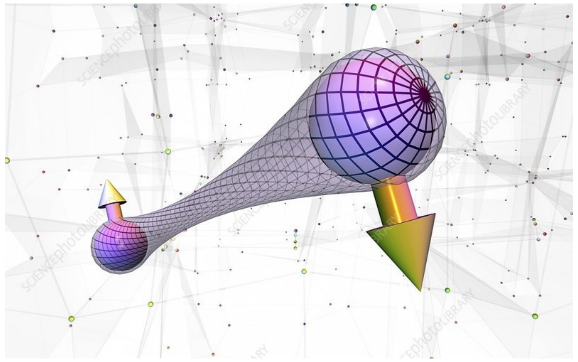
Quantum Entanglement and Bell Inequality Violation at High Energies



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- "Entanglement" between two systems is a pure quantum phenomena
- It is induced by the interaction from which the two systems are produced
- Expected to violates Bell inequalities (set of correlation measurements)
- Violations incompatible with classical physics based on causality and local realism (locality) (EPR paradox, hidden variables theories)

$$a|0\rangle|alive\rangle + b|1\rangle|dead\rangle$$

"About your cat, Mr. Schrödinger—I have good news and bad news."

What is entanglement ?

classical concept of phase space → In QM replaced by → by abstract Hilbert space

makes a gap in the description of composite systems

Consider multipartite system of ***n*** subsystems

- **Classical description** → Cartesian product of ***n*** subsystems → product of the ***n*** separate systems
- **Quantum description** → Hilbert space H → tensorial product of subsystem spaces

$$H = H_1 \otimes H_2 \otimes H_3 \otimes \cdots \otimes H_n$$

superposition principle $$|\psi\rangle = \sum_{\mathbf{i}_n} c_{\mathbf{i}_n} |\mathbf{i}_n\rangle$$

$$|\mathbf{i}_n\rangle = |i_1\rangle \otimes |i_2\rangle \cdots |i_n\rangle$$

$$|\psi\rangle \neq |\psi_1\rangle \otimes |\psi_2\rangle \cdots |\psi_n\rangle$$

→ in general not possible to assign a single state vector to any of n subsystems

giving rise to the phenomenon of entanglement

Local realism

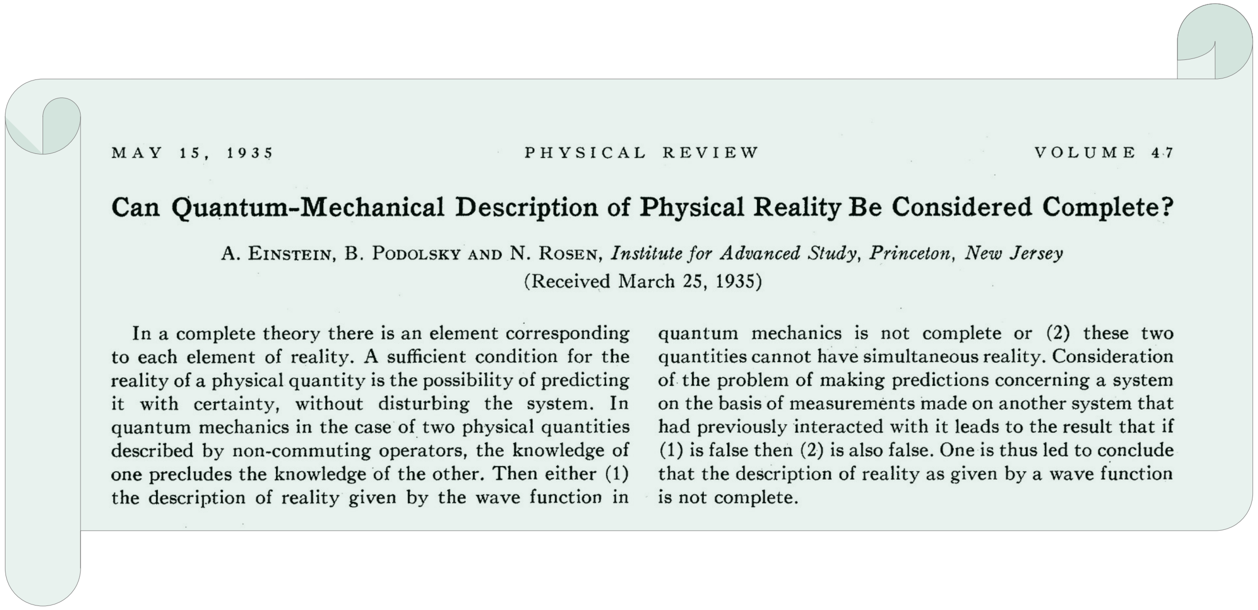
- Based on the (classical physics) idea that objects have definite properties whether or not they are measured

- and that measurements of these properties are not affected by events taking place sufficiently far away

- **Einstein Locality Principle**

Consider two systems **A** and **B** that have interacted in the past and are separated (space-like) far away

The results of a measurement on A is unaffected by operations on the distance system B


MAY 15, 1935 PHYSICAL REVIEW VOLUME 47

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. Einstein, B. Podolsky and N. Rosen, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)

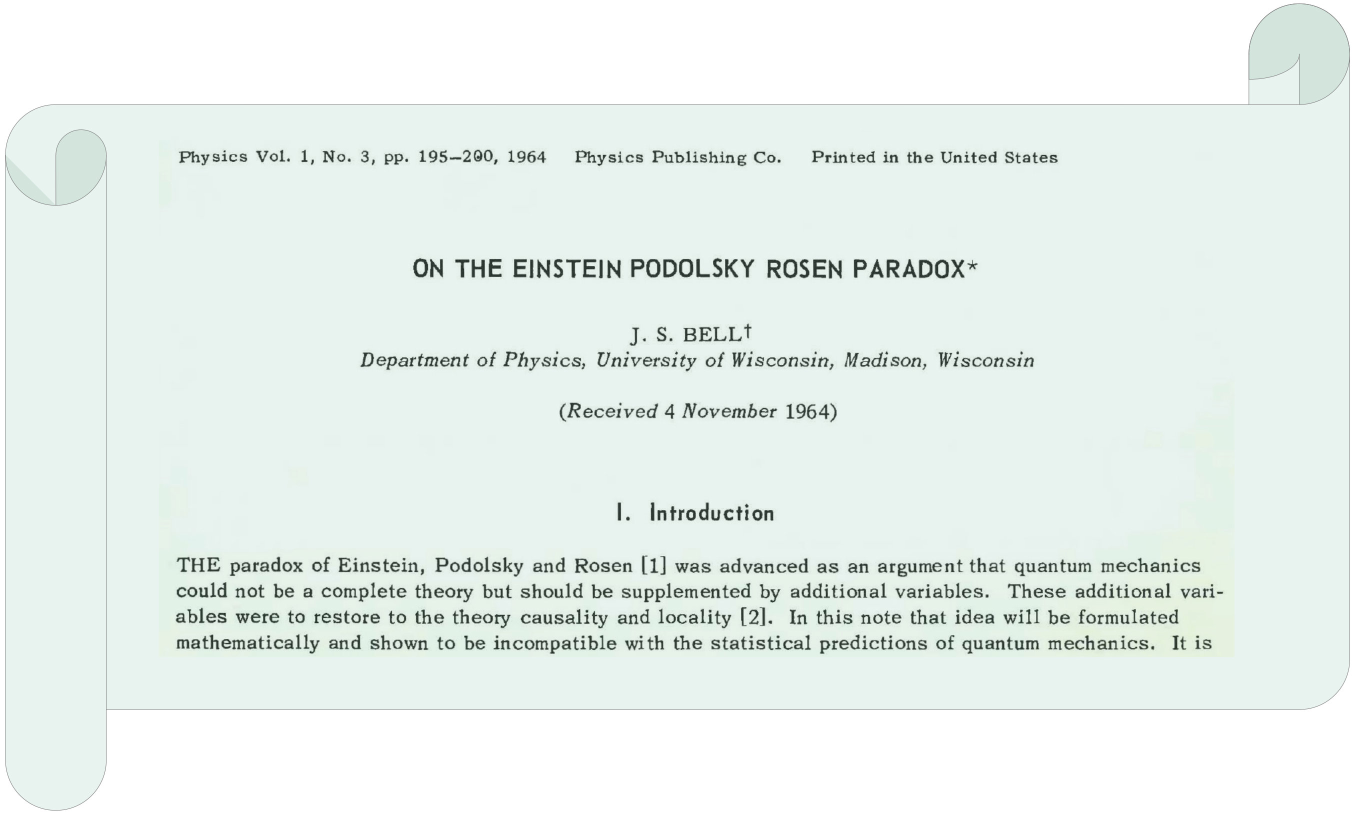
In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not complete.

Based on locality principle
they argue that QM is incomplete

- One may argue that the incompleteness of QM followed from EPR paradox is inherent in the probabilistic interpretation of Quantum Mechanics

- Dynamic behavior at microscopic level appears probabilistic only because some yet unknown parameters (hidden variables) have not been specified

- **Bell inequalities (1964):**
  a test to discriminate between local and non-local (QM) description of Nature



Physics Vol. 1, No. 3, pp. 195–200, 1964 Physics Publishing Co. Printed in the United States

ON THE EINSTEIN PODOLSKY ROSEN PARADOX*

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Department of Physics, University of Wisconsin, Madison, Wisconsin

(*Received* 4 *November* 1964)

I. Introduction

THE paradox of Einstein, Podolsky and Rosen [1] was advanced as an argument that quantum mechanics could not be a complete theory but should be supplemented by additional variables. These additional variables were to restore to the theory causality and locality [2]. In this note that idea will be formulated mathematically and shown to be incompatible with the statistical predictions of quantum mechanics. It is

Quantum Entangled states violate Bell inequalities

example

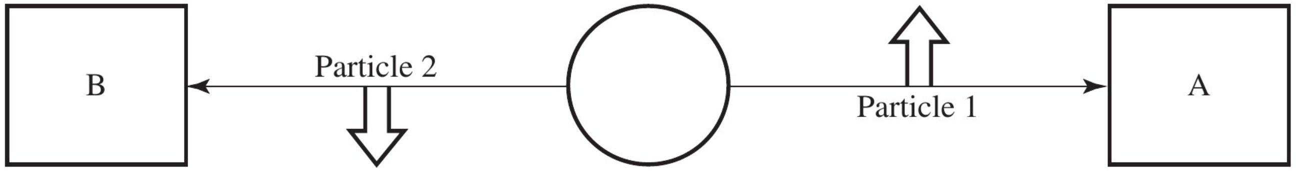
$$H \rightarrow \mu^+ + \mu^-$$

J=0 state

Maximum entangled state

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left[|\mu^+\uparrow;\mu^-\downarrow\rangle - |\mu^+\downarrow;\mu^-\uparrow\rangle\right]$$

Bob



Alice

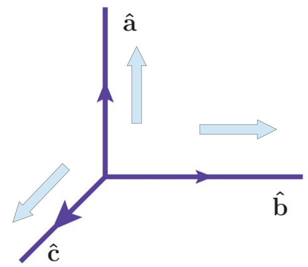
- measurement of spin in particle 1 induces correlation on spin measurement of particle 2

- measuring spin along same directions just test property of angular momentum conservation

- to check departure from Locality → require A and B to perform correlated measurements of spin-projection in **two different** directions

Not necessarily to be orthogonal

$$\left\{\hat{\mathbf{a}},\ \hat{\mathbf{b}},\ \hat{\mathbf{c}}\right\} \Rightarrow \left[\mathbf{S}_{\hat{\mathbf{a}}},\mathbf{S}_{\hat{\mathbf{b}}}\right] \neq 0 \quad \left[\mathbf{S}_{\hat{\mathbf{a}}},\mathbf{S}_{\hat{\mathbf{c}}}\right] \neq 0 \quad \left[\mathbf{S}_{\hat{\mathbf{b}}},\mathbf{S}_{\hat{\mathbf{c}}}\right] \neq 0$$



Bell inequality

Alice Bob

$P(\hat{\mathbf{a}}\uparrow;\hat{\mathbf{b}}\downarrow)$: spin $\uparrow$ along $\hat{\mathbf{a}}$ for particle 1 and spin $\downarrow$ along $\hat{\mathbf{b}}$ for particle 2

- Locality assumption → probability independence

$$P(\hat{\mathbf{a}}\uparrow;\hat{\mathbf{b}}\downarrow) = P(\hat{\mathbf{a}}\uparrow;-)P(-;\hat{\mathbf{b}}\downarrow)$$

- For example: Alice use **a,b** directions and Bob **b,c** directions

- Local deterministic theories (hidden variables) satisfies **Bell inequality**

$$P(\hat{\mathbf{a}}\uparrow;\hat{\mathbf{b}}\uparrow) \leq P(\hat{\mathbf{a}}\uparrow;\hat{\mathbf{c}}\uparrow) + P(\hat{\mathbf{c}}\uparrow;\hat{\mathbf{b}}\uparrow)$$

- Compute these probability correlations in QM for an entangled S=0 state

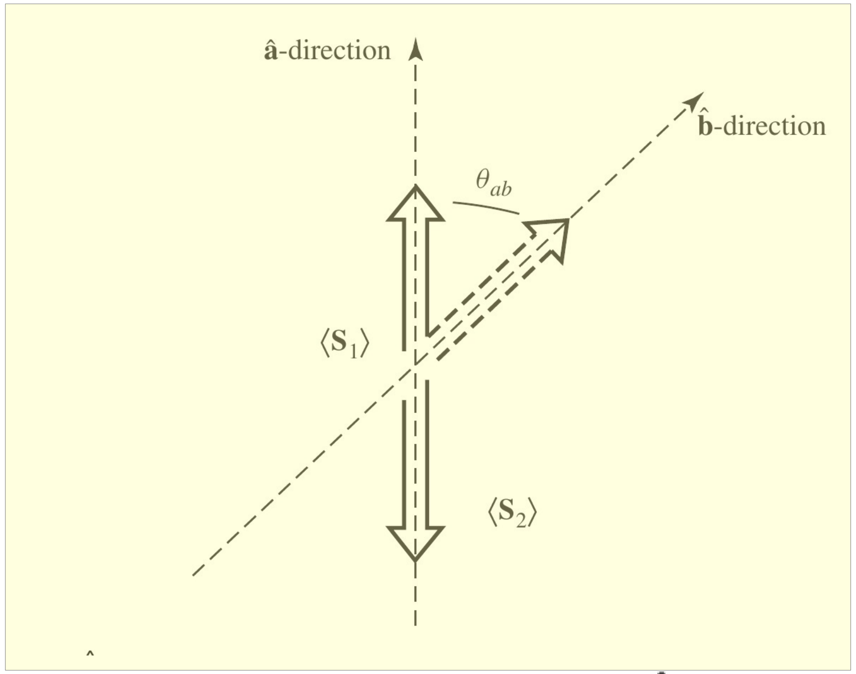
QM predictions

suppose observer A finds $\mathbf{S}_1 \cdot \hat{\mathbf{a}}$ to be positive (+) with certainty

observer B's measurement of $\mathbf{S}_2 \cdot \hat{\mathbf{a}}$ will find it negative (-) with certainty

