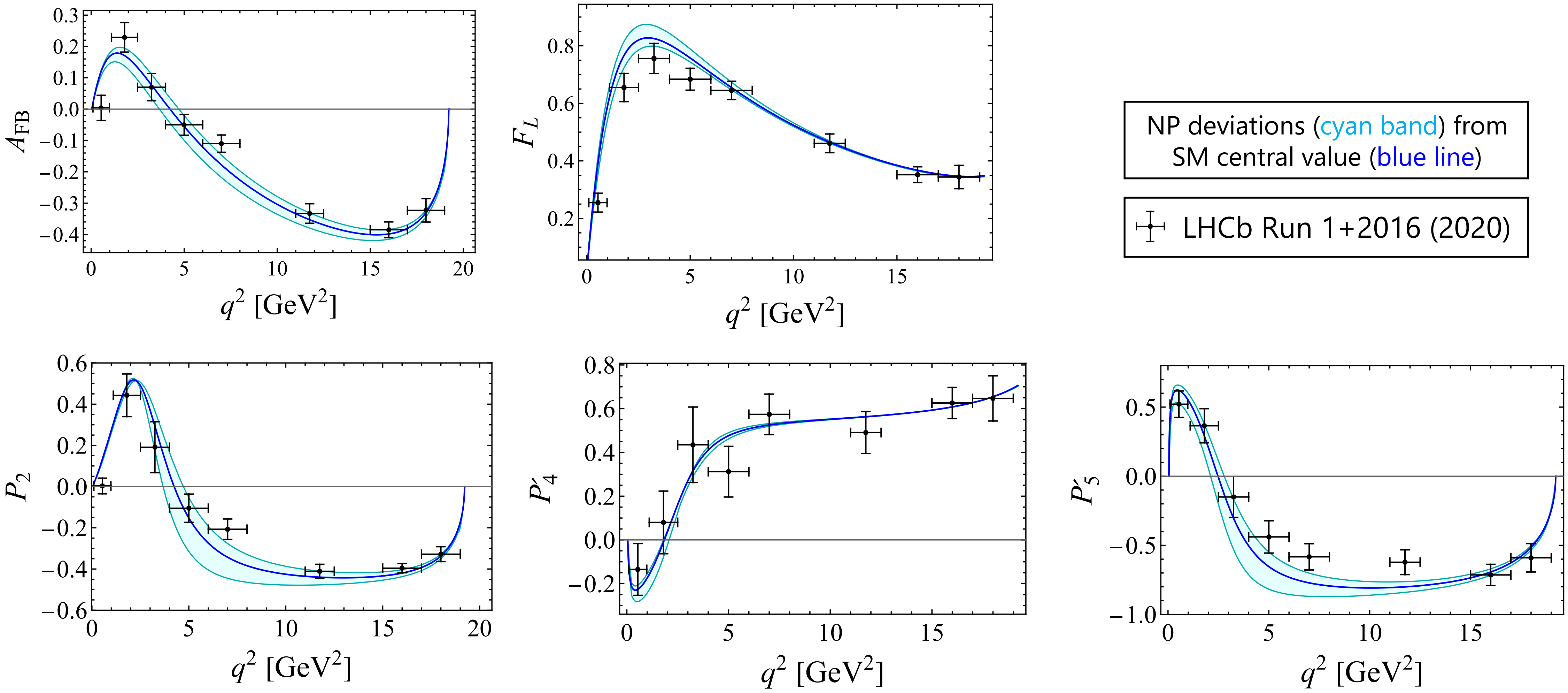
LFV ($b \to s\mu\tau$)



$\mathrm{Re}(C_k^{\mathrm{NP}}) \approx 10\%$ of C_k^{SM} ➜ (small) deviations from SM possible

$\bar{B} \to \bar{K}^* \mu^- \mu^+$ angular analysis

- Starting from angular coefficients several observables can be constructed that are sensitive to NP

