Status of the MUonE theory

F. Piccinini

INFN, Sezione di Pavia and GGI (Italy)

Workshop on "Flavour changing and conserving processes" 2025 (FCCP2025)

Villa Orlandi, Anacapri, 29 September – 1 October 2025

Finanziato dall'Unione europea NextGenerationEU | Ministero dell'Università e della Ricerca | Italiadomani Piano Nazionale di Ripresa e Resilienza | INFN

⋆ G. Abbiendi, C.M. Carloni Calame, U. Marconi, C. Matteuzzi, G. Montagna, O. Nicrosini, M. Passera, F. Piccinini, R. Tenchini, L. Trentadue, G. Venanzoni,
Measuring the leading hadronic contribution to the muon g-2 via μe scattering
Eur. Phys. J. C **77** (2017) no.3, 139 - arXiv:1609.08987 [hep-ph]

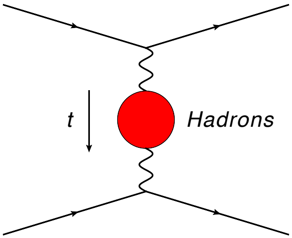
⋆ C. M. Carloni Calame, M. Passera, L. Trentadue and G. Venanzoni,
A new approach to evaluate the leading hadronic corrections to the muon g-2
Phys. Lett. B **746** (2015) 325 - arXiv:1504.02228 [hep-ph]

Master formula

$$a_\mu^{\rm HLO} = \frac{\alpha}{\pi}\int_0^1 dx\,(1-x)\,\Delta\alpha_{\rm had}[t(x)]$$

$$t(x) = \frac{x^2 m_\mu^2}{x-1} < 0$$

e.g. Lautrup, Peterman, De Rafael, Phys. Rept. 3 (1972) 193



⇝ The hadronic VP correction to the running of α enters

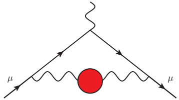
- ⋆ **$\Delta\alpha_{\rm had}(t)$ can be directly measured in a (single) experiment involving a space-like scattering process and $a_\mu^{\rm HLO}$ obtained through numerical integration**

  Carloni Calame, Passera, Trentadue, Venanzoni PLB 746 (2015) 325

- ⋆ **A data-driven evaluation of $a_\mu^{\rm HLO}$, but with space-like data**

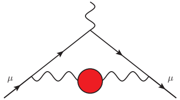
Kernel functions for a_μ^{HVP}

- LO: $\frac{\alpha}{\pi}(1-x)$



Kernel functions for a_μ^{HVP}

- LO: $\frac{\alpha}{\pi}(1-x)$

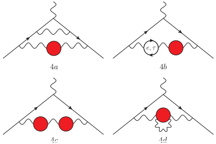


- NLO

E. Balzani, S. Laporta, M. Passera, Phys. Lett. B834 (2022) 137462

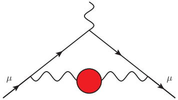
A.V. Nesterenko, J. Phys. G49 (2022) 5, 055001;

J. Phys. G50 (2022) 2, 029401



Kernel functions for a_μ^{HVP}

- LO: $\frac{\alpha}{\pi}(1-x)$

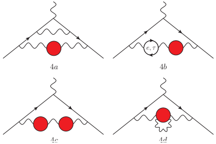


- NLO

  E. Balzani, S. Laporta, M. Passera, Phys. Lett. B834 (2022) 137462

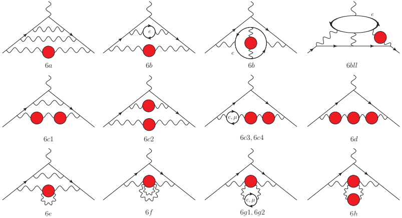
  A.V. Nesterenko, J. Phys. G49 (2022) 5, 055001;