In order to compute $P(\hat{\mathbf{a}}+;\hat{\mathbf{b}}+)$ we must consider a new quantization axis

$\hat{\mathrm{b}}$ that makes an angle θ_{ab} with $\hat{\mathrm{a}}$



The probability that $\mathbf{S}_2 \cdot \hat{\mathbf{b}}$ measurement yields + when particle 2 is known

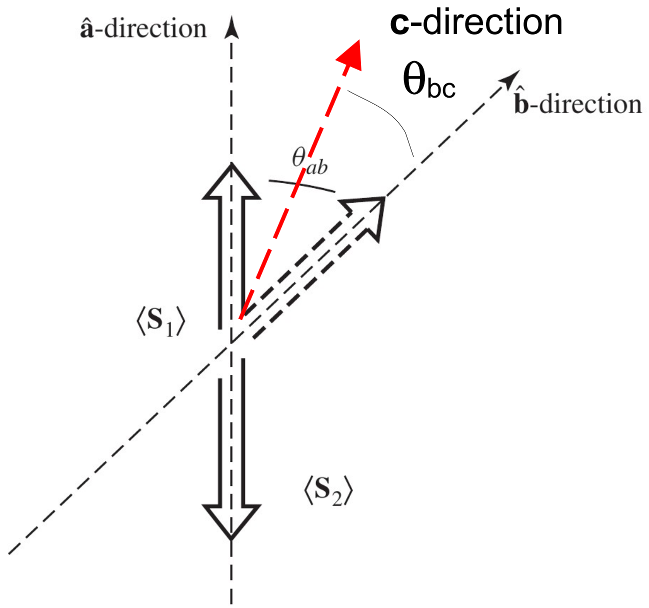
to be in a eigenstate of $\mathbf{S}_2 \cdot \hat{\mathbf{a}}$ with + is =

$$P(\hat{\mathbf{a}}+;\hat{\mathbf{b}}+) = \left(\frac{1}{2}\right)\sin^2\left(\frac{\theta_{ab}}{2}\right)$$

plug in into the Bell inequality and we get...

QM prediction of Bell inequality

$$\sin^2\left(\frac{\theta_{ab}}{2}\right) \leq \sin^2\left(\frac{\theta_{ac}}{2}\right) + \sin^2\left(\frac{\theta_{cb}}{2}\right)$$



choose for example

$$\theta_{ab} = 2\theta, \quad \theta_{ac} = \theta_{cb} = \theta$$

In this case Bell inequality is **violated** for

$$0 < \theta < \frac{\pi}{2}$$

- **optimization problem** → find directions where Bell inequality is maximally violated
- Maximum entangled states violate Bell inequalities but may not provide the maximum violation

Bell inequality violation observed in entangled photons

QM is a non-local theory

measurements in A affects what will be measured in B, even if A and B are space-like separated apart, and no causal exchange of information between them is possible

The Nobel Prize in Physics 2022

© Nobel Prize Outreach. Photo: Stefan Bladh
Alain Aspect
Prize share: 1/3

© Nobel Prize Outreach. Photo: Stefan Bladh
John F. Clauser
Prize share: 1/3

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Anton Zeilinger
Prize share: 1/3

The Nobel Prize in Physics 2022 was awarded jointly to Alain Aspect, John F. Clauser and Anton Zeilinger "for experiments with entangled photons, establishing the violation of Bell inequalities and pioneering quantum information science"

[2] A. Zeilinger *et al.*, Nature **433**, 230 (2005)

[3] S.J. Freedman and J.F. Clauser, Phys. Rev. Lett. **28**, 938 (1972). https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.28.938

[4] A. Aspect, J. Dalibard, and G. Roger, Phys. Rev. Lett. **49**, 1804 (1982). A. Aspect, P. Grangier, and G. Roger, Phys. Rev. Lett. 49, **91** (1982)

[5] G. Weihs, T. Jennewein, C. Simon, H. Weinfurter, A. Zeilinger, Phys. Rev. Lett. 81, 5039 (1998). D. Bouwmeester, J. W. Pan, K. Mattle, M. Eibl, H. Weinfurter, A. Zeilinger, Nature **390**, 575 (1997)

[6] A. Aspect, Physics **8**, 123. (2015)

new challenge: testing entanglement and Bell inequality violation at high energies and in the presence of strong and weak interactions !

Quantifying entanglement and Bell inequality violation

- Requires the knowledge of the **polarization density matrix** of two-particles **A,B** production
- it can be fully reconstructed from the angular distributions of the single **A,B decay products**
- or analogously by measuring the complete set of helicity amplitudes
- **but it can also be computed analytically**
- knowledge of the full polarization density matrix allows to quantify (where possible) entanglement and Bell inequality violations

A. J. Barr, M. Fabbrichesi, R. Floreanini, E. Gabrielli, and L. Marzola, Quantum entanglement and Bell inequality violation at colliders, Prog. Part. Nucl. Phys. **139**, 104134 (2024).

the toolbox

Qubits

spin-1/2 particles, photon

$$\rho(\lambda_1,\lambda_1',\lambda_2,\lambda_2') = \frac{1}{4}\Big[\mathbb{1}\otimes\mathbb{1} + \sum_i B_i^+(\sigma_i\otimes\mathbb{1}) + \sum_j B_j^-(\mathbb{1}\otimes\sigma_j) + \sum_{ij} C_{ij}(\sigma_i\otimes\sigma_j)\Big]$$

σ_i = 2 x 2 Pauli matrices

4 x 4 matrix

Polarization coefficients

spin-correlations coefficients

- **Entanglement**

Concurrence $\mathscr{C}[|\psi\rangle] \equiv \sqrt{2\left(1-\mathrm{Tr}\left[(\rho_A)^2\right]\right)} = \sqrt{2\left(1-\mathrm{Tr}\left[(\rho_B)^2\right]\right)}$

vanishes for separable states,
max value = 1

$$R = \rho\,(\sigma_2\otimes\sigma_2)\,\rho^*\,(\sigma_2\otimes\sigma_2)$$

find → r_i square root of R eigenvalues, i=1,2,3,4 with r_1 the largest one

$$\mathscr{C}[\rho] = \max\left(0, r_1 - r_2 - r_3 - r_4\right)$$

C.H. Bennett, D.P. Divincenzo, J.A. Smolin, W.K. Wootters, Phys. Rev. A 54 (1996) 3824

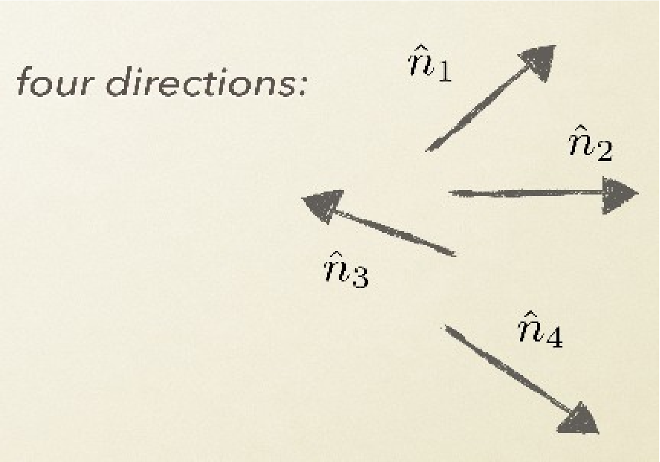
Qubits

- **Bell inequality violation**

$\vec{n}_1,\ \vec{n}_3 \longrightarrow$ for Alice $\Rightarrow (\hat{A}_1,\ \hat{A}_2)$

$\vec{n}_2,\ \vec{n}_4 \longrightarrow$ for Bob $\Rightarrow (\hat{B}_1,\ \hat{B}_2)$

2 outcomes (qubits)



CHSH inequality

$$\mathcal{I}_2 = \langle \hat{A}_1 \hat{B}_1 \rangle + \langle \hat{A}_1 \hat{B}_2 \rangle + \langle \hat{A}_2 \hat{B}_1 \rangle - \langle \hat{A}_2 \hat{B}_2 \rangle \quad \leq 2$$

$$\left| \hat{n}_1 \cdot C \cdot (\hat{n}_2 - \hat{n}_4) + \hat{n}_3 \cdot C \cdot (\hat{n}_2 + \hat{n}_4) \right| \leq 2$$

Clauser-Horne-Schimony-Holt
Phys. Rev. Lett. 24 (1970) 549

$C_{ij} \longrightarrow$ **Correlation matrix**

$M = C^T C \Rightarrow [m_1, m_2, m_3]$ eigenvalues $\Rightarrow$ m_1 and m_2 the largest ones

Horodecki condition $\quad \mathfrak{m}_{12} \equiv m_1 + m_2 > 1 \Rightarrow$ Violation of Bell inequality

R. Horodecki *et al* , Phys. Lett. A200 5 (1995) 340

Qutrits

massive spin-1 particles

$$\rho(\lambda_1, \lambda_1', \lambda_2, \lambda_2') = \left(\frac{1}{9} [\mathbb{1} \otimes \mathbb{1}] + \sum_a f_a [\mathbb{1} \otimes T^a] + \sum_a g_a [T^a \otimes \mathbb{1}] + \sum_{ab} h_{ab} \left[T^a \otimes T^b\right] \right)_{\lambda_1\lambda_1', \lambda_2\lambda_2'}$$

T^a = 3 x 3 Gell-Mann matrices

9 x 9 matrix

Entanglement

F. Mintert, A. Buchleitner, PRL 98 (2007) 140505

Concurrence

$$\left(\mathcal{C}[\rho]\right)^2 \geq \mathscr{C}_2[\rho]$$

$$\begin{aligned}\mathscr{C}_2 \;=\; & 2\max\Big[-\frac{2}{9} - 12\sum_a f_a^2 + 6\sum_a g_a^2 + 4\sum_{ab} h_{ab}^2\,, \\ & -\frac{2}{9} - 12\sum_a g_a^2 + 6\sum_a f_a^2 + 4\sum_{ab} h_{ab}^2\Big]\,,\end{aligned}$$

Lower bound:
witness of Entanglement

Negativity

$$\mathcal{N}(\rho) = \sum_k \frac{|\lambda_k| - \lambda_k}{2}$$

G. Vidal, R.F. Werner, Phys. Rev. A 65 (2002) 032314

vanishes for separable states

only for pure states

Entropy

$$0 \leq \mathscr{E}[\rho] \leq \ln d$$

$$\mathscr{E}[\rho] = -\mathrm{Tr}\left[\rho_A \log \rho_A\right] = -\mathrm{Tr}\left[\rho_B \log \rho_B\right]$$

Qutrits

Bell inequality violation

$$\begin{aligned}\mathcal{I}_3 \;=\; & P(A_1 = B_1) + P(B_1 = A_2 + 1) + P(A_2 = B_2) + P(B_2 = A_1) \\ & - P(A_1 = B_1 - 1) - P(A_1 = B_2) - P(A_2 = B_2 - 1) - P(B_2 = A_1 - 1)\end{aligned}$$

$(A_1\ ,\ A_2)$ $(B_1\ ,\ B_2)$ each can take values $\rightarrow$ $\{0, 1, 2\}$

$$\mathcal{I}_3 = \mathrm{Tr}\left[\rho\,\mathcal{B}\right]$$

$$\mathcal{B} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 2 & 0 & 0 \\ 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 \\ 0 & 0 & 2 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{2}{\sqrt{3}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

▶ **CGLMP** $\mathcal{I}_3 \leq 2$ ⇨ satisfied by deterministic local theories

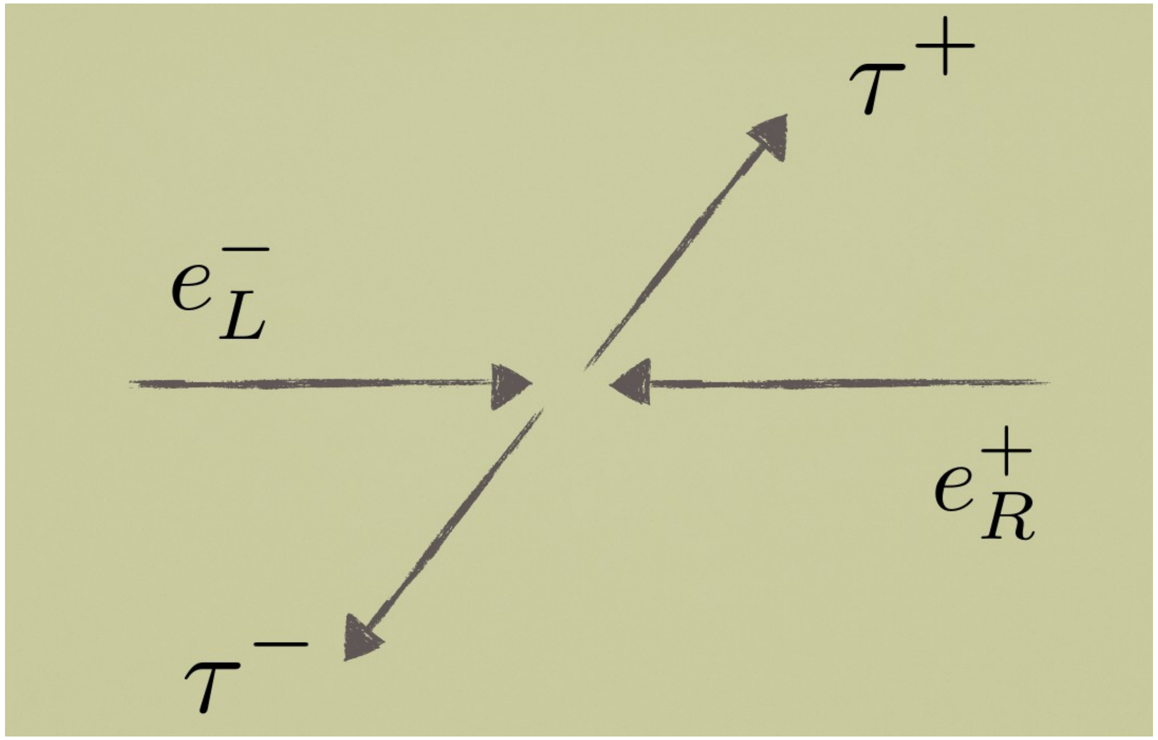
D. Collins, N. Gisin, N. Linden, S. Massar, S. Popescu, Phys. Rev. Lett 88 (2002) 040404

▶ in order to maximize the violation of Bell inequality

$$\mathcal{B} \rightarrow (U \otimes V)^{\dagger} \cdot \mathcal{B} \cdot (U \otimes V)$$