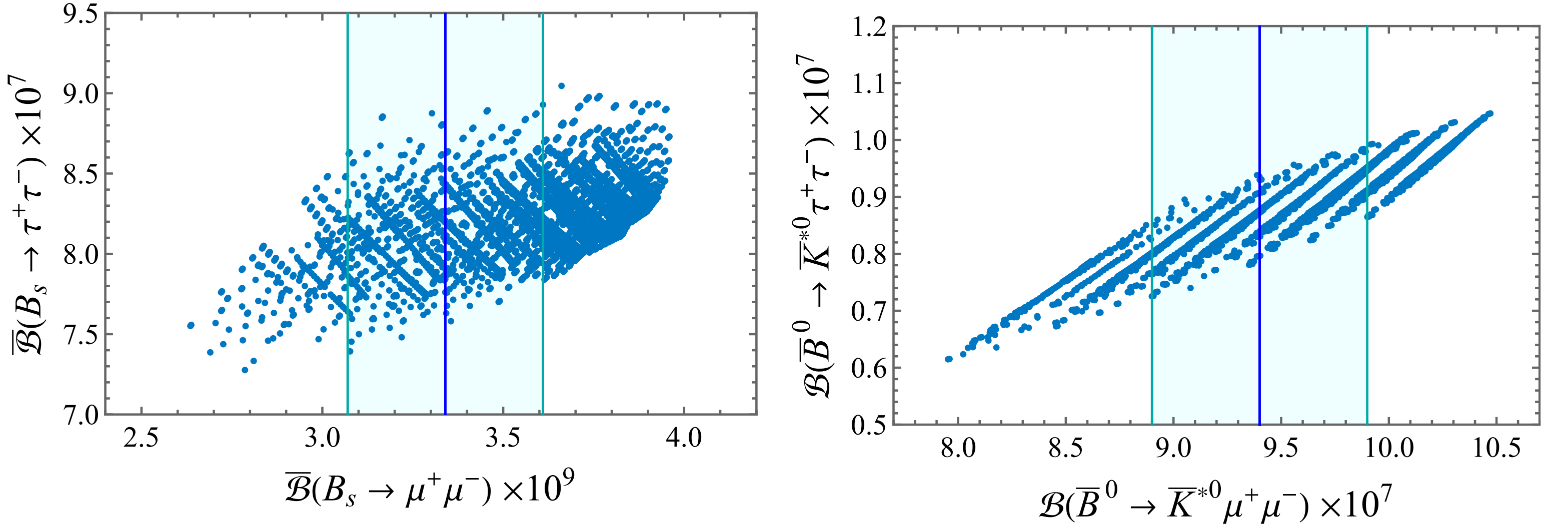


Correlations between LFC & LFV $B_{(s)}$ decay modes

LFC vs LFC

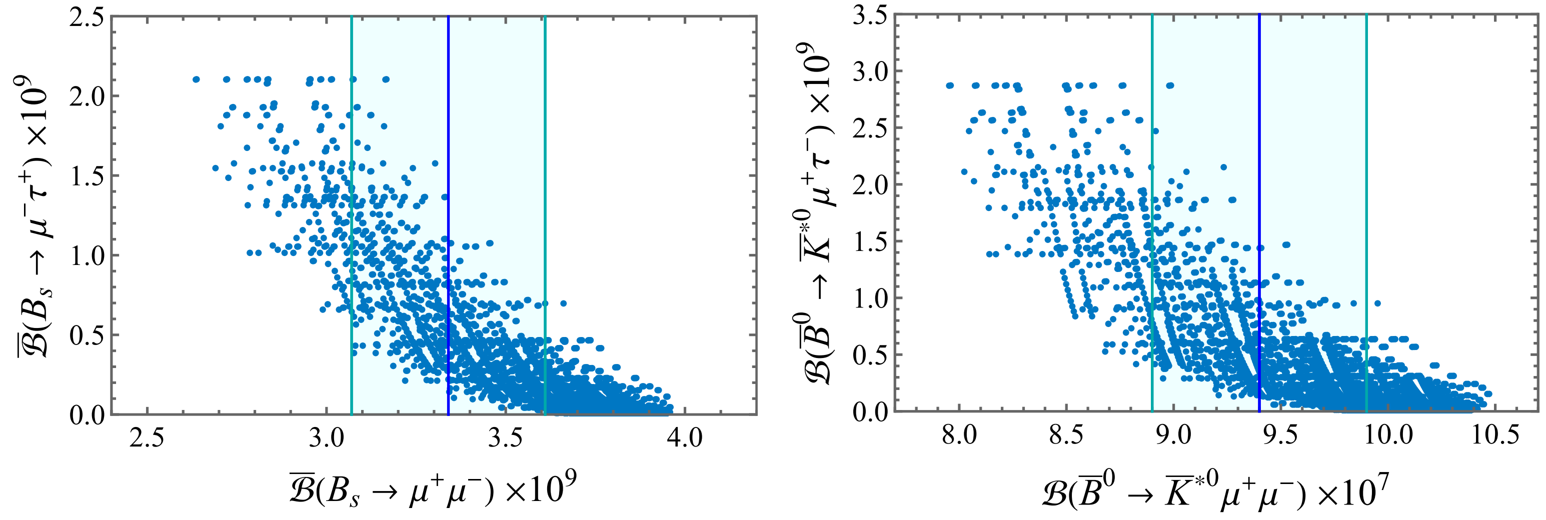
requiring $b \to s\mu\mu$ -induced modes to lie within measured range at 1σ (cyan band):

