

Precise measurement of the $e^+e^- \rightarrow \pi^+\pi^-(\gamma)$ cross section

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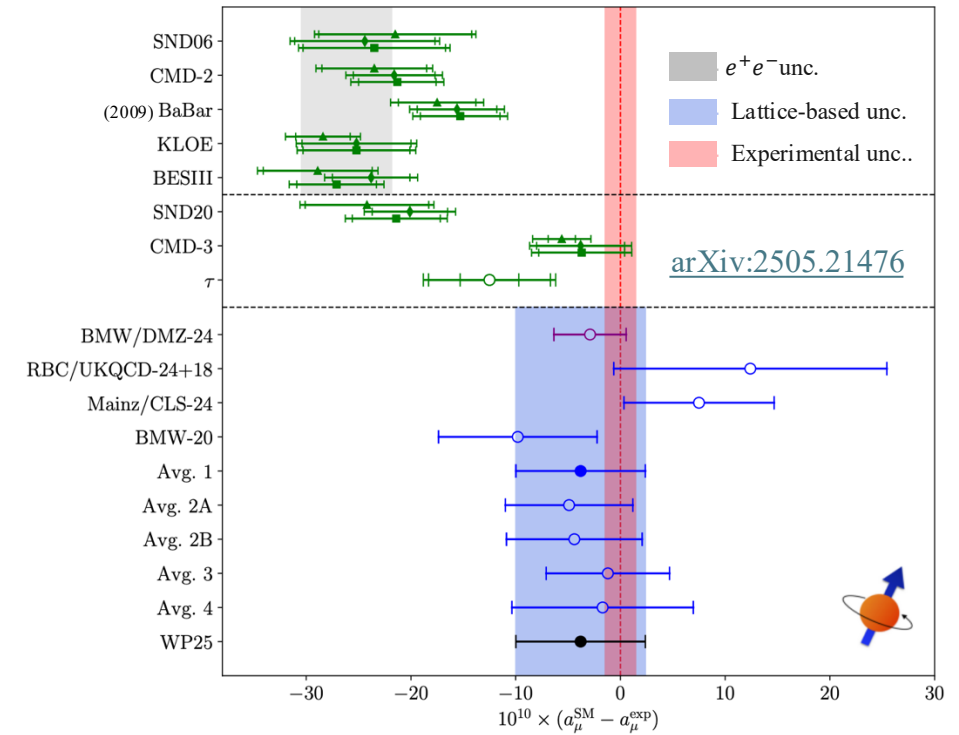
29/09/2025



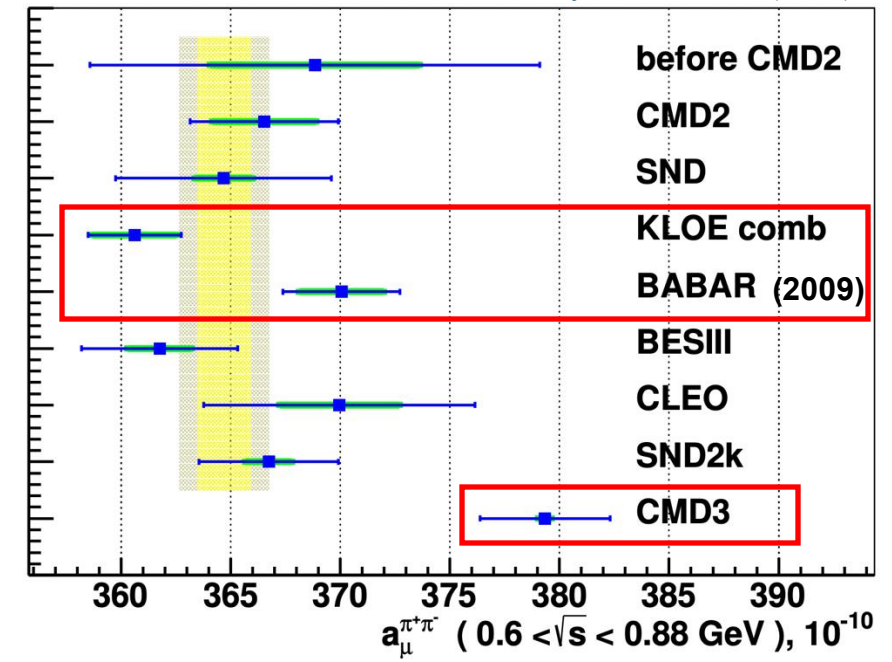
Workshop on Flavour Changing and Conserving Processes (FCCP2025)

Motivation

- The muon anomalous magnetic moment a_μ is sensitive to hadronic vacuum polarization (HVP).
- The dominant input ($\approx 73\%$) comes from $e^+e^- \rightarrow \pi^+\pi^-$.
- Current tensions:
 - Tensions among inputs of the dispersive approach
 - Dispersion approach vs direct experimental measurements.
 - Dispersion approach vs lattice QCD predictions.
- Resolving these discrepancies is crucial for testing the Standard Model.
- BaBar performed a first a_μ measurement in 2009 with **partial data**.
- Need new precise and independent measurements to resolve the situation.

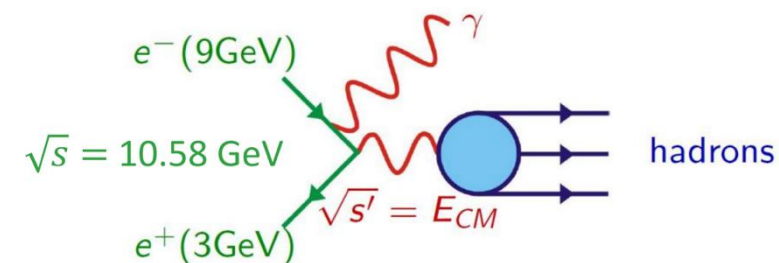
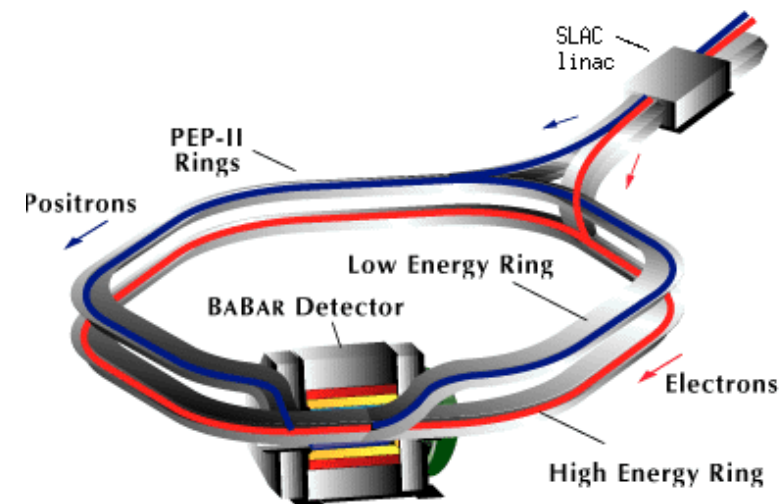
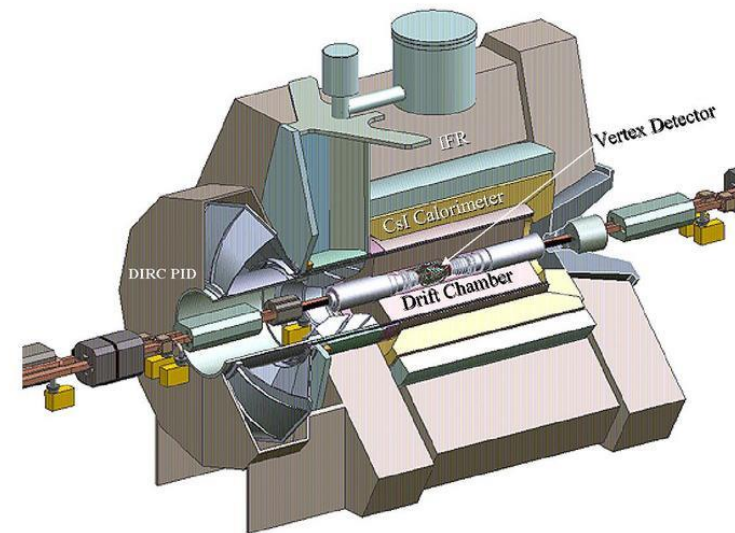


[Phys. Rev. D 109 \(2024\) 112002](https://arxiv.org/abs/2505.21476)



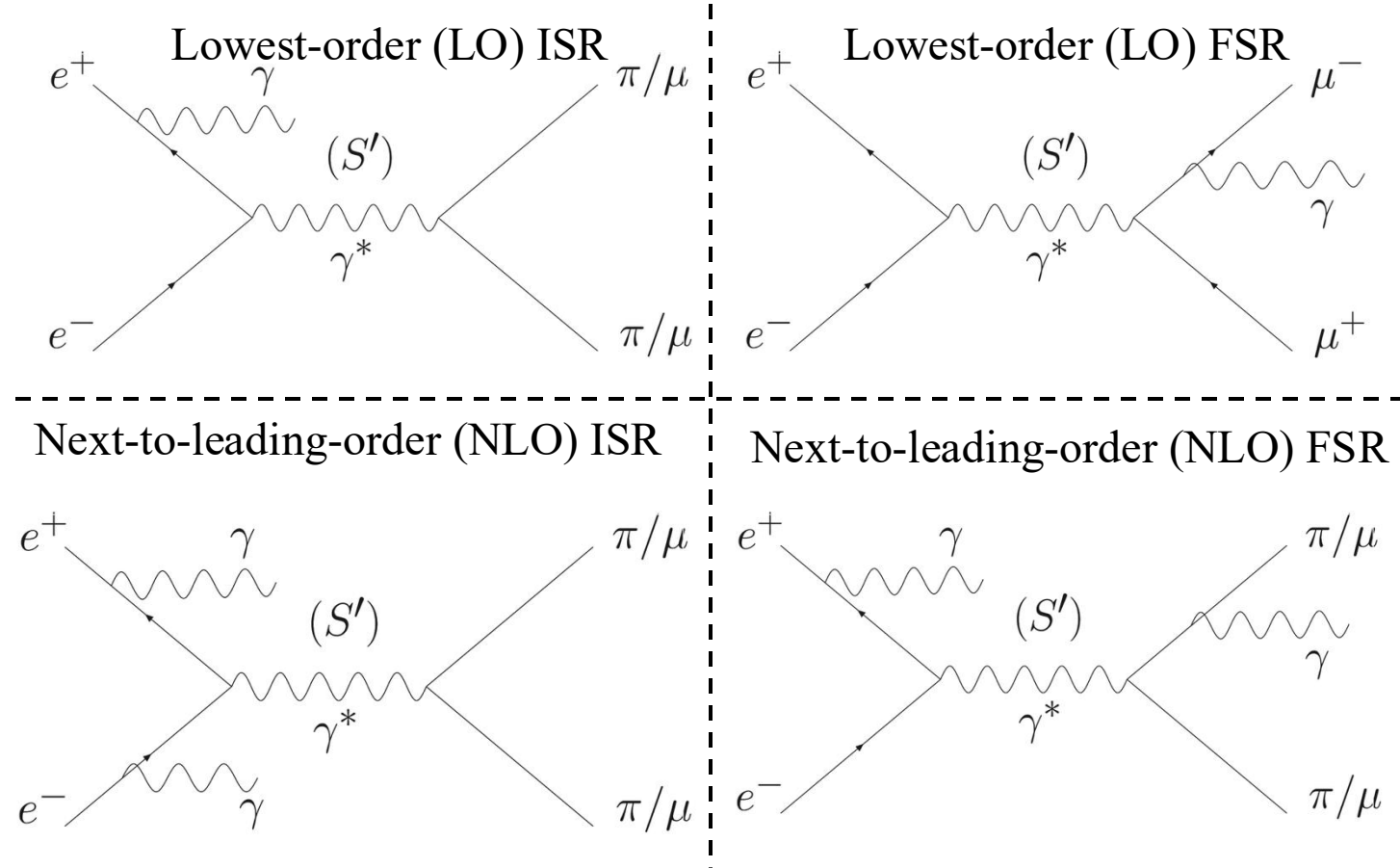
The BaBar experiment

- **PEP-II collider (SLAC, USA):**
 - Asymmetric $e^+(3\text{ GeV}) - e^-(9\text{ GeV})$
 - Operated from 1999 to 2008 at Y(4S/3S/2S) resonance energies
 - Collected **424 fb⁻¹** at ($\sqrt{s} = 10.58\text{ GeV}$) + **36 fb⁻¹** off-resonance
- **Strategy:**
 - Initial State Radiation (ISR) to probe a wide range of effective energies
- **Signals studied:**
 - $e^+e^- \rightarrow \mu^+\mu^-(\gamma_{\text{ISR}})$
 - $e^+e^- \rightarrow \pi^+\pi^-(\gamma_{\text{ISR}})$
 - Simulated with Phokhara9.1
- **Backgrounds:**
 - $e^+e^-\gamma$, $K^+K^-(\gamma_{\text{ISR}})$, $q\bar{q}$ (u, d, s, c), $\tau^+\tau^-$, multipion ISR processes ($X\gamma_{\text{ISR}}$ for $X = n\pi + m\pi^0, \dots$)



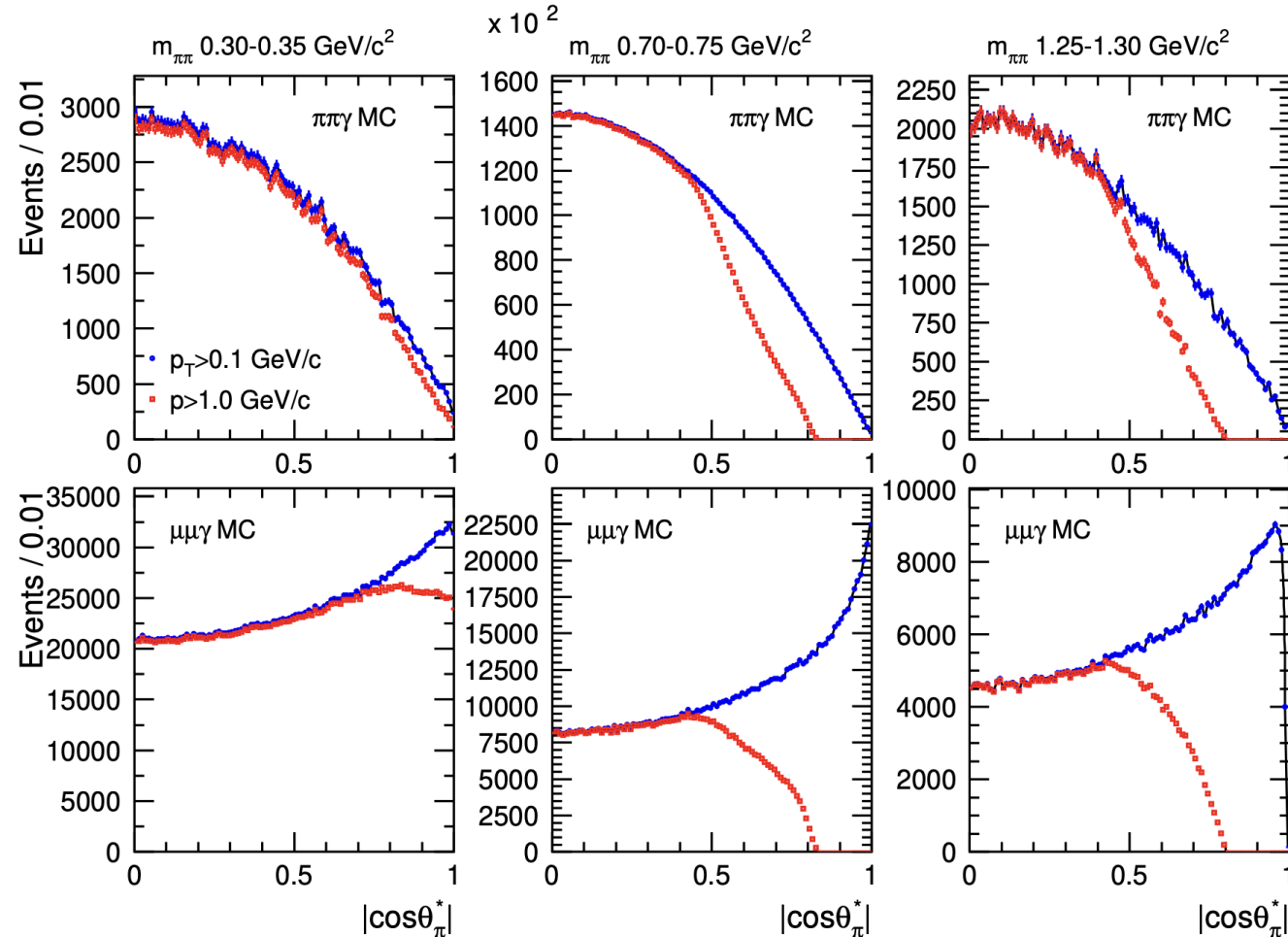
BABAR methods for measuring $e^+e^- \rightarrow \pi^+\pi^-, \mu^+\mu^-$

- **ISR (Initial State Radiation):**
 - Full mass spectrum obtained from one dataset via large-angle ISR photons
 - Good detector acceptance for final states
- **$\pi\pi/\mu\mu$ ratio method:**
 - Cancels common systematics (luminosity, ISR photon efficiency, vacuum polarization)
 - Relies on well-understood $\mu\mu$ QED process as reference
- **Additional radiation (NLO & beyond):**
 - Loose selection includes ISR/FSR photons
 - NLO ISR and FSR measured directly in data
 - NNLO hard contributions studied in 2023 ([Phys. Rev. D 108, L111103 \(2023\)](#))
 - Method largely independent of generator limitations (Phokhara restricted to NLO)