U,V are unitary 3x3 matrices
(depend on the kinematic of the process)

example



in CM frame

massless limit

QM → non separable entangled states

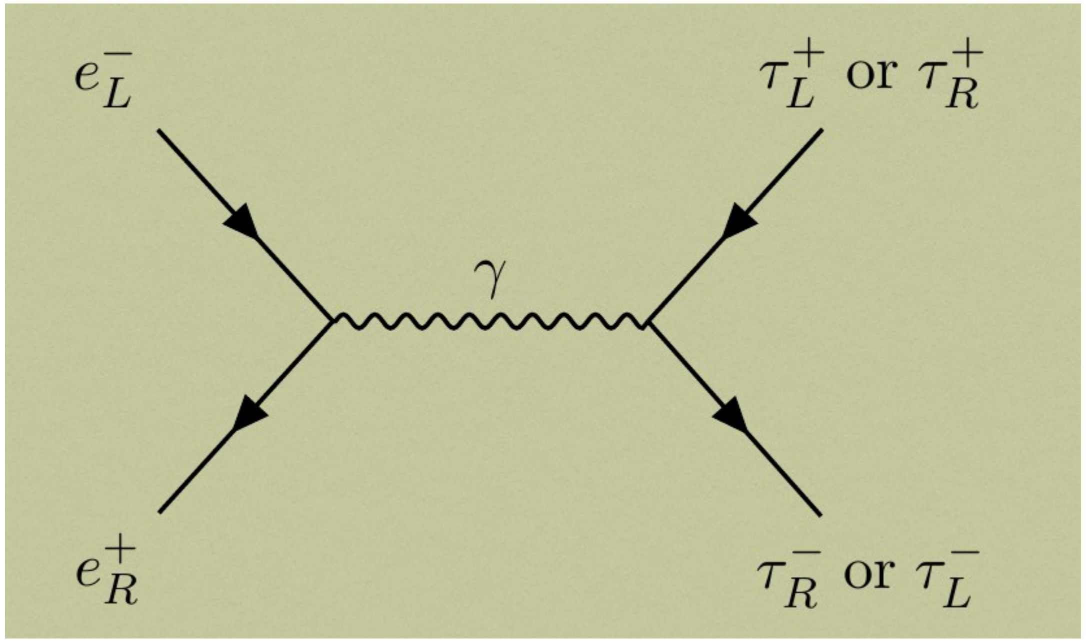
$$|\Psi\rangle = \xi_1|\tau^-_L\rangle|\tau^+_L\rangle + \xi_2|\tau^-_R\rangle|\tau^+_L\rangle + \xi_3|\tau^-_L\rangle|\tau^+_R\rangle + \xi_4|\tau^-_R\rangle|\tau^+_R\rangle$$

$$\left(\sum_i |\xi_i|^2 = 1\right)$$

Deterministic theories → separable states (example)

$$|\Psi\rangle_{\rm cl} = |\tau^-_R\rangle|\tau^+_L\rangle$$

example



Θ scattering angle in the C.M. frame

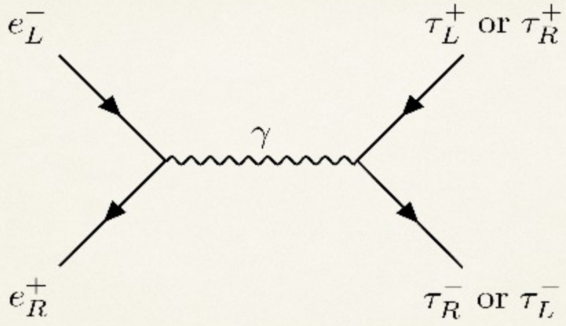
relativistic massless limit

$$|\Psi\rangle = \underbrace{\Big(1+\cos\Theta\Big)}_{\xi_2 = D^{(1)}_{1,1}(\Theta)}|\tau^-_R\rangle|\tau^+_L\rangle + \underbrace{\Big(1-\cos\Theta\Big)}_{\xi_3 = D^{(1)}_{1,-1}(\Theta)}|\tau^-_L\rangle|\tau^+_R\rangle$$

Wigner D-matrix

$J=\pm1 \quad J_z=\pm1 \quad (\Theta=0)$	$\vert\tau^-_R\rangle\,\vert\tau^+_L\rangle$	separable
$J=\pm1 \quad J_z=0 \quad (\Theta=\pi/2)$	$\frac{1}{\sqrt{2}}\Big(\vert\tau^-_R\rangle\,\vert\tau^+_L\rangle+\vert\tau^-_L\rangle\,\vert\tau^+_R\rangle\Big)$	entangled (Bell state)

example

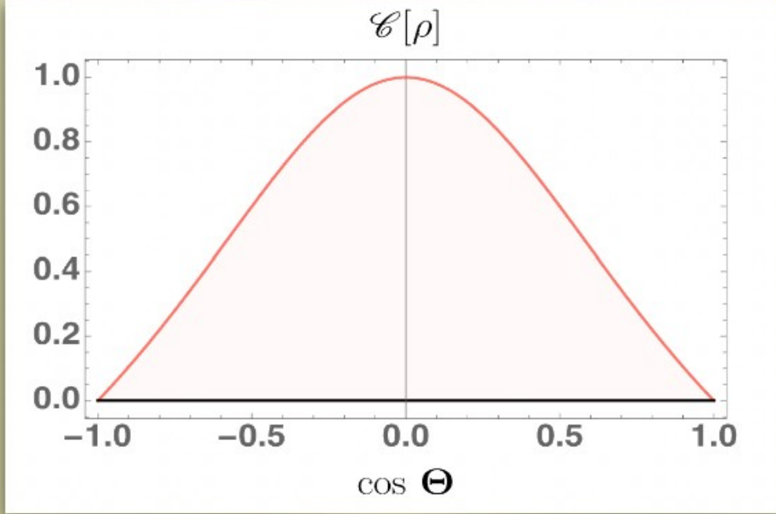


massless limit

$$\Big(1+\cos\Theta\Big)|\tau_R^-\rangle\,|\tau_L^+\rangle + \Big(1-\cos\Theta\Big)|\tau_L^-\rangle\,|\tau_R^+\rangle$$

Concurrence

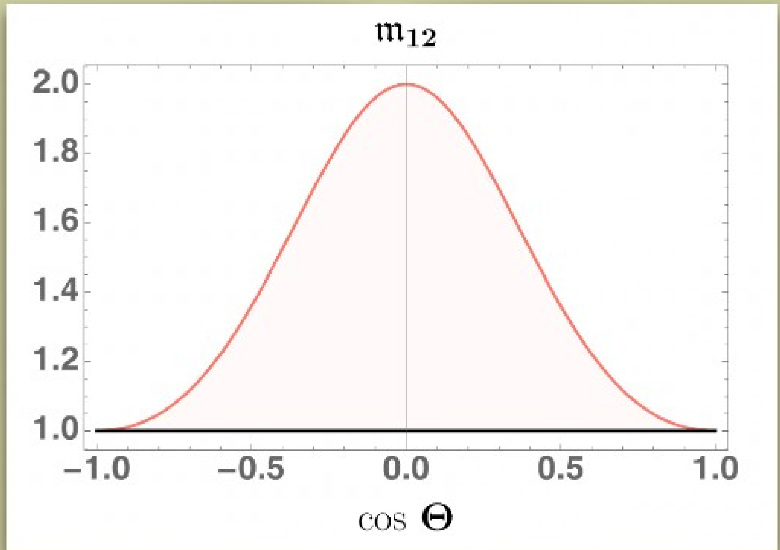
$$\mathcal{C}[\rho] = 2|\zeta_1\zeta_4 - \zeta_2\zeta_3| = \frac{\sin^2\Theta}{1+\cos^2\Theta}$$



$$\Big(1+\cos\Theta\Big)|\tau_R^-\rangle\,|\tau_L^+\rangle + \Big(1-\cos\Theta\Big)|\tau_L^-\rangle\,|\tau_R^+\rangle$$

Horodecki condition $\mathfrak{m}_{12} \equiv m_1 + m_2 > 1$

$$\mathfrak{m}_{12} = 1 + \frac{\sin^4\Theta}{(1+\cos^2\Theta)^2}$$



courtesy of M. Fabbrichesi

Local deterministic models satisfy Bell inequality

Quantum mechanics does not

Both Entanglement and Bell inequality

can be studied at colliders

- high-energy regime
- in the presence of strong and weak interactions
- qubits and qutrits

Where have we already seen
Entanglement or Bell inequality violation
at high energies?

1

Flavor space

$K^0 \bar{K}^0$ oscillations

Probing CPT and T-reversal with entangled neutral Kaons

F. J. Bernabeu, A. Di Domenico, P. Villanueva, JHEP 10 (2015) 13
J. Bernabeu, A. Di Domenico, Phys. Rev. D 105, 116004 (2022)

Bell locality condition

$$p_\lambda(f_1,\tau_1;f_2,\tau_2) = p_\lambda(f_1,\tau_1;-,\tau_2)\ p_\lambda(-,\tau_1;f_2,\tau_2)$$

$\mathcal{P}(f_1,\tau_1;-,\tau_2) \rightarrow$ Probability of finding state f_1 at time τ_1

Bell inequality $\mathcal{P}(f_1,\tau_1;f_2,\tau_2) - \mathcal{P}(f_1,\tau_1;f_4,\tau_2) + \mathcal{P}(f_3,\tau_1;f_2,\tau_2) + \mathcal{P}(f_3,\tau_1;f_4,\tau_2)$

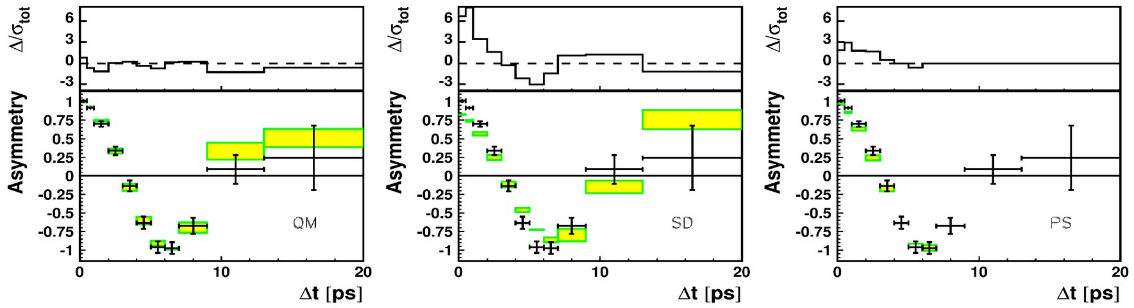
$$\leq \mathcal{P}(f_3,\tau_1;-,\tau_2) + \mathcal{P}(-,\tau_1;f_2,\tau_2)$$

a non vanishing value of epsilon'/epsilon (direct CP violation) implies Bell inequality violation

F. Benatti, R. Floreanini Phys. Rev. D57 (1998); Eur. Phys. J C13 (2000) 267

$B^0 \bar{B}^0$ oscillations

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left[|B^0\rangle_1 \otimes |\overline{B}^0\rangle_2 - |\overline{B}^0\rangle_1 \otimes |B^0\rangle_2\right]$$



Asymmetry

$$A(\Delta t) = (R_{\rm OF} - R_{\rm SF})/(R_{\rm OF} + R_{\rm SF})$$

$R_{\rm OF/SF} =$ rate of Opposite/Same − Flavor

A Go, Belle Collaboration, Phys. Rev. Lett . 99 (2007) 131802

Data favour QM over SD at 13σ and over PS model (locality, hidden variables) at 5.1σ

1

Flavor space

Neutrino oscillations

$$|\nu_\alpha\rangle = \sum_k U^*_{\alpha k} |\nu_k\rangle$$

flavor states

mass states

Leggett-Garg inequality violation

$$C_{ij} \equiv \langle \hat{Q}(t_i)\hat{Q}(t_j)\rangle$$

$$\hat{Q}(t) \equiv \hat{U}^\dagger(t)\hat{Q}\hat{U}(t)$$

$$K_n \equiv \sum_{i=1}^{n-1} C_{i,i+1} - C_{1,n}$$

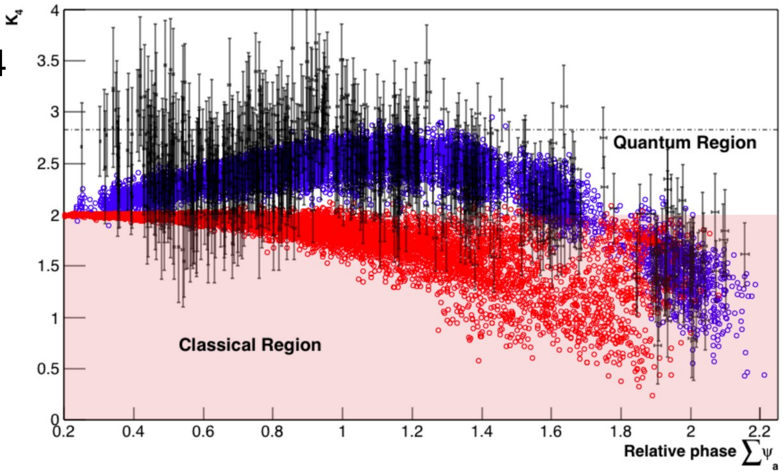
Realism and non-invasive measurements

$$K_n \leq n - 2$$

Minos (6σ)

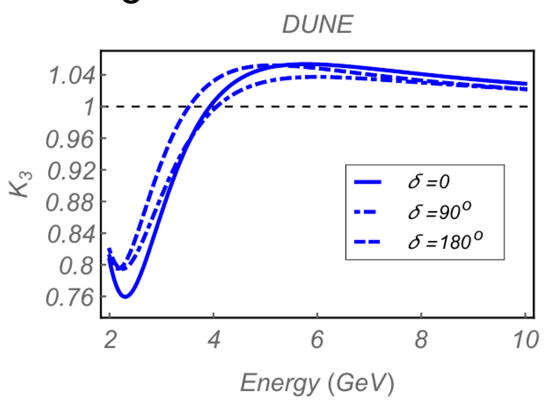
Dune, Nova, T2K

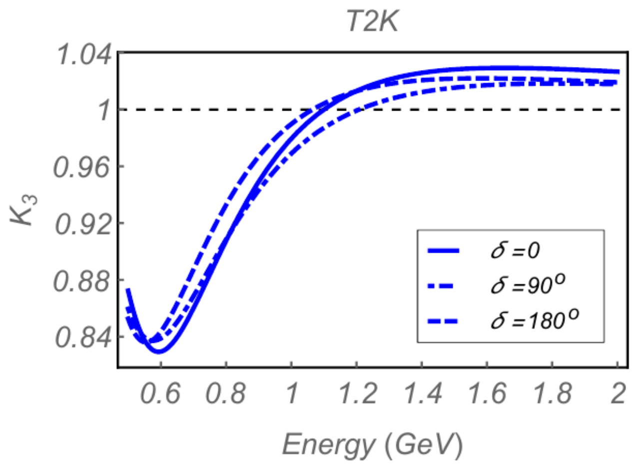
K_4



$K_4 < 2$

$K_3 < 1$





JA Formaggio, DI Kaiser, MM Murskyj and TE Weiss, Phys. Rev. Lett. 117 (2016) 050402

Violation of LG inequality occurs over a distance of 735km.

