

The $X(3872)$ puzzle: insights from its radiative decay

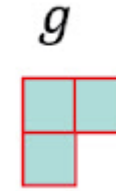
Davide Germani

Exotic Hadrons

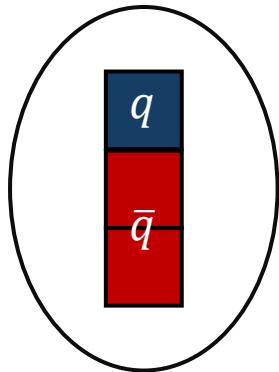
Exotic hadrons are subatomic particles composed of quarks and gluons, but which consist of more than three valence quarks or have an explicit valence gluon content.

R. Jaffe, Phys. Rev. D 15, 267 (1977) DOI: 10.1103/PhysRevD.15.267; H. Fritzsch and M. Gell-Mann, Proceedings of the XVI International Conference on High Energy Physics, Chicago-Batavia (1972).

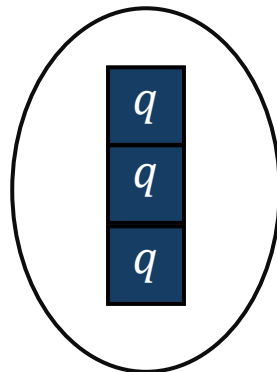
- **Pieces:** quarks, anti-quarks and gluons



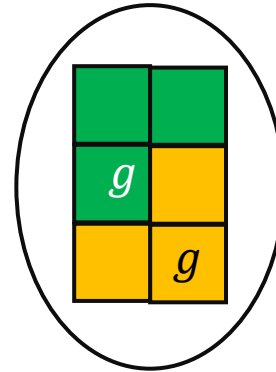
- **$SU(3)_c$ rule:** every combination is allowed as long as it respects confinement i.e. boxes forms columns of three elements



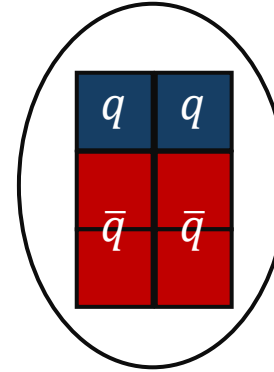
Meson



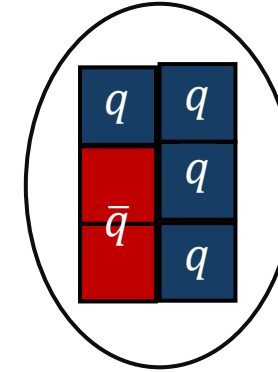
Baryon



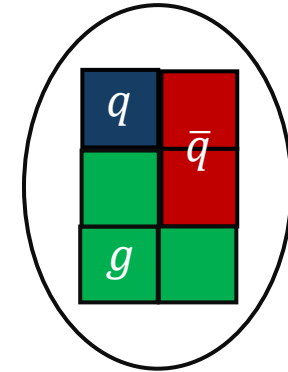
Glueball



Tetraquark



Pentaquark



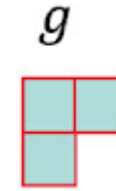
Hybrid

Exotic Hadrons

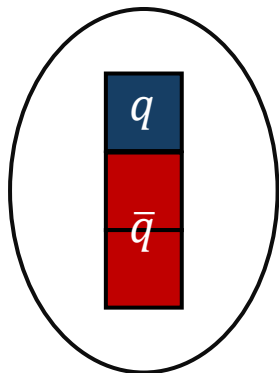
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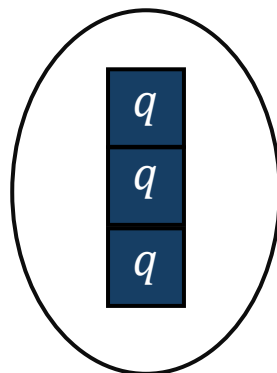
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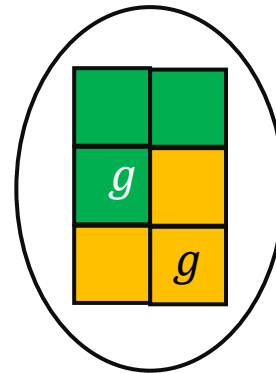
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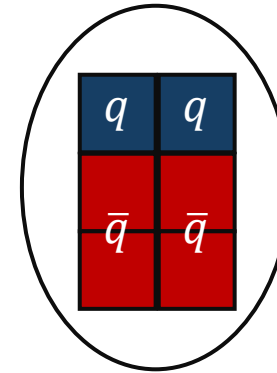
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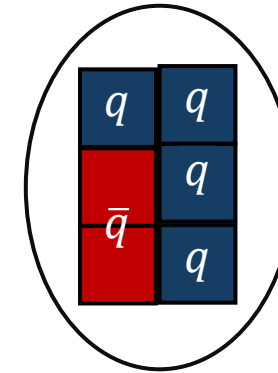
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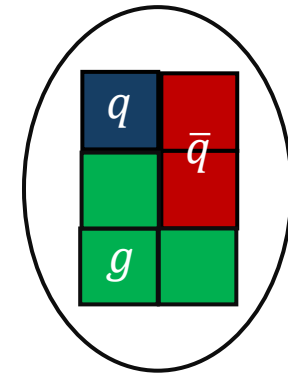
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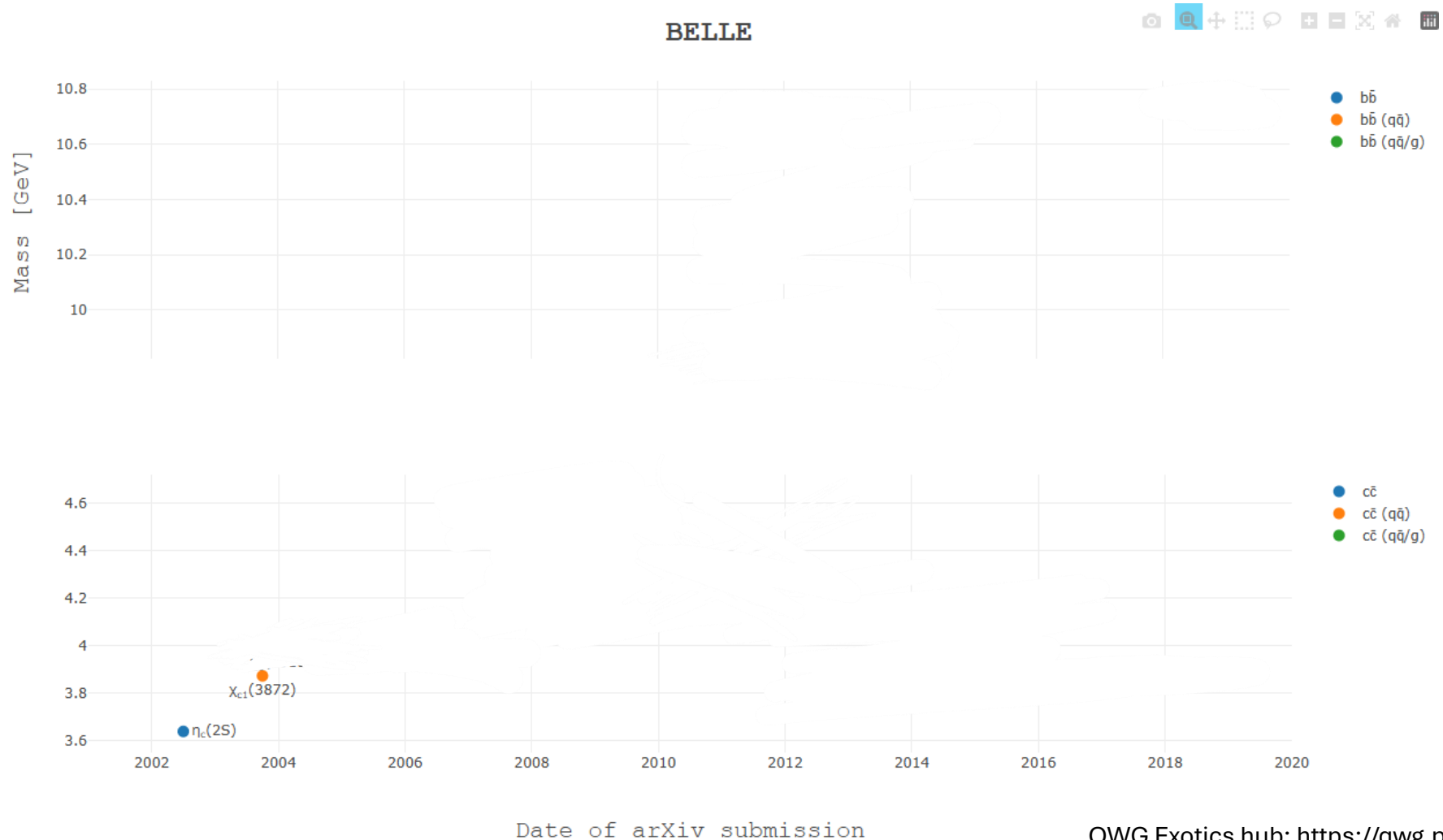


Hybrid

EXOTIC HADRONS

Exotic Hadrons

Although the existence of exotic states had already been theorized in the 1970s, the first exotic particle was discovered in 2003: the ***X(3872)***.



