Heavy quark expansion in QCD and B phenomenology

N. Uraltsev

Introduction; HQE in QCD

The $1/m_Q$ expansion in QCD can be constructed for a few important cases where we now know quite a bit about its terms

The key information is provided by the Small Velocity heavy quark sum rules, including spin sum rules first established for heavy quarks in QCD

need a scheme respecting physical properties (unitarity,...); it is available!

Allowed to predict values of $\overline{\Lambda}$, μ_π^2, ρ_D^3, ... based on one number: the hyperfine mass splitting $M_{B^*} - M_B \approx 47\,\text{MeV}$

The most precision applications have been done for inclusive decays

A similar analysis has been extended motivated by the formfactor $F(0)$ in $B \to D^* \ell\nu$ near zero recoil   Gambino, Mannel, N.U. arXiv:1004.2859 [hep-ph]
arXiv:1206.xxxx [hep-ph]

Model-independent treatment of heavy mesons
The status report   (72 pages...)

Arrived at three apparently isolated, yet linked through the HQE, observations for heavy meson phenomenology

- Large negative *overlap* corrections to $F(0)$ driving it down to $F(0)\approx 0.86$

- Large nonlocal correlators of $\bar{Q}\vec{\pi}^2 Q$ and $\bar{Q}\vec{\sigma}\vec{B}Q$ in B mesons from the hyperfine splitting ΔM^2 in B vs. D

  The enhanced negative corrections in $F(0)$ are related to the 'discrepancy' in the hyperfine splitting ratio between charm and beauty mesons

- Enhanced inclusive yield of *radials* and 'D-waves' in $b \to c\,\ell\nu$

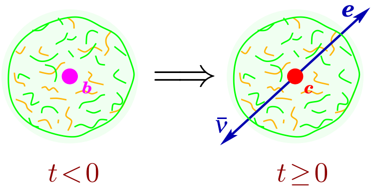
  Resolve '$\frac{1}{2} > \frac{3}{2}$' paradox
  Account for the missing semileptonic channels
  Predict significance of the $\frac{3}{2}^+$ 'D-wave'

- As a byproduct we find significant corrections to the ground-state factorization; relevant for precision inclusive decays

V_{cb} at zero recoil

$$\mathrm{d}w\,(B \to D^* + \ell\bar{\nu}) \;\sim\; G_F^2 \cdot |V_{cb}|^2 \cdot |\vec{p}\,| \cdot |F_{B\to D^*}(\vec{p}\,)|^2$$

$|V_{cb}|$ requires $F_{B\to D^*}(\vec{p}\,)$ – it is shaped by bound-state physics



At $\vec{p}=0$ $(\,\vec{p}_e = -\vec{p}_{\bar{\nu}}\,)$

almost nothing happened!

Without *isotopic* effects (in the heavy quark limit) $F_{\vec{p}=0} = 1$:

$$F_{n/p}(0) \;=\; 1 + \frac{0}{m_{c,b}} + \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{m^2_{c,b}}\right) + \mathcal{O}\left(\frac{\Lambda^3_{\rm QCD}}{m^3_{c,b}}\right) + \ldots$$

No $1/m_{b,c}$-corrections

(cf. Ademollo-Gatto)

1986 Voloshin, Shifman
1990 Luke

Challenge to theory: corrections to $F(0)=1$ are driven by $1/m_c$,
potentially significant!

HQET predicted them to be only about -0.02 prior to 05/1994

In fact, (05/1994) deviations from the symmetry limit in QCD are *considerably larger* in QCD (05/1994) Shifman, N.U., Vainshtein

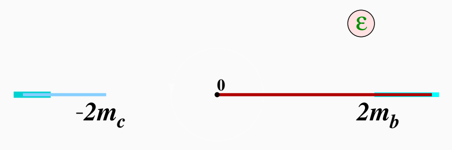
$$\delta_{1/m^2} F \approx -0.09$$

The typical folklore around *sum rules for F_{D^*}* is irrelevant...