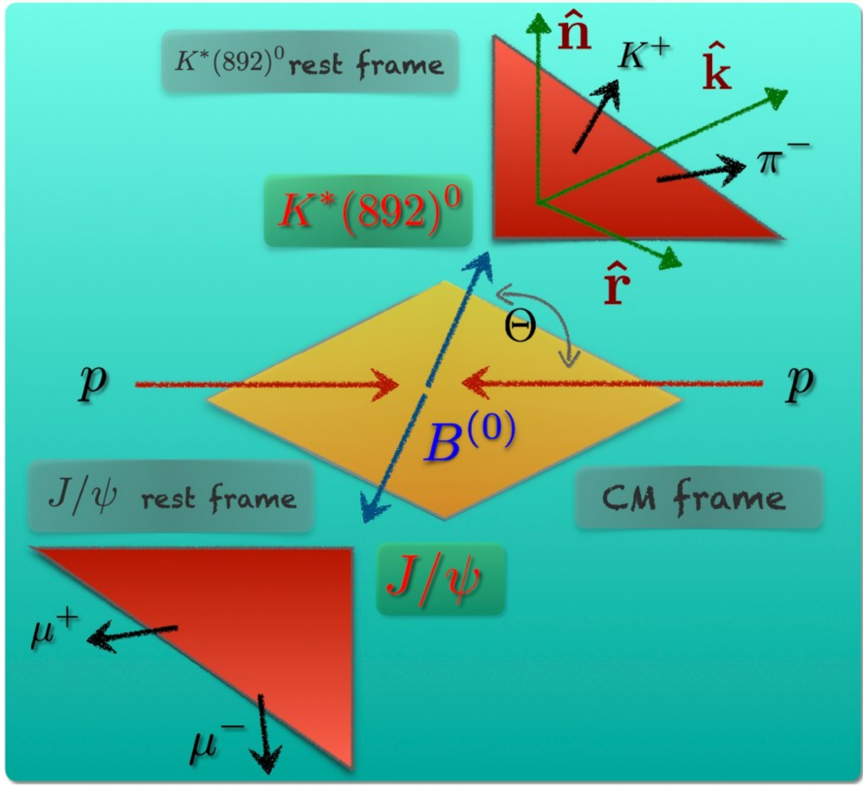
J Naikoo et al, Phys. Rev. D 99 (2019) 095001

2 spin-1 qutrits — B meson decays $B \to M_1 M_2$

$$|\Psi\rangle = \frac{1}{\sqrt{|H|^2}}\Big[h_+\,|\mathbf{V_1}(+)\mathbf{V_2}(-)\rangle + h_0\,|\mathbf{V_1}(0)\mathbf{V_2}(0)\rangle + h_-\,|\mathbf{V_1}(-)\mathbf{V_2}(+)\rangle\Big]$$

h_i = helicity amplitudes

$$|H|^2 = |h_0|^2 + |h_+|^2 + |h_-|^2$$



$$\rho = |\Psi\rangle\langle\Psi| = \frac{1}{|H|^2}\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_+h_+^* & 0 & h_+h_0^* & 0 & h_+h_-^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_0h_+^* & 0 & h_0h_0^* & 0 & h_0h_-^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_-h_+^* & 0 & h_-h_0^* & 0 & h_-h_-^* & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Parameter	Result
$\lvert A_0\rvert^2$	$0.384 \pm 0.007 \pm 0.003$
$\lvert A_\perp\rvert^2$	$0.310 \pm 0.006 \pm 0.003$
$\delta_\parallel$ [rad]	$2.463 \pm 0.029 \pm 0.009$
$\delta_\perp$ [rad]	$2.769 \pm 0.105 \pm 0.011$

	$\lvert A_0\rvert^2$	$\lvert A_\perp\rvert^2$	$\delta_\parallel$	$\delta_\perp$
$\lvert A_0\rvert^2$	1	−0.342	−0.007	0.064
$\lvert A_\perp\rvert^2$		1	0.140	0.088
$\delta_\parallel$			1	0.179
$\delta_\perp$				1

R. Aaij *et al.* [LHCb], Phys. Rev. Lett. **131**, no.17, 171802 (2023) [arXiv:2304.06198 [hep-ex]].

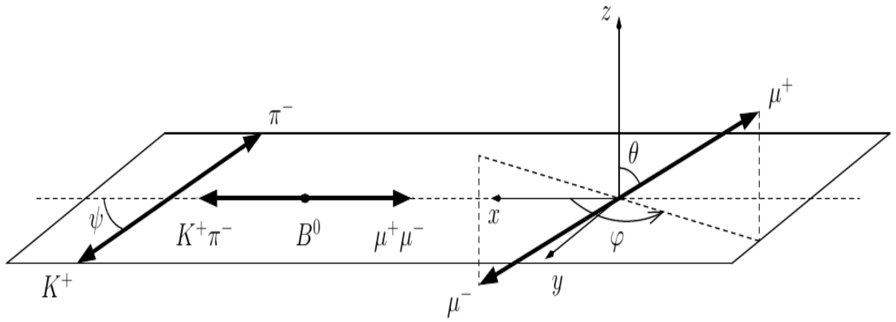
$$\frac{h_0}{|H|} = A_0\,, \quad \frac{h_+}{|H|} = \frac{A_\parallel + A_\perp}{\sqrt{2}} \quad \text{and} \quad \frac{h_-}{|H|} = \frac{A_\parallel - A_\perp}{\sqrt{2}}$$

M. Fabbrichesi, R. Floreanini, EG, L. Marzola, Phys. Rev. D 109 (2024) 3, L031104
EG and L. Marzola, Symmetry 6 (2024) 8, 1036

FIT of coefficients h_k

$$\frac{d^4\Gamma(B^0 \to J/\psi K^{*0})}{dt\, d\Omega} \propto e^{-\Gamma_d t} \sum_{k=1}^{10} h_k f_k(\Omega)$$

$$\Omega = \{\cos\theta, \cos\psi, \varphi\}$$



k	h_k	$f_k(\Omega)$
1	$\boxed{\lvert A_0\rvert^2}$	$\frac{9}{32\pi} 2\cos^2\psi(1-\sin^2\theta\cos^2\varphi)$
2	$\boxed{\lvert A_\parallel\rvert^2}$	$\frac{9}{32\pi}\sin^2\psi(1-\sin^2\theta\sin^2\varphi)$
3	$\boxed{\lvert A_\perp\rvert^2}$	$\frac{9}{32\pi}\sin^2\psi\sin^2\theta$
4	$\boxed{\lvert A_\parallel\rvert\lvert A_\perp\rvert\sin(\delta_\perp-\delta_\parallel)}$	$-\frac{9}{32\pi}\sin^2\psi\sin 2\theta\sin\varphi$
5	$\boxed{\lvert A_0\rvert\lvert A_\parallel\rvert\cos(\delta_\parallel)}$	$\frac{9}{32\pi\sqrt{2}}\sin 2\psi\sin^2\theta\sin 2\varphi$
6	$\boxed{\lvert A_0\rvert\lvert A_\perp\rvert\sin(\delta_\perp)}$	$\frac{9}{32\pi\sqrt{2}}\sin 2\psi\sin 2\theta\cos\varphi$
7	$\lvert A_S\rvert^2$	$\frac{3}{32\pi}2(1-\sin^2\theta\cos^2\varphi)$
8	$\lvert A_\parallel\rvert\lvert A_S\rvert\cos(\delta_\parallel-\delta_S)$	$\frac{3}{32\pi}\sqrt{6}\sin\psi\sin^2\theta\sin 2\varphi$
9	$\lvert A_\perp\rvert\lvert A_S\rvert\sin(\delta_\perp-\delta_S)$	$\frac{3}{32\pi}\sqrt{6}\sin\psi\sin 2\theta\cos\varphi$
10	$\lvert A_0\rvert\lvert A_S\rvert\cos(\delta_S)$	$\frac{3}{32\pi}4\sqrt{3}\cos\psi(1-\sin^2\theta\cos^2\varphi)$

(A_S contribution from non-resonant J/Y K* amplitude)

2

B meson decays

	Entanglement $\mathscr{E}$	Bell inequality $\mathcal{I}_3$	Significance of Bell inequality violation
• $B^0 \to J/\psi\, K^*(892)^0$ [5]	0.756 ± 0.009	2.548 ± 0.015	36σ
• $B^0 \to \phi\, K^*(892)^0$ [20]	$0.707 \pm 0.133^*$	$2.417 \pm 0.368^*$	
• $B^0 \to \rho\, K^*(892)^0$ [21]	$0.450 \pm 0.067^*$	$2.208 \pm 0.129^*$	
• $B_s \to \phi\,\phi$ [22]	$0.734 \pm 0.050^*$	$2.525 \pm 0.084^*$	8.2σ
• $B_s \to J/\psi\,\phi$ [23]	0.731 ± 0.032	2.462 ± 0.080	

Free from locality loophole

M. Fabbrichesi, R. Floreanini, EG, L. Marzola, Phys. Rev. D 109 (2024) 3, L031104
EG and L. Marzola, Symmetry 6 (2024) 8, 1036

K. Chen et al, Eur. Phys. J. C 84 (2024) 580 → $B_c^\pm \to J/\psi\, \rho^\pm$

3 Entanglement in pairs of top quarks

$$D = \frac{1}{3}\,\mathrm{Tr}\,C_{ij} \qquad \mathscr{C}[\rho] = \max[-1 - 3D,\, 0]/2$$

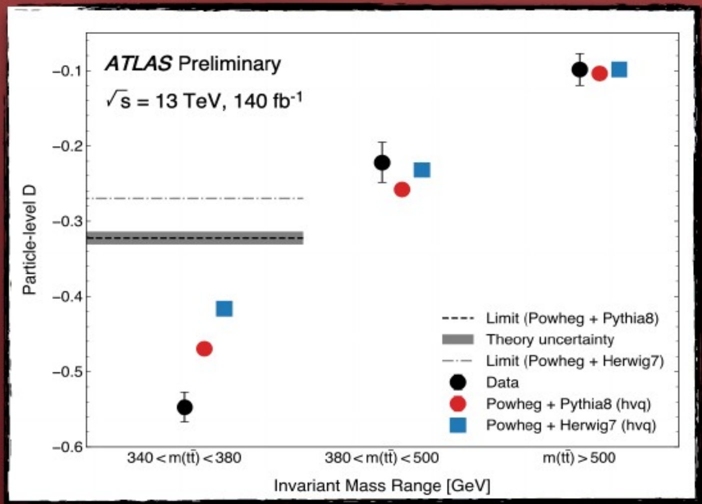
D < -1/3 sufficient condition for entanglement

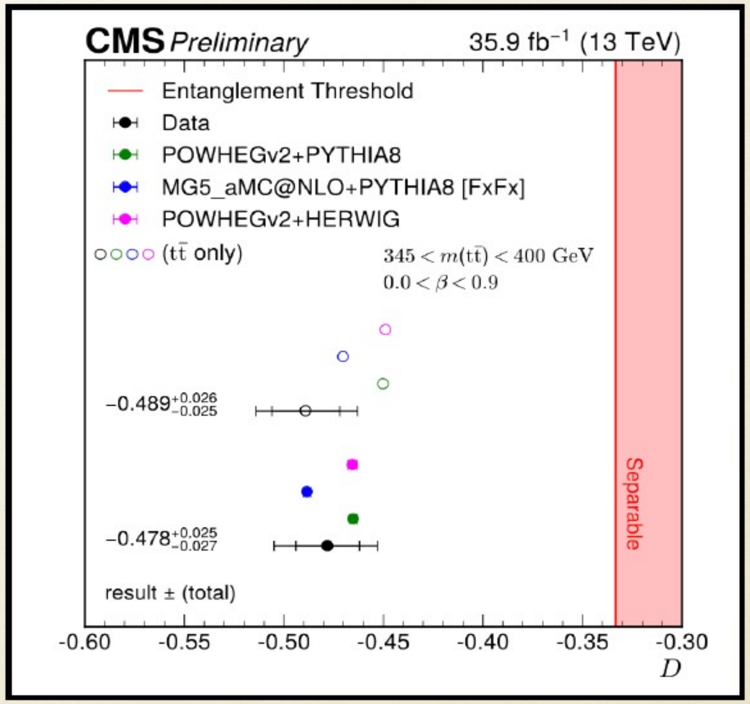
→ also sensitive to Toponium formation

Y. Afik and J.R.M. de Nova, Eur. Phys. J. Plus **136** (2021) 907

$$pp \to t + \bar{t} \to \ell^{\pm}\ell^{\mp} + \text{jets} + E_T^{\text{miss}}$$

$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\phi} = \frac{1}{2}\left(1 - D\cos\phi\right)$$





