Rare and CP violating Kaon Decays SM Prediction and NP Sensitivity

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Kaons: 500 MeV to 10 TeV

Kaon physics involves many different energy scales

M_K: ChiPT, Lattice QCD
m_c: GIM
M_W: SM
M_{NP}: ?

Hierarchy of scales → Potential QCD pollution

clean Observables: ϵ_K & $K \to \pi \nu \nu$

also interesting but no time:
$K \to (\pi) l^+ l^-$, ϵ'/ϵ, Unitarity, K_μ/K_e

Rare and CP violating Kaon Decays

FCNCs which are dominated by top-quark loops:

$b \to s$: $|V_{tb}^* V_{ts}| \propto \lambda^2$

$b \to d$: $|V_{tb}^* V_{td}| \propto \lambda^3$

$s \to d$: $|V_{ts}^* V_{td}| \propto \lambda^5$

CKM suppression: enhanced sensitivity to NP

$$V_{ts}^* V_{td} + V_{cs}^* V_{cd} = -V_{us}^* V_{ud}$$

λ^5 $\quad$ λ $\quad$ λ

how are the light quark suppressed?

Quadratic GIM: $\lambda \frac{m_c^2}{M_W^2}$

CP violation: $\text{Im}(V_{cs}^* V_{cd})$

Potential Operators

modified Z-Penguin for $K \rightarrow \pi \nu \nu$
(& Box type contribution)

$$Q_\nu^{L/R} = (\bar{s}\gamma_\mu d_{L/R})(\bar{\nu}\gamma^\mu \nu_L)$$

ϵ_K: Eight Operators
(Q_1–Q_5 and 3 chirality flipped)

Coefficients constrained by
$|\varepsilon_K| = 2.228(11)\times 10^{-3}$ &
$\Delta M_K = 5.292(9)\,\mathrm{ns}^{-1}$

NP Flavour Problem

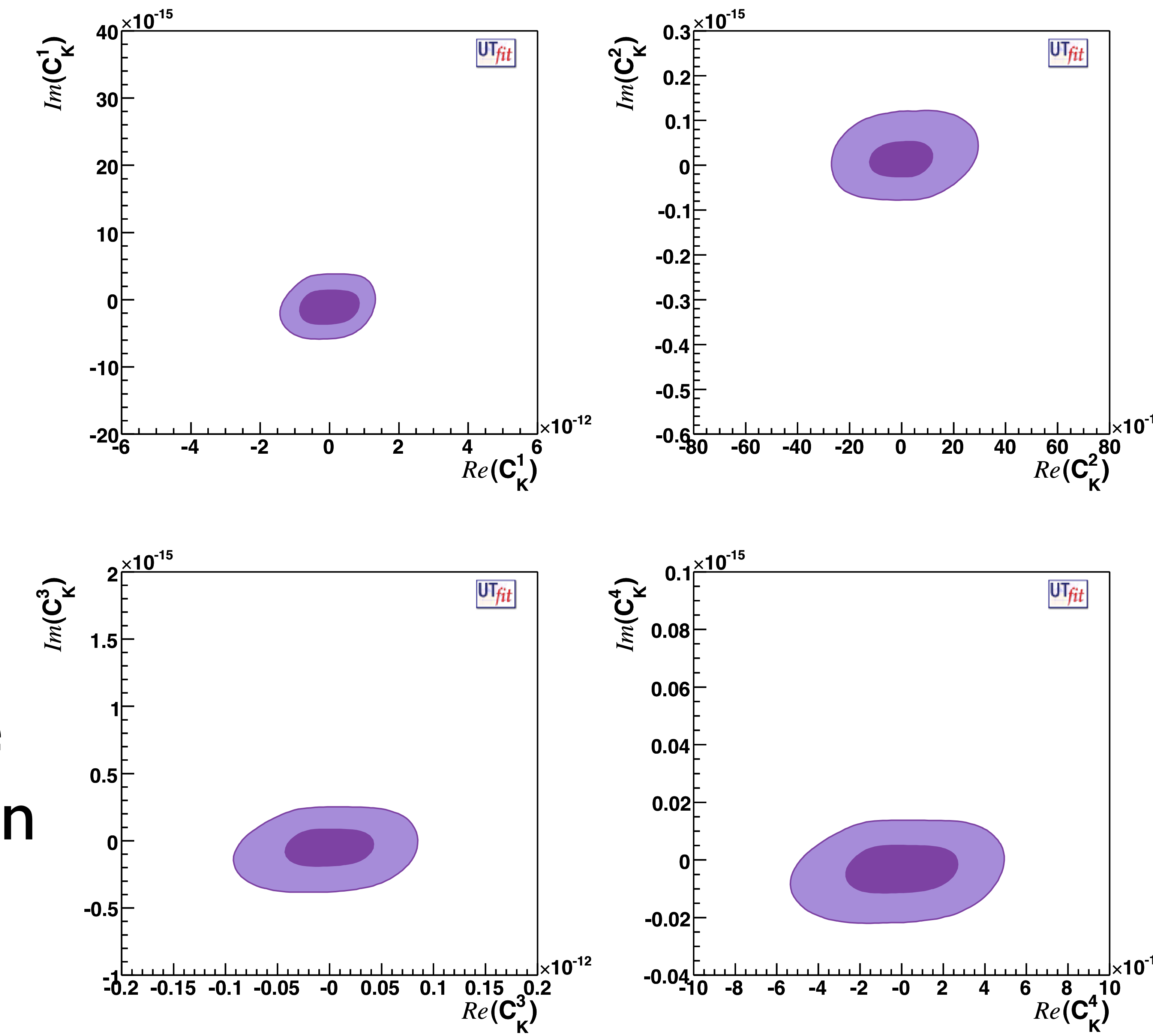
$$O_1 = (\bar{s}\gamma_\mu d_L)(\bar{s}\gamma^\mu d_L)$$
$$O_2 = (\bar{s}^\alpha d_L^\alpha)(\bar{s}^\beta d_L^\beta)$$
$$O_3 = (\bar{s}^\alpha d_L^\beta)(\bar{s}^\beta d_L^\alpha)$$
$$O_4 = (\bar{s}^\alpha d_R^\alpha)(\bar{s}^\beta d_L^\beta)$$
$$O_5 = (\bar{s}^\alpha d_R^\beta)(\bar{s}^\beta d_L^\alpha)$$

Model Independent Constraints

Allowed area quite small

theory & parametric uncertainty → size of area

Constraints might be more severe in concrete realisation of a model.



[UTfit]

Gauged Flavour Models

Example: $SU(3)_Q$x $SU(3)_U$x $SU(3)_D$ [Grinstein et. al `10] (Talk by Carlucci)