The QCD approach

$$T^{\rm zr}(q_0) = \int \mathrm{d}^3x \int \mathrm{d}x_0\; e^{-iq_0x_0} \frac{1}{2M_B} \langle B| \tfrac{1}{3}\; iT\; \{\bar{c}\gamma_k\gamma_5 b(x)\; \bar{b}\gamma_k\gamma_5 c(0)\} |B\rangle$$

$$q_0 = M_B - M_D{}^* - \epsilon$$



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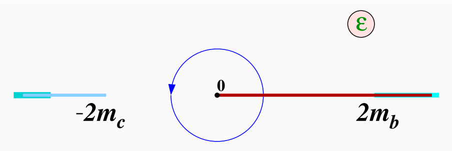

$$I_0(\mu)=-\frac{1}{2\pi i}\oint_{|\epsilon|=\mu} T^{\rm zr}(\epsilon)\,\mathrm{d}\epsilon$$

$T^{\rm zr}(\epsilon)$ can be calculated in the short-distance expansion at $|\epsilon|\gg\Lambda_{\rm QCD}$, expansion parameters are $\frac{\mu_{\rm hadr}}{\epsilon}$, $\frac{\mu_{\rm hadr}}{2m_c+\epsilon}$, $\frac{\mu_{\rm hadr}}{2m_b-\epsilon}$ and α_s at the related scale

hence OPE for $I_0(\mu)$

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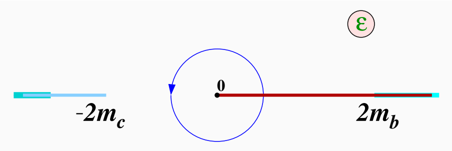

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Using analytic properties of $T^{\rm zr}(\epsilon)$ and the unitarity

$$I_0(\mu) = \frac{1}{\pi}\int_0^\mu \mathrm{Im}\;T^{\rm zr}(\epsilon)\,\mathrm{d}\epsilon = |F_{D^*}|^2 + \sum_{\epsilon<\mu}|F_{B\to n}|^2$$

OPE in QCD

$$\sum_{f\neq D^*}^{\epsilon_f<\mu} \left|F_{B\to f}\right|^2 \;\equiv\; w_{\rm inel}(\mu)$$

$$F_{D^*} = \sqrt{I_0(\mu) - w_{\rm inel}(\mu)} \;<\; \sqrt{I_0(\mu)}$$

OPE:

$$I_0(\mu) = \xi_A^{\rm pert}(\mu) - \Delta_{\frac{1}{m_Q^2}} - \Delta_{\frac{1}{m_Q^3}} - \Delta_{\frac{1}{m_Q^4}} - \ldots$$

$$\Delta_{\frac{1}{m_Q^2}} = \frac{\mu_G^2}{3m_c^2} + \frac{\mu_\pi^2-\mu_G^2}{4}\left(\frac{1}{m_c^2}+\frac{1}{m_b^2}+\frac{2}{3m_c m_b}\right)$$

$$\Delta_{\frac{1}{m_Q^3}} = \frac{\rho_D^3-\frac{1}{3}\rho_{LS}^3}{4m_c^3} + \frac{\rho_D^3+\rho_{LS}^3}{6m_c m_b}\left(\frac{1}{m_c}-\frac{1}{2m_b}\right)$$

...