$$\bar{\mathcal{B}}(B_s \to \tau^+\tau^-) = (8.2 \pm 0.5) \times 10^{-7}$$
$$\mathcal{B}(\bar{B}^0 \to \bar{K}^{*0}\tau^+\tau^-) = (8.6 \pm 1.3) \times 10^{-8}$$



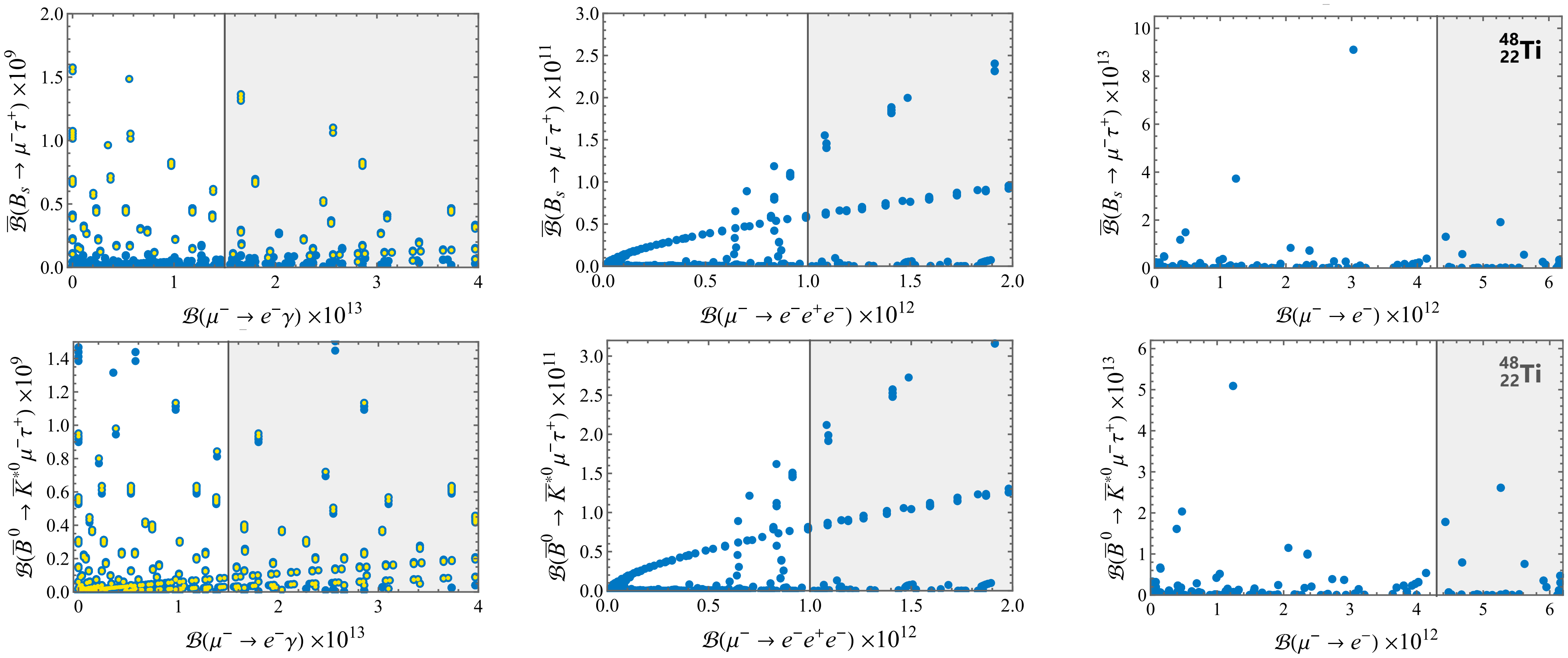
LFC vs LFV

LFV branching ratios bounded by LFC modes to $\mathcal{O}(10^{-9})$



Correlations between LFV $B_{(s)}$ & lepton decay modes

- hadron & lepton decays mutually constrain each other to prevent large deviations from SM
- *$\mu \to e\gamma$, $\mu \to 3e$* and *$\mu \to e$ conversion in nuclei* constrain $B_{(s)}$ branching ratios in hierarchical order



Grey band: excluded region by experimental upper bound *(to be updated by* **MEG II**, **COMET**, **Mu2e**, **Mu3e***)*

Yellow points: extracted requiring the corresponding LFC branching ratio to agree within 1σ with experiment

Summary & future prospects

Summary of results:

Correlations between hadron & lepton decays can be established within ABCD model & used to bound rare & SM-forbidden modes

