THEORY OVERVIEW

Angelo Esposito

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PTOLEMY collaboration meeting, Nov. 21st 2024

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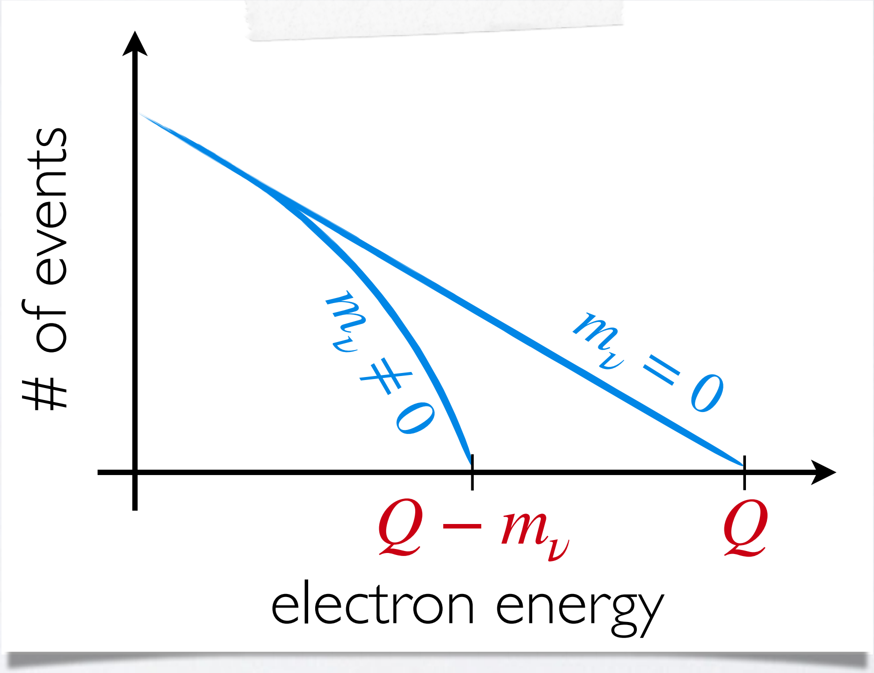

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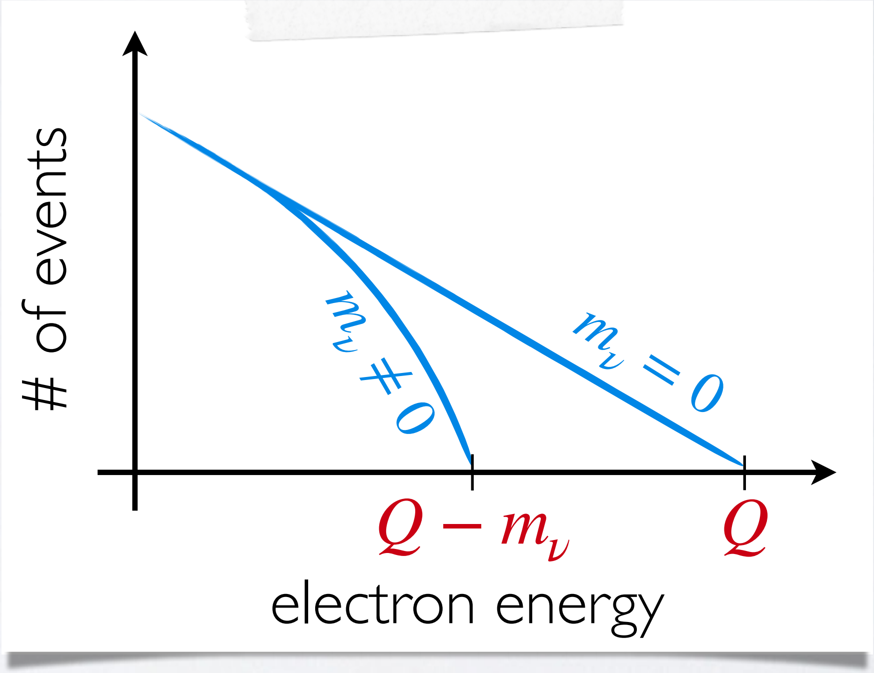

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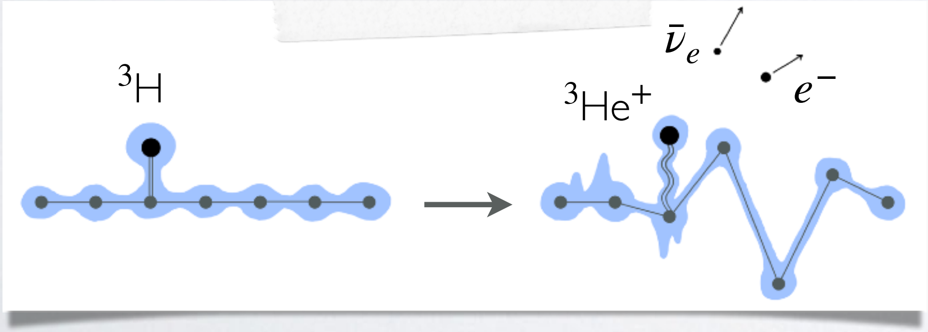