Brief synopsis

$$F_{D^*} = \sqrt{\xi_A^{\rm pert} - \Delta_{\rm power} - w_{\rm inel}}$$

QM: $$F_{D^*} = \sqrt{\xi_A^{\rm pert}} - \langle \delta J^A \rangle - \delta \langle \Psi_{D^*}^\dagger \Psi_B \rangle$$

$$\sqrt{\xi_A^{\rm pert}} \simeq 0.98 \quad \text{at } \mu \simeq 0.8\,\text{GeV}, \qquad -\Delta_{\frac{1}{m_Q^2}} - \Delta_{\frac{1}{m_Q^3}} \simeq -0.13$$

$$F_{D^*} \le 0.92 \quad \text{– upper bound}$$

$w_{\rm inel}$ – *wavefunction overlap deficit* – is more significant than expected

N.U. hep-ph/0312001

Relate it to the hyperfine splitting in B and D: $w_{\rm inel} \gtrsim 0.14$

$$F_{D^*} \approx 0.86 \quad \text{– prediction}$$

GMU 2010; 2012

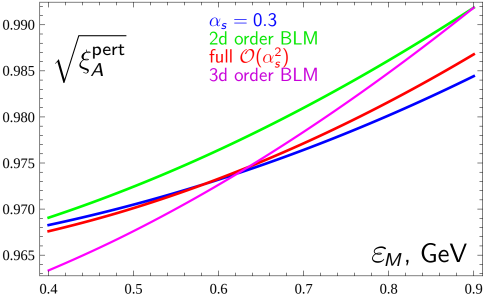
Convergence

How well is the OPE under control?

- Perturbative corrections: small numerically

$$\sqrt{\xi_A} \simeq 1 - 0.022 + (0.005 - 0.004) + 0.0022 - 0.0014 + \ldots$$

applies only to Wilsonian $\xi_A^{\text{pert}}(\mu)$!

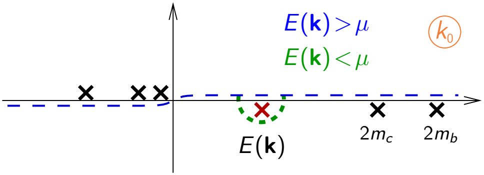


Assume $\mu = \varepsilon_M$ around 0.8 GeV

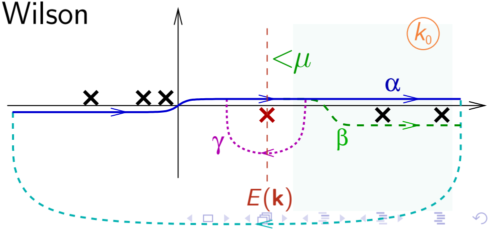
$\sqrt{\xi_A} \simeq 0.98$ at $\alpha_s(m_b) = 0.22$

uncertainty 1% seems reasonably conservative

- The recipe to calculate the whole μ-dependence of the Wilson coefficient is nontrivial:



Allows to obtain $\xi_A(\mu)$ without explicitly considering the power-suppressed operators, yet ensures the cancellation of μ-dependence between the operators and the Wilson coefficient



Convergence

- Power corrections to I_0:

Take the low-end expectation values $\mu_\pi^2 \simeq 0.4\,\text{GeV}^2$, $\rho_D^3 \simeq 0.15\,\text{GeV}^3$:

$-\Delta_{\frac{1}{m_Q^2}} \simeq -0.095$ $\quad -\Delta_{\frac{1}{m_Q^3}} \simeq -0.028$ $\quad -\Delta_{\frac{1}{m_Q^4}} \simeq 0.02$ $\quad -\Delta_{\frac{1}{m_Q^5}} \simeq 0.01$

- α_s-corrections to the coefficients of power-suppressed terms

Calculated for μ_π^2, correction is small (cancellation)
Expect mild effect for μ_G^2; in $1/m_Q^3$ even a 30% renormalization would not produce a significant change

$\sqrt{1-\Delta^A} \lesssim 0.94 \pm 0.01$ the upper bound seems safe at 1% level

Would expect formfactor about 0.92 *if no overlap deficit were there*

Wavefunction overlap deficit in the language of Quantum Mechanics

Required to turn the upper bound for F_{D^*} into an estimate

Model-independent analysis:

$$\mathrm{d}w_{\rm inel}(\omega) = \left(\frac{1}{2m_c}-\frac{1}{2m_b}\right)^2 \frac{\rho_p^{(\frac{1}{2}^+)}(\omega)}{\omega^2} + \left(\frac{1}{2m_c}-\frac{1}{2m_b}\right)\left(\frac{1}{3m_c}+\frac{1}{m_b}\right)\frac{\rho_{pg}^{(\frac{1}{2}^+)}(\omega)}{\omega^2} + \frac{1}{4}\left(\frac{1}{3m_c}+\frac{1}{m_b}\right)^2 \frac{\rho_g^{(\frac{1}{2}^+)}(\omega)}{\omega^2} + \frac{1}{6m_c^2}\frac{\rho_g^{(\frac{3}{2}^+)}(\omega)}{\omega^2}$$

- The four spectral densities $\rho^{(+)}(\omega)$ form a positive set
- Factorization properties
- The $1/m_Q$ correction to the hyperfine splitting is expressed through

$$\rho_{\rm hp} = \int \frac{\mathrm{d}\omega}{\omega}\left(\frac{\rho_g^{(\frac{3}{2}^+)}(\omega)}{2} + 2\rho_{pg}^{(\frac{1}{2}^+)}(\omega) - \frac{2\rho_g^{(\frac{1}{2}^+)}(\omega)}{3}\right)$$

A simple illustration:

$$I_1(\mu) = -\frac{1}{2\pi i}\oint\limits_{|\epsilon|=\mu} T^{\rm zr}(\epsilon)\,\epsilon\,{\rm d}\epsilon = \sum_{\epsilon<\mu}\epsilon_n |F_{B\to n}|^2$$

$$w_{\rm inel}(\mu) = \frac{I_1(\mu)}{\bar\epsilon(\mu)}$$

$\bar\epsilon(\mu)$ – 'average excitation energy' (up to μ)

$I_1(\mu)$ is calculated in the OPE similar to $I_0(\mu)$:

$$I_1 = \underbrace{\frac{-(\rho^3_{\pi G}+\rho^3_A)}{3m_c^2}}_{\text{BPS limit}} + \underbrace{\frac{-2\rho^3_{\pi\pi}-\rho^3_{\pi G}}{3m_c m_b}}_{(\delta\,\text{BPS})^1} + \underbrace{\frac{\rho^3_{\pi\pi}+\rho^3_{\pi G}+\rho^3_S+\rho^3_A}{4}}_{(\delta\,\text{BPS})^2}\left(\frac{1}{m_c^2}+\frac{2}{3m_c m_b}+\frac{1}{m_b^2}\right)+\mathcal{O}\left(\frac{1}{m_Q^3}\right)$$

$$I_1^{\rm (BPS)} = \frac{-(\rho^3_{\pi G}+\rho^3_A)}{3m_c^2}+\mathcal{O}\left(\frac{1}{m_c^3}\right)$$

$(\rho^3_{\pi G}+\rho^3_A)-\rho^3_{LS}$ determines ΔM^2 to order $1/m_Q$

Extract comparing B and D mesons a technical point of the analysis

$\delta^{(2)}I_1$ is positive; $\delta^{(1)}I_1$ comes with small coefficient $1/3m_c m_b$ and the minimum is very shallow:

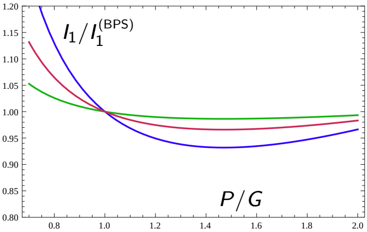
$$I_1/I_1^{\rm (BPS)} = 1 - \underbrace{(1-\nu_{3/2})\frac{m_c^2}{m_b^2}}_{<0.07} + [...]^2 \qquad \nu_{3/2}>0$$