  J. Phys. G50 (2022) 2, 029401



- NNLO

  E. Balzani, S. Laporta, M. Passera, Phys. Lett. B834 (2022) 137462



Main challenge of MUonE: precision on shapes of differential distributions at the 10ppm level

Main challenge of MUonE: precision on shapes of differential distributions at the 10ppm level

Main sources of systematics on the theory side

Main challenge of MUonE: precision on shapes of differential distributions at the 10ppm level

Main sources of systematics on the theory side

- **Radiative corrections to the signal (elastic muon-electron scattering)**

Main challenge of MUonE: precision on shapes of differential distributions at the 10ppm level

Main sources of systematics on the theory side

- **Radiative corrections to the signal (elastic muon-electron scattering)**
- **Predictions for background processes**

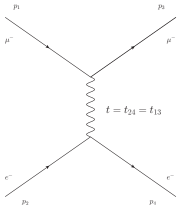
Main challenge of MUonE: precision on shapes of differential distributions at the 10ppm level

Main sources of systematics on the theory side

- **Radiative corrections to the signal (elastic muon-electron scattering)**
- **Predictions for background processes**

High precision Monte Carlo simulation tools required

First step towards precision: QED NLO and MC (2018)

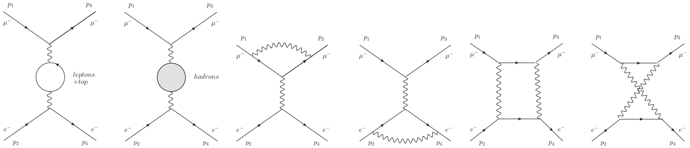


- analytical expression for tree level

$$\frac{d\sigma}{dt} = \frac{4\pi\alpha^2}{\lambda(s, m_\mu^2, m_e^2)} \left[\frac{(s - m_\mu^2 - m_e^2)^2}{t^2} + \frac{s}{t} + \frac{1}{2} \right]$$

- VP gauge invariant subset of NLO rad. corr.
- factorized over tree-level: $\alpha \to \alpha(t)$

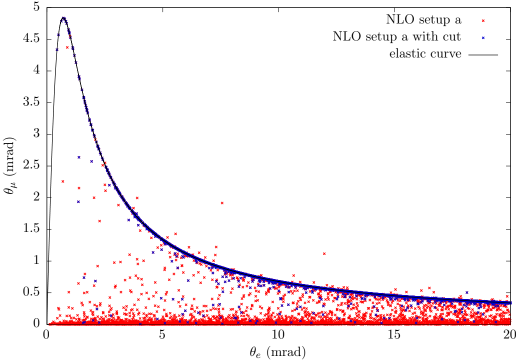
- QED NLO virtual diagrams and real emission diagrams with exact finite m_e and m_μ effects

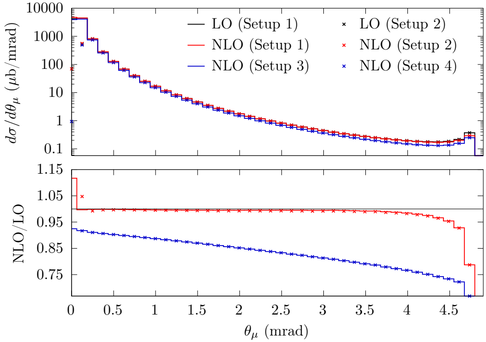


- **tree-level Z-exchange important** at the 10^{-5} level ($\sim tG_\mu / 4\pi\alpha\sqrt{2}$ in the Fermi theory)
- SM weak RCs at most at a few 10^{-6} level, negligible

Alacevich et al. JHEP 02 (2019) 155

First realistic description of scattering events





- many points fall out of the $2 \to 2$ correlation curve $\theta_\mu - \theta_e$ because of the radiative events
- NLO QED radiative corrections at the % level, enhanced by exclusive event selections

