THE SCALES OF SOFT GLUON RESUMMATION

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SUMMARY

- THE PERTURBATIVE ORIGIN OF SOFT RESUMMATION
  (S.F., G.Ridolfi, 2003)
  - FACTORIZATION
  - RESUMMED EXPRESSIONS
  - THE MEANING OF SOFT RESUMMATION
  - THE LANDAU POLE: DIVERGENCE OF THE PERTURBATIVE EXPANSION
- SOFT RESUMMATION IN SCET
  - RESUMMED EXPRESSIONS
  - THE SOFT SCALE
- COMPARING SCET TO PERTURBATIVE QCD
  (M.Bonvini, M.Ghezzi, S.F., G.Ridolfi, 2012)
  - THE MASTER FORMULA
  - PERTURBATIVE PROPERTIES OF THE SCET EXPRESSION
  - THE BECHER-NEUBERT HADRONIC SOFT SCALE
- THE LESSONS LEARNT

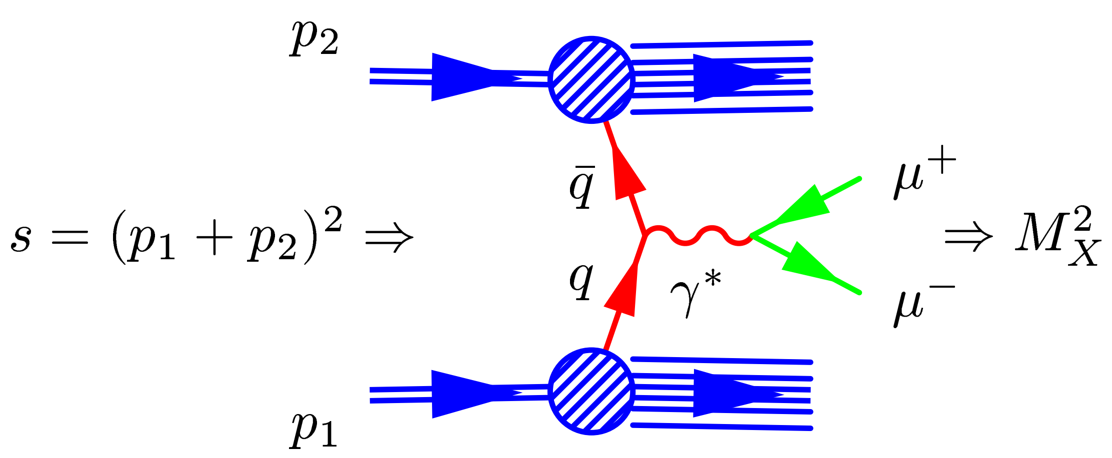
SOFT RESUMMATION IN PERTURBATIVE QCD

FACTORIZATION

$$\sigma(\tau, M^2) = \int_\tau^1 \frac{dz}{z} C(z, M^2)\mathcal{L}\left(\frac{\tau}{z}\right)$$

- $\sigma(\tau, M^2) = \frac{1}{\tau\sigma_0}\frac{d\sigma_{\rm DY}}{dM^2}$ (DIMENSIONLESS HADRONIC CROSS SECTION)
- $\mathcal{L}(\tau) = \sum_{a,b}\int_\tau^1 \frac{dx}{x} f_{a/h_1}(x) f_{b/h_2}(\tau/x)$ (PARTON LUMINOSITY)

DRELL-YAN



- Hadronic c.m. energy: $s = (p_1 + p_2)^2$
- Momentum fractions $x_{1,\,2} = \sqrt{\frac{\hat{s}}{s}} \exp \pm y$;
  Lead. Ord. $\hat{s} = M^2$
- Partonic c.m. energy: $\hat{s} = x_1 x_2 s$
- Invariant mass of final state X (dilepton, Higgs,...):
  $M_W^2 \Rightarrow$ scale of process
- Scaling variable $\tau = \frac{M_X^2}{s}$

N-SPACE FACTORIZATION

$$\sigma(N, M^2) = \int_0^1 d\tau\, \tau^{N-1}\sigma(\tau, M^2) = C(N, M^2)\mathcal{L}(N)$$

MELLIN-SPACE FACTORIZATION $\leftrightarrow$ LONGITUDINAL MOMENTUM CONSERVATION

RESUMMED EXPRESSIONS

- CLOSE TO PARTONIC THRESHOLD, $C(z) \sim \alpha_s^k \frac{\ln^{2k-1}(1-z)}{1-z}_+$
- $\int_0^1 dz\, z^{N-1} \left[\frac{\ln^{p-1}(1-z)}{1-z}\right]_+ \sim \frac{1}{k} \ln^p N$
- $C(N) \sim \alpha_s^k \ln^{2k} N$ AT LARGE N