Nearly a functional relation between $\delta\Delta M^2$ and $w_{\rm inel}$

$$w_{\rm inel} = \frac{0.45\,{\rm GeV}^3 + \tilde\kappa\, 0.35\,{\rm GeV}^3}{3m_c^2\,\epsilon_{\rm rad}} \simeq 14\%$$

Analysis of hyperfine splitting: $|\tilde\kappa| \lesssim 0.15$

A 6% decrease in F_{D^*}



The way to evaluate $\sum|F_{B\to n}|^2$ through I_1 in the 't Hooft model yields almost exact number

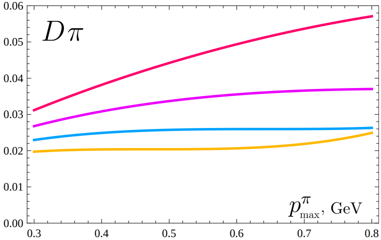
Continuum

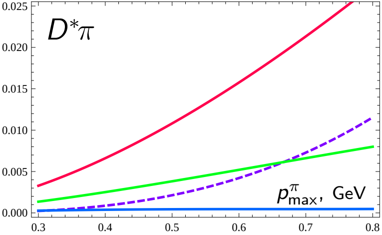
Excited states should predominantly be resonances (*radial* excitations)

Continuum is $1/N_c$-suppressed and is usually smaller

$D^{(*)}\pi$ inelastic contributions can be estimated for soft pions

The complete estimate yields a significant contribution:





$g_{D^*D\pi} = 4.9\,\mathrm{GeV}^{-1} \quad (\Gamma_D = 96\,\mathrm{KeV})$

$g_{B^*B\pi}/g_{D^*D\pi} = 1,\ 0.8,\ 0.6$ and 0.4

$g_{B^*B\pi}/g_{D^*D^*\pi} = 1.3,\ 1,\ 0.8,\ 0.6$

Altogether we expect $D^{(*)}\pi$ piece to yield around 4% in $\sum |F_{B\to n}|^2$;
about a fourth of the resonance-based estimate $\delta F_{D^*} \simeq -2\%$

F_{D^*} summary

QCD lower bound:

$F_{D^*} < 0.92$ $\quad$ $F_{D^*} < 0.9$ including continuum estimate

The unbiased predicted value

$$F_{D^*} \lesssim 0.86$$

The central number has about 2% e-bars it may lower if μ_π^2 turns out larger

Central value goes down for increasing μ_π^2 yet the corrections from higher power terms also increase

$B \to D\,\ell\nu$ near zero recoil

Experimentally challenging theoretically advantageous

$$\langle D(p_2)|\bar{c}\gamma_\nu b|B(p_1)\rangle = f_+(p_1+p_2)_\nu + f_-(p_1-p_2)_\nu \qquad f_\pm \equiv f_\pm(\vec{q}^{\,2})$$

$F_+ \equiv \frac{2\sqrt{M_B M_D}}{M_B+M_D}\, f_+$ has $1/m_Q$ corrections...

$$F_+ = 1 + \left(\frac{\overline{\Lambda}}{2} - \overline{\Sigma}\right)\left(\frac{1}{m_c} - \frac{1}{m_b}\right)\frac{M_B - M_D}{M_B + M_D} - \mathcal{O}\left(\frac{1}{m_Q^2}\right)$$

$$\overline{\Lambda} = M_B - m_b, \quad \overline{\Sigma} = \ldots$$

$$\boxed{\frac{2\sqrt{M_B M_D}}{M_B + M_D}\, f_+(0) = 1.04 \pm 0.01 \pm 0.01}$$

N.U. 2003

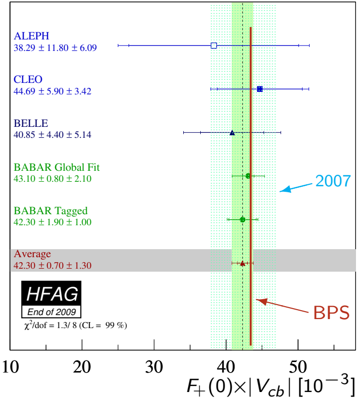
All orders in $1/m$ in 'BPS', to $1/m^2 \cdot 1/\mathrm{BPS}^2$, α_s^1