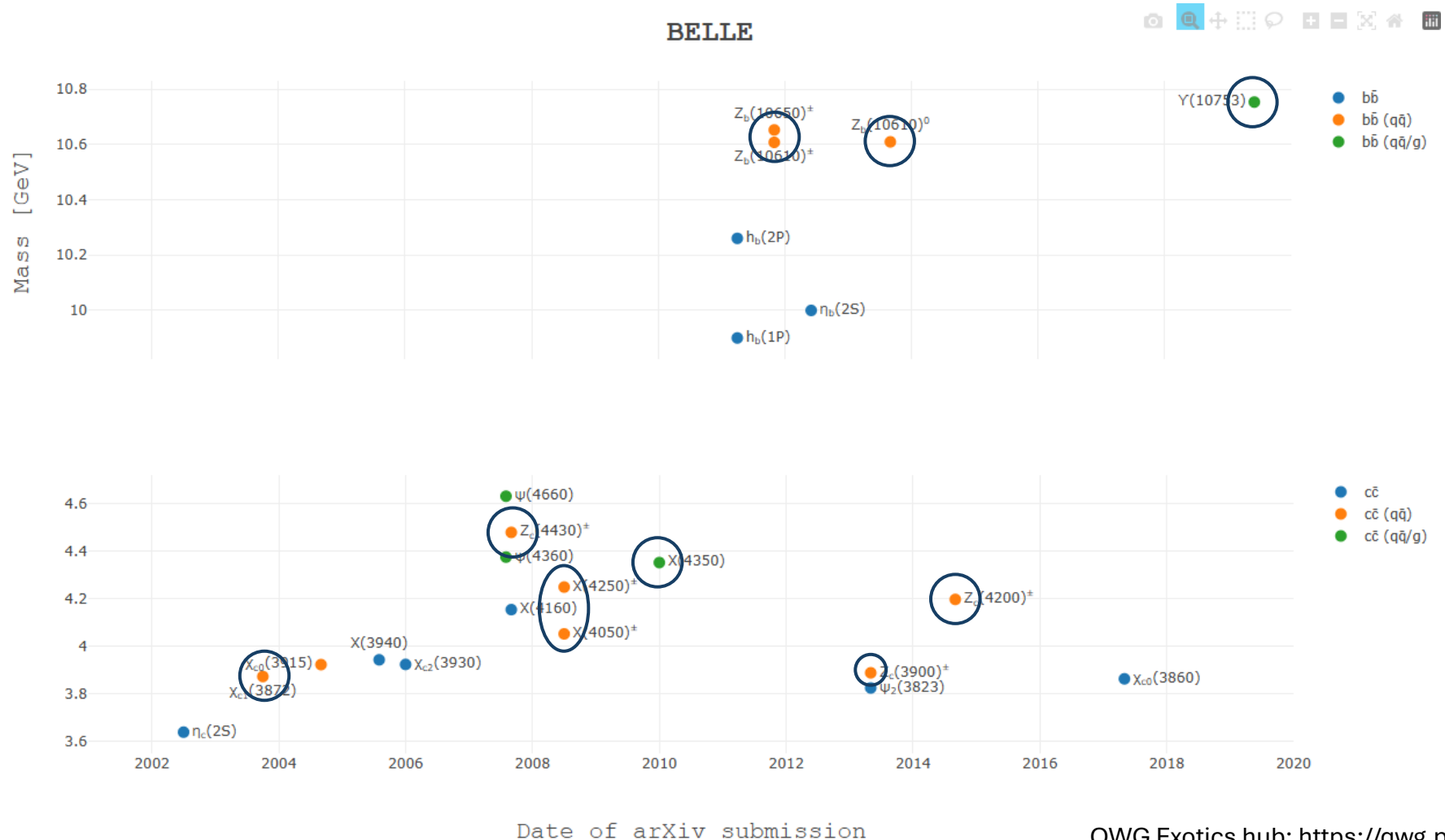
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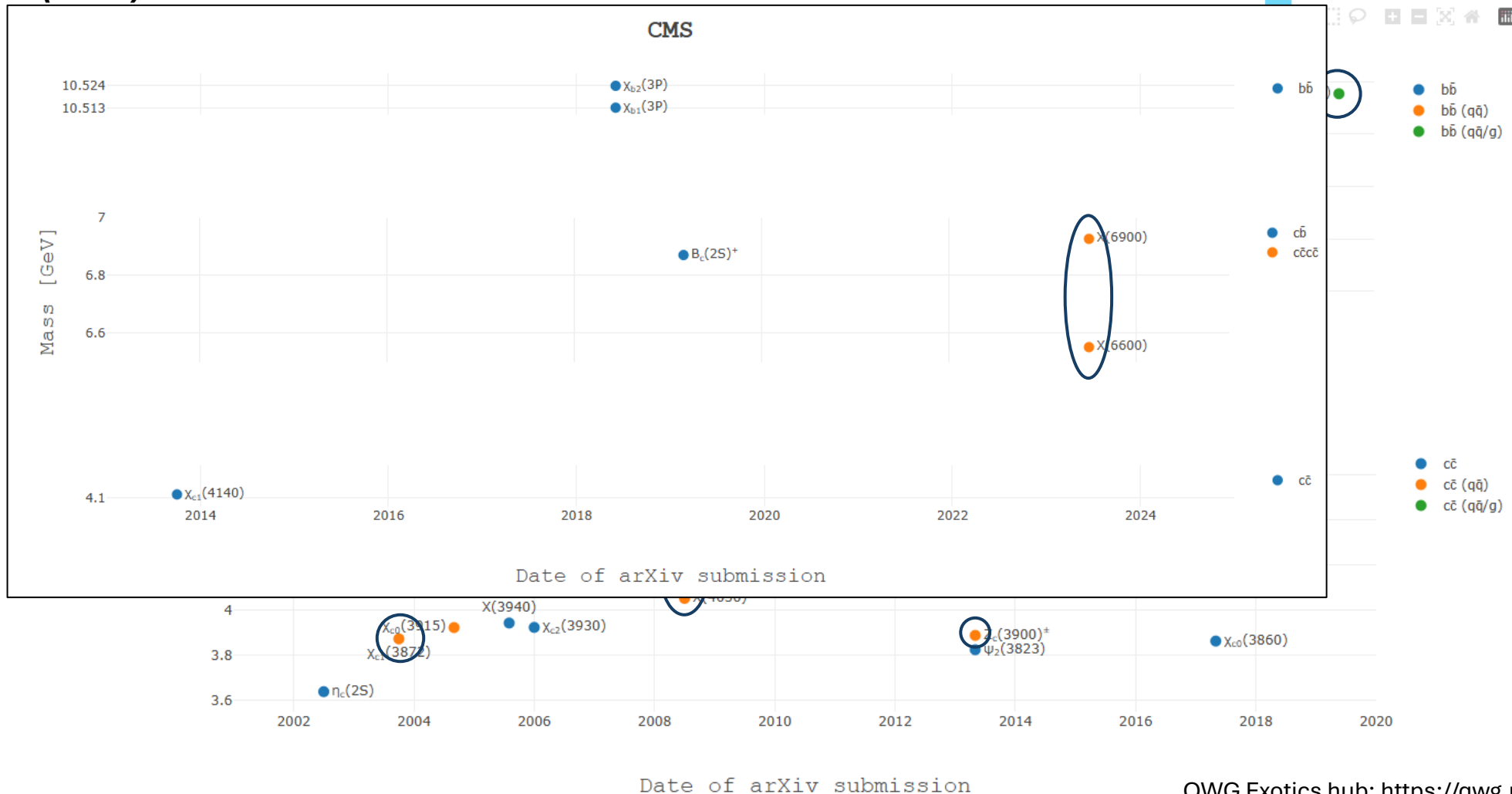
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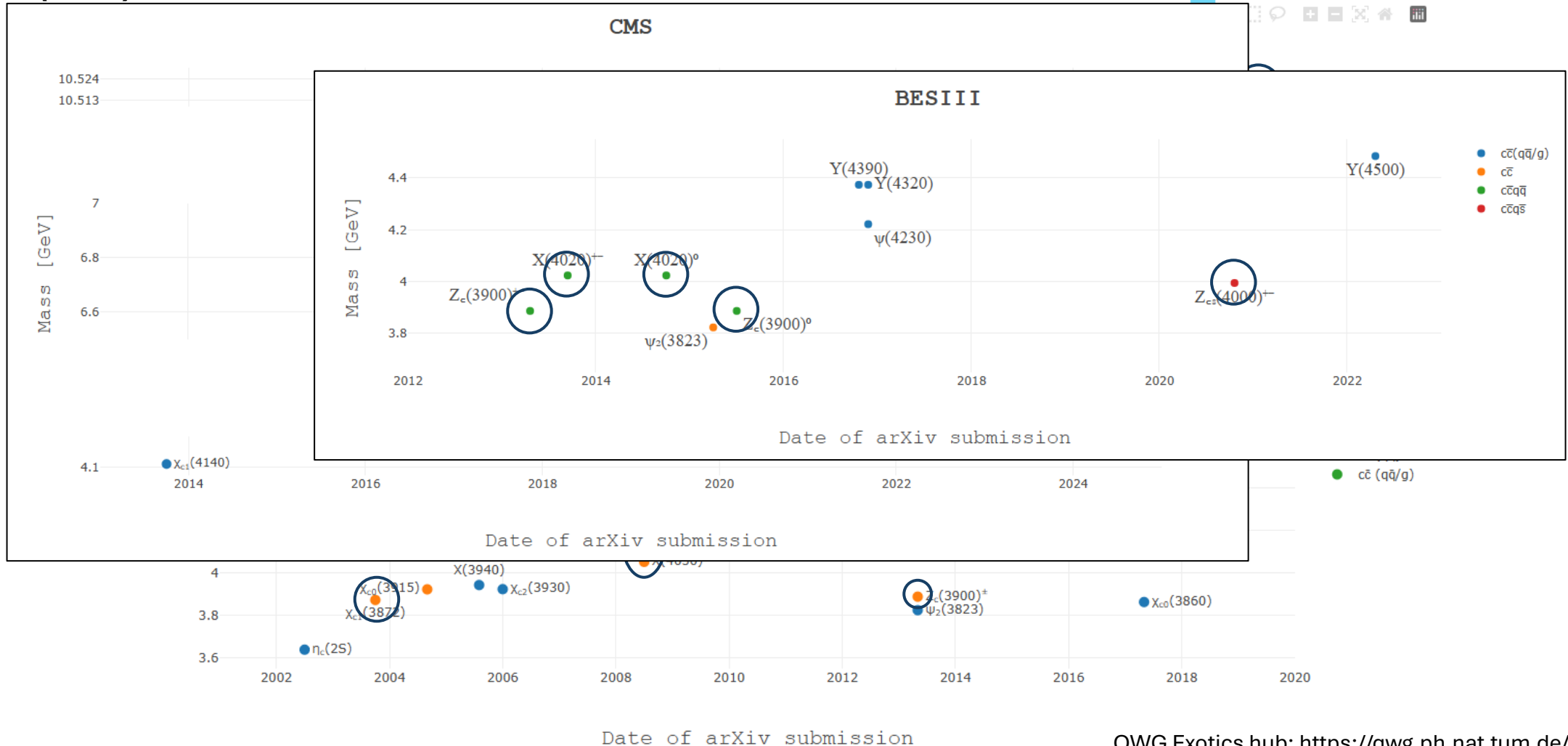
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QWG Exotics hub: <https://qwg.ph.nat.tum.de/exoticshub/>

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Exotic spectroscopy

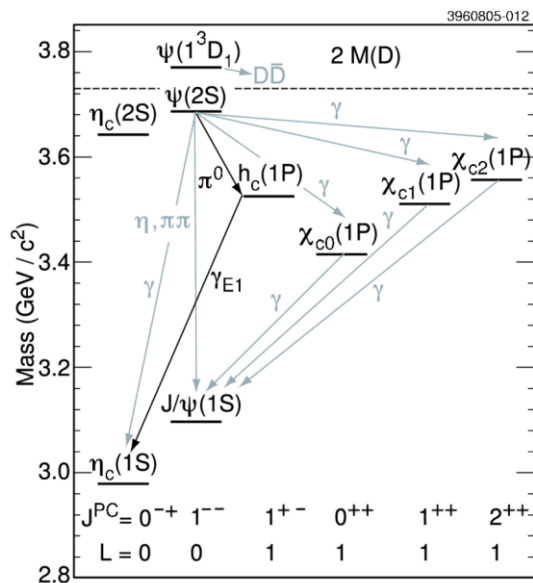
Around **50 exotic hadrons** with at least one heavy (c or b) quark are reported.

N. Hüsken, E. Spadaro Norella, I. Polyakov, Mod.Phys.Lett.A 40 (2025) 17n18, 2530002, arXiv:2410.06923 [hep-ph]

Exotic spectroscopy is a branch of particle physics that focuses on studying the properties of exotic hadrons.

- Provide a theoretical explanation of the experimentally observed properties, grounded as much as possible in first principles (QCD).
- Make predictions about the masses and quantum numbers of exotic hadrons to test hypotheses on the non-confining dynamics of QCD.

Idea



[CLEO Collaboration], **Phys. Rev. D** 72, 092004 (2005)

Reality

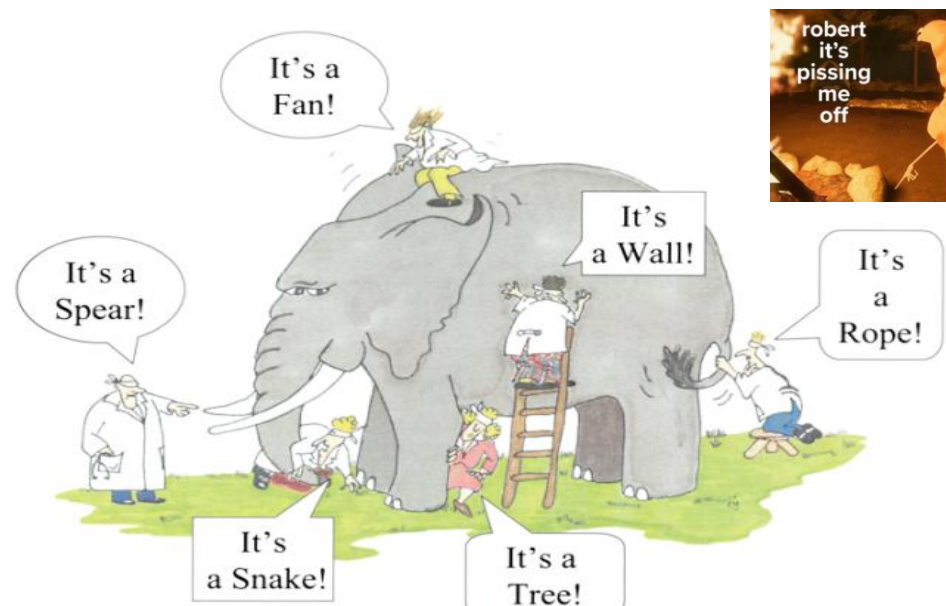


Figure from E. Braaten talk: **Charm 2020 conference**

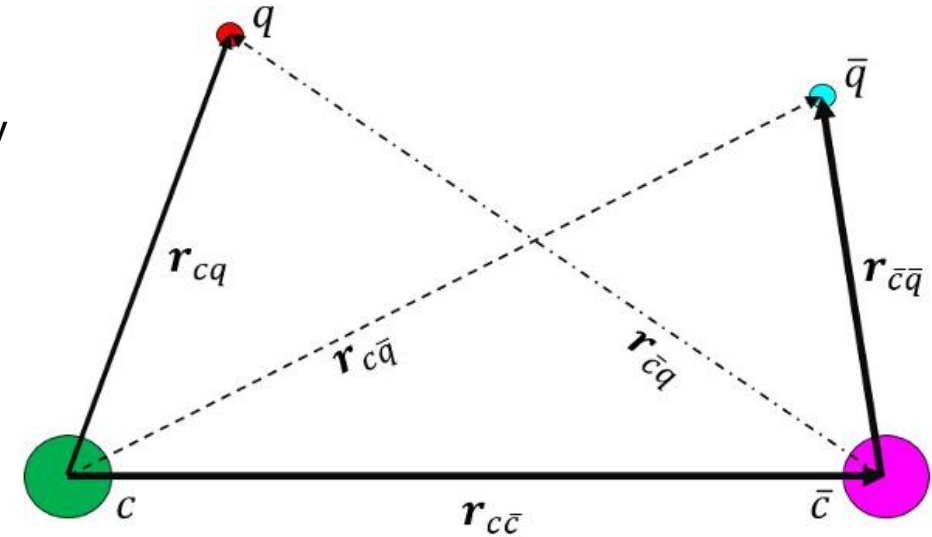
$Q\bar{Q}q\bar{q}$ tetraquarks: Born-Oppenheimer approximation

