Flavor Phenomenology from Lattice QCD

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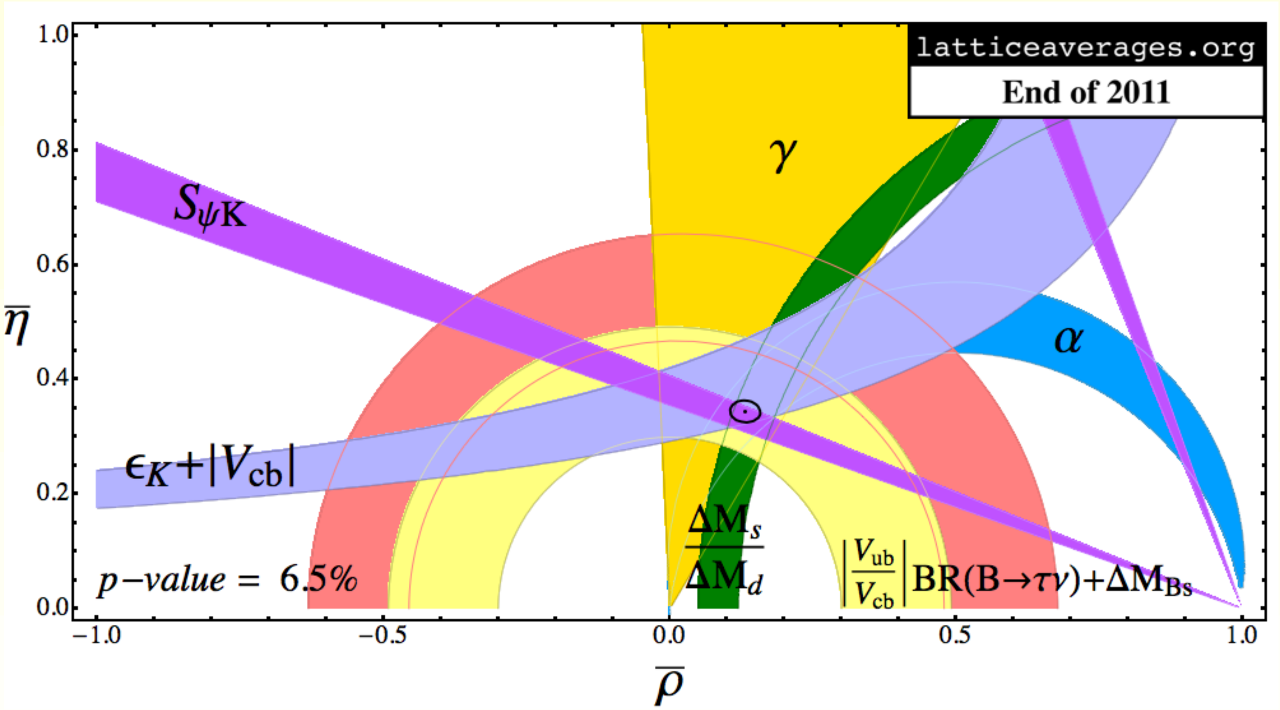
1. Introduction

Lattice QCD can be used to (relying only on first principles.)

* Non-perturbative input for the study of some theory-experiment discrepancies in UT analyses ($\hat{B}_K$, f_B, $f_B\sqrt{B_B}$, ξ ...), processes involving $B^0_{d,s} - \bar{B}^0_{d,s}$ mixing (like-sign dimuon charge asymmetry), heavy-light decay constants ... and also rare decays

Laiho,Lunghi,Van de Water PRD81:034503 (2010)



Error bands are still dominated by theory errors, in particular due to hadronic matrix elements.

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Lattice QCD can be used to (relying only on first principles.)

* Determine fundamental parameters of the SM: quark masses, CKM matrix elements (tensions in inclus.-exclus. determinations of $|V_{ub}|$, $|V_{cb}|$).

$$V = \begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ \pi \to l\nu & K \to l\nu & B \to \tau\nu \\ & K \to \pi l\nu & B \to \pi\tau\nu \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ D \to l\nu & D_s \to l\nu & B \to Dl\nu \\ D \to \pi l\nu & D \to Kl\nu & B \to D^* l\nu \\ |V_{td}| & |V_{ts}| & |V_{tb}| \\ \langle B_d^0|\bar{B}_d^0\rangle & \langle B_s^0|\bar{B}_s^0\rangle & \text{no } t\bar{q} \text{ hadrons} \end{pmatrix} \qquad arg(V_{ub}^*) \quad \langle K^0|\bar{K}^0\rangle$$

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Gold-plated quantities: For stable (or almost stable) hadron, masses and amplitudes with no more then one initial (final) state hadron.

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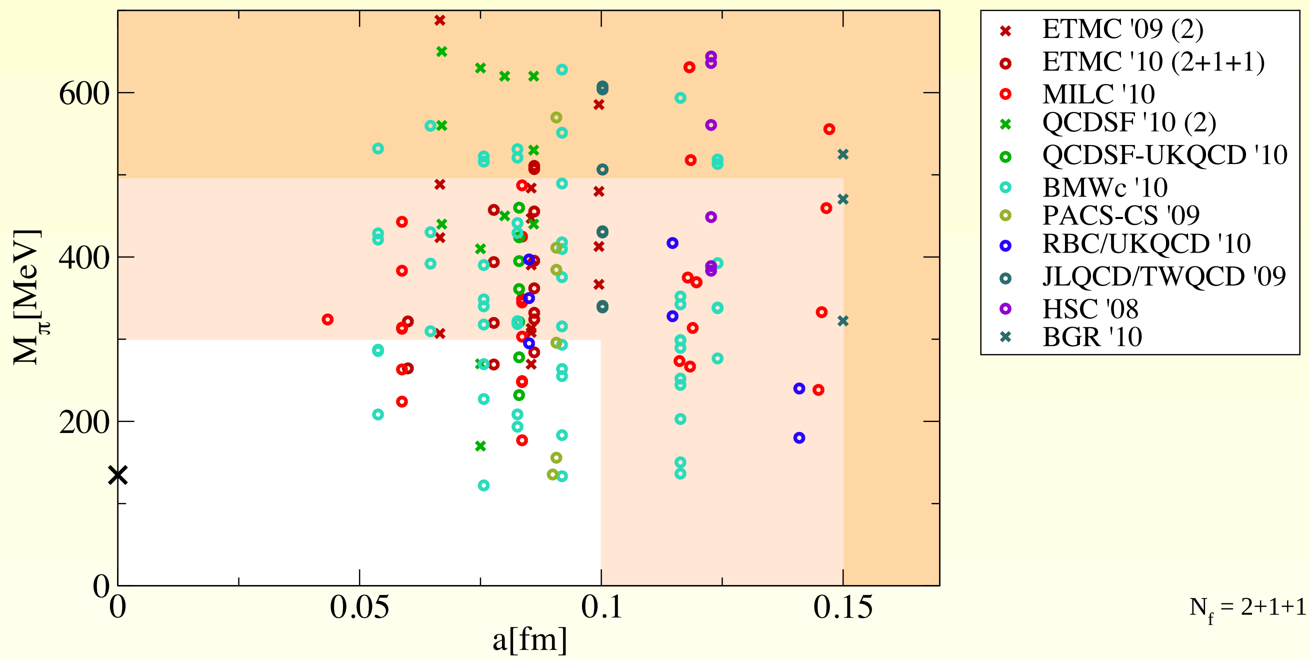
Unquenched calculations

* Quenching the strange quark could have an error as large as 5% and need a $N_f = 2+1$ to have an estimate $\to$ want $N_f = 2+1$

* Neglecting sea charm has effects $\mathcal{O}(1\%)$ (can be estimated with HQET). **Starting to need sea charm effects.**

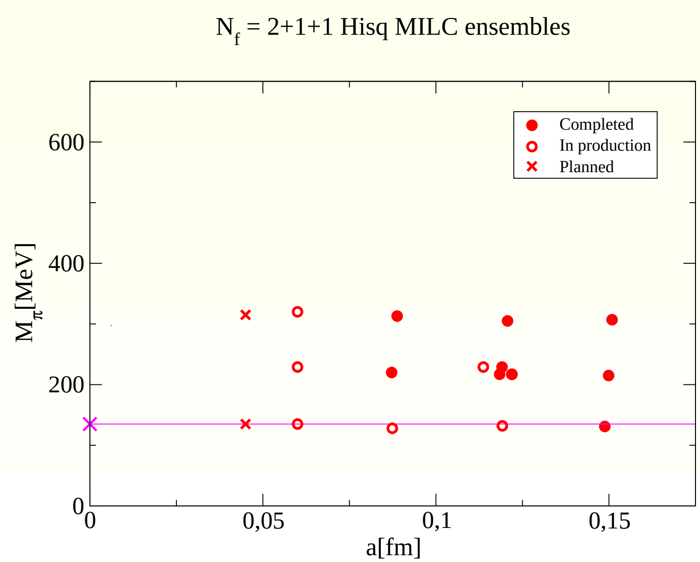
1. **Introduction:** Overview of simulations parameters

Several $N_f = 2+1$ and even $N_f = 2+1+1$, and physical quark masses.



plot by **C. Hoelbling**,

Lattice 2010, 1102.0410



1.1. Introduction: Averaging lattice QCD results

J. Laiho, E. Lunghi, and R. Van de Water (LLV)

Phys.Rev.D81:034503,2010, most updated results in **www.latticeaverages.org**

* Phenomenologically relevant light and heavy quantities + UT fits with lattice inputs.

* Include only $N_f = 2 + 1$.

* Only published results (including proceedings).

Flavianet Lattice Average group: (FLAG)

Eur. Phys. J. C71(2011)1695, updated results in **http://itpwiki.unibe.ch/flag**

* K and π physics, including LEC's.

* Include separate averages for $N_f = 2$ and $N_f = 2 + 1$.

* Only published results with the exception of update proceedings.

Flavor Lattice Averaging Group (FLAG-2): 28 people representing all big lattice collaborations.

* Light and heavy quantities. **First review at the end of 2012**

2. Light quarks matrix elements

2.1. f_K/f_π: Determination of $|V_{us}|$

Decay constants come from simple matrix element
$\langle 0|\bar{q}_1\gamma_\mu\gamma_5 q_2|P(p)\rangle = if_P p_\mu$ $\rightarrow$ precise calculations

* Even higher precision for ratios due to cancellation of statistics and systematics uncertainties

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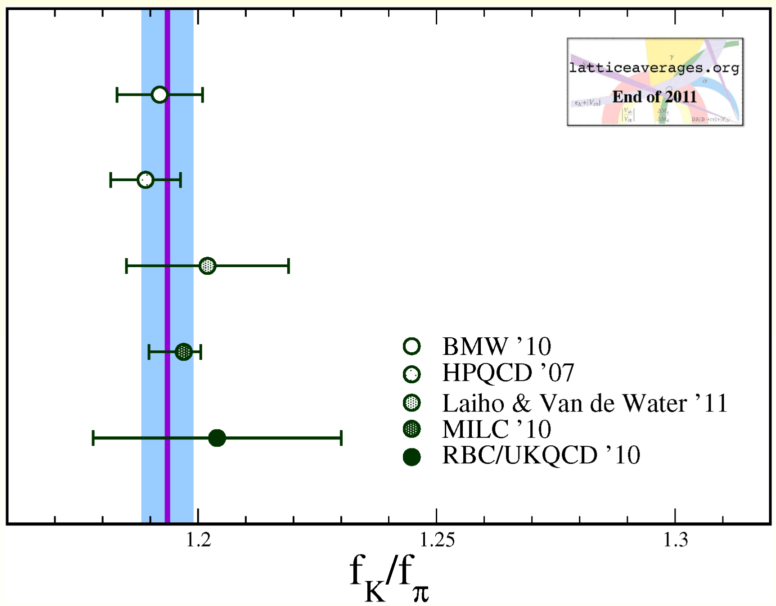
Many $N_f = 2+1$ lattice calculations → good test of lattice QCD



$$f_K/f_\pi^{\mathbf{LLV}} = 1.1936 \pm 0.0053$$

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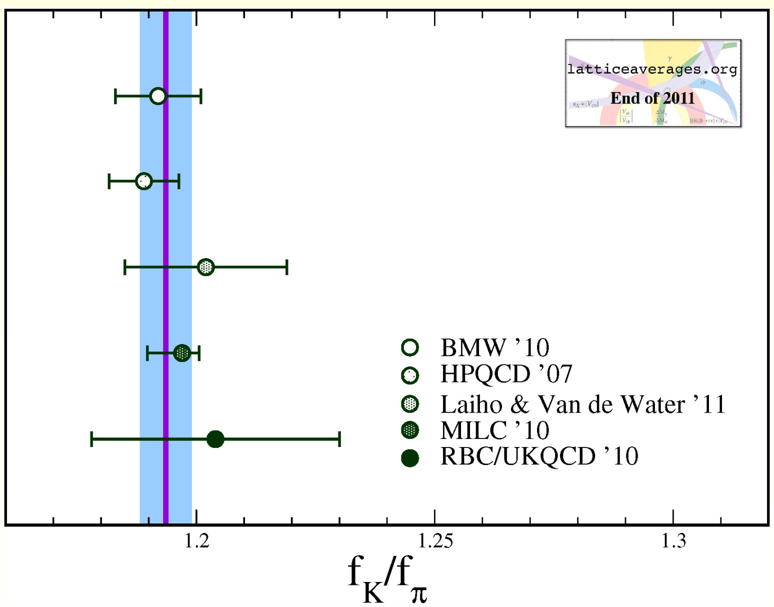