Two completely independent fixed order Monte Carlo codes under development

- **Mesmer** Pavia team
  - approximate NNLO calculation at $\mathcal{O}\left(\left[\frac{\alpha}{\pi}\ln\frac{m_\mu^2}{m_e^2}\right]^2\right)$

github.com/cm-cc/mesmer

- **McMule** PSI/Bern/Liverpool...
  - more refined approximation to NNLO: only terms of $\mathcal{O}(m_e^2/Q^2)$ neglected

gitlab.com/mule-tools/mcmule

see backup slides for details and references

On the phenomenological side

- **NNLO corrections at the $10^{-4} - 10^{-3}$ level**

On the phenomenological side

- **NNLO corrections at the $10^{-4} - 10^{-3}$ level**

- **eventually fixed order calculations need to be matched to resummation of higher order corrections, through PS techniques (e.g. BaBayaga) or YFS techniques (e.g. KKMC/SHERPA)**

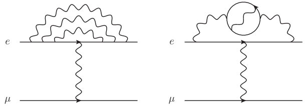
On the phenomenological side

- **NNLO corrections at the $10^{-4} - 10^{-3}$ level**
- **eventually fixed order calculations need to be matched to resummation of higher order corrections, through PS techniques (e.g. BaBayaga) or YFS techniques (e.g. KKMC/SHERPA)**
- **also next perturbative order should be estimated**

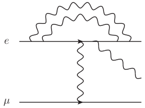
Towards N^3LO on the electron line

Y. Ulrich, N^3LO kick-off workstop/thinkstart, Durham, 3-5 August 2022

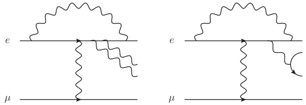
- All virtual (three loops)



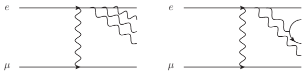
- Single real emission (two loops)



- Double real emission (one loops)



- Triple real



M. Fael, MUonE Collaboration Meeting, 16/05/2023, CERN

- **this contribution will allow improved perturbative predictions and more reliable theoretical uncertainty estimates**

Recent progress

- **the three-loop form factor with finite fermion mass is now available**

M. Fael, F. Lange, K. Schönwald, M. Steinhauser, Phys. Rev. Lett 128 (2022) 172003

M. Fael, F. Lange, K. Schönwald, M. Steinhauser, Phys. Rev.D 106 (2022) 034029

M. Fael, F. Lange, K. Schönwald, M. Steinhauser, Phys. Rev.D 107 (2023) 094017

- **All order subtraction scheme FKS$^\ell$ availale**

T. Engel, A. Signer, Y. Ulrich, JHEP 01 (2020) 085

- **very recent generalisation of the LBK theorem to multi-photon emission** $\Longrightarrow$ extension of next-to-soft stabilisation to multiple radiation

T. Engel, JHEP 03 (2024) 004

- **real-virtual-virtual corrections recently recalculated with** $m_e \to 0$

S. Badger, J. Krys, R. Moodle, S. Zoia, JHEP 11 (2023) 041

V.S. Fadin, R.N. Lee, JHEP 11 (2023) 148; T. Dave, W.J. Torres Bobadilla, arXiv:2411.07063

All the previous discussion applies to the scattering of a μ on a free electron

Fixed target experiment $\Longrightarrow$ bound electron effects

Fixed target experiment $\Longrightarrow$ bound electron effects

- **recent estimates**

R. Plestid and M.B. Wise, arXiv:2403.12184

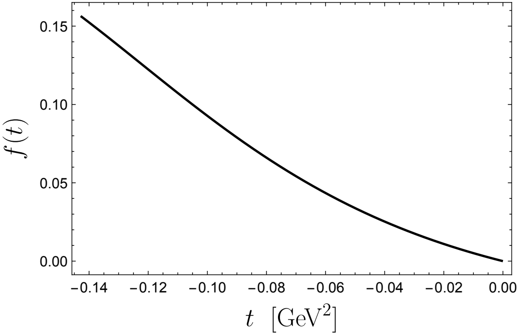
- **recent estimates**

R. Plestid and M.B. Wise, arXiv:2403.12184

- for C

$$\frac{1}{\sigma}\frac{d\sigma}{dt} = \frac{1}{\sigma^0}\frac{d\sigma^0}{dt}\,(1 - K f(t))$$