	LFV mode	upper limit on branching ratio: constrained by $\mu \to e\gamma$	constrained by $\mu \to 3e$	constrained by $\mu \to e$ in ${}^{48}_{22}\mathrm{Ti}$	experiment
$M_{Z'} = 1$ TeV	$B_s \to \mu^- \tau^+$	$(0.00 \div 1.60)\times 10^{-9}$	$(0.00 \div 1.20)\times 10^{-11}$	$(0.00 \div 9.20)\times 10^{-13}$	$< 4.2 \times 10^{-5}$
	$\bar{B} \to \bar{K}^* \mu^- \tau^+$	$(0.00 \div 1.15)\times 10^{-9}$	$(0.00 \div 1.60)\times 10^{-11}$	$(0.00 \div 5.10)\times 10^{-13}$	$< 1.0 \times 10^{-5}$
$M_{Z'} = 3$ TeV	$B_s \to \mu^- \tau^+$	$(0.00 \div 0.14)\times 10^{-9}$	$(0.00 \div 1.10)\times 10^{-11}$	$(0.00 \div 1.05)\times 10^{-13}$	$< 4.2 \times 10^{-5}$
	$\bar{B} \to \bar{K}^* \mu^- \tau^+$	$(0.00 \div 0.20)\times 10^{-9}$	$(0.00 \div 1.53)\times 10^{-11}$	$(0.00 \div 1.42)\times 10^{-13}$	$< 1.0 \times 10^{-5}$

Future prospects in ABCD model:

- Explore new scenarios
- Study other rare & SM-forbidden decays

Thank you

Backup

Angular coefficients

Angular coefficients are expressed in terms of transversity amplitudes[2] $(A_{\perp,\parallel,0,t})_{L/R}$:

- $$I_1^s = \frac{\lambda_\ell + 2[q^4 - (m_i^2 - m_j^2)^2]}{4q^2}\Big[|A_{\perp L}|^2 + |A_{\parallel L}|^2 + (L \to R)\Big] + \frac{4m_i m_j}{q^2}\,\mathrm{Re}\{A_{\perp L}A_{\perp R}^* + A_{\parallel L}A_{\parallel R}^*\}$$
- $$I_1^c = \frac{q^4 - (m_i^2 - m_j^2)^2}{q^4}\Big[|A_{0L}|^2 + |A_{0R}|^2\Big] + \frac{8m_i m_j}{q^2}\,\mathrm{Re}\{A_{0L}A_{0R}^* - A_{tL}A_{tR}^*\} - 2\,\frac{(m_i^2 - m_j^2)^2 - q^2(m_i^2 + m_j^2)}{q^4}\Big[|A_{tL}|^2 + |A_{tR}|^2\Big]$$
- $$I_2^s = \frac{\lambda_\ell}{4q^4}\Big[|A_{\perp L}|^2 + |A_{\parallel L}|^2 + (L \to R)\Big]$$
- $$I_2^c = -\frac{\lambda_\ell}{q^4}\Big[|A_{0L}|^2 + (L \to R)\Big]$$
- $$I_3 = \frac{\lambda_\ell}{2q^4}\Big[|A_{\perp L}|^2 - |A_{\parallel L}|^2 + (L \to R)\Big]$$
- $$I_4 = \frac{\lambda_\ell}{\sqrt{2}q^4}\Big[\mathrm{Re}\{A_{0L}A_{\parallel L}^*\} + (L \to R)\Big]$$
- $$I_5 = \frac{\sqrt{2}\lambda_\ell^{1/2}}{q^2}\Big[\mathrm{Re}\{A_{0L}A_{\parallel L}^* - (L \to R)\} - \frac{m_i^2 - m_j^2}{q^2}\mathrm{Re}\{A_{tL}A_{\parallel L}^* + (L \to R)\}\Big]$$
- $$I_6^s = \frac{2\lambda_\ell^{1/2}}{q^2}\Big[\mathrm{Re}\{A_{\parallel L}A_{\perp L}^*\} - (L \to R)\}\Big]$$

[2] Altmannshofer, Ball, Bharucha, Buras, Straub, Wick [JHEP 01(2009)019]

Transversity amplitudes

- $A_{\perp L,R} = \mathcal{N}\sqrt{2}\lambda_H^{1/2}\left[2(m_b+m_s)C_7\frac{T_1(q^2)}{q^2} + (C_9^+ \mp C_{10}^+)\frac{V(q^2)}{M_B+M_{K^*}}\right]$
- $A_{\parallel L,R} = -\mathcal{N}\sqrt{2}(M_B^2 - M_{K^*}^2)\left[2(m_b-m_s)C_7\frac{T_2(q^2)}{q^2} + (C_9^- \mp C_{10}^-)\frac{A_1(q^2)}{M_B-M_{K^*}}\right]$
- $$A_{0L,R} = \frac{-\mathcal{N}}{2M_{K^*}\sqrt{q^2}}\left\{2(m_b-m_s)C_7\left[(M_B^2+3M_{K^*}^2-q^2)T_2(q^2) - \frac{\lambda_H T_3(q^2)}{M_B^2-M_{K^*}^2}\right]\right.$$ $$\left. + (C_9^- \mp C_{10}^-)\left[(M_B^2-M_{K^*}^2-q^2)(M_B+M_{K^*})A_1(q^2) - \frac{\lambda_H A_2(q^2)}{M_B+M_{K^*}}\right]\right\}$$
- $A_{tL,R} = \frac{-\mathcal{N}}{\sqrt{q^2}}\lambda_H^{1/2}(C_9^- \mp C_{10}^-)A_0(q^2)$

where: $\mathcal{N} = V_{tb}^* V_{ts}\left[\frac{G_F^2\alpha^2}{3\times 2^{10}\pi^5 M_B^3}\lambda_H^{1/2}\lambda_\ell^{1/2}\right]^{1/2}$

with:
$$\lambda_H = \lambda(q^2, M_B^2, M_{K^*}^2)$$
$$\lambda_\ell = \lambda(q^2, m_i^2, m_j^2)$$
(Källén function)

and: $C_k^\pm = C_k \pm C_k'$

$A_{0,1,2}(q^2)$, $V(q^2)$, $T_{1,2,3}(q^2)$ are *$B \to K^*$ form factors*

$B \to K^*$ hadronic matrix elements

$B \to K^*$ hadronic matrix elements (standard parametrization):