2009 analysis vs 2025 analysis

- **2009 Analysis** ([Phys.Rev.Lett. 103 \(2009\) 231801](#))
 - Data: partial 232 fb^{-1} (Y(4S))
 - π/μ separation via **particle ID (PID)** $\rightarrow$ main source of systematics
 - Track cut: $p > 1 \text{ GeV}/c$
 - Signal MC: AfkQED
 - Systematic uncertainty $\approx 0.5\%$
- **2025 Analysis**
 - Data: full 460 fb^{-1} **New method** (angular fit on $|\cos\theta^*|$) $\rightarrow$ no PID required
 - Track cut: $p_T > 0.1 \text{ GeV}/c$, higher statistics
 - Signal MC: Phokhara9.1
 - Blind analysis with reduced systematics
- **Main improvements (2025):**
 - $\times 2$ larger luminosity
 - Lower momentum threshold
 - Different, more robust π/μ separation method



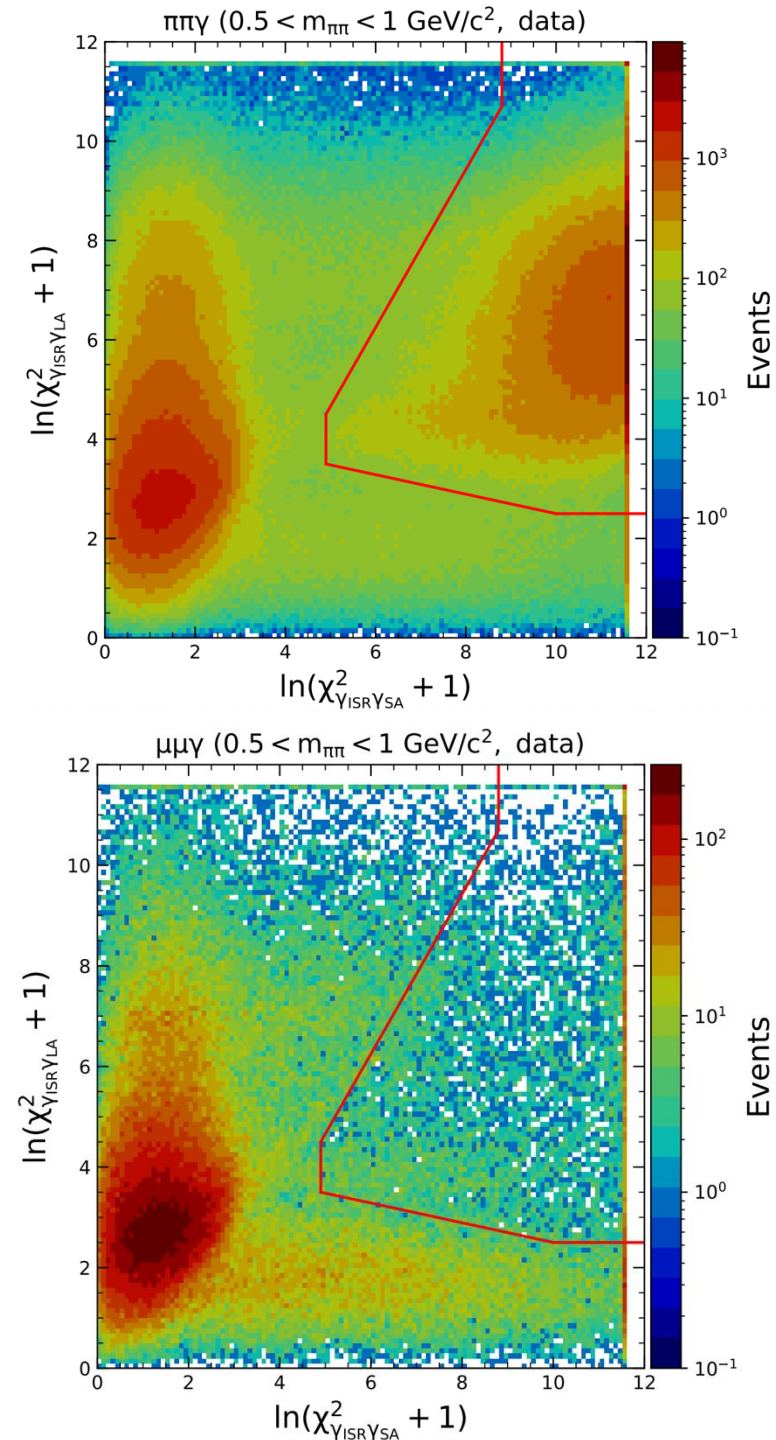
θ^* : angle between negative charged track and γ_{ISR} in 2-track CM frame

Signal event selection

- **ISR photon:** $0.35 < \theta_\gamma < 2.4$ rad, $E_\gamma > 3$ GeV, $E_\gamma^* > 4$ GeV
- **Two opposite-charge “super-good” tracks:**
 - $p_T > 0.1$ GeV/c, $0.4 < \theta < 2.45$ rad
 - ≥ 15 DCH hits, DOCA < 5 mm, $|d_z| < 6$ cm
- Allow additional photons and other quality tracks per event.
- Vertex constraint: $V_{xy} < 0.5$ cm
- Veto electron contamination
- Two NLO radiation fits per event (small-angle ISR $\gamma_{\text{ISR}}\gamma_{\text{SA}}$ & large-angle ISR/FSR $\gamma_{\text{ISR}}\gamma_{\text{LA}}$)

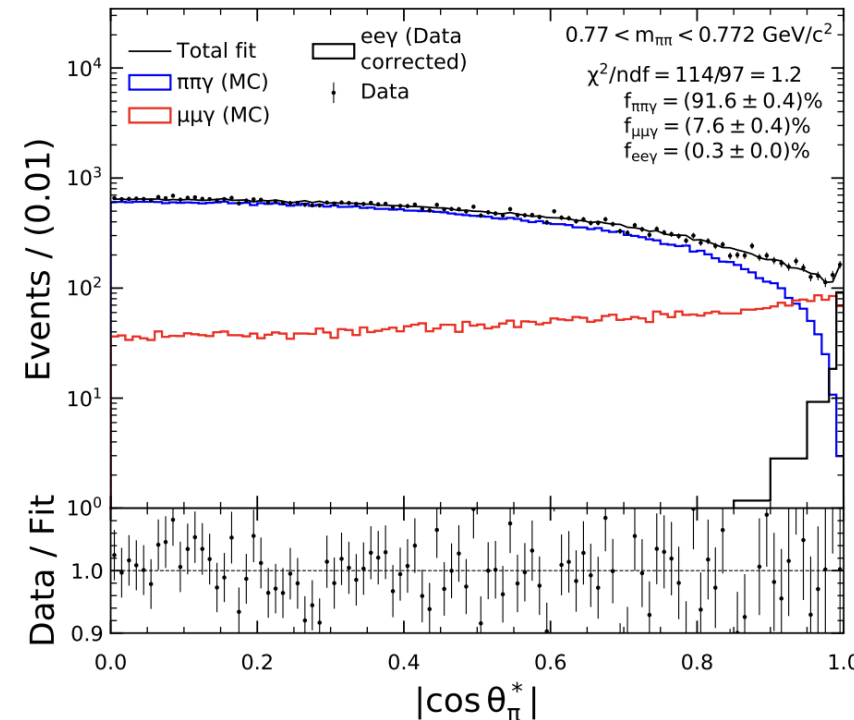
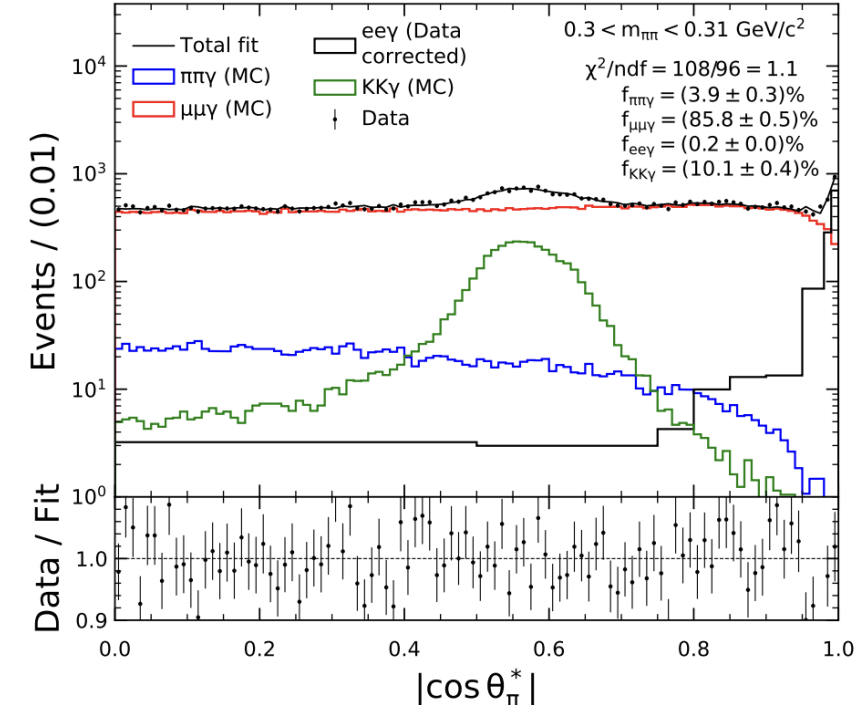
Background Control and Suppression

- Optimized 2D $-\chi^2$ selection using BDT in three mass regions
- With all selections, residual background $< 5\%$
- Dedicated studies for background normalization: $2\pi\pi^0$, uds , and $\tau\tau$
- Other backgrounds estimated from MC simulations normalized to BaBar cross-section measurements



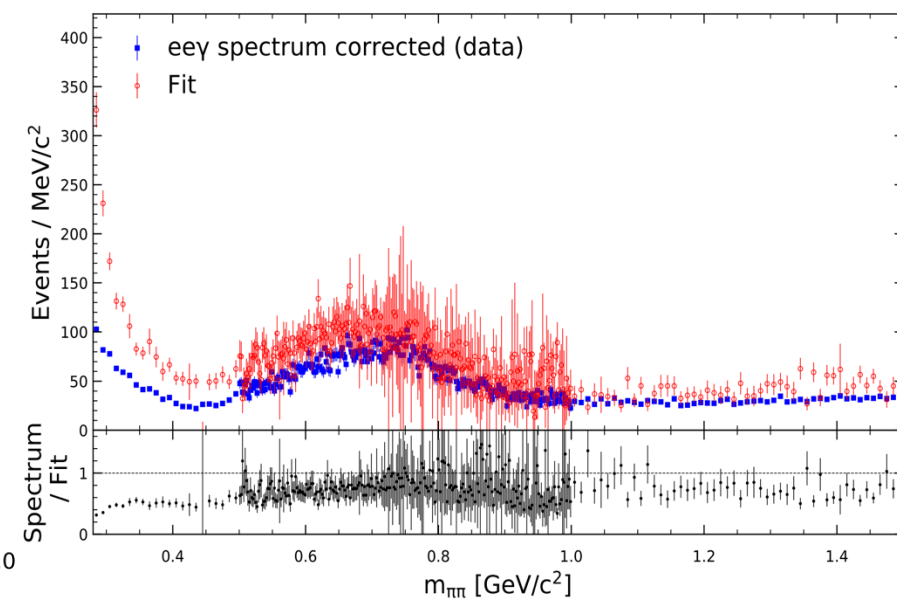
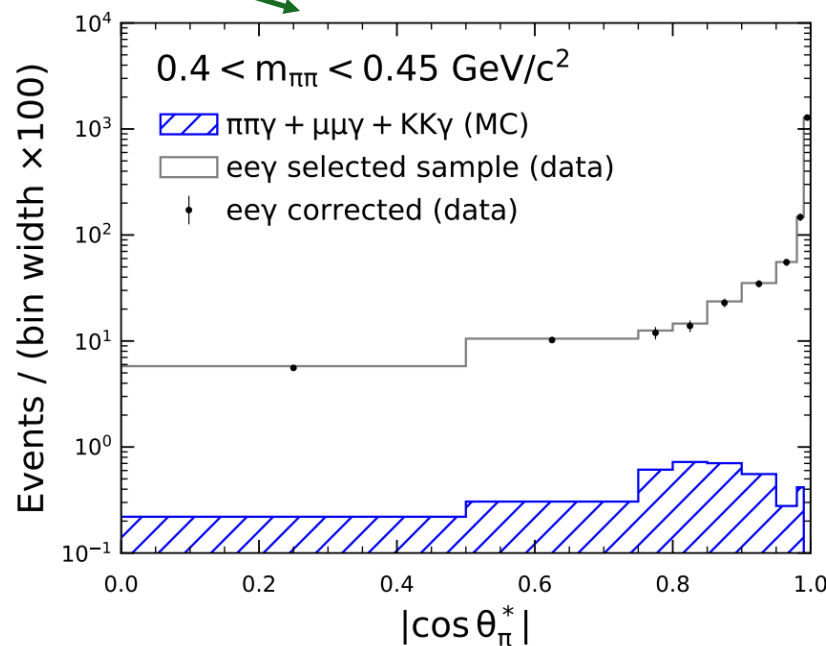
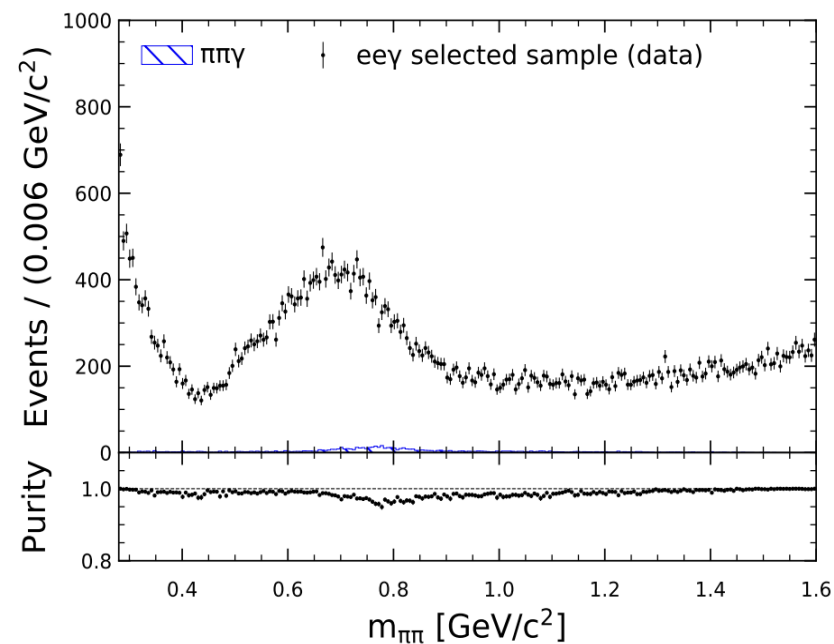
Template fits to separate $\mu\mu/\pi\pi$ final states

- $|\cos\theta_\pi^*|$ distributions on background-subtracted data are fitted using **templates**:
 - $\pi\pi\gamma$, $\mu\mu\gamma$ and $KK\gamma$ obtained from MC signal + data/MC corrections
 - $ee\gamma$ obtained from Data-driven (cut-based and BDT selections)
- **Templates need to be corrected for**:
 - V_{xy} selection efficiency
 - 2D- χ^2 selection efficiency
 - Trigger and tracking efficiencies
- Trigger and tracking corrections are **blinded** until the end of the analysis.
- Fits done twice: pion mass ($m_{\pi\pi}$) & muon mass ($m_{\mu\mu}$) charged track hypotheses
- Data $|\cos\theta_\pi^*|$ distribution is adjusted by linear combination of templates in >300 bins
- For each mass bin, the fit strategy is as follows:
 - $|\cos\theta_\pi^*| \in [0.9, 1.0]$: to obtain $ee\gamma$ normalization (template from data)
 - $|\cos\theta_\pi^*| \in [0.0, 0.9]$: separate $\pi\pi$ / $\mu\mu$ (and KK for low mass)
 - Extrapolate to $|\cos\theta_\pi^*| = 1 \rightarrow$ full $\pi\pi/\mu\mu$ yields
- Fit fractions ($f_{\pi\pi}$, $f_{\mu\mu}$, f_{KK} , $f_{ee\gamma}$) extracted from template shapes
- Mass spectra **blinded**, obtained after extrapolation of final fit results to $|\cos\theta_\pi^*| = 1$



Data-driven $ee\gamma$ template distribution

- The Monte Carlo prediction is found to be unreliable $\rightarrow$ the $ee\gamma$ templates are derived using a data-driven method.
- Pure $ee\gamma$ background selected from data using cut- and BDT-based selections.
- Event yields vs. $m_{\pi\pi}$ used to build template normalization.
- Angular distribution ($|\cos\theta_{\pi}^*|$) extracted as the **template shape**.
- $ee\gamma$ yields from template fits vs mass compared with the initial spectrum.
- Validation of data driven templates.



Corrections to Templates and Mass Distributions

- Detailed and dedicated analysis studies of **data/MC differences** on PID-selected muon and pion samples: triggers, tracking, V_{xy} , and 2D- χ^2 selections.
- Efficiency corrections applied:

$$\varepsilon = \varepsilon_{\text{MC}} \left(\frac{\varepsilon_{\text{trigger}}^{\text{data}}}{\varepsilon_{\text{trigger}}^{\text{MC}}} \right) \left(\frac{\varepsilon_{\text{tracking}}^{\text{data}}}{\varepsilon_{\text{tracking}}^{\text{MC}}} \right) \left(\frac{\varepsilon_{V_{xy}}^{\text{data}}}{\varepsilon_{V_{xy}}^{\text{MC}}} \right) \left(\frac{\varepsilon_{\chi^2}^{\text{data}}}{\varepsilon_{\chi^2}^{\text{MC}}} \right)$$

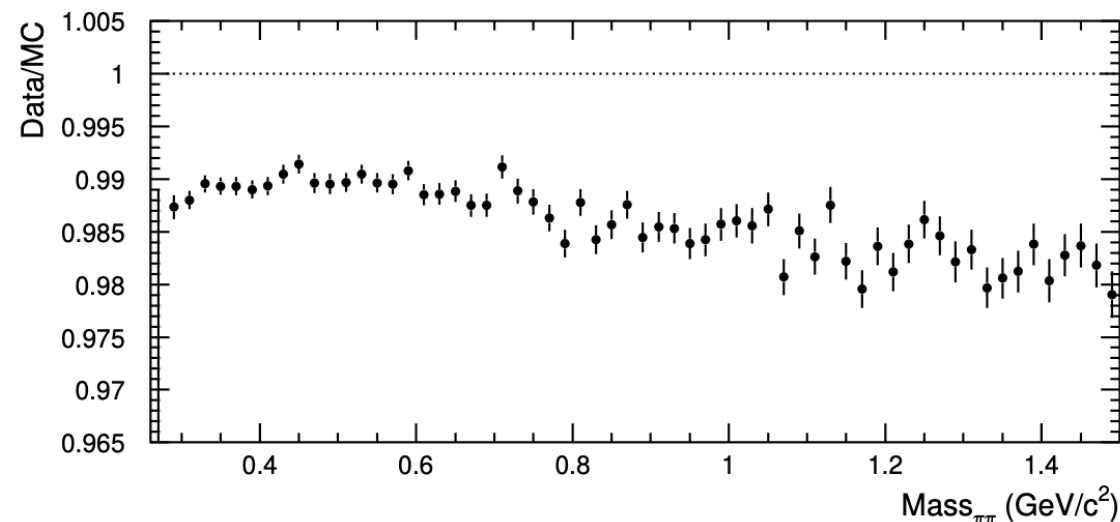
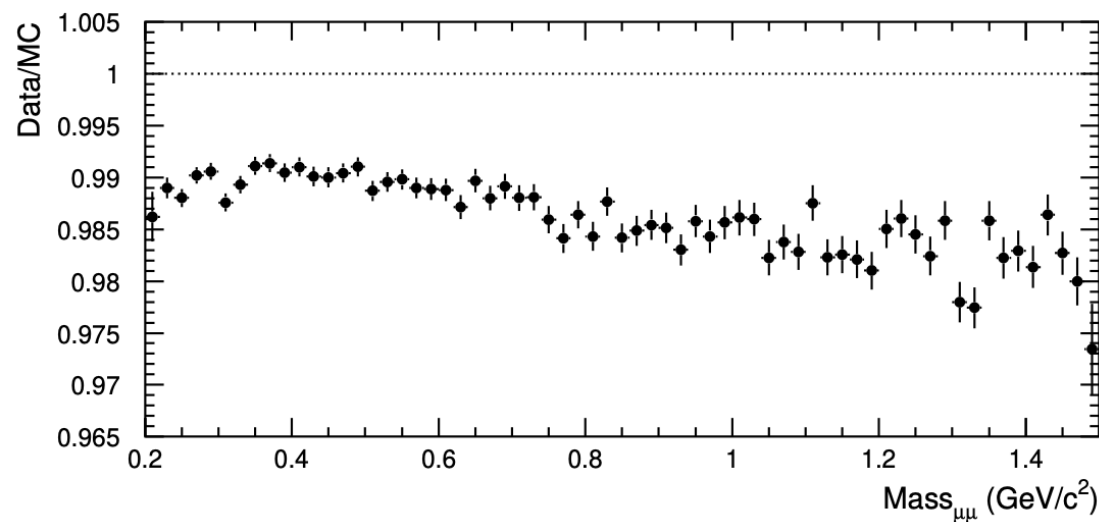
- Additional corrections for pions: fake photons and secondary interactions
- Each correction has two components:
 - Template shape corrections (affect $\pi\pi/\mu\mu$ separation)
 - Mass dependence corrections (affect $\pi\pi$ and $\mu\mu$ spectra)
- **Blinded procedure:** trigger and tracking corrections initially offset (2 offsets for each process)

2D- χ^2 Selection Efficiency

- For muons **clean channel** $\rightarrow$ background negligible.
- For pions $\rightarrow$ background ($ee\gamma, 2\pi\pi^0, uds, KK\gamma, etc$) too large.
- Use **μ 2D- χ^2 efficiencies $\epsilon_{\chi^2}^{\mu\mu\gamma}$ to obtain** the pion 2D- χ^2 efficiency $\epsilon_{\chi^2}^{\pi\pi\gamma}$:

$$\epsilon_{\chi^2}^{\pi\pi\gamma, \text{data}} = \epsilon_{\chi^2}^{\mu\mu\gamma, \text{data}} + \epsilon_{\chi^2}^{\pi\pi\gamma, \text{MC}} - \epsilon_{\chi^2}^{\mu\mu\gamma, \text{MC}}$$

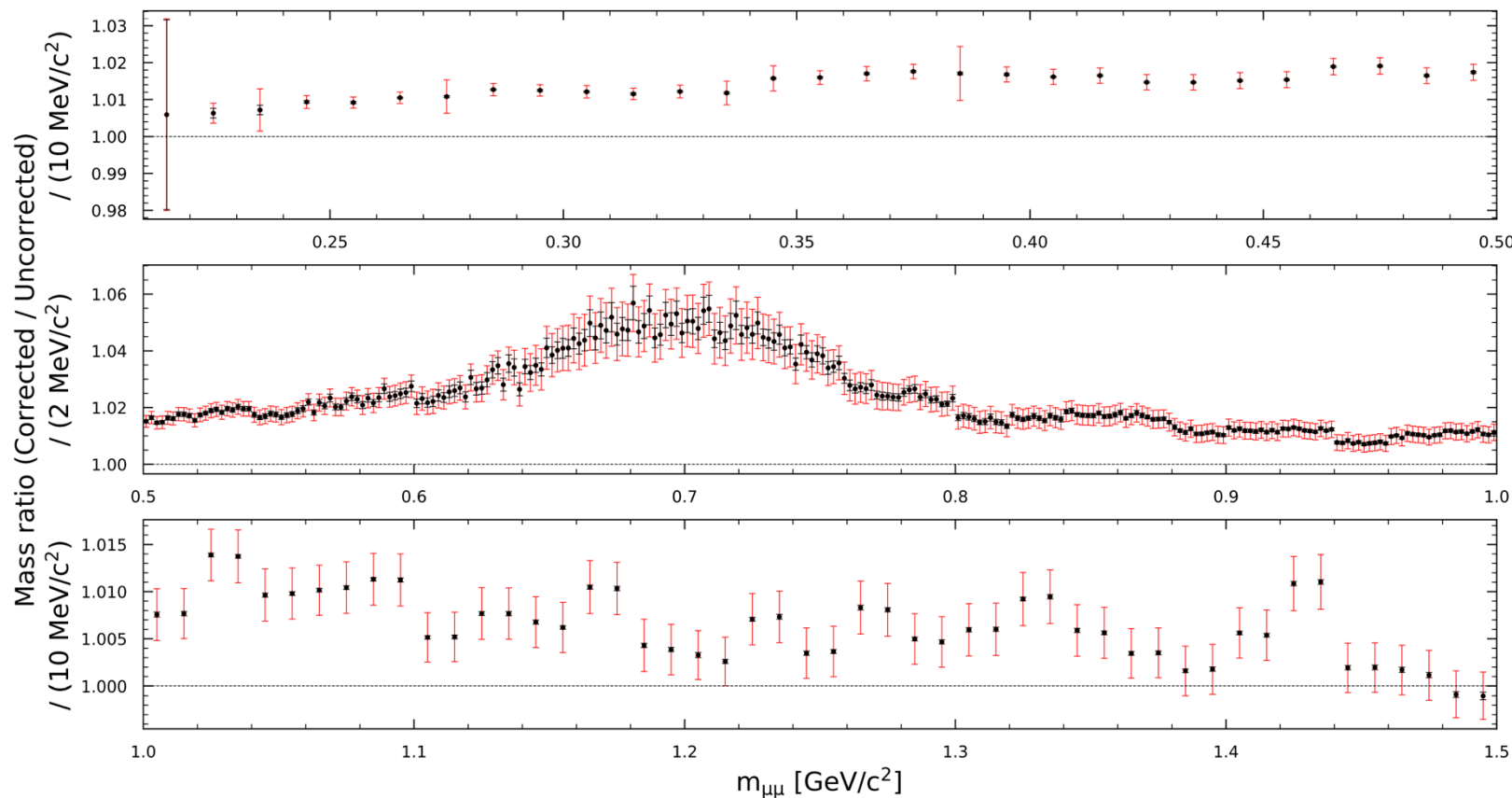
- Extra correction for pion: Secondary interactions +fake photons are accounted for by an additional correction derived from data/MC comparisons of pion interactions



Impact of Tracking Data/MC Corrections

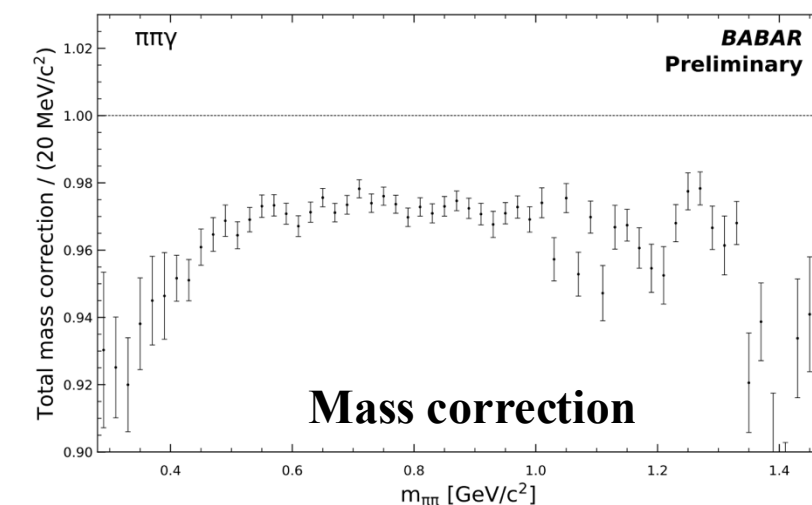
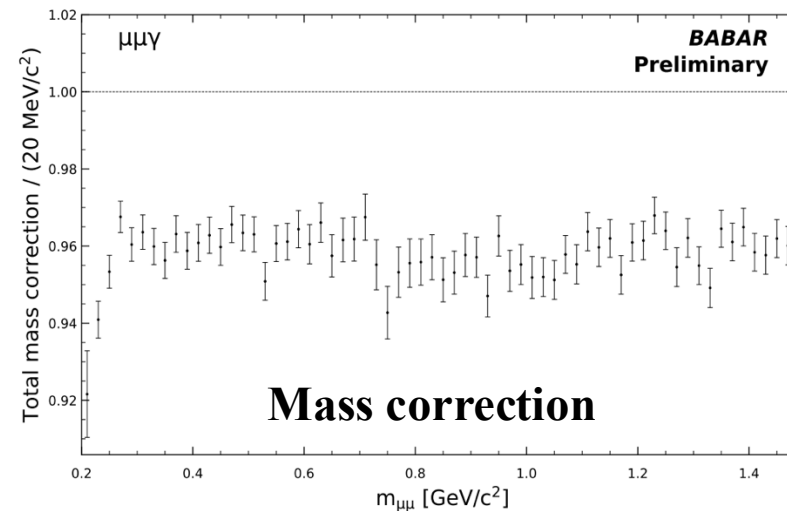
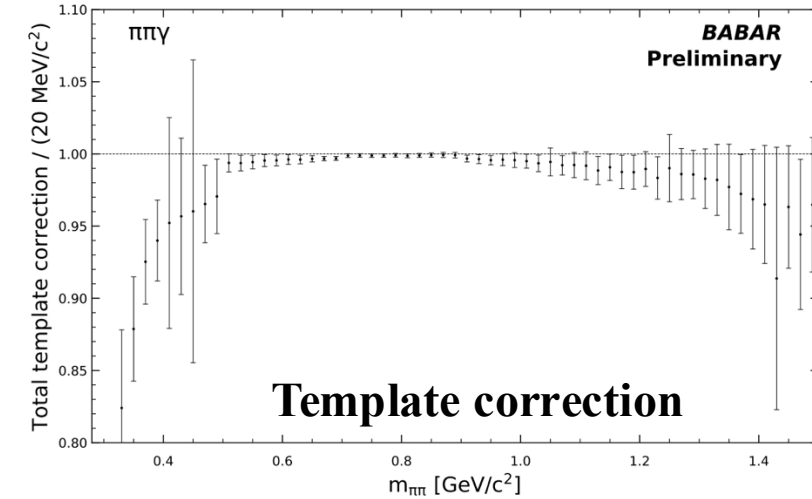
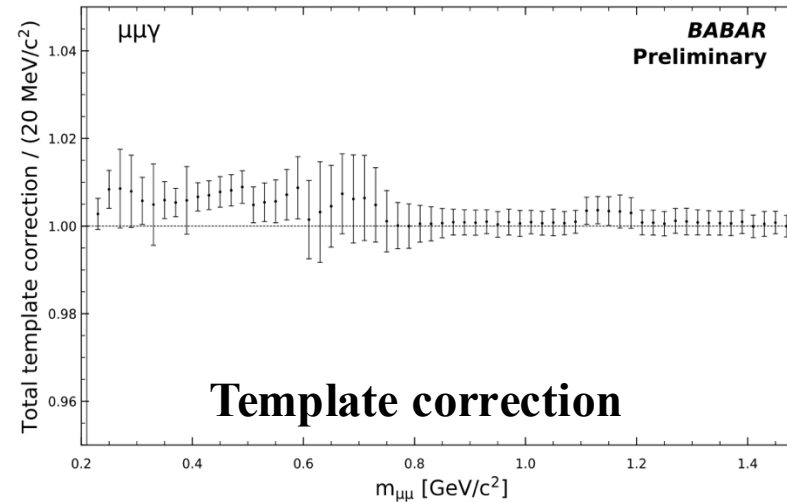
- **Tag-and-probe method** used as in 2009.
- **Correction factor** applied to account for data/MC differences
- $\mu\mu$ fraction is sensitive to small effects in $\pi\pi$ templates (especially near the ρ peak)

- Black error bars = statistical error (pseudo experiments with 1000 toys)
- Red error bars = uncertainties from corrections
- Tracking = **largest correction applied**
- Other corrections (e.g. 2D- χ^2) show compensating effects



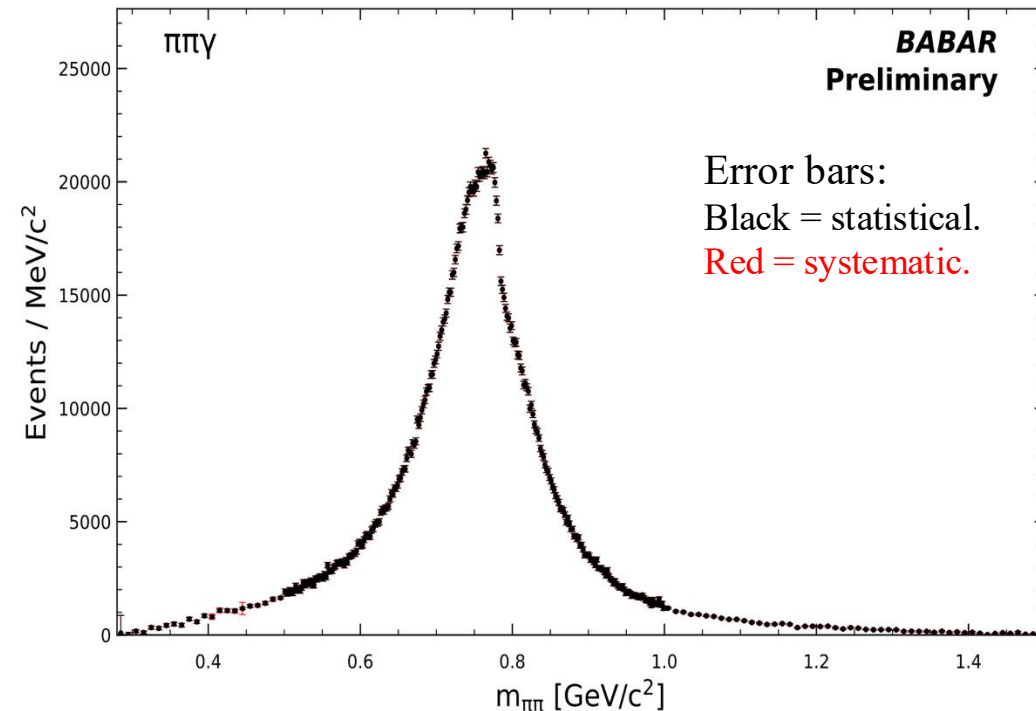
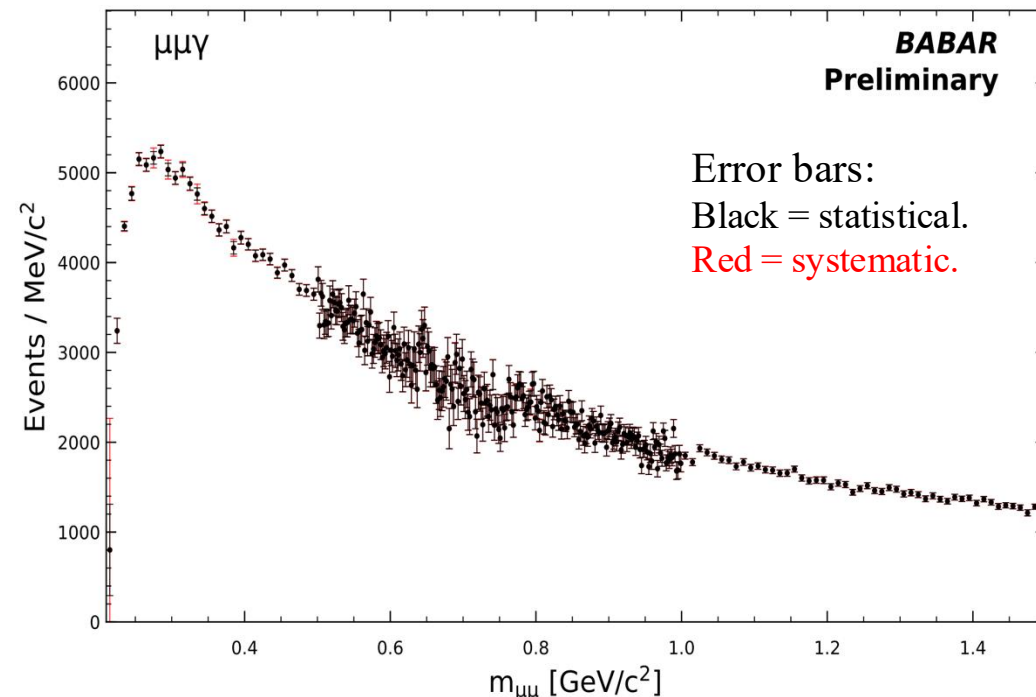
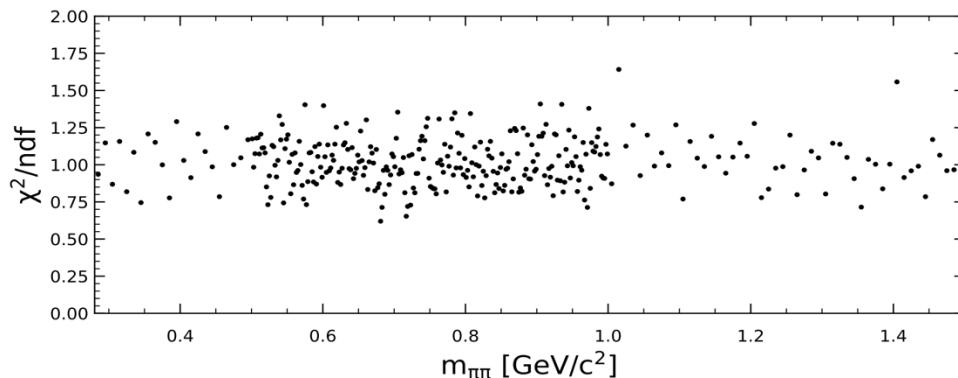
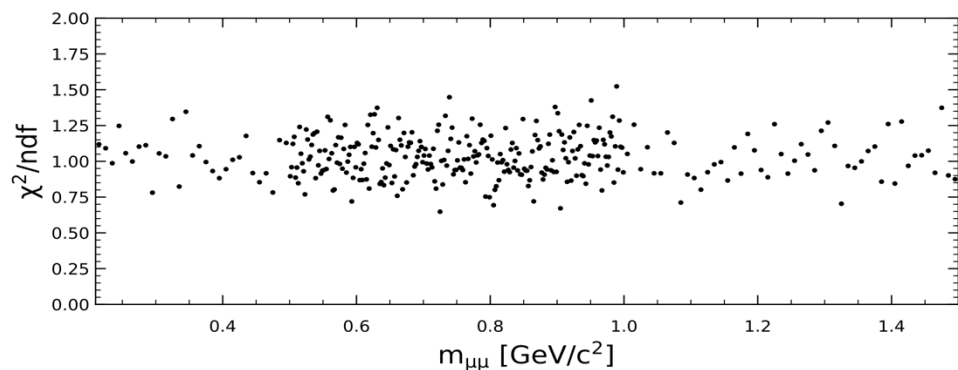
Corrections for the Mass Spectrum

- Combined corrections applied to **Templates** and **Mass spectrum**
- Corrections are fairly flat vs $m_{\mu\mu/\pi\pi}$ showing no strong mass dependence
- Large effect of corrections on mass spectra (below $\pm 5\%$) that tends to cancel out when combined
- Corrections remain close to unity $\rightarrow$ small overall impact
- Corrections under control, ensuring reliable m_{XX} spectrum



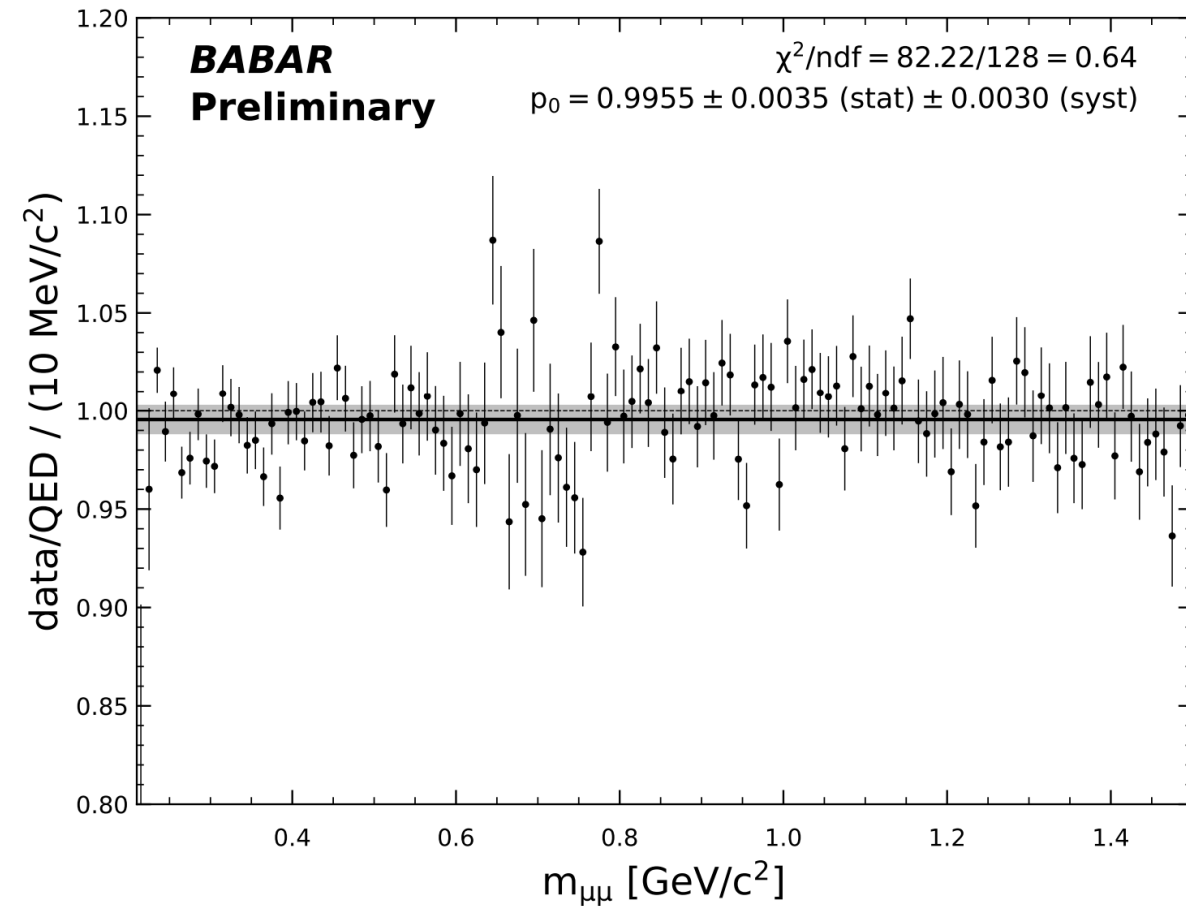
Fitted Mass Spectrum

- Mass spectra obtained from angular fits with blinded absolute corrections (3rd offset).
- Fits performed in:
 - 2 MeV/c² bins for 0.5–1.0 GeV/c².
 - 10 MeV/c² bins elsewhere.
- Mass spectra (muon & pion) initially **blinded** with an offset.



Muon Mass Spectrum vs QED

- $\mu\mu\gamma$ data spectrum can be compared to QED prediction, obtained by correcting the simulated Phokhara spectrum for:
 - ISR photon efficiency
 - NLO–NNLO acceptance correction (<0.03%)
 - Improved vacuum polarization contribution
- Agreement with QED within **0.7%**.
- More precise than 2009 result ($\pm 1.1\%$)
- Confirms robustness of $\pi\pi/\mu\mu$ separation, no major systematic bias

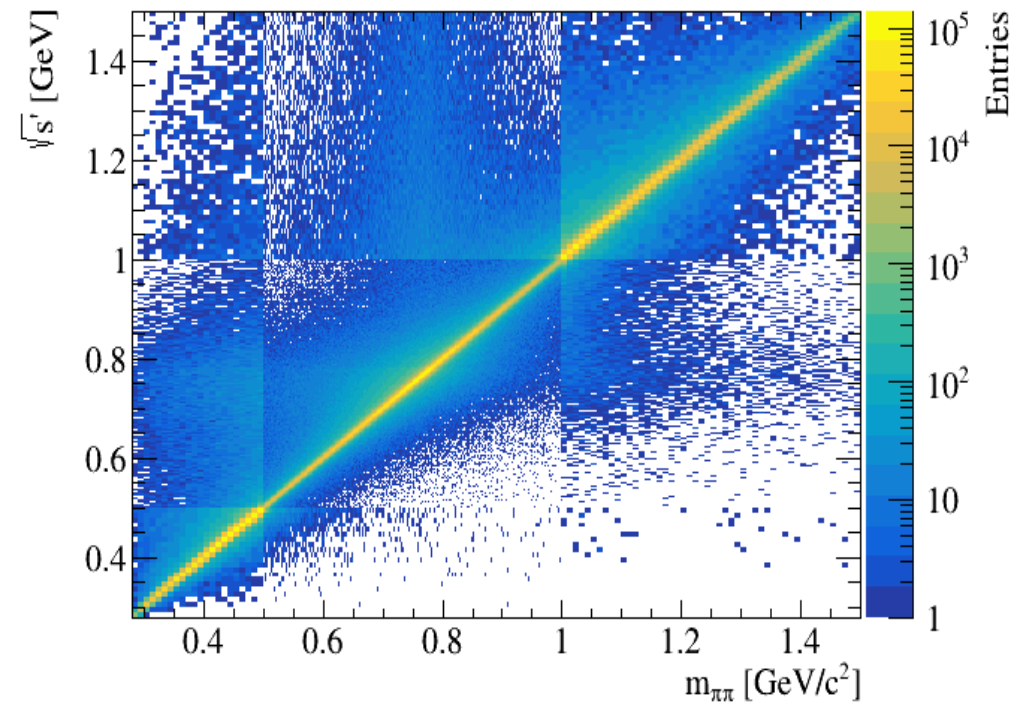
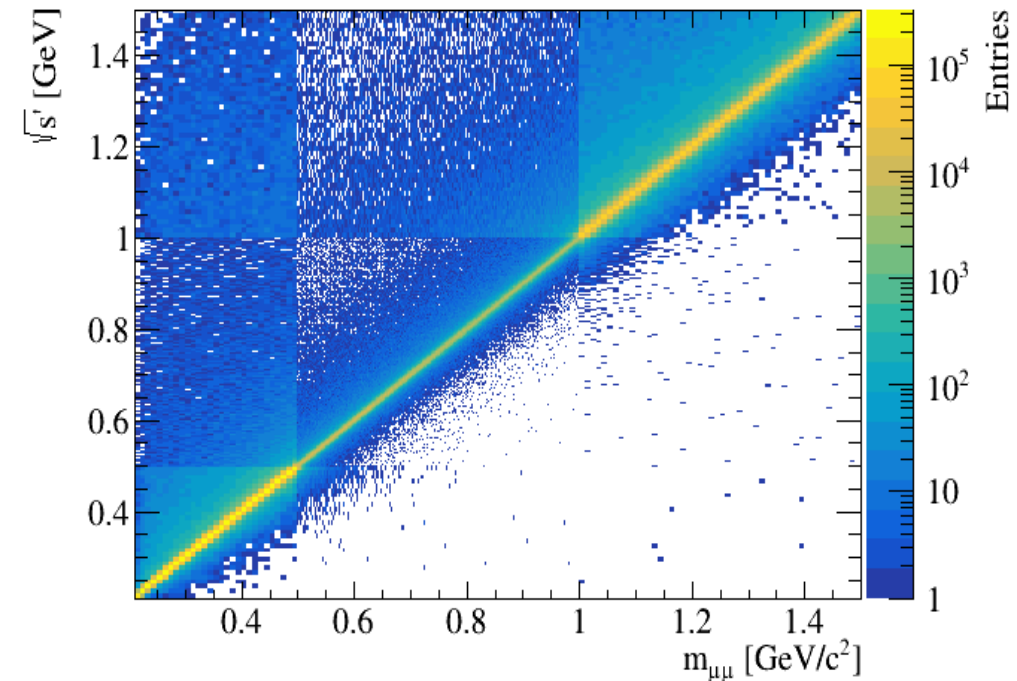


$$\begin{array}{cccc}
 2025: R_{\mu\mu} = 0.9955 \pm 0.0035_{\text{stat}} \pm 0.0030_{\text{syst}} \pm 0.0033_{\gamma \text{ ISR}} \pm 0.0043_{\text{lumi ee}} \\
 \underbrace{\hspace{1.5cm}} & \underbrace{\hspace{1.5cm}} & \underbrace{\hspace{1.5cm}} & \underbrace{\hspace{1.5cm}} \\
 \text{data + stat} & \text{Syst. related} & \text{ISR photon} & \text{error on} \\
 \text{errors on} & \text{to } \pi/\mu & \text{data/MC} & \text{e}^+ \text{e}^- \\
 \text{corrections} & \text{separation} & \text{efficiency} & \text{luminosity} \\
 \underbrace{\hspace{1.5cm}} & \underbrace{\hspace{1.5cm}} & \underbrace{\hspace{1.5cm}} & \underbrace{\hspace{1.5cm}} \\
 2009: R_{\mu\mu} = 1.0040 \pm 0.0019_{\text{stat}} \pm 0.0043_{\text{syst}} \pm 0.0034_{\gamma \text{ ISR}} \pm 0.0094_{\text{lumi ee}}
 \end{array}$$

Unfolding procedure

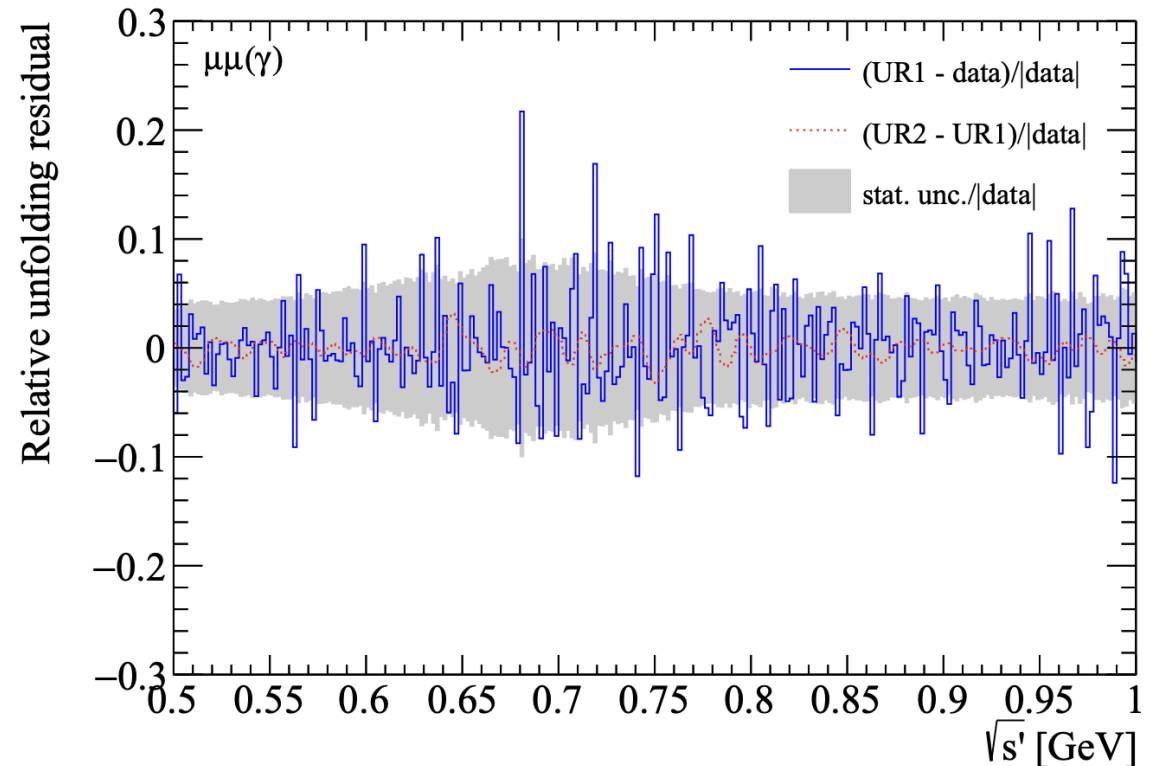
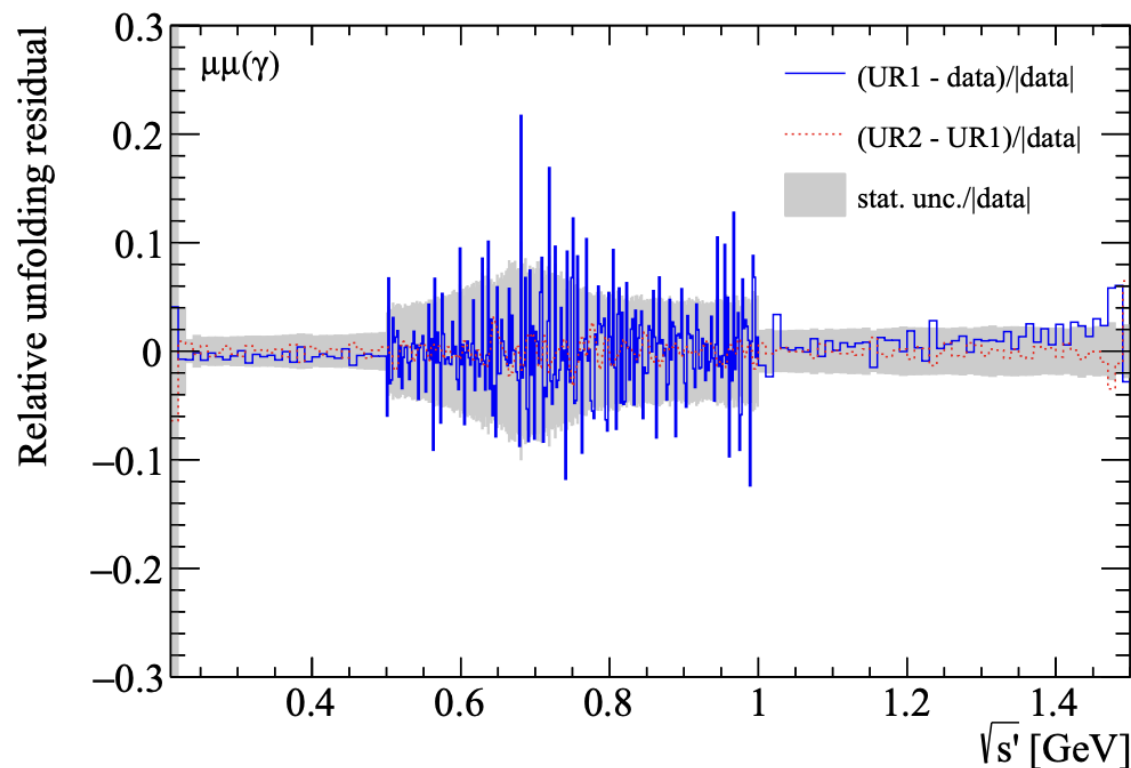
- Implementation of [An iterative, dynamically stabilized method of data unfolding](#) to correct data/MC shape differences.
- Corrects for:
 - Detector resolution.
 - Effect of Final State Radiation (FSR) $\rightarrow$ shifts $m_{XX} \rightarrow \sqrt{s'}$.
- To perform the unfolding a migration matrix from simulation is constructed using the reconstructed invariant mass m_{XX} and the reduced energy $\sqrt{s'}$ defined as:

$$\sqrt{s'} = \begin{cases} m_{XX} & \text{if. } \theta_{\min}^{\text{true}} > 20^\circ \\ m_{XX\gamma} & \text{if. } \theta_{\min}^{\text{true}} \leq 20^\circ \end{cases}$$
- With $\theta_{\min}^{\text{true}}$ the angle between the FSR/ISR LA photon and the nearest charged track.
- Data-driven studies show systematic uncertainties are small.
- Systematic related to the response matrix ($\theta_{\min}^{\text{true}}$ cut) are included by unfolding the spectrum with different matrices.
- Systematics uncertainties are propagated by $\pm 1\sigma$ variations



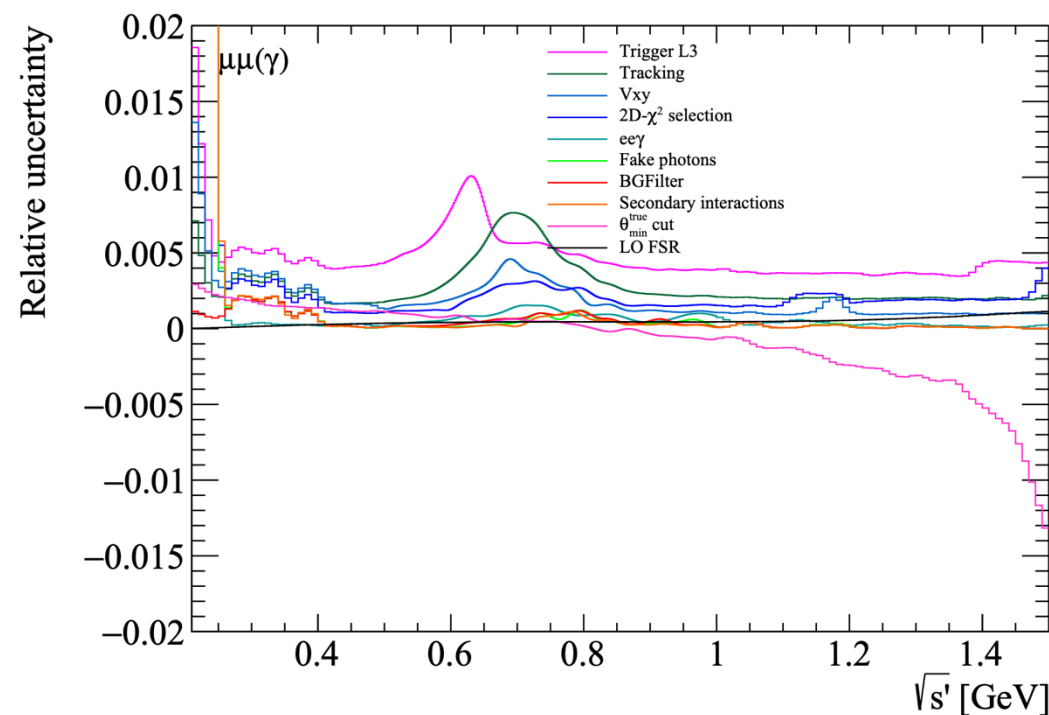
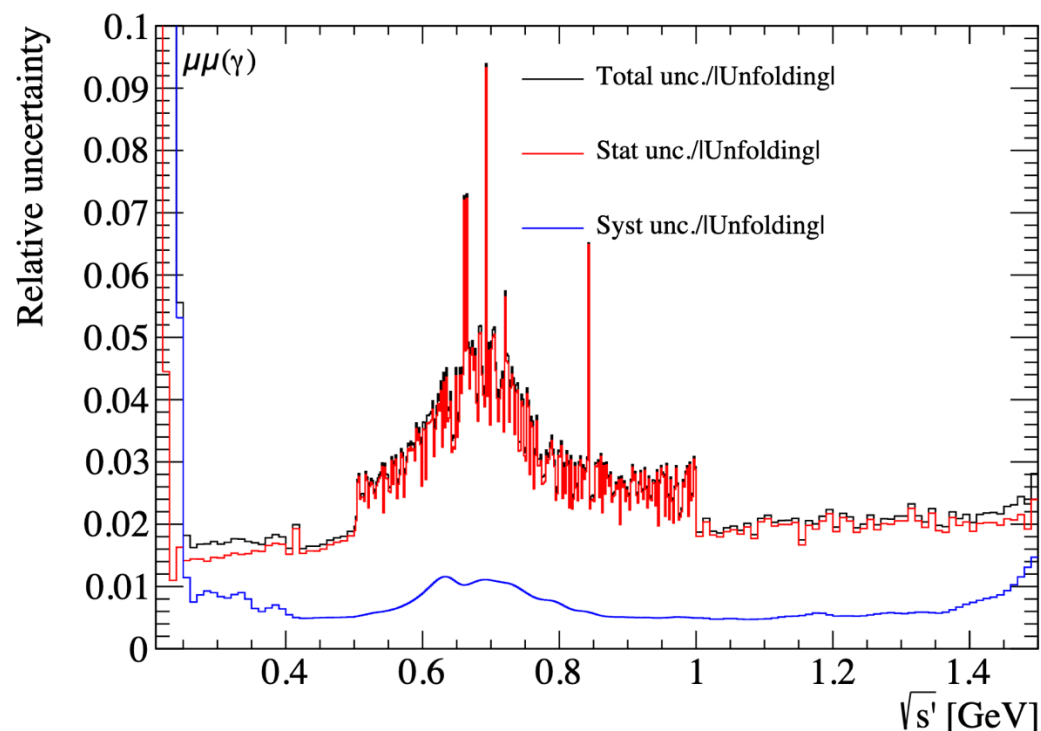
Muon unfolding

- Unfolding results with original MC (UR1) and with one iteration of truth–MC reweighting (UR2) show to be close to the initial data.
- Further iterations have negligible effect, and the unfolded spectrum remains stable, indicating a minimal systematic impact from mismodeling the generated mass distribution.
- MC reweighted (UR2) is used as the nominal unfolding result.



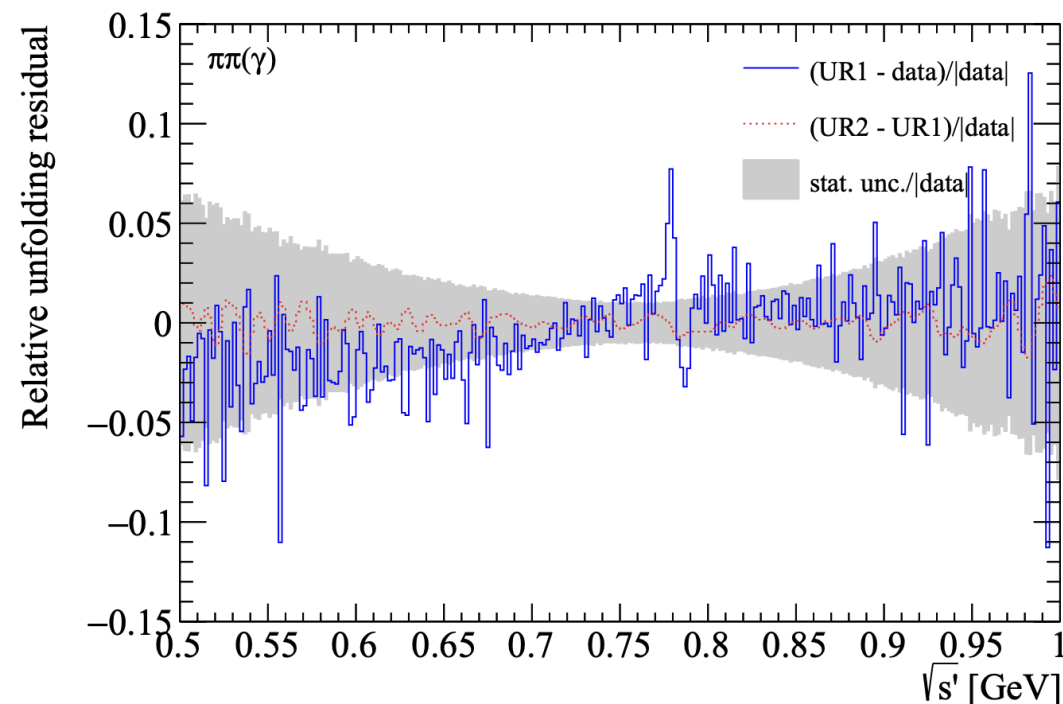
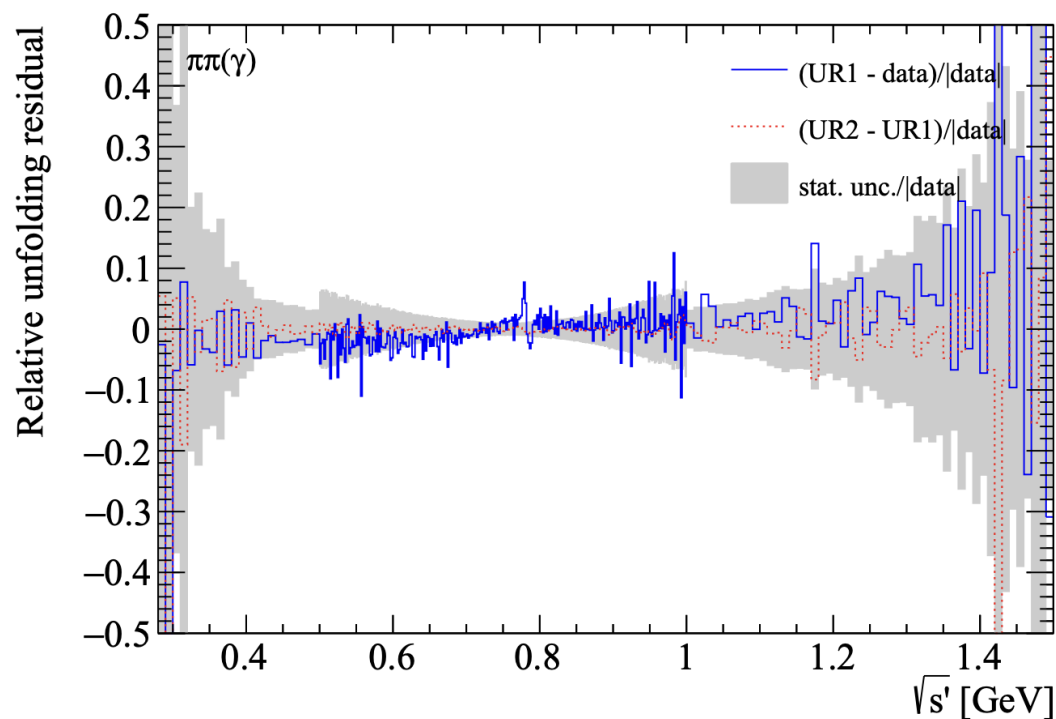
Muon uncertainties after unfolding

- Statistical uncertainties are estimated by **coherent Poisson** pseudo experiments of the data spectrum and the migration matrix.
- Systematic uncertainties propagated through coherent shifts of all bins for a given systematic source + unfolding + comparison with nominal spectrum ($\pm 1\sigma$ variations).
- Systematics are smoothed using a Gaussian kernel.
- Dominated by Trigger L3, tracking, vertex, 2D $-\chi^2$ selection, etc.
- Statistical uncertainty below 5% under the ρ peak.



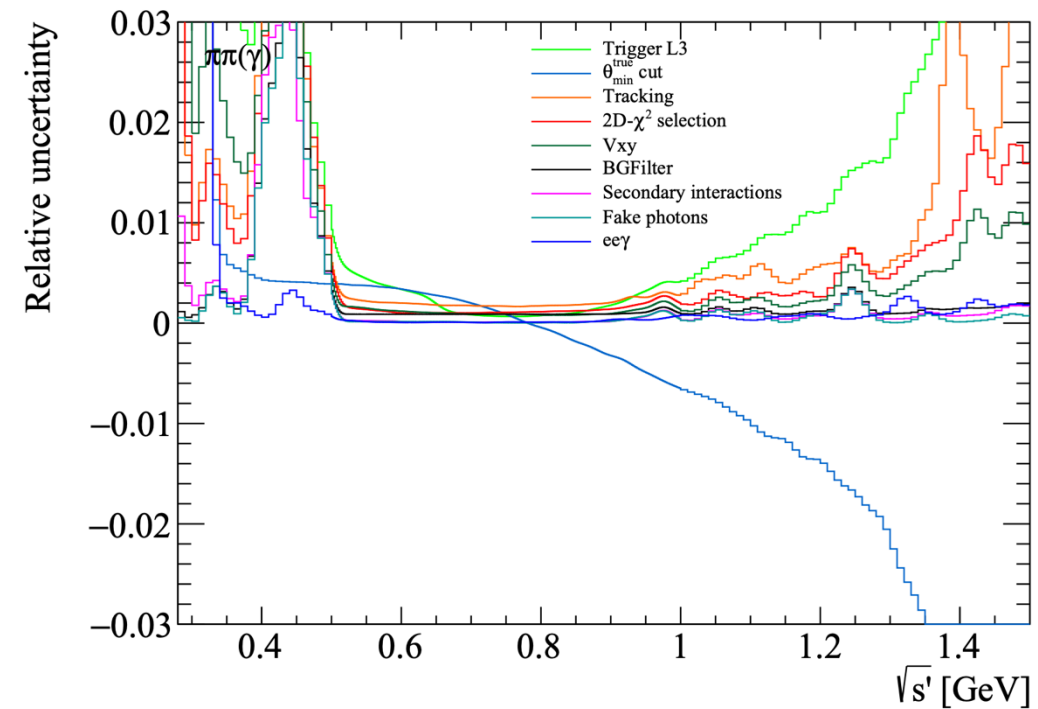
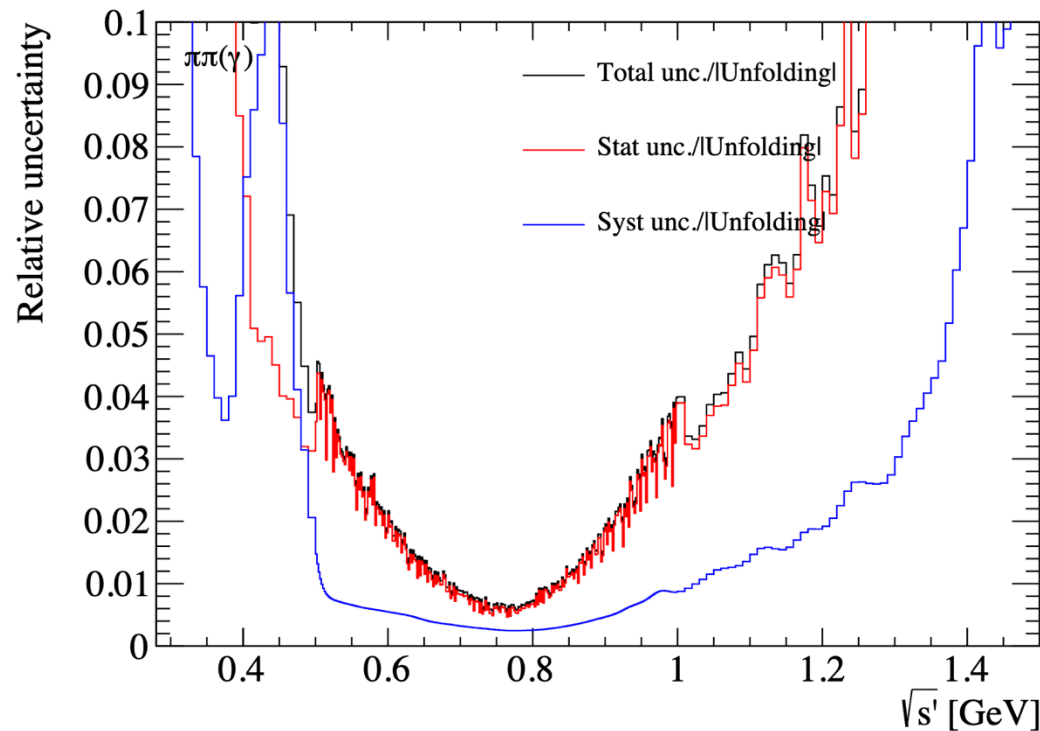
Pion unfolding

- Unfolding results (UR1 and UR2) show to be close to the initial data with more significant corrections in the $\rho - \omega$ interference region.
- Further iterations have negligible effect, and the unfolded spectrum remains stable, indicating a minimal systematic impact from mismodeling the generated mass distribution.
- MC reweighted (UR2) is used as the nominal unfolding result.