  - $K = 4.5 \cdot 10^{-4}$, scaling as $1/Z_A$



- interactions between outgoing electrons and the residual charged debris in the final state

R. Plestid and M.B. Wise, arXiv:2407.21752

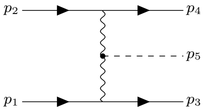
Backgrounds

Backgrounds

- **pion pair production forbidden** kinematically with the available $\sqrt{s}$

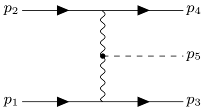
Backgrounds

- **pion pair production forbidden** kinematically with the available $\sqrt{s}$
- **single π^0 production possible**



Backgrounds

- **pion pair production forbidden** kinematically with the available $\sqrt{s}$
- **single π^0 production possible**

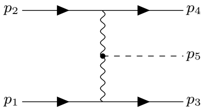


- **π^0 production** calculated and shown to be **well below 10^{-5}** w.r.t. $\mu e \to \mu e$

E. Budassi et al., PLB 829 (2022) 137138

Backgrounds

- **pion pair production forbidden** kinematically with the available $\sqrt{s}$
- **single π^0 production possible**



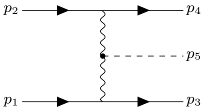
- **π^0 production** calculated and shown to be **well below 10^{-5}** w.r.t. $\mu e \to \mu e$

E. Budassi et al., PLB 829 (2022) 137138

- **lepton pair production**

Backgrounds

- **pion pair production forbidden** kinematically with the available $\sqrt{s}$
- **single π^0 production possible**



- **π^0 production** calculated and shown to be **well below 10^{-5}** w.r.t. $\mu e \to \mu e$

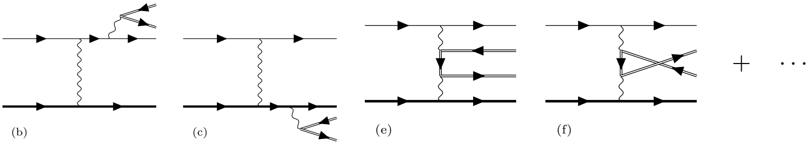
E. Budassi et al., PLB 829 (2022) 137138

- **lepton pair production**
  - $\mu^\pm e^- \to \mu^\pm e^- \ell^+ \ell^-$
  - $\mu^\pm N \to \mu^\pm N \ell^+ \ell^-$

$\mu^\pm e^- \to \mu^\pm e^- \ell^+ \ell^-$ (same pert. order as virtual pairs)

E. Budassi et al., JHEP 11 (2021) 098

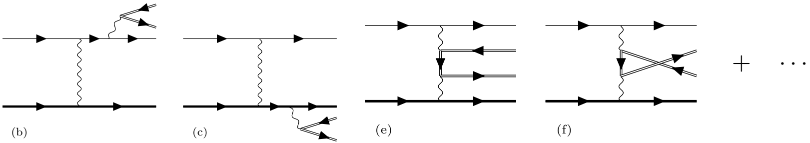
- it also contributes at NNLO accuracy w.r.t. $\mu e \to \mu e$



$\mu^\pm e^- \to \mu^\pm e^- \ell^+ \ell^-$ (same pert. order as virtual pairs)