D. G., B. Grinstein and A. D. Polosa, JHEP 04 (2025) 004, arXiv:2501.13249 [hep-ph].

We focus on tetraquarks made of two heavy quarks (in particular $c\bar{c}$) and two light quarks:

- This is close to the H_2 molecule, where the nuclei correspond to the heavy quarks and the electrons to the light ones

$$\Psi_T(\mathbf{r}_{cq}, \mathbf{r}_{\bar{c}\bar{q}}, \mathbf{r}_{c\bar{c}}) = \Psi_{c\bar{c}}(\mathbf{r}_{c\bar{c}}) \Psi_{q\bar{q}}(\mathbf{r}_{cq}, \mathbf{r}_{\bar{c}\bar{q}}, \mathbf{r}_{c\bar{c}})$$



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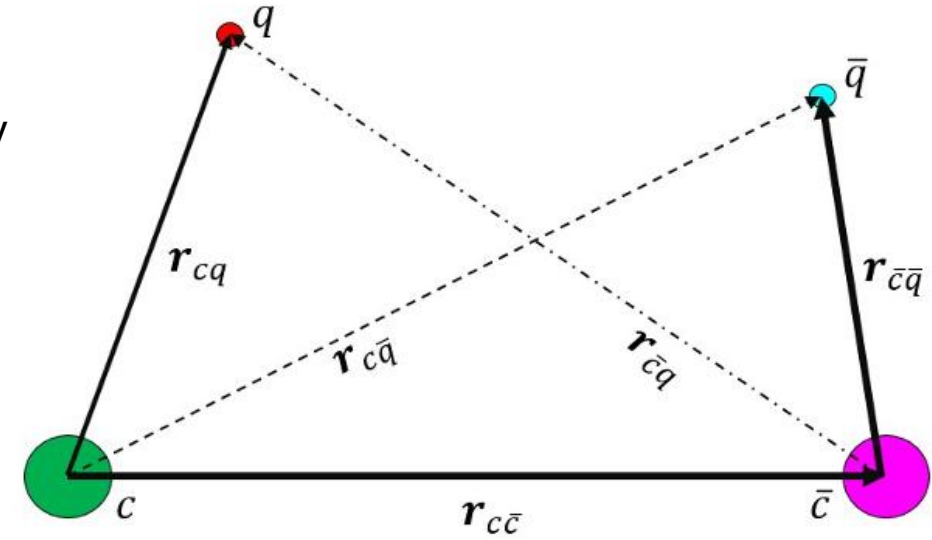
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- The wave function $\Psi_{q\bar{q}}$ is a solution of the Schrödinger equation

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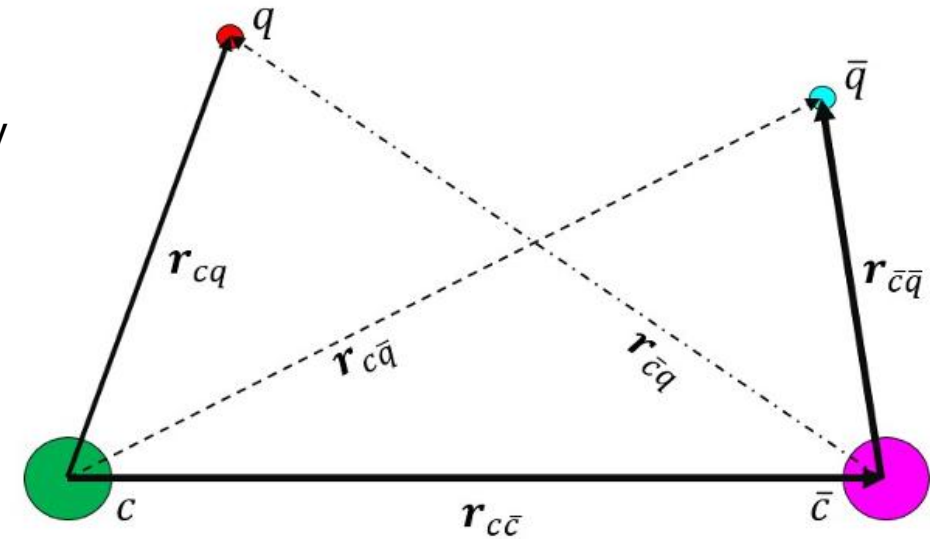
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$$(H_{c\bar{c}} + \underbrace{\Delta E(r_{c\bar{c}})}_{\text{BO potential}}) \Psi_{c\bar{c}}(\mathbf{r}_{c\bar{c}}) = E \Psi_{c\bar{c}}(\mathbf{r}_{c\bar{c}})$$



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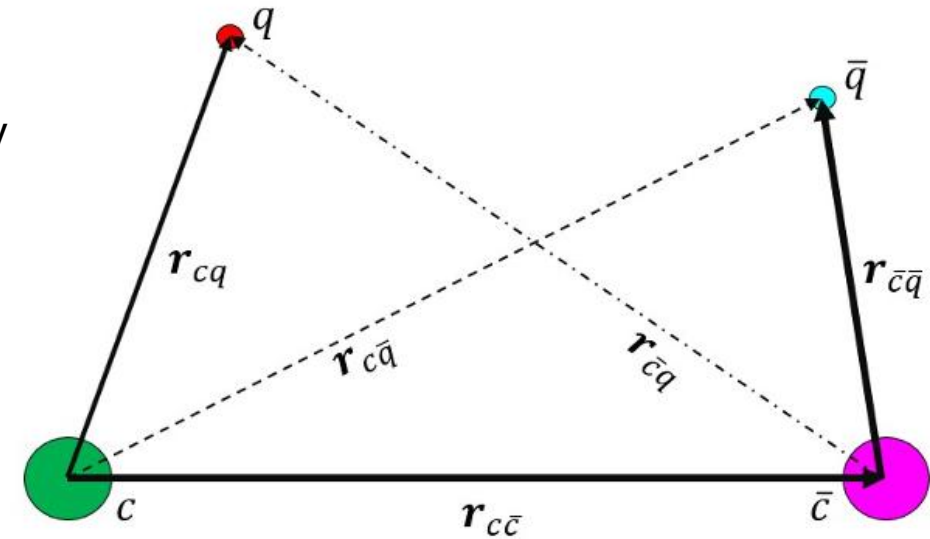
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DONE

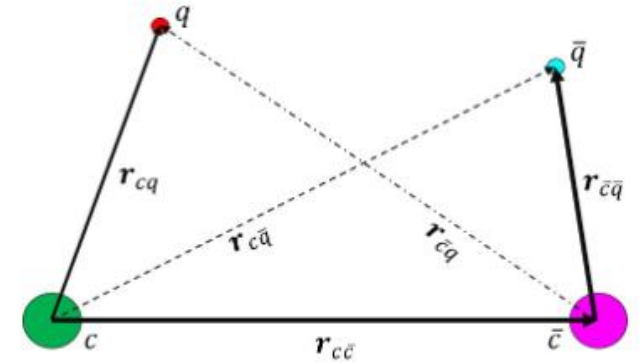


Born-Oppenheimer approximation: first step

- **Light dof: BO potential**

$$H_{q\bar{q}} \Psi_{q\bar{q}}(\mathbf{r}_{cq}, \mathbf{r}_{\bar{c}\bar{q}}, \mathbf{r}_{c\bar{c}}) = \Delta E(r_{c\bar{c}}) \Psi_{q\bar{q}}(\mathbf{r}_{cq}, \mathbf{r}_{\bar{c}\bar{q}}, \mathbf{r}_{c\bar{c}})$$

In $H_{q\bar{q}}$ we have a **Coulomb-like** term, a **confinement term**, and a **spin-dependent** contribution



This equation cannot be solved analytically. We employ a sort of ‘Linear combination of atomic orbitals’ (LCAO) method:

- **Ansatz**

$$\Psi_{q\bar{q}}(\mathbf{r}_{cq}, \mathbf{r}_{\bar{c}\bar{q}}, \mathbf{r}_{c\bar{c}}) = a_1(r_{c\bar{c}}) \psi_D(\mathbf{r}_{cq}) \psi_D(\mathbf{r}_{\bar{c}\bar{q}}) + a_2(r_{c\bar{c}}) \psi_M(\mathbf{r}_{c\bar{q}}) \psi_M(\mathbf{r}_{\bar{c}q})$$

$$\psi_c(\zeta) = \sqrt{\frac{c^3}{\pi}} e^{-c\zeta}$$

- D: the q ($\bar{q}$) quark orbits around the c ($\bar{c}$) quark
- M: the q ($\bar{q}$) quark orbits around the $\bar{c}$ (c) quark

D. G., B. Grinstein and A. D. Polosa, JHEP 04 (2025) 004, arXiv:2501.13249 [hep-ph].

Goal:

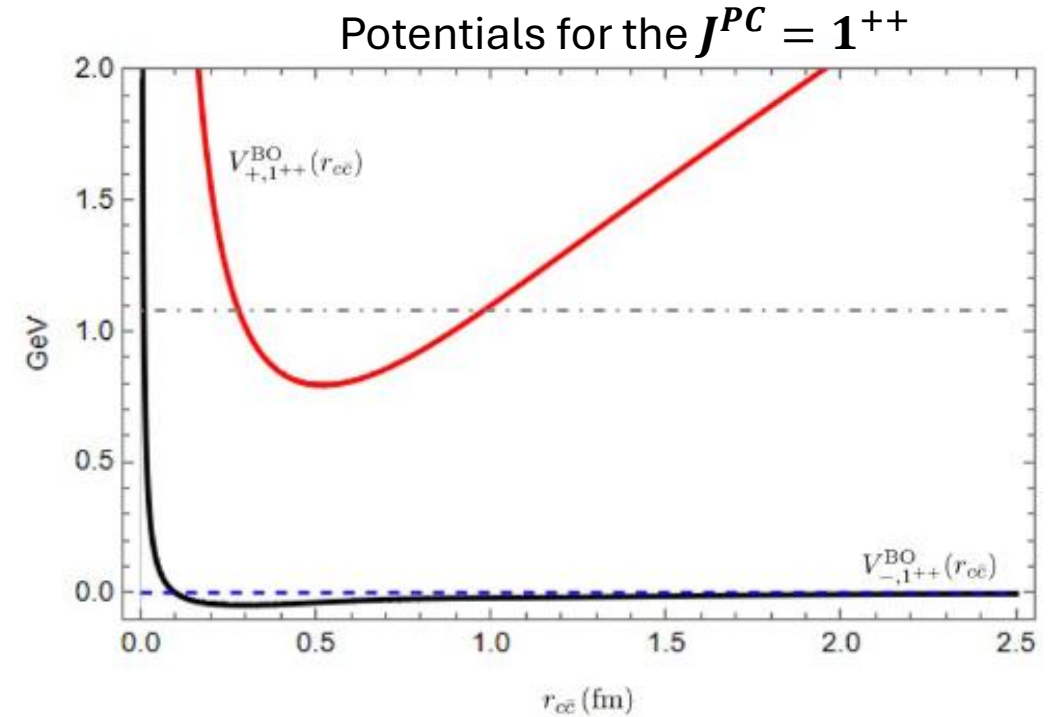
Find a_i by minimizing the expectation value of the hamiltonian $H_{q\bar{q}}$

Born-Oppenheimer approximation: second step

- **Heavy dof**

$$(H_{c\bar{c}} + \Delta E(r_{c\bar{c}})) \Psi_{c\bar{c}}(\mathbf{r}_{c\bar{c}}) = E \Psi_{c\bar{c}}(\mathbf{r}_{c\bar{c}})$$

This equation can be solved numerically to obtain the **wavefunction** of the ground state and the **mass spectrum**