The bulk 3% is the perturbative factor, only 1% comes from power terms

Taking V_{cb} from $\Gamma_{\rm sl}(B)$:

$$F_+(0) \simeq 1.021 \pm 0.019 \pm 0.041 \pm \delta_{\rm incl}$$

Good agreement with the dynamic heavy quark expansion in QCD



Using $|V_{cb}|$ from $\Gamma_{\rm sl}(B)$ I predicted

$$|V_{cb}|F_+(0) = 43.\not{7}\cdot 10^{-3}$$
5

There is no tension between *inclusive* and *exclusive* V_{cb}

rather a remarkable agreement

Estimated D^{**} contribution around 15% of the D^* at zero recoil

These are 'radial' excitations ($j^P = \frac{1}{2}^+$) or 'D-wave' states ($j^P = \frac{3}{2}^+$)

Prediction: $\gtrsim 7 \div 8\%$ of $\Gamma_{\rm sl}$ over the full phase space Why?

The dominant amplitude is not the HQET one $\propto \mathbf{v}^2 \ll 1$, but $1/m_c$!

$$A(B \to D^{(n)^+}) = G_F V_{cb} \left[r\,\mathbf{v}^2 + P_c \frac{\mu_{\rm hadr}}{m_c} + P_b \frac{\mu_{\rm hadr}}{m_b} + \ldots \right]$$

Sum of the squared for the latter is fixed by $w_{\rm inel}$ at zero recoil

$$\mathrm{BR}_{\text{'rad'}} \propto w_{\rm inel} = \xi_A^{\rm pert} - F_{D^*}^2 - \Delta_A$$

We can evaluate $w_{\rm inel}$ well enough and do not need a phenomenological extraction

Using a certain trick radically simplifies the inclusive calculation

'Radials'

Production of concrete excited mesons is unreliable: heavy quark symmetry works poorly for excited charmes states

The total yield, in particular summed over the 'radials' and 'D-waves' is stable and well constrained

$$\frac{\Gamma_{\rm sl}(B \to \mathcal{R}^{(1)}) + \Gamma_{\rm sl}(B \to \mathcal{D}^{(1)})}{\Gamma_{\rm sl}} \gtrsim 7\%$$

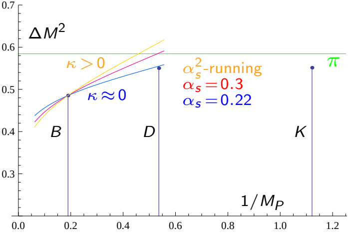
Probably the charm mesons with $M \simeq 2.6\,\mathrm{GeV}$ are $\frac{1}{2}^+$ and with $M \simeq 2.75\,\mathrm{GeV}$ are $\frac{3}{2}^+$

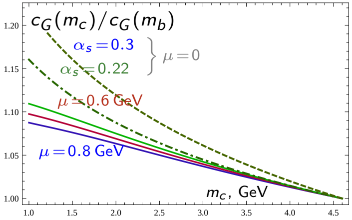
We have already encountered: the second negative correction to $F_{D^*}(0)$

$$M_{B^*}^2 - M_B^2 \simeq M_{D^*}^2 - M_D^2 \simeq M_{K^*}^2 - M_K^2 \simeq M_\rho^2 - M_\pi^2$$

If these were exact and if perturbative corrections could be discarded

$$-(-\rho_{LS}^3 + \rho_{\pi G}^3 + \rho_A^3) \simeq 2\overline{\Lambda}\mu_G^2\,(1+\kappa)$$





The final outcome: $\kappa \approx -0.2$ and $-(\rho_{\pi G}^3 + \rho_A^3) \approx 0.45\,\text{GeV}^3$ – large!