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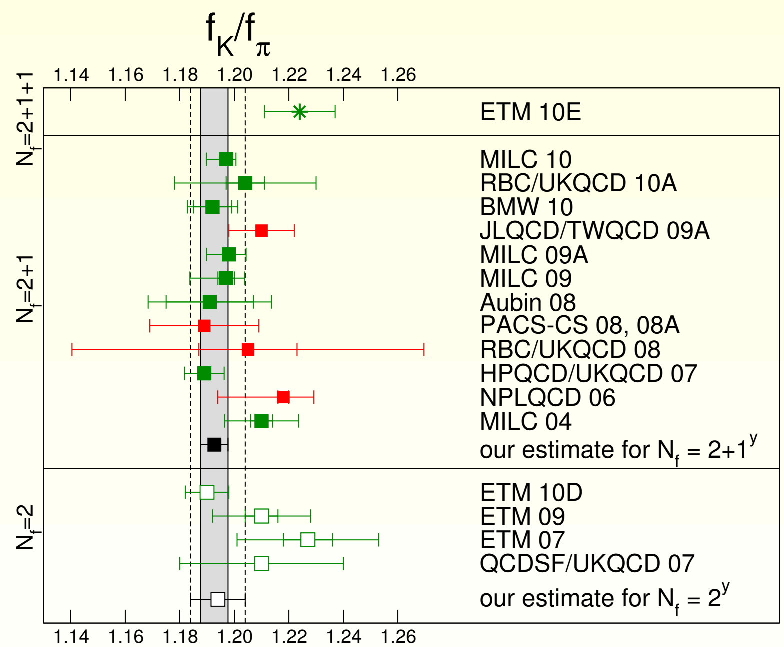

$f_K/f_\pi^{\mathbf{FLAG}_{N_f=2+1}} = 1.193 \pm 0.005$

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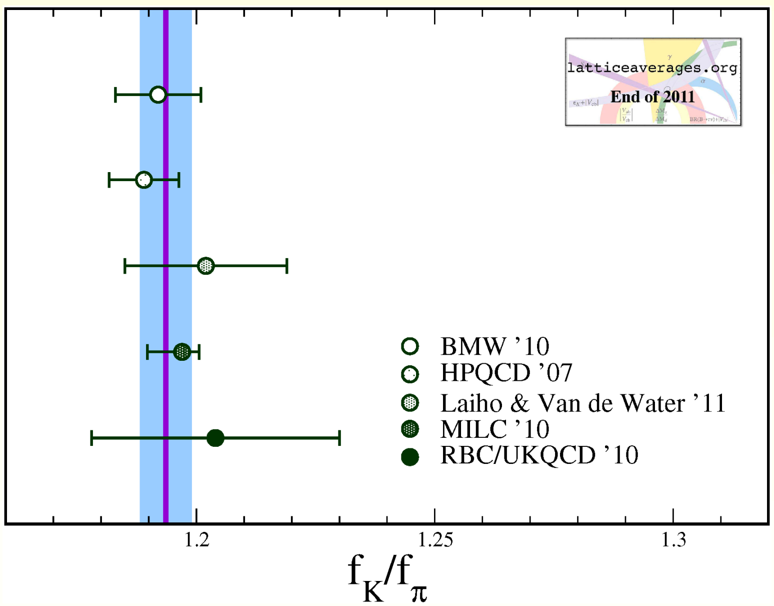

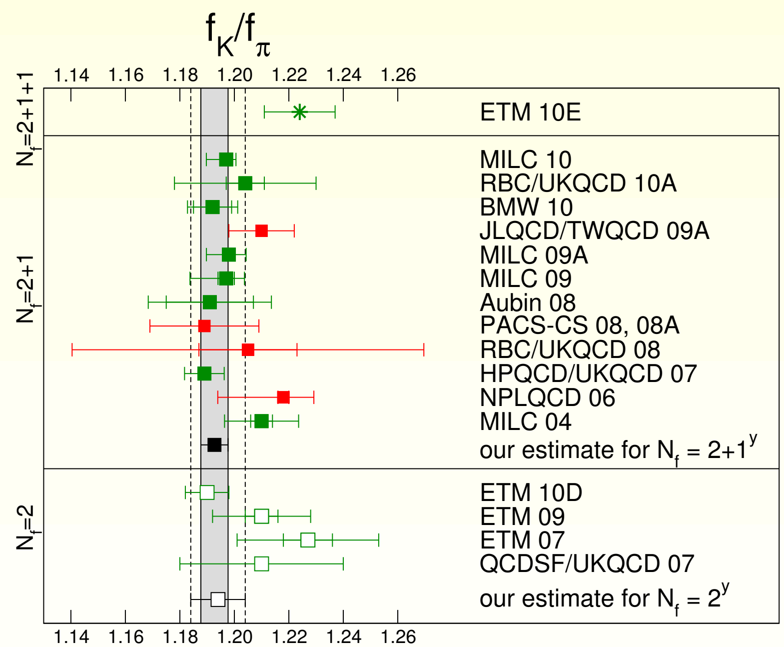

$f_K/f_\pi^{\mathbf{LLV}} = 1.1936 \pm 0.0053$

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Physical light quark masses results this summer **FNAL/MILC, BMW, PAC-CS ...**

2.2. $K \to \pi l \nu$: Determination of $|V_{us}|$

$|V_{us}|$ can also be extracted from K_{l3} decay rates via

$$\Gamma[K \to \pi l \nu_l(\gamma)] = \frac{G_F^2}{192\pi^3} C^2 I_K^l S_{EW} (1 + \delta_K^l) |V_{us}|^2 f_+^2(0)$$

using $f_+(0)$ as calculated with lattice QCD from the 3-point function

$$\langle \pi^-(p') | \bar{s}\gamma_\mu u | K^0(p) \rangle = (p + p')_\mu f_+(t) + (p - p')_\mu f_-(t)$$

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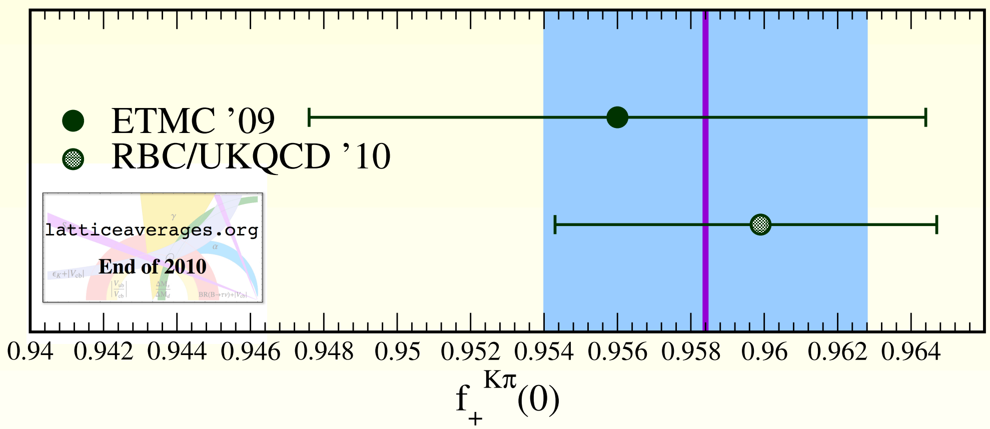

$$f_+(0)^{\mathbf{LLV}} = 0.9584 \pm 0.0044$$

$N_f = 2$ ETMC result included in average because they calculate quenching effects at NLO in ChPT and estimate NNLO effects

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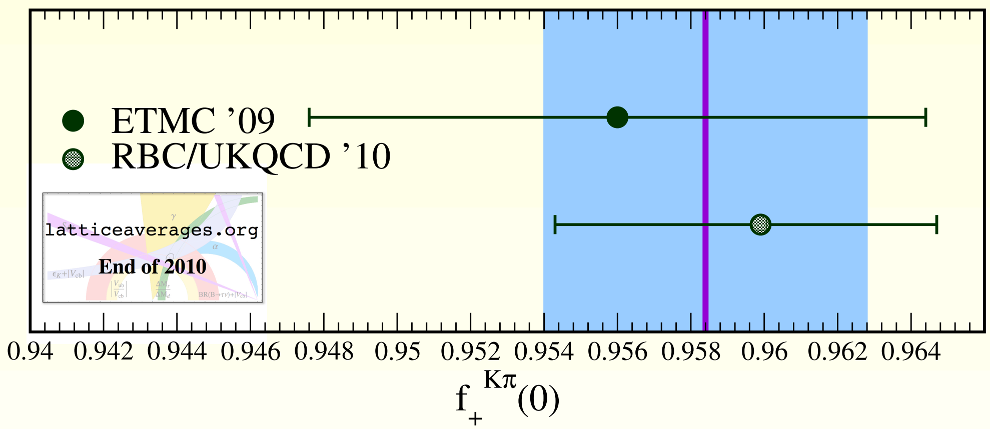

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Limitations of current calculations: one single lattice spacing, $N_f = 2$, ChPT only at NLO ... $\to$ room for improvemet

2.2. $K \to \pi l \nu$: Determination of $|V_{us}|$

In progress:

* $N_f = 2+1$ staggered calculation on **MILC** lattices with twisted boundary conditions at several lattice spacings
  FNAL/MILC POS(Lattice 2010)306

* $N_f = 2+1+1$ staggered calculation on **MILC** lattices with twisted boundary conditions at several lattice spacings and physical quark masses.

* $N_f = 2+1$ overlap calculation: **JLQCD** POS(Lattice 2011)284

(Preliminary) results with physical quark masses this summer: **FNAL/MILC**

2.1. and 2.2. Test of Unitarity in the first row

Experimental averages: **M. Antonelli et al.**, 1005.2323

$$|V_{us}|/|V_{ud}| \times f_K/f_\pi = 0.2758(5) \qquad |V_{us}|f_+(0)^{K\to\pi} = 0.2163(5)$$

and $$\frac{|V_{us}|^2}{|V_{ud}|^2} \times \frac{f_K^2}{f_\pi^2} \propto \frac{\Gamma(K\to\mu\bar{\nu}_\mu(\gamma))}{\Gamma(\pi\to\mu\bar{\nu}_\mu(\gamma))}$$ **Marciano** 2004

* Check unitarity in the first row of CKM matrix.

$$\Delta_{CKM} = |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = -0.0001(6)$$ **M. Antonelli et al**

fits to K_{l3}, K_{l2} exper. data and lattice results for $f_+(0)^{K\to\pi}$ and f_K/f_π

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Look for new physics effects in the comparison of $|V_{us}|$ from helicity suppressed $K_{\mu 2}$ versus helicity allowed K_{l3}

$$R_{\mu 23} = \left(\frac{f_K/f_\pi}{f_+^{K\pi}(0)}\right) \times \mathrm{experim.\,data\,on}\, K_{\mu 2}\pi_{\mu 2}\, \mathrm{and}\, K_{l3}$$

* In the SM $R_{\mu 23} = 1$. Not true for some BSM theories (for exam., charged Higgs)

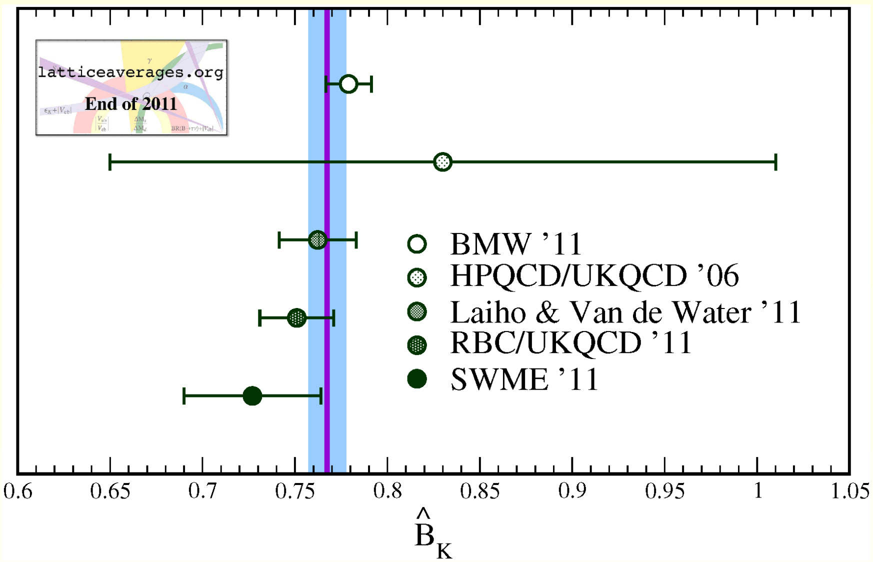
* Current value $R_{\mu 23} = 0.999(7)$, limited by lattice inputs.

2.3. $K^0 - \bar{K}^0$ mixing

One of the most stringent constraints in UT analyses.

$$|\epsilon_K| \quad = e^{i\phi_\epsilon} \kappa_\epsilon C_\epsilon \hat{B}_K |V_{cb}|^2 \lambda^2 \eta \left(|V_{cb}|^2 (1-\bar{\rho}) + \eta_{tt} S_0(x_t) + \eta_{ct} S_0(x_c, x_t) - \eta_{cc} x_c \right)$$

Great success of lattice QCD: reducing $\hat{B}_K$ errors to $\sim 1.3\%$



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* Several fermion formulations and configuration sets.

* Good agreement with $\hat{B}_K^{N_f=2} = 0.729(30)$ **ETMC**, 1009.5606.

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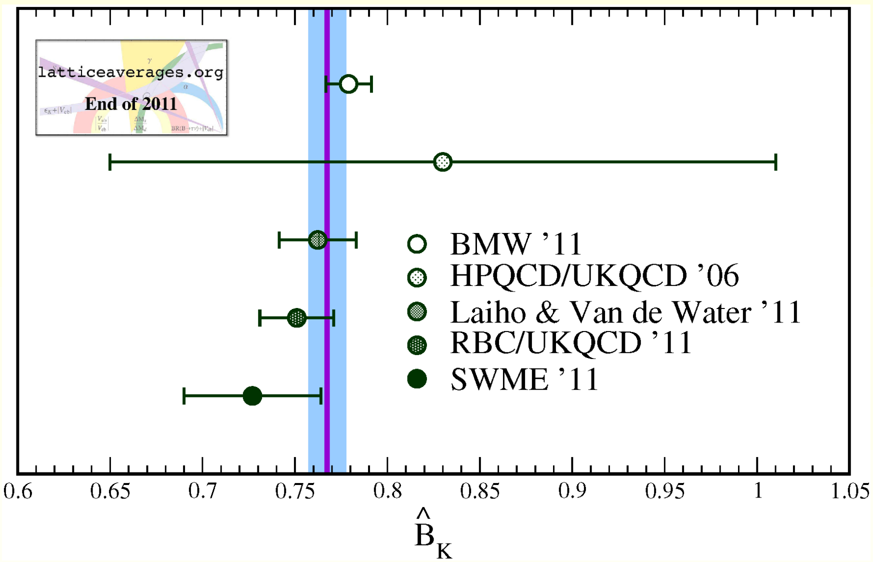

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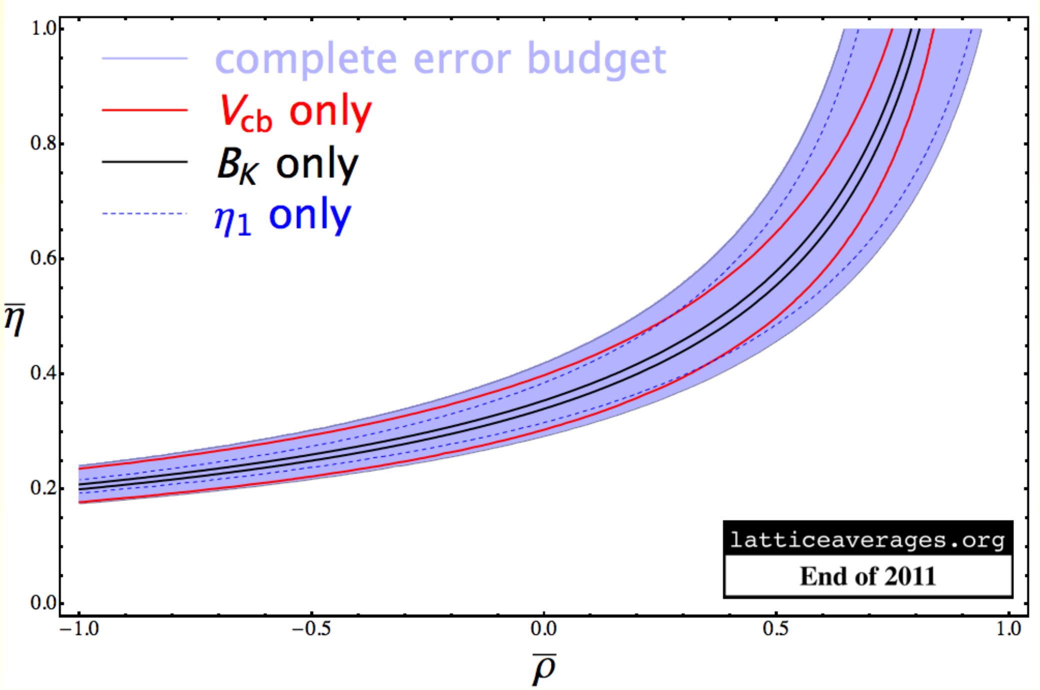
* $\hat{B}_K$ is no longer the dominant source of uncertainty in neutral K mixing.

2.3. $K^0 - \bar{K}^0$ mixing

* Need to include subleading effects: $\kappa_\epsilon = 0.94 \pm 0.02$

(long-distance contributions and $\phi_\epsilon \neq \pi/4$)

$$|\epsilon_K| \quad = \text{known}\ \kappa_\epsilon \hat{B}_K |V_{cb}|^2 \left(|V_{cb}|^2 (1-\bar{\rho}) + \eta_{tt} S_0(x_t) + \eta_{ct} S_0(x_c, x_t) - \eta_{cc} x_c \right)$$



Dominant errors

$|V_{cb}|$: lattice excl. semil.

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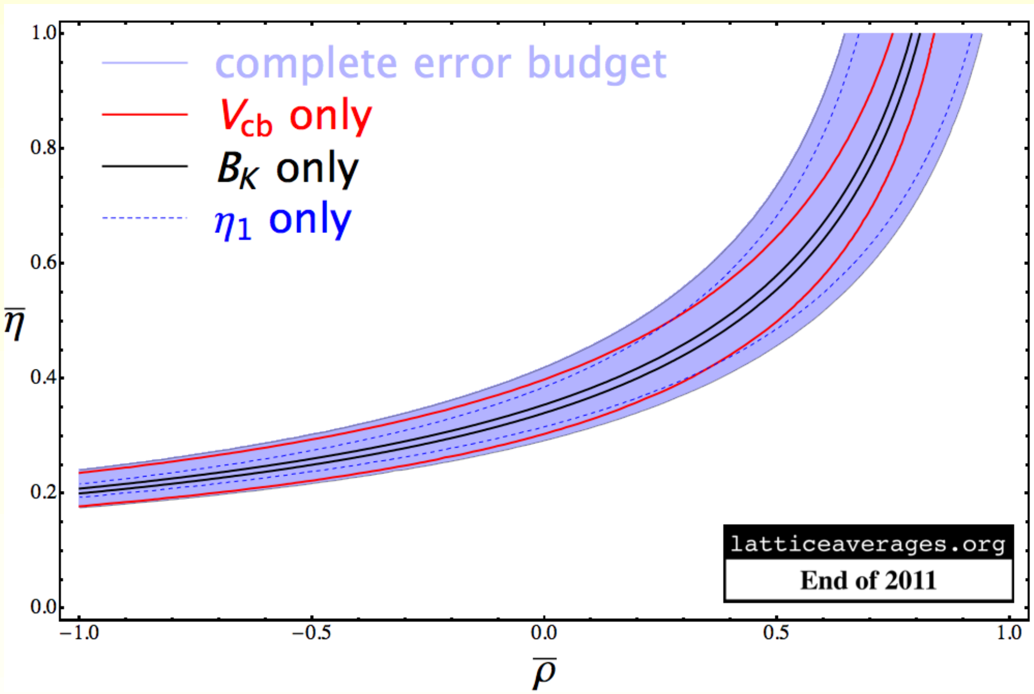

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\# ETMC is calculating the contribution to $K - \bar{K}$ mixing from a complete set of $\Delta S = 2$ effective operators (including BSM) with $N_f = 2 + 1 + 1$ configurations

2.4. $K \to \pi\pi$ and $\varepsilon'_K/\varepsilon_K$

Going beyond gold-plated quantities.

$\Delta I = 3/2$ contribution:

* RBC: First quantitative results at the 20% level from a direct calculation at a small pion mass.

arXiv:1111.1699,1111.4889

* Laiho and Van de Water: New method developed based on combining χPT (indirect) and direct methods.

arXiv:1011.4524

$\Delta I = 1/2$ contribution:

* RBC: First calculation using the direct method on small volume and large pion mass with a 25%. Feasibility study.

arXiv:1111.1699

3. Heavy quark phenomenology

Problem is discretization errors ($\simeq m_Q a, (m_Q a)^2, \cdots$) if $m_Q a$ is large.

* **Effective theories:** Need to include multiple operators matched to full QCD (NRQCD,HQET,RHQ,static). `B-physics` $\surd$

* **Relativistic (improved) formulations:**

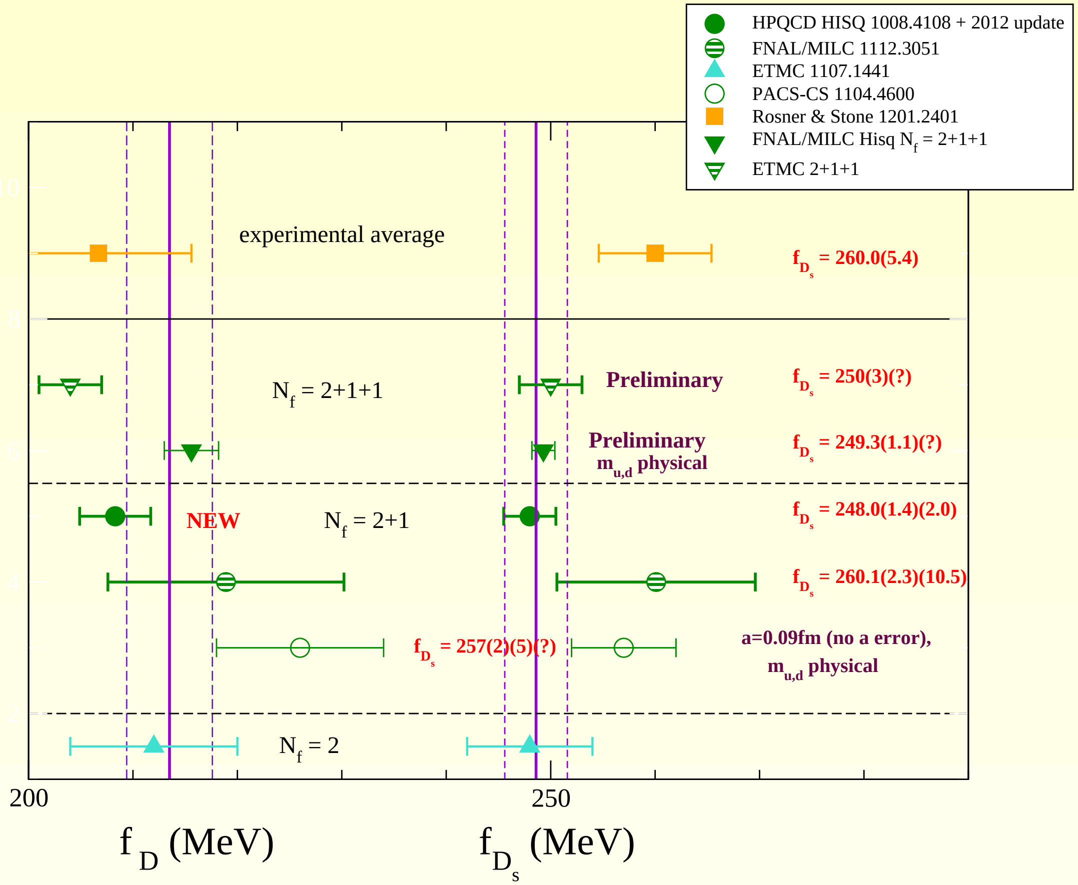
  ** Allow accurate results for `charm` (especially twisted mass, Hisq (Highly improved staggered quarks)).

  ** Advantages of having the same formulation for light and charm: ratios light/charm, PCAC for heavy-light, ... Also simpler tuning of masses.

  One could get the same precision for D as for K

  ** Also for `bottom`: Results for $m_c - \sim m_b$ and extrapolation to m_b (twisted mass, HISQ).

3.1. D and D_s meson decay constants



Experiment: Average of **CLEO, BaBar, Belle** (use $|V_{cs}|^{\text{unit.}} = 0.97345(22)$)

BES will improve this measurement.

$$f_D^{\mathbf{LLV}} = (213.5 \pm 4.1)\ MeV \qquad f_{D_s}^{\mathbf{LLV}} = (248.6 \pm 3.0)\ MeV$$

$f_{D_s}^{\mathbf{exp}} = (260.0 \pm 5.4)\ MeV \rightarrow$ again $\sim 2\sigma$ discrepancy.

3.2. B and B_s meson decay constants

The measured value of $Br(B \to \tau\nu)$ suffers from a tension with the SM at the $2-3\sigma$ level **Laiho,Lunghi,Van de Water**, 1204.0791

* Direct comparison of experiment with f_B^{lat} is difficult because we need $|V_{ub}|$.

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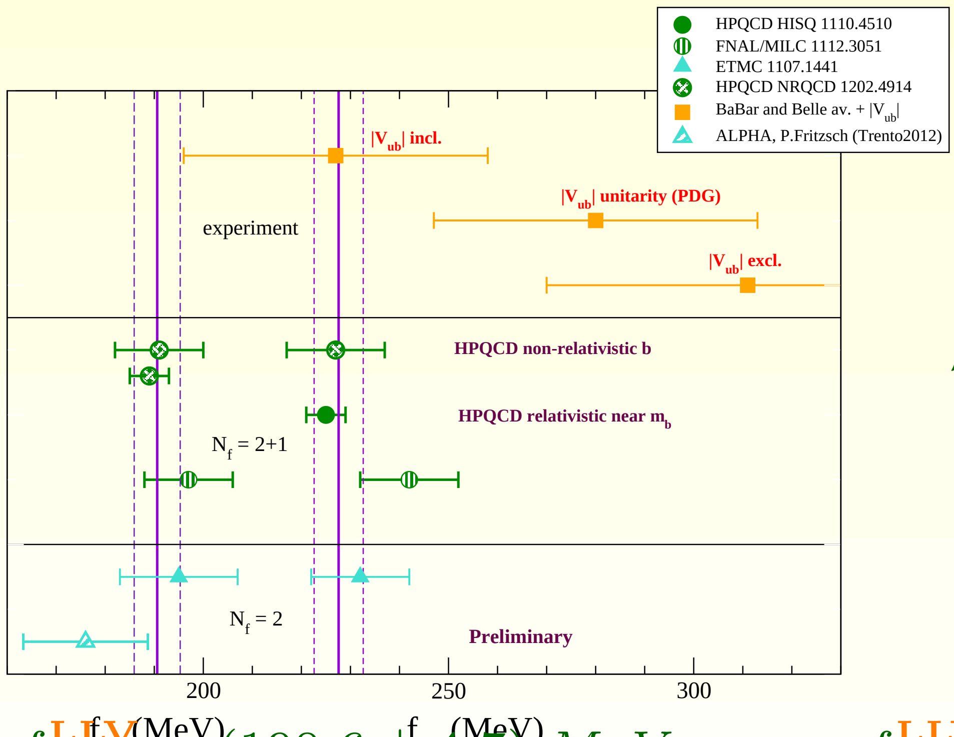


Experimental average

Rosner and Stone, 1201.2401

$$\mathcal{B}r(B \to \tau\nu) = (1.68 \pm 0.31) \times 10^{-4}$$

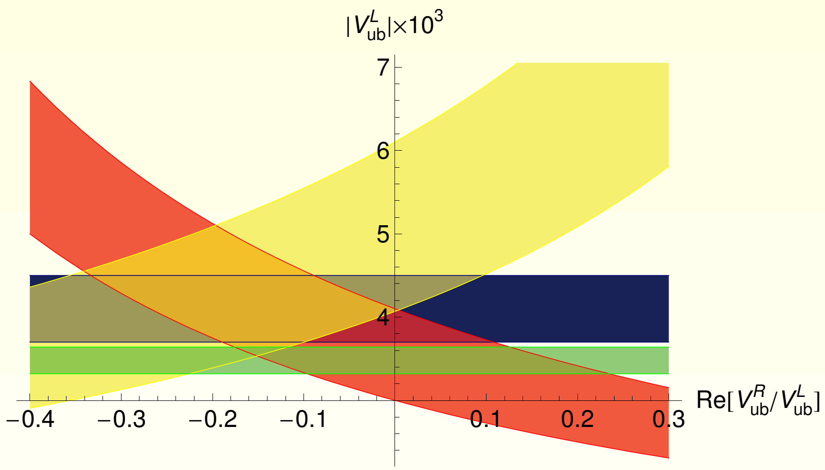
$$f_B^{\mathbf{LLV}} = (190.6 \pm 4.7)\ MeV \qquad f_{B_s}^{\mathbf{LLV}} = (227.6 \pm 5.0)\ MeV$$

In progress: **RBC and UKQCD** ($N_f = 2+1$, domain wall+RHQ).

3.2. B and B_s meson decay constants

$B^- \to \tau^- \bar{\nu}_\tau$ is a sensitive probe of effects from charged Higgs bosons.

$\mathcal{L} = V_{ub}^L \bar{u}_L \not{W} b_L + V_{ub}^R \bar{u}_R \not{W} b_R$



* Leptonic: $(4.95 \pm 0.55) \times 10^{-3}$

$$|V_{ub}| = |V_{ub}^L - V_{ub}^R|$$

using $f_B^{\mathbf{LLV}}$ and experimental average for $Br(B \to \tau)$ **Rosner and Stone**, 1201.2401

* Exclusive: $(3.12 \pm 0.26) \times 10^{-3}$

Laiho,Lunghi,Van de Water,

www.latticeaverages.org

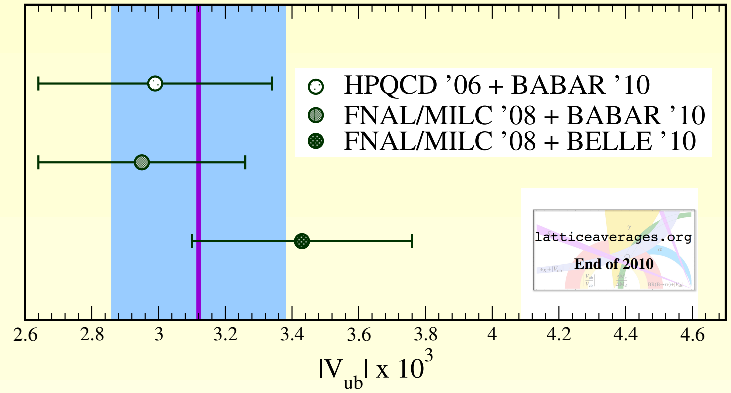
$$|V_{ub}| = |V_{ub}^L + V_{ub}^R|$$

* Inclusive: $(4.41 \pm 0.23) \times 10^{-3}$

Vera Luth, FPCP12

$$|V_{ub}| \approx |V_{ub}^L|$$

3.3. $B \to \pi l \nu$: Exclusive determination of $|V_{ub}|$



* No new calculations since 2010.

Combined fit of lattice and experimental data from different q^2 regions using z-expansion.

$$|V_{ub}^{exc.}|^{\mathbf{LLV}} = (3.12 \pm 0.26) \times 10^{-3}$$

In progress:

* FNAL/MILC Similar methodology as used before but many more data, smaller lattice spacings, improvements on parametrization of shape ...

** Also work in progress for $B_s \to K l \nu$.

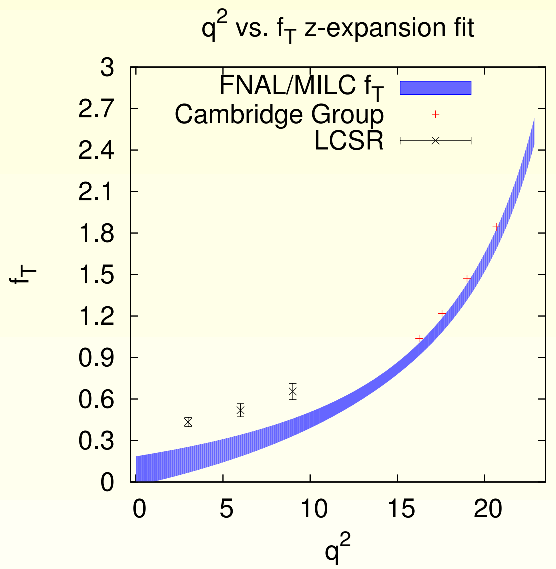
* RBC/UKQCD $(N_f = 2+1)$ and ALPHA $(N_f = 2)$: Similar methodology as for f_B, f_{B_s} calculations.

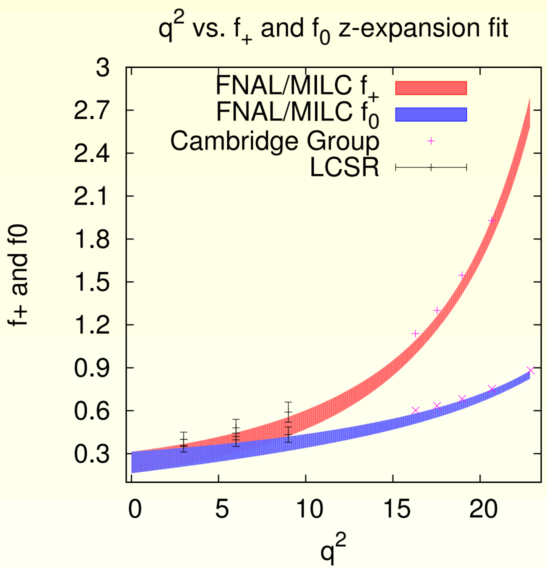
3.3. Form factors for $B \to K l^+ l^-$

Preliminary results from FNAL/MILC, 1111.0981 and Cambridge group, 1010.2726

* Same light formalism and configurations, but different heavy quark formulations.

* Need three form factors (vector, scalar, tensor).





* FNAL/MILC shape from `z-expansion` and systematic errors included.

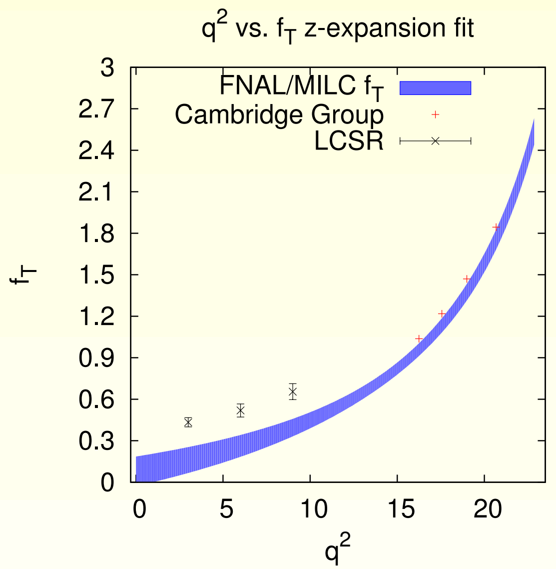
.

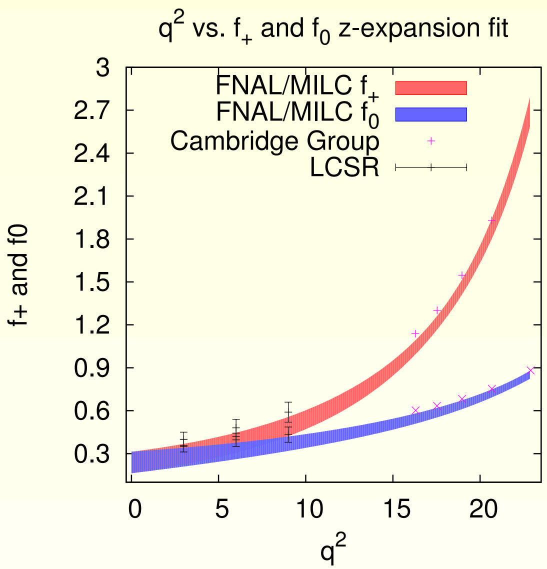
3.3. Form factors for $B \to K l^+ l^-$

Preliminary results from FNAL/MILC, 1111.0981 and Cambridge group, 1010.2726

* Same light formalism and configurations, but different heavy quark formulations.

* Need three form factors (vector, scalar, tensor).





* FNAL/MILC shape from `z-expansion` and systematic errors included.

End of the summer: Near final results from FNAL/MILC.

End of the summer: Updates from HPQCD for $B \to K(K^*) l\nu$

3.3. Exclusive determination of $|V_{cb}|$

Extraction from exclusive B decays ($w = v \cdot v'$ is the velocity transfer):

$$\frac{d\Gamma(B \to D^* l\nu)}{dw} = (\text{known}) \times |V_{cb}|^2 \times (w^2 - 1)^{1/2} |\mathcal{F}(w)|^2$$

$$\frac{d\Gamma(B \to D l\nu)}{dw} = (\text{known}) \times |V_{cb}|^2 \times (w^2 - 1)^{3/2} |\mathcal{G}(w)|^2$$

Updated 2010 **FNAL/MILC** determination of $\mathcal{F}$ at zero recoil (blind anlysis) + **BaBar** and **Belle** measurements: **Will be updated Lattice2012**

$$|V_{cb}|_{excl} = (39.7 \pm 0.7_{exp} \pm 0.7_{LQCD}) \times 10^{-3}$$

* 2σ tension with inclusive determination $|V_{cb}|_{incl} \times = (41.9 \pm 0.8) \times 10^{-3}$

Vera Luth, FPCP12

3.3. Exclusive determination of $|V_{cb}|$

At zero recoil HFAG 2010

$$|V_{cb}|\mathcal{F}(1) = (36.04 \pm 0.52) \times 10^{-3} \qquad |V_{cb}|\mathcal{G}(1) = (42.3 \pm 1.5) \times 10^{-3}$$

$\Longrightarrow$ Need $B \to Dl\nu$ form factors at non-zero recoil to match $B \to D^* l\nu$ precision in the determination of $|V_{cb}|$.

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$\Longrightarrow$ Need $B \to Dl\nu$ form factors at non-zero recoil to match $B \to D^* l\nu$ precision in the determination of $|V_{cb}|$.

Calculation of non-zero recoil form factors $B \to D^{(*)} l\nu$ in progress

FNAL/MILC, arXiv:1111.0677.

$\to$ will allow complementary extraction of $|V_{cb}|$.

3.4. D semileptonic decays

Extraction of the CKM matrix elments $|V_{cd(cs)}|$
still dominated by lattice determination of the relevant form factors.

$$\frac{d}{dq^2}\Gamma(D \to K(\pi)l\nu) \quad \propto \quad |V_{cs(cd)}|^2 \, |f_+^{D\to K(\pi)}(q^2)|^2$$

Testing lattice QCD: shape of the form factors
$\to$ use same methodology for other processes like $B \to \pi l\nu$ or $B \to Kl\bar{l}$

Correlated signals of NP to those in leptonic decays.

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New method to calculate $f_+(0)$ for semileptonic decays when using the same fermion formulation for all quark flavours HPQCD, Phys.Rev.D82:114506(2010)

* Use Ward identity to relate $f_0(q^2)$ to three-point functions with a scalar (vs. vector) insertion $\boxed{f_+(0) = f_0(0) = \frac{m_c - m_q}{m_D^2 - m_\pi^2}\langle D|S|K\rangle}$

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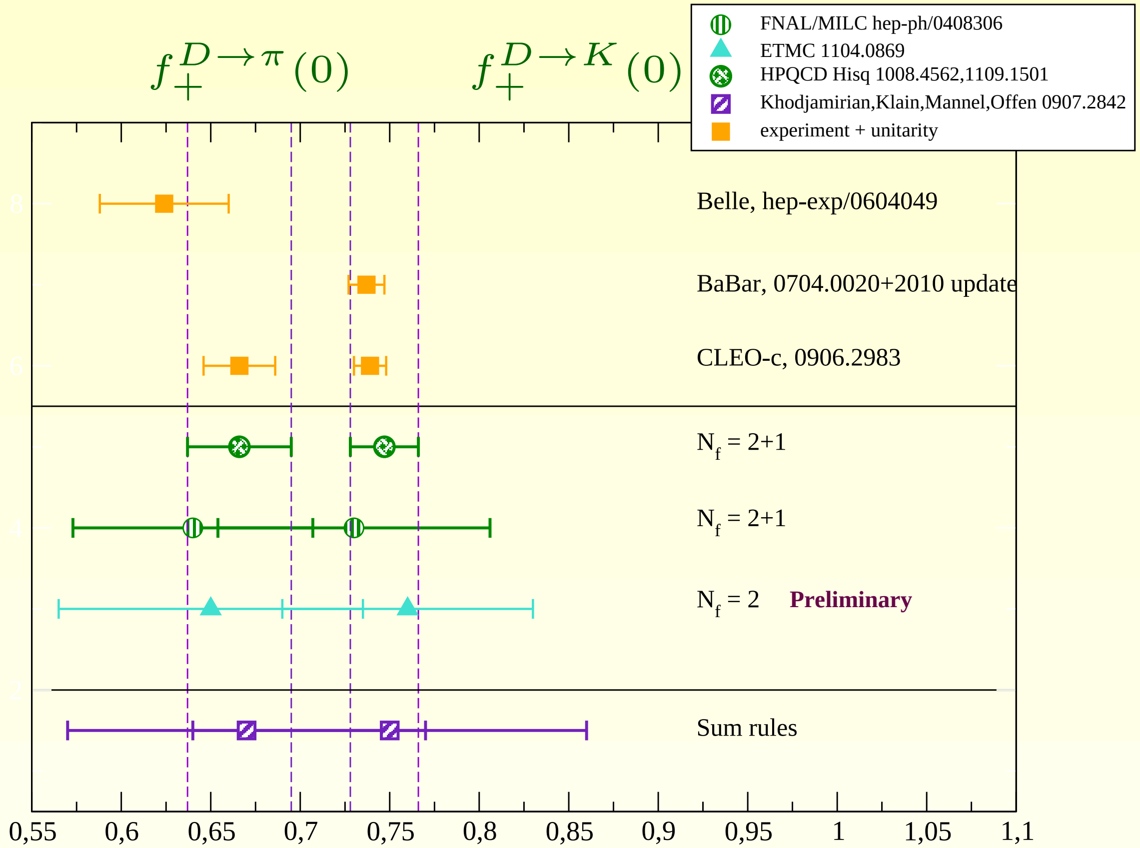
* Very precise determination of $|V_{cq}|$, but can not get the shape of $f_+(q^2)$. Only $f_0(q^2)$.

3.4. D semileptonic decays



error $f_+^{D\to K}$: 11% → 2.5%.

error $f_+^{D\to\pi}$: 10% → 5%.

$|V_{cs}| = 0.961(11)_{exp}(24)_{lat}$ compatible with unitarity value $|V_{cs}|^{unit.} = 0.97345(16)$

$|V_{cd}| = 0.225(6)_{exp}(10)_{lat}$ compatible with unitarity value $|V_{cd}|^{unit.} = 0.2252(7)$

* competitive with ν scattering determination $|V_{cd}|^{\nu} = 0.230(11)$

3.4. D semileptonic decays: form factors at $q^2 \neq 0$

* Global fit in SM + experiment $\rightarrow$ $|V_{cs(cd)}|$ and $f_+^{D\to K(\pi)}(q^2)$

* New results from several lattice groups by the summer/end 2012.

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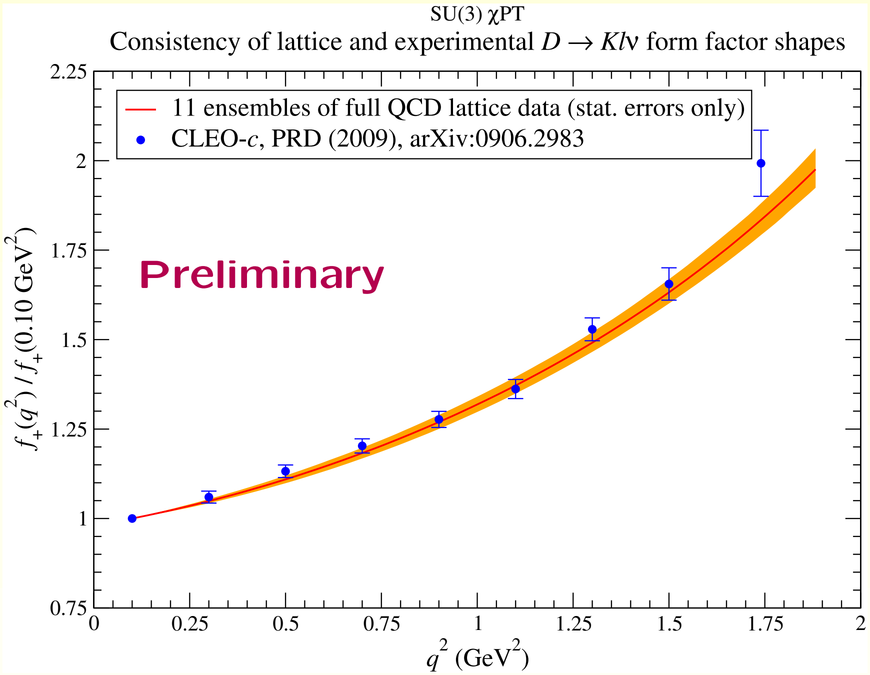
Jon Bailey, FNAL/MILC 2012



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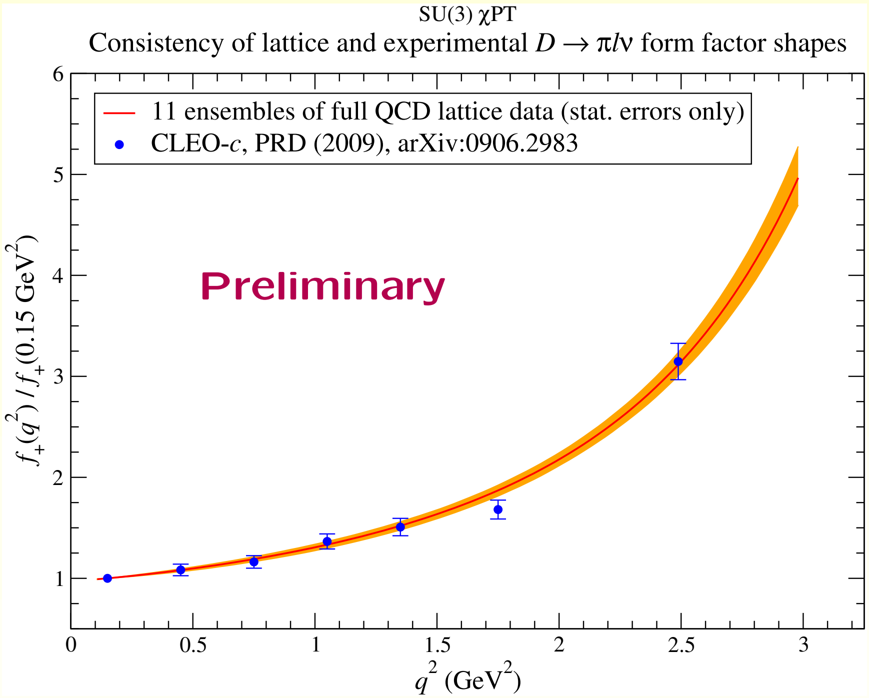

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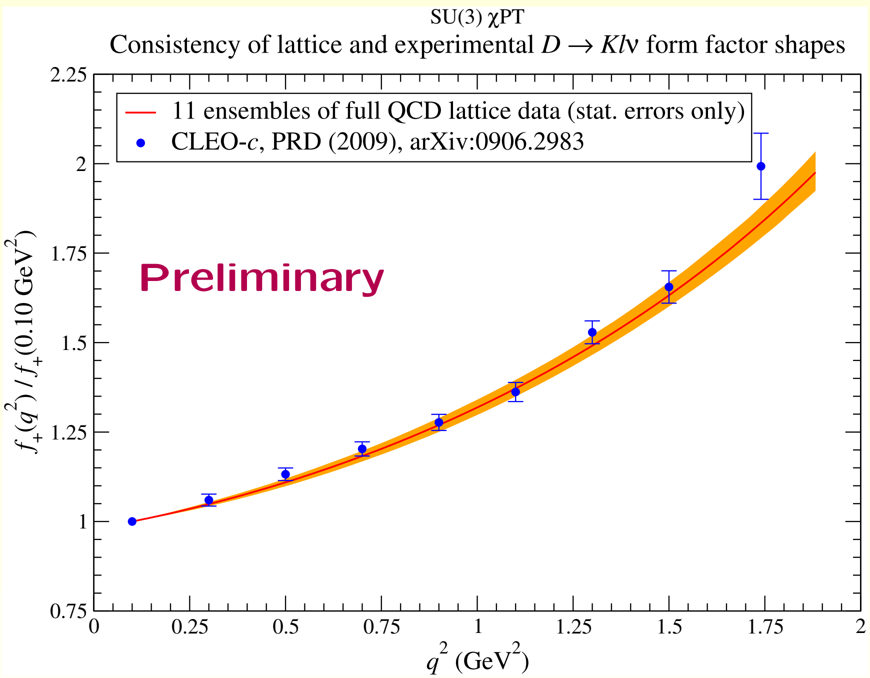
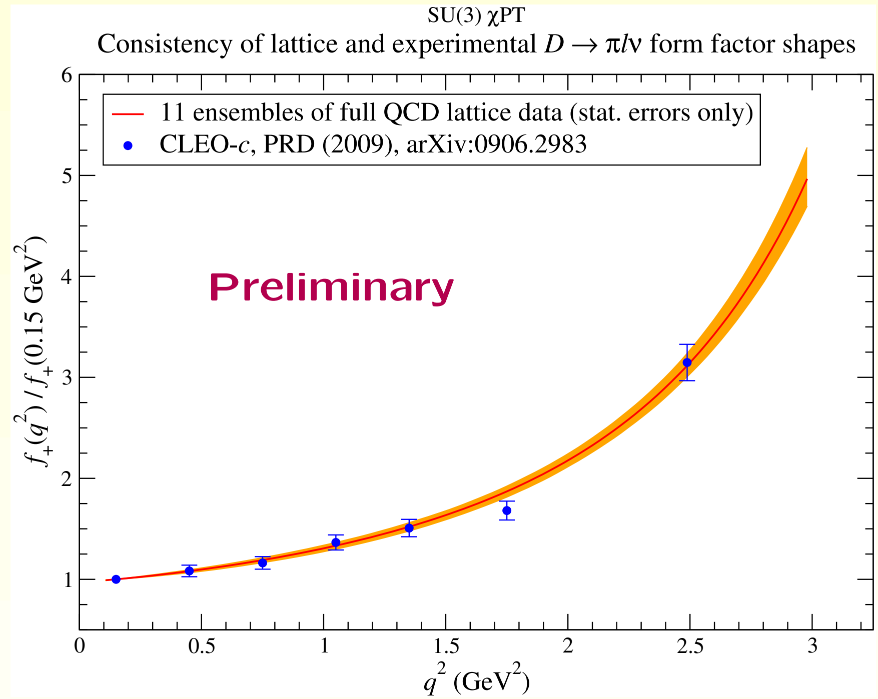


Jon Bailey, FNAL/MILC 2012

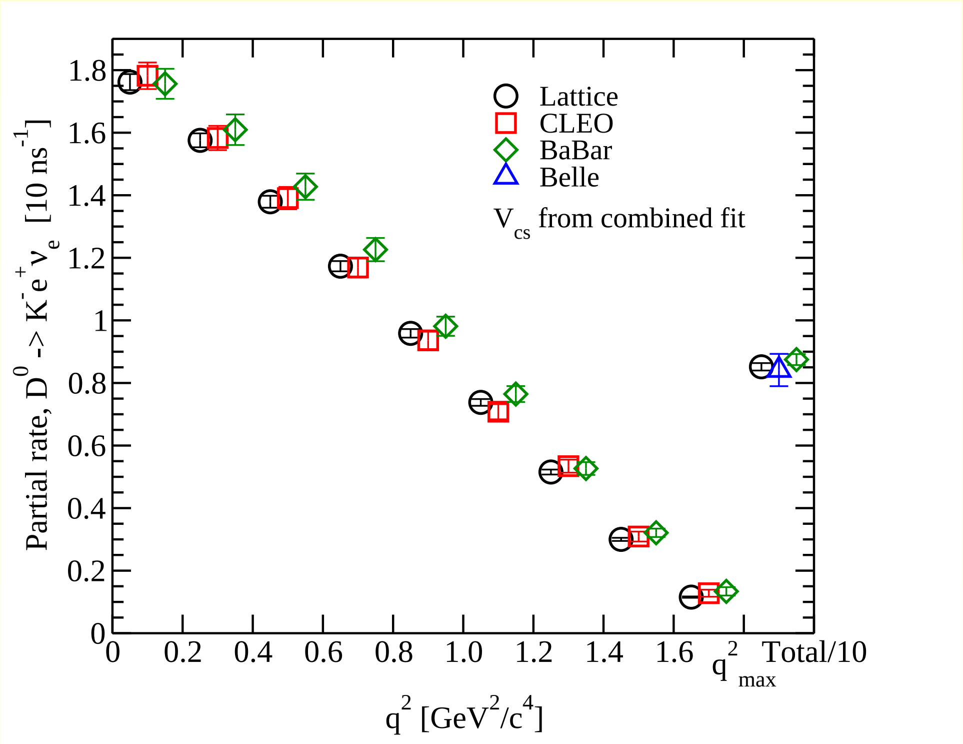


* **In progress** FNAL/MILC: Study of $D \rightarrow K(\pi)l\nu$ form factors with $N_f = 2+1+1$ Hisq MILC ensembles with physical light quark masses.

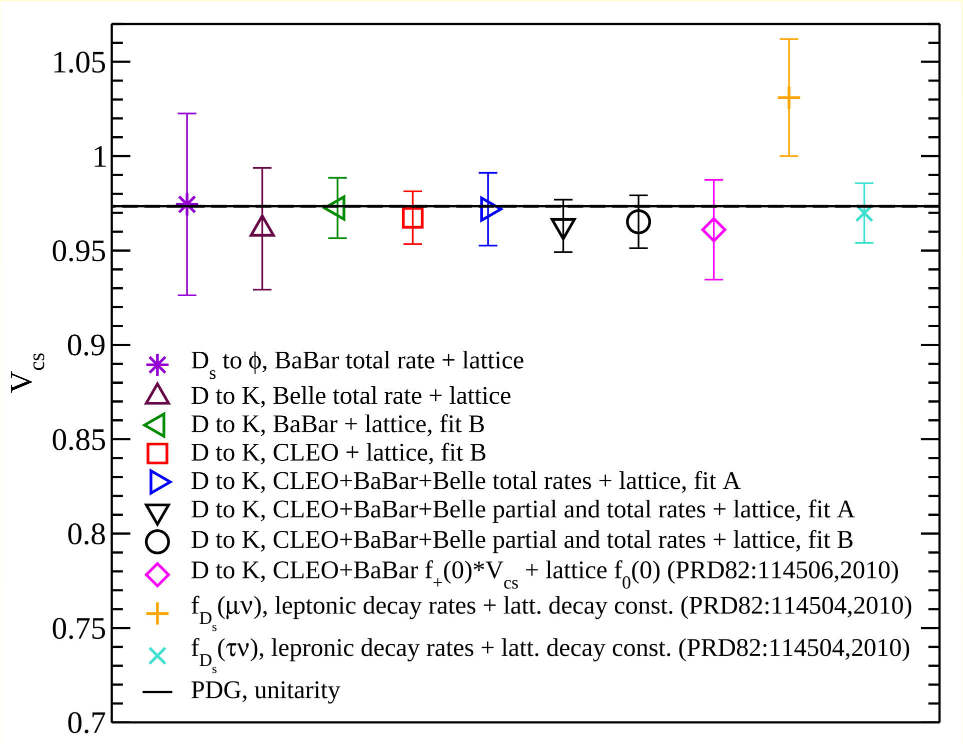
3.4. D semileptonic decays: form factors at $q^2 \neq 0$

PRELIMINARY

Jonna Koponen, HPQCD 2012



Jonna Koponen, HPQCD 2012



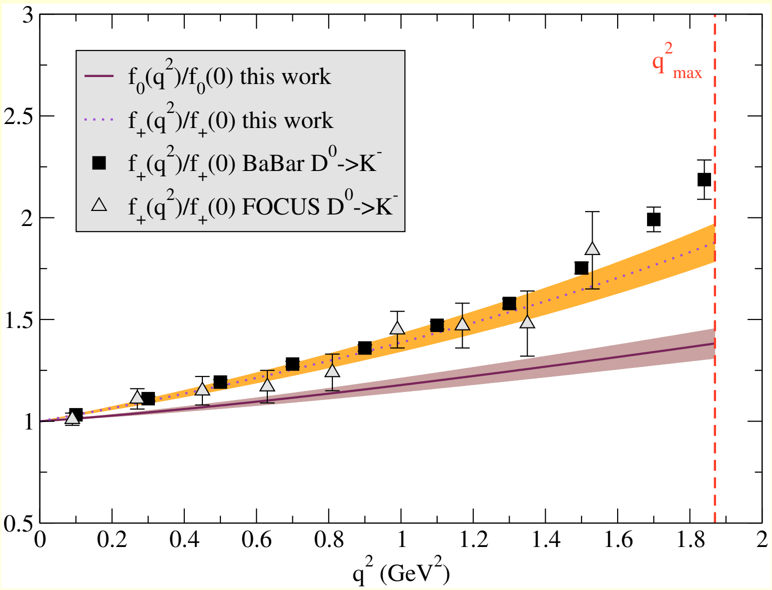
* Form factors for $D \to K l \nu$ with 1.6% accuracy.

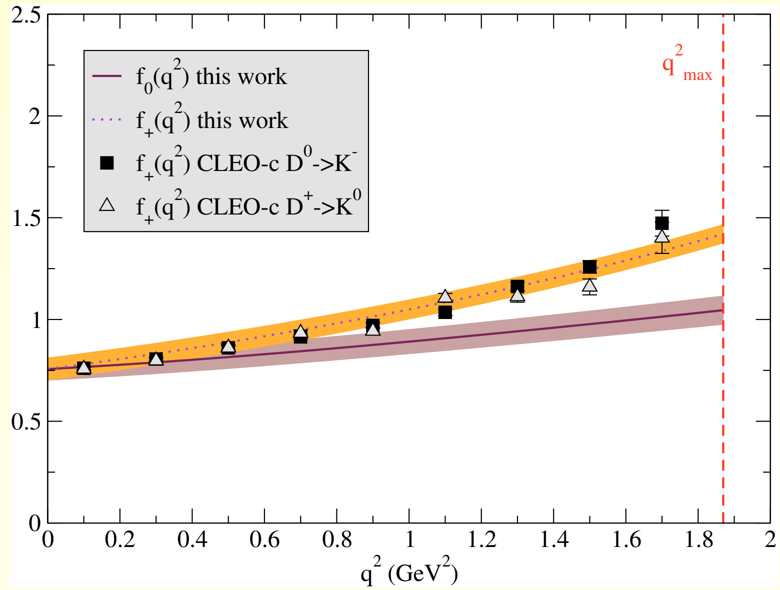
* Best **preliminary** value: $|V_{cs}| = 0.965(14)$ with 1.4% error from all available experimental data.

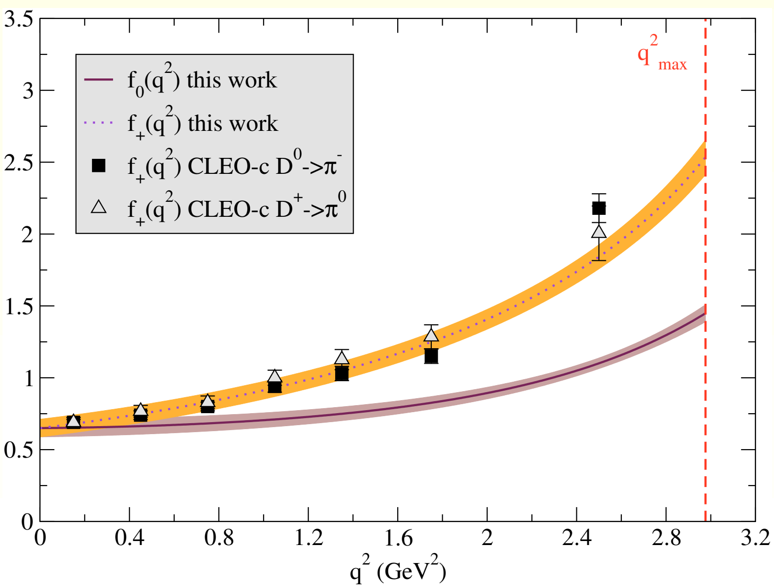
Working also on $D \to \pi l \nu$ and $D_s \to K l \nu$.

3.4. D semileptonic decays: form factors at $q^2 \neq 0$

PRELIMINARY







ETMC, 11104.0869

Good agreement with experiment

3.5. Neutral B-meson mixing

Hints of NP in neutral B-meson mixing at the $(2-3)\sigma$ level: UTfit 1010.5089, CKMfitter 1203.0238, like-sign dimuon charge asymmetry 1106.6308 + UT tensions

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Hints of NP in neutral B-meson mixing at the $(2-3)\sigma$ level: UTfit 1010.5089, CKMfitter 1203.0238, like-sign dimuon charge asymmetry 1106.6308 + UT tensions

Effective Hamiltonian describing neutral $B-$meson mixing.

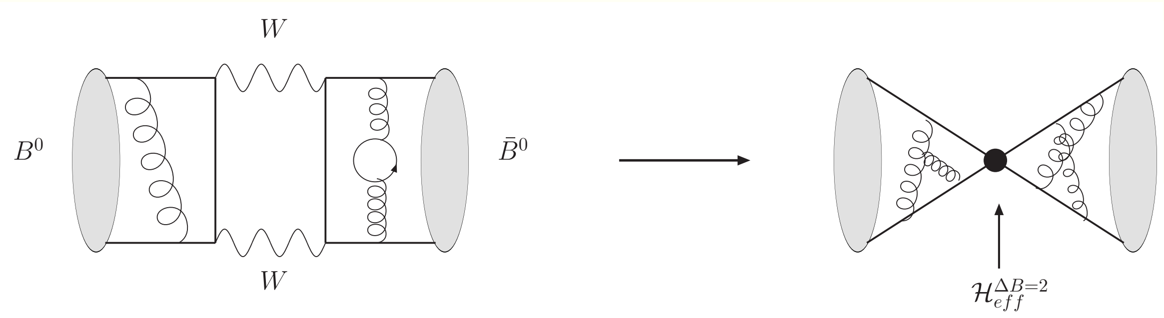
$$\mathcal{H}_{eff}^{\Delta F=2} = \sum_{i=1}^{5} C_i Q_i + \sum_{i=1}^{3} \widetilde{C}_i \widetilde{Q}_i$$

$$Q_1^q = \left(\bar{\psi}_f^i \gamma^\nu (\mathrm{I} - \gamma_5) \psi_q^i\right) \left(\bar{\psi}_f^j \gamma^\nu (\mathrm{I} - \gamma_5) \psi_q^j\right) \qquad \textbf{SM}$$

$$Q_2^q = \left(\bar{\psi}_f^i (\mathrm{I} - \gamma_5) \psi_q^i\right) \left(\bar{\psi}_f^j (\mathrm{I} - \gamma_5) \psi_q^j\right) \qquad Q_3^q = \left(\bar{\psi}_f^i (\mathrm{I} - \gamma_5) \psi_q^j\right) \left(\bar{\psi}_f^j (\mathrm{I} - \gamma_5) \psi_q^i\right)$$

$$Q_4^q = \left(\bar{\psi}_f^i (\mathrm{I} - \gamma_5) \psi_q^i\right) \left(\bar{\psi}_f^j (\mathrm{I} + \gamma_5) \psi_q^j\right) \qquad Q_5^q = \left(\bar{\psi}_f^i (\mathrm{I} - \gamma_5) \psi_q^j\right) \left(\bar{\psi}_f^j (\mathrm{I} + \gamma_5) \psi_q^i\right)$$

$$\tilde{Q}_{1,2,3}^q = Q_{1,2,3}^q \text{ with the replacement } (\mathrm{I} \pm \gamma_5) \rightarrow (\mathrm{I} \mp \gamma_5)$$



3.5. Neutral B-meson mixing

\# In the Standard Model

* The mass differences $\Delta M_{s(d)}$ depend on a single matrix element.

$$\Delta M_q|_{SM} = \frac{G_F^2 M_W^2}{6\pi^2} |V_{tq}^* V_{tb}|^2 \eta_2^B S_0(x_t) M_{B_s} f_{B_q}^2 \hat{B}_{B_q}$$

** Non-perturbative input

$$\frac{8}{3} f_{B_q}^2 B_{B_q}(\mu) M_{B_q}^2 = \langle \bar{B}_q^0 | O_1 | B_q^0 \rangle (\mu) \quad \text{with} \quad O_1 \equiv [\overline{b^i}\, q^i]_{V-A} [\overline{b^j}\, q^j]_{V-A}$$

* $\Delta\Gamma_{s(d)}$ depend on $\langle O_1 \rangle$ and $\langle O_3 \rangle$ (or, alternatively, $\langle O_1 \rangle$ and $\langle O_2 \rangle$).

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* $\Delta\Gamma_{s(d)}$ depend on $\langle O_1 \rangle$ and $\langle O_3 \rangle$ (or, alternatively, $\langle O_1 \rangle$ and $\langle O_2 \rangle$).

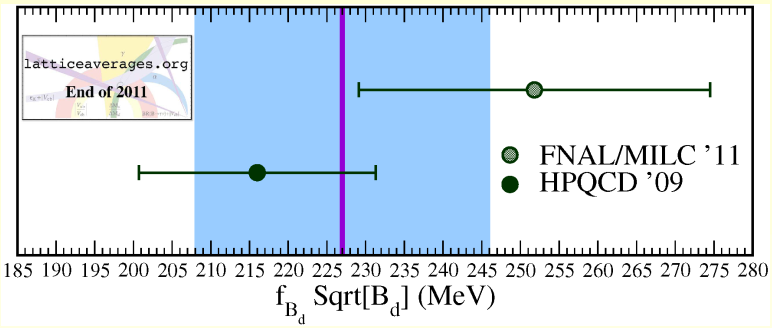
Most interesting for phenomenology (UT analyses):

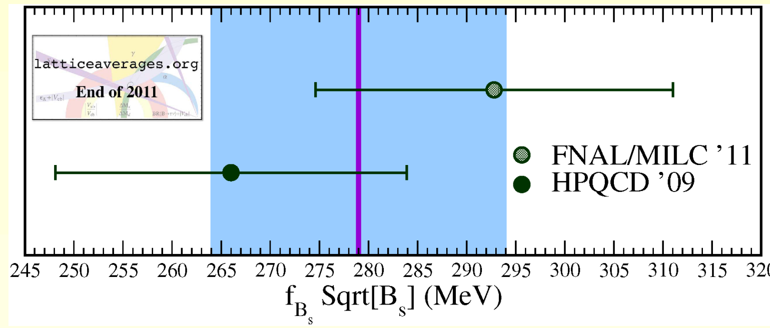
$$f_{B_q} \sqrt{\hat{B}_{B_q}}\ ^{*} \qquad\qquad \xi = \frac{f_{B_s}\sqrt{B_{B_s}}}{f_{B_d}\sqrt{B_{B_d}}}$$

* In particular as the role of $|V_{cb}|$ (reaching is ultimate theoretical accuracy) in UT analyses is being replaced by ΔM_{B_s} and $B \to \tau\nu$.

3.5. Neutral B-meson mixing: SM

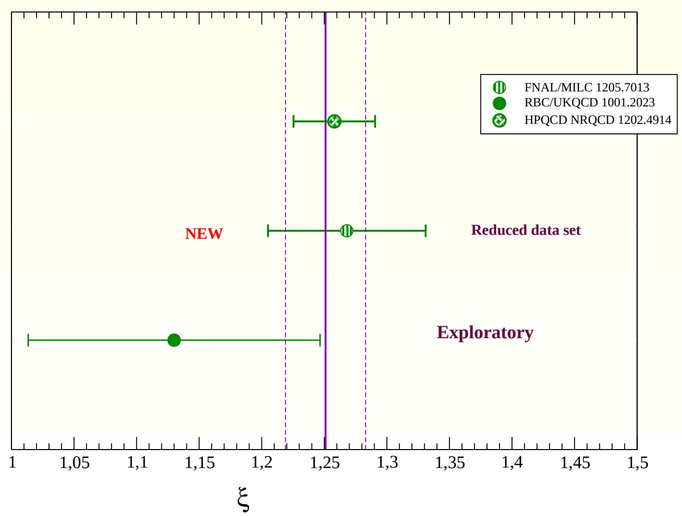
Two results for $\sqrt{f_B \hat{B}_B}$ using **MILC** $N_f = 2+1$ but different description of heavy quarks.





$$f_{B_s}\sqrt{\hat{B}_{B_s}}^{\,\mathrm{LLV}} = 279(15)\mathrm{MeV} \qquad f_{B_d}\sqrt{\hat{B}_{B_d}}^{\,\mathrm{LLV}} = 227(19)\mathrm{MeV}$$

Results for $\xi = \dfrac{f_{B_s}\sqrt{B_{B_s}}}{f_{B_d}\sqrt{B_{B_d}}}$



$$\xi^{\mathrm{lat}} = 1.251 \pm 0.032$$

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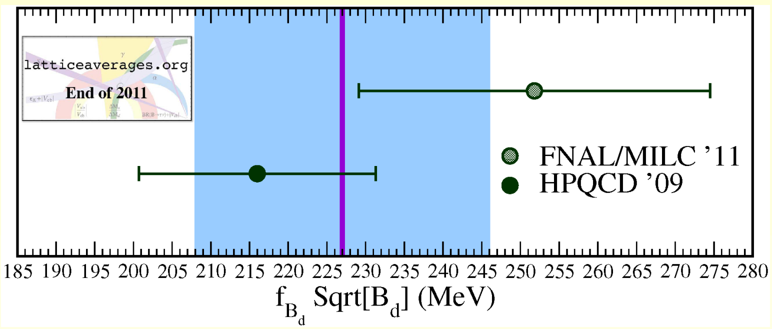
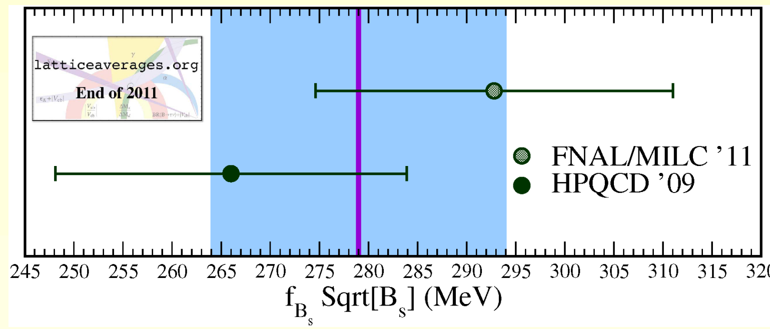
Two results for $\sqrt{f_B \hat{B}_B}$ using MILC $N_f = 2+1$ but different description of heavy quarks.





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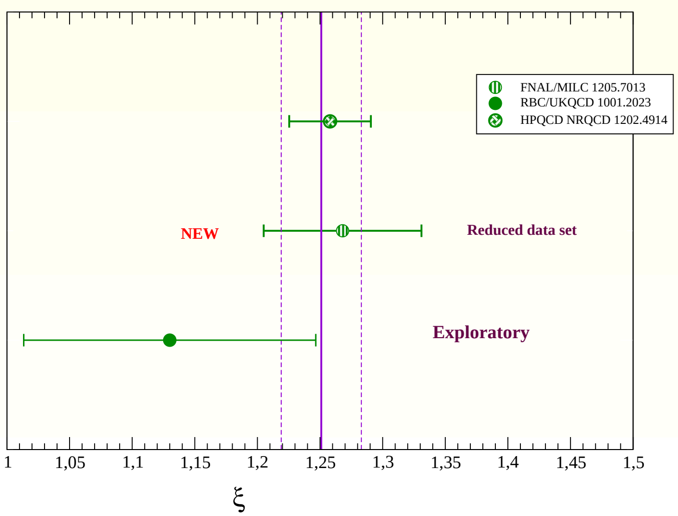

$$\boxed{\xi^{\mathbf{lat}} = 1.251 \pm 0.032}$$

* FNAL/MILC calculation with the same choice of actions but improved statistics, discret. errors, and analysis techniques is **in progress.** New (better) results by end of summer

3.5. Neutral B-meson mixing: BSM

SM predictions + BSM contributions = experiment

→ constraints on BSM building **Dobrescu and Krnjaic**, 1104.2893; **Altmannshofer and Carena**, 1110.0843; **Buras and Girrbach**, 1201.1302 ...

* Need matrix elements of all the operators in $\mathcal{H}_{eff}^{\Delta B=2}$

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* Need matrix elements of all the operators in $\mathcal{H}_{eff}^{\Delta B=2}$

⟹ **FNAL/MILC** will have final results by the end of the year

	B_d^0		B_s^0	
[GeV^2]	BBGLN	BJU	BBGLN	BJU
$f_{B_q}^2 B_{B_q}^{(1)}$	0.0411(75)		0.0559(68)	
$f_{B_q}^2 B_{B_q}^{(2)}$	0.0574(92)	0.0538(87)	0.086(11)	0.080(10)
$f_{B_q}^2 B_{B_q}^{(3)}$	0.058(11)	0.058(11)	0.084(13)	0.084(13)
$f_{B_q}^2 B_{B_q}^{(4)}$	0.093(10)		0.135(15)	
$f_{B_q}^2 B_{B_q}^{(5)}$	0.127(15)		0.178(20)	

* $\langle Q_1\rangle, \langle Q_3\rangle$ will also allow new prediction for $\Delta\Gamma_s$.

$\Delta\Gamma_s^{exp} = (0.116 \pm 0.019) ps^{-1}$ **LHCb**, Moriond 2012

$\Delta\Gamma_s^{SM} = (0.087 \pm 0.021) ps^{-1}$ **Lenz,Nierste**, 1102.4274

3.6. Rare decays $\mathcal{B}r(B_{s(d)} \to \mu^+\mu^-)$

\# Bag parameters $B_{B_{s,d}}$ describing $B-\mathtt{meson}$ mixing in the SM can be can be used for theoretical prediction of $\mathcal{B}r(B \to \mu^+\mu^-)$

$$\frac{\mathcal{B}r(B_q \to \mu^+\mu^-)}{\Delta M_q} = \tau(B_q)\, 6\pi \frac{\eta_Y}{\eta_B} \left(\frac{\alpha}{4\pi M_W sin^2\theta_W} \right)^2 m_\mu^2 \frac{Y^2(x_t)}{S(x_t)} \frac{1}{\hat{B}_q}$$

* Using **HPQCD** determinations of $\hat{B}_q$ **Gámiz et al.**, 0902.1815

$\mathcal{B}r(B_s \to \mu^+\mu^-) = (3.19 \pm 0.19) \times 10^{-9}$ and $\mathcal{B}r(B_d \to \mu^+\mu^-) = (1.02 \pm 0.09) \times 10^{-10}$

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* Improved $f_{B_{s,d}}^{lattice}$ makes direct theoretical calculation competitive.

Buras and Girrbach,1204.5064

$$\mathcal{B}r(B_s \to \mu^+\mu^-) = (3.1 \pm 0.2) \times 10^{-9} \text{ and } \mathcal{B}r(B_d \to \mu^+\mu^-) = (1.0 \pm 0.1) \times 10^{-10}$$

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Buras and Girrbach,1204.5064

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Very small in the SM and potentially sensitive to NP

$\to$ subject of active search at LHC and Tevatron.

* Most stringest experimental bounds **LHCb, 1203.4493**:

$$\mathcal{B}r(B_s \to \mu^+\mu^-) < 4.5 \times 10^{-9} \qquad \mathcal{B}r(B_d \to \mu^+\mu^-) < 8.1 \times 10^{-10}$$

3.6. Rare decays $\mathcal{B}r(B_{s(d)} \to \mu^+\mu^-)$

Hadron colliders measure $\mathcal{B}r(B_s \to \mu^+\mu^-)$ using a normalization channel

$$\mathcal{B}r(B_s \to \mu^+\mu^-) = \mathcal{B}r(B_d \to X)\,\frac{f_d}{f_s}\,\frac{\varepsilon_X}{\varepsilon_{\mu\mu}}\,\frac{N_{\mu\nu}}{N_X}$$

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Fleischer, Serra, Tuning, 1004.3982 proposed a new strategy to determine f_s/f_d: use the hadronic decay ratio $\mathcal{B}r(\bar{B}^0_s \to D_s^+\pi^-)/\mathcal{B}r(\bar{B}^0 \to D^+K^-)$

and factorization

$$\frac{f_s}{f_d} = 0.0743 \times \frac{\tau_{B^0}}{\tau_{B^0_s}} \times \left[\frac{\varepsilon_{DK}}{\varepsilon_{D_s\pi}}\,\frac{N_{D_s\pi}}{\varepsilon_{DK}}\right] \times \frac{1}{\mathcal{N}_a\mathcal{N}_F} \quad \text{with} \quad \mathcal{N}_a = \left[\frac{a_1^{(s)}(D_s^+\pi^-)}{a_1^{(d)}(D^+K^-)}\right]^2$$

3.6. Rare decays $\mathcal{B}r(B_{s(d)} \to \mu^+\mu^-)$

Hadron colliders measure $\mathcal{B}r(B_s \to \mu^+\mu^-)$ using a normalization channel

$$\mathcal{B}r(B_s \to \mu^+\mu^-) = \mathcal{B}r(B_d \to X)\, \frac{f_d}{f_s}\, \frac{\varepsilon_X}{\varepsilon_{\mu\mu}}\, \frac{N_{\mu\nu}}{N_X}$$

Fleischer, Serra, Tuning, 1004.3982 proposed a new strategy to determine f_s/f_d: use the hadronic decay ratio $\mathcal{B}r(\bar{B}_s^0 \to D_s^+\pi^-)/\mathcal{B}r(\bar{B}^0 \to D^+K^-)$

and factorization

$$\frac{f_s}{f_d} = 0.0743 \times \frac{\tau_{B^0}}{\tau_{B_s^0}} \times \left[\frac{\varepsilon_{DK}}{\varepsilon_{D_s\pi}}\, \frac{N_{D_s\pi}}{\varepsilon_{DK}}\right] \times \frac{1}{\mathcal{N}_a \mathcal{N}_F} \quad \text{with} \quad \mathcal{N}_a = \left[\frac{a_1^{(s)}(D_s^+\pi^-)}{a_1^{(d)}(D^+K^-)}\right]^2$$

and $\mathcal{N}_F$ can be obtained from the scalar form factors of $B_s \to D_s l\nu$ and $B \to Dl\nu$ at non-zero momentum transfer.

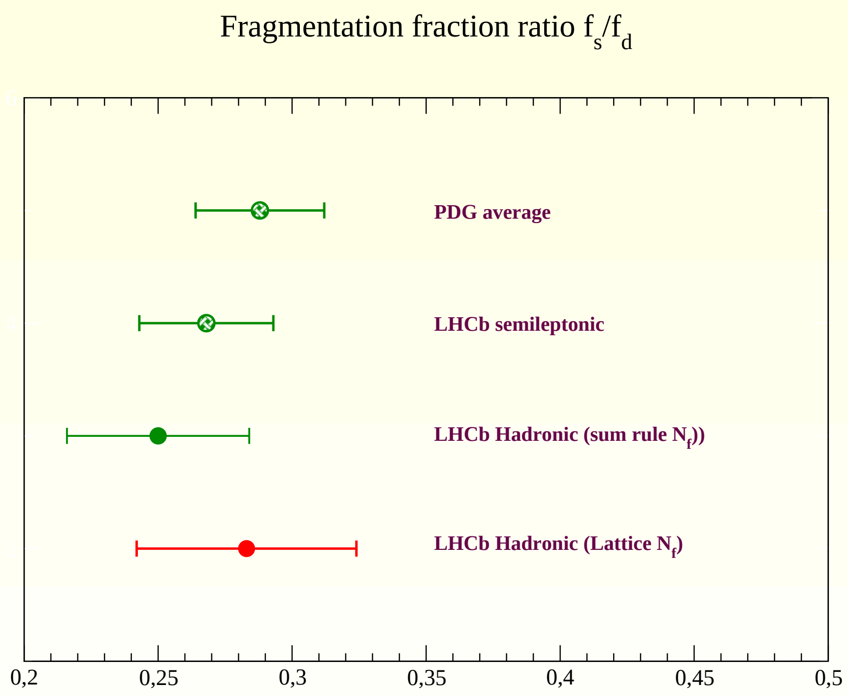
$$\mathcal{N}_F = \left[\frac{f_0^{(s)}(M_\pi^2)}{f_0^{(d)}(M_K^2)}\right]^2$$

3.6. Rare decays $\mathcal{B}r(B_{s(d)} \to \mu^+\mu^-)$

Calculate the ratio of form factors on the lattice.

$$\frac{f_0^{(s)}(M_\pi^2)}{f_0^{(d)}(M_K^2)} = 1.046(44)(15)$$ **FNAL/MILC** 1202.6346

from a subset of the full **MILC** data set used in the extraction of $|V_{cb}|$ from non-zero recoil $B \to Dl\nu$ decays



Results from the full **MILC** data set by the end of the summer

+ $|V_{cb}|$ from non-zero recoil $B \to Dl\nu$

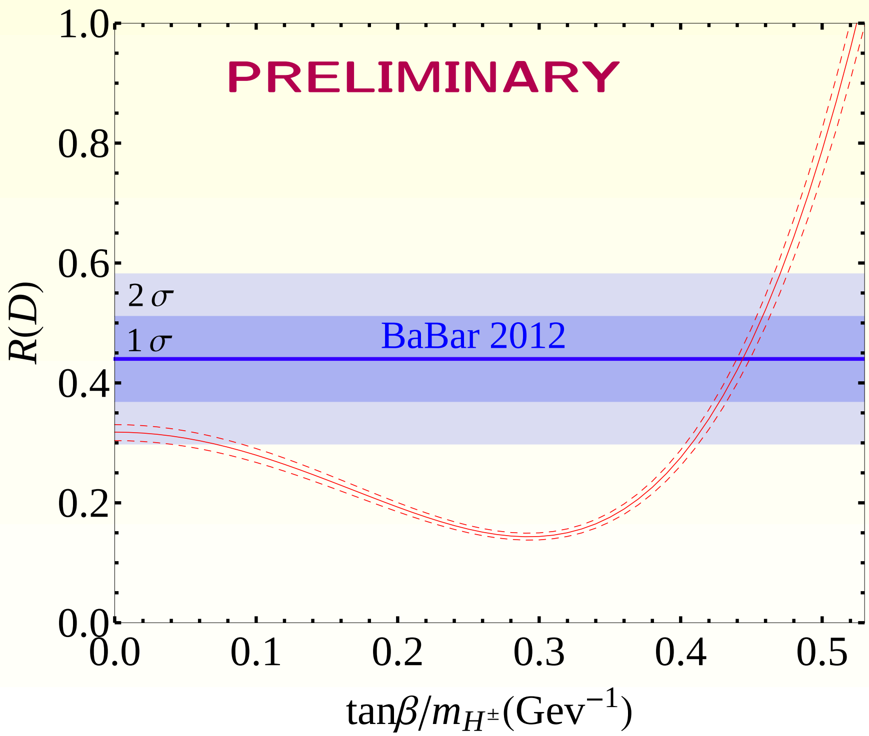
3.7. More on D semileptonic decays NEW

\# **BaBar** recently measured the ratio of branching fractions

$$R(D) = \frac{\mathcal{B}r(B \to D\tau\nu)}{\mathcal{B}r(B \to Dl\nu)} = 0.440(71) \qquad \text{1205.5442}$$

* Using the form factors calculated at non-zero recoil on the lattice (**FNAL/MILC** reduced data set) we can get a prediction for that ratio

D. Du, Fermilab theory seminar, **A. El-Kadra**, FPCP2012



$R(D)$ from SM and experiment differ by 2σ

$R(D) + R(D^*)$ measurement give $(3.4)\sigma$ exclusion of SM 1205.5442

2HDM is excluded when combining $R(D)$ with $R(D^*)$

2HDM prediction from **Tanaka and Watanabe**,1005.4306 + **FNAL/MILC** form factors

4. Conclusions and outlook

Important progress in lattice calculations including **sea quarks** ($N_f = 2 + 1$)

- * Light quarks: Results from many collaborations
  $\rightarrow$ excellent checks.
- * Heavy quarks: Currently dominated by **HPQCD** and **FNAL/MILC**, but precision results from other groups will be available soon: **ETMC**, **RBC**
- * Need averages: **LLV**, **FLAG-1**, and **FLAG-2** soon.
- * Approaching the physical light quark masses.

4. Conclusions and outlook

Expected in the next year:

* Results from simulations performed at the physical light quark masses. Three collaborations have ensembles with physical light quark masses: **MILC, PAC-CS, BMW**.

* Need to include effects that are currently subdominant:

  ** isospin breaking.

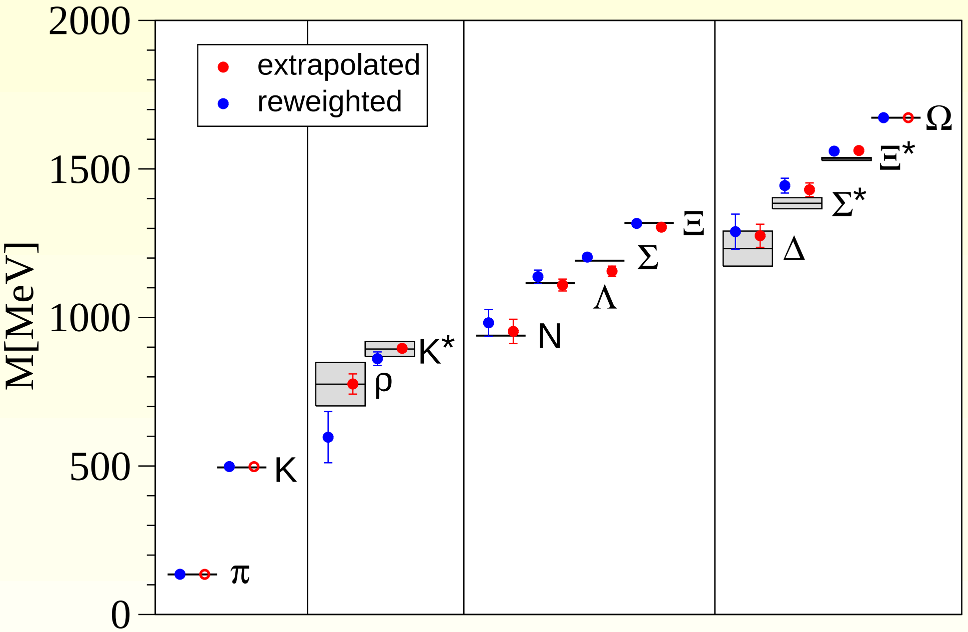
  ** electromagnetic effects.

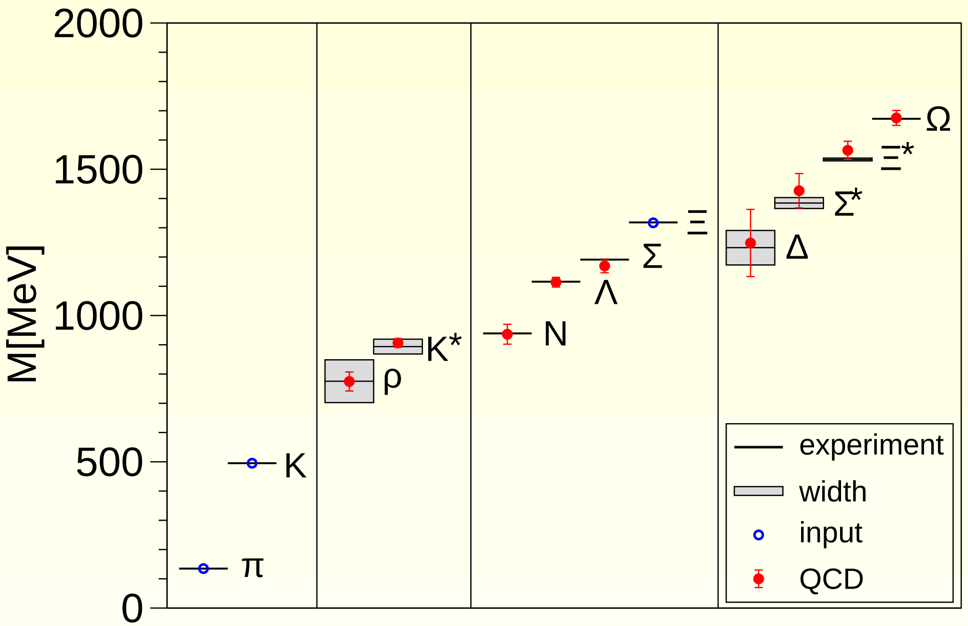
  ** charm sea quarks.

* Develop methods to reliably calculate quantities that are beyond easy.

A.1. Spectrum of light hadrons: test of lattice QCD

Good agreement between $N_f = 2 + 1$ lattice calculations and the experimentally measured light spectrum.

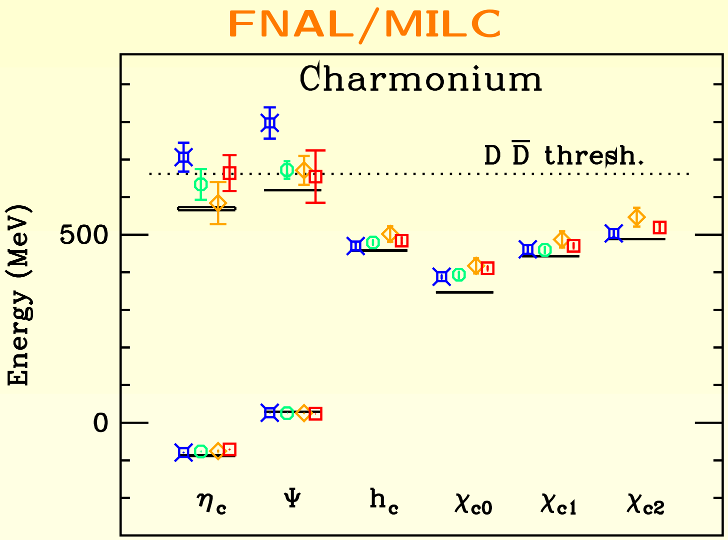


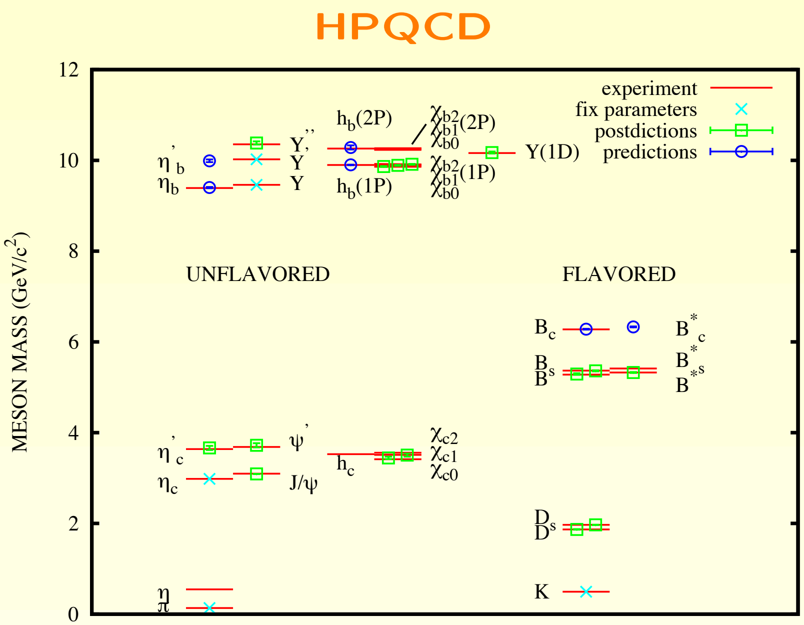


PACS-CS 0807.1661,0911.2561

BMW 0906.3599

A.2. Spectrum of heavy hadrons





\# Some post/predictions with NRQCD b (**S. Meinel**, 1007.3966, 1010.0889)

$(m_\Upsilon - m_{\eta_b})(1S) = (60.3 \pm 7.7)$ MeV $((m_\Upsilon - m_{\eta_b})(1S)^{exp} = 69.3 \pm 2.9)$

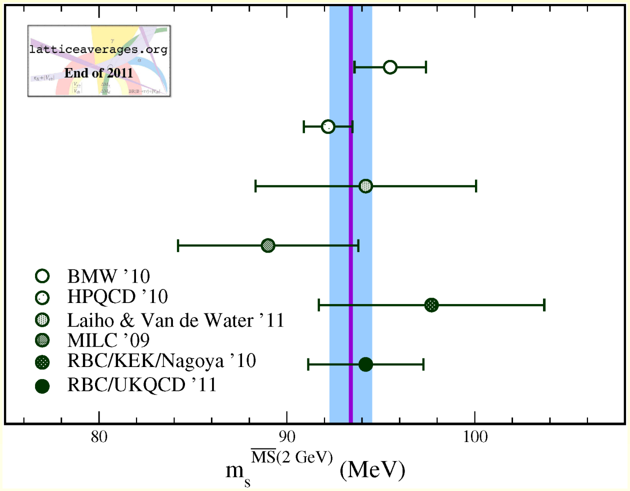
$(m_\Upsilon - m_{\eta_b})(2S) = (23.5 \pm 4.7)$ MeV

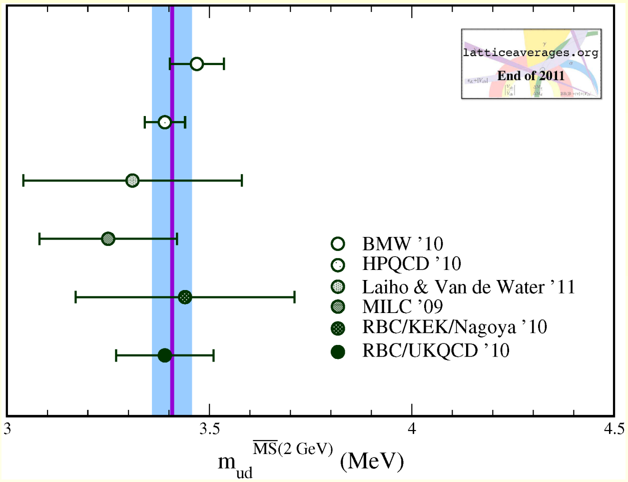
$m_{\Omega_{bbb}} = (14.371 \pm 0.012)$ GeV

\# Prediction for $\boxed{m_{B_c^*} = 6.3330(6)(2)(6) \text{ GeV}}$

B.1. Light quark masses

Determination of m_s with around $1-5\%$ errors from several $N_f = 2+1$ collaborations.





$$m_s^{\mathbf{LLV},\overline{MS}}(2\text{GeV}) = (93.4 \pm 1.1)\ \text{MeV}; \quad m_{ud}^{\mathbf{LLV},\overline{MS}}(2\text{GeV}) = (3.408 \pm 0.047)\ \text{MeV}$$

B.2. Heavy quark masses

Heavy masses from current-current correlators **HPQCD**, PRD82(2010) ($N_f = 2+1$)

$$m_c(3\ \mathrm{GeV}, n_f = 4) = 0.986(6)\ \mathrm{GeV}$$

$$m_b(10\ \mathrm{GeV}, n_f = 5) = 3.617(25)\ \mathrm{GeV}$$

$N_f = 2+1$ NRQCD b quarks **A. Hart et al.**, Pos(Lat2010)223

$$m_b(m_b) = 4.25(12)\ \mathrm{GeV}$$

$N_f = 2$ twisted mass calculation from **ETMC**, Pos(Lat2010)239

$$\bar{m}_c(\bar{m}_c) = 1.28(4)\ \mathrm{GeV}$$

$$\bar{m}_b \bar{m}_b = 4.3(2)\ \mathrm{GeV}$$

$N_f = 2$ twisted mass calculation from **ALPHA**, Trento2012?

$$\bar{m}_b \bar{m}_b = 4.288(76)(43)(14)\ \mathrm{GeV}$$

2.1. and 2.2. Test of Unitarity in the first row

$|V_{us}|$ from leptonic decays using $f_K/f_\pi^{\mathbf{LLV}} = 1.1936 \pm 0.0053$:

$$\frac{|V_{us}|^2}{|V_{ud}|^2} \times \frac{f_K^2}{f_\pi^2} \propto \frac{\Gamma(K \to \mu \bar{\nu}_\mu(\gamma))}{\Gamma(\pi \to \mu \bar{\nu}_\mu(\gamma))}$$

Marciano 2004 $\Longrightarrow$ $|V_{us}| = 0.2252(11)^*$

* Using $|V_{us}|/|V_{ud}| \times f_K/f_\pi = 0.2758(5)$ **M. Antonelli et al.**, 1005.2323 and $|V_{ud}| = 0.97425(22)$ **Hardy and Towner**, PRC79(2009) update

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$|V_{us}|$ from semileptonic decays using $f_+(0)^{\mathbf{LLV}} = 0.9584 \pm 0.0044$

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CKM unitarity test in the first row at the 0.1% level (error dominated by la

$$|V_{us}|^{unitarity} = 0.22545(22)$$ **M. Antonelli et al.**, 1005.2323