E. Budassi et al., JHEP 11 (2021) 098

- it also contributes at NNLO accuracy w.r.t. $\mu e \to \mu e$

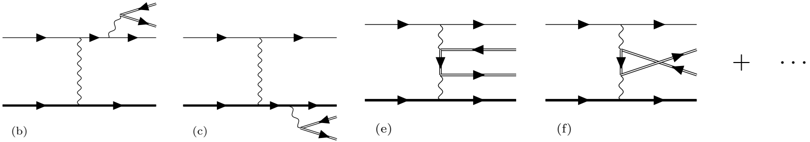


- the emission of an extra electron pair $\mu e \to \mu e\, e^+ e^-$ is potentially a dramatically large background, because of the presence of "peripheral" diagrams which develop powers of collinear logarithms upon integration

G. Racah, Il Nuovo Cimento 14 (1937) 83-113; L.D. Landau, E.M. Lifschitz, Phys. Z. Sowjetunion 6 (1934) 244; H.J. Bhabha, Proc. Roy. Soc. Lond. A152 (1935) 559;

R.N. Lee, A.A. Lyubyakin, V.A. Smirnov, Phys. Lett. B 848 (2024) 138408

- it also contributes at NNLO accuracy w.r.t. $\mu e \to \mu e$



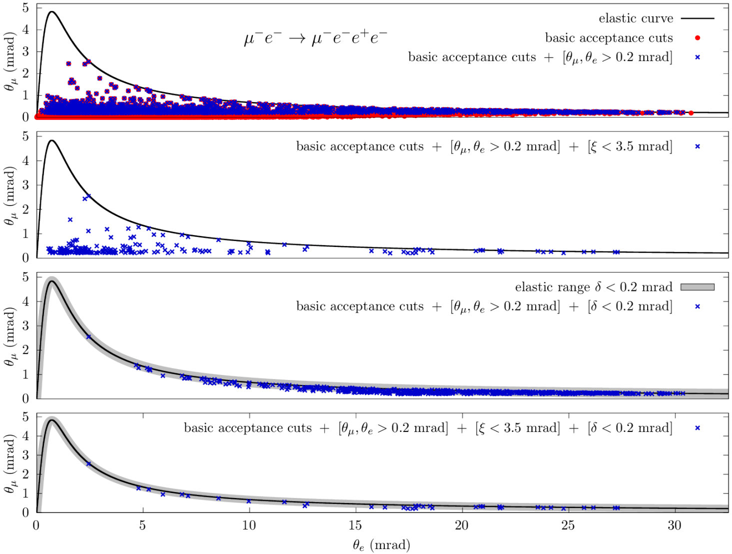
- the emission of an extra electron pair $\mu e \to \mu e\, e^+ e^-$ is potentially a dramatically large background, because of the presence of "peripheral" diagrams which develop powers of collinear logarithms upon integration

G. Racah, Il Nuovo Cimento 14 (1937) 83-113; L.D. Landau, E.M. Lifschitz, Phys. Z. Sowjetunion 6 (1934) 244; H.J. Bhabha, Proc. Roy. Soc. Lond. A152 (1935) 559;

R.N. Lee, A.A. Lyubyakin, V.A. Smirnov, Phys. Lett. B 848 (2024) 138408

- **$\mu^\pm e^- \to \mu^\pm e^- \ell^+ \ell^-$ calculated with finite mass effects and implemented in `Mesmer`**

simulation of $5 \cdot 10^5$ points of $\mu^\pm e^- \to \mu^\pm e^- \ell^+ \ell^-$



Real pair emission from scattering on nucleus: $\mu^{\pm} N \to \mu^{\pm} N \ell^{+} \ell^{-}$

G. Abbiendi et al., Phys. Lett B854 (2024) 138720

- **it can mimic the signal if one particle is not reconstructed or two tracks overlap within resolution**

Real pair emission from scattering on nucleus: $\mu^\pm N \to \mu^\pm N \ell^+ \ell^-$

G. Abbiendi et al., Phys. Lett B854 (2024) 138720

- **it can mimic the signal if one particle is not reconstructed or two tracks overlap within resolution**
- **cross section scaling $\sim Z^2$**