$$\rho^3_{\pi\pi} + \rho^3_S = -(\rho^3_{\pi G} + \rho^3_A) + \int \frac{\mathrm{d}\omega}{\omega} \left[\rho_p^{\frac{1}{2}+} - 2\rho_{pg}^{\frac{1}{2}+} + \rho_g^{\frac{1}{2}+}\right] > -(\rho^3_{\pi G} + \rho^3_A)$$

Equality is attained in the BPS limit

We can use the existing precision fits to semileptonic moments to determine $\rho^3_{\pi\pi} + \rho^3_S$; the preliminary result is

$$\rho^3_{\pi\pi} + \rho^3_S \gtrsim (0.33 \pm 0.17)\,\text{GeV}^3$$

Another intriguing consistency of the completely different analyses

Summary

Comprehensive heavy quark expansion can say much about nonperturbative effects in certain cases

Inequalities or positivity properties are essential, require a physical renormalization scheme – it is available!

Heavily exploit physical behavior of the correlators in Minkowski domain

- $F_{D^*} \approx 0.86$; uncertainty about 2% at known μ_π^2, ρ_D^3, plus effect of higher-order power terms. Central value goes down for larger μ_π^2, yet higher power corrections become significant
- Large *overlap deficit* decreasing F_{D^*} by 6%

  Close to the BPS estimate
- Large nonlocal correlators $\rho^3_{...}$ from the hyperfine splittings
  'Discrepancy' in the splitting for B and D is settled
- Large inclusive yield of 'radials' plus 'D-waves', $7\div 10\%$ of $\Gamma_{\rm sl}$
  Resolve $\frac{1}{2}$ vs. $\frac{3}{2}$ problem
  May provide missing semileptonic channels


RECYCLE

Lattice $F_{D^*}(0)$

$F_{D^*} = 0.924 \pm 0.012 \pm 0.019$ FNAL: arXiv:0710.1111 [hep-lat]

GMU: arXiv:1004.2859 [hep-ph]

$F_{D^*} = 0.908 \pm 0.005 \pm 0.016$ FNAL: arXiv:1011.2166 [hep-lat] PRELIMINARY

0.902 A.Kronfeld: 2012, undocumented

This is not a first-principle calculation of F_{D^*} – it is considered in an effective theory very different from the QCD nonrelativistic theory

All effects $1/m_Q^k$ are different, driven by different masses rather than by a single m_Q; renormalization is $\mathcal{O}(1)$ rather than $\mathcal{O}(\alpha_s)$ in QCD

'Matching' is done only at tree level! lattice $\alpha_s(m_c)$...

only for $1/m_Q^2$ the hardest scale

The FNAL lattice evaluation of F_{D^*} seemed to violate unitarity once we apply the $1/m_Q$ expansion of the scattering amplitude and consider known inelastic channels does it?

Lattice $F_{D^*}(0)$

FNAL lattice			OPE in QCD	
effect	accuracy	allocated error bar	accuracy	magnitude
$\eta_{\rm pert}$	α_s @BLM	± 0.003	$\alpha_s^2 + \alpha_s^3$ (BLM)	-0.004
$1/m_Q^2$ in overlap	tree	± 0.005	tree	-0.06
$1/m_Q^2$ in J_μ	tree (?)	± 0.009	α_s	± 0.012
$1/m_Q^3$ in J_μ	ad hoc	± 0.009	tree	-0.015
$1/m_Q^4$ in J_μ	ad hoc	0	tree	0.012
$1/m_Q^5$ in J_μ	ad hoc	0	tree	0.006
$1/m_Q^{3+}$ in overlap	ad hoc	0	–	?

Simply adding systematics linearly more than doubles the error bar!

If we applied the same error rules we would have ended up with

$$F_{D^*}(0) = 0.85 \pm 0.009 \pm 0.005$$

or even smaller error bars (there is nearly no model-dependence to order $1/m_Q^2$)