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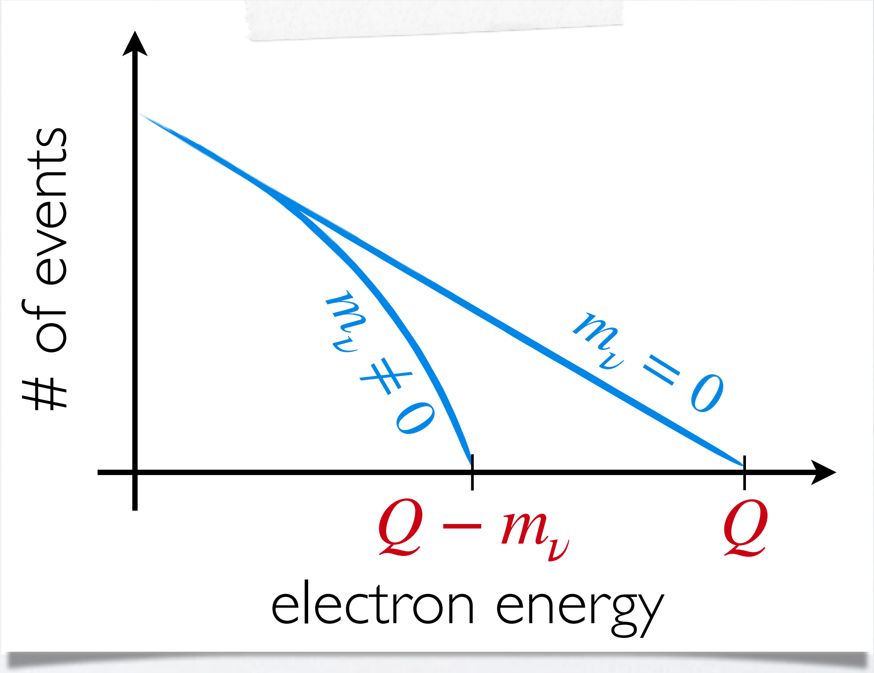

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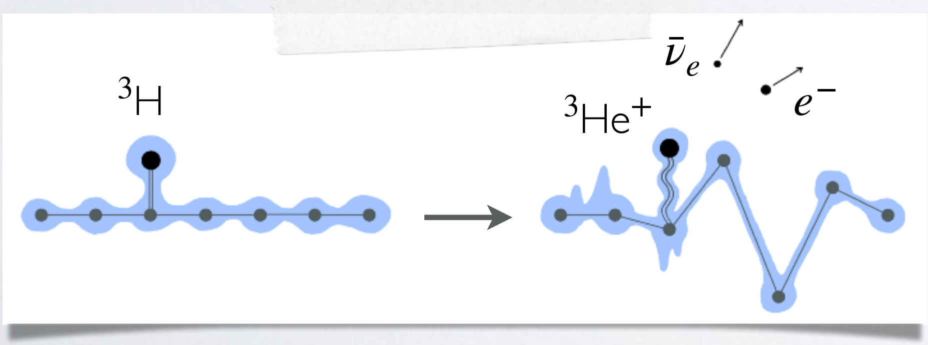

$$^3\text{H} \to {}^3\text{He}^+ + e^- + \bar{\nu}_e + \text{extra stuff}$$

ROADMAP

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Forecast/measurement of m_ν

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Accurate model of $dN(m_\nu)/dE_e$

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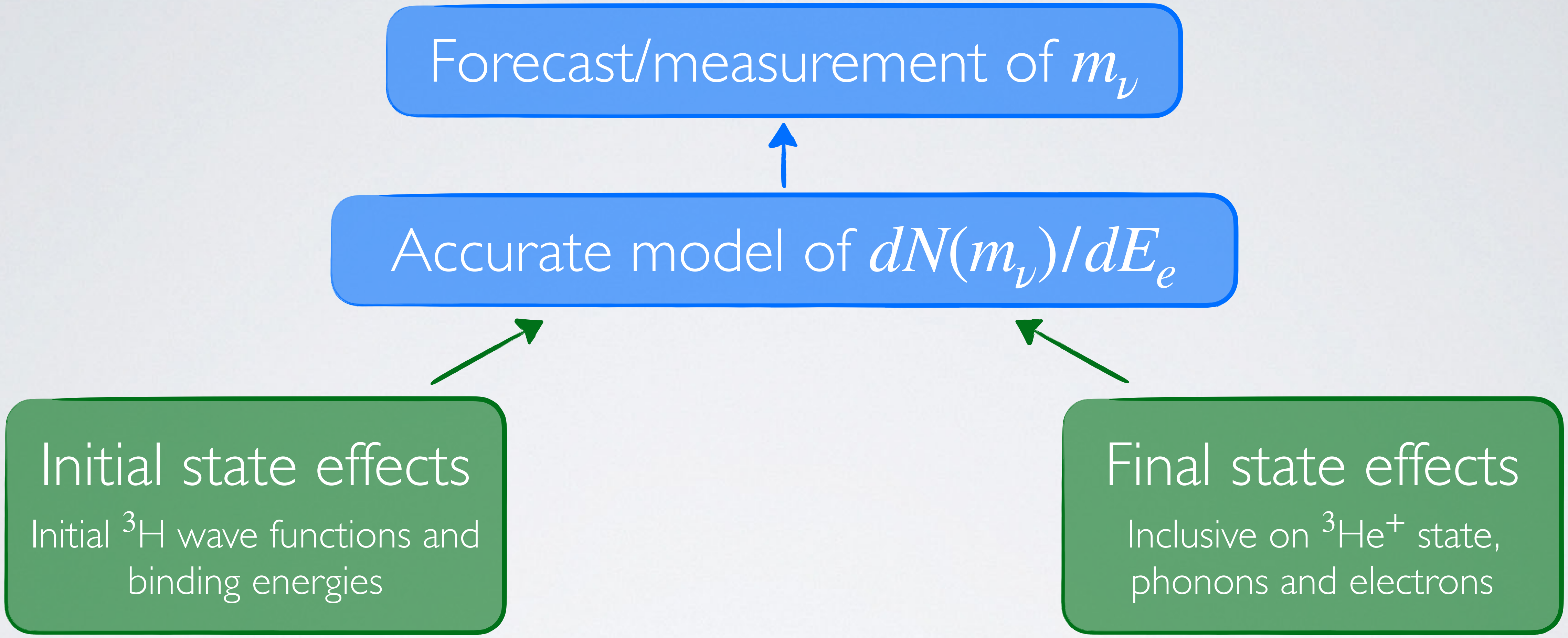
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Initial ^{3}H wave functions and binding energies
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Inclusive on $^3He^+$ state, phonons and electrons

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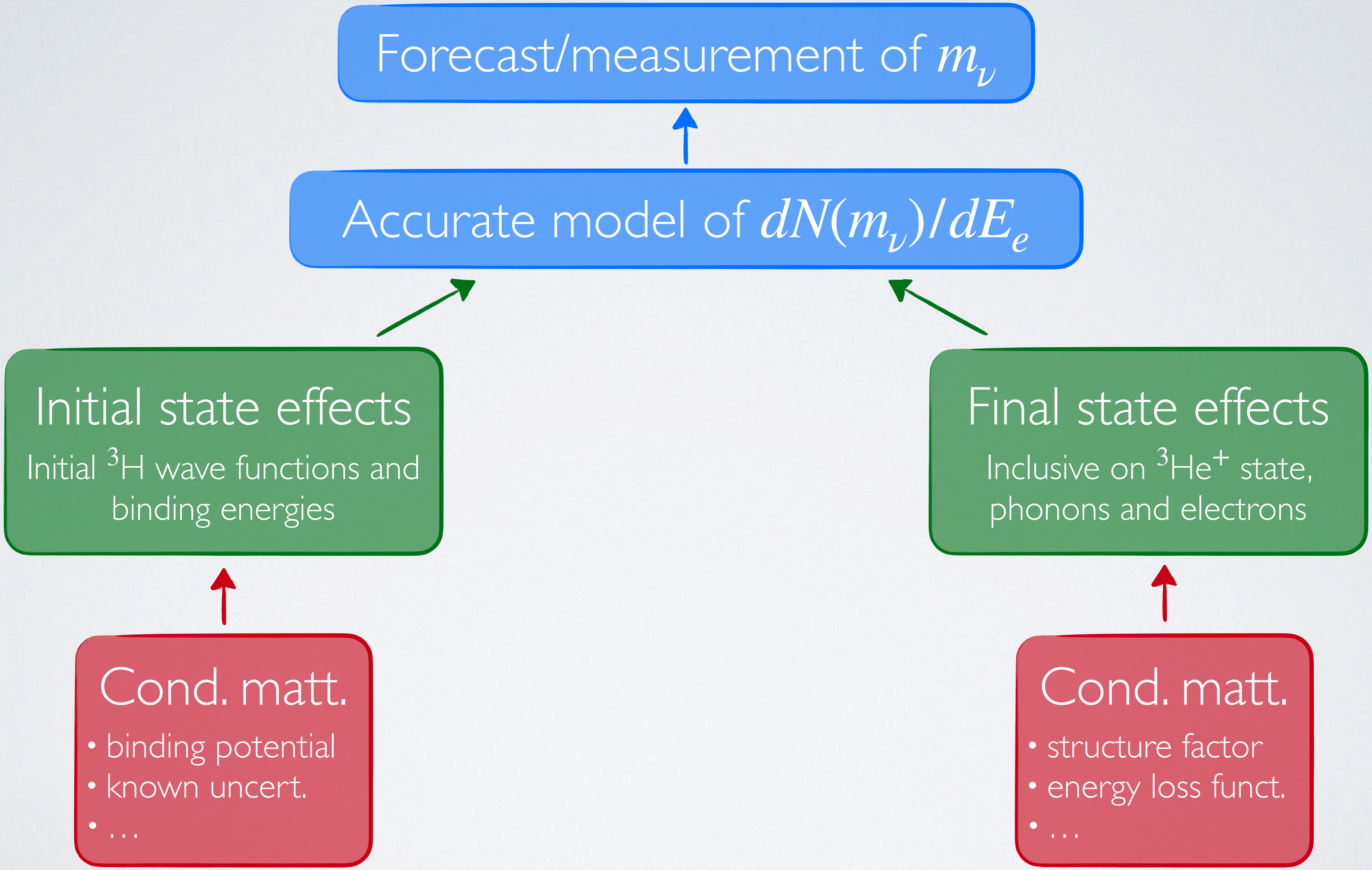

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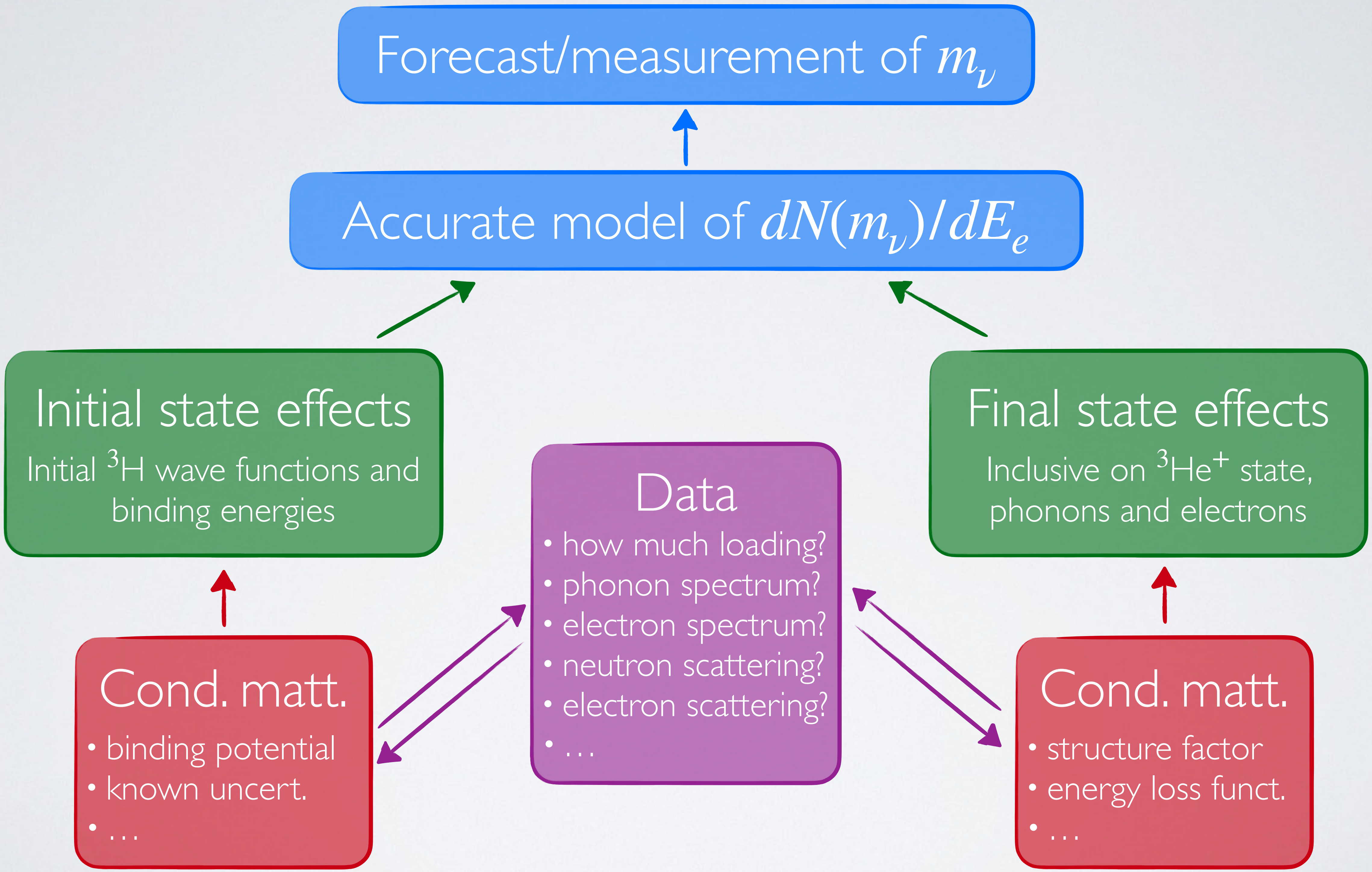
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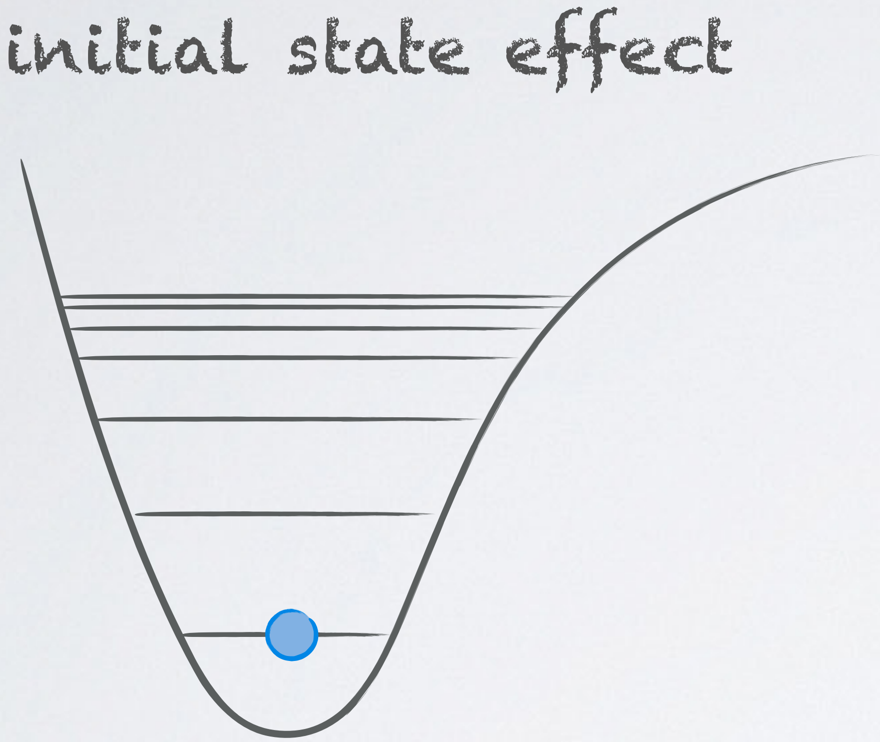

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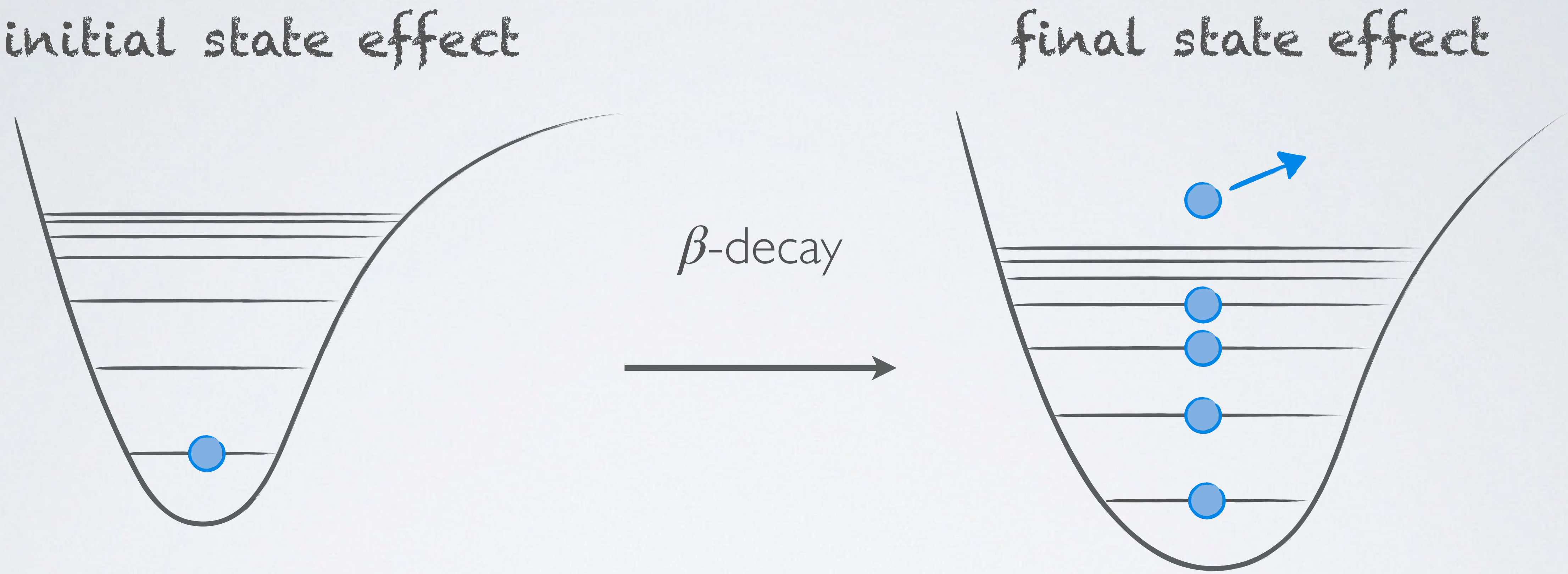

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$^3He^+$ can end up in any eigenstate of the *final* potential

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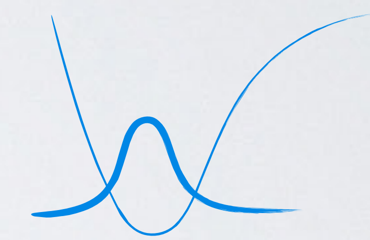

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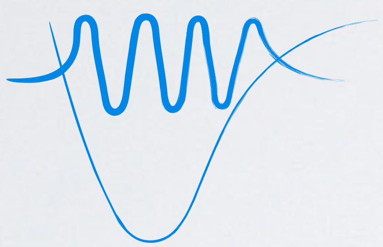

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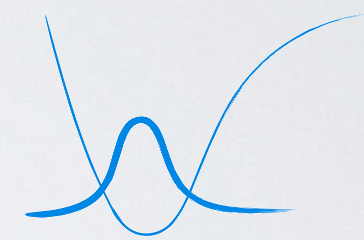

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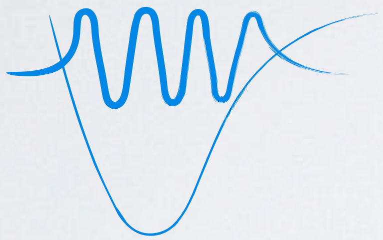

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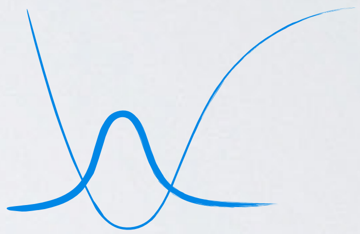

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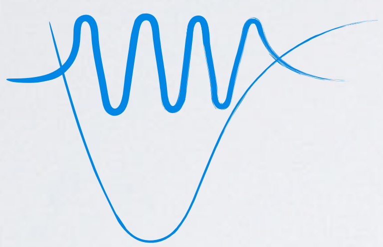

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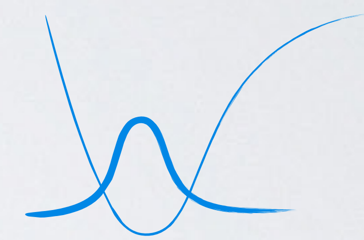

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- How do the potential and wave functions look like?

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- To go from this to the "He potential" turns out to be an ill posed question…

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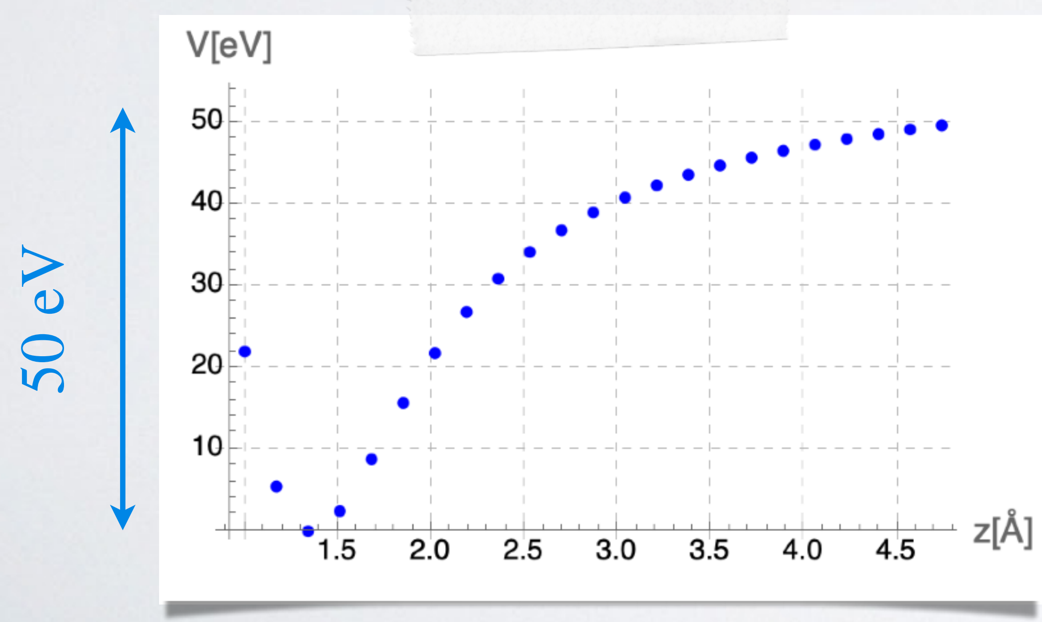

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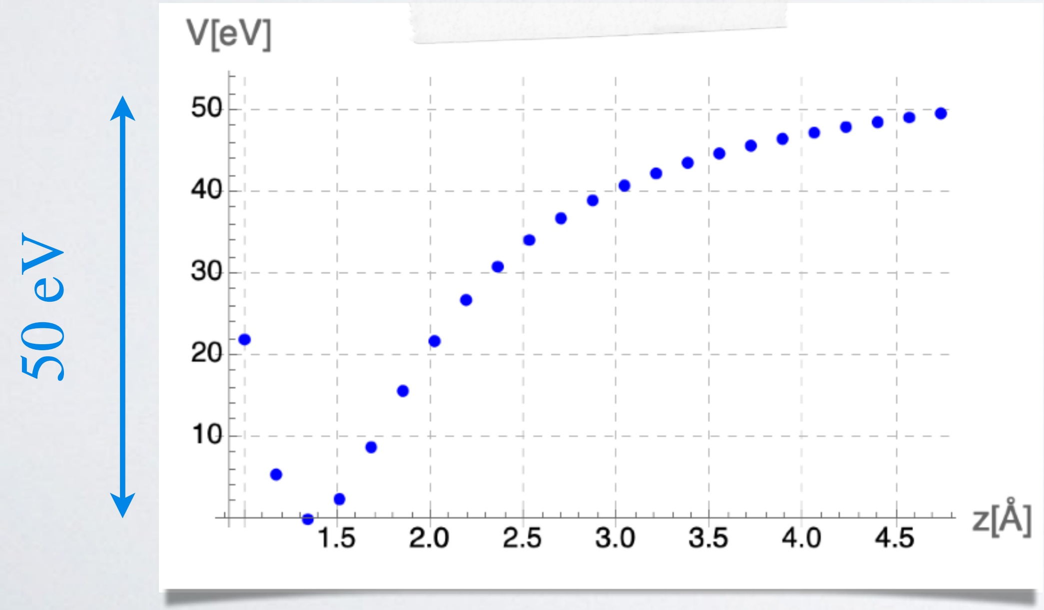

This should be correct near the minimum.

For larger distances the electron density should have time to rearrange.

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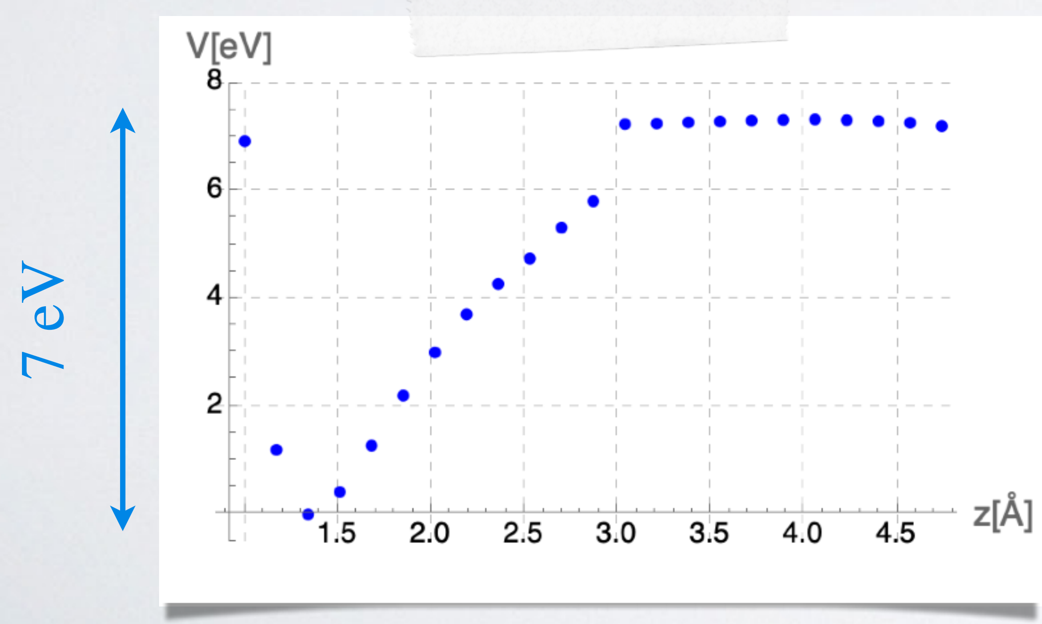

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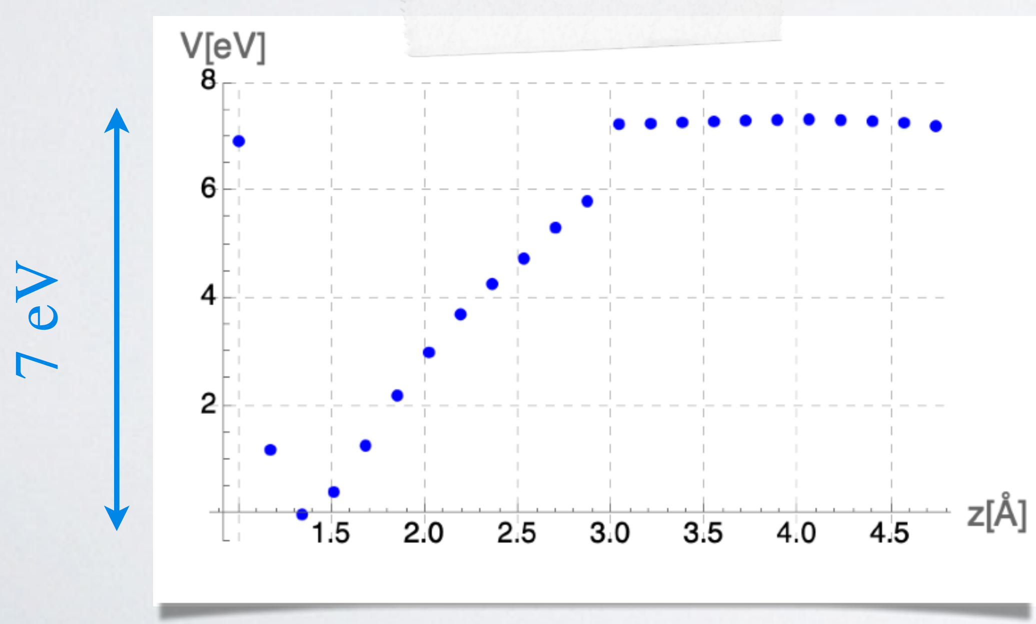

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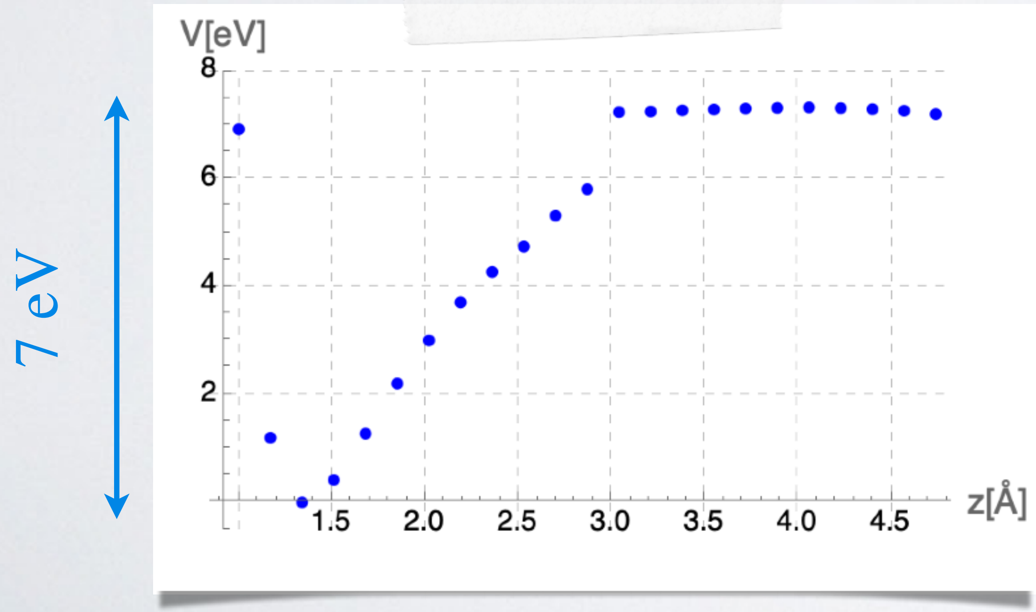

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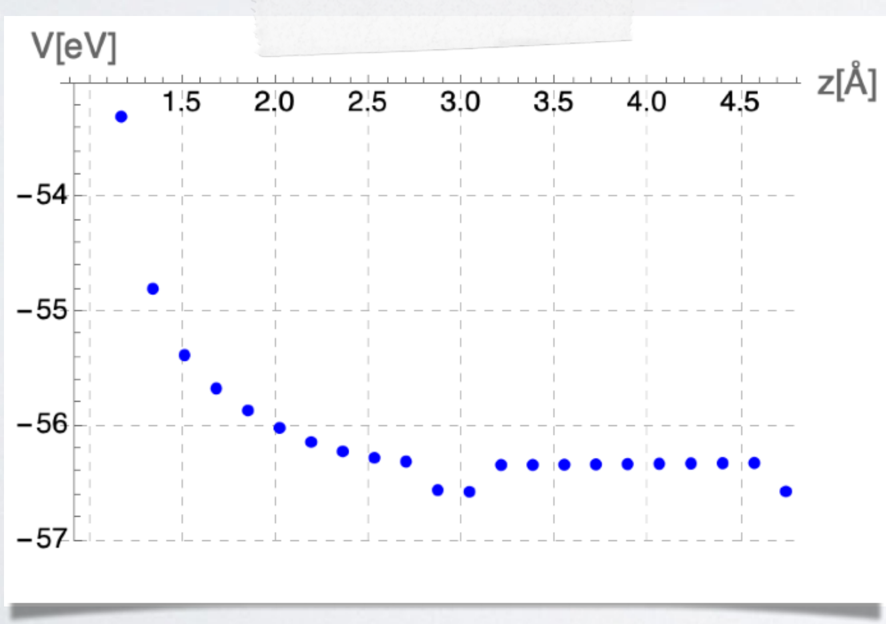

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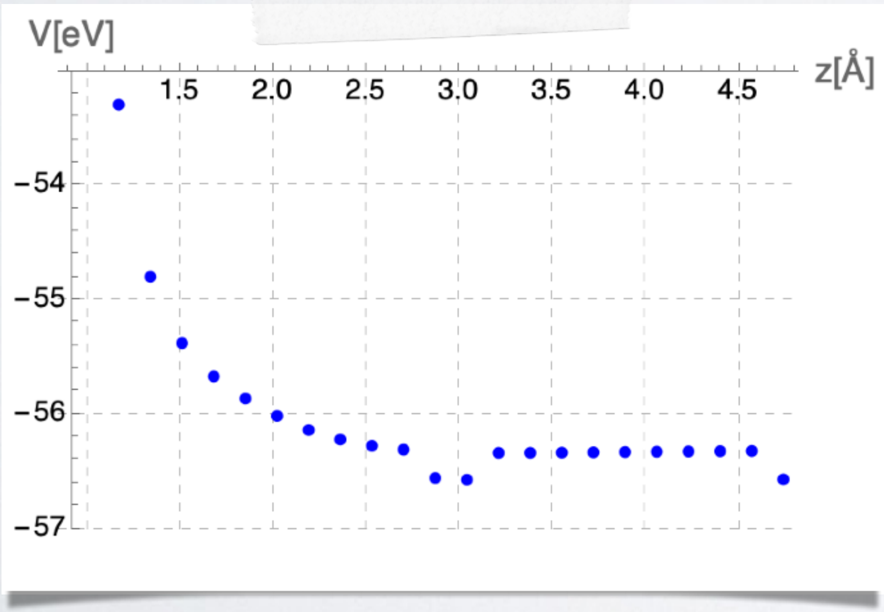

This is likely a good description only at very large distances, where the whole system had enough time to relax.

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Thank you for the attention!