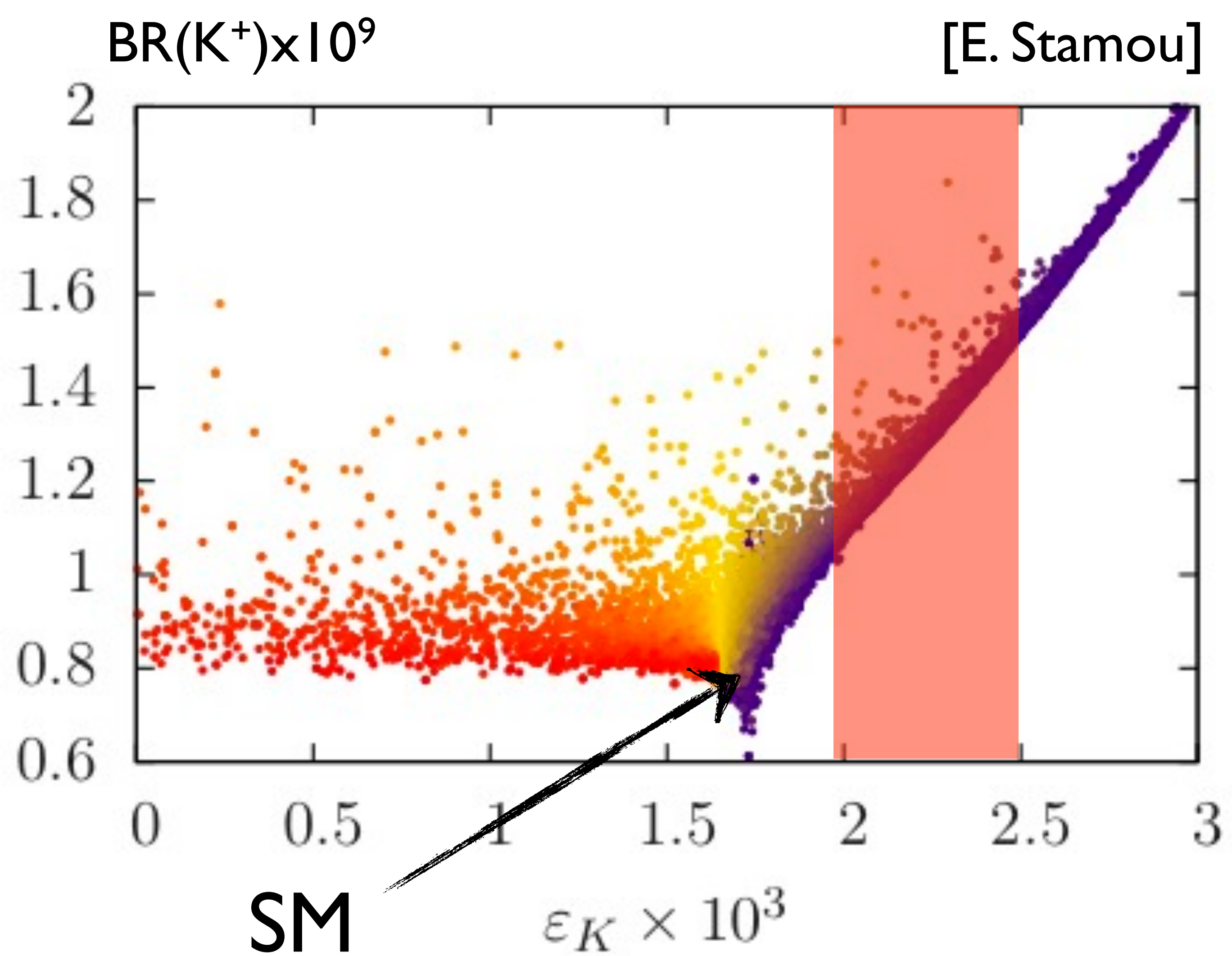
Flavour violation of extra gauge bosons suppressed for Kaons

Mixing of vector like fermions (t–t´) contributes to ϵ_K & $K \to \pi \nu\nu$

Using results for arbitrary perturbative theories
[Brod, Casagrande, MG in preperation]

we find a strong correlation between ϵ_K & $K \to \pi \nu\nu$

Can study minimal extensions of vectors, fermions & scalars

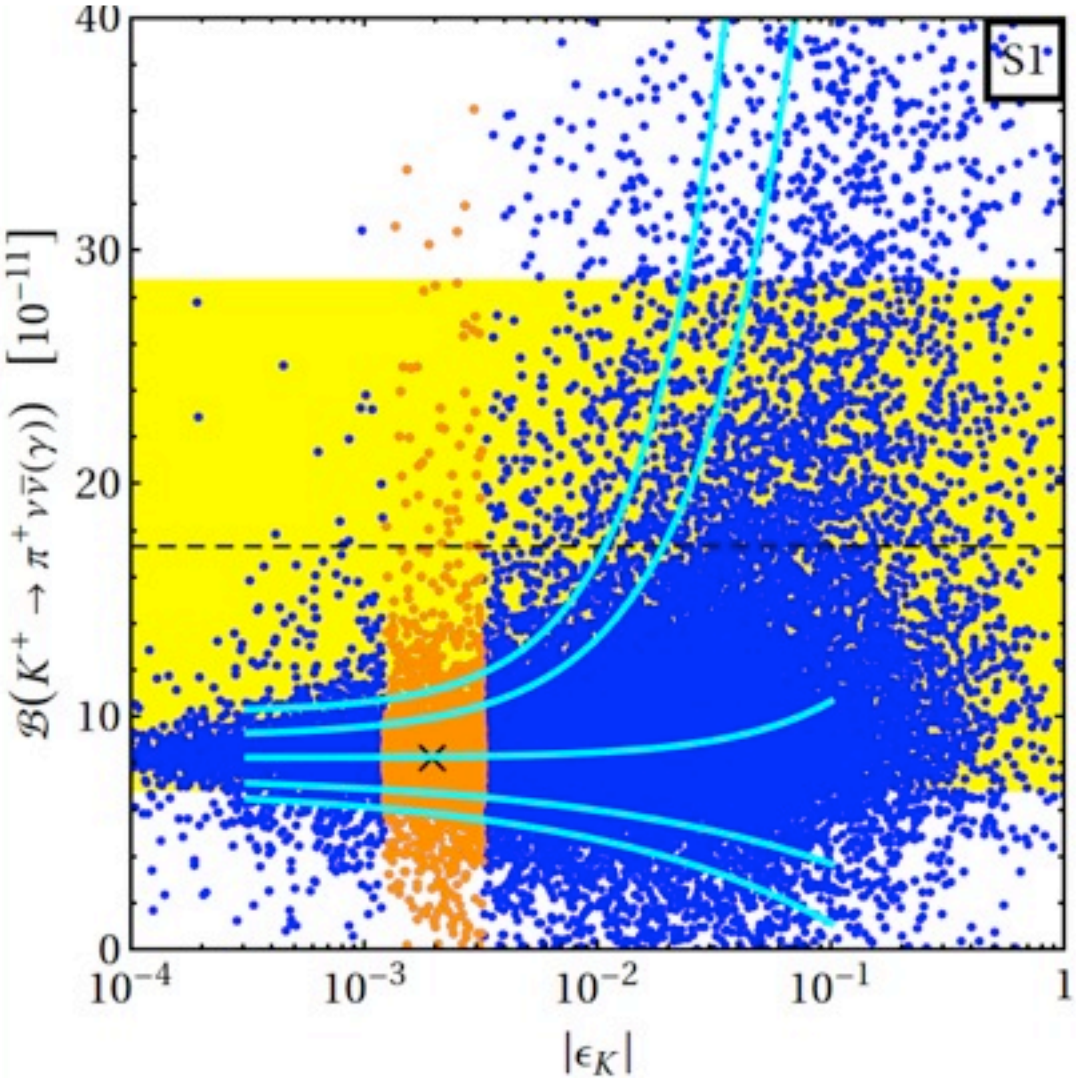


Constraints from ϵ_K in RS

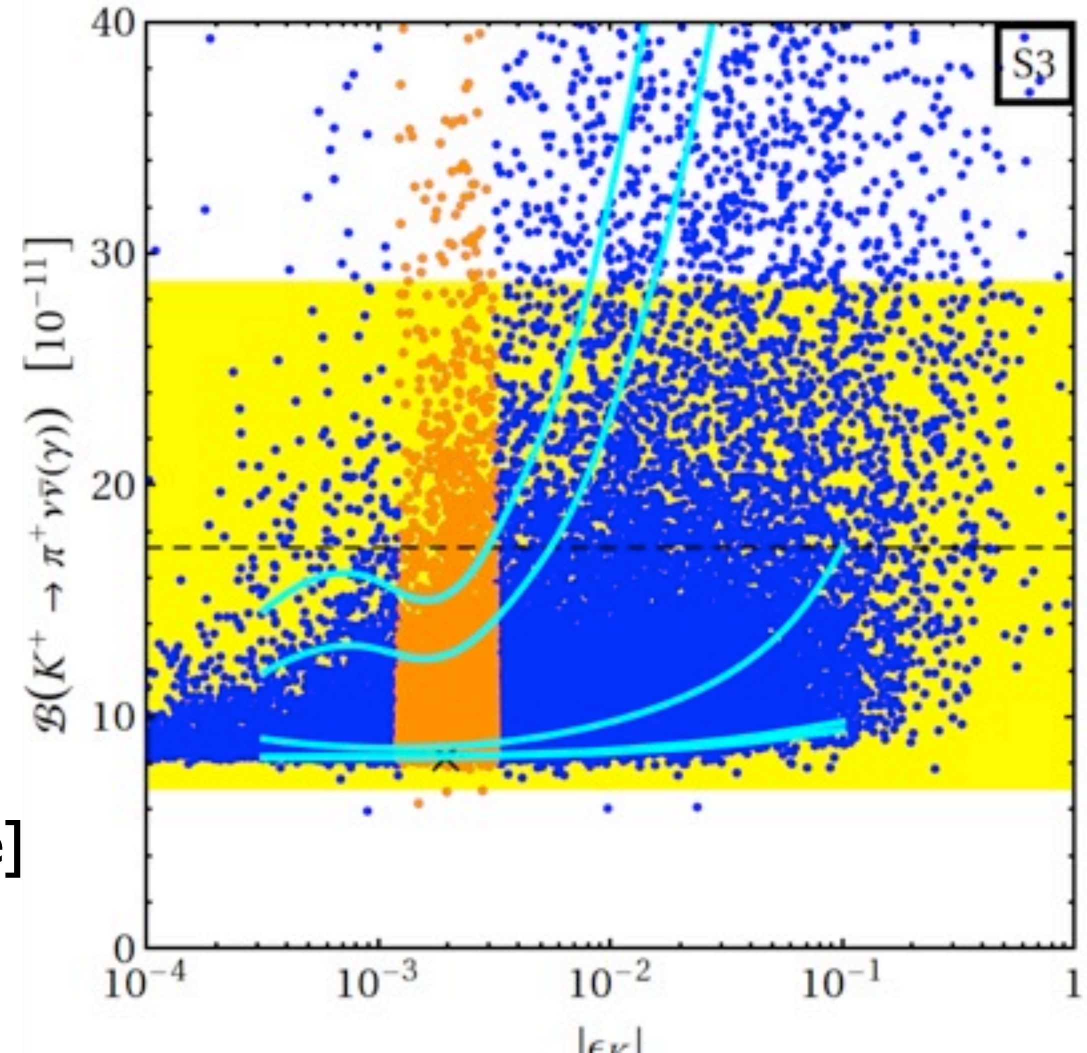
Randall-Sundrum: KK vector bosons flavour violating & intergenerational couplings of vector-like quarks: Z, KK gluon flavour violation

No simple correlation, but ϵ_K constrains size of typical effects.

[Analysis by Bauer, Casagrande, Haisch Neubert `09]



[common down-type bulk mass]



[Casagrande]

Standard Model

To improve the NP sensitivity improve the SM prediction

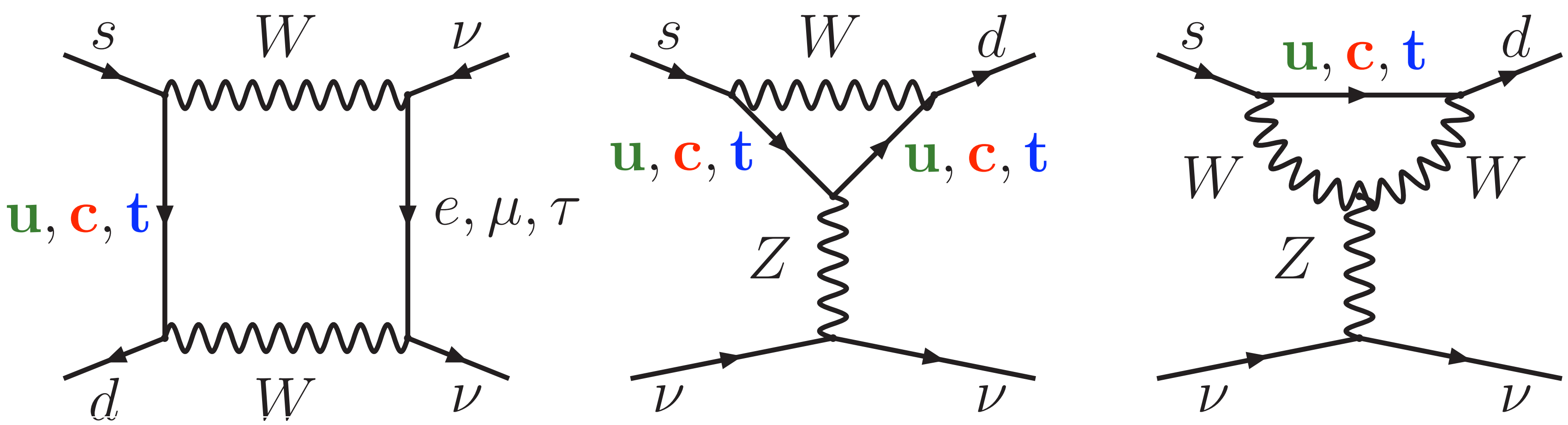
In our case:

Perturbative calculation
Matching (M_W)
RGE ($M_W \to m_c$)
integrating out the charm quark

Non-perturbative calculation (Lattice & ChiPT)
Matrix elements
Higher dimensional operators
(non-local terms)

with Lattice
in ??
years

$K^+ \rightarrow \pi^+ \bar{\nu} \nu$ at M_W



$$\sum_i V_{is}^* V_{id} F(x_i) = V_{ts}^* V_{td}(F(x_t) - F(x_u)) + V_{cs}^* V_{cd}(F(x_c) - F(x_u))$$

Quadratic GIM: $\lambda^5 \frac{m_t^2}{M_W^2}$ $\quad \lambda \frac{m_c^2}{M_W^2} \ln \frac{M_W}{m_c}$ $\quad \lambda \frac{\Lambda^2}{M_W^2}$

Matching (NLO +EW):

[Misiak, Urban; Buras, Buchalla; Brod, MG, Stamou]

Operator Mixing (RGE)

$$Q_\nu = (\bar{s}_L \gamma_\mu d_L)(\bar{\nu}_L \gamma^\mu \nu_L)$$

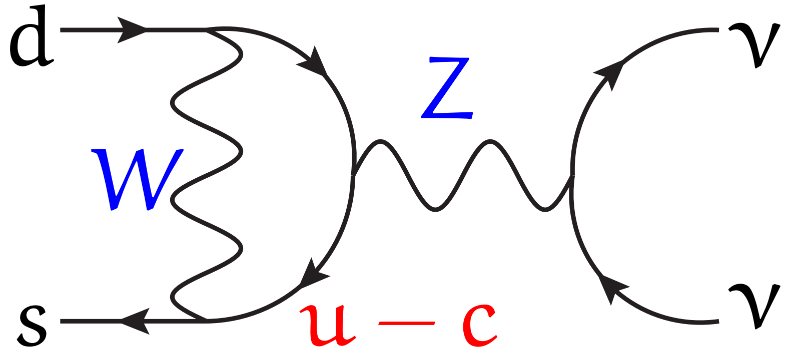
Matrix element from K_{l3} decays

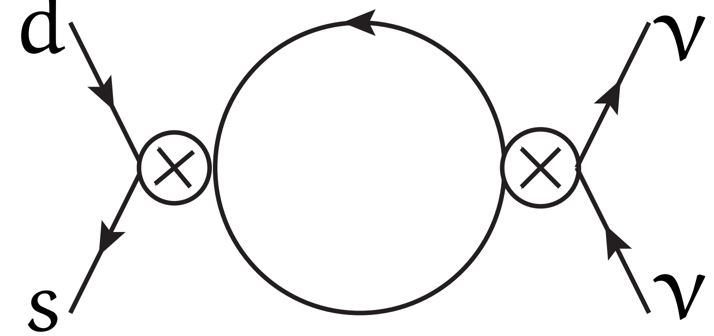
(Isospin symmetry: $K^+ \rightarrow \pi^0 e^+ \nu$)

[Mescia, Smith]

GIMnastics at m_c

Quadratic GIM suppresses light quark contribution





$m_c^2 - m_u^2$

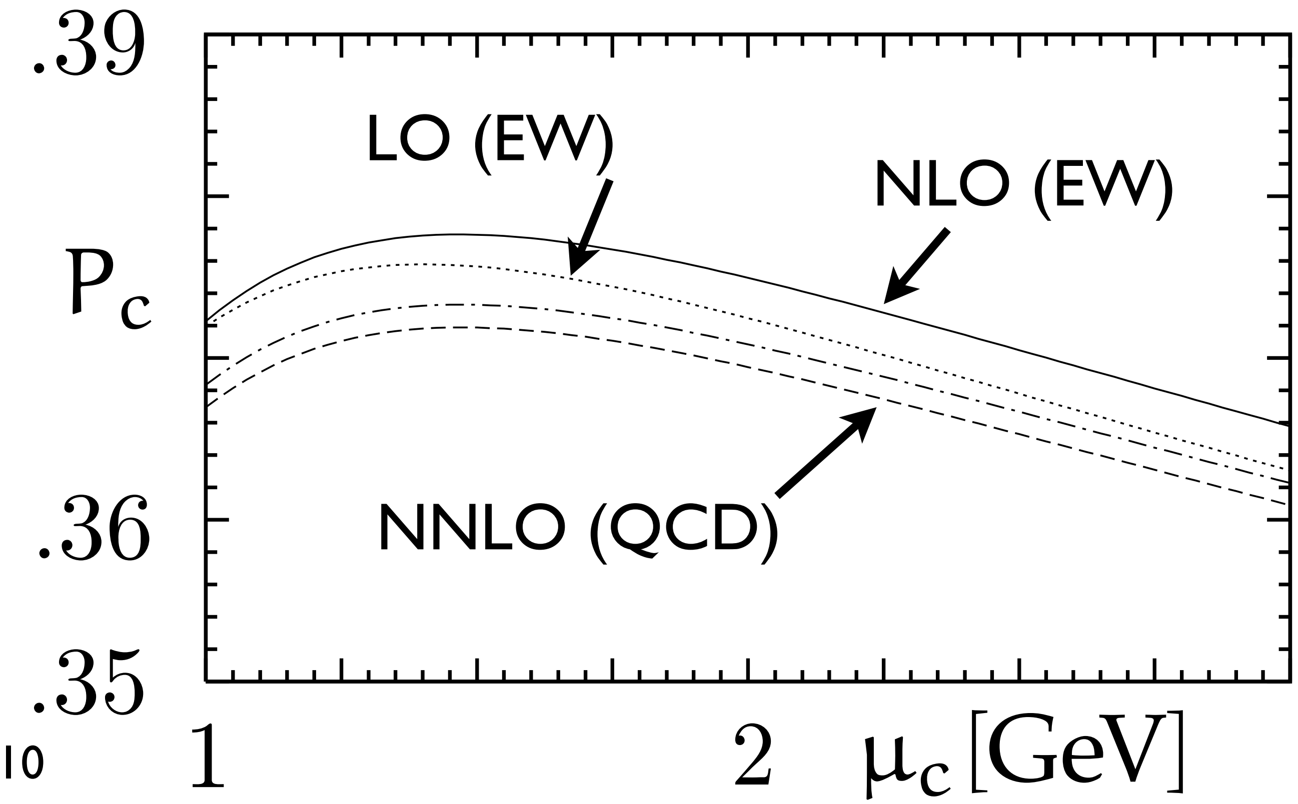
$m_c^2 G_F^2 \log \frac{m_c}{M_W}$

NNLO+EW

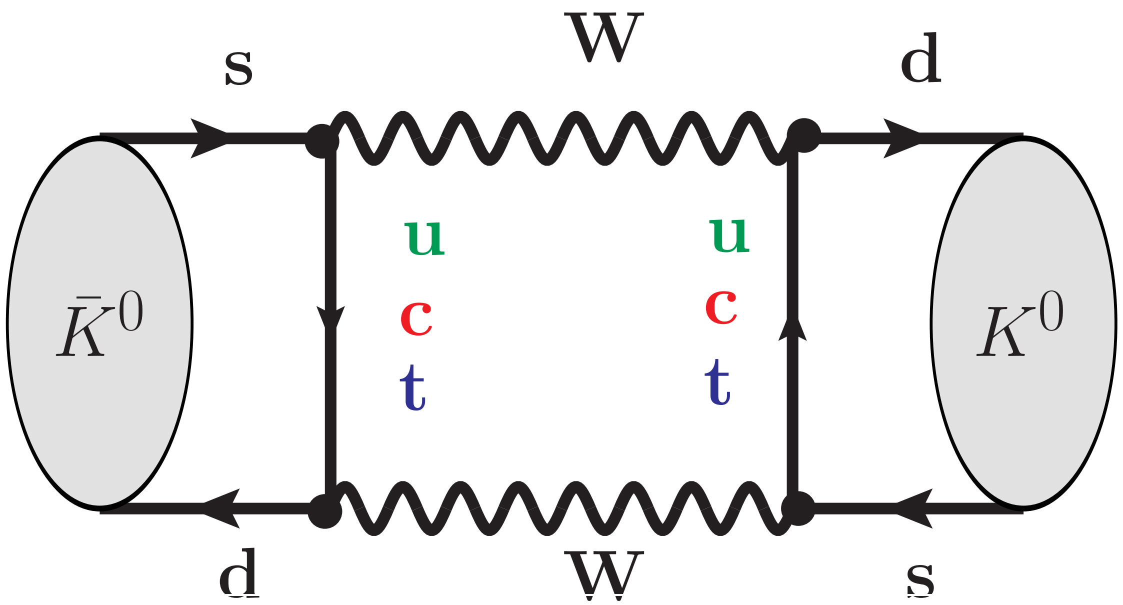
[Buras, MG, Haisch, Nierste; Brod MG]

P_c: charm quark contribution to $K^+ \to \pi^+ \bar{\nu} \nu$ (30% to BR)

P_c calculation works remarkably well (±2.5% uncertainty)



M_{12} at M_W



$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$x_i = \frac{m_i^2}{M_W^2} \qquad \lambda_i = V_{id}^* V_{is}$$

Three CKM factors: $\lambda_t = O(\lambda^5 e^{i\delta})$, $\lambda_c = O(\lambda + i\lambda^5)$ and $\lambda_u = O(\lambda)$

$$x_t \lambda_t \lambda_t + x_c \log(x_c) \lambda_c \lambda_t + x_c \lambda_c \lambda_c + \frac{\Lambda_{QCD}^2}{M_W^2} \lambda_c \lambda_{t/c}$$

$$2M_K M_{12} = \langle K^0 | H^{|\Delta S|=2} | \bar{K}^0 \rangle - \frac{i}{2} \int d^4x \, \langle K^0 | H^{|\Delta S|=1}(x) \, H^{|\Delta S|=1}(0) | \bar{K}^0 \rangle$$

dispersive part

M_{12}, Kaon Mixing & ϵ_K

CP violation in mixing $\mathrm{Re}(\epsilon_K)$ and interference $\mathrm{Im}(\epsilon_K)$

$$\epsilon_K = e^{i\phi_\epsilon} \sin\phi_\epsilon \left(\frac{\mathrm{Im}(M_{12})}{\Delta m_K} + \frac{\mathrm{Im}(A_0)}{\mathrm{Re}(A_0)} \right) \qquad A_I = \langle (\pi\pi)_I | K^0 \rangle$$

Δm_K, ϕ_ε: Directly from experiment:

$\mathrm{Im}(A_0)/\mathrm{Re}(A_0)$: from ϵ'/ϵ

$$\mathrm{Im}(M_{12}) = \mathrm{Im}(M_{12})_{SD} + \mathrm{Im}(M_{12})_{D=8} + \mathrm{Im}(M_{12})_{\text{Non Local}}$$

Factorize short and long distance: $H^{|\Delta S|=2} = C(\mu)\tilde{Q}$

From lattice: $\hat{B}_K = \frac{3b(\mu)}{2f_K^2 M_K^2} \langle K^0 | \tilde{Q} | \bar{K}^0 \rangle \qquad \left(\tilde{Q} = (\bar{s}_L \gamma_\mu d_L)(\bar{s}_L \gamma^\mu d_L) \right)$

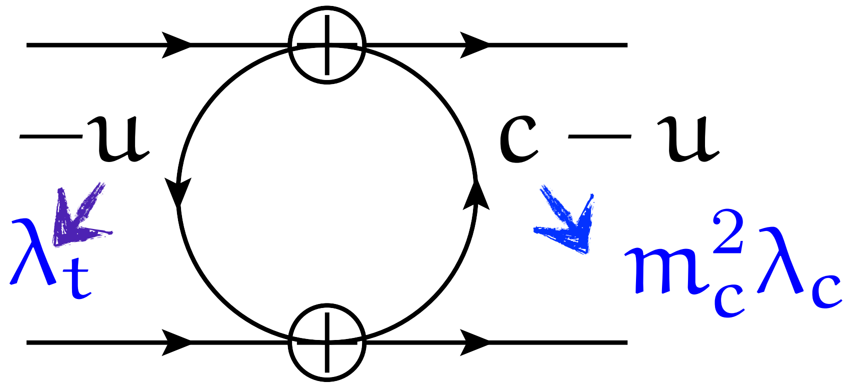
Perturbative Calculation

	CKM	$\not{CP}$	LO Logs ($n\geq 0$)	ϵ_K	State
η_{tt}: A(t-u,t-u)	$\lambda_t\lambda_t$	$O(\lambda^{10})$	$x_t\ (\alpha_s \log x_c)^n$	75(±1)%	NLO
η_{ct}: A(t-u,c-u)	$\lambda_t\lambda_c$	$O(\lambda^{6})$	$x_c \log x_c\ (\alpha_s \log x_c)^n$	40(±10)%	NNLO
η_{cc}: A(c-u,c-u)	$\lambda_c\lambda_c$	$O(\lambda^{6})$	$x_c\ (\alpha_s \log x_c)^n$	15(±50)%	NNLO

Lattice results quoted in Renormalisation Group Invariant scheme

in RGI: multiply by $b^{-1}(\mu) = \alpha_s^{2/9}(\mu)\left(1 - 1.9\frac{\alpha_s(\mu)}{4\pi} + 16.4\frac{\alpha_s^2(\mu)}{16\pi^2}\right)$

$\eta_{ct} : \alpha_s^{2/9} \lambda_t \lambda_c x_c \log x_c (\alpha_s \log x_c)^n + ..$



$\rightarrow \lambda_t \lambda_c x_c \log x_c$

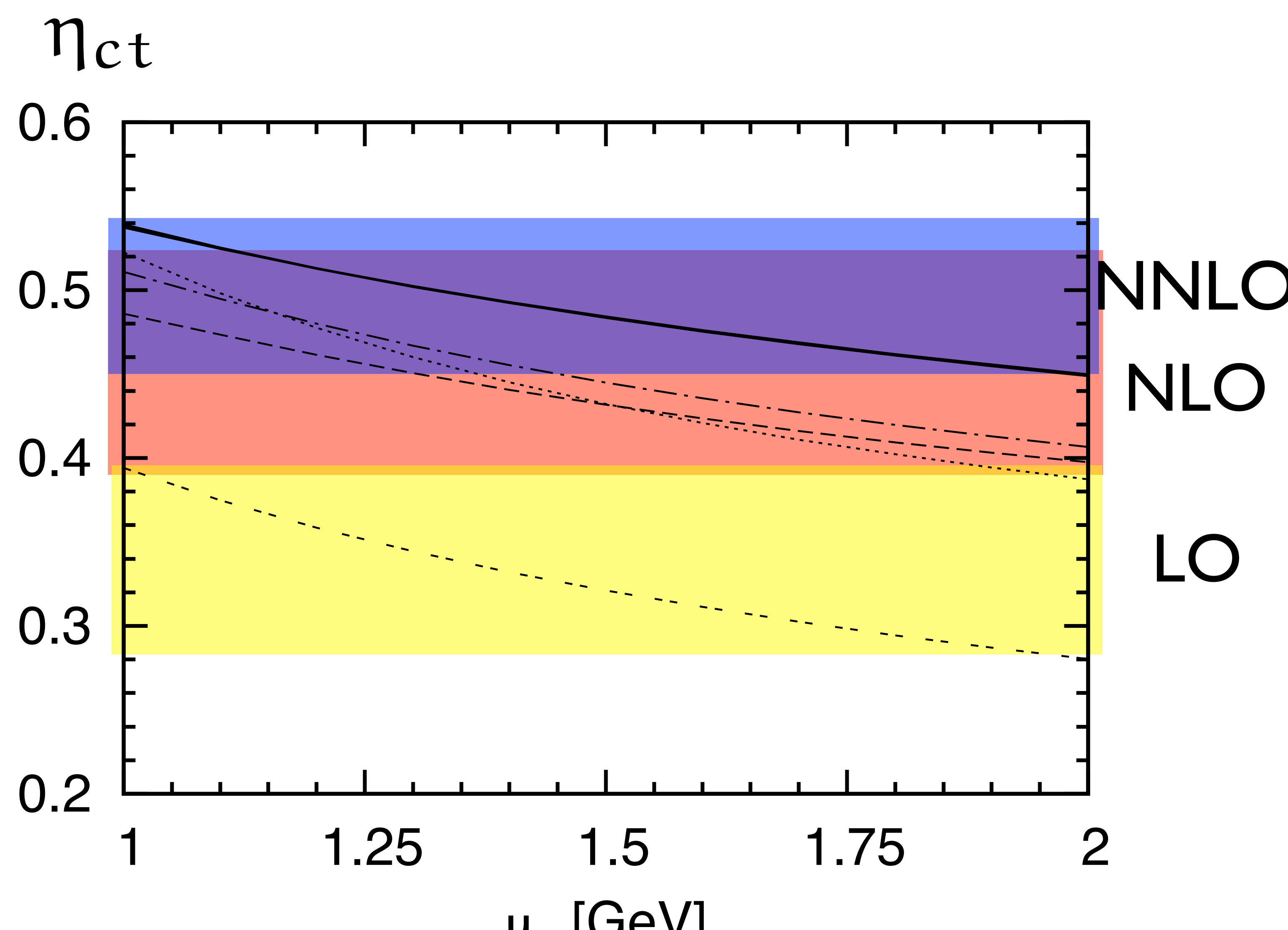
NNLO: 2 loop Matching at μ_W & μ_c
and 3 loop running [Brod, MG]

Perturbation theory works

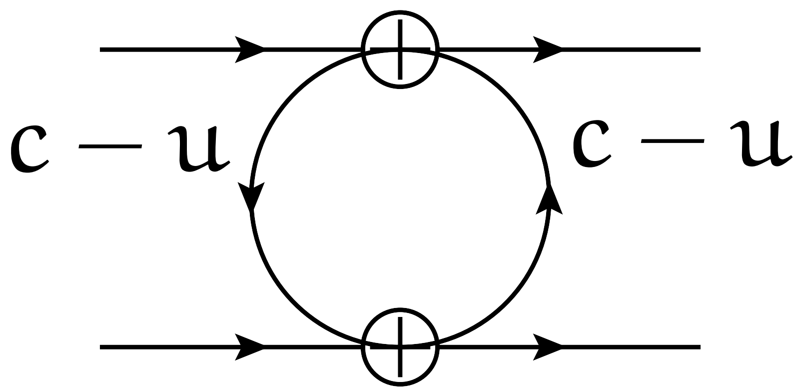
$\eta_{ct}^{NNLO} = 0.496(46)$

(not as well as $K^+ \rightarrow \pi^+ \bar{\nu} \nu$)

involves the charm scale
and
a large anomalous dimension



$\eta_{cc}: \alpha_s^{2/9}\lambda_c^2 x_c(\alpha_s \log x_c)^n + ..$



$\rightarrow \lambda_c^2 x_c$

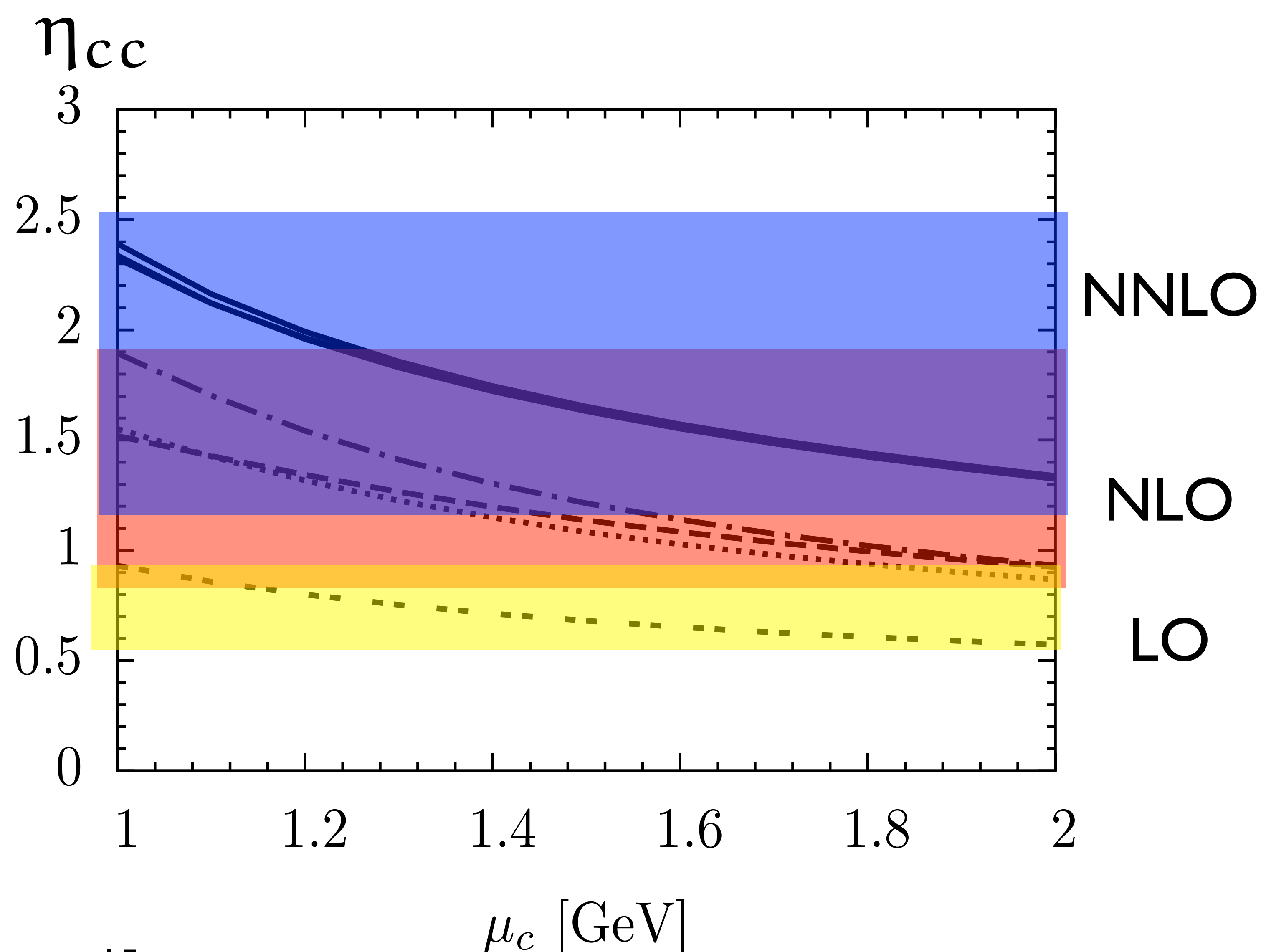
NNLO:
3 loop running of current-current Operators
and 3 loop Matching at μ_c [Brod, MG]

LO → NLO → NNLO shift
of similar size

bad convergence

scale uncertainty and shift of
similar size: add in quadrature

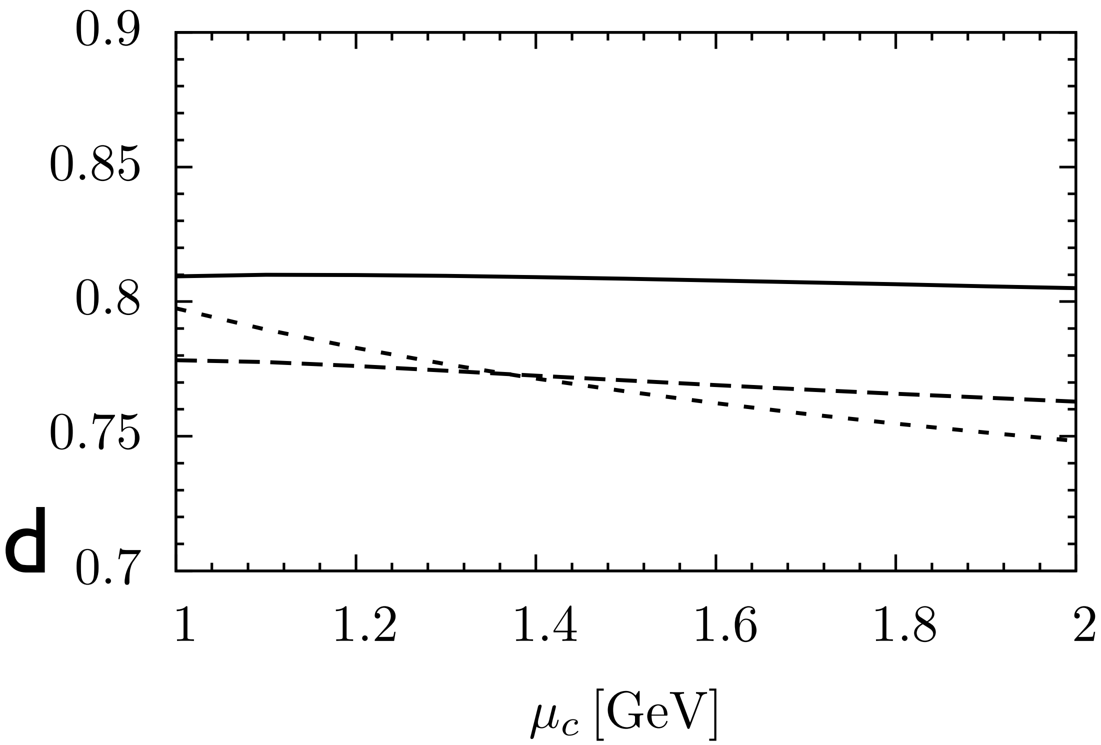
$\eta_{cc}^{NNLO} = 1.87(76)$



Why the large shift?

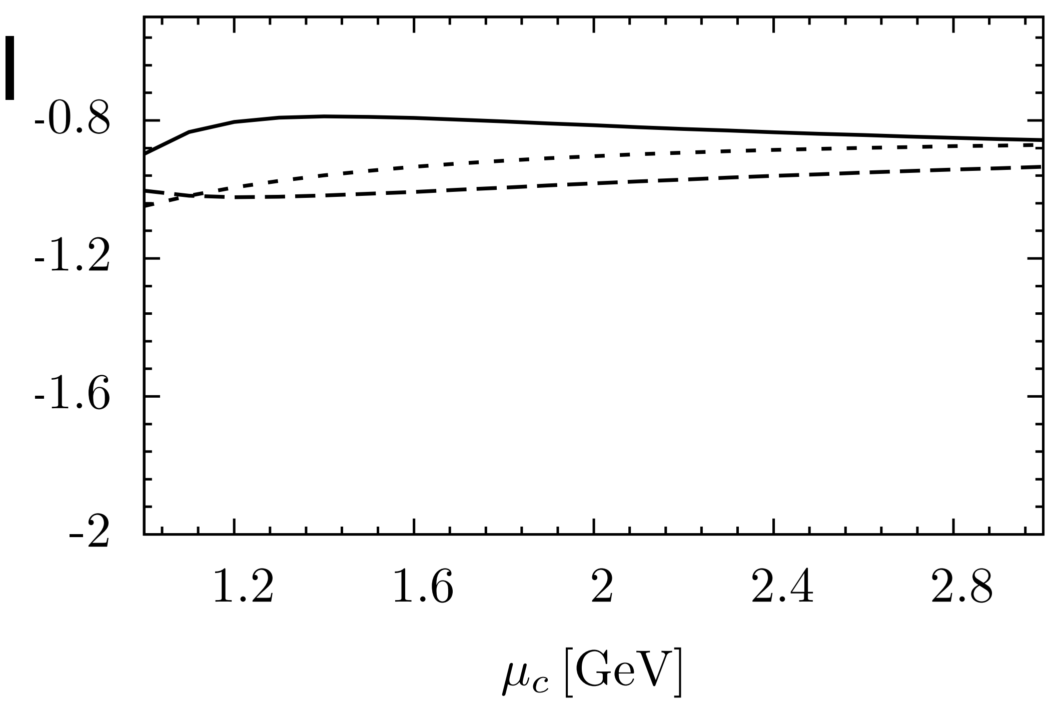
Large ADMs and large finite corrections at μ_c

If the matrix element at μ_c would contain only logs
scale dependence would reduce nicely



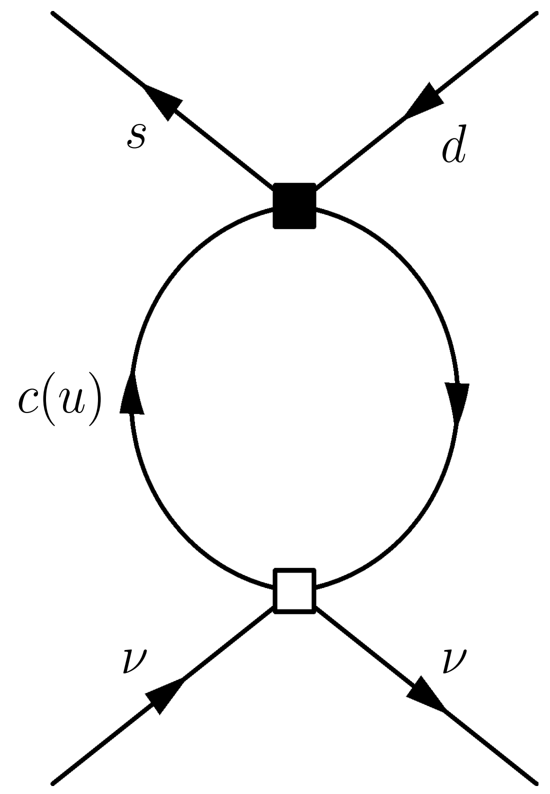
If the matrix element could be calculated on the lattice including charm quarks:

A (RI-MOM) scheme change would cancel the current-current μ_c dependence



RI-MOM matching of B_K does not seem to change the perturbative expansion

Long Distance $K^+ \to \pi^+ \bar{\nu}\nu$



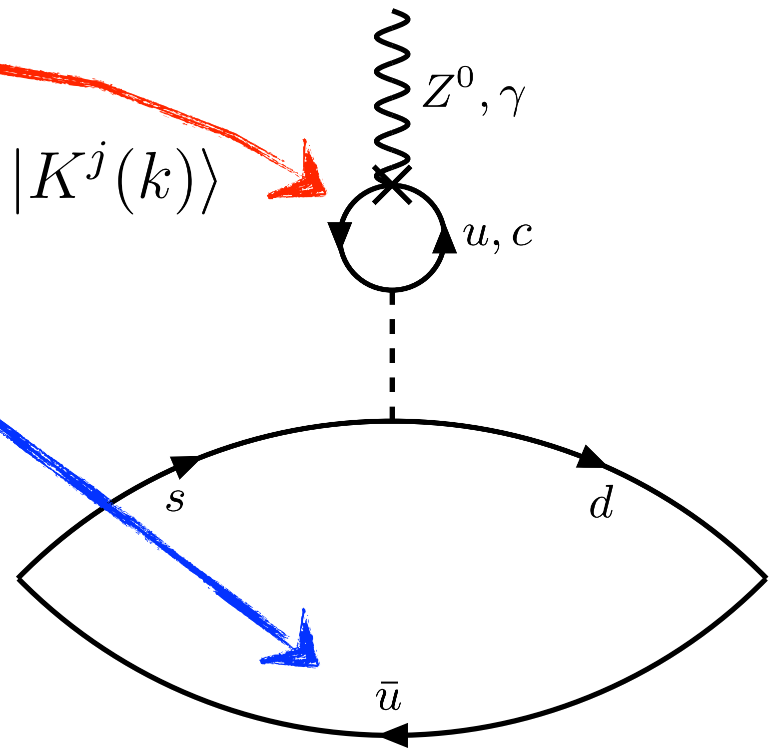
No GIM below the charm quark mass scale

higher dimensional operators UV scale dependent

One loop ChiPT calculation approximately cancels this scale dependence $\delta P_{c,u} = 0.04 \pm 0.02$

[Isidori, Mescia, Smith `05]

$$(\mathcal{T}_i^j)^\mu_{\rm em}(q^2) \;=\; -i\int d^4x\, e^{-i\,q\cdot x}\,\langle \pi^j(p)|T\left\{J^\mu_{\rm em}(x)\left[Q_i^u(0)-Q_i^c(0)\right]\right\}|K^j(k)\rangle$$

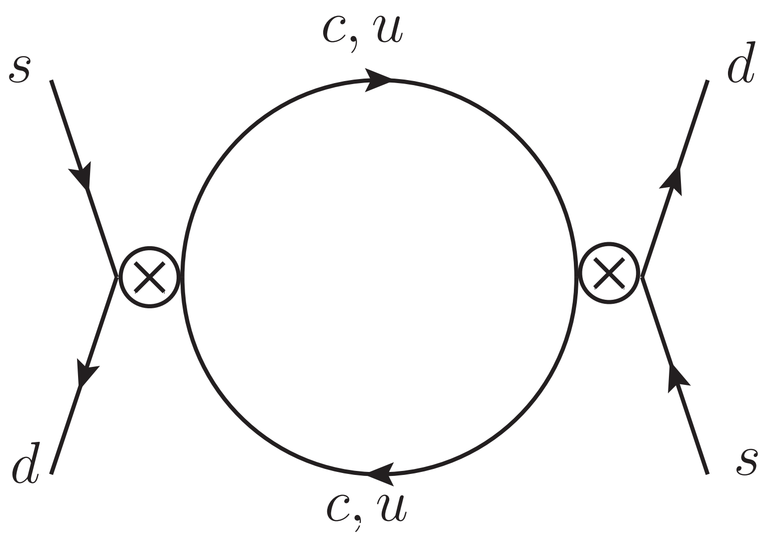


Could be calculated on the lattice

[Isidori, Martinelli, Turchetti `06]

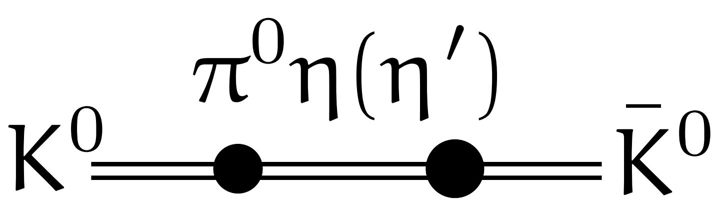
GIM to cancel $1/a^2$ divergence

Long Distance ϵ_K



$$\int d^4x \, \langle K^0 | \, H^{|\Delta S|=1}(x) \, H^{|\Delta S|=1}(0) \, | \bar{K}^0 \rangle$$

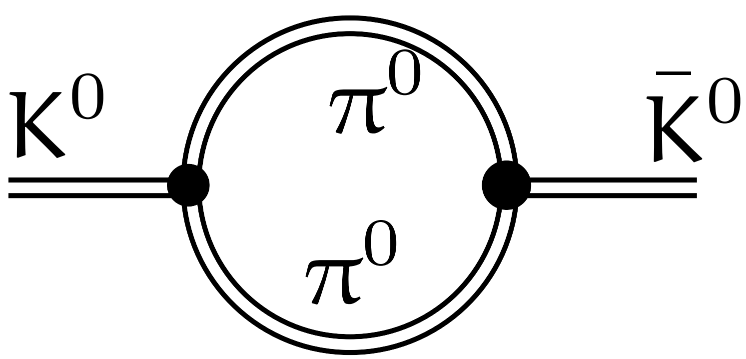
Higher dimensional operator [Cata Peris`04]



Light quark loops in CHPT:

π^0,η tree level vanishes (Gell-Mann-Okuba)

η´comes with zero phase [Gerard et.al. `05]



1-loop diagram divergent:

estimate from $\ln(m_\pi/m_\rho)$ [Buras et.al. `10]

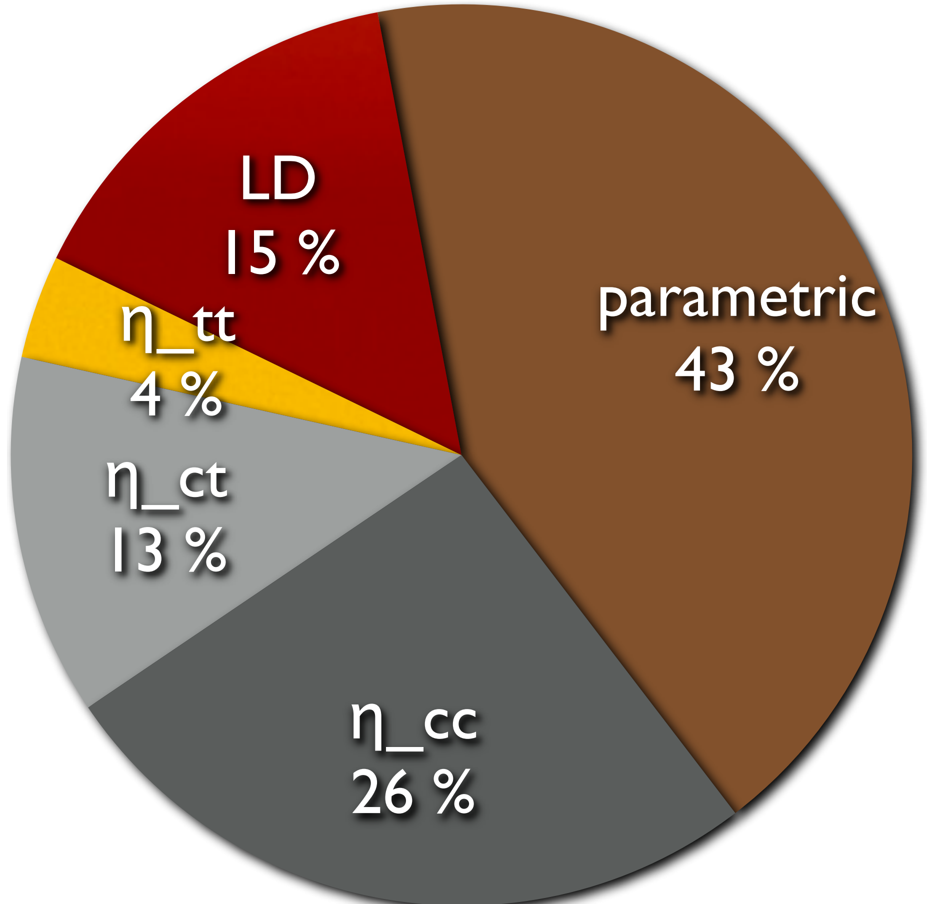
$$\epsilon_K = e^{i\phi_\epsilon} \sin\phi_\epsilon \left(\frac{\mathrm{Im}(M_{12}^K)}{\Delta M_K} + \frac{\mathrm{Im}(A_0)}{\mathrm{Re}(A_0)} \right)$$

absorptive part

estimated form ϵ'

Future: Lattice

[N. Christ]

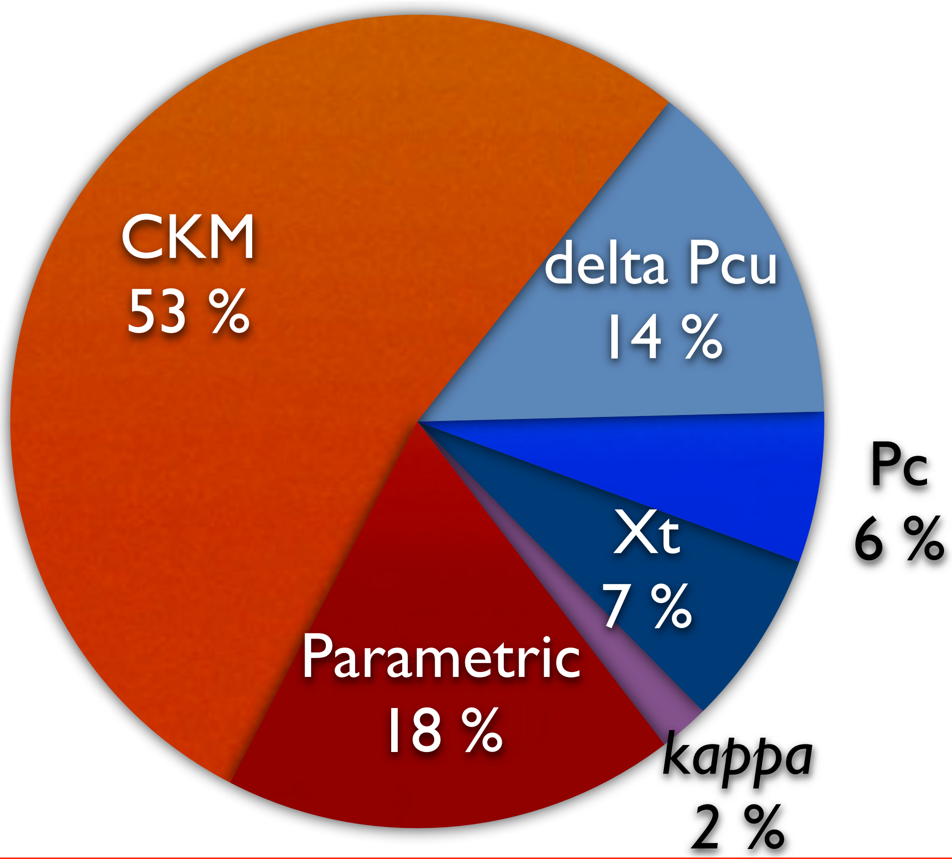


ϵ_K : SM prediction

$$|\epsilon_K| = 1.81(28) \times 10^{-3}$$

$$|\epsilon_K| \overset{\text{exp.}}{=} 2.228(11) \times 10^{-3}$$

$$|V_{cb}| = 406(13) \times 10^{-4}$$



$K^+ \to \pi^+ \bar{\nu} \nu$: SM prediction

$$\mathcal{B}_{K^+} = 0.822(69)(29) \times 10^{-10}$$

$$\mathcal{B}r_{K^+} \overset{\text{exp.}}{=} (1.73^{+1.15}_{-1.05}) \times 10^{-10}$$

NA62 aims at 10% uncertainty

Conclusions

Kaons sensitive to deviations from minimal flavour violation
(EW precision, Lepton Universality)

O(1) NP contribution to the clean $K^+ \to \pi^+ \bar{\nu} \nu$ decay possible

Perturbative calculation for ϵ_K at the limit
(improvement could come from lattice)