Significance > 5σ ✔

$$D = -0.478^{+0.025}_{-0.027}$$

$$D = -0.547 \pm 0.002\ [\text{stat}] \pm 0.021\ [\text{syst}]$$

ATLAS Collaboration, Nature 633 (2024) 542

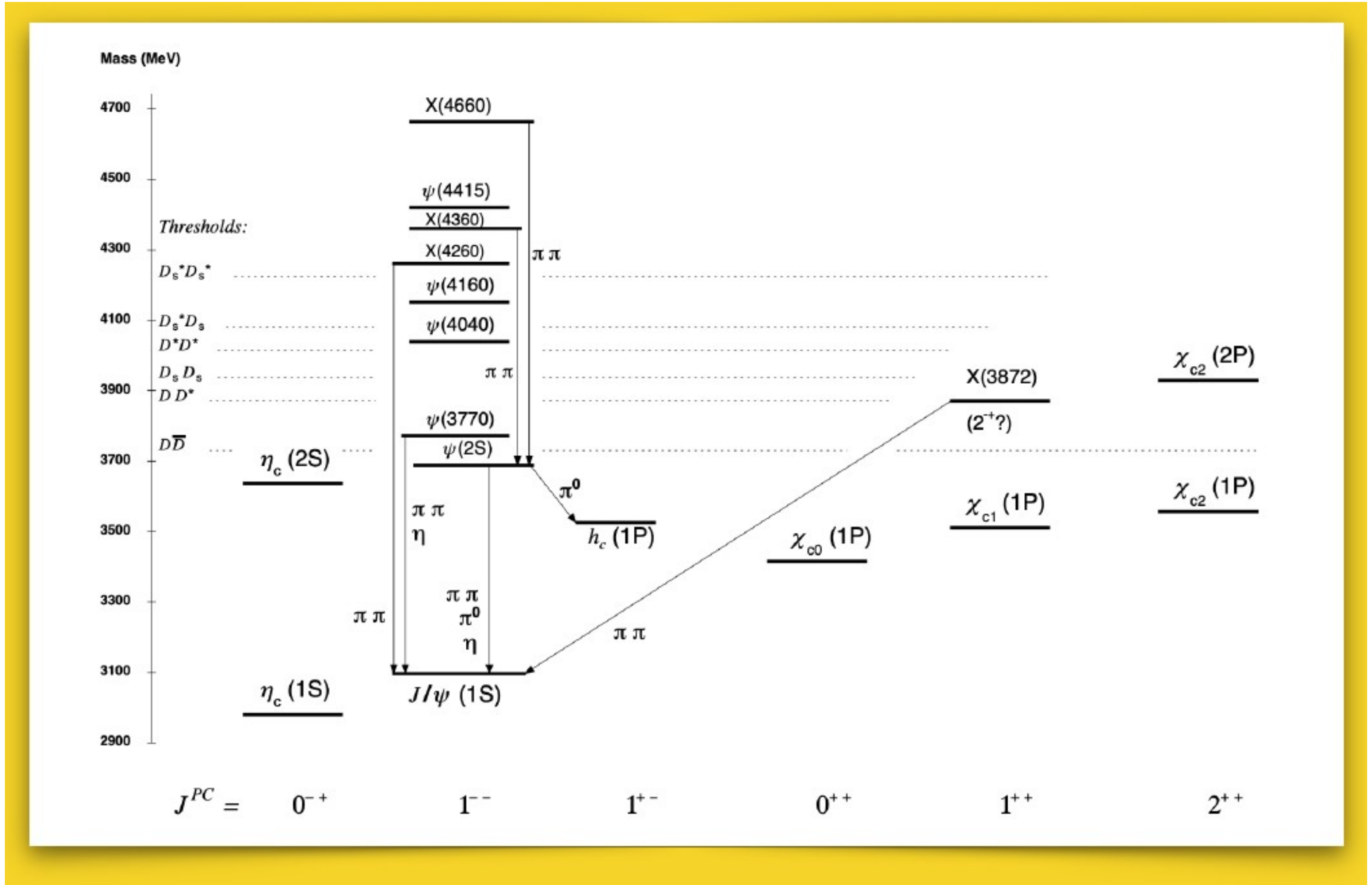
CMS Collaboration, arXiv:2406.03976 (2024)
CMS Collaboration, arXiv:2409.11067 (2024)

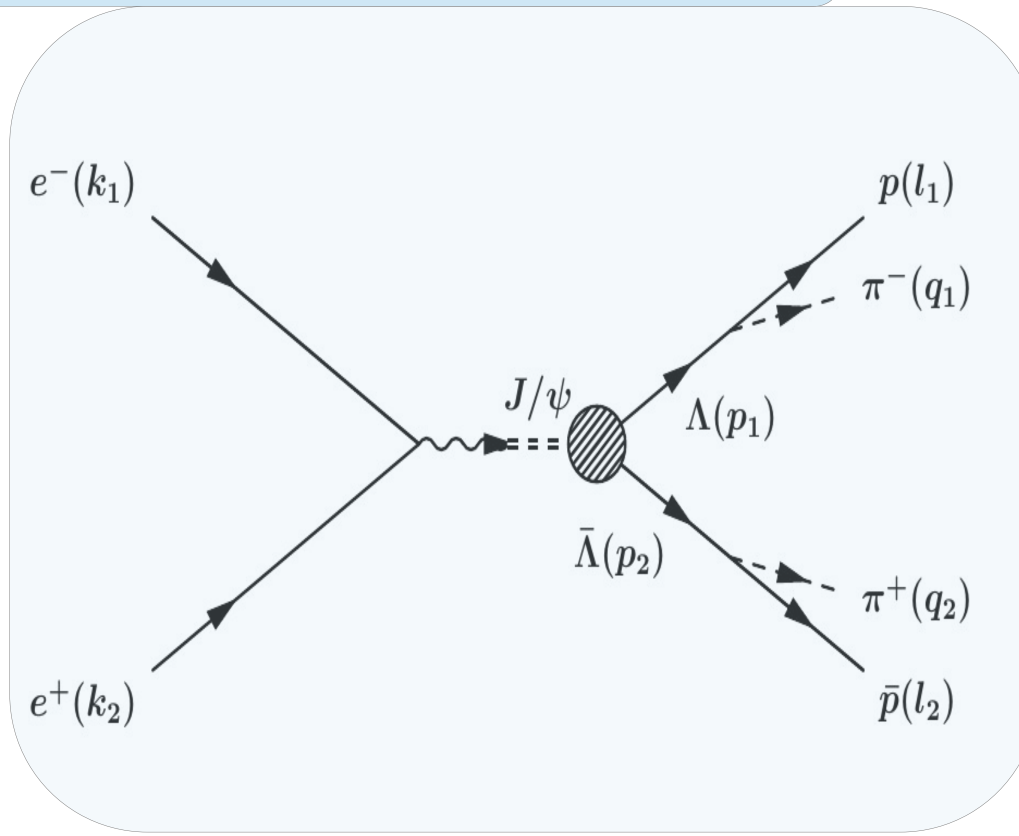
ϕ is the angle between the respective leptons as computed in the rest frame of the decaying top and anti-top

$$340\,\text{GeV} < m_{t\bar{t}} < 380\,\text{GeV}$$

4 qubits, qutrits

Charmonium





$$\mathbf{p}_1 = -\mathbf{p}_2 = \mathbf{p} \qquad \mathbf{k}_1 = -\mathbf{k}_2 = \mathbf{k}$$

$$\cos\Theta = \hat{\mathbf{p}} \cdot \hat{\mathbf{k}}$$

scattering angle

helicity amplitudes decomposition

$$\rho_{\lambda_1\lambda_2,\lambda_1'\lambda_2'} \propto w_{\lambda_1\lambda_2} w^*_{\lambda_1'\lambda_2'} \sum_k D^{(J)*}_{k,\lambda_1-\lambda_2}(0,\Theta,0) D^{(J)}_{k,\lambda_1'-\lambda_2'}(0,\Theta,0)$$

Wigner rotation D-matrix

M. Fabbrichesi, R. Floreanini, EG, L. Marzola, Phys. Rev. D 110 (2024) 3, 053008
see, also: S. Wu et al. Phys Rev. D110 (2024) 054012

Qubits (spin ½)

$$\Lambda(\rightarrow p\pi^-)\bar{\Lambda}(\rightarrow \bar{p}\pi^+)$$

$$\mathrm{d}\sigma \propto \mathcal{W}(\boldsymbol{\xi})\,\mathrm{dcos}\,\theta\,\mathrm{d}\Omega_1\mathrm{d}\Omega_2$$

$$\boldsymbol{\xi} = (\theta, \Omega_1, \Omega_2)$$

$$\begin{aligned}
\mathcal{F}_0(\boldsymbol{\xi}) &= 1\\
\mathcal{F}_1(\boldsymbol{\xi}) &= \sin^2\theta \sin\theta_1 \sin\theta_2 \cos\phi_1 \cos\phi_2 + \cos^2\theta \cos\theta_1 \cos\theta_2\\
\mathcal{F}_2(\boldsymbol{\xi}) &= \sin\theta \cos\theta (\sin\theta_1 \cos\theta_2 \cos\phi_1 + \cos\theta_1 \sin\theta_2 \cos\phi_2)\\
\mathcal{F}_3(\boldsymbol{\xi}) &= \sin\theta \cos\theta \sin\theta_1 \sin\phi_1\\
\mathcal{F}_4(\boldsymbol{\xi}) &= \sin\theta \cos\theta \sin\theta_2 \sin\phi_2\\
\mathcal{F}_5(\boldsymbol{\xi}) &= \cos^2\theta\\
\mathcal{F}_6(\boldsymbol{\xi}) &= \cos\theta_1 \cos\theta_2 - \sin^2\theta \sin\theta_1 \sin\theta_2 \sin\phi_1 \sin\phi_2. \qquad (6.56)
\end{aligned}$$

$$\begin{aligned}
\mathcal{W}(\boldsymbol{\xi}) =& \mathcal{F}_0(\boldsymbol{\xi}) + \alpha\mathcal{F}_5(\boldsymbol{\xi})\\
&+\alpha_1\alpha_2\left(\mathcal{F}_1(\boldsymbol{\xi}) + \sqrt{1-\alpha^2}\cos(\Delta\Phi)\mathcal{F}_2(\boldsymbol{\xi}) + \alpha\mathcal{F}_6(\boldsymbol{\xi})\right)\\
&+\sqrt{1-\alpha^2}\sin(\Delta\Phi)\left(\alpha_1\mathcal{F}_3(\boldsymbol{\xi}) + \alpha_2\mathcal{F}_4(\boldsymbol{\xi})\right),
\end{aligned}$$

$$w_{\frac{1}{2}\,\frac{1}{2}} = w_{-\frac{1}{2}\,-\frac{1}{2}} = \frac{\sqrt{1-\alpha}}{\sqrt{2}}$$

$$w_{\frac{1}{2}\,-\frac{1}{2}} = w_{-\frac{1}{2}\,\frac{1}{2}} = \sqrt{1+\alpha}\,\exp[-i\Delta\Phi]$$

maximum likelihood fit

$$\alpha = 0.4748 \pm 0.0022|_{\mathrm{stat}} \pm 0.0031|_{\mathrm{syst}}$$

$$\Delta\Phi = 0.7521 \pm 0.0042|_{\mathrm{stat}} \pm 0.0066|_{\mathrm{syst}}$$

to extract helicity amplitudes

$$\Theta \equiv \theta$$

$$\rho_{\lambda_1\lambda_2,\lambda_1'\lambda_2'} \propto \boxed{w_{\lambda_1\lambda_2} w^*_{\lambda_1'\lambda_2'}} \sum_k D^{(J)*}_{k,\lambda_1-\lambda_2}(0,\Theta,0)\, D^{(J)}_{k,\lambda_1'-\lambda_2'}(0,\Theta,0)$$

Charmonium spin-0 states

Qubits (spin ½)

$$\eta_c \to \Lambda + \bar\Lambda \quad \textbf{and} \quad \chi_c^0 \to \Lambda + \bar\Lambda$$

$$|\psi_0\rangle \propto w_{\frac{1}{2}\,-\frac{1}{2}}\,|\tfrac{1}{2},\tfrac{1}{2}\rangle \otimes |\tfrac{1}{2},-\tfrac{1}{2}\rangle + w_{-\frac{1}{2}\,\frac{1}{2}}\,|\tfrac{1}{2},-\tfrac{1}{2}\rangle \otimes |\tfrac{1}{2},\tfrac{1}{2}\rangle$$

$$\rho_{\Lambda\,\Lambda} = |\psi_0\rangle\langle\psi_0| = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & \pm 1 & 0 \\ 0 & \pm 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Concurrence **Horodecki condition**

$$\mathscr{C}[\rho] = 1 \qquad \mathfrak{m}_{12} = 2$$

- maximum violation of Bell inequality
- data not yet available to assess significance

N.A. Tornqvist, Phys. 11 (1981) 171-177
N.A. Tornqvist, Phys. Lett. A 117 (1986) 1 4
S.P. Baranov, Phys. G 35 (2008) 075002

Qutrits (spin 1)

$$\chi_c^0 \to \phi + \phi$$

$$|\Psi\rangle = w_{-1\,-1}\,|-1,\,-1\rangle + w_{0\,0}\,|0\,0\rangle + w_{1\,1}\,|1,\,1\rangle$$

$$\left|\frac{w_{1,1}}{w_{0\,0}}\right| = 0.299 \pm 0.003|_{\text{stat}} \pm 0.019|_{\text{syst}}$$

BesIII Collaboration, JHEP 05 (2023) 069 [arXiv:2301.12922]

$$\rho_{\phi\phi} = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & |w_{-1\,-1}|^2 & 0 & w_{-1\,-1}w^*_{0\,0} & 0 & w_{-1\,-1}w^*_{1\,1} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & w_{0\,0}w^*_{-1,-1} & 0 & |w_{0\,0}|^2 & 0 & w_{0\,0}w^*_{1\,1} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & w_{1\,1}w^*_{-1\,-1} & 0 & w_{1\,1}w^*_{0\,0} & 0 & |w_{1\,1}|^2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

Entropy

$$\mathscr{E}[\rho] = 0.531 \pm 0.040 \qquad (13{,}3\ \sigma)$$

CGLMP $\mathcal{I}_3$

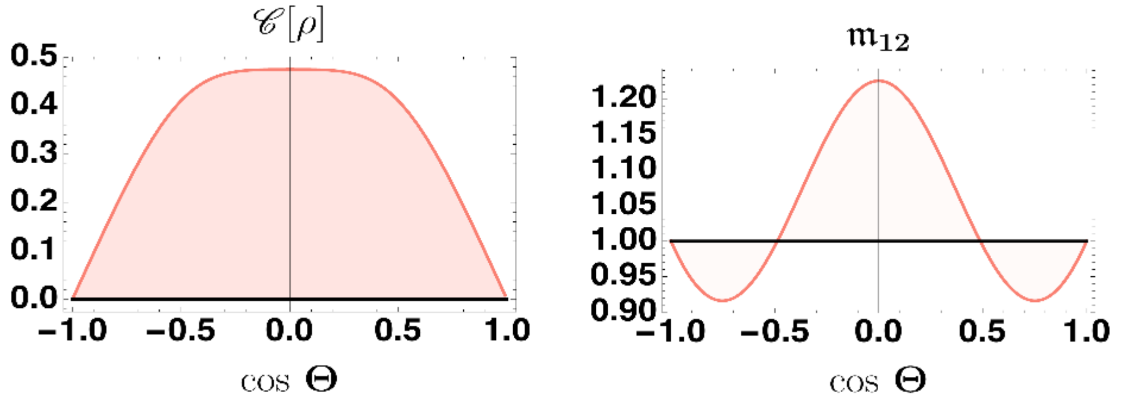
$$\text{Tr}\,\rho_{\phi\phi}\,\mathscr{B} = 2.296 \pm 0.034 \quad (8{,}8\ \sigma)$$