Real pair emission from scattering on nucleus: $\mu^{\pm} N \to \mu^{\pm} N \ell^{+} \ell^{-}$

G. Abbiendi et al., Phys. Lett B854 (2024) 138720

- **it can mimic the signal if one particle is not reconstructed or two tracks overlap within resolution**
- **cross section scaling $\sim Z^2$**

- **GEANT4: "for the process of e^+e^- pair production the muon deflection is neglected"**

A.G. Bogdanov et al., IEEE transactions on nuclear science, 53, n. 2, April 2006

Real pair emission from scattering on nucleus: $\mu^\pm N \to \mu^\pm N \ell^+ \ell^-$

G. Abbiendi et al., Phys. Lett B854 (2024) 138720

- **it can mimic the signal if one particle is not reconstructed or two tracks overlap within resolution**
- **cross section scaling $\sim Z^2$**

- **GEANT4: “for the process of e^+e^- pair production the muon deflection is neglected”**

A.G. Bogdanov et al., IEEE transactions on nuclear science, 53, n. 2, April 2006

$\Longrightarrow$ **a dedicated calculation implemented in the Monte Carlo generator `Mesmer`**

- **approximation: scattering on the external nucleus field**

- **approximation: scattering on the external nucleus field**
- **finite extension of the nucleus through a form factor**

$$F_Z(q) = \frac{1}{Ze} \int_0^\infty dr\, r^2 \rho_Z(r) \frac{\sin(qr)}{qr}$$

  - q : momentum transferred to the nucleus
  - ρ_Z : nuclear charged density

- **approximation: scattering on the external nucleus field**
- **finite extension of the nucleus through a form factor**

$$F_Z(q) = \frac{1}{Ze}\int_0^\infty dr\, r^2 \rho_Z(r) \frac{\sin(qr)}{qr}$$

  - q : momentum transferred to the nucleus
  - ρ_Z : nuclear charged density

- **different models for charge density**

J. Heeck, R. Szafron, Y. Uesaka, PRD 105 (2022) 053006

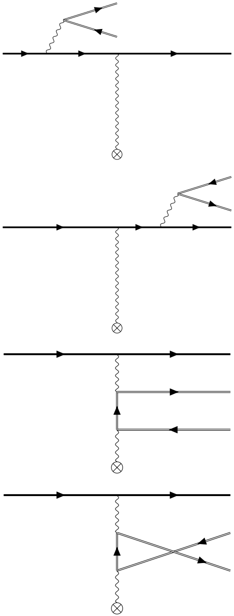
  - $F_Z(q) = 1$ (conservative)
  - 1 parameter Fermi model (1pF)

$$\rho_Z(r) = \frac{\rho_0}{1 + \exp\frac{r-c}{z}}$$

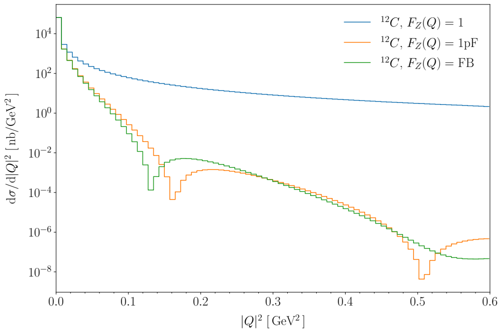
  - Fourier Bessel expansion (FB)

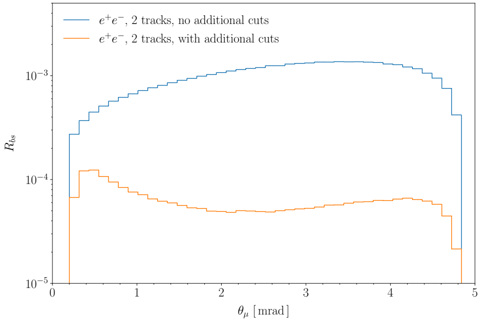
$$\begin{aligned} \rho_Z(r) \quad &= \quad \sum_k^n a_k\, j_0\left(\frac{k\pi r}{R}\right), \quad r \geq R \\ &= \quad 0 \qquad\qquad\qquad\qquad > R \end{aligned}$$