LOG COUNTING: CROSS SECTION

$$C_{\rm QCD}(N,M^2) = g_0(\alpha_s) \exp \mathcal{S}_{\rm QCD}\left(\bar\alpha L, \bar\alpha\right);$$
$$\mathcal{S}_{\rm QCD}(\bar\alpha L, \bar\alpha) = \frac{1}{\alpha_s} g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \alpha_s^2 g_4(\alpha_s L) + \dots$$
WHERE $\alpha_s = \alpha_s(M^2)$; $L \equiv \ln \frac{1}{N^2}$

STRUCTURE OF LOGS: "PHYSICAL" ANOMALOUS DIMENSION

$$\gamma(\alpha_s(M^2),N) = \frac{\partial \ln \sigma(M^2,N)}{\partial \ln M^2} = \int_1^N \frac{dn}{n}\, g_1 \alpha_s(M^2/n) + g_2 \alpha_s^2(M^2/n) + \dots$$

STRUCTURE OF LOGS: CROSS SECTION

$$C_{\rm QCD}(N,M^2) = \exp \int_1^{N^2} \frac{dn}{n} \int_{\mu^2}^{M^2} \frac{dk^2}{k^2} g_1(\alpha_s L) + \dots$$
$$\overline{\rm MS} \Rightarrow \mu^2 = nM^2$$
$$\mathcal{S}_{\rm QCD}\left(M^2, \frac{M^2}{N^2}\right) = \int_{M^2}^{M^2/\bar N^2} \frac{d\mu^2}{\mu^2}\, \bar\gamma\left(\alpha_s(\mu^2), \frac{M^2}{N^2\mu^2}\right)$$
$$\bar\gamma\left(\alpha_s(\mu^2), \frac{M^2}{N^2\mu^2}\right) = \left[-A\left(\alpha_s(\mu^2)\right) \ln\left(\frac{M^2}{N^2}/\mu^2\right) + D_2 \alpha_s^2(\mu^2)\right]$$

$A\left(\alpha_s(\mu^2)\right)$ "cusp anomalous dimension": coeff of + distribn in GLAP anomalous dimension

THE MEANING OF SOFT RESUMMATION

THE STRUCTURE OF SOFT RESUMMATION IS DETERMINED BY THREE INGREDIENTS: IN THE SOFT LIMIT (I.E. UP TO $\frac{1}{N}$)

- LOGS FROM REAL EMISSION TERMS CAN ONLY COME FROM UPPER LIMIT OF PHASE-SPACE INTEGRAL:

$$k^0_{\max} = \frac{\sqrt{s}}{2}\left(1 - \frac{M^2}{s}\right) = \sqrt{\frac{M^2(1-x)^2}{4x}}$$

- VIRTUAL EMISSIONS ONLY CONTRIBUTE A FUNCTION OF M^2 ("HARD" FUNCTION)
  TECHNICALLY, THE d-DIMENSIONAL COEFFICIENT FUNCTION HAS THE FACTORIZED STRUCTURE (α_0 bare coupling)

$$C(M^2, N; \alpha_0) = C^l\left(\frac{M^2}{N^2}, \alpha_0; \epsilon\right) C^c(M^2, \alpha_0; \epsilon)$$

  WHERE $C^l\left(\frac{M^2}{N^2}, \alpha_0; \epsilon\right) = \sum_k C_k(\epsilon)\left(\frac{M^2}{N^2}\right)^{-k\epsilon}$

- RG INVARIANCE
  REGULARIZED FCTN OF α_s AND SINGLE SCALE M^2 CAN ONLY DEPEND ON $\alpha_s(M^2)$:

$$\gamma\left(\alpha(M^2)\right) = \gamma^{(c)}\left(\frac{M^2}{\mu^2}, \alpha(\mu^2)\right) + \gamma^{(l)}\left(\frac{M^2}{\mu^2 N^2}, \alpha(\mu^2)\right);$$

RGI: $\mu^2 \frac{\partial}{\partial \mu^2}\gamma = 0 \Rightarrow \mu^2 \frac{\partial}{\partial \mu^2}\gamma^c = -g(\alpha(\mu^2)) = -\mu^2 \frac{\partial}{\partial \mu^2}\gamma^l$

$$\Rightarrow \gamma^l = \int_{M^2}^{M^2/N^2} \frac{d\mu^2}{\mu^2} g(\alpha(\mu^2))$$