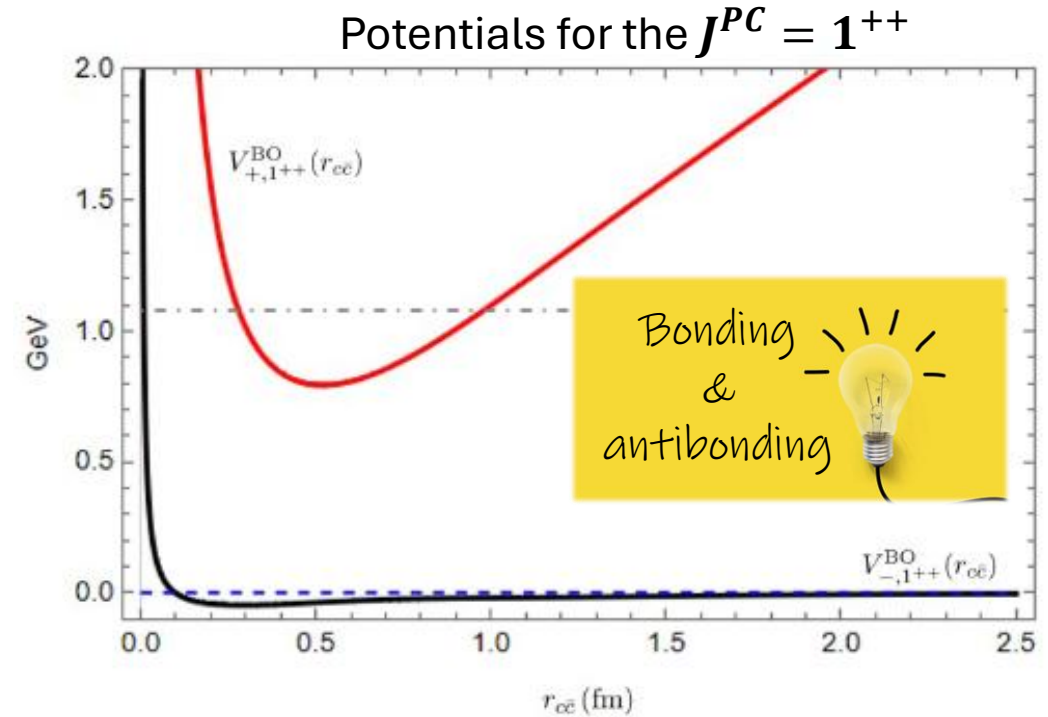
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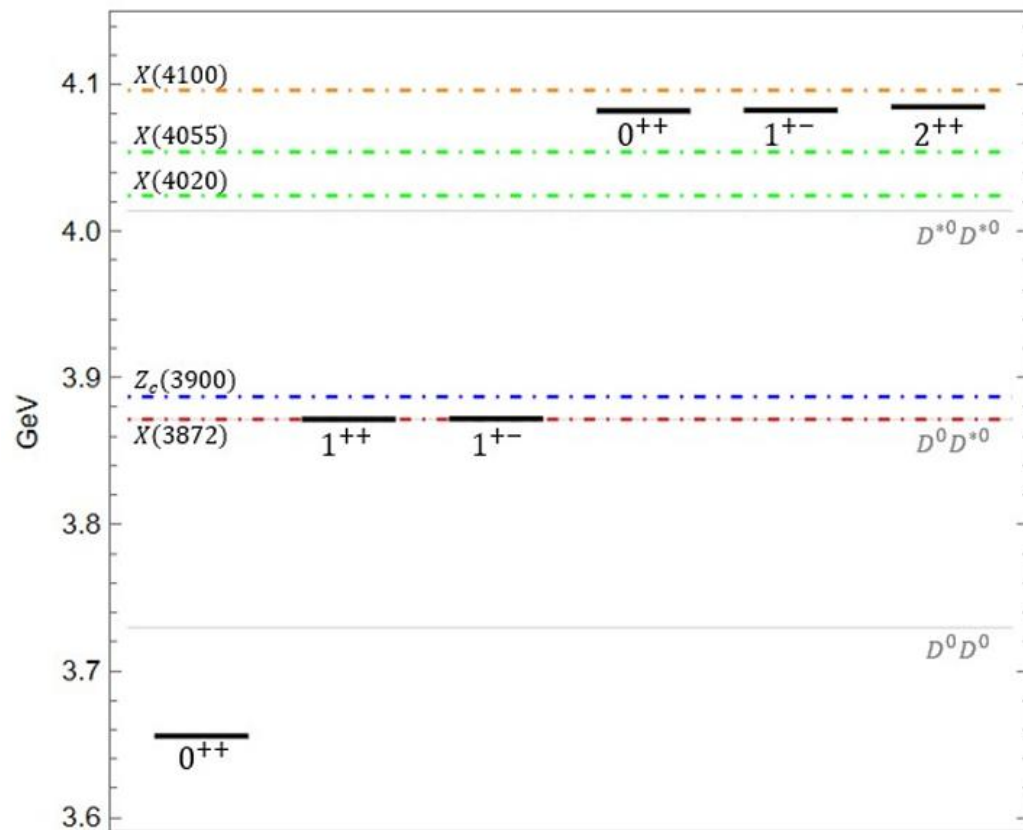
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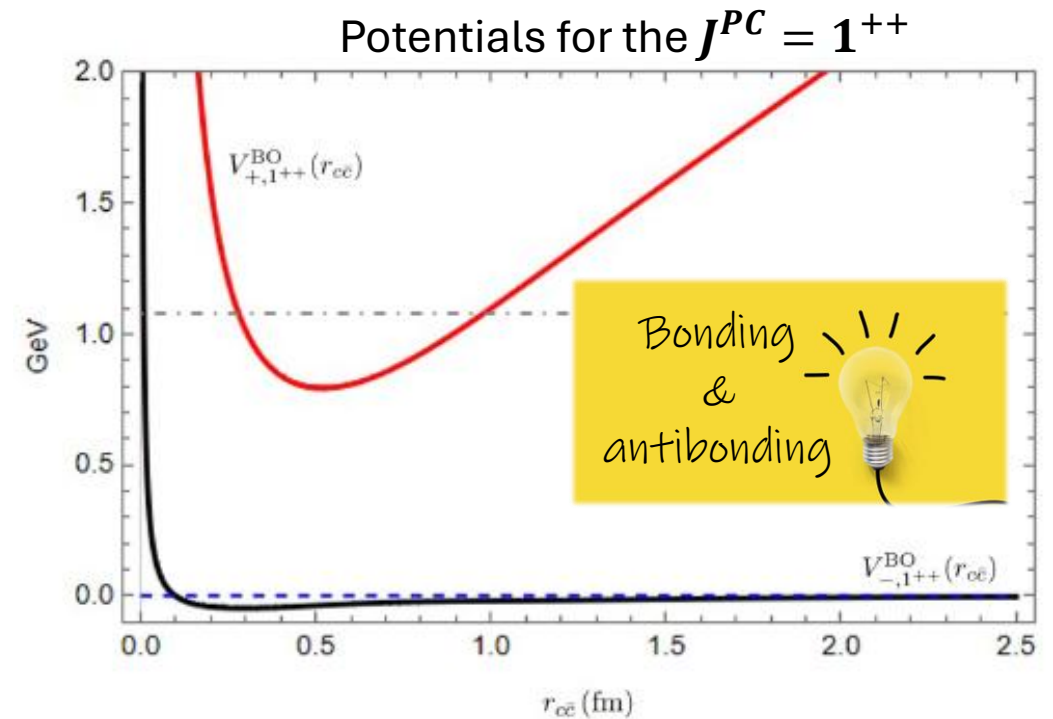
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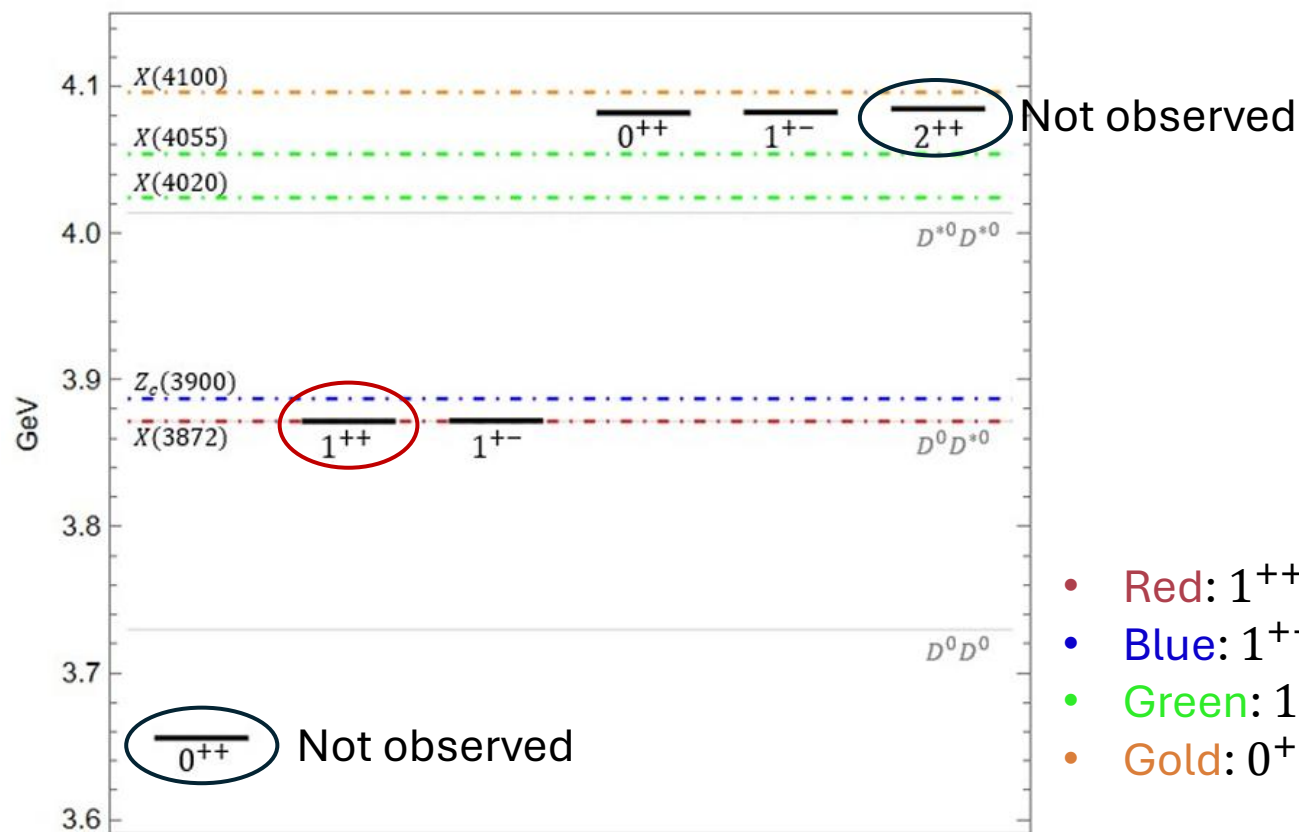
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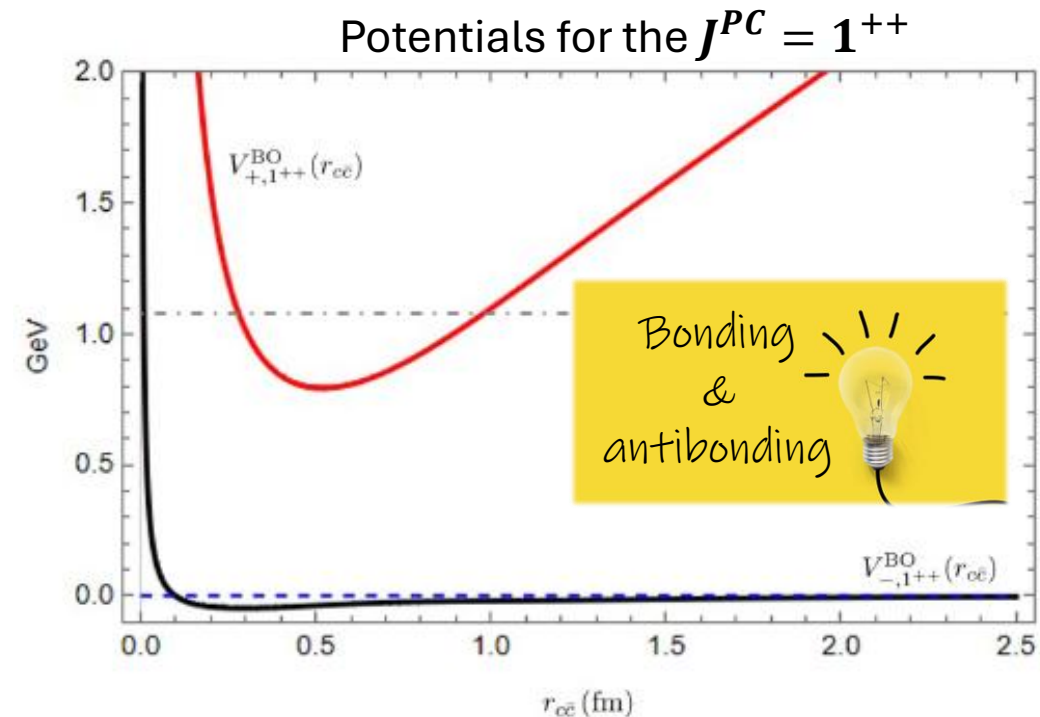
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- Red: 1^{++}
- Blue: 1^{+-}
- Green: 1^{+-}
- Gold: 0^{++}



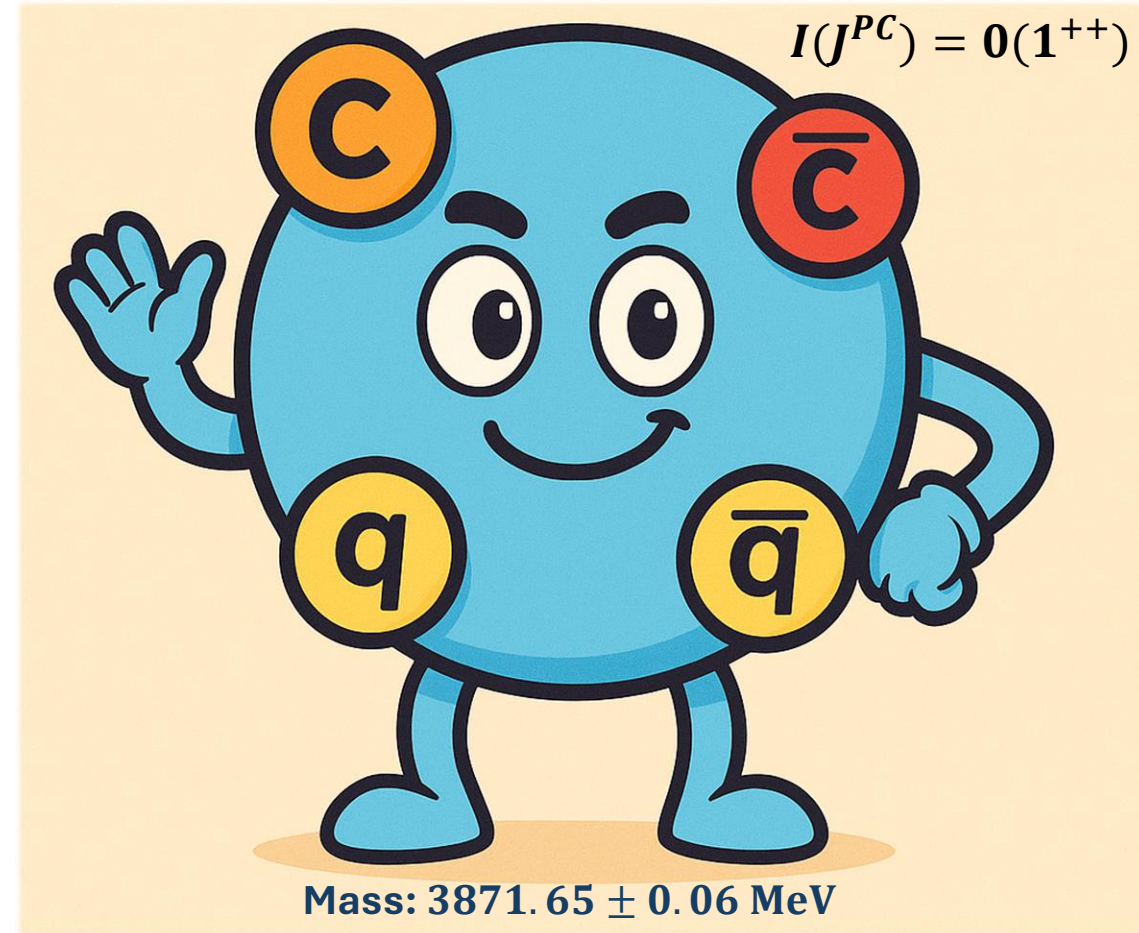
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Let's meet the $X(3872)$

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Exotic Meson $[c\bar{c}q\bar{q}]$

$$I(J^{PC}) = 0(1^{++})$$



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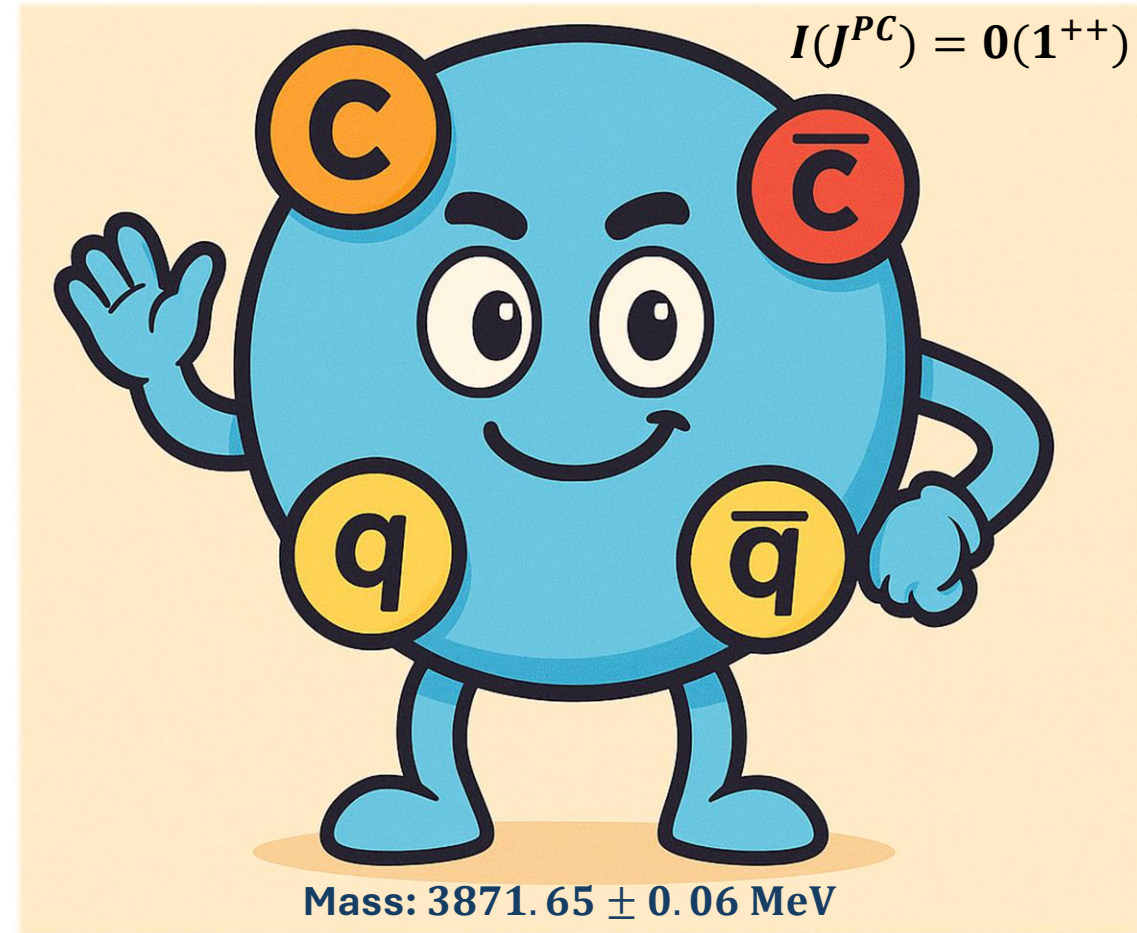
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$$(m_D + m_{D^*}) - m_X \sim O(\text{keV})$$

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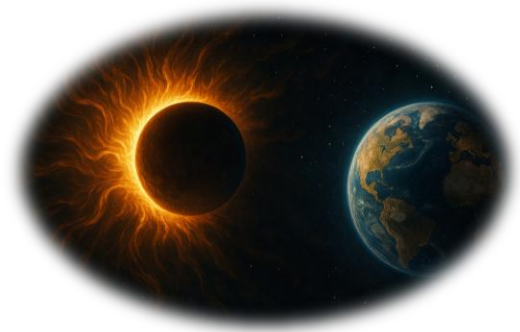


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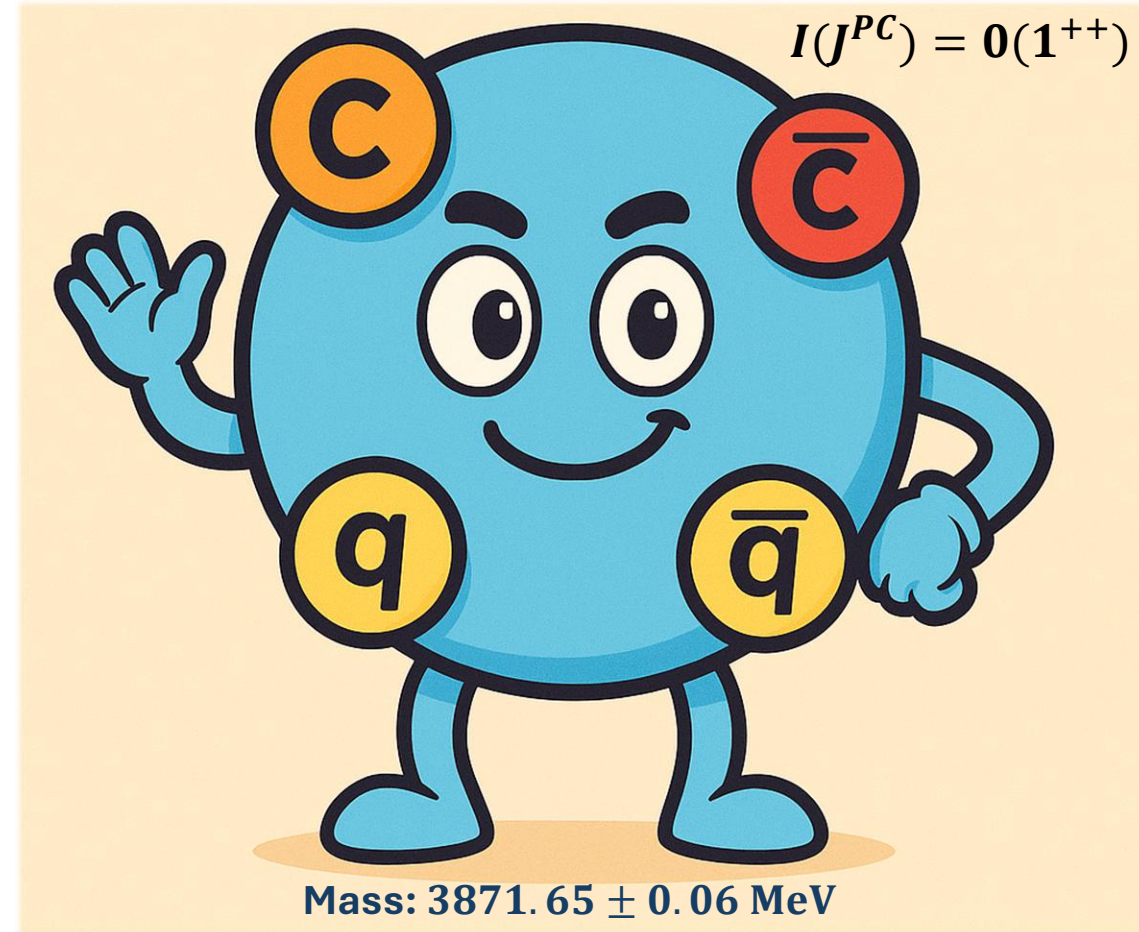
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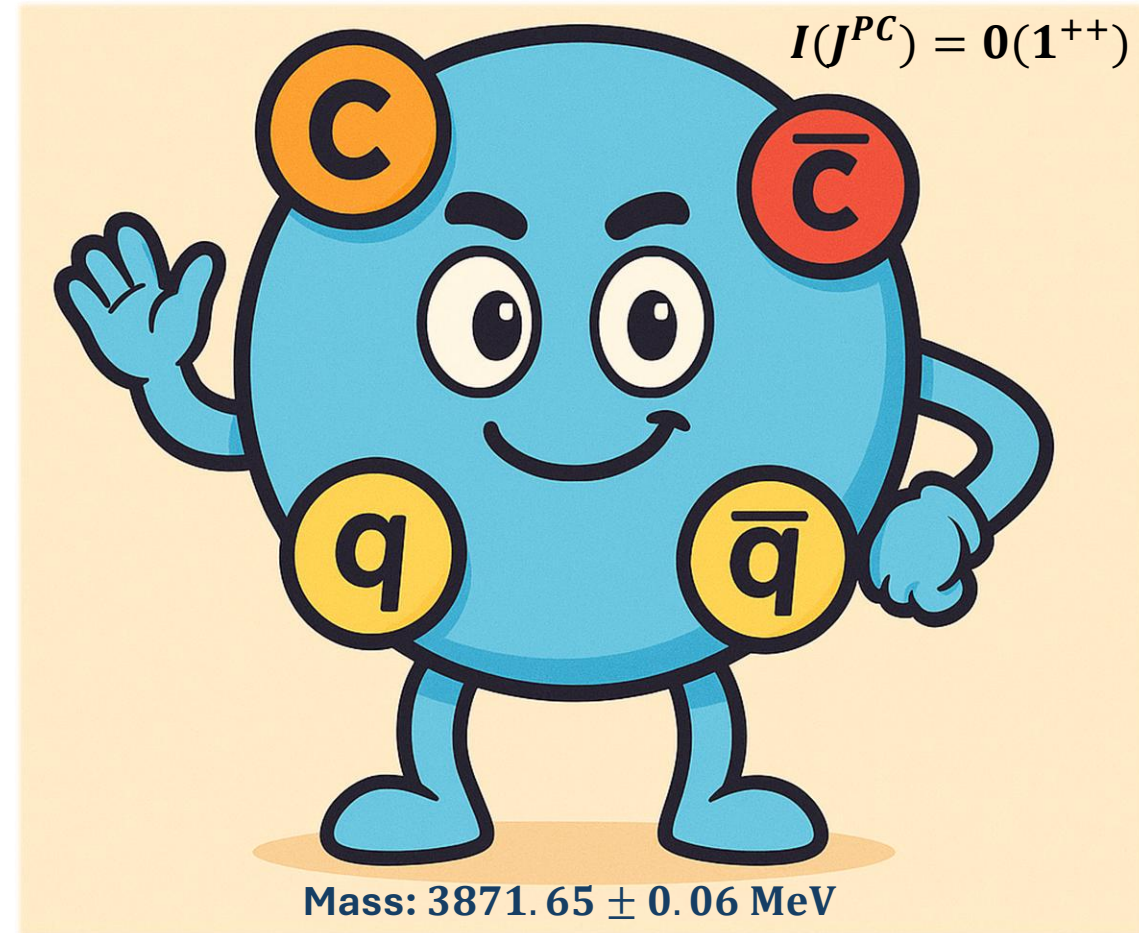
- The main decay channel is $D^0\bar{D}^0\pi^0$, primarily mediated by $X \rightarrow D^0\bar{D}^{*0}$ followed by $\bar{D}^{*0} \rightarrow \bar{D}^0\pi^0$

$$\text{Br}(X(3872) \rightarrow D^0\bar{D}^0\pi^0) = (55 \pm 28)\%$$

Particle Data Group Collaboration, S. Navas et al. Phys. Rev. D **110** (2024), no. 3 030001.