Pion uncertainties after unfolding

- Systematic uncertainties dominate at low and high mass while in the central region the statistical uncertainty dominates
- Systematic uncertainty is mainly dominated by Trigger L3, $\theta_{\min}^{\text{true}}$ cut, tracking, etc.
- Statistical uncertainty below 1% around the $\rho - \omega$ interference.



Effective ISR luminosity determination

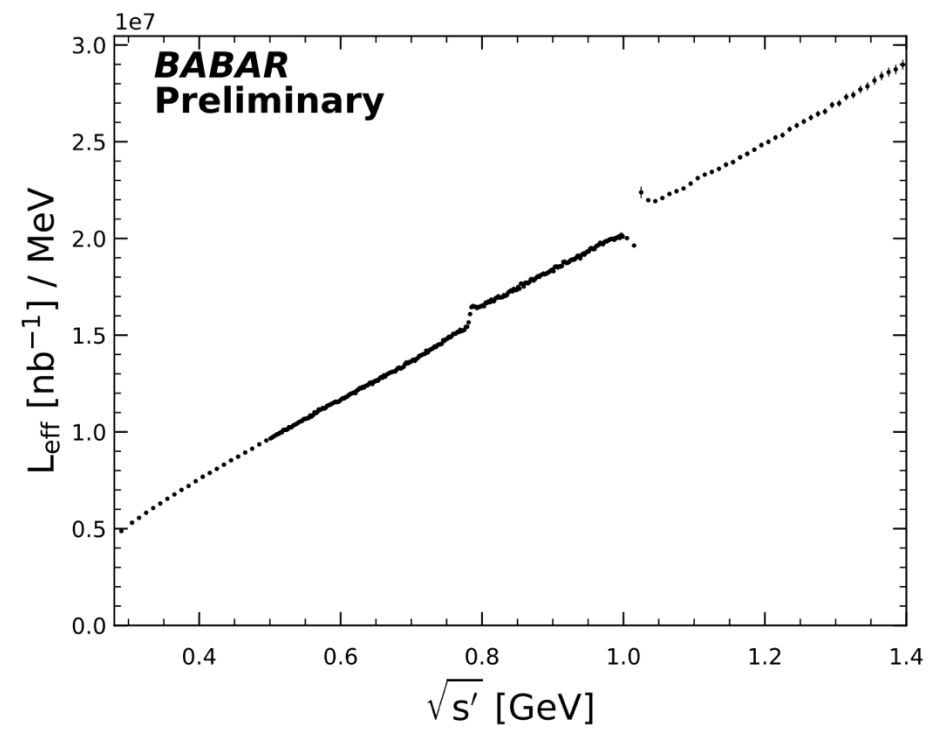
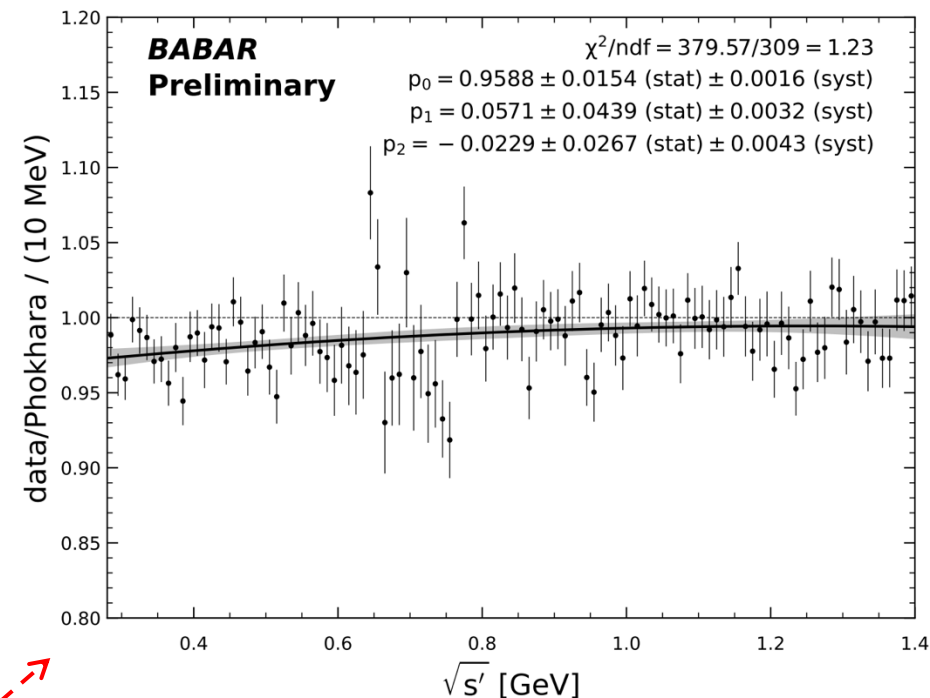
- Effective ISR luminosity needed to extract $\pi\pi$ cross sections. Derived from the **muon channel**.

$$\frac{dL_{\text{ISR}}^{\text{eff}}}{d\sqrt{s'}} = \frac{dN_{\mu\mu}^{\text{ISR}}/d\sqrt{s'}}{\epsilon_{\mu\mu}(\sqrt{s'}) \sigma_{\mu\mu}^0(\sqrt{s'})}$$

- where:
 - $\epsilon_{\mu\mu}$: acceptance of the selection for muons
 - $\sigma_{\mu\mu}^0$: Bare cross section $e^+e^- \rightarrow \mu^+\mu^-$
 - $dN_{\mu\mu}^{\text{ISR}}/d\sqrt{s'}$: Unfolded muon spectrum

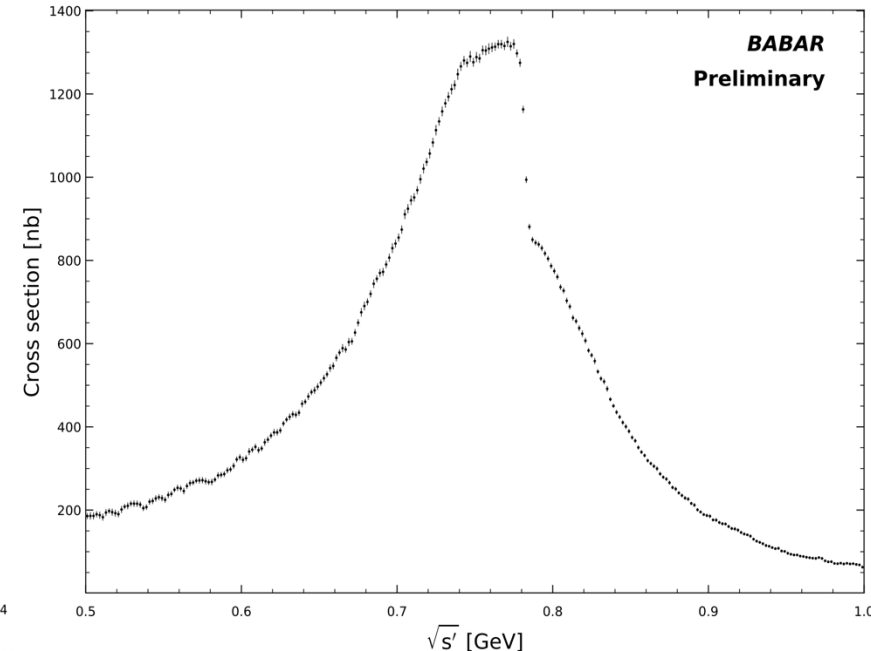
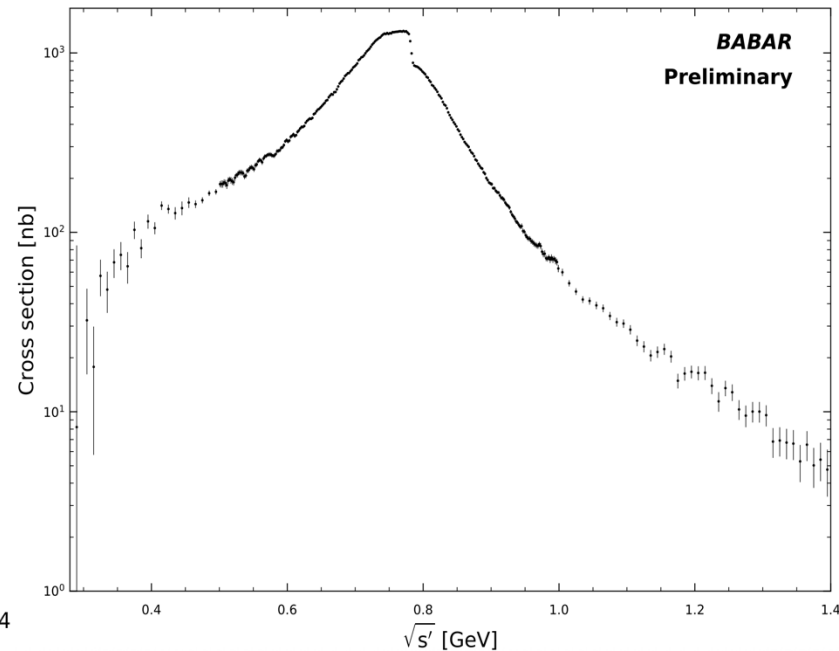
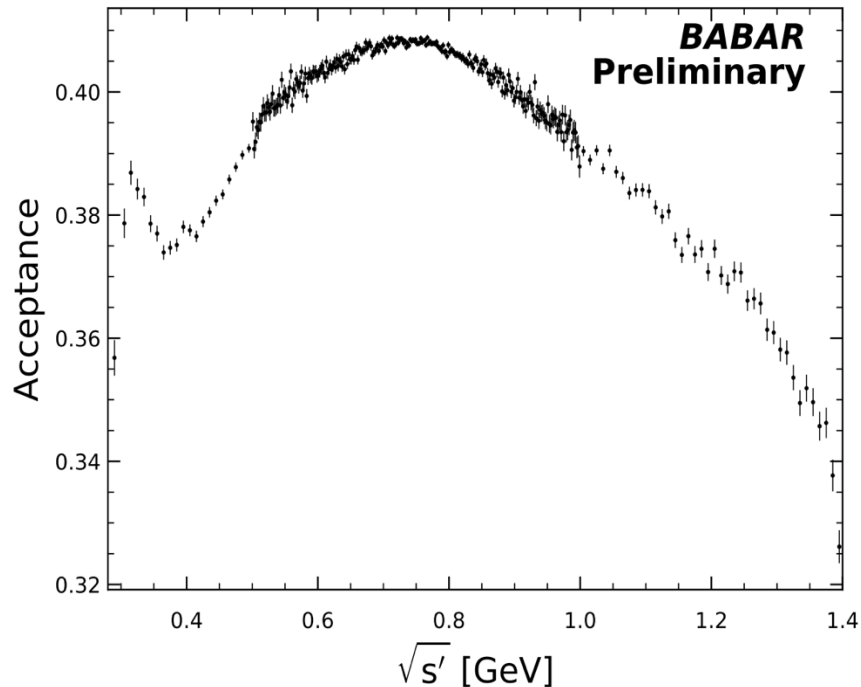
- Ratio $(dN_{\mu\mu}^{\text{ISR}}/d\sqrt{s'})/\epsilon_{\mu\mu}$ is replaced by

$$\frac{dN_{\mu\mu}^{\text{ISR}}/d\sqrt{s'}}{\epsilon_{\mu\mu}(\sqrt{s'})} = \underbrace{\frac{dN_{\mu\mu}^{\text{MC gen}}}{d\sqrt{s'}}}_{\text{Phokhara spectrum at generation level}} \times \underbrace{(1 - f_{\text{LO FSR}})}_{\text{removes the LO FSR contribution}} \times \underbrace{f_{\mu\mu}(\sqrt{s'})}_{\text{2nd order polynomial fit to data/MC}}$$



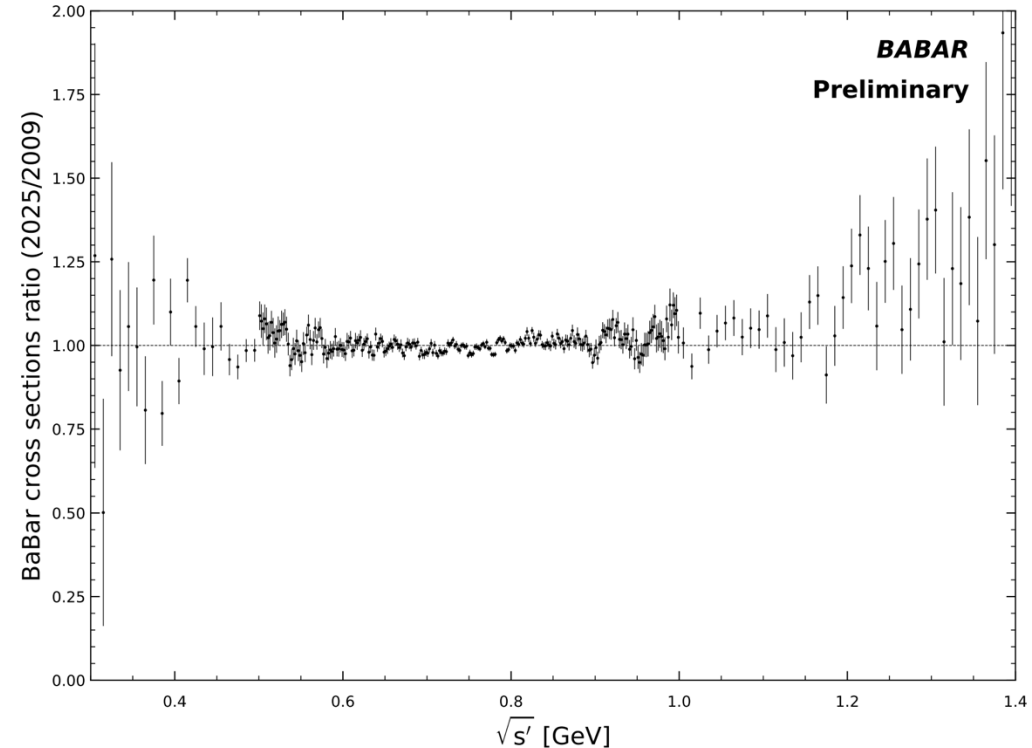
$$\sigma_{\pi\pi}^0(\sqrt{s'}) = \frac{\frac{dN_{\pi\pi}}{d\sqrt{s'}}}{\epsilon_{\pi\pi}(\sqrt{s'}) \frac{dL_{\text{ISR}}^{\text{eff}}}{d\sqrt{s'}}}$$

- Using the unfolding pion spectrum, detector acceptance vs $\sqrt{s'}$ and the effective ISR luminosity (from $\mu\mu$ channel) we obtain the cross section $\sigma_{\pi\pi}^0(\sqrt{s'})$.
- Reduced systematic uncertainties due to cancelling error sources or corrections common to $\mu\mu\gamma$ and $\pi\pi\gamma$.
- Clear $\rho - \omega$ interference structure



$\pi\pi$ contribution to a_μ

- 2025 results are in **excellent agreement** with 2009
- Best precision achieved in the $\rho - \omega$ interference region.
- Larger uncertainties at low/high masses due to muon background



Energy range [GeV]	2025 ($a_\mu^{\pi\pi} \pm \text{stat} \pm \text{syst} [10^{-10}]$)	2009 ($a_\mu^{\pi\pi} \pm \text{stat} \pm \text{syst} [10^{-10}]$)
Below 0.5	$58.0 \pm 5.5 \pm 1.7$	$57.6 \pm 0.6 \pm 0.6$
0.5 – 1.4	$456.2 \pm 2.2 \pm 1.7$	$455.6 \pm 2.1 \pm 2.6$

Energy range [GeV]	2025–2009 average (preliminary) $a_\mu^{\pi\pi} [10^{-10}]$
Below 0.5	58.2 ± 0.8
0.5 – 1.4	455.9 ± 2.1
Below 1.4	514.1 ± 2.5
Below 1.8 (1.4 – 1.8 from 2009)	514.4 ± 2.5

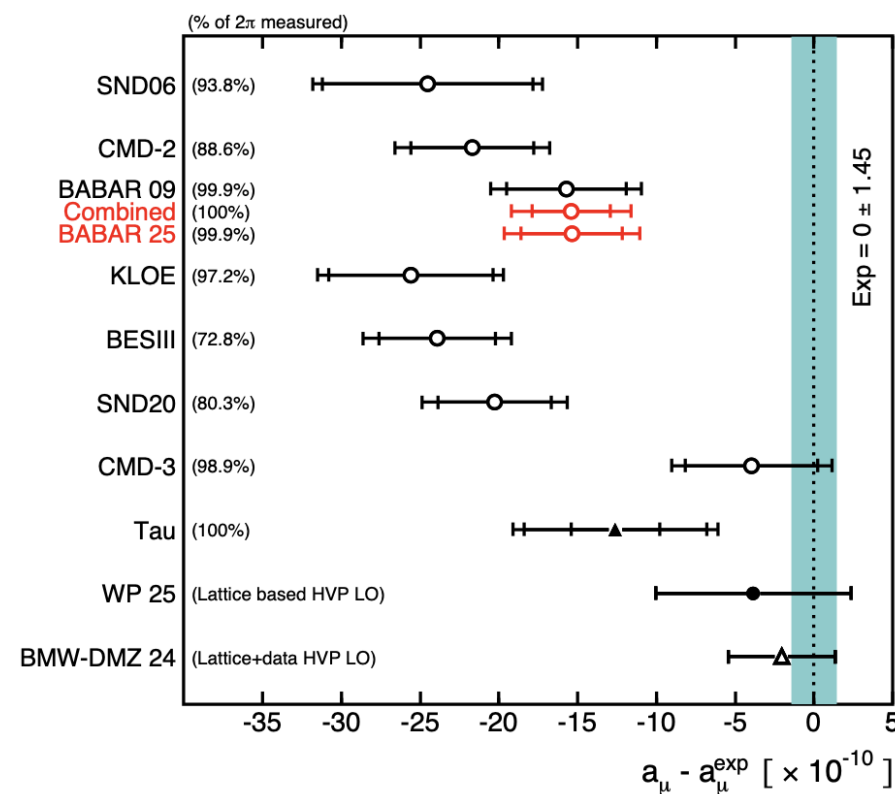
Summary and outlook

- New **blind BaBar analysis** (460 fb^{-1}) confirms the $\pi^+\pi^-$ contribution to a_μ .
- Independent method (angular fits, no PID) removes dominant 2009 systematic.
- Unblinded $\mu\mu\gamma$ spectrum agrees with QED, validating the approach.
- $\pi\pi$ cross section consistent with 2009, with **reduced systematics** in 0.5–1.4 GeV.
- Results:
 - Below 0.5 GeV: $a_\mu^{\pi\pi} = (58.0 \pm 5.5 \text{ (stat.)} \pm 1.7 \text{ (syst.)}) \times 10^{-10}$
 - 0.5–1.4 GeV: $a_\mu^{\pi\pi} = (456.2 \pm 2.2 \text{ (stat.)} \pm 1.7 \text{ (syst.)}) \times 10^{-10}$
- Robustness shown by excellent agreement with 2009.

More information in Lepton Photon 2025 talks:

[Léonard's talk](#) : New precise measurement of the $e^+e^- \rightarrow \pi^+\pi^-(\gamma)$ cross section with BaBar

[Zhiqing's talk](#) : Review of HVP calculations via e^+e^- measurements



BACKUP

Getting a_μ from $e^+e^- \rightarrow \pi^+\pi^-(\gamma)$ cross section

- Cross section of $e^+e^- \rightarrow \pi^+\pi^-(\gamma)$ at reduced energy $\sqrt{s'} = m_{XX}$ (X = any final state) from measurement of $e^+e^- \rightarrow X\gamma_{ISR}$ with $E_{\gamma_{ISR}}$ energy in center of mass (CM) frame.
- Measuring the yield $N_{X\gamma_{ISR}}$ gives the bare cross section $\sigma_X^0(\sqrt{s'})$ (excluding vacuum polarization)

$$\frac{dN_{X\gamma}}{d\sqrt{s'}} = \frac{dL_{ISR}^{eff}}{d\sqrt{s'}} \varepsilon_{X\gamma}(\sqrt{s'}) \sigma_X^0(\sqrt{s'}) \quad (1)$$

- $\varepsilon_{X\gamma}$ = detection efficiency in acceptance $\rightarrow$ from simulation with data corrections.
- L_{ISR}^{eff} = effective ISR luminosity $\rightarrow$ from $X = \mu\mu(\gamma_{FSR})$ in (1) and $\sigma_X^0(\sqrt{s'})$ taken from QED computation
- Ratio of $\pi\pi$ and $\mu\mu$ mass spectra $\Rightarrow$ cancellation of VP $\Rightarrow$ ratio of (1)

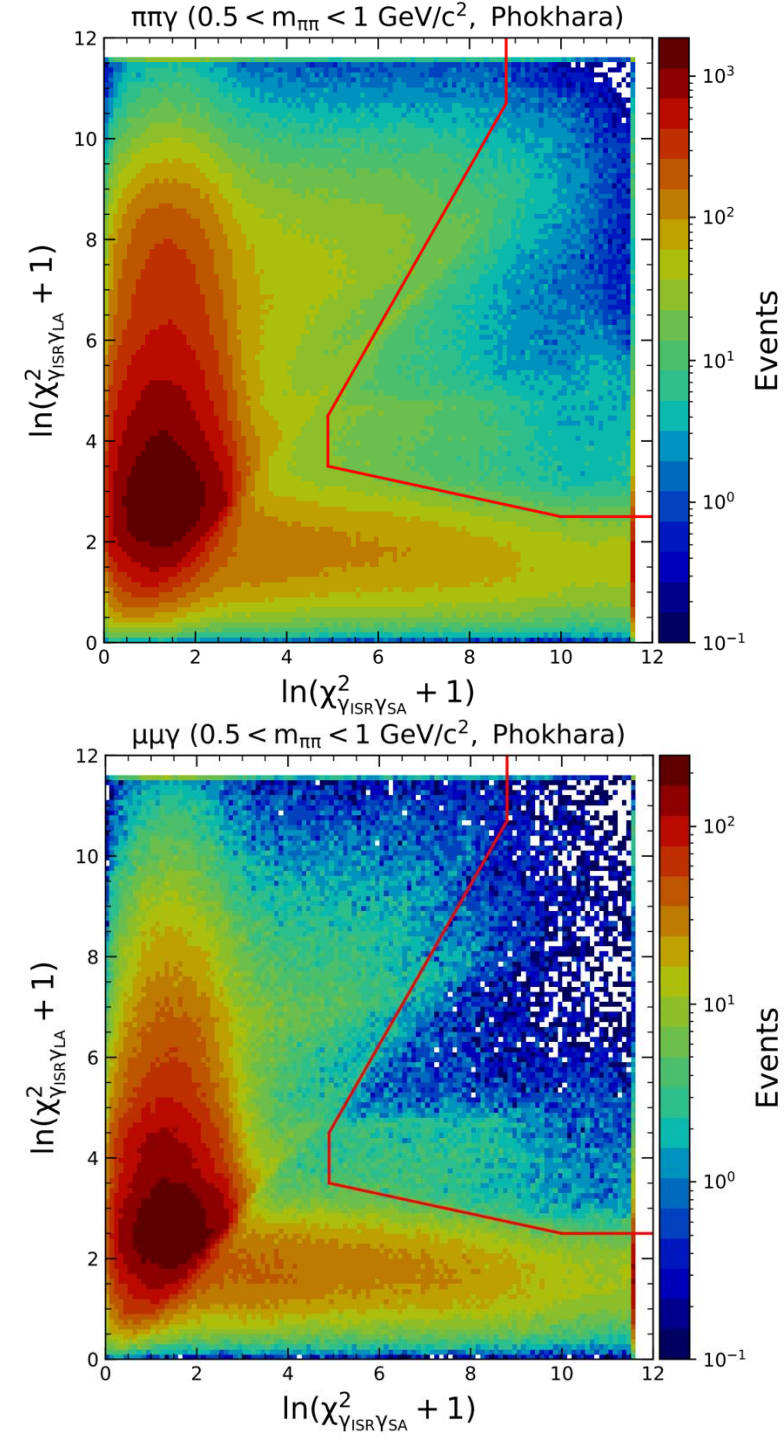
$$\frac{\sigma_{\pi\pi(\gamma_{FSR})}^0(\sqrt{s'})}{\sigma_{pt}(\sqrt{s'})(1 + \delta_{FSR}^{\mu\mu})(1 + \delta_{add. FSR}^{\mu\mu})} \quad (2)$$

- $\sigma_{pt}(\sqrt{s'}) = 4\pi\alpha^2/3s'$: cross section for point-like charged fermions
- $(1 + \delta_{(add.) FSR}^{\mu\mu})$: corrections for lowest-order(additional) FSR contributions.
- Dispersion relation with QED kernel $K(s')$:

$$a_\mu^{\pi\pi(\gamma_{FSR}), LO} = \frac{1}{4\pi^3} \int_{4m_\pi^2}^{\infty} ds' K(s') \sigma_{\pi\pi(\gamma_{FSR})}^0(s') \quad (3)$$

NLO fits description

- Use measured ISR energy/direction + momenta/angles of both tracks.
- **$\gamma_{\text{ISR}}\gamma_{\text{LA}}$ fit:** additional large angle (LA) γ with respect to the beams (0.35 – 2.45 rad).
- **$\gamma_{\text{ISR}}\gamma_{\text{SA}}$ fit:** additional small angle (SA) γ fitted assuming collinearity with one of the beams.
- **NLO LA sample:** $\chi_{\text{LA}}^2 < \chi_{\text{SA}}^2$, $E_{\gamma,\text{LA}} > 200$ MeV.
- **NLO SA sample:** $\chi_{\text{LA}}^2 > \chi_{\text{SA}}^2$, $E_{\gamma,\text{SA}}^* > 200$ MeV.
- **LO sample:** events below the thresholds.
- Larger background in $\pi\pi\gamma$ process, suppressed with
- optimized BDT-based 2D – χ^2 selection (98-99% efficiency).



Cut-based selection of $e e \gamma$ events

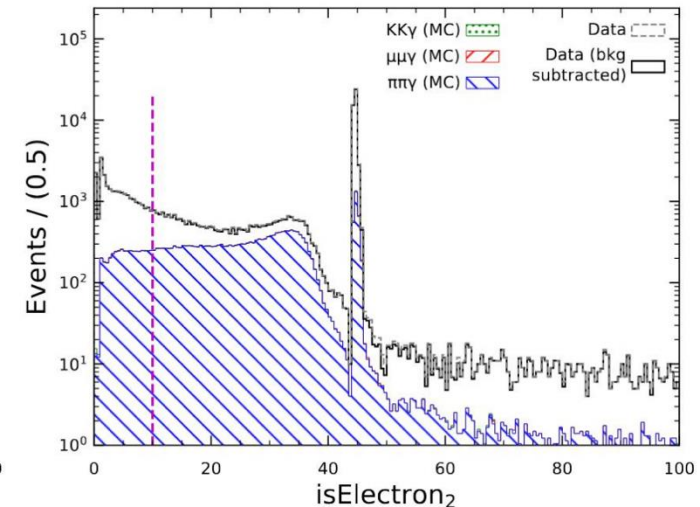
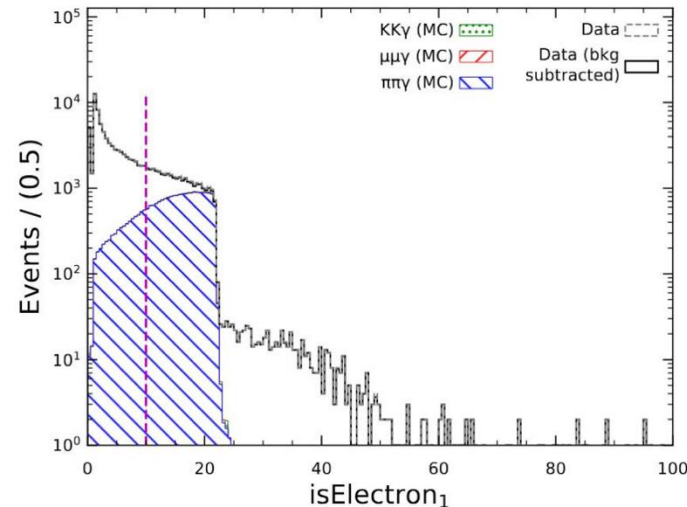
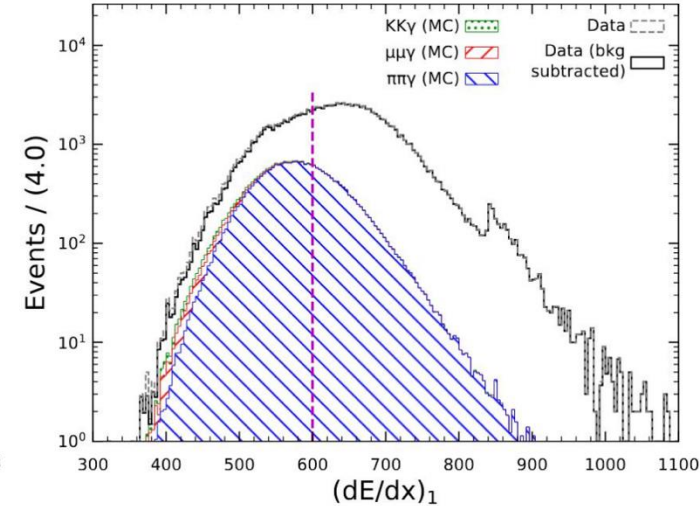
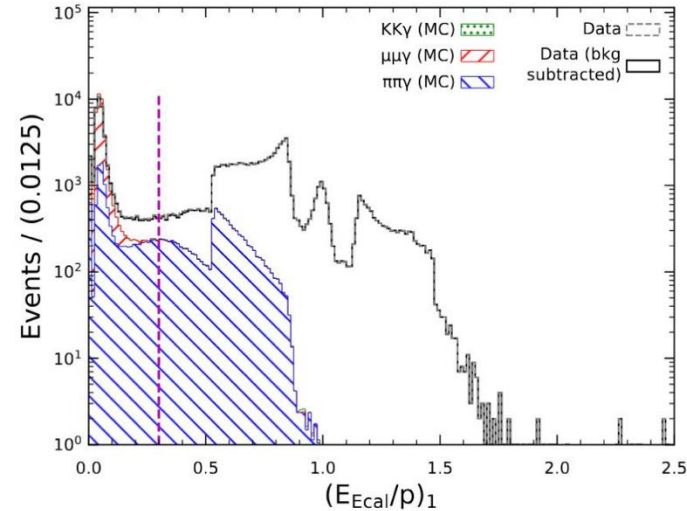
- Cut-based selection studied in a previous analysis (2015):

$$\left(\frac{E_{\text{cal}}}{p}\right)_1 > 0.5, \quad \left(\frac{dE}{dx}\right)_1 > 600,$$

$$\left(\frac{dE}{dx}\right)_2 > 550 + 60 \times p_2 + 8.9 \times p_2^2,$$

$$\text{isElectron}_{1/2} = \left(\frac{E_{\text{cal}}/p-1}{0.15}\right)^2 + \left(\frac{dE/dx-690}{150}\right)^2 < 10$$

- For 1 (2) the track with the highest (lowest) momentum.

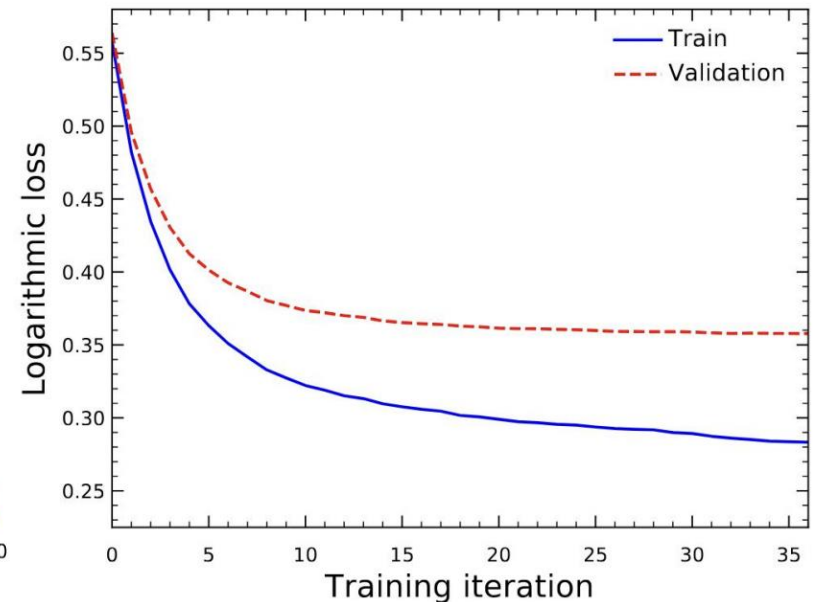
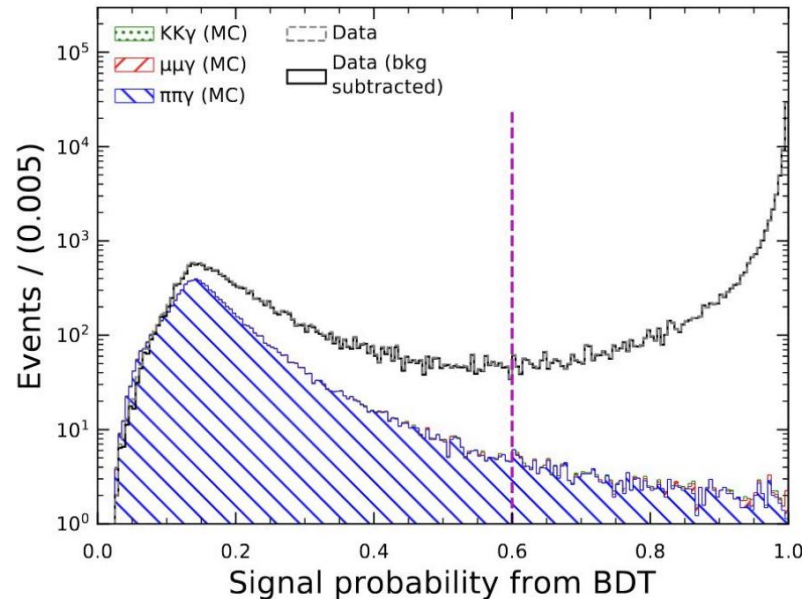
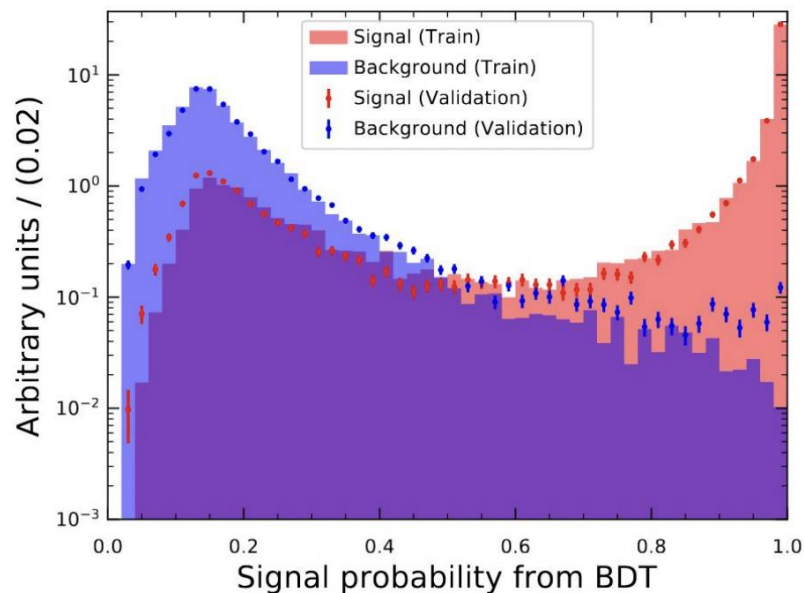


BDT-based selection of $ee\gamma$ events

- Additional selection performed using a BDT implemented with the XGBoost library, trained on the following variables:

$$(E_{\text{Ecal}}/p)_1, (dE/dx)_{1/2}, p_{1/2}, \text{isElectron}_{1/2}, V_{xy}, m_{\pi\pi}, m_{\mu\mu}$$

- Training is carried out on events that pass the cut-based selection, with real data used as signal and Monte Carlo as background.
- BDT hyperparameters (tree depth and learning rate) are optimized to maximize classification accuracy.
- To avoid bias, two independent BDTs are trained on separate halves of the dataset, with each applied to the opposite half.
- The training uses the logarithmic loss function as the evaluation metric, and early stopping is applied to prevent overfitting.
- The output of the classifier is the probability that an event corresponds to signal (i.e., $ee\gamma$ -like).

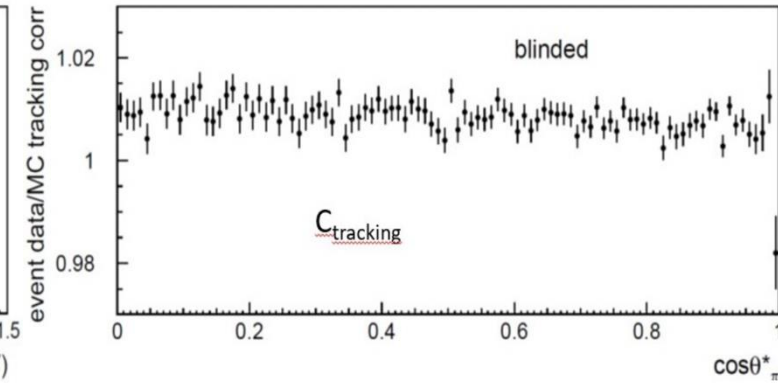
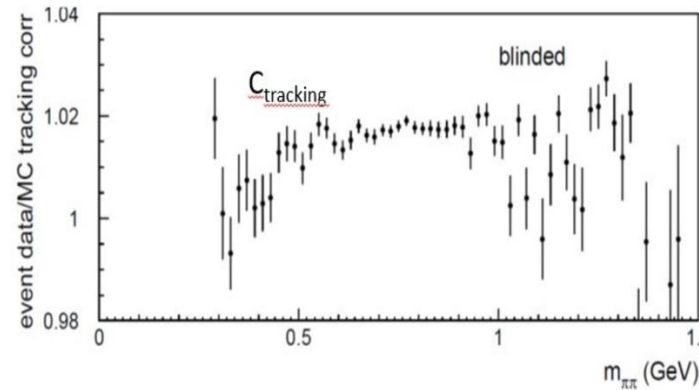
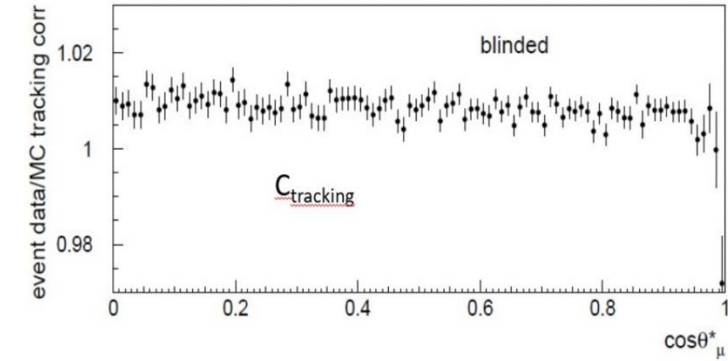
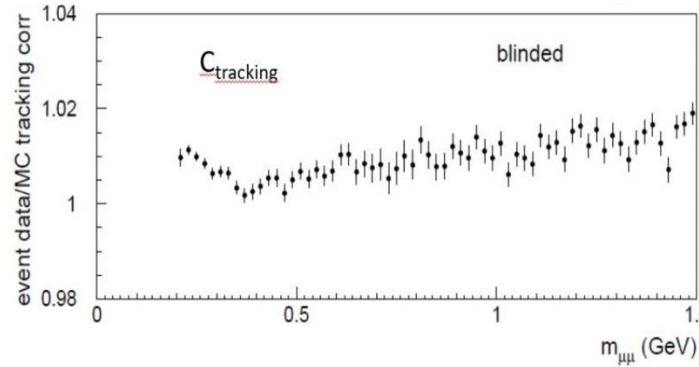


Tracking efficiency corrections on muons

- **2009 formalism:** tag-and-probe method with single track reconstructed + second track predicted from 1-constraint kinematic fit (method 1).
- Poor resolution on predicted parameters of low-momentum tracks and/or at edge of acceptance: alternative, 4-constraint fit (method 2), ignoring predicted probe track parameters.
- Tracking correction:

$$C_{\text{tracking}} = \frac{[(1 - f_0 - f_3)\epsilon^2]_{\text{data}}}{[(1 - f_0 - f_3)\epsilon^2]_{\text{MC}}}$$

- ϵ : average track efficiency
- f_0 : 2-track correlated loss probability
- f_3 : extra reco track loss probability



Angular fit

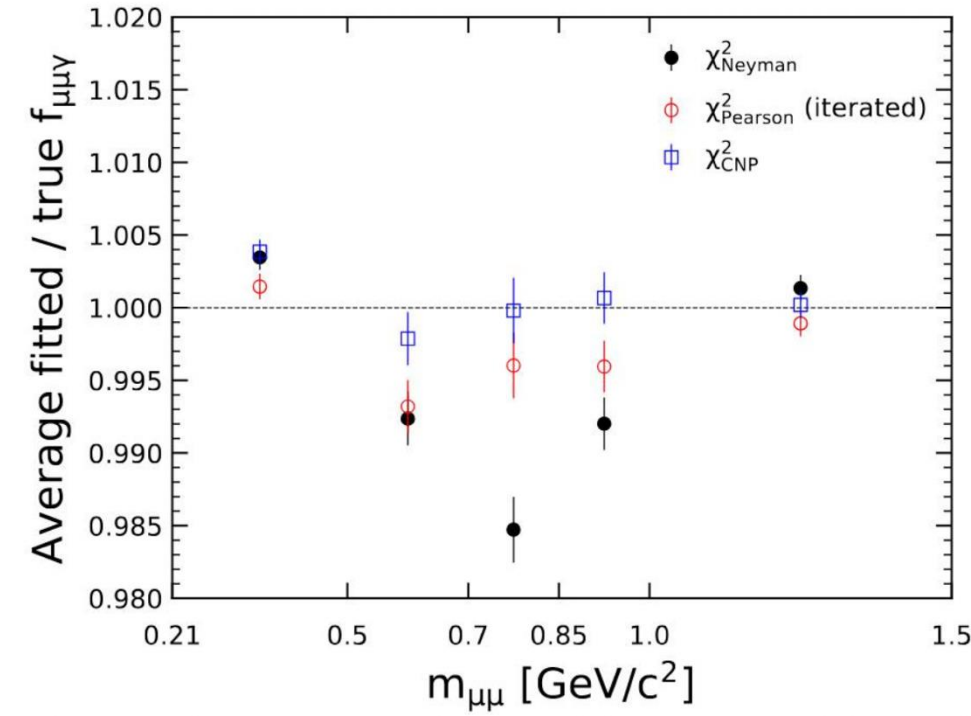
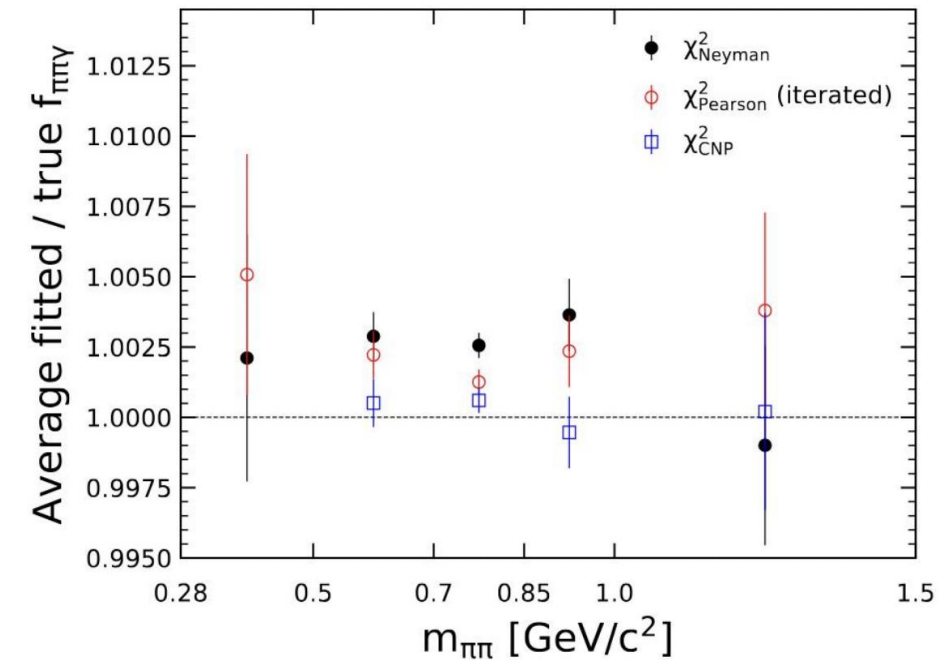
- Neymann and Pearson χ^2 were also considered, CNP was retained.
- Carried out by a Combined Neymann-Pearson χ^2
 - j : $|\cos\theta^*|$ bin.
 - M_j : observed events in a given bin (data).
 - N_j : probability model (predicted events).
 - ΔN_j : Statistical error in N_j .

$$\chi_{\text{CNP}}^2 = \sum_j \frac{(M_j - N_j)^2}{3 / \left(\frac{1}{M_j + (\Delta N_j)^2} + \frac{2}{N_j + (\Delta N_j)^2} \right)}$$

- Where the probability model is a linear combination of templates

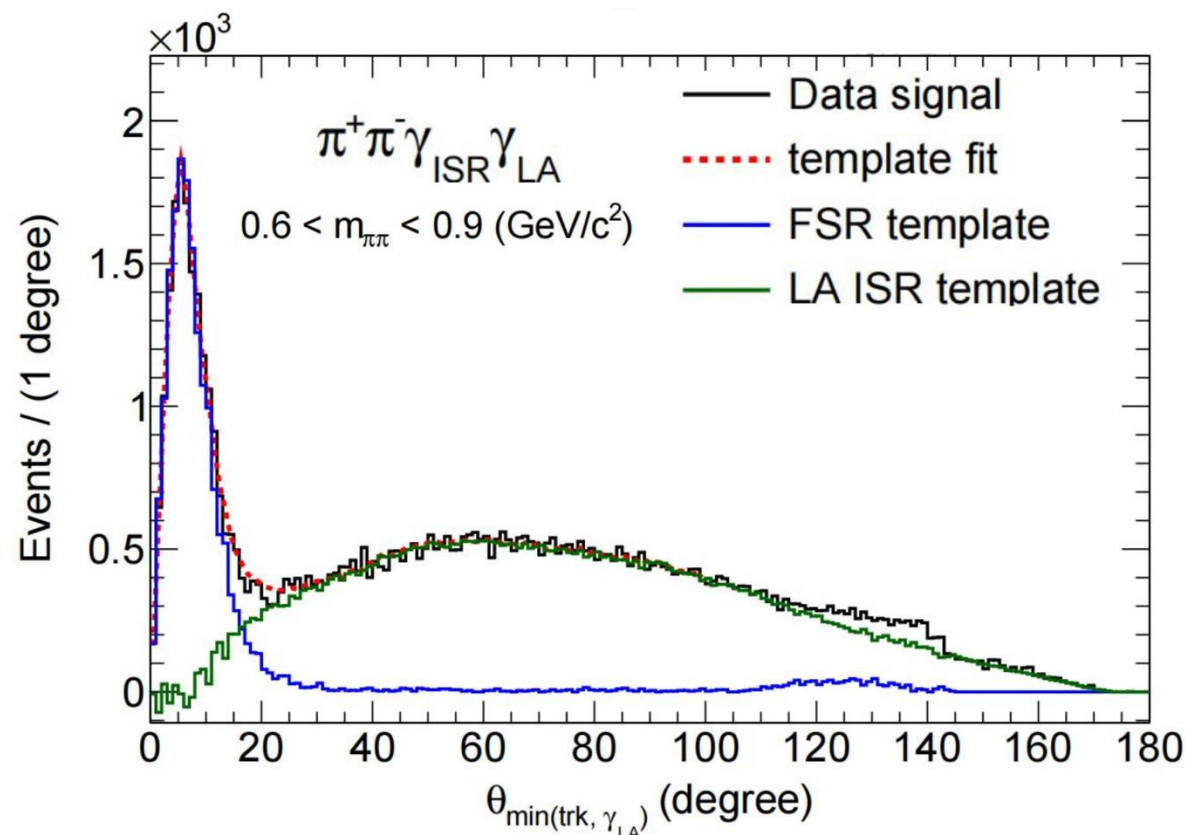
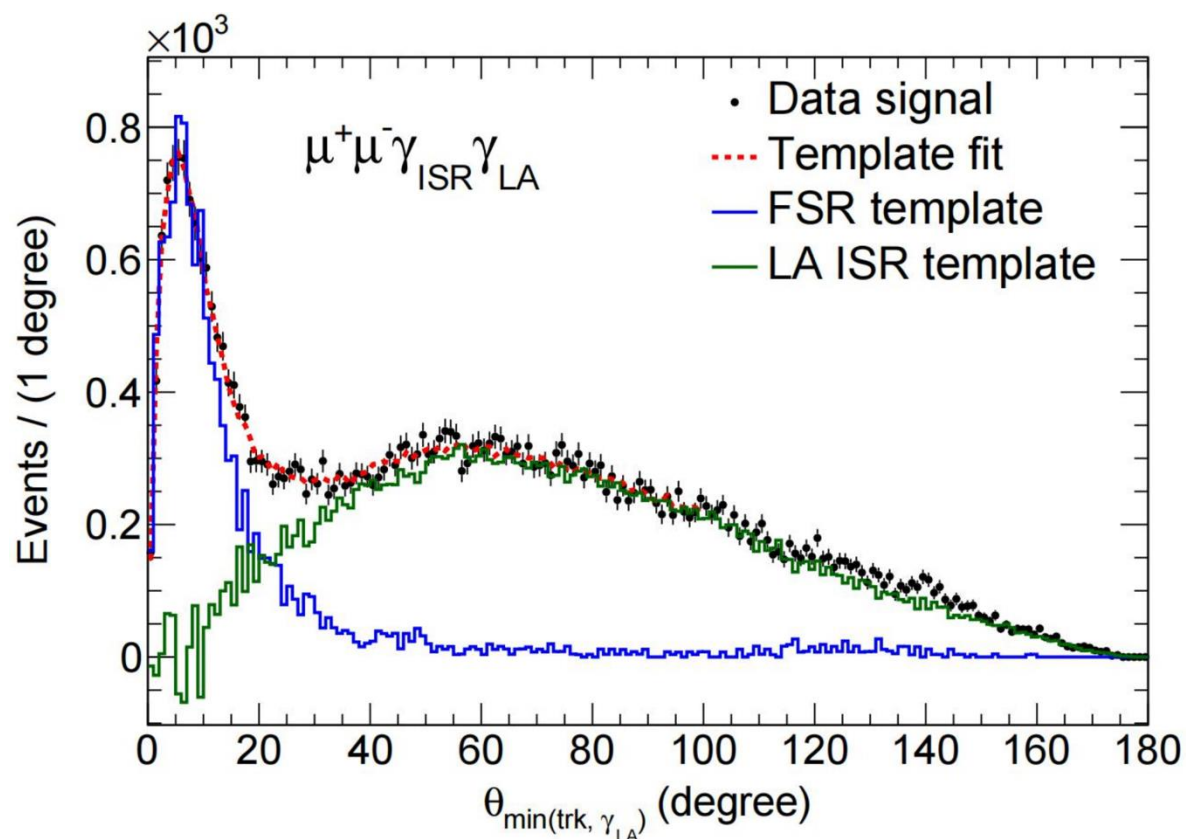
$$N = M \left[\sum_{i=1}^{k-1} f_i x_i + \left(1 - \sum_{i=1}^{k-1} f_i \right) x_k \right] \pm M \sqrt{\sum_{i=1}^{k-1} (f_i \Delta x_i)^2 + \left[\left(1 - \sum_{i=1}^{k-1} f_i \right) \Delta x_k \right]^2}$$

- M : integral over $|\cos\theta^*|$ of the fitted data distribution in a mass bin,
- $f_i \in [0,1]$: parameters of fit, scale factors of templates with $\sum_i f_i = 1$



FSR region

- BaBar has a large acceptance to measure ISR Large Angle (LA) photons.
- We define the “nominal” using $\theta_{\min}^{\text{true}} < 20^\circ$.
- Different angle cuts were considered: $\theta_{\min}^{\text{true}} = [10, 15, 20, 25]$



$m_{\pi\pi}$ vs Unfolding spectrum