PERTURBATIVE DIVERGENCE

THE LANDAU POLE

γ DEPENDS ON $\alpha_s(M^2/N^2) \Rightarrow$ SINGULARITY AT LARGE N:

$$\gamma_{\rm LL}(\alpha_s(M^2),N) = g_1 \int_1^N \frac{dn}{n}\, \alpha_s(M^2/n) = -\frac{g_1}{\beta_0} \ln\left(1+\beta_0\alpha_s(M^2)\ln\frac{1}{N}\right),$$

CUT ON REAL N AXIS FOR $N \geq N_L \equiv e^{\frac{1}{\beta_0\alpha_s(M^2)}} \Rightarrow$ CANNOT BE A MELLIN

DIVERGENCE: ORDER-BY-ORDER INVERSE MELLIN: $P_{\rm LL}(\alpha_s(M^2),x) = \frac{g_1}{\beta_0}\left[\frac{R(\alpha_s(M^2),x)}{1-x}\right]_+$

$$R(\alpha_s(M^2),x) = \lim_{K\to\infty}\sum_{n=0}^{K} \Delta^{(n)}(1)\left[-\beta_0\alpha_s(M^2(1-x))\right]^{n+1};$$

$$\frac{1}{\Gamma(z)} = \sum_{n=0}^{\infty}(-1)^n \frac{\Delta^{(n)}(1)}{n!} z^n$$

- DIVERGES FACTORIALLY!
- DIVERGENCE ONLY KICKS IN AT VERY HIGH ORDER, EG FOR $\alpha_s \sim 0.1$ & $\tau \sim 0.1$, BEYOND THE 20^{th} ORDER
- A FACT OF LIFE
- COULD BE REMOVED BY HIGHER TWIST TERMS (BOREL PRESCRIPTION) OR EXPONENTIALLY SUPPRESSED TERMS (MINIMAL PRESCRIPTION)

SCET AND SOFT RESUMMATION

RESUMMED EXPRESSIONS

SCET PROVIDES A RESUMMED EXPRESSION THROUGH MULTI-SCALE FACTORIZATION:

$$C_{\rm SCET}(z, M^2, \mu_s^2) = H(M^2) U(M^2, \mu_s^2) S(z, M^2, \mu_s^2)$$

- $H(M^2)$ IS ANALOGOUS TO THE PREVIOUS $g_0(M^2)$ (HARD FUNCTION)

- $S(z, M^2, \mu_s^2) = \tilde{s}_{\rm DY}\left(\ln \frac{M^2}{\mu_s^2} + \frac{\partial}{\partial \eta}, \mu_s\right) \frac{1}{1-z} \left(\frac{1-z}{\sqrt{z}}\right)^{2\eta} \frac{e^{-2\gamma\eta}}{\Gamma(2\eta)}$

  PERTURBATIVE MATCHING FUNCTION, WITH $\eta = \int_{M^2}^{\mu_s^2} \frac{d\mu^2}{\mu^2} A(\alpha_s(\mu^2))$,
  $A(\alpha_s) \Rightarrow$ SAME AS IN THE QCD EXPRESSION (CUSP AN. DIM.)

- $U(M^2, \mu_s^2) = \exp\left\{-\int_{M^2}^{\mu_s^2} \frac{d\mu^2}{\mu^2} \left[\Gamma_{\rm cusp}\left(\alpha_s(\mu^2)\right) \ln \frac{\mu^2}{M^2} - \gamma_W\left(\alpha_s(\mu^2)\right)\right]\right\}$

  CONTAINS ALL THE DEP. ON THE SOFT SCALE μ_s^2 (RESUMMATION)

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  CONTAINS ALL THE DEP. ON THE SOFT SCALE μ_s^2 (RESUMMATION)

COMMENTS

- RESUMMED RESULT DEPENDS ON A SOFT SCALE μ_s, WHOSE CHOICE DETERMINES WHAT IS BEING RESUMMED
- THE DISTRIBUTION STRUCTURE COMES FROM ANALYTIC CONTINUATION
  $(1-z)^{2\eta-1} = \left[(1-z)^{2\eta-1}\right]_+ + \frac{1}{2\eta}\delta(1-z)$
- THE NOMINAL LOG ACCURACY OF THE SCET AND PERTURBATIVE QCD EXPRESSIONS IS NOT THE SAME:

LOG APPROX.	QCD ACCURACY:	SCET ACCURACY: $\alpha_s^n L^k$
LL	$k = 2n$	$k = 2n$
NLL	$2n-2 \le k \le 2n$	$2n-1 \le k \le 2n$
NNLL	$2n-4 \le k \le 2n$	$2n-3 \le k \le 2n$

accuracy at the xsect level, Becher-Neubert version of SCET, mileage may vary with use

THE SOFT SCALE

- THE SCET RESULT DEPENDS ON THE CHOICE OF SOFT SCALE
- INEQUIVALENT FORMS OF THE SCET RESULT ARE OBTAINED BY
  - CHOOSING A PARTONIC OR HADRONIC SOFT SCALE
  - PERFORMING MELLIN TRANSFORM BEFORE OR AFTER CHOOSING THE SCALE
- IN ORDER TO COMPARE SCET TO QCD WE DETERMINE THE N-SPACE FORM OF THE SCET RESULY

THE SCET RESULT IN MELLIN SPACE

$$C_{\rm SCET}(N, M^2, \mu_s^2) = H(M^2) E(N, M^2, \mu_s^2) \exp \mathcal{S}_{\rm SCET}(M^2, \mu_s^2)$$

- THE HARD FUNCTION IS THE SAME AS IN PERT. QCD: $H(M^2) = g_0(M^2)$
- $\hat{\mathcal{S}}_{\rm SCET}(M^2, \mu_s^2) = \int_{M^2}^{\mu_s^2} \frac{d\mu^2}{\mu^2} \bar{\gamma}\left(\alpha_s(\mu^2), \frac{M^2}{N^2\mu^2}\right)$, WITH $\bar{\gamma}\left(\alpha_s(\mu^2), \frac{M^2}{N^2\mu^2}\right)$ SAME AS IN QCD EXPRESSION
- $E(N, M^2, \mu_s^2)$ IS A MATCHING FUNCTION OF THE FORM $E = 1 + e^0 \alpha_s(\mu_s^2) \left(\ln \frac{M^2}{N^2\mu_s^2}\right)^2$

NOTE THE MELLIN TRANSFORM IS COMPUTED AT FIXED μ_s !

COMPARING SCET TO PERTURBATIVE QCD

THE MASTER FORMULA

SCET RESULT DIFFERS FROM THE QCD EXPRESSION TRHOUGH

- THE REPLACEMENT OF $\frac{Q^2}{N^2}$ WITH THE SOFT SCALE μ_s^2:
  $\exp \int_{M^2}^{M^2/\bar{N}^2} \frac{d\mu^2}{\mu^2} \to \exp \int_{M^2}^{\mu_s^2} \frac{d\mu^2}{\mu^2}$
- THE PRESENCE OF A MATCHING FUNCTION

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- THE PRESENCE OF A MATCHING FUNCTION

$$\boxed{C_{\rm QCD}(N, M^2) = C_r(N, M^2, \mu_s^2) C_{\rm SCET}(N, M^2, \mu_s^2)}$$

$$C_r(N, M^2, \mu_s^2) = \frac{\exp \int_{\mu_s^2}^{M^2/\bar{N}^2} \frac{d\mu^2}{\mu^2} \bar{\gamma}\left(\alpha_s(\mu^2), \frac{M^2}{N^2\mu^2}\right)}{E(N, M^2, \mu_s^2)}$$
$$= e^{\left[\int_{\mu_s^2}^{M^2/\bar{N}^2} \frac{d\mu^2}{\mu^2} \bar{\gamma}\left(\alpha_s(\mu^2), \frac{M^2}{N^2\mu^2}\right) - \gamma_E\left(\alpha_s(\mu^2), \frac{M^2}{N^2\mu^2}\right)\right]}$$

- RESUMMATION $\bar{\gamma}\left(\alpha_s(\mu^2), \frac{M^2}{N^2\mu^2}\right) = \left[-A\left(\alpha_s(\mu^2)\right) \ln(\frac{M^2}{N^2}/\mu^2) + D_2\alpha_s^2(\mu^2)\right]$
- MATCHING $\gamma_E\left(\alpha_s(\mu^2), \frac{M^2}{N^2\mu^2}\right) = \frac{A_1\alpha_s(\mu^2)}{4} \ln(\frac{M^2}{N^2}/\mu^2) - \frac{A_1}{8}\beta(\alpha_s(\mu^2)) \ln^2(\frac{M^2}{N^2}/\mu^2)$

EFFECT OF MATCHING FUNCTION $\Rightarrow$ REMOVE THE $O(\alpha_s)$ CONTRIBUTION TO $A(\alpha_s)$

THE SCET EXPRESSION: PERTURBATIVE PROPERTIES

$$\boxed{C_{\rm QCD}(N,M^2)=C_r(N,M^2,\mu_s^2)C_{\rm SCET}(N,M^2,\mu_s^2)}$$

$$C_r(N,M^2,\mu_s^2)=e^{\left[\int_{\mu_s^2}^{M^2/\bar N^2}\frac{d\mu^2}{\mu^2}\bar\gamma\left(\alpha_s(\mu^2),\frac{M^2}{N^2\mu^2}\right)-\gamma_E\left(\alpha_s(\mu^2),\frac{M^2}{N^2\mu^2}\right)\right]}$$

- $\mu_s=M^2/\bar N^2 \Rightarrow$ MELLIN-SPACE QCD & SCET EXPRESSIONS COINCIDE
- μ_s KEPT FIXED (DOES NOT DEPEND ON PARTON KINEMATICS)
  $\Rightarrow C_{\rm SCET}(N,M^2,\mu_s^2)$ DOES ADMIT A MELLIN INVERSE (IT IS A MELLIN TRANSF!).
  $\Rightarrow$ DIVERGENCE OF $C_{\rm QCD}(N,M^2)$ CONTAINED IN $C_r(N,M^2,\mu_s^2)$
- MASTER FORMULA MAY BE MELLIN-INVERTED PERTURBATIVELY ORDER BY ORDER:

$$\boxed{C_{\rm QCD}(z,M^2)=\int_z^1\frac{dy}{y}C_r\left(\frac{y}{z},M^2,\mu_s^2\right)C_{\rm SCET}\left(y,M^2,\mu_s^2\right),}$$

- WITH $\mu_s^2=M^2(1-z)^2$, SCET EXPRESSION COINCIDES WITH THE z SPACE PERTURBATIVE QCD ONE FOR $z<1$
- FOR $z=1$ FACTORIAL DIVERGENCE (DUE TO MOMENTUM-NONCONSERVATION, UNRELATED TO LANDAU POLE) ALREADY IN PERTURBATIVE QCD, WORSE IN SCET

A HADRONIC SCALE CHOICE (Becher, Neubert 2006)

$$\mu_s^2 = M^2(1-\tau)^2$$

- RESUMMATION ESPRESSED IN TERMS OF A PHYSICAL (HADRONIC) SCALE $\Rightarrow$ NO LANDAU POLE
- PERTURBATIVE FACTORIZATION SPOILED:
  PARTON XSECT DEPS ON HADRON KINEMATICS
  $$\sigma_{\rm SCET}(\tau, M^2) = \int_\tau^1 \frac{dz}{z}\, C_{\rm SCET}\left(z, M^2, M^2(1-\tau)^2\right) \mathcal{L}\left(\frac{\tau}{z}\right)$$
  DOES NOT FACTORIZE UPON MELLIN TRANSFORM
- WHAT IS THE NATURE OF THE FACTORIZATION BREAKING, DIVERGENCE CANCELLING TERMS?
  CAN COMPARE ONLY AT THE LEVEL OF CROSS-SECTIONS

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AN EXPLICIT COMPARISON

$$C_{\rm QCD}(z, M^2) = C_r \otimes C_{\rm SCET}:$$

$$C_r(N, M^2, \mu_s^2) = 1 + \alpha_s^2(M^2)\left(C_r^{2,3}\ln^3\frac{M^2}{N^2\mu_s^2} + C_r^{2,2}\ln^2\frac{M^2}{N^2\mu_s^2} + C_r^{2,1}\ln\frac{M^2}{N^2\mu_s^2}\right) + \mathcal{O}(\alpha_s^3)$$

- C_r EXPONENTIAL $\Rightarrow C_r = 1 + \Delta C_r \Rightarrow C_{\rm QCD} = C_{\rm SCET} + \Delta C \Rightarrow$
  $\sigma_{\rm QCD} = \sigma_{\rm SCET} + \Delta\sigma$
- $\Delta\sigma$ DETERMINED AS CONVOLUTION OF POWERS OF LOG WOTH $\sigma_{\rm SCET}$
- CAN DETERMINE IN TERMS OF A GENERATING FUNCTIONAL

  $$\ln^n \frac{1}{N} = \frac{d^n}{d\xi^n}\Delta(\xi)\int_0^1 dz\, z^{N-1}\left[\ln^{\xi-1}\frac{1}{z}\right]_+\Bigg|_{\xi=0} + \delta_{n0},$$

$$\sigma_{\rm QCD}(\tau, M^2) = \sigma_{\rm SCET}(\tau, M^2) + \alpha_s^2(M^2)\left(C_r^{2,3}\frac{\partial^3}{\partial\xi^3} + C_r^{2,2}\frac{\partial^2}{\partial\xi^2} + C_r^{2,1}\frac{\partial}{\partial\xi}\right)(1-\tau)^\xi\Sigma(\tau,\xi)\Big|_{\xi=0}$$

POWER COUNTING...

$$\sigma_{\rm QCD}(\tau, M^2) = \sigma_{\rm SCET}(\tau, M^2) + \alpha_s^2(M^2)\left(C_r^{2,3}\frac{\partial^3}{\partial\xi^3} + C_r^{2,2}\frac{\partial^2}{\partial\xi^2} + C_r^{2,1}\frac{\partial}{\partial\xi}\right)(1-\tau)^{\xi}\Sigma(\tau,\xi)\Big|_{\xi=0}$$

- $\Sigma \sim (1-\tau)^{-\xi}$ SO $(1-\tau)^{\xi}\Sigma$ POLYNOMIAL IN ξ
- $\Delta\sigma$ GENERATES NO NEW LOGS ON TOP OF THOSE CONTAINED IN $\sigma_{\rm SCET}$
- NNLO $\Delta\sigma$ OF ORDER
  $\alpha_s^3 \ln(1-\tau) \times \alpha_s^k \ln^{2k}(1-\tau) \times \ln^p(1-\tau) = \alpha_s^h \ln^{2h-5+p}(1-\tau); \quad h \equiv k+3,$
  $p \Rightarrow$ power of log in the parton lumi

...AND ITS CONSEQUENCES

- IF $p = 0$ (NO LOGS IN PARTON LUMI) $\Rightarrow$ DIVERGENCE REMOVED BY DOWNGRADING QCD ACCURACY TO "SCET" ACCURACY:
  NNLO FAILS AT ORDER $\alpha_s^k \ln^{2k-3+p}(1-\tau)$;
- IF $p > 0$ (LOGS IN PDFS) $\Rightarrow$ ACCURACY OF RESULT SPOILED TO (POTENTIALLY) ANY ACCURACY

POWER COUNTING...

$$\sigma_{\rm QCD}(\tau,M^2)=\sigma_{\rm SCET}(\tau,M^2)+\alpha_s^2(M^2)\left(C_r^{2,3}\frac{\partial^3}{\partial\xi^3}+C_r^{2,2}\frac{\partial^2}{\partial\xi^2}+C_r^{2,1}\frac{\partial}{\partial\xi}\right)(1-\tau)^\xi\Sigma(\tau,\xi)\Big|_{\xi=0}$$

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WHAT IF $\tau<<1$?

- IN PHENOMENOLOGICALLY RELEVANT CASES (HIGGS @ LHC?)
  DOMINANT PARTONIC z REGION $<<$ HADRONIC τ
- $\Rightarrow M(1-\tau)\sim M$, THOUGH RESUMMATION RELEVANT
- IN THIS CASE $\mu_s^2\sim M^2$ SO
  $C_r(N,M^2,\mu_s^2)=1+\alpha_s^2(M^2)\left(C_r^{2,3}\ln^3\frac{M^2}{N^2\mu_s^2}+\dots\right)\sim 1+\alpha_s^2(M^2)\left(C_r^{2,3}\ln^3\frac{1}{N^2}+\dots\right)$
  $\Rightarrow$ SCET RESUMMATION ALREADY FAILS AT NLL!

THE LESSON LEARNT

- THE PERTURBATIVE EXPANSION OF RESUMMED EXPRESSIONS IN PERTURBAITVE QCD DIVERGES
- THE DIVERGENCE STEMS FROM THE FACT THAT RESUMMATION RG IMPROVES THE SCALE OF PARTON RADIATION

  $\Rightarrow$ PRESUMABLY CANCELLED BY PHYSICS BEYOND THE LEADING-TWIST PERTURBATIVE EXPANSION
- ATTEMPT TO CUT OFF THE DIVERGENCE BY HADRONIC SCALE INTRODUCES AT BEST SPURIOUS LOG-SUPPRESSED TERMS