✔

Charmonium spin-1 states

Qubits (spin ½)

$$J/\psi \to \Lambda + \bar{\Lambda} \quad \textbf{and} \quad \psi(3686) \to \Lambda + \bar{\Lambda}$$



$$|\psi_\uparrow\rangle \propto w_{\frac{1}{2}\frac{1}{2}} |\tfrac{1}{2}\,\tfrac{1}{2}\rangle \otimes |\tfrac{1}{2}\,\tfrac{1}{2}\rangle$$

$$|\psi_\downarrow\rangle \propto w_{-\frac{1}{2}-\frac{1}{2}} |\tfrac{1}{2}\,-\tfrac{1}{2}\rangle \otimes |\tfrac{1}{2}\,-\tfrac{1}{2}\rangle$$

$$|\psi_0\rangle \propto w_{\frac{1}{2}-\frac{1}{2}} |\tfrac{1}{2}\,\tfrac{1}{2}\rangle \otimes |\tfrac{1}{2}\,-\tfrac{1}{2}\rangle + w_{-\frac{1}{2}\frac{1}{2}} |\tfrac{1}{2}\,-\tfrac{1}{2}\rangle \otimes |\tfrac{1}{2}\,\tfrac{1}{2}\rangle$$

$$w_{\frac{1}{2}\frac{1}{2}} = w_{-\frac{1}{2}-\frac{1}{2}} = \frac{\sqrt{1-\alpha}}{\sqrt{2}} \quad \text{and} \quad w_{\frac{1}{2}-\frac{1}{2}} = w_{-\frac{1}{2}\frac{1}{2}} = \sqrt{1+\alpha}\,\exp[-i\Delta\Phi]$$

$$\alpha = 0.4748 \pm 0.0022|_{\text{stat}} \pm 0.0031|_{\text{syst}}$$

$$\Delta\Phi = 0.7521 \pm 0.0042|_{\text{stat}} \pm 0.0066|_{\text{syst}}$$

BesIII Collaboration, Phys. Rev. Lett 129 (2022) n. 13 131801 [arXiv:2204.11058]

Concurrence

$$\mathscr{C}[\rho] = 0.475 \pm 0.004 \quad (118{,}7\sigma)$$

Horodecki condition

$$\mathfrak{m}_{12} = 1.225 \pm 0.004 \; (56{,}3\sigma)$$

✔

Bell inequality violation

decay	$\mathfrak{m}_{12}$	significance
$J/\psi \to \Lambda\bar{\Lambda}$	1.225 ± 0.004	56.3
$\psi(3686) \to \Lambda\bar{\Lambda}$	1.476 ± 0.100	4.8
$J/\psi \to \Xi^-\bar{\Xi}^+$	1.343 ± 0.018	19.1
$J/\psi \to \Xi^0\bar{\Xi}^0$	1.264 ± 0.017	15.6
$\psi(3686) \to \Xi^-\bar{\Xi}^+$	1.480 ± 0.095	5.1
$\psi(3686) \to \Xi^0\bar{\Xi}^0$	1.442 ± 0.161	2.7
$J/\psi \to \Sigma^-\bar{\Sigma}^+$	1.258 ± 0.007	36.9
$\psi(3686) \to \Sigma^-\bar{\Sigma}^+$	1.465 ± 0.043	10.8
$J/\psi \to \Sigma^0\bar{\Sigma}^0$	1.171 ± 0.007	24.4
$\psi(3686) \to \Sigma^0\bar{\Sigma}^0$	1.663 ± 0.065	10.2

ongoing work

$e^+e^- \to \tau^+\tau^-$

A

Belle II, data already available

Analysis under way

$pp \to t\bar{t}$

B

LHC, data already available

Analysis under way

$pp \to H \to ZZ^*$

C

LHC, data already available

Analysis under way

While waiting — let us see some simulations

Entanglement and Bell inequality violation at Belle II

$$e^+ + e^- \to \tau^- + \tau^+$$

$\sqrt{s} = 10$ GeV at SuperKEKB

$$\mathscr{C}[\rho] = \frac{(s - 4\,m_\tau^2)\sin^2\Theta}{4\,m_\tau^2\sin^2\Theta + s\,(\cos^2\Theta + 1)}$$

$$\rho_{\tau\bar{\tau}} = \lambda\rho^{(+)} + (1-\lambda)\,\rho^{(1)}_{\rm mix} \quad \text{with} \quad \lambda = \frac{\beta_\tau^2}{2-\beta_\tau^2}$$

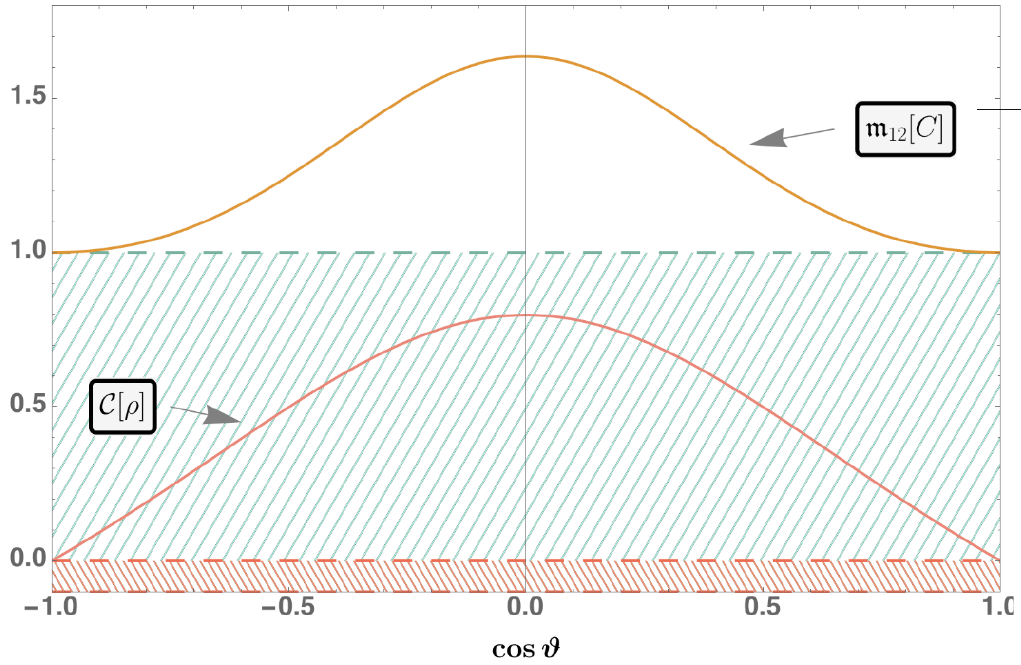
$$\tilde{\rho}^{(2)}_{\rm mix} = \frac{1}{2}\Big(|\mathrm{RR}\rangle\langle\mathrm{RR}| + |\mathrm{LL}\rangle\langle\mathrm{LL}|\Big)$$

$$\tilde{\rho}^{(+)} = |\tilde{\psi}^{(+)}\rangle\langle\tilde{\psi}^{(+)}|\,, \qquad |\tilde{\psi}^{(+)}\rangle = \frac{1}{\sqrt{2}}\Big(|+-\rangle + |-+\rangle\Big)$$

$$\mathfrak{m}_{12} = 1 + \left(\frac{(s - 4\,m_\tau^2)\sin^2\Theta}{4\,m_\tau^2\sin^2\Theta + s\,(\cos^2\Theta + 1)}\right)^2$$

- at threshold $\beta_\tau \simeq 0$ the state is a mixed one, with no quantum correlations
- at relativistic regime $\beta_\tau \to 1$ the state is maximally entangled

Entanglement



Bell inequality violation

$e^+ + e^- \to \tau^- + \tau^+$ Montecarlo simulations for Belle II

Assuming data set of about 200million of events. Analysis based on six decay channels

$$\pi^+\pi^-,\ \pi^\pm\rho^\mp,\ \pi^\pm a_1^\mp,\ \rho^+\rho^-,\ \rho^\pm a_1^\mp\ \ a_1^+ a_1^-$$

Spin orientation reconstructed using the polarimeter vector method

S. Jadach, J. H. Kühn, and Z. Was, "TAUOLA: a library of Monte Carlo programs to simulate decays of polarized tau leptons," *Comput. Phys. Commun.* **64** (1990) 275.

V. Cherepanov and C. Veelken, "The polarimeter vector for $\tau \to 3\pi\nu_\tau$ decays," arXiv:2311.10490 [hep-ex].

Decay channel	$\mathcal{C}[\rho]$	$\mathfrak{m}_{12}[\mathbf{C}]$
$\pi^+\pi^-$	0.7079 ± 0.0071	1.483 ± 0.011
$\pi^\pm\rho^\mp$	0.7113 ± 0.0029	1.482 ± 0.008
$\pi^\pm a_1^\mp$	0.6762 ± 0.0028	1.388 ± 0.009
$\rho^+\rho^-$	0.7111 ± 0.0032	1.495 ± 0.007
$\rho^\pm a_1^\mp$	0.6798 ± 0.0026	1.402 ± 0.008
$a_1^+ a_1^-$	0.6386 ± 0.0060	1.294 ± 0.018
All channels	0.6905 ± 0.0014	1.444 ± 0.004

Events passing selection cuts $|\cos(\vartheta)| < 0.40$

Observation of Quantum entanglement and Bell inequality violation expected with a significance well above 5σ

K. Ehataht, M. Fabbrichesi, L. Marzola, C. Veelken, Phys. Rev. D. 109 (2024) 3, 032005; [arXiv: 2311.17555]

Entanglement at work for New Physics search at Belle II

$$\Gamma^\mu(\tau) = \left[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m_\tau}F_2(q^2) + \frac{\sigma^{\mu\nu}\gamma_5 q_\nu}{2m_\tau}F_3(q^2)\right]$$

→ EM tau-vertex

$$a_\tau = F_2(0) \quad \text{and} \quad d_\tau = \frac{e}{2m_\tau}F_3(0)$$

$$\mathcal{L} = e[\bar\tau\Gamma^\mu\tau]A_\mu$$

- NP can arise from the following 3 contact-interactions (CI) dim. 5 operators

$$\hat{O}_1 = e\frac{c_1}{m_\tau^2}\bar\tau\gamma^\mu\tau D^\nu F_{\mu\nu}$$

$$\hat{O}_2 = e\frac{c_2\, v}{2m_\tau^2}\bar\tau\sigma^{\mu\nu}\tau F_{\mu\nu}$$

$$\hat{O}_3 = e\frac{c_3\, v}{2m_\tau^2}\bar\tau\sigma^{\mu\nu}\gamma_5\tau F_{\mu\nu}$$

$$F_1(q^2) = 1 + c_1\frac{q^2}{m_\tau^2} + \dots$$

$$F_{2,3}(0) = 2\, c_{2,3}\frac{v}{m_\tau}$$

- Three observables $\mathscr{O}_i(a_\tau, d_\tau, c_1)$ employed to constrain NP

$$\mathscr{C}_{odd} = \frac{1}{2}\sum_{\substack{i,j\\ i<j}}\left|\mathrm{C}_{ij} - \mathrm{C}_{ji}\right|$$

Concurrence $\mathscr{C}[\rho]$

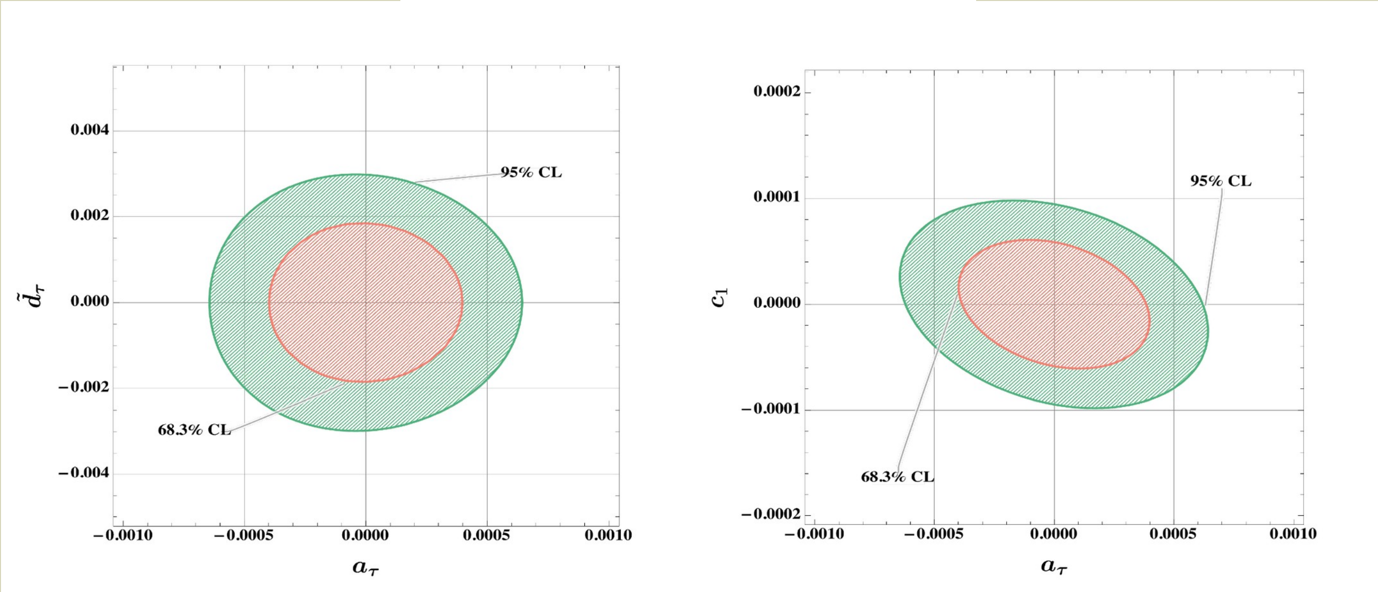
Total cross section

Entanglement at work for New Physics search at Belle II

$$\sum_i \left[\frac{\mathscr{O}_i(a_\tau, d_\tau, c_1) - \mathscr{O}_i(0,0,0)}{\sigma_i}\right]^2 \leq (2.30)\ 5.99$$