  - modified-harmonic oscillator model



Background/signal ratio





G. Abbiendi, E. Budassi, C.M. Carloni Calame, A. Gurgone, F.P., Phys.Lett.B 854 (2024) 138720

For a safe background subtraction

For a safe background subtraction

- **QED corrections to lepton pair production will be important**

For a safe background subtraction

- **QED corrections to lepton pair production will be important**

- **Interaction with an additional nucleus is of the same order, enhanced by Z**

Possible New Physics contamination in the $\Delta\alpha(t)$ determination?

A. Masiero, P. Paradisi and M. Passera, Phys. Rev. D102 (2020) 075013

P.S.B. Dev, W. Rodejohann, X.-J. Xu and Y. Zhang, JHEP 05 (2020) 053

Possible New Physics contamination in the $\Delta\alpha(t)$ determination?

A. Masiero, P. Paradisi and M. Passera, Phys. Rev. D102 (2020) 075013

P.S.B. Dev, W. Rodejohann, X.-J. Xu and Y. Zhang, JHEP 05 (2020) 053

- Effects of **heavy** $(M_{NP} \gg 1$ GeV$)$ NP mediators investigated through EFT with dim-6 operators

Possible New Physics contamination in the $\Delta\alpha(t)$ determination?

A. Masiero, P. Paradisi and M. Passera, Phys. Rev. D102 (2020) 075013

P.S.B. Dev, W. Rodejohann, X.-J. Xu and Y. Zhang, JHEP 05 (2020) 053

- Effects of **heavy** $(M_{NP} \gg 1 \text{ GeV})$ NP mediators investigated through EFT with dim-6 operators
  - excluded (at the 10^{-5} level) by existing data

Possible New Physics contamination in the $\Delta\alpha(t)$ determination?

A. Masiero, P. Paradisi and M. Passera, Phys. Rev. D102 (2020) 075013

P.S.B. Dev, W. Rodejohann, X.-J. Xu and Y. Zhang, JHEP 05 (2020) 053

- Effects of **heavy** $(M_{NP} \gg 1$ GeV$)$ NP mediators investigated through EFT with dim-6 operators
  - excluded (at the 10^{-5} level) by existing data
- Effects of **light** $(M_{NP} \leq 1$ GeV$)$ NP mediators investigated with spin-dependent general models

Possible New Physics contamination in the $\Delta\alpha(t)$ determination?

A. Masiero, P. Paradisi and M. Passera, Phys. Rev. D102 (2020) 075013

P.S.B. Dev, W. Rodejohann, X.-J. Xu and Y. Zhang, JHEP 05 (2020) 053

- Effects of **heavy** $(M_{NP} \gg 1$ GeV$)$ NP mediators investigated through EFT with dim-6 operators
  - excluded (at the 10^{-5} level) by existing data
- Effects of **light** $(M_{NP} \leq 1$ GeV$)$ NP mediators investigated with spin-dependent general models
  - spin-0 NP mediators (ALPs)
  - spin-1 NP mediators (Dark Photons, light Z' vector bosons)

Possible New Physics contamination in the $\Delta\alpha(t)$ determination?

A. Masiero, P. Paradisi and M. Passera, Phys. Rev. D102 (2020) 075013

P.S.B. Dev, W. Rodejohann, X.-J. Xu and Y. Zhang, JHEP 05 (2020) 053

- Effects of **heavy** $(M_{NP} \gg 1$ GeV$)$ NP mediators investigated through EFT with dim-6 operators
  - excluded (at the 10^{-5} level) by existing data
- Effects of **light** $(M_{NP} \leq 1$ GeV$)$ NP mediators investigated with spin-dependent general models
  - spin-0 NP mediators (ALPs)
  - spin-1 NP mediators (Dark Photons, light Z' vector bosons)
  - excluded (at the 10^{-5} level) by existing data

Possible New Physics contamination in the $\Delta\alpha(t)$ determination?

