Calculational Tools in the Era of Super-Flavour Factories

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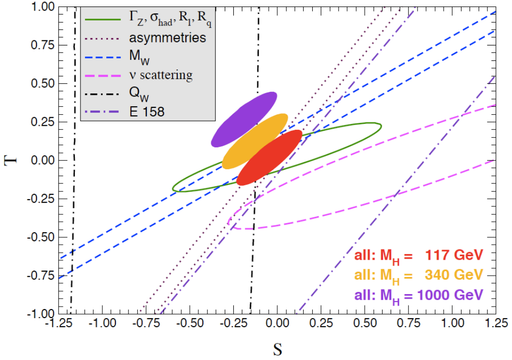
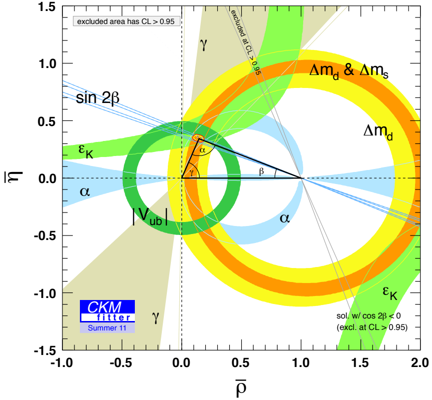
Contents

The Need for Precision Flavour Physics

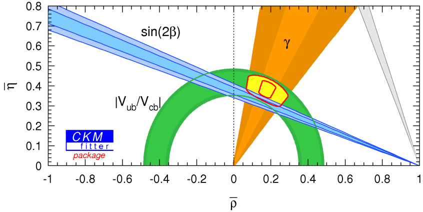
- The Standard Model passed all tests
  up to the 100 GeV Scale:
- LEP: test of the gauge Structure
- Flavour factories: test of the Flavour Sector



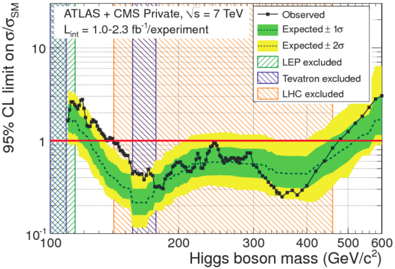


No significant deviation has been found (yet)!

... only a few "tensions"
(= Observables off by 2σ
or even less)



LHC will perform a direct test of the TeV Scale



What can Flavour tell us?

- Flavour Physics $\leftrightarrow$ No new physics at the TeV scale with a generic flavour structure
- Parametrization of new physics:
  Higher Dimensional Operators:

$$\mathcal{L} = \mathcal{L}_{\mathrm{SM}} + \frac{1}{\Lambda}\mathcal{L}^{(5)} + \frac{1}{\Lambda^2}\mathcal{L}^{(6)} + \cdots \qquad \mathcal{L}^{(n)} = \sum_j C_j O_j^{(n)}$$

- Λ: New Physics scale
- $O_j^{(n)}$: Local Operators of dimension n

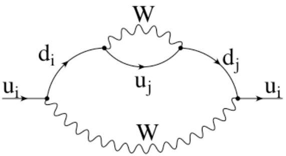
- Some of the $O_j^{(n)}$ may mediate flavour transitions: e.g.

$$
\begin{aligned}
O_1^{(6)} &= (\bar{s}_L \gamma_\mu d)(\bar{s}_L \gamma^\mu d) && \text{(Kaon Mixing)} \\
O_2^{(6)} &= (\bar{b}_L \gamma_\mu d)(\bar{b}_L \gamma^\mu d) && (B_d \text{ Mixing}) \\
O_3^{(6)} &= (\bar{b}_L \gamma_\mu 2)(\bar{b}_L \gamma^\mu s) && (B_s \text{ Mixing}) \\
O_4^{(6)} &= (\bar{c}_L \gamma_\mu u)(\bar{c}_L \gamma^\mu u) && (D \text{ Mixing})
\end{aligned}
$$

- $\Lambda \sim 1000$ TeV from Kaon mixing ($C_i = 1$)
- $\Lambda \sim 1000$ TeV from D mixing
- $\Lambda \sim 400$ TeV from B_d mixing
- $\Lambda \sim 70$ TeV from B_s mixing

Another Peculiarity of SM Flavour Parametrization: CP

- Strong CP remains mysterious
- Flavour diagonal CP Violation is well hidden:
  e.g electric dipole moments:
  For quarks at least three loops (Shabalin)



$$d_e^{\text{quark}} \sim e\, \frac{\alpha_s}{\pi} \frac{G_F^2}{(16\pi^2)^2} \frac{m_t^2}{M_W^2} \mathrm{Im}\Delta\, \mu^3$$

$$\sim 10^{-32} e\, cm \quad \text{with } \mu \sim 0.3\,\mathrm{GeV}$$

$$d_{\exp}^{\text{Neutron}} \leq 3.0 \times 10^{-26} e\, cm$$

Composite objects can have larger edm's ("loopless") :

$$d^{\text{Neutron}} \sim 10^{-31} e\, cm$$

TM, Uraltsev 2012

Theory Tools for Precision Flavour Physics

The Toolbox

- QCD based effective field theories
  - Chiral Perturbation Theory, including heavy hadron χPT
  - Heavy Quark Effective Theory
  - Heavy Quark OPE for inclusive processes
  - Soft Collinear Effective Theory
- QCD Sum Rules
  - Fixed point sum rules
  - Light-Cone Sum rules
- (Approximate) Flavour Symmetries
  - ... including SU(3) breaking
  - ... supplemented by “diagrammatic considerations”
- Lattice

Models have become (almost) obsolete!

Example 1: The V_{ub} Problem

... this requires almost the whole toolset:

- Exclusive V_{ub} from $B \to \pi\ell\nu$:
  Lattice and Light-Cone QCD Sum Rules
- Inclusive V_{ub}: HQE and SCET

Exclusive determination of: V_{ub}

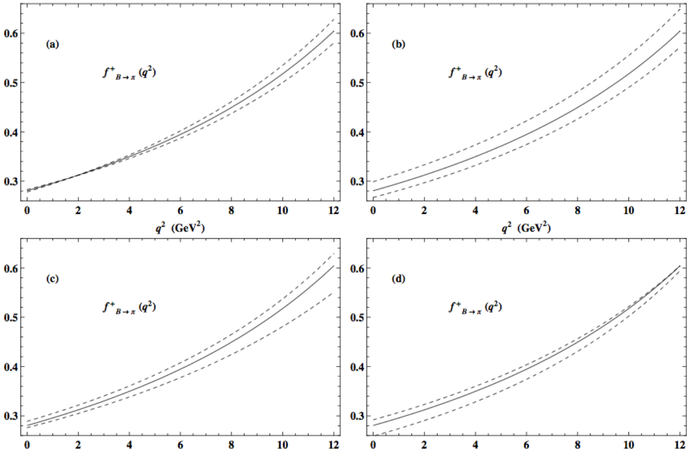
- Recent improvement of the calculations for $B \to \pi\ell\bar{\nu}$
- LCQCDSR Calculation of

$$\Delta\zeta\,(0, q^2_{max}) = \frac{1}{|V_{ub}|^2 \tau_{B^0}} \int_0^{q^2_{max}} dq^2 \frac{d\mathcal{B}(B \to \pi\ell\nu_\ell)}{dq^2},$$

- ... including
  - Full $\mathcal{O}(\alpha_s)$ QCD corrections
  - Subleading twists
  - a_2 and a_4 corrections to the pion DA, fitted from the electromagnetic pion form factor

Khodjamirian, Klein, T.M., Wang

LCQCDSR Result for the from factor, $0 \leq q^2 \leq 12$ GeV^2

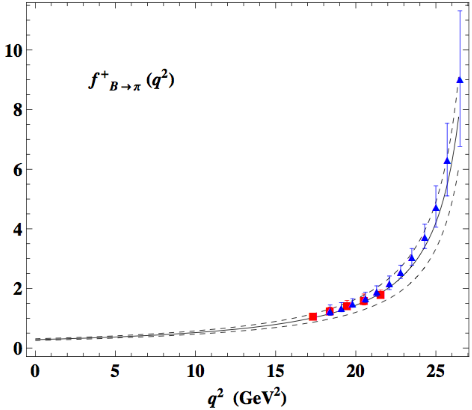


(a): a_2^π, a_4^π, (b): μ_π, (c): μ, (d): M^2, s_0^B

Linking high q^2 with low q^2

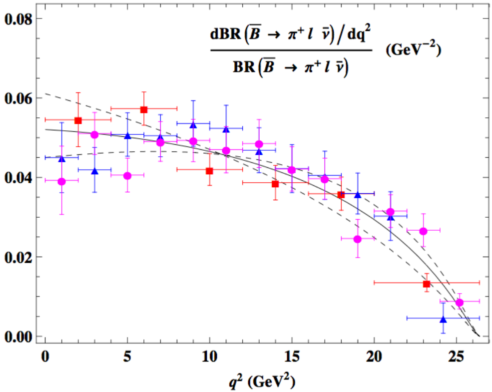
- LCQCDSR are limited to “small” values of q^2
- Complementary to lattice calculations
- We have QCD based calculations / estimates of the from factors f_+ and f_0 in the full kinematic region
- Uncertainties become controllable and are already quite small !
- May become the most accurate way to determine V_{ub}

Linking high q^2 with low q^2: z parametrization



The vector form factor $f^+_{B\pi}(q^2)$ calculated from LCSR and fitted to the BCL parameterization (solid) with uncertainties (dashed), compared with the HPQCD [4] (triangles) and FNAL/MILC [5] (squares) results.

Theory vs. Experiment



(colour online) The normalized q^2-distribution in $B \to \pi l\nu$ obtained from LCSR and extrapolated with the z-series parameterization (central input- solid, uncertainties -dashed). The experimental data points are from BABAR: (red) squares [1], (blue) triangles [2] and Belle [3]: (magenta) full circles.

Value of V_{ub} from Khodjamirian et al.:

$$|V_{ub}| = (3.50^{+0.38}_{-0.33}\big|_{th.} \pm 0.11\big|_{exp.}) \times 10^{-3}$$

Lattice $\otimes$ LCQCDSR has reached 10% th. uncertainty in $V_{ub,excl}$!

This is to be compared to the inclusive value:

$$|V_{ub}| = (4.41 \pm 0.15 \, {}^{+\,0.15}_{-\,0.19}) \times 10^{-3}$$

We have to find out what is going on here!

The Role of $B \to \tau\bar{\nu}$

- $B \to \tau\bar{\nu}$ depends crucially on f_B

$$\mathcal{B}(B^- \to \tau\bar{\nu}_\tau) = \frac{G_F^2}{8\pi}|V_{ub}|^2 m_\tau^2 m_B \left(1 - \frac{m_\tau^2}{m_B^2}\right)^2 f_B^2 \tau_{B^-}$$

- The extracted V_{ub} value is quite large ...
- However, if the data are right, QCD (or the SM) must have a problem: Define Khodjamirian, Klein, T.M., Wang

$$R_{s/l}(q_1^2, q_2^2) \equiv \frac{\Delta\mathcal{B}_{B\to\pi\ell\nu_\ell}(q_1^2, q_2^2)}{\mathcal{B}(B \to \tau\nu_\tau)} \left(\frac{\tau_{B^-}}{\tau_{B^0}}\right)$$

$$= \frac{\Delta\zeta(q_1^2, q_2^2)}{(G_F^2/8\pi)m_\tau^2 m_B(1 - m_\tau^2/m_B^2)^2 f_B^2}$$

Exp.	$\Delta\mathcal{B}(10^{-4})$ [Ref.]	$\mathcal{B}(B \to \tau\nu_\tau)(10^{-4})$ [Ref.]	$R_{s/l}$
BABAR	0.32 ± 0.03 [1] $0.33 \pm 0.03 \pm 0.03$ [2]	1.76 ± 0.49 [36, 37]	$0.20^{+0.08}_{-0.05}$
Belle	0.398 ± 0.03 [3]	$1.54^{+0.38+0.29}_{-0.37-0.31}$ [38]	$0.28^{+0.13}_{-0.07}$

QCD	$\Delta\zeta(\mathrm{ps}^{-1})$ [Ref.]	$f_B(\mathrm{MeV})$ [Ref.]	$R_{s/l}$
HPQCD	2.02 ± 0.55 [4]	190 ± 13 [34]	0.52 ± 0.16
FNAL/MILC	$2.21^{+0.47}_{-0.42}$ [5]	212 ± 9 [35]	0.46 ± 0.10

$R_{s/l}$ for the region 16 GeV$^2 < q^2 <$ 26.4 GeV^2

Exp.	$\Delta\mathcal{B}(10^{-4})$ [Ref.]	$\mathcal{B}(B \to \tau\nu_\tau)(10^{-4})$ [Ref.]	$R_{s/l}$
BABAR	0.88 ± 0.06 [1] $0.84 \pm 0.03 \pm 0.04$ [2]	1.76 ± 0.49 [36, 37]	$0.52^{+0.20}_{-0.12}$

QCD	$\Delta\zeta$ [Ref.]	f_B(MeV) [Ref.]	$R_{s/l}$
LCSR/QCDSR	$4.59^{+1.00}_{-0.85}$ [this work]	210 ± 19 [41]	$0.97^{+0.28}_{-0.24}$

$R_{s/l}$ for the region $0\ \text{GeV}^2 < q^2 < 12.0\ \text{GeV}^2$

Some clarification is needed here ...

Example 2: Calculation of $B \to K^{(*)}\ell\ell$

- Theory of $B \to K^{(*)}\ell^+\ell^-$ is substantially different from the one for $B \to D\ell\bar{\nu}$:
- Effective Interaction:

$$H_{eff} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i(\mu) O_i(\mu) \, ,$$

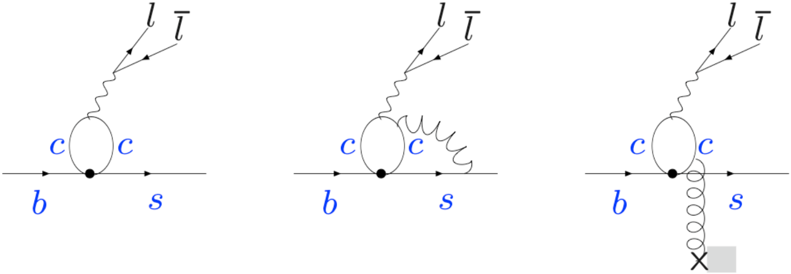
- Dominant $b \to s$ effective operators: $O_{7,9,10}$
  $C_7(m_b) \simeq -0.3$, $C_9(m_b) \simeq 4.4$, $C_{10}(m_b) \simeq= -4.7$
- ... can be expressed in terms of form factors

$$O_{7,9,10} \propto \langle K^{(*)}(p) | \bar{s} \Gamma b | B(p+q) \rangle$$

Charm Loops

Buchalla, Isidori, Feldmann, Khodjamirin, TM, Pivovarov, Wang

- Charm-loop effect: a combination of the $(\bar{s}c)(\bar{c}b)$ weak interaction ($O_{1,2}$) and e.m.interaction $(\bar{c}c)(\bar{\ell}\ell)$



- new hadronic matrix elements, not a form factor

- Light cone expansion of the charm loop
- Expansion parameter $\frac{\Lambda_{\rm QCD}^2}{(4m_c^2 - q^2)}$
- Leads to a non-local operator ("shape-function-like" operator)

$$\widetilde{\mathcal{O}}_\mu(q) = \int d\omega \, I_{\mu\rho\alpha\beta}(q, m_c, \omega) \bar{s}_L \gamma^\rho \left(\delta[\omega - \frac{(in_+\mathcal{D})}{2}] \widetilde{G}_{\alpha\beta} \right) b_L \ ,$$