χ^2 test with 3 dof

$$e^+e^- \to \tau^+\tau^-$$



benchmark
Luminosity 1ab^{-1}

$$a_\tau^{\rm SM} = 1.17721(5) \times 10^{-3}$$

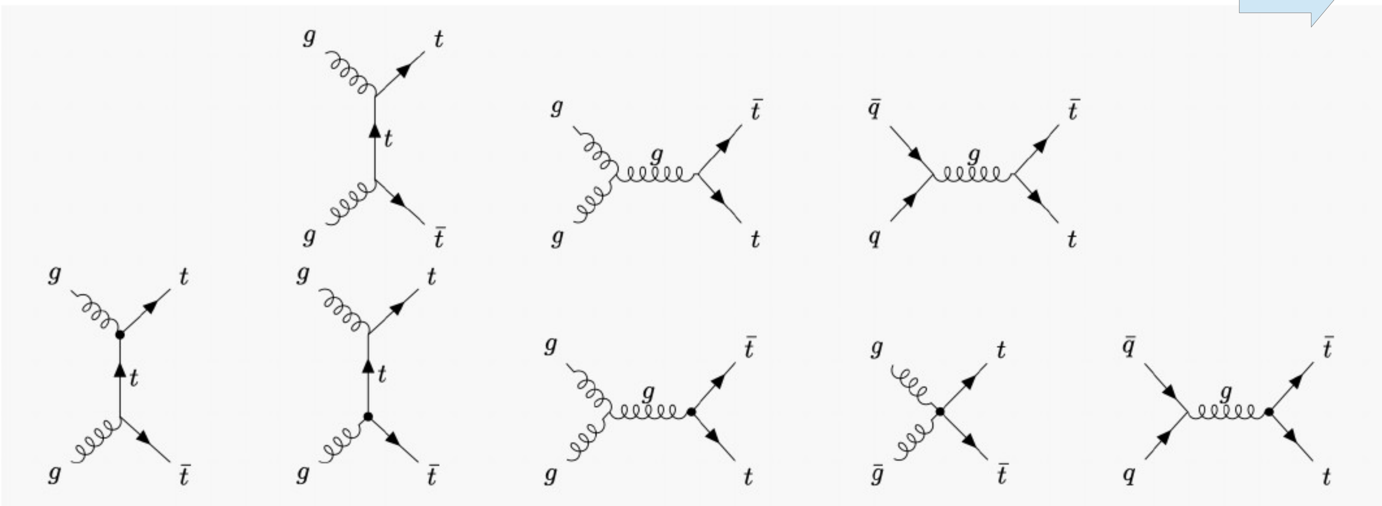
PDG (2022)	*This work*
$-1.9 \times 10^{-17} \leq d_\tau \leq 6.1 \times 10^{-18}$ e cm	$\lvert d_\tau \rvert \leq 1.7 \times 10^{-17}$ e cm
$-5.2 \times 10^{-2} \leq a_\tau \leq 1.3 \times 10^{-2}$	$\lvert a_\tau \rvert \leq 6.3 \times 10^{-4}$
$\Lambda_{\rm C.I.} \geq 7.9$ TeV	$\lvert c_1 \rvert \leq 9.5 \times 10^{-5}$, $\Lambda_{\rm C.I.} \geq 2.6$ TeV

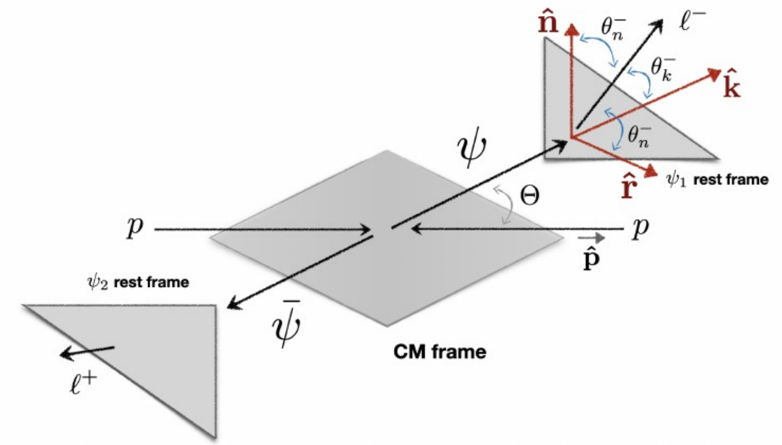
Limits @ 95% C.L.

M. Fabbrichesi, L. Marzola, Phys. Rev. D 109 (2024) 9, 095026; [arXiv:2401.04449]

K. Ehataht M. Fabbrichesi, L. Marzola, C. Veelken, Phys. Rev. D. 109 (2024) 3, 032005; [arXiv: 2311.17555]

Bell inequality violation in top-antitop production at LHC



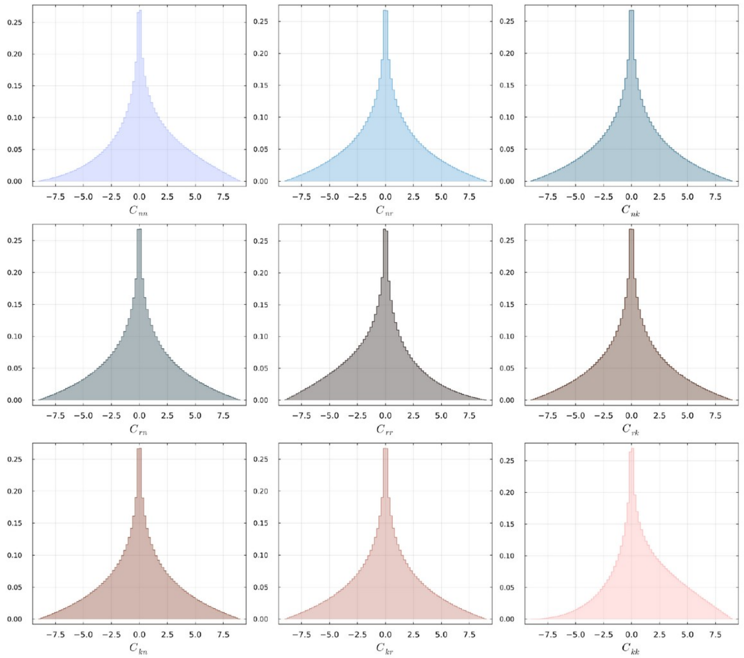


$C_{ij}[m_{t\bar{t}},\ \Theta]$ can be extracted by fitting the double angle distribution

$$\frac{1}{\sigma}\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta_i^+\,\mathrm{d}\cos\theta_j^-} = \frac{1}{4}\left(1 + C_{ij}\ \cos\theta_i^+\ \cos\theta_j^-\right)$$

$$\hat{\mathbf{n}} = \frac{1}{\sin\Theta}\left(\hat{\mathbf{p}} \times \hat{\mathbf{k}}\right)$$

$$\hat{\mathbf{r}} = \frac{1}{\sin\Theta}\left(\hat{\mathbf{p}} - \cos\Theta\hat{\mathbf{k}}\right)$$



a and $b \in \{\mathrm{k}, \mathrm{n}, \mathrm{r}\}$

$$\cos\theta_-^b = \hat{\ell}_- \cdot \hat{b}$$

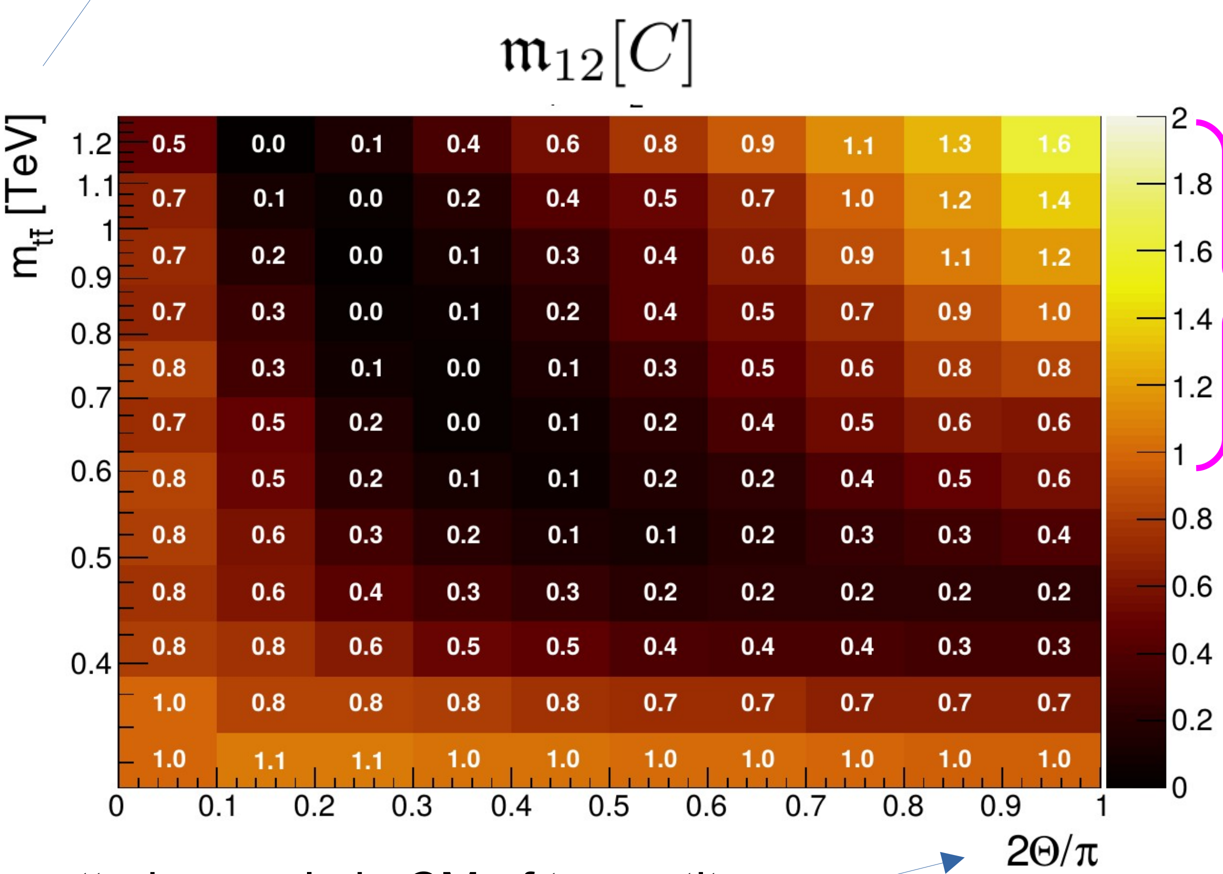
$$\cos\theta_+^a = \hat{\ell}_+ \cdot \hat{a}$$

angles computed in the corresponding rest frame of the decaying top or antitop

Montecarlo simulations

M. Fabbrichesi, R. Floreanini, G. Panizzo, Phys. Rev. Letters 127 (2021), 2102.11883 [hep-ph]

invariant mass of top-antitop



scattering angle in CM of top-antitop

First analysis of Bell inequalities where correlation matrix C_{ij} is extracted from event simulation, including ATLAS detector resolution (DELPHES), acceptance, migration and efficiency effects.

Bell inequality violation
Horodecki condition

$$\mathfrak{m}_{12}[C] > 1$$

Violation of null hypothesis can be assessed:

- at 2 σ level with present Run 2 Luminosity
- at 4 σ with projected full Run 3 Luminosity

$$\mathcal{L}_{\rm dipole} = \frac{c_{tG}}{\Lambda^2}\left(\mathcal{O}_{tG} + \mathcal{O}^{\dagger}_{tG}\right) \quad \text{with} \quad \mathcal{O}_{tG} = g_s\,(\bar{Q}_L\,\sigma^{\mu\nu}\,T^a\,t_R)\,\tilde{H}G^a_{\mu\nu}$$

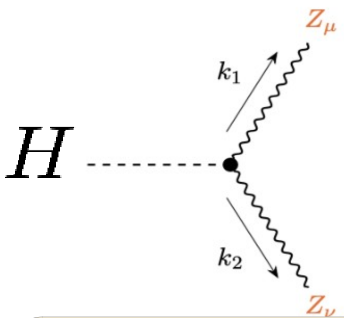
Sensitivity to **NP** (EFT) studied in

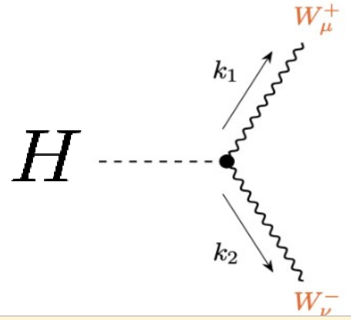
R. Aoude, E. Madge, F. Maltoni, L. Mantani, Phys. Rev. D 106 (2022) 5, 055007; [arXiv:2203.05619]
C. Severi, E. Vryonidou, JHEP 01 (2023) 148; [arXiv:2210.09339]

M. Fabbrichesi, EG, R. Floreanini, EPJC 83 (2023) 2,162; [arXiv:2208.11723]

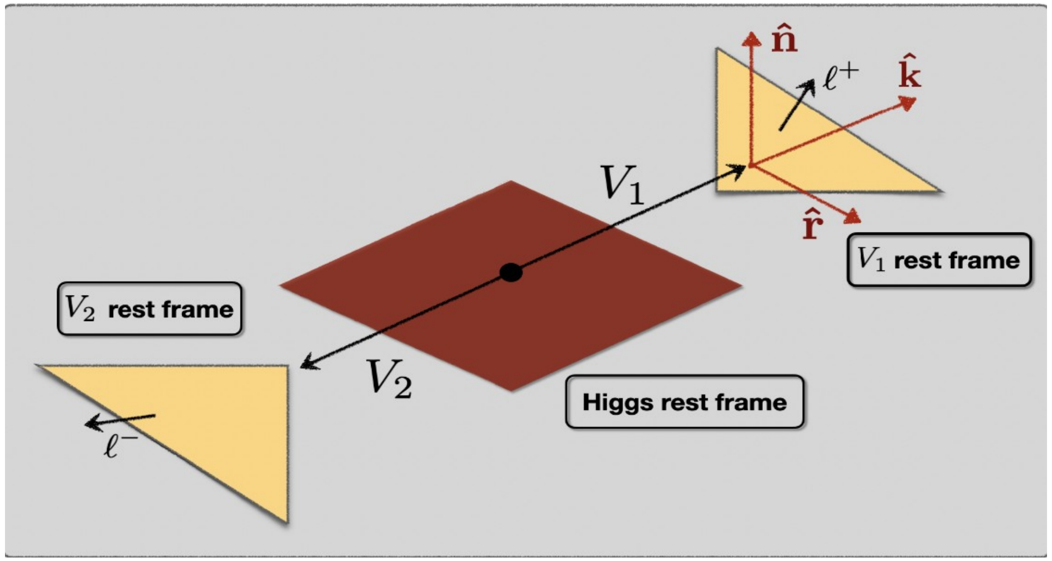
Entanglement and Bell inequality violation in Higgs → ZZ*

$$H \to V(k_1,\lambda_1)\, V^*(k_2,\lambda_2)$$





V=Z,W



$$|\Psi\rangle = \frac{1}{\sqrt{2+\varkappa^2}}\left[|+-\rangle - \varkappa\,|0\,0\rangle + |-+\rangle\right]$$

$$\varkappa = 1 + \frac{m_H^2 - (1+f)^2 M_V^2}{2 f M_V^2}$$

$$M_V^* = f\, M_V$$

maximum entanglement for χ=1 (ZZ* both at rest)

$$\rho_H = 2\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & h_{44} & 0 & h_{16} & 0 & h_{44} & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & h_{16} & 0 & 2\,h_{33} & 0 & h_{16} & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & h_{44} & 0 & h_{16} & 0 & h_{44} & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix},$$

$$\rho_H = |\Psi\rangle\langle\Psi|$$

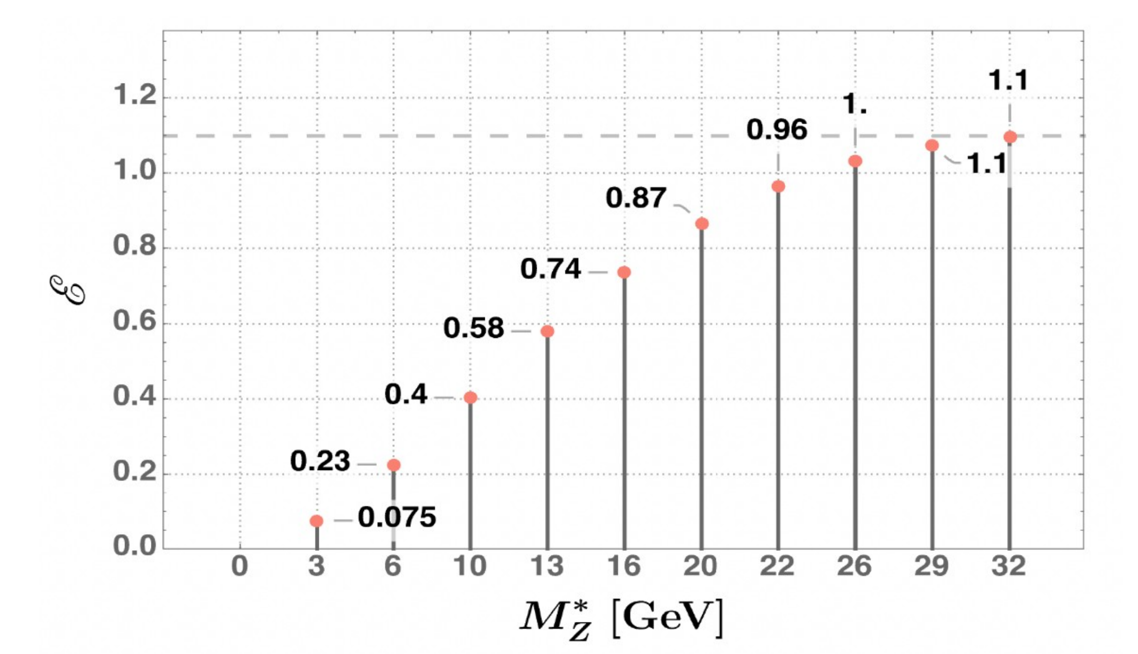
$$\rho_H^2 = \rho_H$$ Pure state

M. Fabbrichesi, EG, R. Floreanini, L. Marzola EPJC 83 (2023) 9,823; [arXiv:2302.00683]

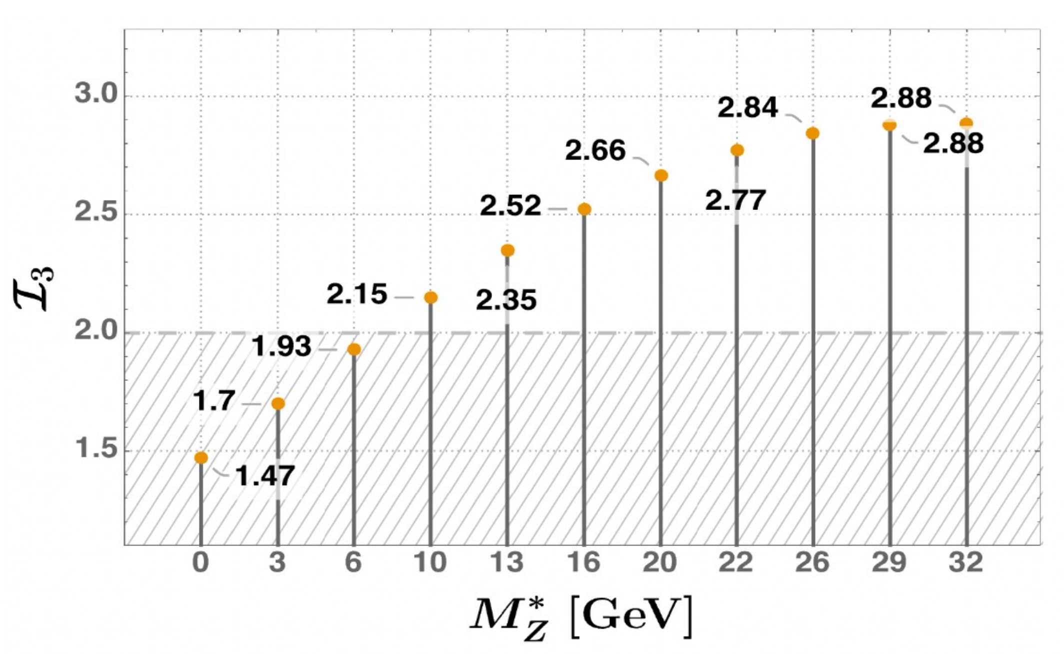
$H \to Z\ Z^*$ SM expectations

M. Fabbrichesi, EG, R. Floreanini, L. Marzola EPJC 83 (2023) 9,823

Quantum entanglement (Entropy)



Bell inequality violation ($\mathcal{I}_3$ >2)



Montecarlo simulation

MadGraph5_aMC@NLO

Luminosity of 3ab^{-1} (Hi-Lumi at LHC)

Expected significance for observing the Bell inequality violation is 4.5σ

J.A. Aguilar-Saavedra, A. Bernal, J.A. Casas, J.M. Moreno, Phys . Rev. D 107 1 (2023) 016012, [arXiv:2209.13441] → tensor basis

R. Ashby-Pickering, A.J. Barr, A. Wierzchucka, JHEP 05 (2023) 020; [arXiv:2209.13990] → Gell-Mann basis

Entanglement at work for Higgs anomalous couplings

$$\mathcal{L}_{HVV} = g\, M_W W^+_\mu W^{-\mu} H + \frac{g}{2\cos\theta_W} M_Z Z_\mu Z^\mu H - \frac{g}{M_W}\left[\frac{a_W}{2} W^+_{\mu\nu} W^{-\mu\nu} + \frac{\widetilde{a}_W}{2} W^+_{\mu\nu} \widetilde{W}^{-\mu\nu} + \frac{a_Z}{4} Z_{\mu\nu} Z^{\mu\nu} + \frac{\widetilde{a}_Z}{4} Z_{\mu\nu} \widetilde{Z}^{\mu\nu}\right] H$$

$$\mathscr{E}_{ent} = -\mathrm{Tr}\left[\rho_A \log \rho_A\right]$$

$$\mathscr{C}_2$$

$$\mathscr{C}_{odd} = \frac{1}{2} \sum_{\substack{a,b \\ a<b}} \left| h_{ab} - h_{ba} \right|$$

→ **3 observables**

$$\sum_i \left[\frac{O_i(a_V, \widetilde{a}_V) - O_i(0,0)}{\sigma_i}\right]^2 \leq 5.991$$

χ^2 test with 3 dof

@95C.L.

dedicated Montecarlo to estimate uncertainties

$$f_{g2} = \frac{\sigma_2}{\sigma} \left|a_V\right|^2 , \quad \text{and} \quad f_{g3} = \frac{\sigma_3}{\sigma} \left|\widetilde{a}_V\right|^2$$

ours

$$f^Z_{g2} < 7.8 \times 10^{-6}, \quad f^Z_{g3} < 1.5 \times 10^{-5}$$

CMS

$$f^V_{g2} < 3.4 \times 10^{-3}, \quad f^V_{g3} < 1.4 \times 10^{-2}$$

@ 95%C.L.

our limits are mostly idealized whereas CMS includes statistical, systematics uncertainties+ background

M. Fabbrichesi, R. Floreanini, EG, L. Marzola, JHEP 09 (2023) 195; [arXiv:2304.02403]

Backup slides

Closing the locality loophole (LL)

- One must consider decays in which the produced particles are identical as in the **B** → ϕ ϕ

  (so their life time is also the same)

we need to check how many events satisfies the space-like condition ⇨

$$\frac{|t_1 - t_2|\, c}{(t_1 + t_2)\, v} < 1$$

$t_{1,2}$ → time of decays

decay times follow the PDF distribution P(t) ~ Exp[- γ β t]

β → the velocity in unit of c
γ → the Lorentz factor

- For **B** → ϕ ϕ we find that almost 90% of events satisfies this condition

- the two bases used in measuring the polarization are arbitrarily chosen (U V diagonalization)

- → provides a set-up where orientations of polarimeters can be freely and arbitrarily chosen

- So locality loophole can be closed!

M. Fabbrichesi, R. Floreanini, EG, L. Marzola, Phys. Rev. D 109 (2024) 3, L031104
EG and L. Marzola, Symmetry 6 (2024) 8, 1036

Closing the detection loophole (DL)

- DL exploits the fact that detectors are not 100% efficient
- Already for qubit the DL is closed if efficiency is more than 80%
- This requirement above is even lower for states belonging to larger Hilbert space as qutrits
- The efficiency of LHCb detector for pion, Kaons, and muons is more than 90%

- So also detection loophole is closed for LHCb !

How to Extract density matrix of Two-Qutrits from data

W W

$$p\,p \to V_1 + V_2 + X \to \ell^+\ell^- + \text{jets} + E_T^{\text{miss}}$$

Differential cross section

$$\frac{1}{\sigma}\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega^+\,\mathrm{d}\Omega^-} = \left(\frac{3}{4\pi}\right)^2 \mathrm{Tr}\left[\rho_{V_1V_2}\left(\Gamma_+ \otimes \Gamma_-\right)\right]$$

depend on the invariant mass m_{VV} (or velocity β) and scattering angle Θ in the V_1V_2 cm frame

R. Rahaman, R.K. Singh, NPB 984 (2022) 115984, [arXiv:2109.09345]

$$\mathrm{d}\Omega^{\pm} = \sin\theta^{\pm}\mathrm{d}\theta^{\pm}\,\mathrm{d}\phi^{\pm} \longrightarrow$$

polar angle $\ell^{\pm}$ azimuthal angle $\ell^{\pm}$

phase space written in terms of the spherical coordinates (with independent polar axis) for the momenta of the final charged leptons in the respective rest frames of the decaying spin-1 particles

$\rho_{V_1V_2}$ = density matrix of V_1V_2

$\Gamma_{\pm}$ $\longrightarrow$ Density matrices that describe the polarization of the two decaying W into final leptons (the charged ones assumed to be massless)

these are projectors in the case of the W-bosons because of their chiral couplings to leptons

R. Ashby-Pickering, A.J. Barr, A. Wierzchucka, JHEP 05 (2023) 020; [arXiv:2209.13990] can be computed by rotating to an arbitrary polar axis the spin states of gauge bosons from the ones given in the **k-direction** quantization axis

$$\Gamma_\pm = \frac{1}{3}\,\mathbb{1} + \sum_{i=1}^{8} \mathfrak{q}_\pm^a\, T^a$$

→ Density matrices for W-bosons

$\mathfrak{q}_\pm^a$ (Wigner q-symbols) are functions of the corresponding spherical coordinates

set of polynomials of spherical coordinates (see backup slide)

$$h_{ab} = \frac{1}{\sigma}\int\int \frac{d\sigma}{d\Omega^+\, d\Omega^-}\,\mathfrak{p}_+^a\,\mathfrak{p}_-^b\, d\Omega^+ d\Omega^-$$

$$f_a = \frac{1}{\sigma}\int \frac{d\sigma}{d\Omega^+}\,\mathfrak{p}_+^a\, d\Omega^+$$

$$g_a = \frac{1}{\sigma}\int \frac{d\sigma}{d\Omega^-}\,\mathfrak{p}_-^a\, d\Omega^-$$

$\mathfrak{p}_\pm^n$ a particular set of orthogonal functions (see next slide)

$$\left(\frac{3}{4\,\pi}\right)\int \mathfrak{p}_\pm^n\,\mathfrak{q}_\pm^m\, d\Omega^\pm = \delta^{nm}$$

For **ZZ** case, the set of functions are linear combinations of $\mathfrak{q}_\pm^a$ → see backup slides

Wigner's *Q symbols*

$$
\begin{aligned}
\mathfrak{q}^1_\pm &= \frac{1}{\sqrt{2}} \sin\theta^\pm \Big(\cos\theta^\pm \pm 1 \Big) \cos\phi^\pm \,, \\
\mathfrak{q}^2_\pm &= \frac{1}{\sqrt{2}} \sin\theta^\pm \Big(\cos\theta^\pm \pm 1 \Big) \sin\phi^\pm \,, \\
\mathfrak{q}^3_\pm &= \frac{1}{8} \Big(1 \pm 4\cos\theta^\pm + 3\cos 2\theta^\pm \Big) \,, \\
\mathfrak{q}^4_\pm &= \frac{1}{2} \sin^2\theta^\pm \cos 2\,\phi^\pm \,, \\
\mathfrak{q}^5_\pm &= \frac{1}{2} \sin^2\theta^\pm \sin 2\,\phi^\pm \,, \\
\mathfrak{q}^6_\pm &= \frac{1}{\sqrt{2}} \sin\theta^\pm \Big(-\cos\theta^\pm \pm 1 \Big) \cos\phi^\pm \,, \\
\mathfrak{q}^7_\pm &= \frac{1}{\sqrt{2}} \sin\theta^\pm \Big(-\cos\theta^\pm \pm 1 \Big) \sin\phi^\pm \,, \\
\mathfrak{q}^8_\pm &= \frac{1}{8\sqrt{3}} \Big(-1 \pm 12\cos\theta^\pm - 3\cos 2\theta^\pm \Big) \,,
\end{aligned}
$$

$$
\begin{aligned}
\mathfrak{p}^1_\pm &= \sqrt{2} \sin\theta^\pm \Big(5\cos\theta^\pm \pm 1 \Big) \cos\phi^\pm \,, \\
\mathfrak{p}^2_\pm &= \sqrt{2} \sin\theta^\pm \Big(5\cos\theta^\pm \pm 1 \Big) \sin\phi^\pm \,, \\
\mathfrak{p}^3_\pm &= \frac{1}{4} \Big(5 \pm 4\cos\theta^\pm + 15\cos 2\theta^\pm \Big) \,, \\
\mathfrak{p}^4_\pm &= 5\sin^2\theta^\pm \cos 2\,\phi^\pm \,, \\
\mathfrak{p}^5_\pm &= 5\sin^2\theta^\pm \sin 2\,\phi^\pm \,, \\
\mathfrak{p}^6_\pm &= \sqrt{2} \sin\theta^\pm \Big(-5\cos\theta^\pm \pm 1 \Big) \cos\phi^\pm \,, \\
\mathfrak{p}^7_\pm &= \sqrt{2} \sin\theta^\pm \Big(-5\cos\theta^\pm \pm 1 \Big) \sin\phi^\pm \,, \\
\mathfrak{p}^8_\pm &= \frac{1}{4\sqrt{3}} \Big(-5 \pm 12\cos\theta^\pm - 15\cos 2\theta^\pm \Big) \,.
\end{aligned}
$$