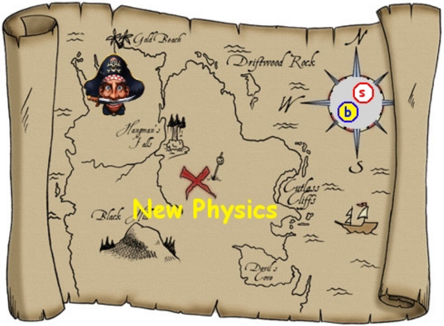
An exploration of $B_s \to J/\psi\, s\,\bar{s}$

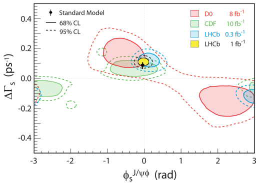
Rob Knegjens



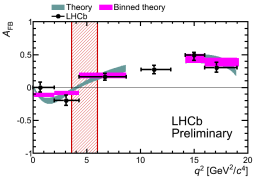
NIKHEF

FOM

The **search** for **New Physics**



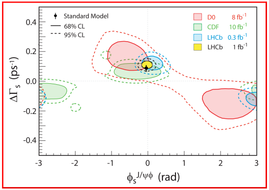
CP observables → SM predictions

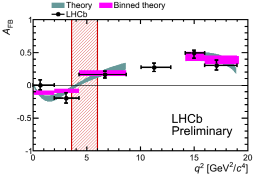


- Possible to distinguish **smallish New Physics**?
- Can we complement the current **flagship** analyses?

The **search** for New Physics

CP observables → SM predictions



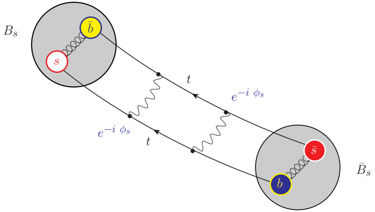


- Possible to distinguish **smallish New Physics**?
- Can we complement the current **flagship** analyses?

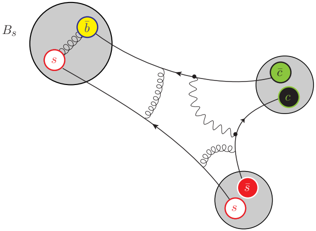
- **In pursuit of new physics with $B_s \to K^+K^-$** R. Fleischer, RK (arXiv:1011.1096)
- **Anatomy of $B^0_{s,d} \to J/\psi\ f_0(980)$** R. Fleischer, RK, G. Ricciardi (arXiv:1109.1112)
- **Effective lifetimes of B_s decays and their constraints on the B^0_s–$\bar{B}^0_s$ mixing parameters** R. Fleischer, RK (arXiv:1109.5115)
- **Exploring CP Violation and η–η' Mixing with the $B^0_{s,d} \to J/\psi\eta^{(\prime)}$ Systems** R. Fleischer, RK, G. Ricciardi (arXiv:1110.5490)

New Physics in B_s^0–$\bar{B}_s^0$ mixing?

Mixing



Decay



$\Delta M_s,\ \Delta\Gamma_s \equiv \Gamma_{\rm L} - \Gamma_{\rm H}$

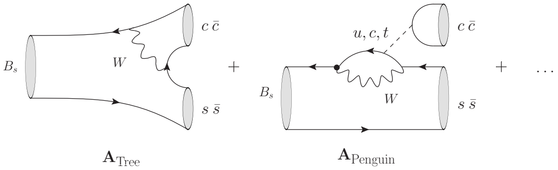
ϕ_s }

$\Delta\phi,\ C$ (direct CPV)

hadronic physics }

$$\frac{\Gamma\left(B_s(t) \to f\right) - \Gamma(\bar{B}_s(t) \to f)}{\Gamma(B_s(t) \to f) + \Gamma(\bar{B}_s(t) \to f)} = \text{function}\left(\Delta M_s,\ \Delta\Gamma_s,\ \boxed{\phi_s + \Delta\phi},\ C\right)$$

$B_s \to \bar{c}c\bar{s}\,s$



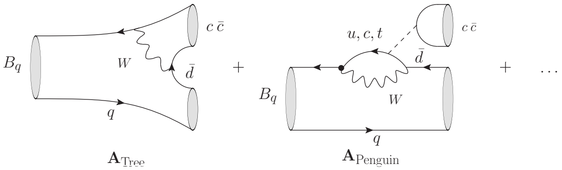
$$A(B_s \to \bar{c}c\bar{s}\,s) = A_{\mathrm{T}}\,V_{cb}^{*}V_{cs} + A_{\mathrm{P}}^{u}\,V_{ub}^{*}V_{us} + A_{\mathrm{P}}^{c}\,V_{cb}^{*}V_{cs} + A_{\mathrm{P}}^{u}\,V_{tb}^{*}V_{ts} + \ldots$$

$$= \mathcal{A}\left[1 + \underbrace{\epsilon}_{0.05}\, e^{i\gamma}\, b\, e^{i\theta}\right], \qquad \left(\epsilon \equiv \frac{\lambda^2}{1-\lambda^2}\right)$$

$$\boxed{b\,e^{i\theta} = \underbrace{\left(\tfrac{1}{\lambda} - \tfrac{\lambda}{2}\right)\left|\tfrac{V_{ub}}{V_{cb}}\right|}_{R_b}\left(\frac{A_{\mathrm{P}}^{(ut)}}{A_{\mathrm{T}} + A_{\mathrm{P}}^{(ct)}}\right)}$$

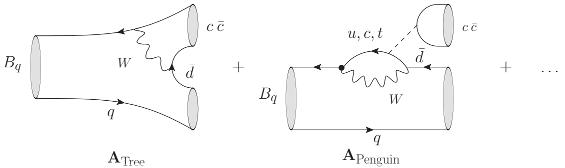
$$\begin{cases} C & = -2\,\epsilon\, b \sin\gamma \sin\theta + \mathcal{O}(\epsilon^2) \\ \Delta\phi & = \underbrace{2\,\epsilon}_{6^\circ}\, b \sin\gamma \cos\theta + \mathcal{O}(\epsilon^2) \end{cases}$$

Penguin control via flavour symmetry



$$A(B_q \to \bar{c}c\bar{d}\,q) \overset{SU(3)_F}{=} -\lambda\,\mathcal{A}\left[1 - \underbrace{\epsilon\!\!\!/}_{1}\; e^{i\gamma}\, b\, e^{i\theta}\right]$$

Penguin control via flavour symmetry



$$A(B_q \to \bar{c} c \bar{d}\, q) \overset{SU(3)_F}{=} -\lambda\, \mathcal{A} \left[1 - \underbrace{\epsilon\!\!\!/}_{1}\, e^{i\gamma}\, b\, e^{i\theta} \right]$$

- Example: $B_d \to J/\psi\, K^0$ to $B_d \to J/\psi\, \pi^0$:

  $b \in [0.15, 0.67]$, $\quad \theta \in [174°, 212°] \quad \Longrightarrow \quad \Delta\phi^d_{J/\psi\, K^0} = [-3.9°, -0.8°]$

  S. Faller, R. Fleischer, M. Jung, T. Mannel, PRD 79 014030 (2009)

  $\Delta S_{B_d \to J/\psi K_s} = 0.00 \pm 0.02 \quad \Longrightarrow \quad \Delta\phi^d_{J/\psi\, K^0} = (0.0 \pm 1.5)°$

  M. Ciuchini, M. Pierini, L. Silvestrini, PRL 95 221804 (2005)/arXiv:1102.0392

- Soon also $B_s \to J/\psi\, K^0$

  K. De Bruyn, R. Fleischer, P. Koppenburg, Eur.Phys.J. C70 (2010) 1025-1035

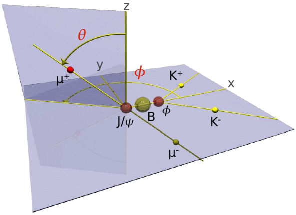
Which $B_s \to J/\psi\ s\bar{s}$?

- The **flagship** decay (Vector-Vector):

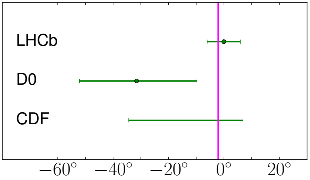
$$B_s \to J/\psi\,\phi$$

requires a time-dependent,

  - **tagged**,
  - **angular analysis** ☹



$\phi_s + \Delta\phi^{\lambda}_{J/\psi\,\phi}$



Which $B_s \to J/\psi\ s\bar{s}$?

- The **flagship** decay (Vector-Vector):

$$B_s \to J/\psi\,\phi$$

requires a time-dependent,
  - **tagged**,
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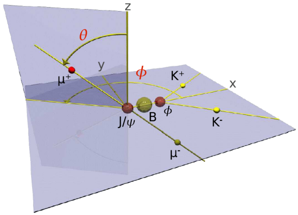
- (Vector-Pseudoscalar):

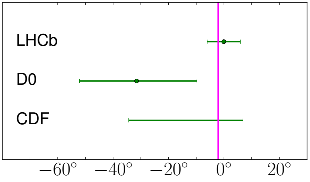
$$B_s \to J/\psi\,\eta^{(\prime)}$$

- (Vector-Scalar):

$$B_s \to J/\psi\,f_0(980)$$



$$\phi_s + \Delta\phi^{\lambda}_{J/\psi\,\phi}$$



$B_s \to J/\psi\eta^{(\prime)}$

$$|\eta\rangle = \cos\phi_P \frac{1}{\sqrt{2}}\left(|u\bar{u}\rangle + |d\bar{d}\rangle\right) - \sin\phi_P |s\bar{s}\rangle$$

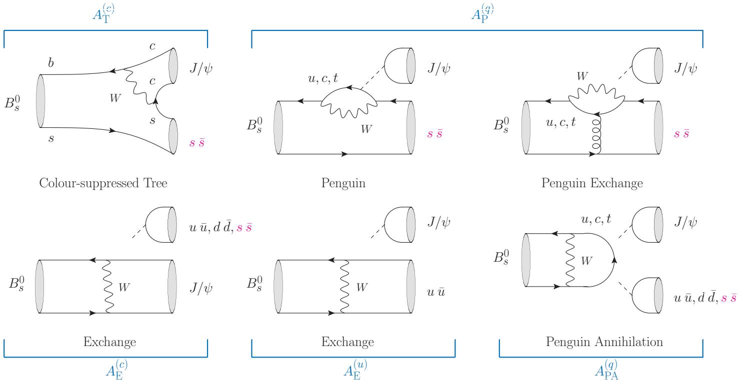
$$|\eta'\rangle = \cos\phi_G \sin\phi_P \frac{1}{\sqrt{2}}\left(|u\bar{u}\rangle + |d\bar{d}\rangle\right) + \cos\phi_G \cos\phi_P |s\bar{s}\rangle\rangle + \sin\phi_G |gg\rangle$$

$$30^\circ \lesssim \phi_P \lesssim 45^\circ \quad \text{and} \quad |\phi_G| \sim 20^\circ$$

C. Di Donato, G. Ricciardi, I. Bigi, Phys.Rev. D85 013016 (2012)

A.S. Dighe, M. Gronau, J.L.Rosner, Phys.Lett. B367 (1996) 357-361

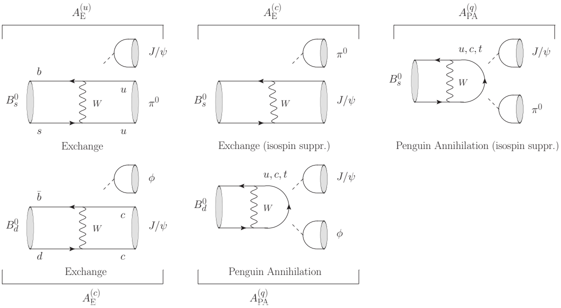
Decay topologies



Control: consider $B_d \to J/\psi\eta^{(\prime)}$:

- hadronic parameters: different ϕ_P, ϕ_G dependence
- Possible if $\boxed{A_{\rm T}, A_{\rm P} \gg A_{\rm E}, A_{\rm PA}}$

Topological hierarchies from flavour symmetry



$$\left| \frac{A_{\mathrm{E}}^{(ct)} + A_{\mathrm{PA}}^{(ct)}}{A_{\mathrm{T}}} \right| \sim \frac{1}{\lambda} \sqrt{\frac{\mathrm{BR}(B_d \to J/\psi\phi)}{\mathrm{BR}(B_d \to J/\psi K^{*0})}} \lesssim 0.1$$

Belle, Phys.Rev. D78 (2008) 011106

Constrain $A_{\mathrm{E}}^{(ut)}$ with $\mathrm{BR}(B_s \to J/\psi\pi^0)$ (last update L3 1997)

$B_s \to J/\psi\eta^{(\prime)}$ experimental progress

- Belle results:

$$\mathrm{BR}(B_s \to J/\psi\eta) = 5.10^{+1.30}_{-0.97} \times 10^{-4} \quad \mathrm{BR}(B_d \to J/\psi\eta) = 12.3^{+1.9}_{-1.8} \times 10^{-6}$$
$$\mathrm{BR}(B_s \to J/\psi\eta') = 3.71^{+1.00}_{-0.85} \times 10^{-4} \quad \mathrm{BR}(B_d \to J/\psi\eta') < 7.4 \times 10^{-6}$$

M. C. Chang et al, Phys. Rev. D85 (2012) 091102

Belle Collaboration, Phys. Rev. Lett. 108 (2012) 181808

- Difficult LHC signature:

$$\eta^{(\prime)} \overset{\text{prominently}}{\longrightarrow} \gamma, \pi^0$$

- Better suited for Super "flavour" factories!

Determining the $\eta^{(\prime)}$ mixing angles

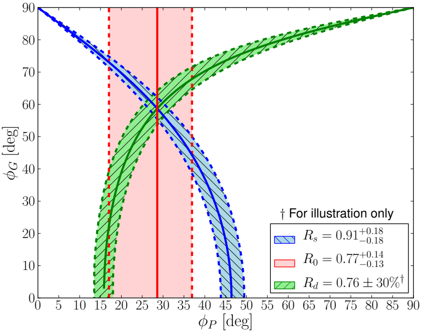
Assuming $A_\mathrm{T}, A_\mathrm{P} \gg A_\mathrm{E}, A_\mathrm{PA}$:

$$R_s \equiv \frac{\mathrm{BR}(B_s \to J/\psi\eta')}{\mathrm{BR}(B_s \to J/\psi\eta)} \left(\frac{\Phi_s^\eta}{\Phi_s^{\eta'}}\right)^3 = \frac{\cos^2\phi_G}{\tan^2\phi_P} = 0.91 \pm 0.18$$

Φ_q^P : phase space

$$R_0 \equiv \frac{\mathrm{BR}(B_d \to J/\psi\eta)}{\mathrm{BR}(B_d \to J/\psi\pi^0)} \left(\frac{\Phi_d^\pi}{\Phi_d^{\eta}}\right)^3 = \cos^2\phi_P = 0.77 \pm 0.14$$

$$R_d \equiv \frac{\mathrm{BR}(B_d \to J/\psi\eta')}{\mathrm{BR}(B_d \to J/\psi\eta)} \left(\frac{\Phi_d^\eta}{\Phi_d^{\eta'}}\right)^3 = \cos^2\phi_G \tan^2\phi_P = \textbf{??}$$



$B_s \to J/\psi\, f_0(980)$

S. Stone and L. Zhang, Phys. Rev. D 79 (2009)

$$B_s \;/\; \bar{B}_s \;\to\; J/\psi\ \not{\phi}\ f_0(980)$$

$f_0(980)$ [a] $I^G(J^{PC}) = 0^+(0^{++})$ **$f_0(980)$ Section References**

See also the minireview on scalar mesons .

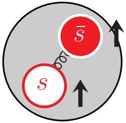
Mass m = (980 ± 10) MeV
Full width Γ = 40 to 100 MeV

$f_0(980)$ DECAY MODES

Γ_i	Mode	Fraction (Γ_i / Γ)	p (MeV/c)
Γ_1	$\pi\,\pi$	dominant	471
Γ_2	$K\,\bar{K}$	seen	-1
Γ_3	$\gamma\,\gamma$	seen	490
Γ_4	$e^+\,e^-$		490

What is the $f_0(980)$?

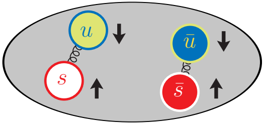
Quark-antiquark picture



$S = 1, \quad \mathbf{L = 1} \quad \rightarrow \quad J = 0$

$$f_0(980) = \cos\varphi_M \; s\,\bar{s} \; + \; \frac{\sin\varphi_M}{\sqrt{2}} \left(u\,\bar{u} \; + \; d\,\bar{d}\right)$$

Tetraquark picture

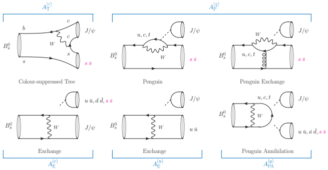


$\mathbf{L = 0}$

$$f_0(980) = \frac{[s\,u][\bar{s}\,\bar{u}] \; + \; [s\,d][\bar{s}\,\bar{d}]}{\sqrt{2}}$$

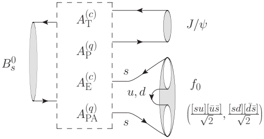
Or. . .

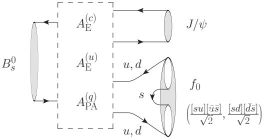
Decay topologies



Quark-antiquark $f_0(980)$

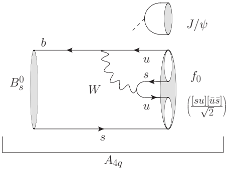
- φ_M mixing angle dependence





Tetraquark $f_0(980)$

- new topology: A_{4q}



Proposed control channel: $B_d \to J/\psi\, f_0(980)$

- **Useful** if :

$$\boxed{A_{\rm T}, A_{\rm P} \gg A_{\rm E}, A_{\rm PA}, A_{4q}}$$

- Constrain A_{4q} with $\mathrm{BR}(B_s \to J/\psi\, a_0^0(980))$ if

$$a_0^0(980) = \frac{1}{\sqrt{2}}([su][\bar{s}\bar{u}] - [sd][\bar{s}\bar{d}])$$

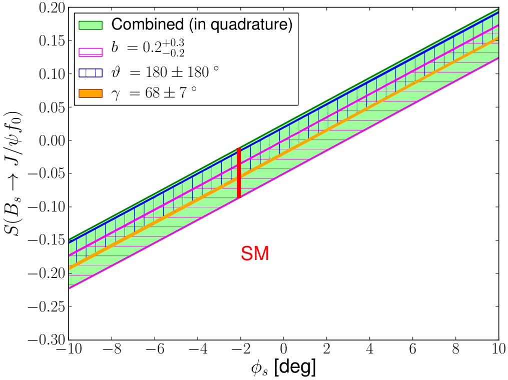
- Assuming $\gamma = 68^\circ$ and $A_{\rm T} > A_{\rm others}$:

$$\Delta\phi_{J/\psi\, f_0} \in [-3^\circ, 3^\circ], \quad C_{J/\psi\, f_0} \leq 0.05$$

- **Predict**:

$$\mathrm{BR}(B_d \to J/\psi\, f_0; f_0 \to \pi^+\pi^-) \sim (1-3)\times 10^{-6} \times \begin{cases} \left[\frac{\tan\varphi_M}{\tan 35^\circ}\right]^2 & : \quad q\bar{q} \\ 1 & : \quad \text{tetraquark} \end{cases}$$

$$S_{J/\psi\, f_0} = \sqrt{1 - C_{J/\psi\, f_0}{}^2}\, \sin(\phi_s + \Delta\phi_{J/\psi\, f_0})$$



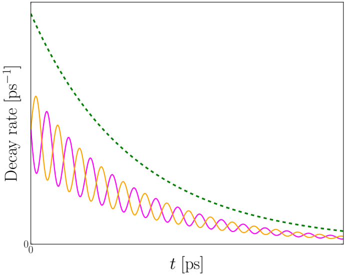
What about **untagged** measurements?

CDF, Phys.Rev. D84 (2011) 052012

$$\tau_{J/\psi\, f_0} = 1.70^{+0.12}_{-0.11} \pm 0.03 \text{ ps}$$

Effective lifetimes

$$\langle\Gamma\rangle \equiv \Gamma(B_s(t) \to f) \ + \ \Gamma(\bar{B}_s(t) \to f)$$
$$\propto e^{-\Gamma_s t}\,[\cosh(\Delta\Gamma_s\, t) + \mathcal{A}_{\Delta\Gamma} \sinh(\Delta\Gamma_s\, t)]$$



Fitting $\frac{1}{\tau_{\text{eff}}} e^{-t/\tau_{\text{eff}}}$ gives:

$$\tau_{\text{eff}} \equiv \frac{\int_0^\infty \ t \ \langle\Gamma\rangle \ dt}{\int_0^\infty \ \langle\Gamma\rangle \ dt}$$
$$= \frac{\tau_{B_s}}{1 - {y_s}^2}\left(\frac{1 + 2\,\mathcal{A}_{\Delta\Gamma}\, y_s + {y_s}^2}{1 + \mathcal{A}_{\Delta\Gamma}\, y_s}\right)$$

$$\text{where :} \quad y_s \equiv \frac{\Delta\Gamma_s}{2\Gamma_s}$$

What about **untagged** measurements?

CDF, Phys.Rev. D84 (2011) 052012

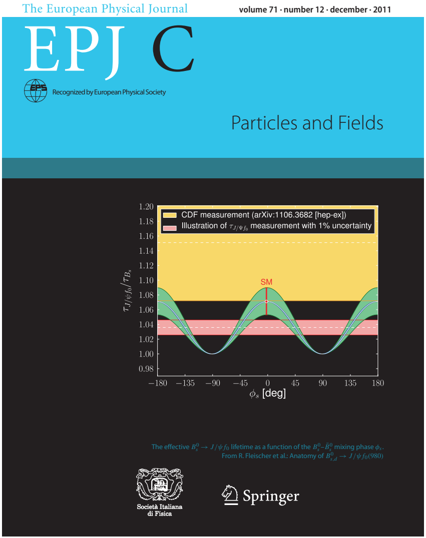
$$\tau_{J/\psi\, f_0} = 1.70^{+0.12}_{-0.11} \pm 0.03 \text{ ps}$$

Using:

$$\frac{\Delta\Gamma_s^{\mathrm{Th}}}{\Gamma_s} = 0.133 \pm 0.032$$

A. Lenz and U. Nierste, arxiv:1102.4274

$\tau_{J/\psi\, f_0}$ **deviates** $1\,\sigma$ from SM expectation



Contours in the ϕ_s–$\Delta\Gamma_s$ plane

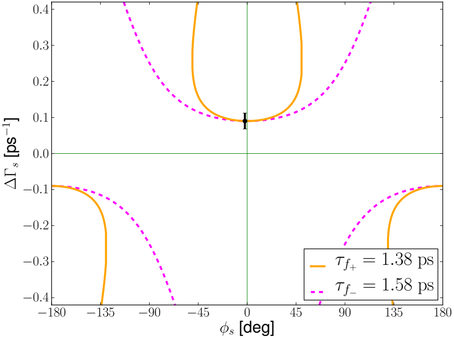
$$y_s{}^3 + \left(\frac{\tau_{B_s} - \tau_{\text{eff}}}{\tau_{\text{eff}}\,\mathcal{A}_{\Delta\Gamma}}\right) y_s{}^2 + \left(\frac{2\,\tau_{B_s} - \tau_{\text{eff}}}{\tau_{\text{eff}}}\right) y_s + \left(\frac{\tau_{B_s} + \tau_{\text{eff}}}{\tau_{\text{eff}}\,\mathcal{A}_{\Delta\Gamma}}\right) = 0$$

$$\boxed{\mathcal{A}^f_{\Delta\Gamma} = -\eta\,\sqrt{1 - C_f^2}\,\cos(\phi_s + \Delta\phi_f)}$$

Assuming:

$$\Delta\phi_f = 0, \;\; C_f = 0$$

$$\mathcal{A}^f_{\Delta\Gamma} = \begin{cases} -\cos\phi_s & : \quad f_{\text{even}} \\ +\cos\phi_s & : \quad f_{\text{odd}} \end{cases}$$



CP odd and even ingredients

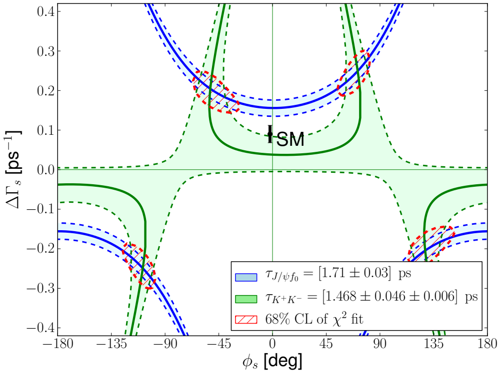
- **CP Odd**: $B_s \to J/\psi\, f_0(980)$ *LHCb, 1204.5675*

$$\tau_{J/\psi\, f_0} = 1.71 \pm 0.03\ \text{ps}, \quad \Delta\phi_{J/\psi\, f_0} \in [-3^\circ, 3^\circ], \quad C_{J/\psi\, f_0} \leq 0.05$$

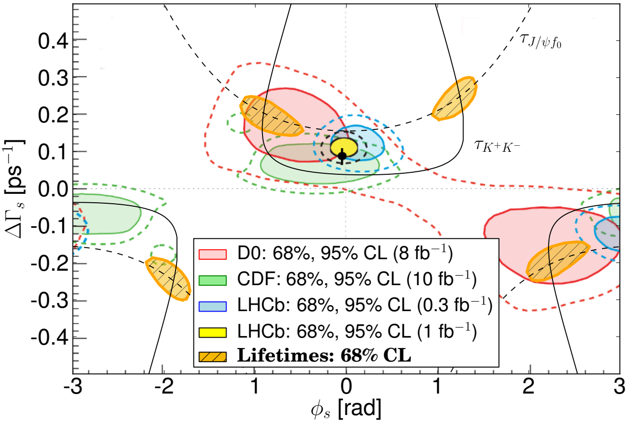
- **CP Even**: $B_s \to K^+K^-$ *LHCB-CONF-2012-001*

$$\tau_{K^+K^-} = 1.468 \pm 0.046\ \text{ps}, \quad \Delta\phi_{K^+K^-} = -\left(10.5^{+3.1}_{-2.8}\right)^\circ, \quad C_{K^+K^-} = 0.09$$

R. Fleischer and RK, Eur.Phys.J. C71 (2011) 1532



Untagged determination of B_s mixing parameters



Note : $\phi_s + \Delta\phi^{\lambda}_{J/\psi\phi} \neq \phi_s + \Delta\phi_{J/\psi f_0(980)}$

Summary

- CP observables $\rightarrow$ SM predictions

Disentangle New Physics 

from **SM Hadronic Physics** 

- $B_s \rightarrow J/\psi\, \bar{s}s$ good decay mode for mixing-induced ϕ_s determinations, but...
  - Left uncontrolled, i.e. $A_{\rm others} \sim A_{\rm T}$, in SM:

$$\Delta\phi \in [-3^\circ, 3^\circ] \quad \text{(particularly for } f_0(980))$$

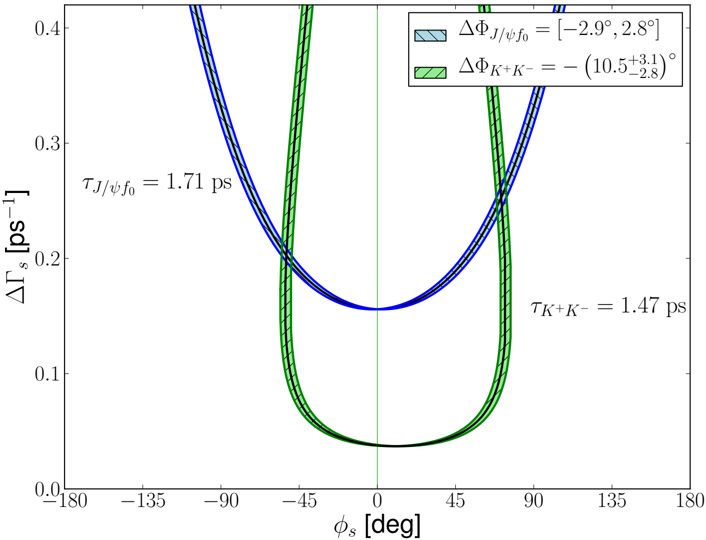
- Taking $\eta, \eta', f_0(980)$ for $s\bar{s}$ meson avoids angular analysis
  - Control possible with B_d counteparts if:

$$A_{\rm T}, A_{\rm P}, \gg A_{\rm E}, A_{\rm PA}, (A_{4q})$$

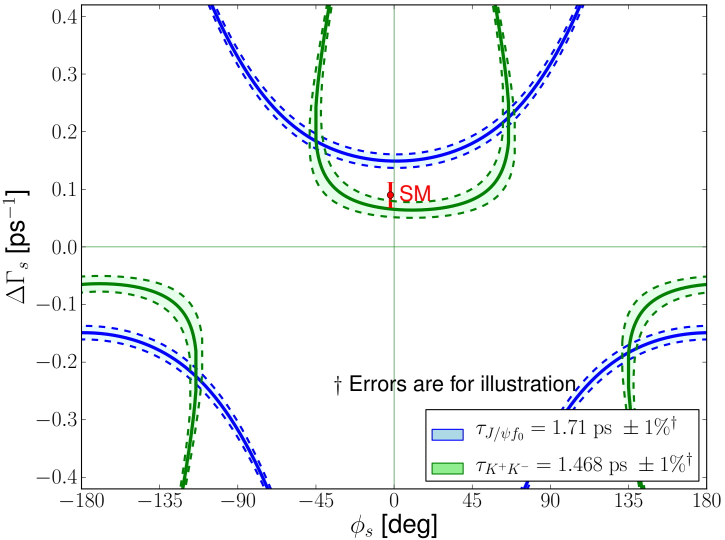
- Can constrain ϕ_s, $\Delta\Gamma_s$ with **pair of effective lifetimes** for **CP even** and **odd** final states

Backup

Hadronic uncertainties



Future precision



Fitting an effective lifetime

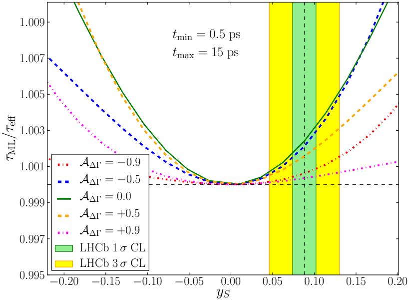
$$f_{\rm true}(t) \equiv \frac{A(t)\,\langle\Gamma(t)\rangle}{\int_0^\infty A(t)\,\langle\Gamma(t)\rangle\,dt}, \qquad f_{\rm fit}(t;\tau) \equiv \frac{A(t)\,e^{-t/\tau}}{\int_0^\infty A(t)\,e^{-t/\tau}\,dt}$$

Minimise : $$-\log L(\tau) = -n\int_0^\infty dt\ f_{\rm true}(t)\log\left[f_{\rm fit}(t;\tau)\right]$$

$$\frac{\int_0^\infty t\,A(t)e^{-t/\tau}\,dt}{\int_0^\infty A(t)e^{-t/\tau}\,dt} = \frac{\int_0^\infty t\,A(t)\langle\Gamma(t)\rangle\,dt}{\int_0^\infty A(t)\langle\Gamma(t)\rangle\,dt}$$

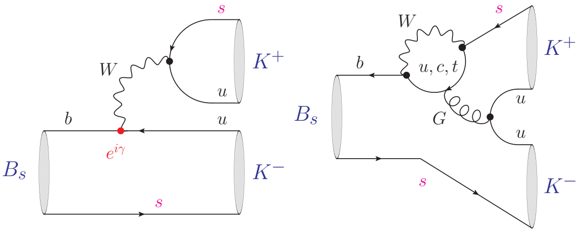
Limit that $A(t) = 1$:

$$\tau = \frac{\int_0^\infty t\,\langle\Gamma(t)\rangle\,dt}{\int_0^\infty \langle\Gamma(t)\rangle\,dt} \equiv \tau_{\rm eff},$$



Controlling the **CP Even** Decay Mode

$$B_s \to K^+ K^-$$



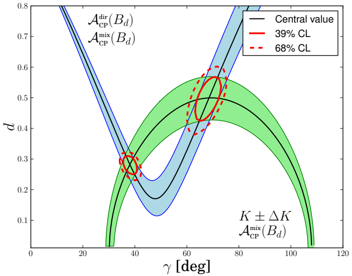
$$A(B_s^0 \to K^+ K^-) = \lambda \mathcal{C} \left[e^{i\gamma} + \frac{1}{\epsilon} d\, e^{i\theta} \right]$$

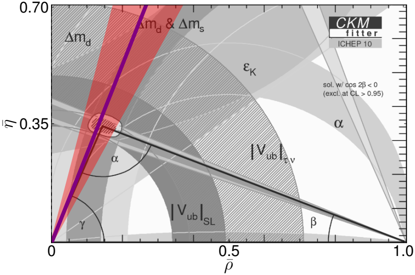
- Use *U*-**spin** flavour symmetry:

interchange $s \leftrightarrow d$ quarks

Related to $B_d \to \pi^+ \pi^-$

U-spin determination of hadronic parameters and γ





$$\gamma = (68 \pm 7)^\circ, \quad d = 0.50^{+0.12}_{-0.11}, \quad \theta = \left(154^{+11}_{-14}\right)^\circ$$

$$\boxed{\Delta\phi_{K^+K^-} = -\left(10.5^{+3.1}_{-2.8}\right)^\circ}, \quad C_{K^+K^-} = 0.09 \pm 0.05$$

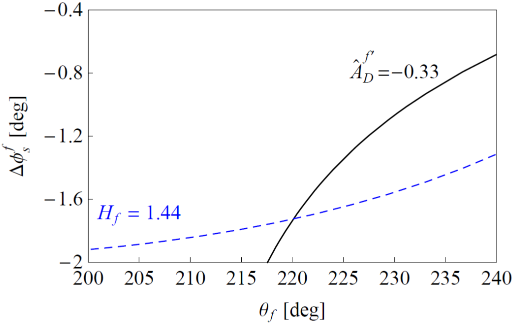
R. Fleischer and RK, Eur.Phys.J. C71 (2011) 1532

$B_s \to J/\psi\phi$ hadronic uncertainties

$$\text{Measure :} \quad \phi_s + \Delta\phi^f_{J/\psi\phi}$$

- **Numerical example** compatible with $\Delta\phi_d$ analysis

  S. Faller, R. Fleischer and T. Mannel, Phys.Rev. D79 (2009) 014005



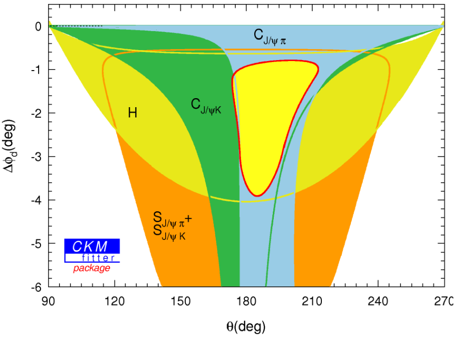
- Future control channels: $B_s \to J/\psi \bar{K}^{*0}$ and $B_d \to J/\psi \rho^0$

Hadronic uncertainty of B_d^0–$\bar{B}_d^0$ mixing

$$\text{Measure :} \quad 2\beta + \Delta\phi_d$$

Probe using $B_d \to J/\psi\, K_S$ and $B_d \to J/\psi\, \pi$

S. Faller, R. Fleischer, M. Jung, T. Mannel, Phys.Rev. D79 (2009) 014030



See also: *Extracting gamma and Penguin Topologies through CP Violation in* $B_s^0 \to J/\psi K_S$, *K. De Bruyn, R. Fleischer and P. Koppenburg* (arXiv:1010.0089)

Tetraquarks

- diquark–antidiquark (colour) bound states

$$\sigma = [ud][\bar{u}\bar{d}]$$
$$\kappa = [su][\bar{u}\bar{d}]; [sd][\bar{u}\bar{d}] \quad (+\mathrm{c.d})$$
$$f_0 = \frac{[su][\bar{s}\bar{u}] + [sd][\bar{s}\bar{d}]}{\sqrt{2}}$$
$$a_0 = [su][\bar{s}\bar{d}]; \frac{[su][\bar{s}\bar{u}] - [sd][\bar{s}\bar{d}]}{\sqrt{2}}; [sd][\bar{s}\bar{u}]$$

diquark $\equiv [q_1\, q_2]$, colour $\overline{\mathbf{3}}$, flavour $\overline{\mathbf{3}}$, $S = 0$

- Issues: $f_0 \to \pi\pi$ coupling too small, $a_0 \to \eta\pi$ too large.
- Solved by adding *instanton-induced effects*

A Theory of Scalar Mesons, G. 't Hooft, G. Isidori, A.D Polosa, V. Riquer,

(arXiv:0801.2288)

Notation

The CP asymmetry:

$$\frac{\Gamma\,(B_s(t) \to f) - \Gamma(\bar{B}_s(t) \to f)}{\Gamma(B_s(t) \to f) + \Gamma(\bar{B}_s(t) \to f)} = \frac{C \cos(\Delta M_s\, t) - S\, \sin(\Delta M_s\, t)}{\cosh(\Delta\Gamma_s\, t) + \mathcal{A}_{\Delta\Gamma} \sinh(\Delta\Gamma_s\, t)}$$

Observables for $\mathcal{CP}|f\rangle = \eta\,|f\rangle$:

$$\xi_f \equiv \frac{q}{p}\, \frac{A(\overline{B}_s^0 \to f)}{A(B_s^0 \to f)} = -\eta\; e^{-i\phi_s}\, \sqrt{\frac{1-C}{1+C}}\, e^{-i\Delta\phi}$$

$$\mathcal{A}_{\Delta\Gamma} - i\,S = \frac{2\;\xi_f}{1+|\xi_f|^2} = \boxed{-\eta\,\sqrt{1-C^2}\; e^{-i(\phi_s+\Delta\phi)}}$$

Untagged observable: General Formalism

$$\xi = -\eta\, e^{-i\phi_s} \left[\frac{e^{-i\varphi_1} + e^{-i\varphi_2}\, h\, e^{i\delta}}{e^{i\varphi_1} + e^{i\varphi_2}\, h\, e^{i\delta}} \right]$$

$$\boxed{\frac{2\,\xi}{1+|\xi|^2} = -\eta\,\sqrt{1-C^2}\, e^{-i(\phi_s+\Delta\phi)}}$$

$$C = \frac{2\,h\,\sin\delta\,\sin(\varphi_1-\varphi_2)}{1+2\,h\,\cos\delta\cos(\varphi_1-\varphi_2)+h^2}$$

$$\Delta\Phi = \arctan\left(\frac{\sin 2\varphi_1 + 2\,h\,\cos\delta\,\sin(\varphi_1+\varphi_2) + h^2\,\sin 2\varphi_2}{\cos 2\varphi_1 + 2\,h\,\cos\delta\cos(\varphi_1+\varphi_2) + h^2\,\cos 2\varphi_2}\right)$$

$$\mathcal{A}_{\Delta\Gamma} = -\eta\cos\phi_s \quad \rightarrow \quad \boxed{\mathcal{A}_{\Delta\Gamma} = -\eta\,\sqrt{1-C^2}\,\cos(\phi_s+\Delta\phi)}$$