- Matrix element can be calculated in a LCSR for $q^2 \leq 0$

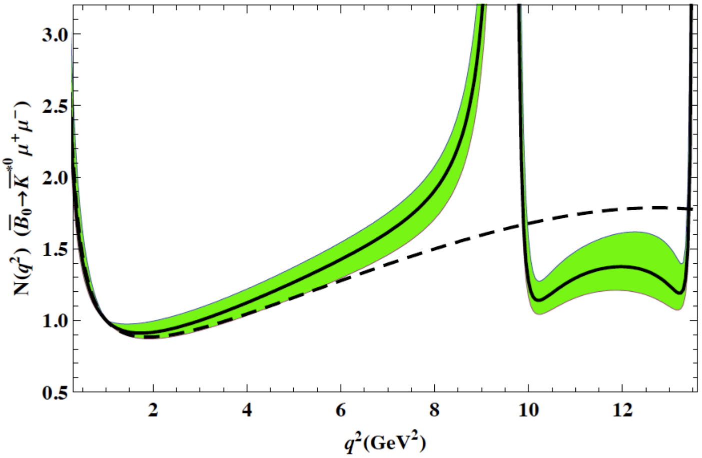
Accessing the region $q^2 > 0$

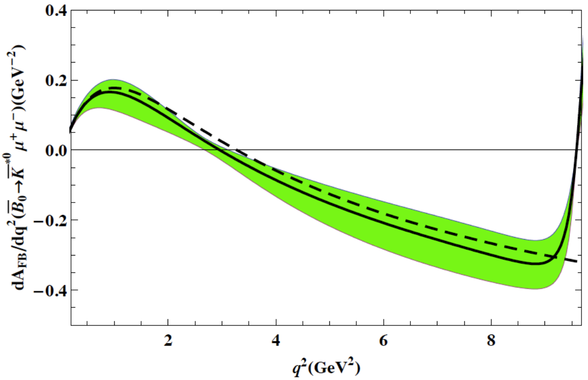
- Hadronic Dispersion Relation:

$$\mathcal{H}^{(c,s,b)}_{(BK)}(q^2)\mathcal{H}^{(c,s,b)}_{(BK)}(0)$$
$$+q^2\Big[\sum_{\psi=J/\psi,\psi(2S),..} \frac{f_\psi A_{B\psi K}}{m_\psi^2(m_\psi^2 - q^2 - i m_\psi \Gamma_\psi^{tot})}$$
$$+\int_{4m_D^2}^{\infty} ds \frac{\rho(s)}{s(s-q^2-i\epsilon)}\Big]$$

- Residues from $BR(B \to \psi K)$ and $BR(\psi \to \ell\ell)$
- FSI Phase attributed to $A_{B\psi K}$
- Fit to the result at $q^2 < 0$

Results on $B \to K^* \ell^+ \ell^-$





Problem to compute above the charm threshold?

Problem also below charm theshold: $B \to K\phi \to K\ell^+\ell^-$

... currently under consideration Khodjamirian, Wang, TM

Detailed Analysis of $B \to K\ell\ell$

- Form factors and Light-cone distribution amplitudes are better known for K
- Current work: A detailed analysis ... (Khodjamirian, T.M., Wang)

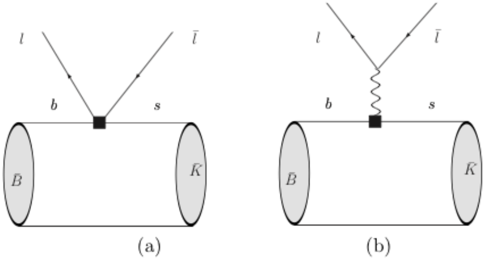


Figure 1: *Dominant contributions to* $B \to K\ell^+\ell^-$ *amplitude due to the effective operators* $O_{9,10}$ *(a) and* O_7 *(b) denoted as black squares.*

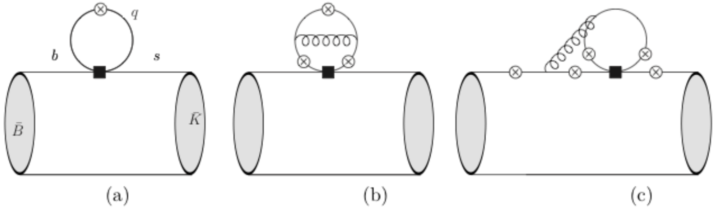


Figure 2: *Factorizable quark-loop contributions to* $B \to K\ell^+\ell^-$ *amplitudes due to four-quark effective operators* $O^c_{1,2}$ *and* O_{3-6}. *Crossed circles denote the possible emission points of the virtual photon.*

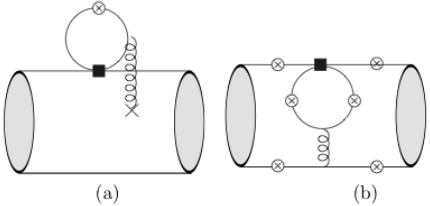


Figure 3: *Nonfactorizable quark-loop contributions to* $B \to K\ell^+\ell^-$*: (a) with soft gluon (denoted by crossed line) and (b) with hard-gluon.*

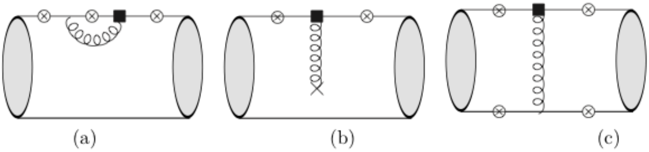


Figure 4: *Contributions of the* O_{8g} *operator to* $B \to K\ell^+\ell^-$ *amplitude: (a) factorizable and nonfactorizable with (b) soft-gluon and (c) hard -gluon.*

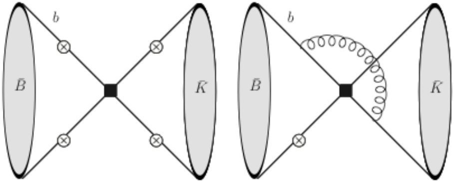
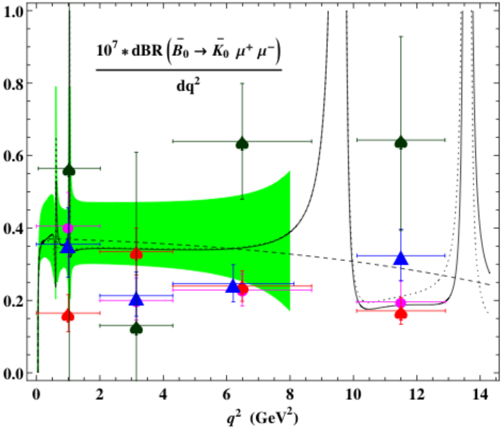


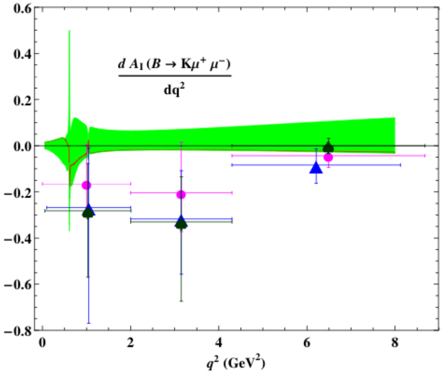
Figure 5: *Weak annihilation contribution to* $B \to K\ell^+\ell^-$ *amplitude: (a) in LO and (b) one of the NLO hard-gluon exchange diagrams.*

Preliminary Results for $B \to K\ell\ell$



Data: BaBar (magenta), Belle (blue), CDF (red), LHCb (cyan)
Solid and dotted: Two different ansaetze for the dispersion relation
Dashed: Without the nonlocal contributions

$$\frac{dA_I^{(0-)}(q^2)}{dq^2} = \frac{d\Gamma(\bar{B}_0 \to \bar{K}_0\ell^+\ell^-)/dq^2 - d\Gamma(B^- \to K^-\ell^+\ell^-)/dq^2}{d\Gamma(\bar{B}_0 \to \bar{K}_0\ell^+\ell^-)/dq^2 + d\Gamma(B^- \to K^-\ell^+\ell^-)/dq^2}$$



VERY Preliminary, Uncertainties to be discussed

Outlook

- Theory tools for many processes are ready for higher precision
- Leptonic and Semileptonic is quite mature
- Nonleptonic still remains problematic