- $\langle K^*(p',\epsilon)|\bar{\mathbf{s}}\gamma_\mu(1-\gamma_5)\mathbf{b}|B(p)\rangle = \epsilon_{\mu\nu\alpha\beta}\epsilon^{*\nu}p^\alpha p'^\beta \frac{2V(q^2)}{M_B+M_{K^*}} - i\left\{\epsilon^*_\mu(M_B+M_{K^*})A_1(q^2) - (p+p')_\mu(\epsilon^*\cdot p)\frac{A_2(q^2)}{M_B+M_{K^*}} - q_\mu(\epsilon^*\cdot q)\frac{2M_{K^*}}{q^2}\left[A_3(q^2)-A_0(q^2)\right]\right\}$
- $\langle K^*(p',\epsilon)|\bar{\mathbf{s}}\sigma_{\mu\nu}q^\nu(1+\gamma_5)\mathbf{b}|B(p)\rangle = i\epsilon_{\mu\nu\alpha\beta}\epsilon^{*\nu}p^\alpha p'^\beta T_1(q^2) + i\left[\epsilon^*_\mu(M_B-M_{K^*})A_1(q^2) - (p+p')_\mu(\epsilon^*\cdot q)\right]T_2(q^2) + (\epsilon^*\cdot q)\left[q_\mu - \frac{q^2}{M_B^2-M_{K^*}^2}(p+p')_\mu\right]T_3(q^2)$

*K** polarization 4-vector

$A_{0,1,2,3}(q^2)$, $V(q^2)$, $T_{1,2,3}(q^2)$ are *form factors* (FF). They are not all independent, since:

- $A_3(q^2) = \frac{M_B+M_{K^*}}{2M_K^*}A_1(q^2) - \frac{M_B-M_{K^*}}{2M_K^*}A_2(q^2)$
- $A_3(0) = A_0(0)$
- $T_1(0) = T_2(0)$

$B \to K^*$ form factors (BSZ)

Independent form factors $F = V, A_0, A_1, A_{12}, T_1, T_2, T_{23}$ parametrized according to **BSZ**[3]:

$$F(q^2) = \frac{1}{1 - q^2/M_{B^*}^2} \sum_{k=0}^{2} \alpha_k(F)\left[z(q^2) - z(0)\right]^k$$

mass of lightest resonance

$$z(t) = \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}}$$

with: $\begin{cases} t_\pm = (M_B \pm M_{K^*})^2 \\ t_0 = t_+(1 - \sqrt{1 - t_-/t_+}) \end{cases}$

F	M_{B^*} (GeV)	$\alpha_0(F)$	$\alpha_1(F)$	$\alpha_2(F)$
V	5.415	0.38 ± 0.03	-1.17 ± 0.26	2.42 ± 1.53
A_0	5.366	0.37 ± 0.03	-1.37 ± 0.26	0.13 ± 1.63
A_1	5.829	0.30 ± 0.03	0.39 ± 0.19	1.19 ± 1.03
A_{12}	5.829	0.27 ± 0.02	0.53 ± 0.13	0.48 ± 0.66
T_1	5.415	0.31 ± 0.03	-1.01 ± 0.19	1.53 ± 1.64
T_2	5.829	0.31 ± 0.03	0.50 ± 0.17	1.61 ± 0.80
T_{23}	5.829	0.67 ± 0.06	1.32 ± 0.22	3.82 ± 2.20

Finally A_2 & T_3 are obtained from:

$$A_{12} = \frac{(M_B + M_{K^*})^2(M_B^2 - M_{K^*}^2 - q^2)A_1(q^2) - \lambda(q^2, M_B^2, M_{K^*}^2)A_2(q^2)}{16 M_B M_{K^*}(M_B + M_{K^*})}$$

$$T_{23} = \frac{(M_B - M_{K^*})^2(M_B^2 + 3M_{K^*}^2 - q^2)T_2(q^2) - \lambda(q^2, M_B^2, M_{K^*}^2)T_3(q^2)}{8 M_B M_{K^*}(M_B - M_{K^*})}$$

[3] Bharucha, Straub, Zwicky [JHEP 08(2016)098]