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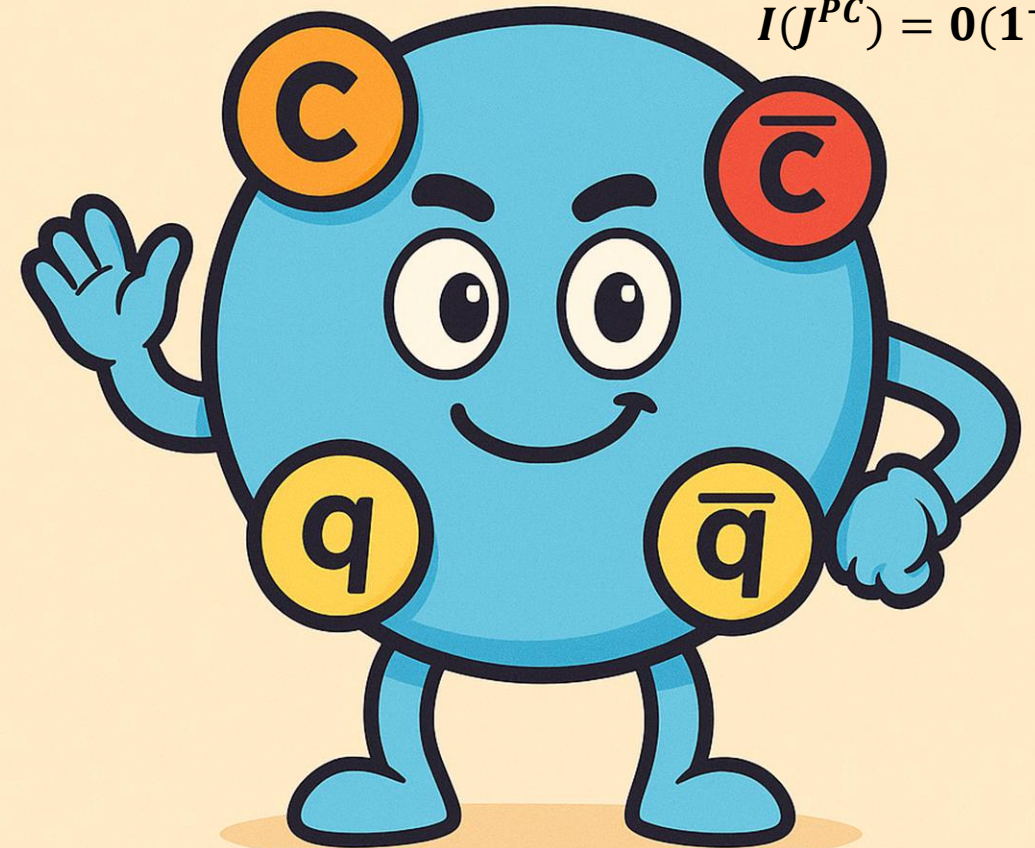
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The proximity to the $D^0\bar{D}^{*0}$ threshold and the strong coupling to these two particles make it difficult to determine whether it is a **compact** tetraquark or a **molecular** state

Exotic Meson $[c\bar{c}q\bar{q}]$

$$I(J^{PC}) = 0(1^{++})$$



Mass: $3871.65 \pm 0.06 \text{ MeV}$

Compact= the constituent quarks interact within a region smaller than the QCD confinement scale ($\sim 1 \text{ fm}$)

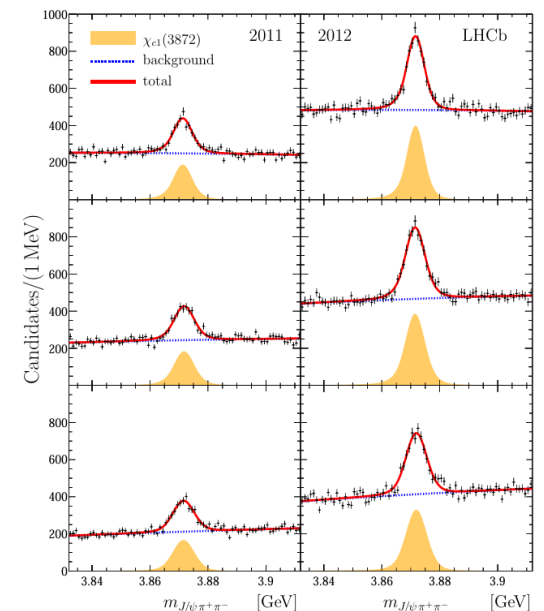
Overview of the current status

- Lattice QCD ab initio calculations usually assume that the heavy quarks are static, and only the light ones are treated dynamically. They tend to be more reliable for exotic states made entirely of heavy quarks;

P. Bicudo, Phys.Rept. **1039** (2023) 1-49, arXiv:2212.07793 [hep-lat]

- Many phenomenological models describe the $X(3872)$ as either a compact tetraquark or a molecular state. Some studies, on the other hand, suggest it could be a pure charmonium $c\bar{c}$, or even a mixed state with both compact and charmonium components; N. Brambilla et al., Phys.Rev.Lett. **135** (2025) 13, 13, arXiv:2411.14306 [hep-ph]; T. Ji et al., arXiv:2502.04458 [hep-ph]; P. Colangelo et al., Phys.Rev.D **111** (2025) 7, 074014, arXiv:2501.15888 [hep-ph]

- The lineshape of the $X(3872)$ measured by the LHCb Collaboration has not provided a conclusive result. At this stage, neither the molecular nor the compact interpretation can be ruled out with certainty. LHCb, Phys.Rev.D **102** (2020) 9, 092005, arXiv:2005.13419 [hep-ex]



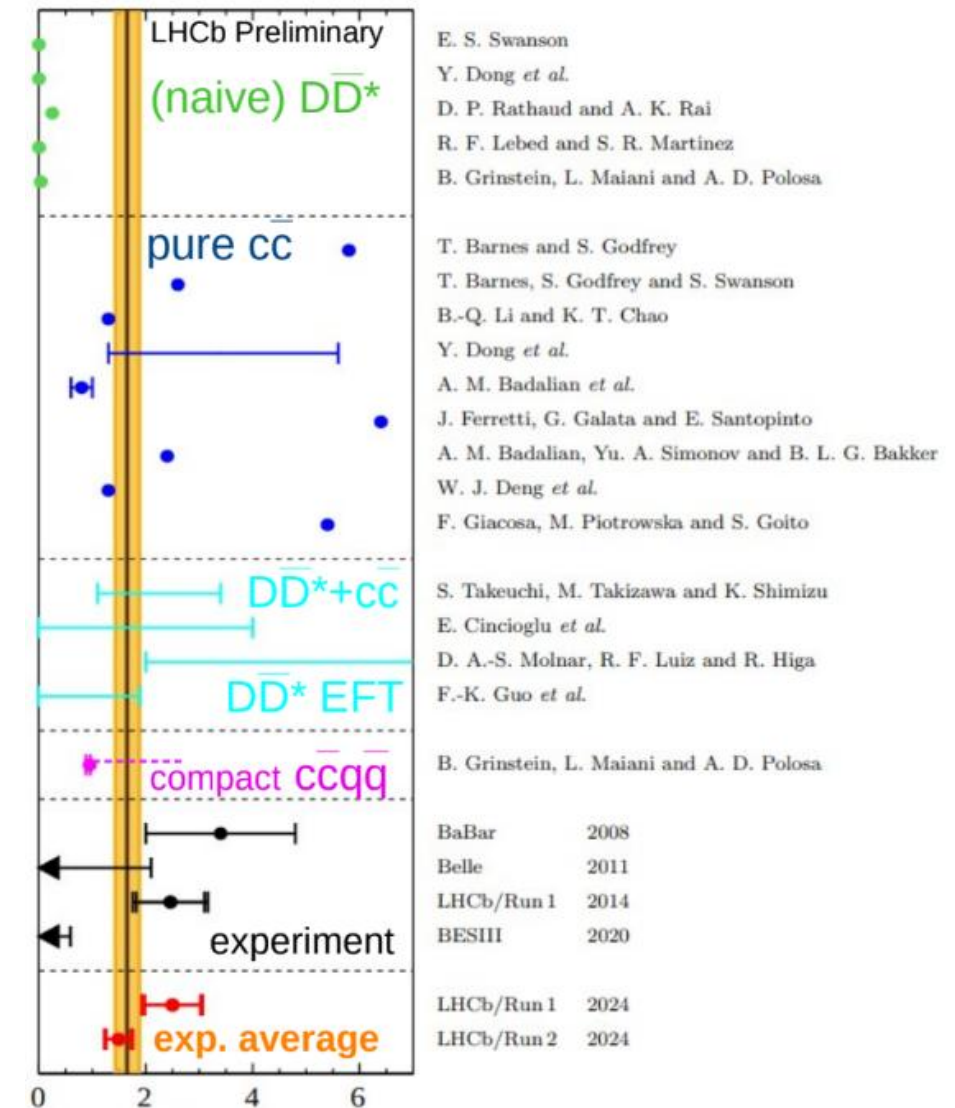
Why study the radiative decays of the $X(3872)$?

- $X(3872) \rightarrow J/\psi \gamma, \psi' \gamma$

$$\mathcal{R}_{\text{exp}} = \frac{\text{Br}(X \rightarrow \psi' \gamma)}{\text{Br}(X \rightarrow \psi \gamma)} = 1.67 \pm 0.21 \pm 0.12 \pm 0.04$$

I. Bezshyiko et al., JHEP11 (2024), 121, 2406.17006.

If the decay dynamics of the X into ψ' and J/ψ were the same, we would expect a value less than 1, due to an obvious phase space argument.



I. Polyakov on behalf of the LHCb collaboration, CERN LHC Seminar 11 June 2024

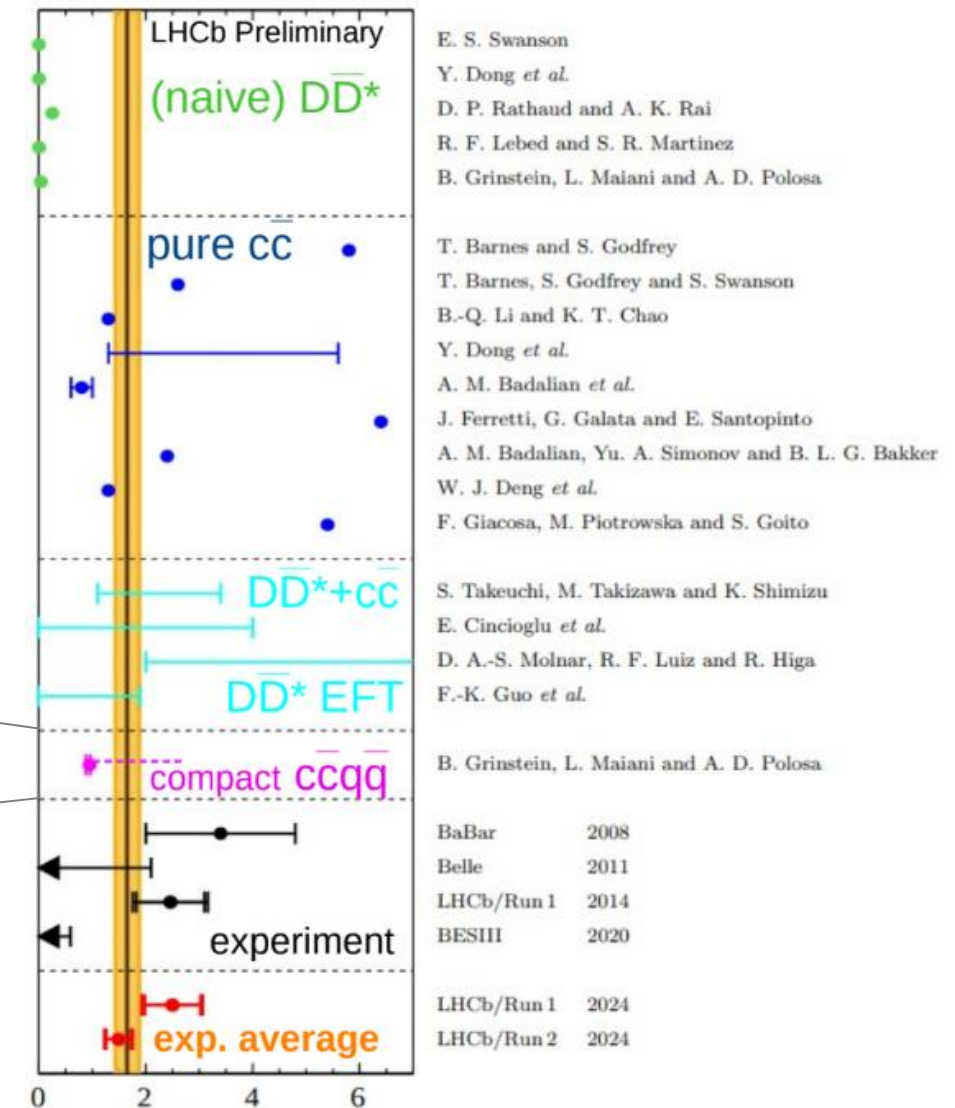
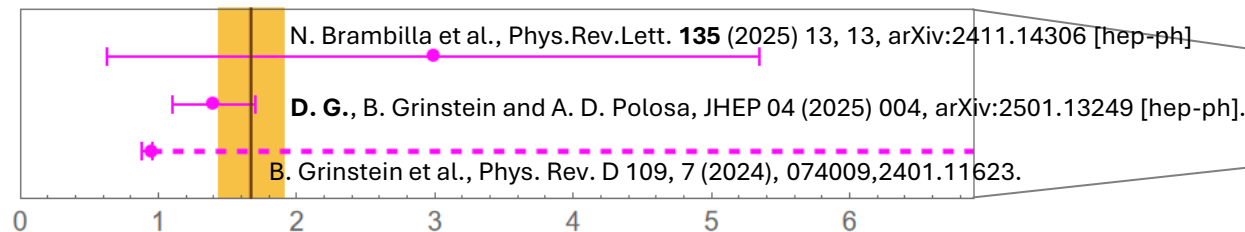
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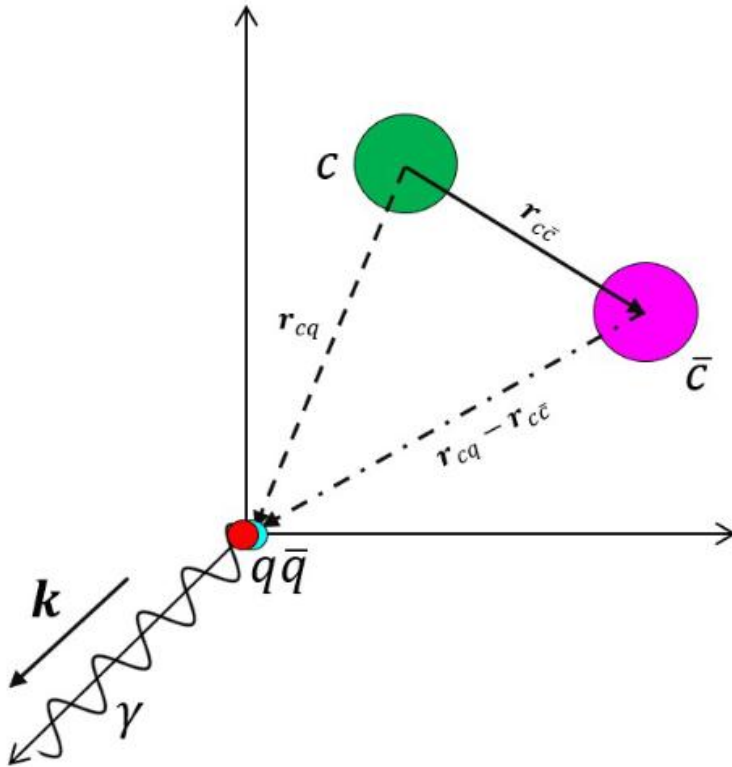
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Radiative decays: non relativistic approach

Annihilation of the $q\bar{q}$ pairs



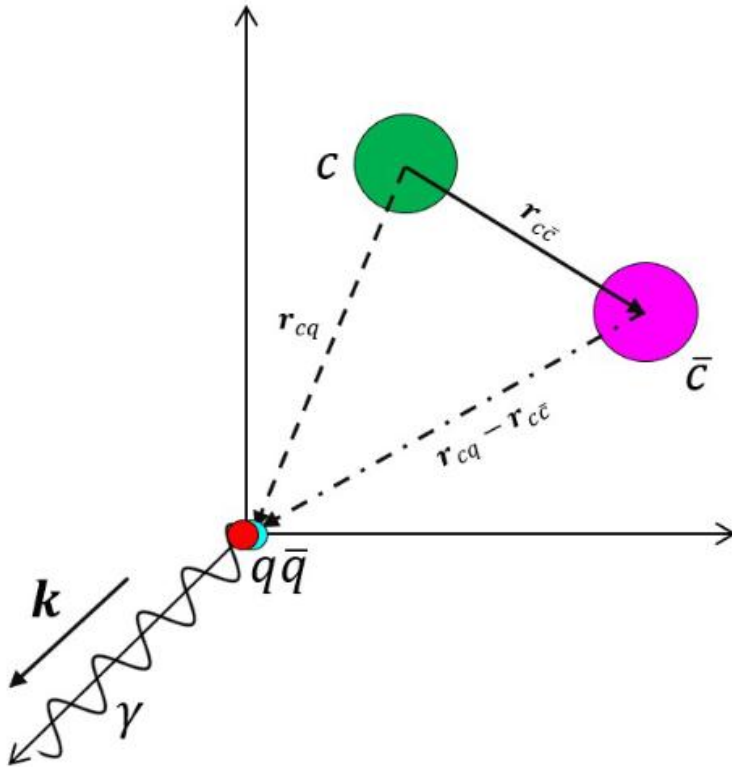
Transition amplitude

$$A\left(X \rightarrow \psi^{(l)} \gamma\right)=\mathcal{F} \int_{\mathbf{r}_{c \bar{c}}, \mathbf{r}_{c q}} e^{-i \mathbf{k} \cdot\left(\frac{\mathbf{r}_{c \bar{c}}}{2}-\mathbf{r}_{c q}\right)} \psi\left(\left|\mathbf{r}_{c \bar{c}}\right|\right) \Psi_{c \bar{c} q \bar{q}}\left(\mathbf{r}_{c \bar{c}}, \mathbf{r}_{c q}\right)$$

D. G., B. Grinstein and A. D. Polosa, JHEP 04 (2025) 004, arXiv:2501.13249 [hep-ph].

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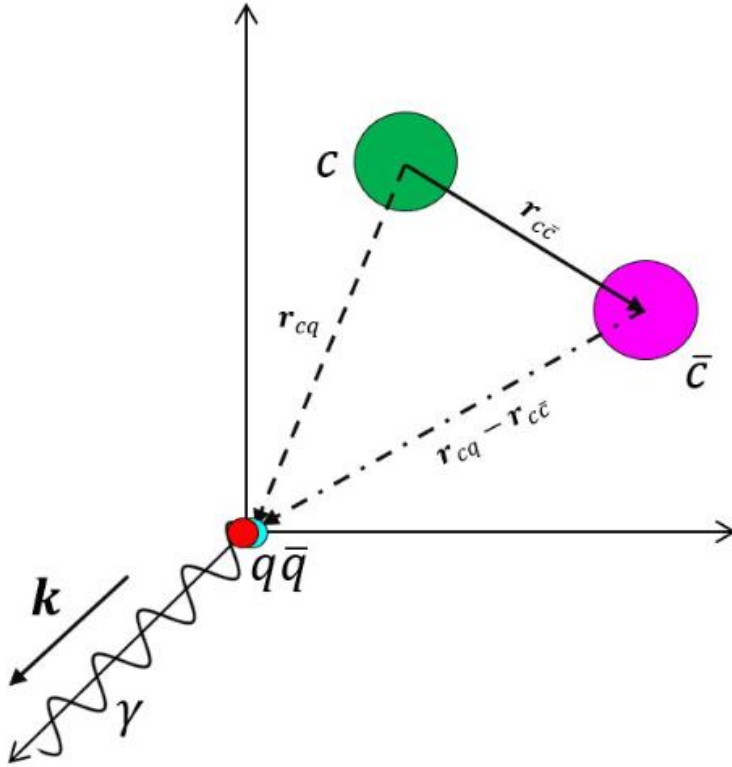
$$A(X \rightarrow \psi^{(\prime)} \gamma) = \mathcal{F} \int_{r_{c\bar{c}}, r_{cq}} e^{-ik \cdot \left(\frac{r_{c\bar{c}}}{2} - r_{cq} \right)} \psi(|r_{c\bar{c}}|) \Psi_{c\bar{c}q\bar{q}}(r_{c\bar{c}}, r_{cq})$$

Recoil effect

D. G., B. Grinstein and A. D. Polosa, JHEP 04 (2025) 004, arXiv:2501.13249 [hep-ph].

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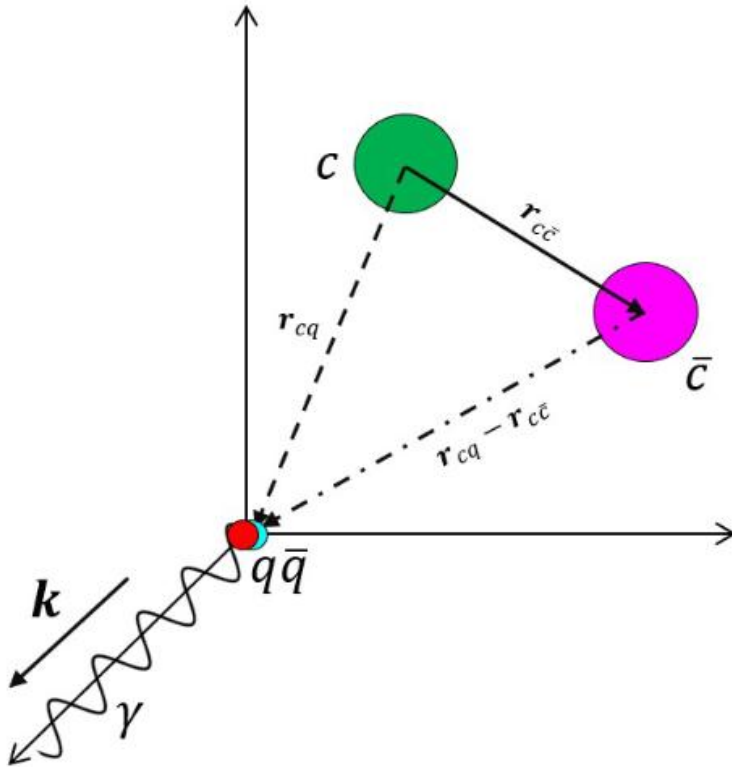
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Final charmonia wavefunction

D. G., B. Grinstein and A. D. Polosa, JHEP 04 (2025) 004, arXiv:2501.13249 [hep-ph].

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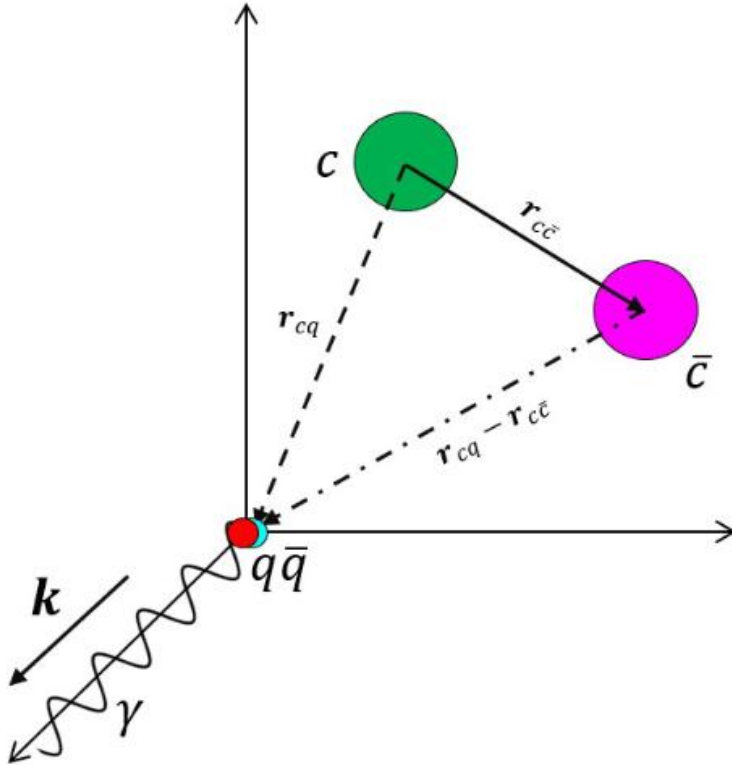
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B. Grinstein et al., Phys. Rev. D 109, 7 (2024), 074009,2401.11623.

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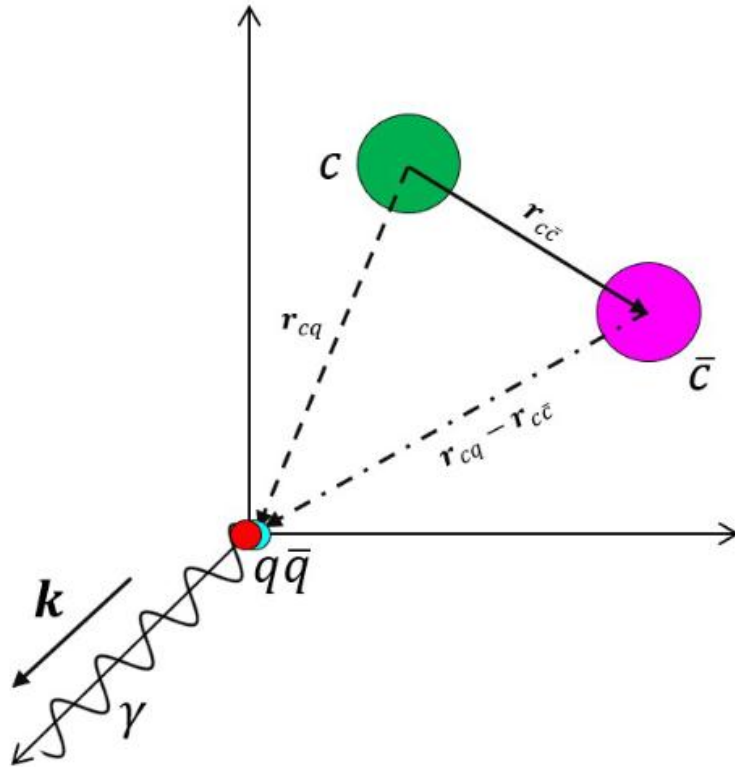
Sum over polarizations: 0.98

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Born-Oppenheimer approximation

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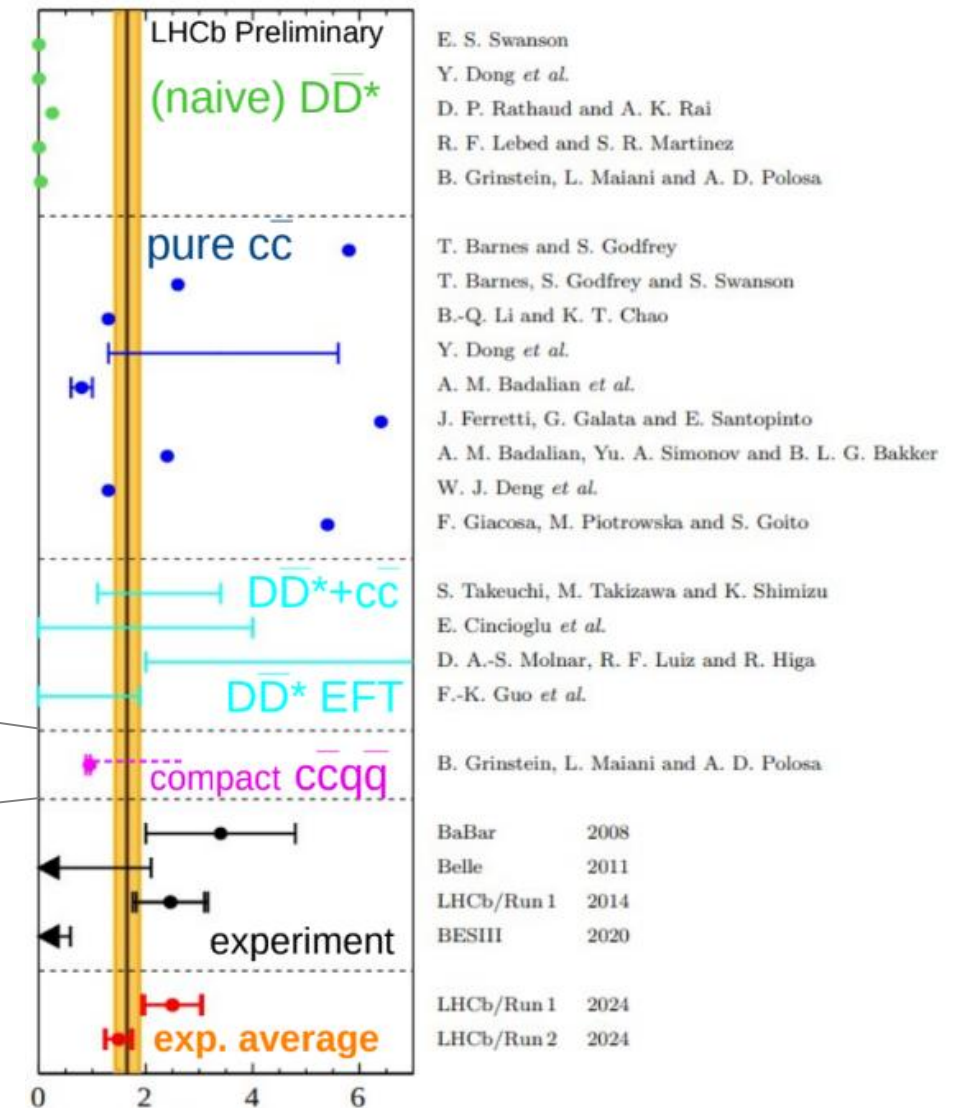
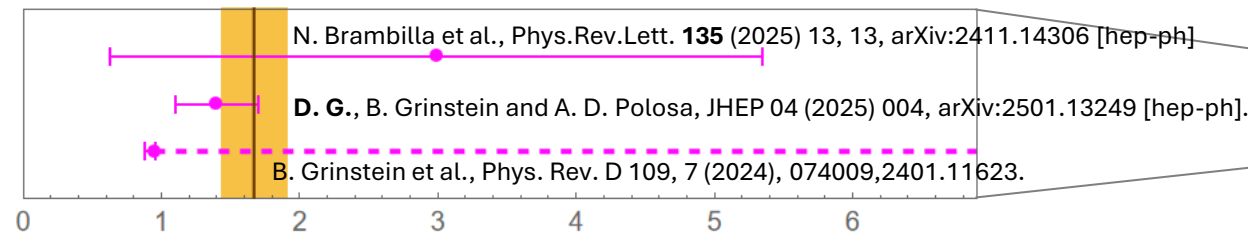
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Thank you for your attention

BACKUP

$Q\bar{Q}q\bar{q}$ tetraquarks: Born-Oppenheimer approximation

$$H_{q\bar{q}} = -\frac{\nabla_{\mathbf{r}_{cq}}^2}{2m_{cq}} - \frac{\nabla_{\mathbf{r}_{\bar{c}\bar{q}}}^2}{2m_{c\bar{q}}} + \boxed{V^{\text{coul}}} + \boxed{V^{\text{conf}}} + \boxed{V^{\text{spin}}}$$

$$\boxed{V^{\text{coul}} = -\frac{1}{3}\alpha_s \frac{1}{r_{cq}} - \frac{1}{3}\alpha_s \frac{1}{r_{\bar{c}\bar{q}}} - \frac{7}{6} \frac{1}{r_{c\bar{q}}} - \frac{7}{6}\alpha_s \frac{1}{r_{\bar{c}q}} + \frac{1}{6}\alpha_s \frac{1}{|\mathbf{r}_{cq} - \mathbf{r}_{\bar{c}\bar{q}} - \mathbf{r}_{c\bar{c}}|}}$$

$$\begin{aligned} \langle \psi_D(\mathbf{r}_{cq}) \psi_D(\mathbf{r}_{\bar{c}\bar{q}}) | V^{\text{conf}} | \psi_D(\mathbf{r}_{cq}) \psi_D(\mathbf{r}_{\bar{c}\bar{q}}) \rangle &= 2k J_2^D \\ \langle \psi_M(\mathbf{r}_{c\bar{q}}) \psi_M(\mathbf{r}_{\bar{c}q}) | V^{\text{conf}} | \psi_M(\mathbf{r}_{c\bar{q}}) \psi_M(\mathbf{r}_{\bar{c}q}) \rangle &= 2k J_2^M \\ \langle \psi_D(\mathbf{r}_{cq}) \psi_D(\mathbf{r}_{\bar{c}\bar{q}}) | V^{\text{conf}} | \psi_M(\mathbf{r}_{c\bar{q}}) \psi_M(\mathbf{r}_{\bar{c}q}) \rangle &= 2k S_{DM}(r_{c\bar{c}}) J_2^{MD}(r_{c\bar{c}}) \end{aligned}$$

$$\boxed{V^{\text{spin}} = \sum_{\text{pairs}} -\frac{\lambda_{ij}}{m_i m_j} \frac{8\pi}{3} \alpha_s \delta^{(3)}(\mathbf{r}_i - \mathbf{r}_j) \mathbf{S}_i \cdot \mathbf{S}_j}$$

Born-Oppenheimer approximation: first step

Ansatz

$$\Psi_{q\bar{q}} = \sum_{i=1}^N a_i \Phi_i \quad \longrightarrow \quad \frac{d}{da_i} \langle \Psi_{q\bar{q}} | H_{q\bar{q}} | \Psi_{q\bar{q}} \rangle = 0$$



$$\sum_{j=1}^N (\langle \Phi_i | H_{q\bar{q}} | \Phi_j \rangle - \Delta E(r_{c\bar{c}}) \langle \Phi_i | \Phi_j \rangle) a_j = 0$$



$$\begin{pmatrix} H_D - \Delta E(r_{c\bar{c}}) & H_{DM} - S_{DM}^2(r_{c\bar{c}}) \Delta E(r_{c\bar{c}}) \\ H_{DM} - S_{DM}^2(r_{c\bar{c}}) \Delta E(r_{c\bar{c}}) & H_M - \Delta E(r_{c\bar{c}}) \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = 0$$

Born-Oppenheimer approximation: first step

$$T = \underset{\text{Initial state}}{|(c\bar{c})_8(q\bar{q})_8\rangle_1} \left\{ \begin{array}{l} T_D = \sqrt{\frac{2}{3}} |(cq)_{\bar{3}}(\bar{c}\bar{q})_3\rangle - \sqrt{\frac{1}{3}} |(cq)_6(\bar{c}\bar{q})_{\bar{6}}\rangle \longrightarrow V_D(\zeta) = -\frac{1}{3}\alpha_s \frac{1}{\zeta} + k\zeta \\ T_M = \sqrt{\frac{8}{9}} |(c\bar{q})_1(\bar{c}q)_1\rangle - \sqrt{\frac{1}{9}} |(c\bar{q})_8(\bar{c}q)_8\rangle \longrightarrow V_M(\zeta) = -\frac{7}{6}\alpha_s \frac{1}{\zeta} + k\zeta \end{array} \right.$$

$$E_{\mathcal{C}} = \left\langle \psi_{\mathcal{C}}(\zeta) \left| -\frac{\nabla^2}{2m_{cq}} + V_{\mathcal{C}}(\zeta) \right| \psi_{\mathcal{C}}(\zeta) \right\rangle = \frac{\mathcal{C}^2}{2m_{cq}} + \lambda_{\mathcal{C}}\alpha_s \mathcal{C} + \frac{3k}{2\mathcal{C}}$$

$$\lambda(S) = \frac{1}{2}(C(S) - C(R_1) - C(R_2)) \longrightarrow \lambda_{cq} = \lambda_{\bar{c}\bar{q}} = \frac{2}{3} \left(\frac{1}{2} \left(\frac{4}{3} - \frac{8}{3} \right) \right) + \frac{1}{3} \left(\frac{1}{2} \left(\frac{10}{3} - \frac{8}{3} \right) \right) = -\frac{1}{3}$$

Radiative decays: non relativistic approach