A. Masiero, P. Paradisi and M. Passera, Phys. Rev. D102 (2020) 075013

P.S.B. Dev, W. Rodejohann, X.-J. Xu and Y. Zhang, JHEP 05 (2020) 053

- Effects of **heavy** $(M_{NP} \gg 1$ GeV$)$ NP mediators investigated through EFT with dim-6 operators
  - excluded (at the 10^{-5} level) by existing data
- Effects of **light** $(M_{NP} \leq 1$ GeV$)$ NP mediators investigated with spin-dependent general models
  - spin-0 NP mediators (ALPs)
  - spin-1 NP mediators (Dark Photons, light Z' vector bosons)
  - excluded (at the 10^{-5} level) by existing data

HVP determination with MUonE data will be robust against New Physics

Possible New Physics studies with MUonE (in complementary regions to $\Delta\alpha_h$)

- **interesting proposals for NP searches at MUonE (new light mediators) in $2 \to 3$ processes**
  - invisibly decaying light Z' in $\mu e \to \mu e Z'$

    Asai et al., Phys. Rev. D106 (2022) 5
    - a relevant background can be $\mu e \to \mu e \pi^0$, in addition to $\mu e \to \mu e \gamma$
  - long-lived mediators with displaced vertex signatures $\mu e \to \mu e A' \to \mu e e^+ e^-$

    Galon et al., Phys.Rev.D 107 (2023) 095003
  - through scattering off the target nuclei $\mu N \to \mu N X \to \mu N e^+ e^-$

    Grilli di Cortona and E. Nardi, Phys. Rev. D105 (2022) L111701

Summary

- **Given its precision requirements, MUonE represents a challenge (on the theory side) for**
  - **QED corrections**
  - **background calculation**

- **at present we have two independent Monte Carlo tools, `Mesmer` and `McMule` featuring**
  - NLO QED corrections
  - NNLO QED corections from single lepton legs
  - YFS inspired approximation to the full NNLO QED in **`Mesmer`**
  - full NNLO QED with electron "massification" in **`McMule`**
  - pair production in **`Mesmer`**
    - $\mu^{\pm}e^{-} \to \mu^{\pm}e^{-}\ell^{+}\ell^{-}$
    - $\mu^{\pm}N \to \mu^{\pm}N\ell^{+}\ell^{-}$

- **efforts for N^{3}LO started**
- **for the near future**
  - more refined studies of background calculations
  - matching with higher order QED corrections for signal

A collection of references on dedicated calculation developments

- Carloni Calame et al., PLB 746 (2015), 325
- Abbiendi et al., EPJ C77 (2017), 139
- Mastrolia et al., JHEP 11 (2017) 198
- Di Vita et al., JHEP 09 (2018) 016
- Alacevich et al., JHEP 02 (2019) 155
- Fael and Passera, PRL 122 (2019) 19, 192001
- Fael, JHEP 02 (2019) 027
- Engel et al., JHEP 02 (2019) 118
- Engel et al., JHEP 01 (2020) 085
- Carloni Calame et al., JHEP 11 (2020) 028
- Banerjee et al., SciPost Phys. 9 (2020), 027
- Banerjee et al., EPJC 80 (2020) 6, 591
- Budassi et al., JHEP 11 (2021) 098
- Balzani et al., PLB 834 (2022) 137462
- Bonciani et al., PRL 128 (2022) 2, 022002
- Budassi et al., PLB 829 (2022) 137138
- Engel et al., JHEP 04 (2022) 097
- Fael et al., PRL 128 (2022) 172003
- