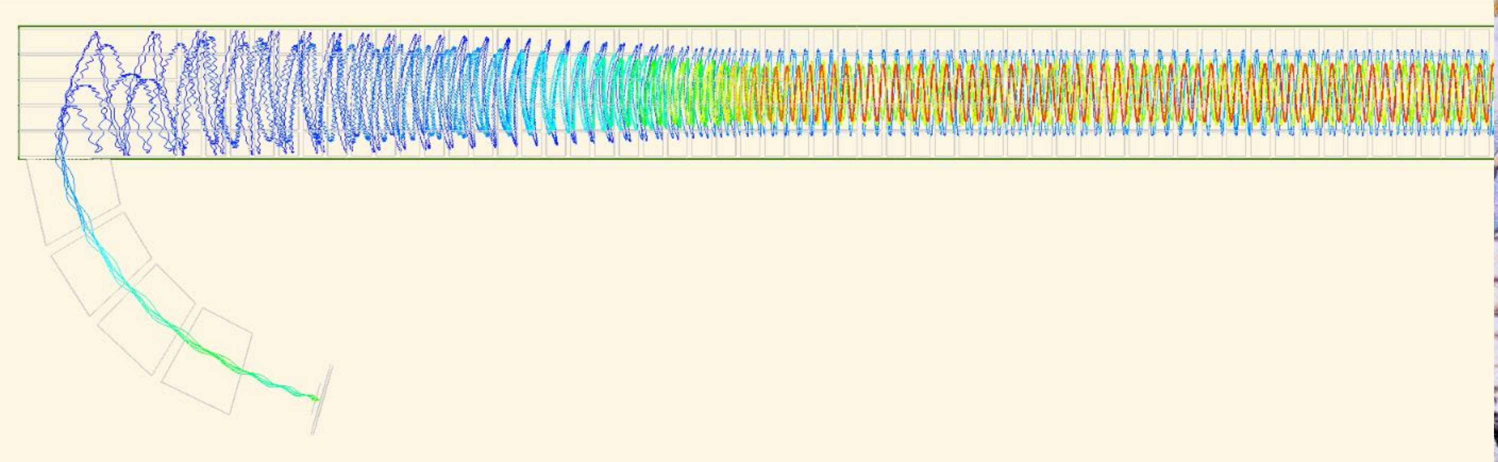
Method of

Magnetic Adiabatic Drift Collimation and Transmission (MADCAT)

for the Determination of the Neutrino Mass
at the Tritium Endpoint



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Genoa

Permanent Magnet Target

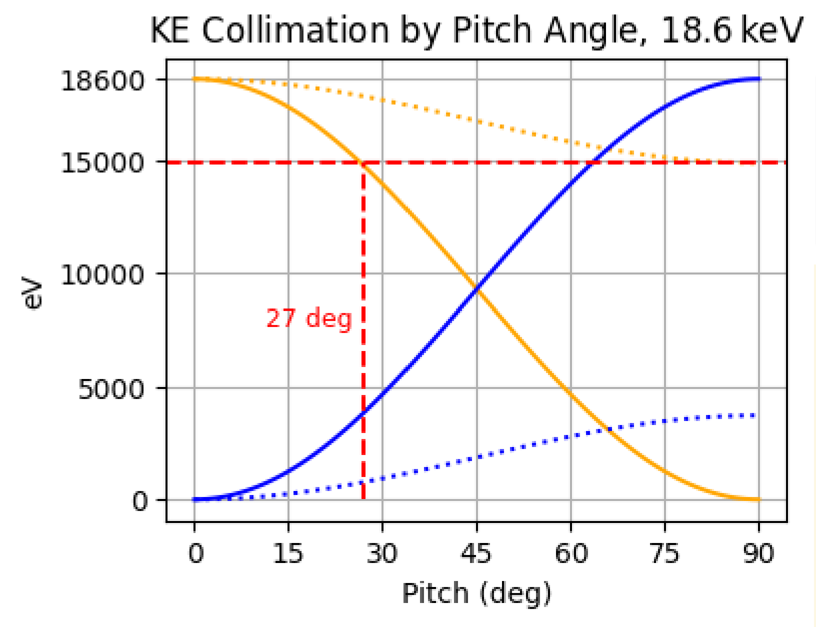
$$\mu = \frac{T_\perp}{B}$$

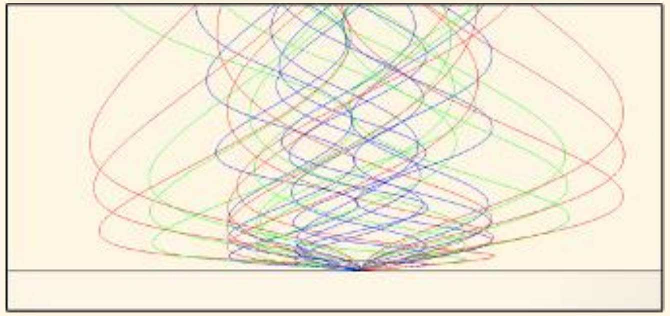
$$\theta_{col} = \theta_{orig} \cdot \frac{B_{ext}}{B_{perm}}$$

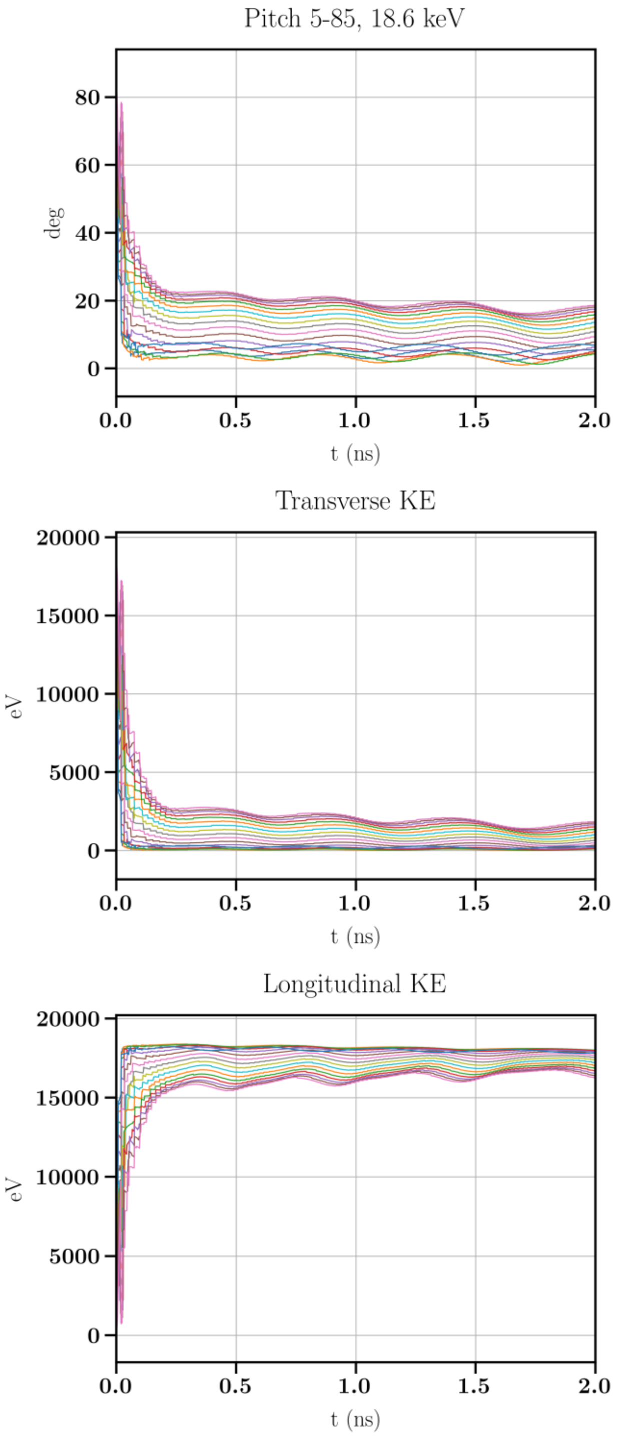
$$B_{ext} \approx 0.1$$

$$B_{perm} \approx 0.5$$

- Unconstrained trajectories follow field lines
- Transverse momentum collimated into longitudinal
  - Linear “collimation factor” in B via adiabatic invariant
- Effectively a pitch range compression (90 → 27)



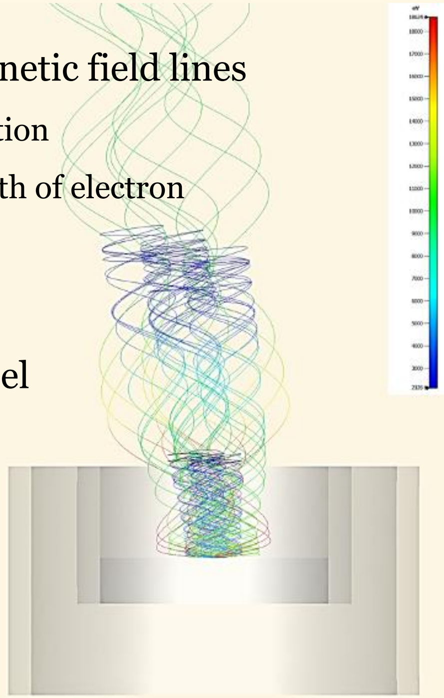


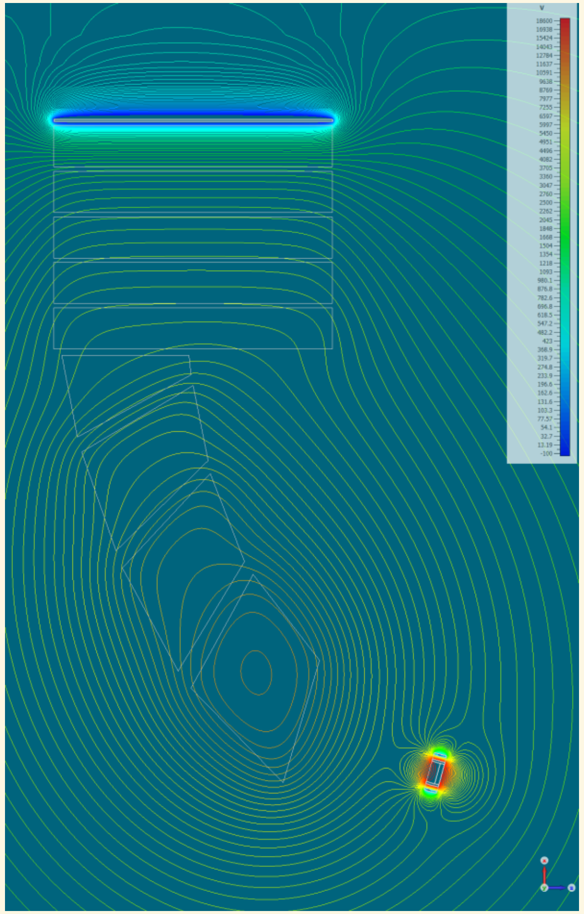


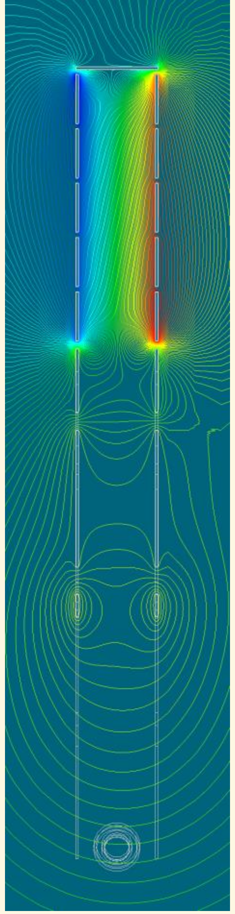
Collimated Injection

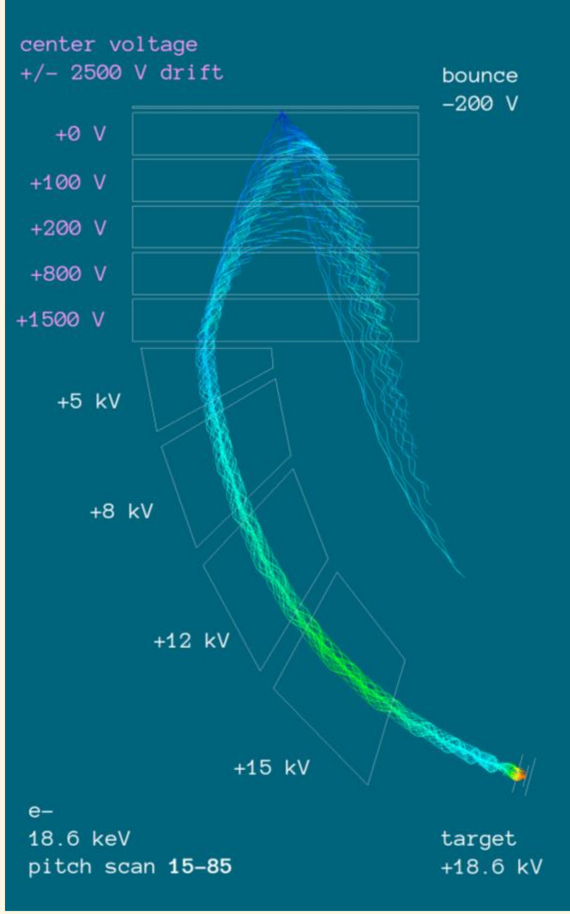
- Shape electrodes to follow the magnetic field lines
  - Well-parameterized by ellipse equation
  - Aligns potential gradient against path of electron
  - Drains parallel momentum

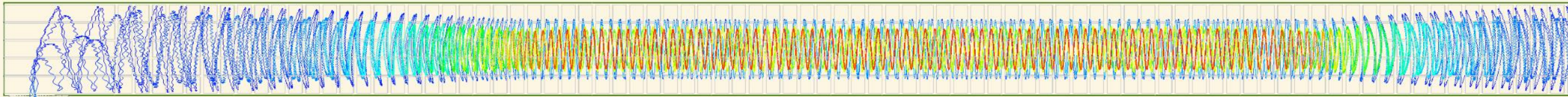
- Once electrons enter central channel
  - Apply fast drift in z
  - Contain bounce motion
  - Accelerate to uniform region



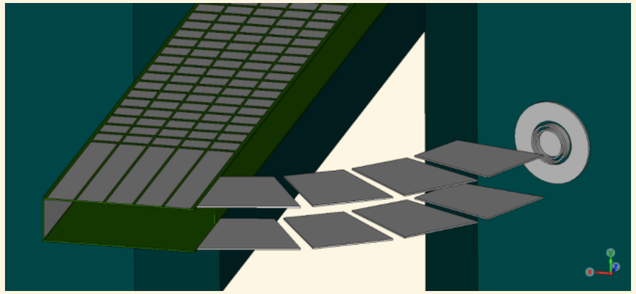


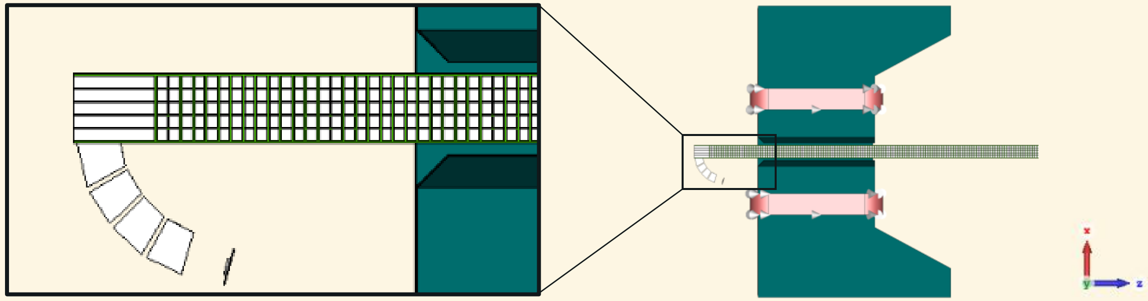






18.6 keV electrons
pitch 5, 25, 45, 65, 85





Method of Adiabatic Drift Collimation

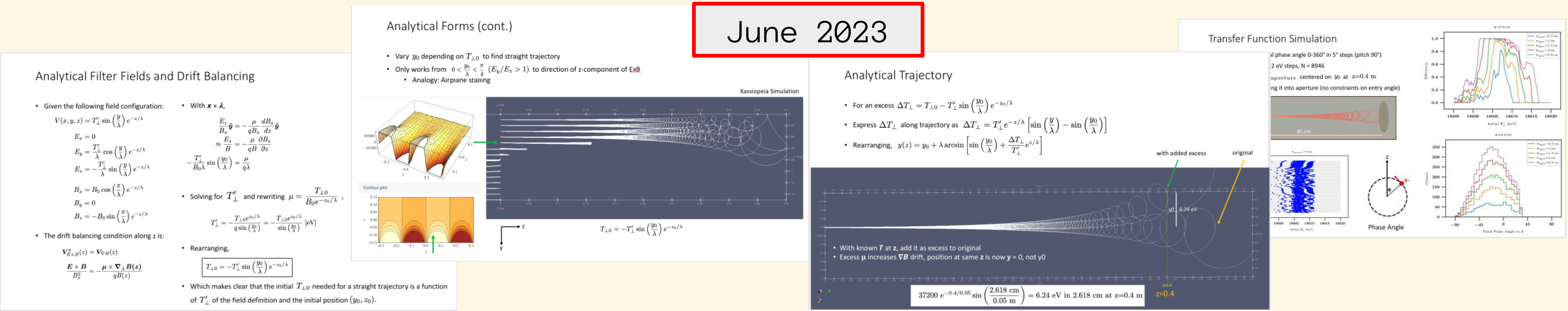
λ_B fixed by magnet, λ_E still variable

Intentionally introduce imbalances at injection and filter entry

Exploit:

- Low-pitch electrons have lower transverse KE
- ➢ <u>Lower grad-B drift (smaller y-excursion) for same z-excursion</u>

- Can be calculated/predicted analytically (see spring 2023 slides)
- ➢ <u>Make trajectories intersect/converge at desired z position across wide pitch range</u>



June 2023

Analytical Filter Fields and Drift Balancing

- Given the following field configuration:

$$V(x,y,z) = T'_\perp \sin\left(\frac{y}{\lambda}\right) e^{-z/\lambda}$$
$$E_x = 0$$
$$E_y = \frac{T'_\perp}{\lambda} \cos\left(\frac{y}{\lambda}\right) e^{-z/\lambda}$$
$$E_z = -\frac{T'_\perp}{\lambda} \sin\left(\frac{y}{\lambda}\right) e^{-z/\lambda}$$
$$B_x = B_0 \cos\left(\frac{x}{\lambda}\right) e^{-z/\lambda}$$
$$B_y = 0$$
$$B_z = -B_0 \sin\left(\frac{x}{\lambda}\right) e^{-z/\lambda}$$

- The drift balancing condition along z is:

$$\boldsymbol{V}^y_{E\times B}(z) = \boldsymbol{V}_{\nabla B}(z)$$
$$\frac{\boldsymbol{E}\times\boldsymbol{B}}{B_x^2} = -\frac{\boldsymbol{\mu}\times\boldsymbol{\nabla}_\perp \boldsymbol{B}(\boldsymbol{z})}{qB(z)}$$

- With ***x* « *λ***,

$$\frac{E_z}{B_x}\hat{\boldsymbol{y}} = -\frac{\mu}{qB_x}\frac{dB_x}{dz}\hat{\boldsymbol{y}}$$
$$\approx \frac{E_z}{B} = -\frac{\mu}{qB}\frac{\partial B_x}{\partial z}$$
$$-\frac{T'_\perp}{B_0\lambda}\sin\left(\frac{y_0}{\lambda}\right) = \frac{\mu}{q\lambda}$$

- Solving for $T'_\perp$ and rewriting $\mu = \frac{T_{\perp 0}}{B_0 e^{-z_0/\lambda}}$,

$$T'_\perp = -\frac{T_{\perp 0} e^{z_0/\lambda}}{q\sin\left(\frac{y_0}{\lambda}\right)} = -\frac{T_{\perp 0} e^{z_0/\lambda}}{\sin\left(\frac{y_0}{\lambda}\right)} \text{ [eV]}$$

- Rearranging,

$$\boxed{T_{\perp 0} = -T'_\perp \sin\left(\frac{y_0}{\lambda}\right) e^{-z_0/\lambda}}$$

- Which makes clear that the initial $T_{\perp 0}$ needed for a straight trajectory is a function of $T'_\perp$ of the field definition and the initial position (y_0, z_0).

Analytical Forms (cont.)

- Vary y_0 depending on $T_{\perp 0}$ to find straight trajectory
- Only works from $0 < \frac{y_0}{\lambda} < \frac{\pi}{4}$ $(E_y/E_z > 1)$ to direction of z-component of ExB
  - Analogy: Airplane stalling

Contour plot

Kassiopeia Simulation

$$T_{\perp 0} = -T'_\perp \sin\left(\frac{y_0}{\lambda}\right) e^{-z_0/\lambda}$$

Analytical Trajectory

- For an excess $\Delta T_\perp = T_{\perp 0} - T'_\perp \sin\left(\frac{y_0}{\lambda}\right) e^{-z_0/\lambda}$
- Express $\Delta T_\perp$ along trajectory as $\Delta T_\perp = T'_\perp e^{-z/\lambda}\left[\sin\left(\frac{y}{\lambda}\right) - \sin\left(\frac{y_0}{\lambda}\right)\right]$
- Rearranging, $y(z) = y_0 + \lambda \arcsin\left[\sin\left(\frac{y_0}{\lambda}\right) + \frac{\Delta T_\perp}{T'_\perp} e^{z/\lambda}\right]$

with added excess

original

- With known ***T*** at **z**, add it as excess to original
- Excess **μ** increases **∇*B*** drift, position at same **z** is now **y** = 0, not y0

$$37200\ e^{-0.4/0.05} \sin\left(\frac{2.618\text{ cm}}{0.05\text{ m}}\right) = 6.24\text{ eV in }2.618\text{ cm at z=0.4 m}$$

Transfer Function Simulation

l phase angle 0-360° in 5° steps (pitch 90°)

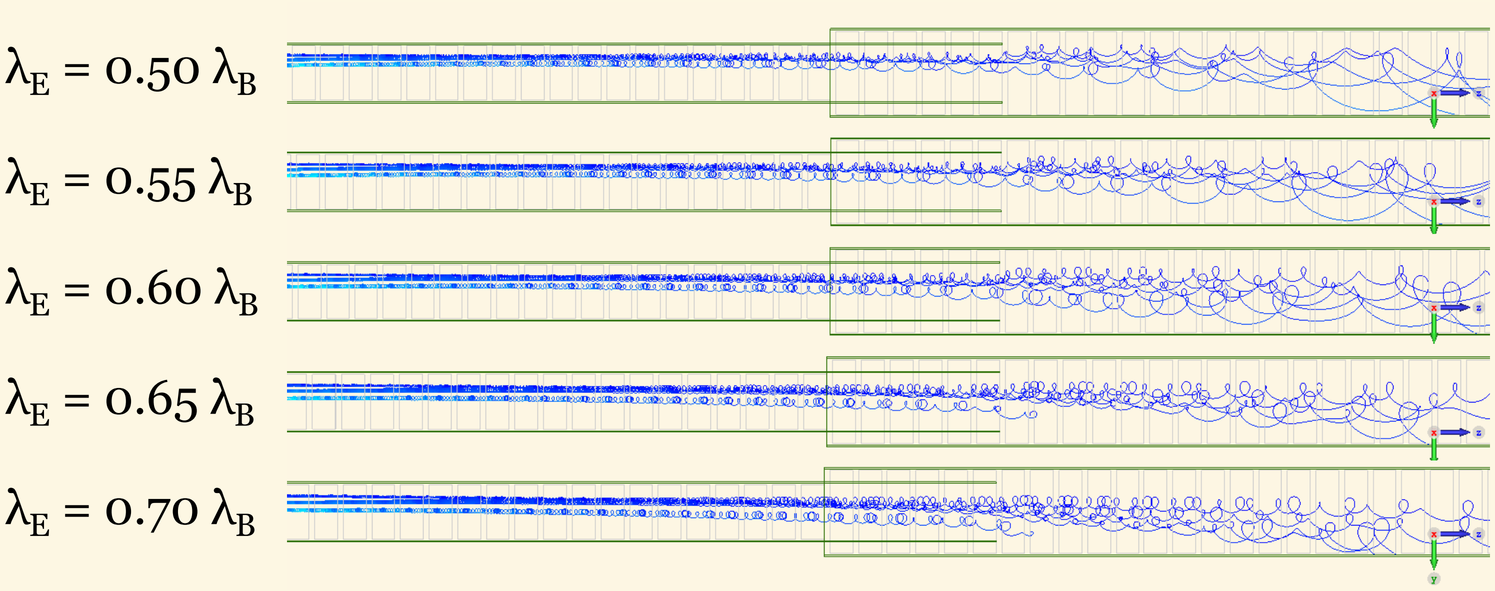
2 eV steps, N = 8946

aperture centered on y_0 at z=0.4 m

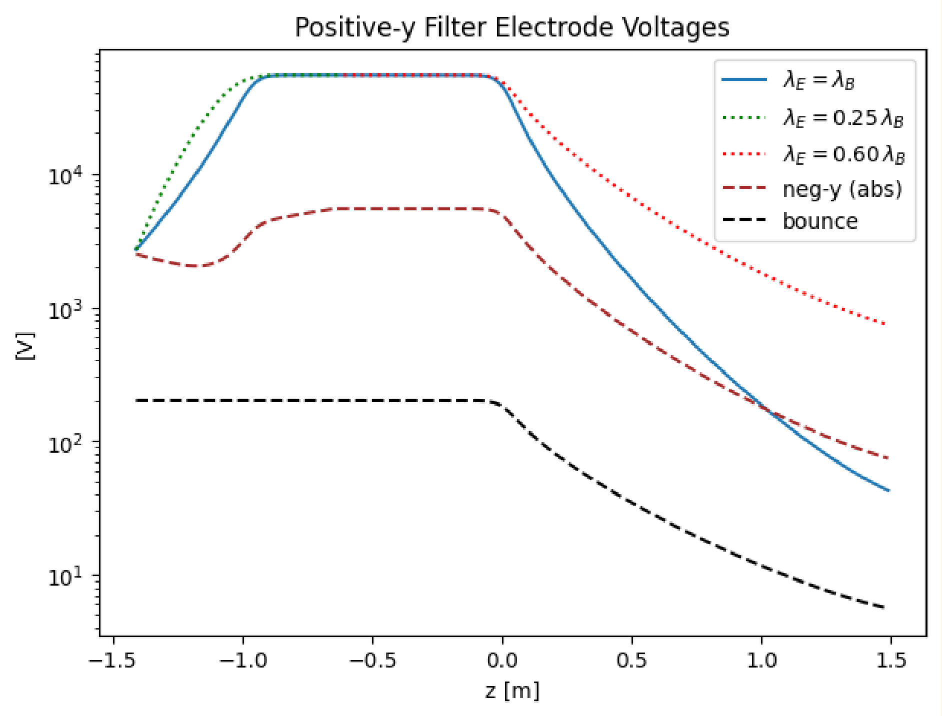
ng it into aperture (no constraints on entry angle)

Phase Angle

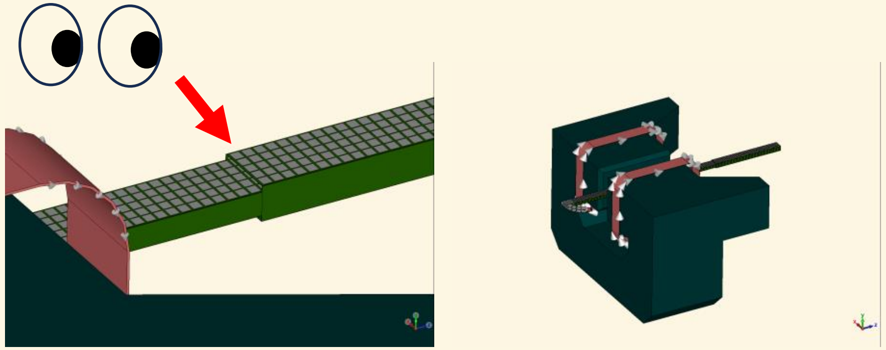
Shifting λ_E as a multiple of λ_B



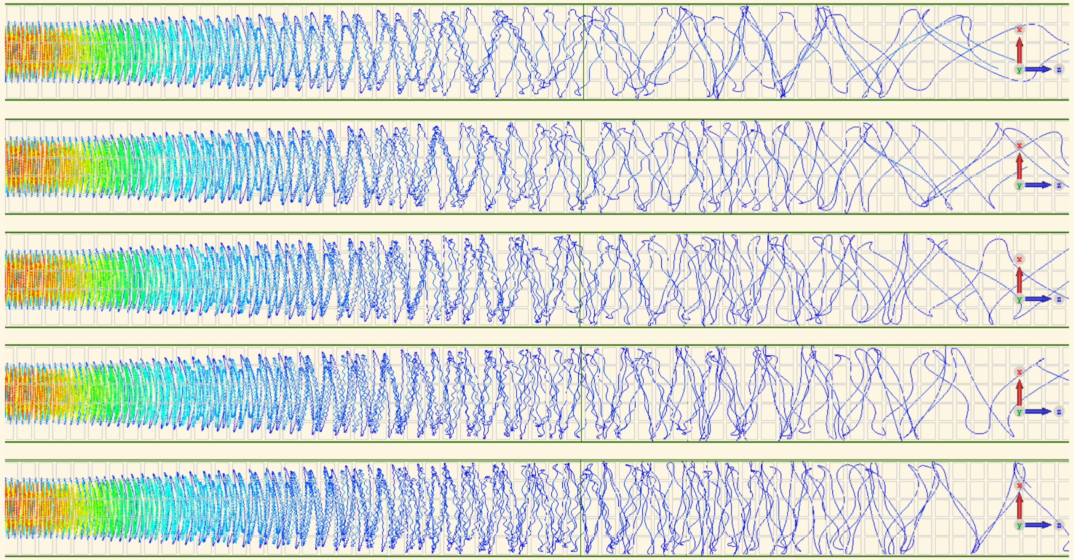
single electrons, pitch 5-85 (20 deg steps), endpoint energy



- Voltage transitions connected smoothly from injection electrodes to uniform region (see neg-y electrodes)
- Bounce voltages constant from injection through uniform region
- All adjustments made w.r.t normalized sampled B_x field
- Separate lambda shift during injection/acceleration, $\lambda_E = 0.25\ \lambda_B$



Tradeoff: Runaway ExB drift



$\lambda_E = 0.50\ \lambda_B$

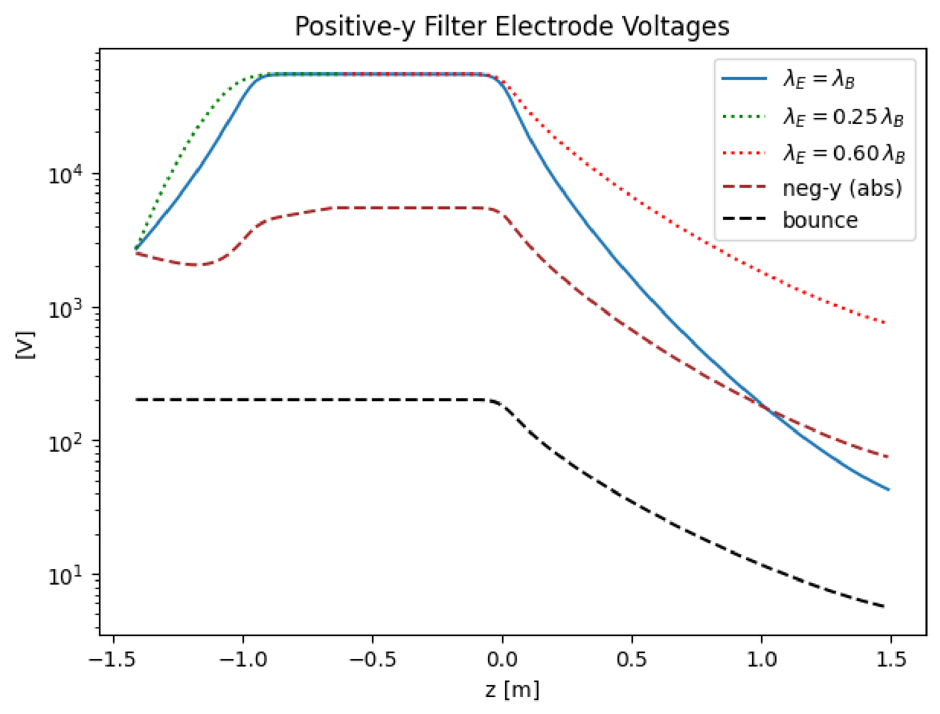
$\lambda_E = 0.55\ \lambda_B$

$\lambda_E = 0.60\ \lambda_B$

$\lambda_E = 0.65\ \lambda_B$

$\lambda_E = 0.70\ \lambda_B$

single electrons, pitch 5-85 (20 deg steps), endpoint energy



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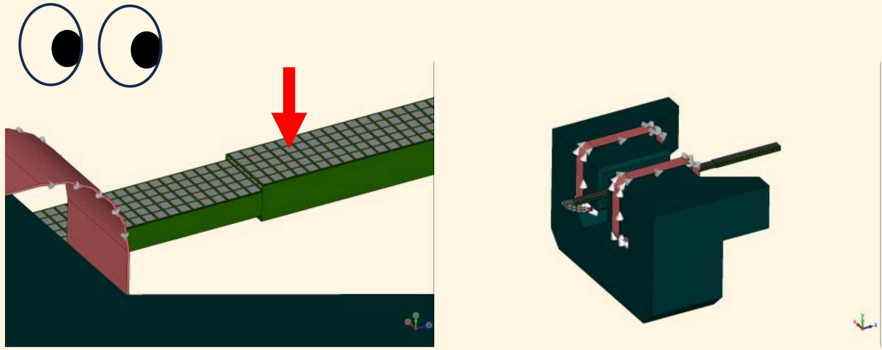
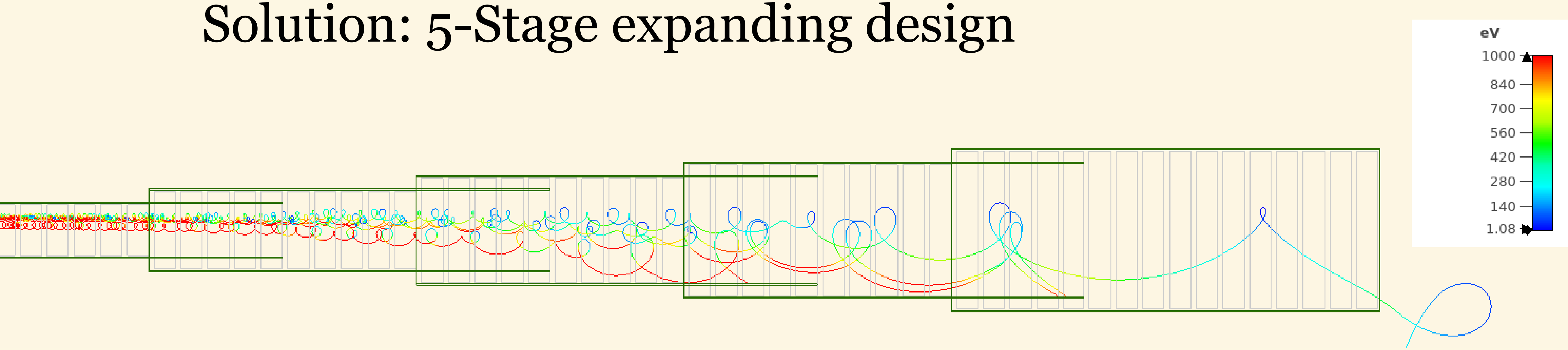
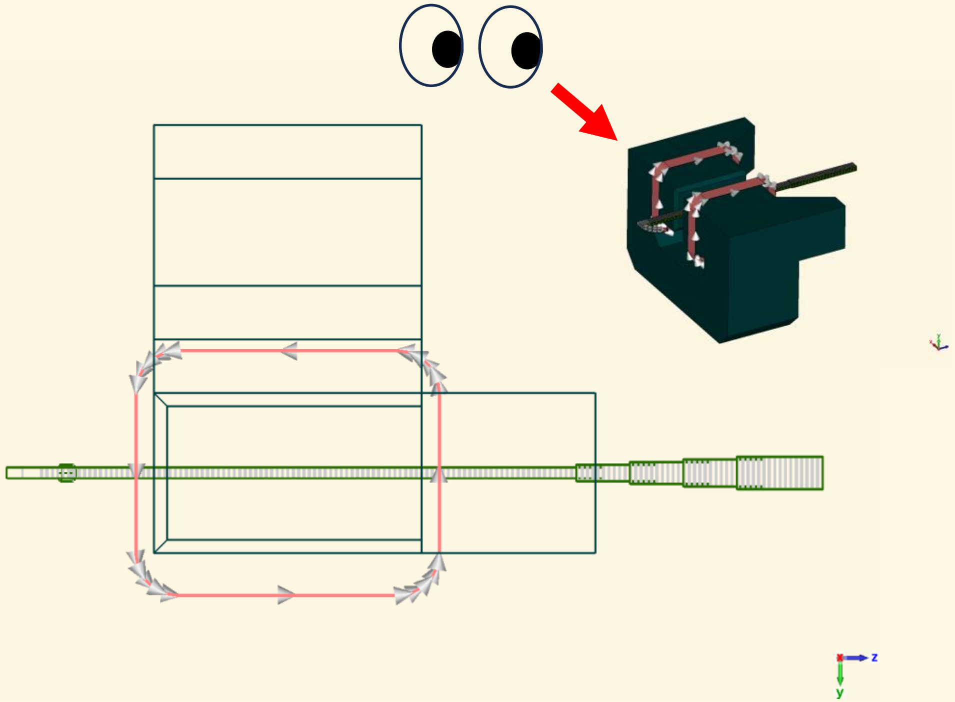


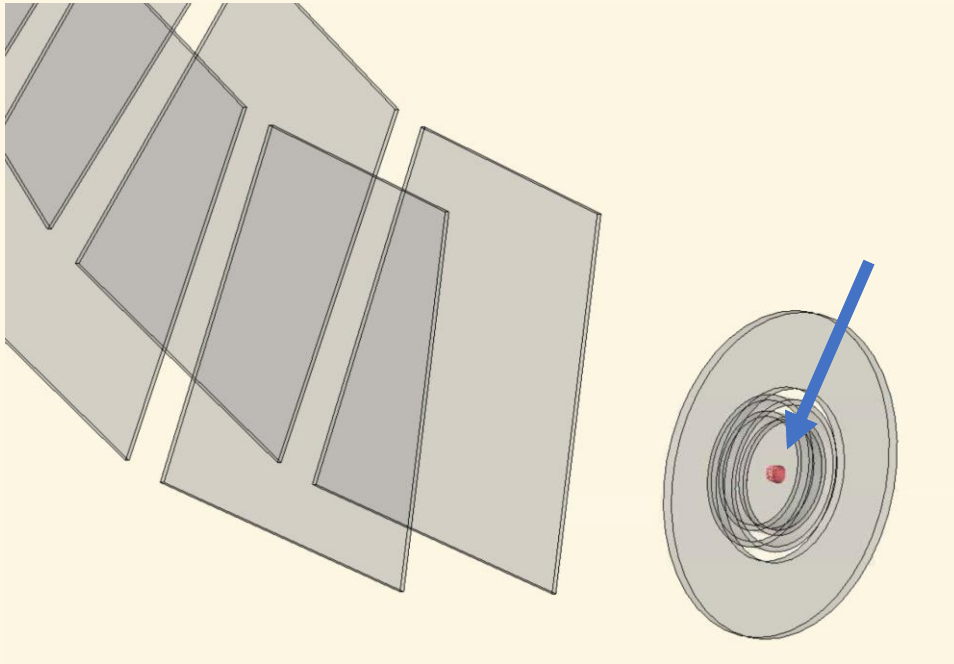
Solution: 5-Stage expanding design

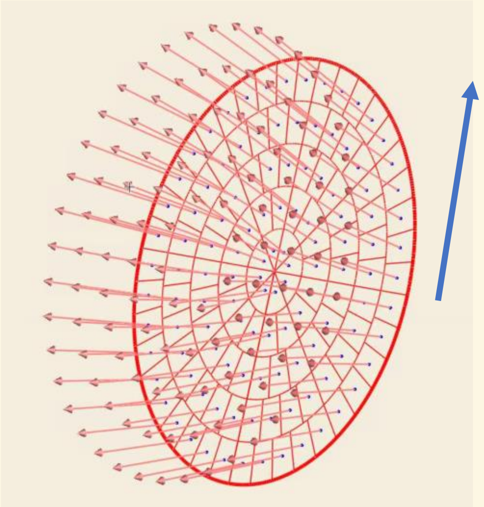


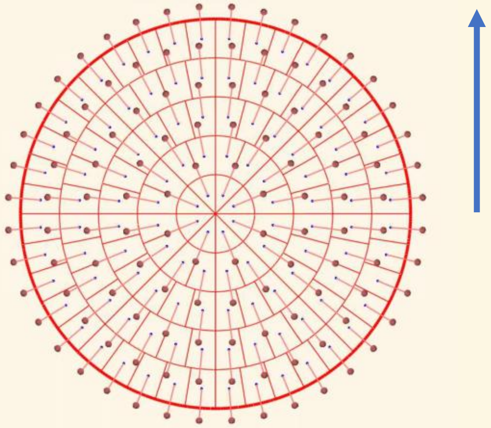
- Increased volume for cyclotron radius containment
- Decreased ExB drift due to larger spacing
- Higher aspect ratio amenable to residual parallel momentum draining
- Pick $\lambda_E = 0.60\, \lambda_B$ for now, run first transmission curves



Transmission Function Setup

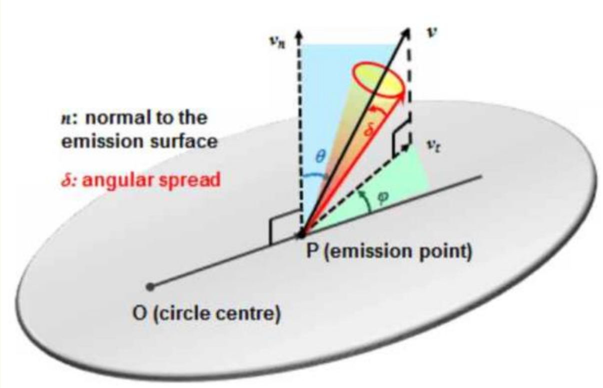






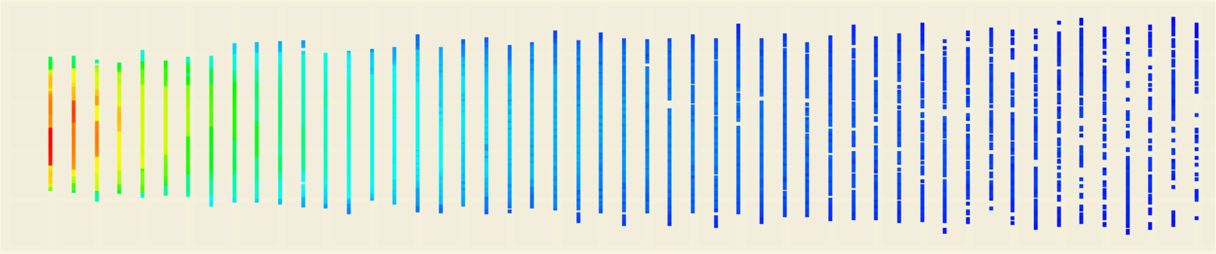
- 1mm radius circular area split into 50 rings (5 shown)
- Set pitch (theta) and emission phase angle (phi)

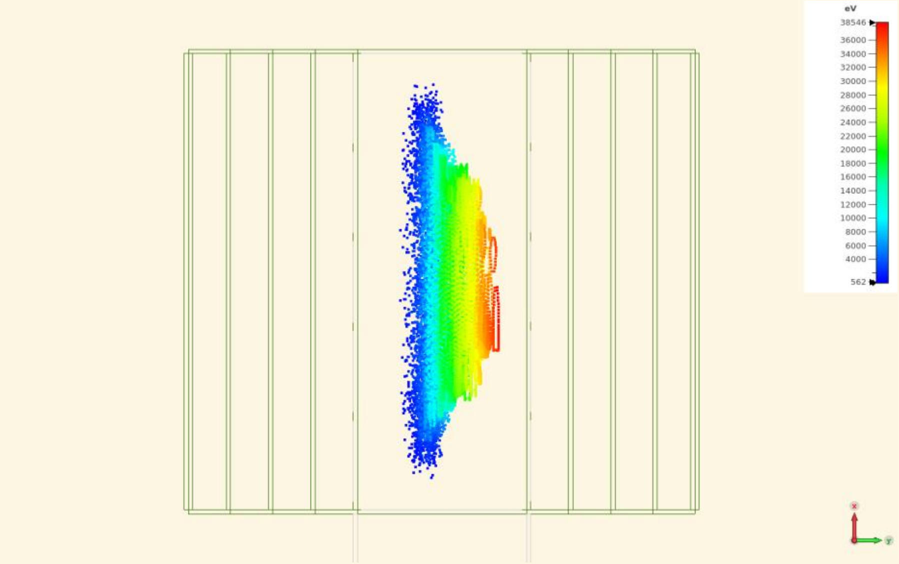
- Blocks of 10 deg pitch (5-15, 15-25, …, 75-85 deg)
  - Uniform distribution of +/- 5 deg about 10, 20, … , 80 deg pitches
  - Per pitch block: 8x fixed phi blocks in 45 deg steps (0, 45, … , 270, 315 deg)
  - 8 pitch blocks x 8 phi blocks = 64 blocks
  - 50 rings = 10,200 particles per block
  - Total N = 652,800 endpoint electrons

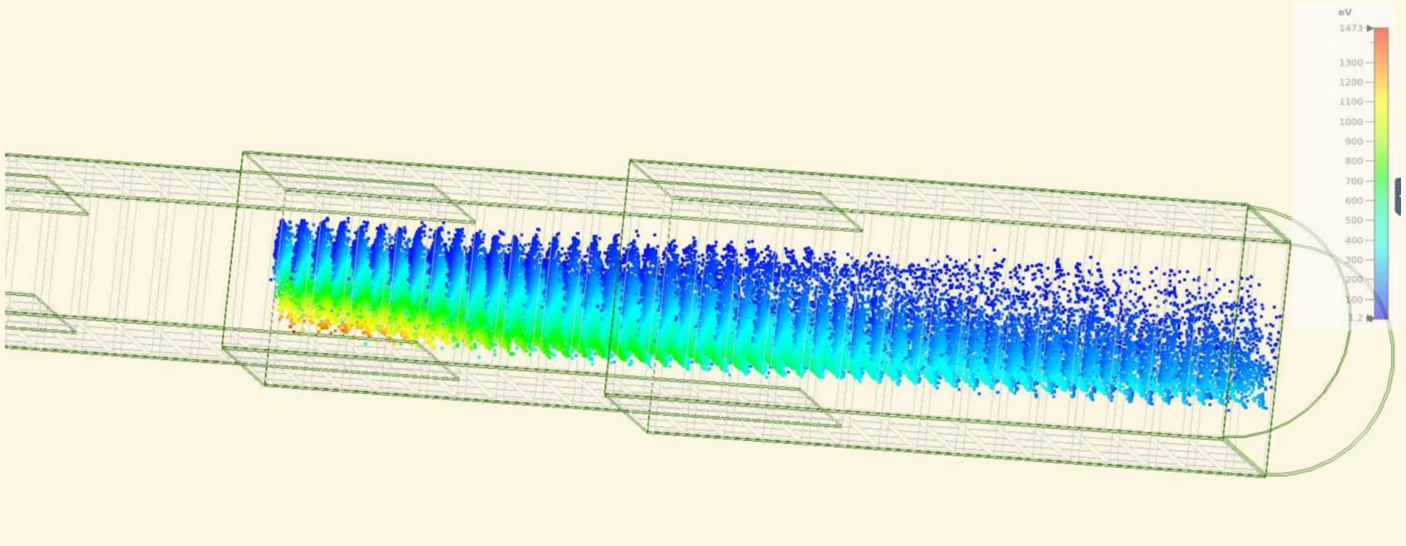


Transmission Count Method

- 2D plane monitors every 1 cm
- Count unique particle hits per plane
- Average energy over all hits (incl. duplicates)

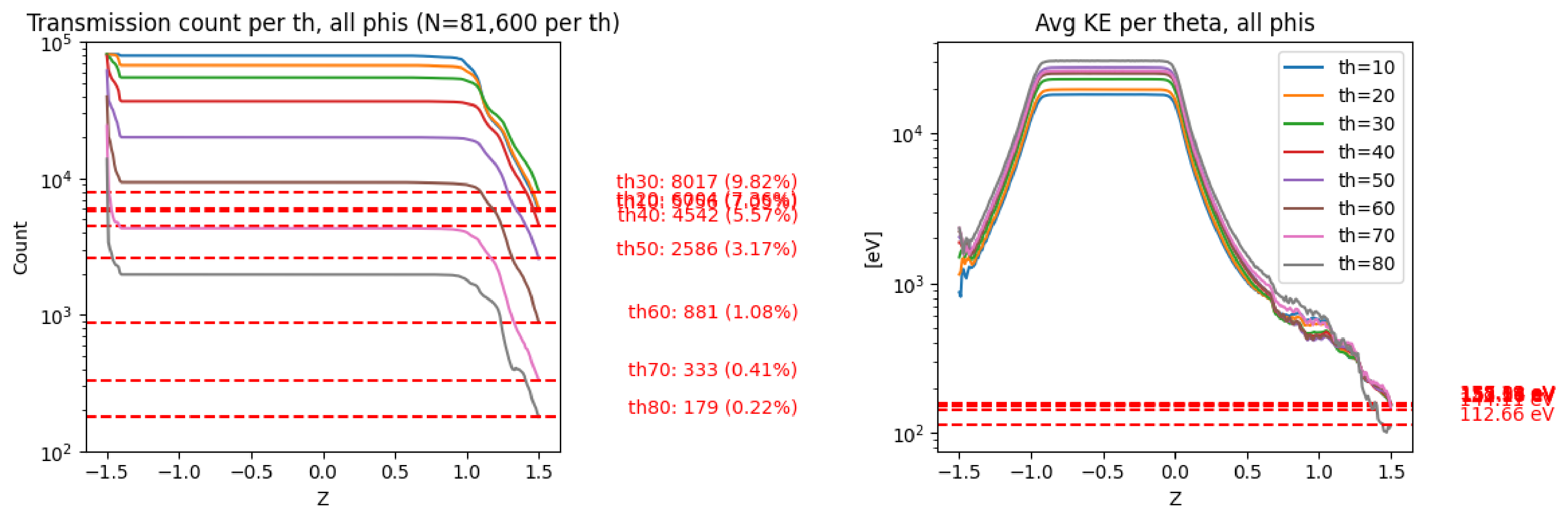


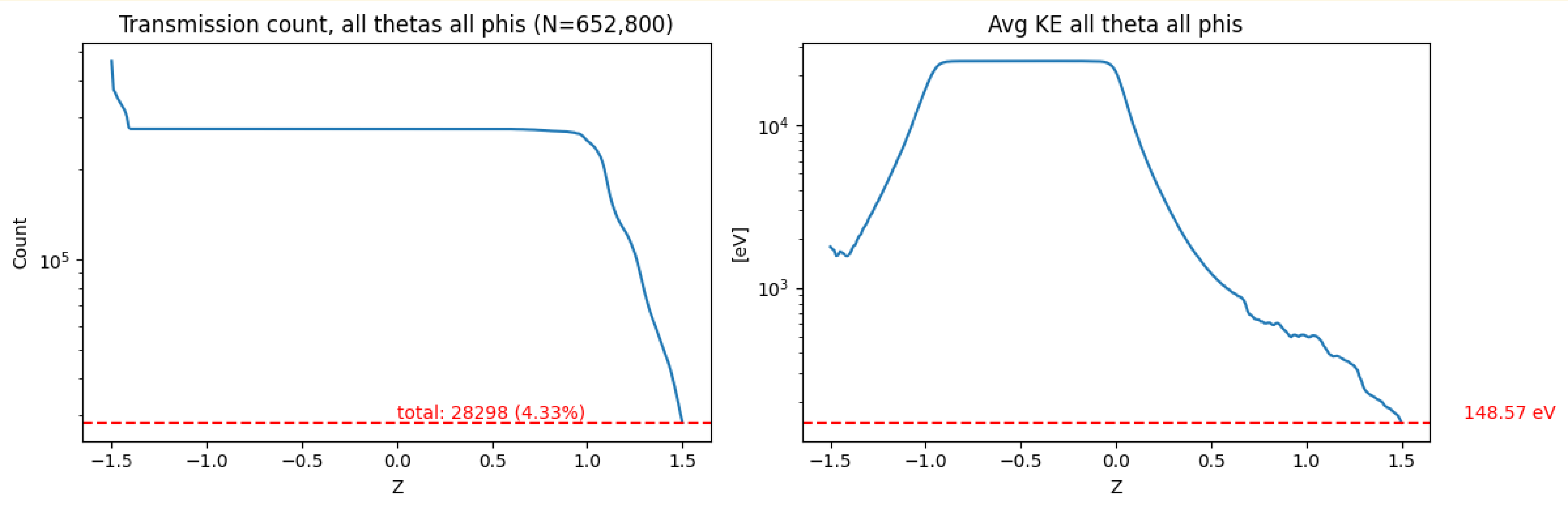




First Transmission Plots

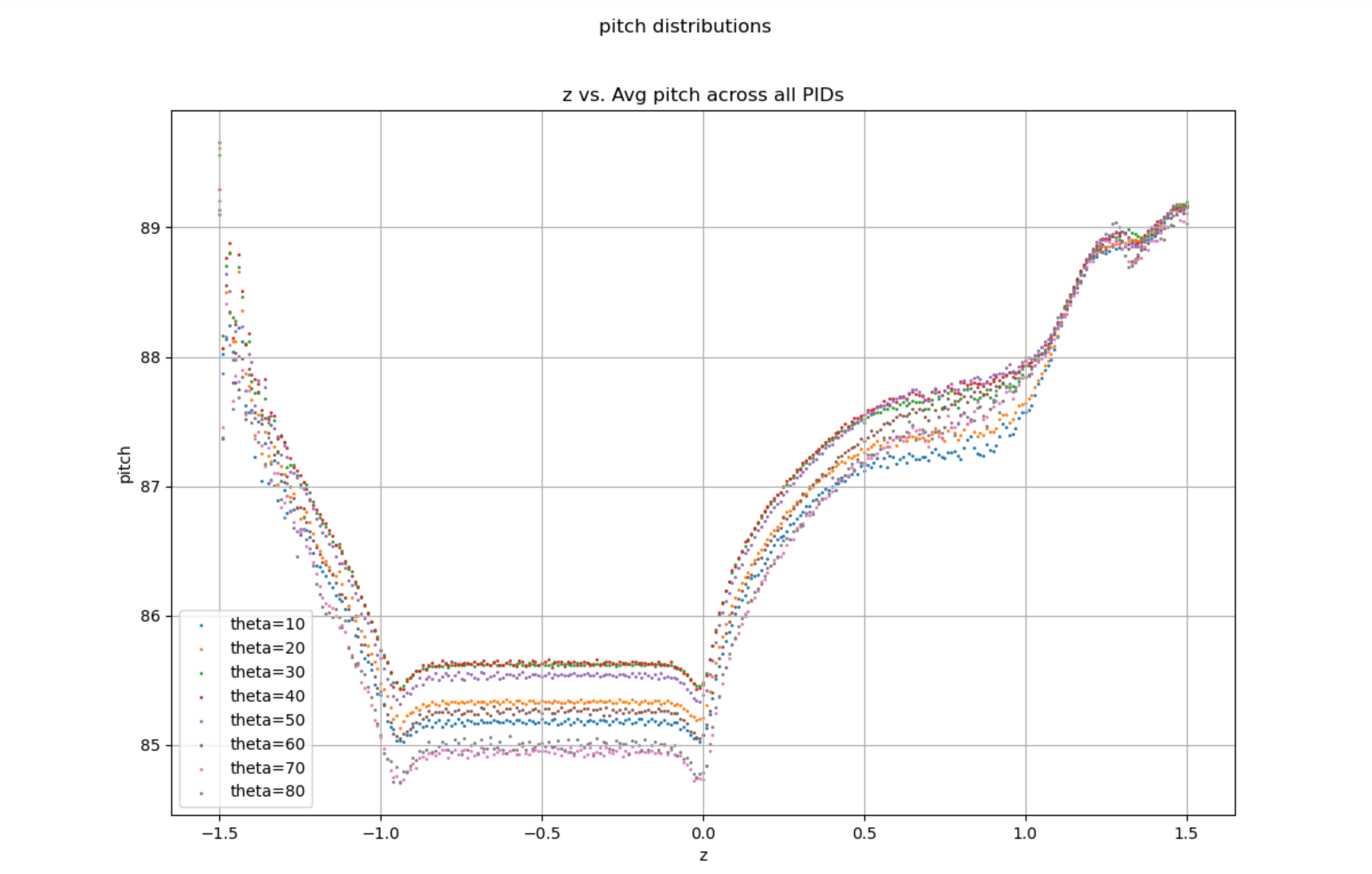
- Entirely static configuration
- No optimization at all
- Total energy filter idea appears to work
- Drop in efficiency around 75-100 cm likely due to weak bounce voltages

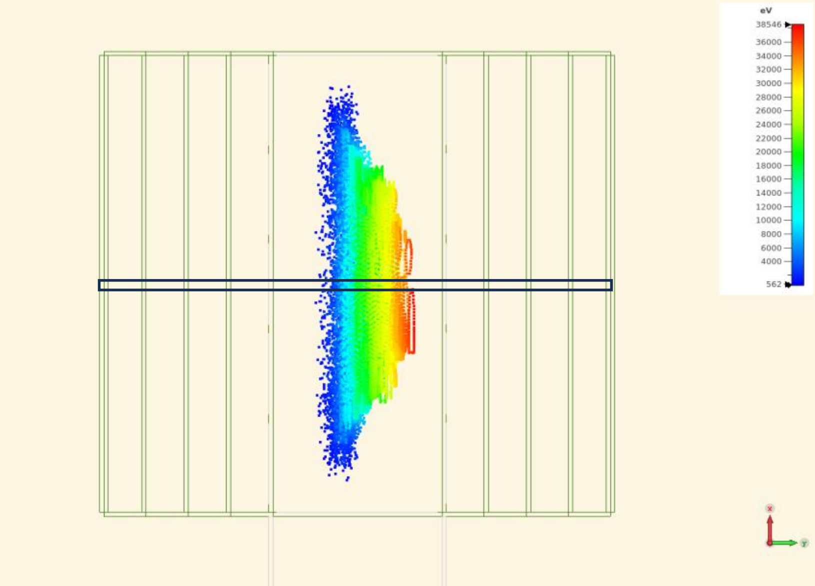




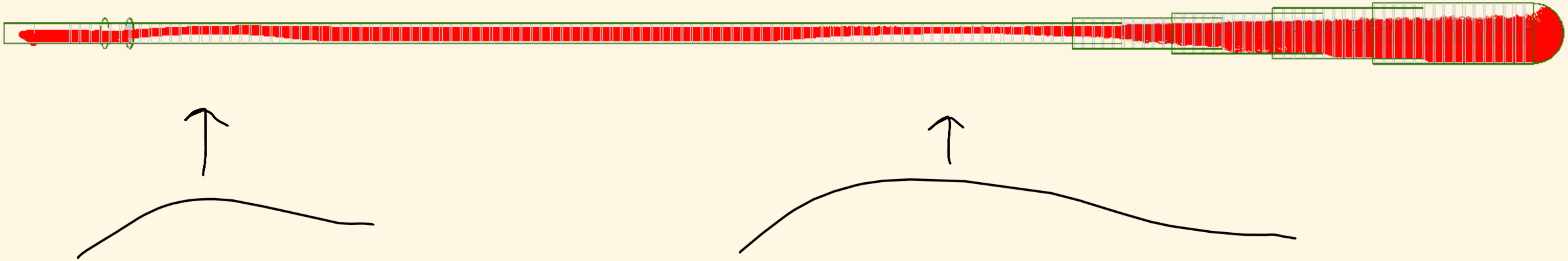
Effective Pitch

- Apply cut in x-position (within 5mm of center) to estimate pitch





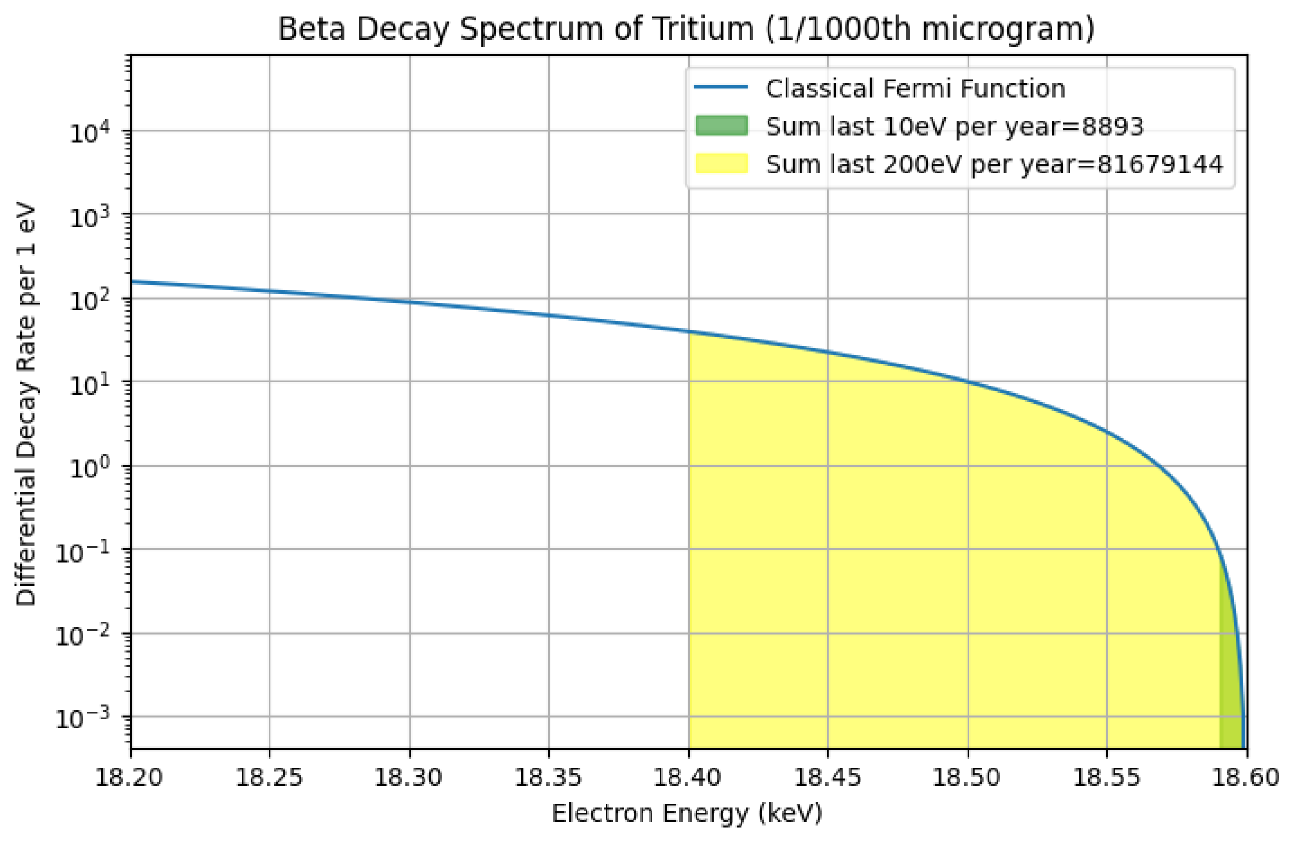
A picture is worth a thousand words…



- Drift envelopes directly reflect adjustments to lambda
- Silhouette provides a clear <u>visual guide to optimize</u> lambda adjustments and voltages
- Adjust to opt for more acceptance - ExB drift may not matter

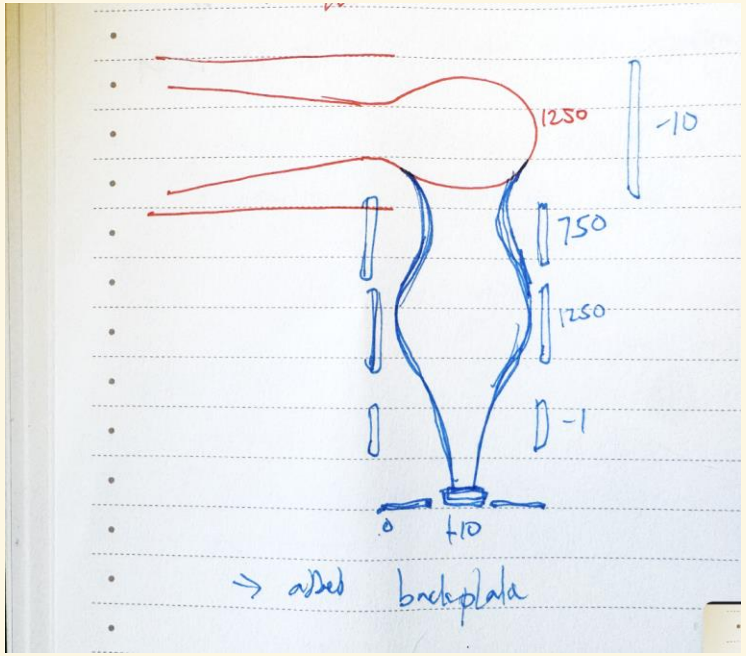
Estimated event rate

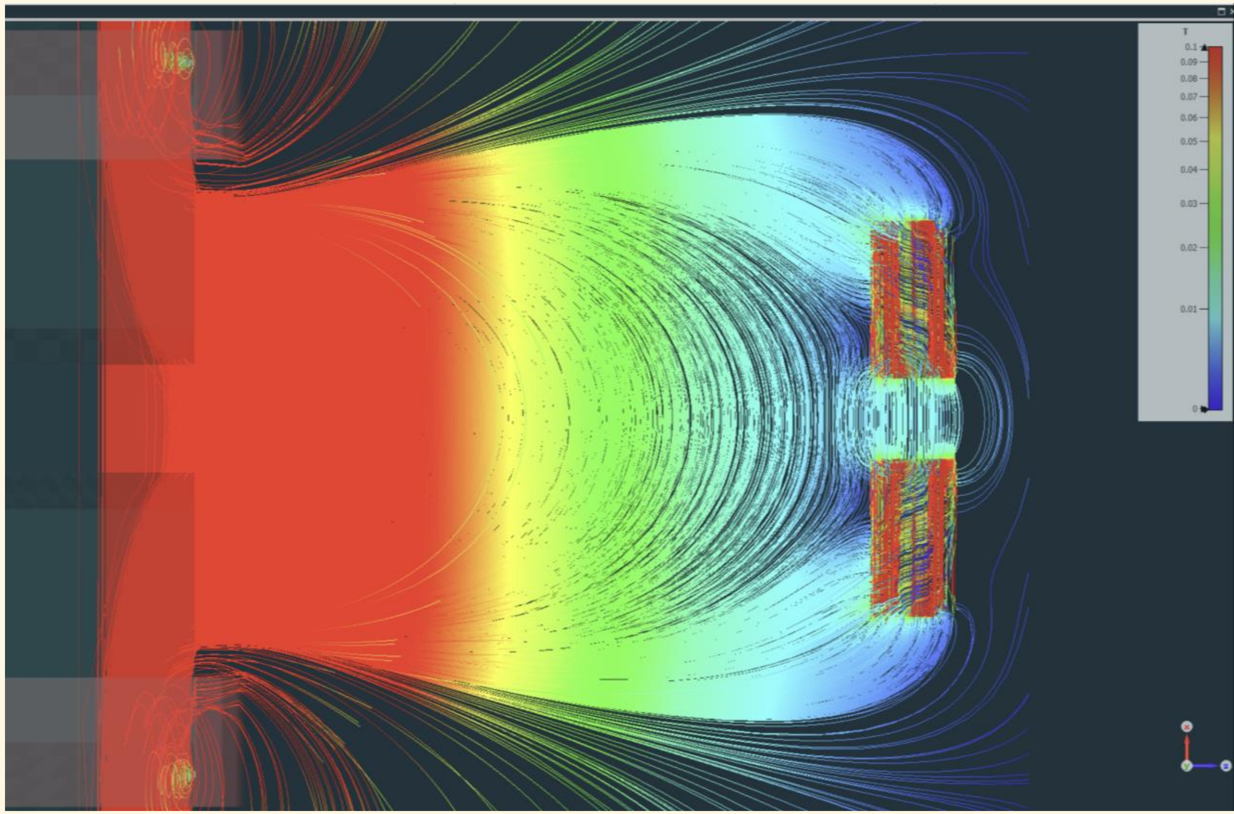
- Using classical Fermi function
- 1 nanogram of tritium
- Assuming 10% efficiency from filter to microcalorimeter…
  - ~1% from target to microcalorimeter
  - Last 10 eV ~100 events/yr
  - RF will be needed to validate a high confidence measurement (background/signal coincidence – e.g. listening to a dinner conversation in a loud restaurant)



Filter to Microcalorimeter Transport

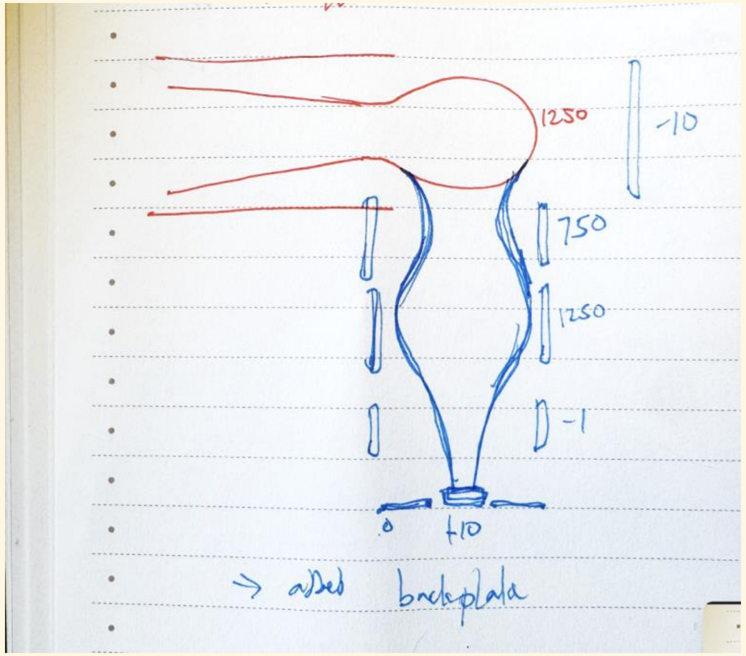
- Introduce cylindrical “mu-metal”(Iron) shielding, cancellation coils
- Idea is to go for “second collimation” effect
- Eventually want potential contours to look like:



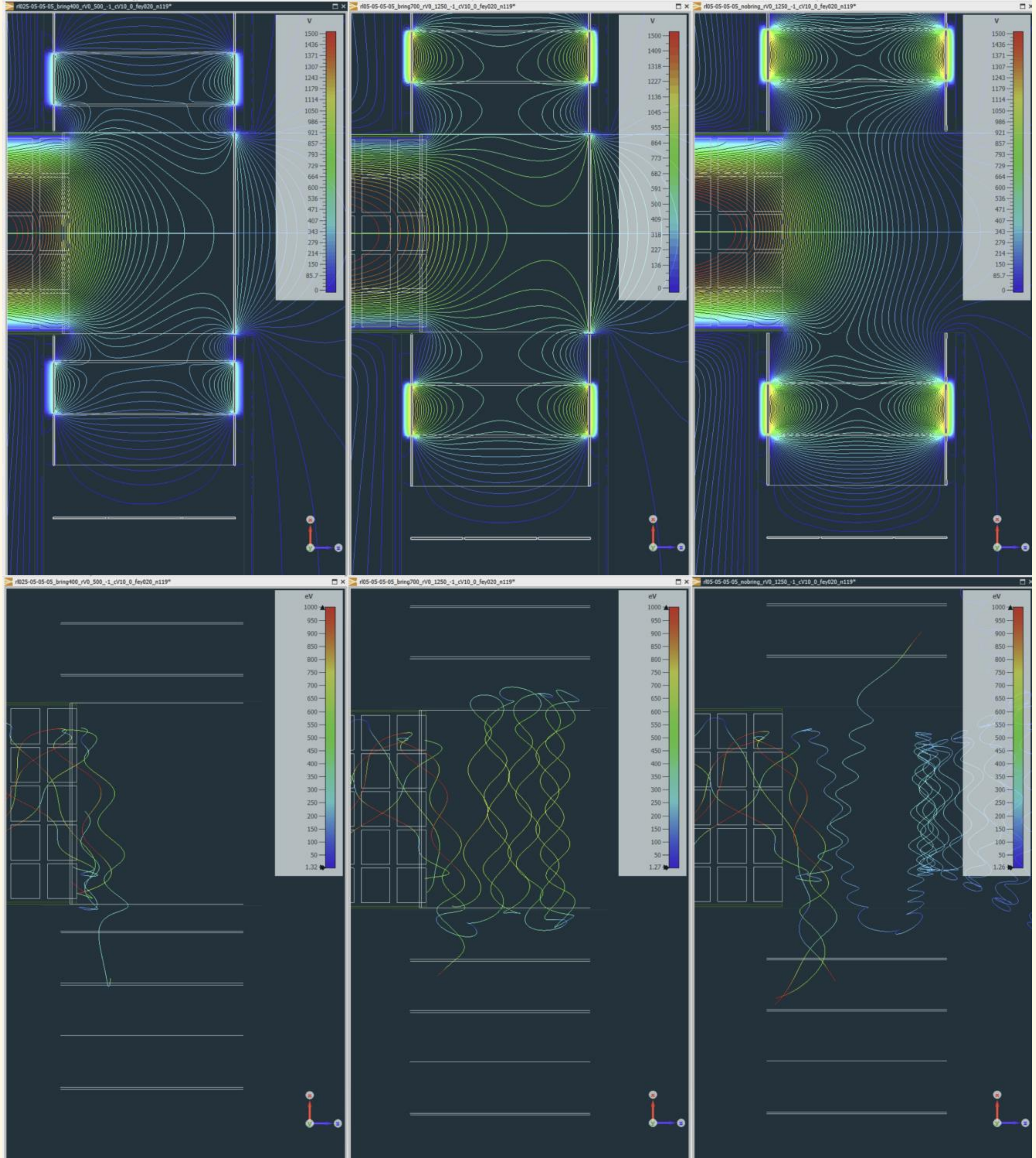


First Attempts

(z=100cm)

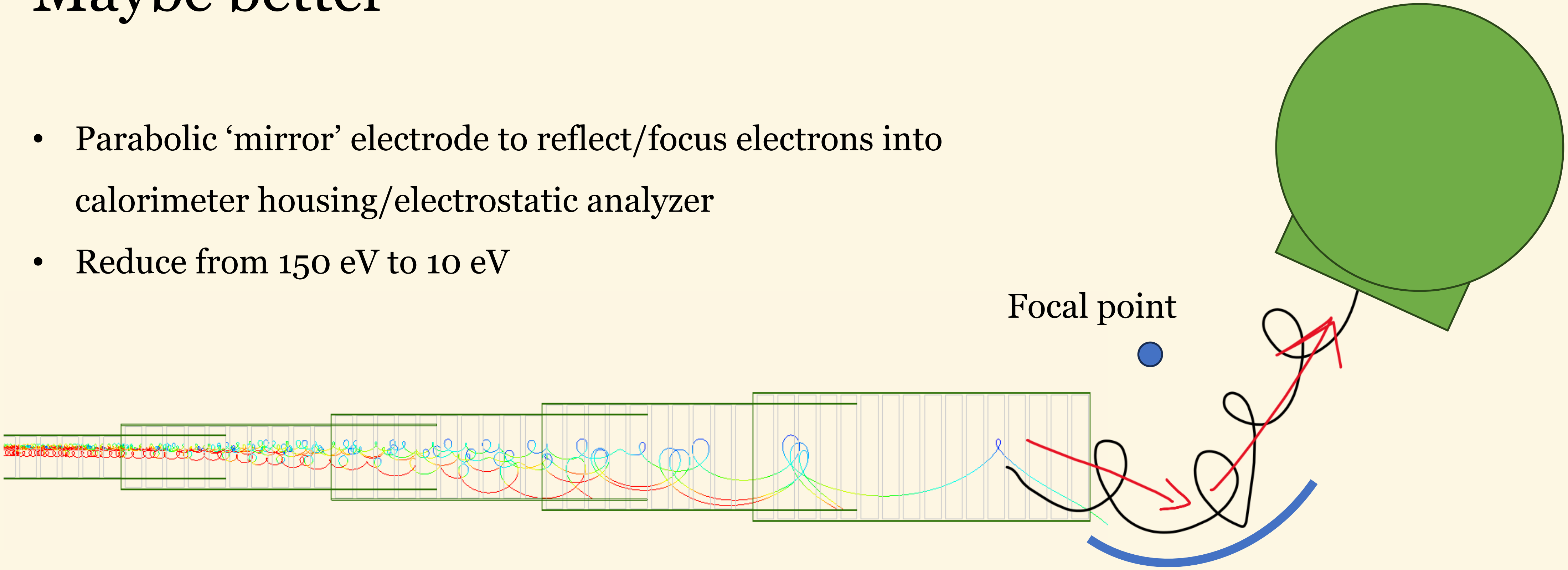


→ added backplate



Maybe better

- Parabolic ‘mirror’ electrode to reflect/focus electrons into calorimeter housing/electrostatic analyzer
- Reduce from 150 eV to 10 eV



- More to come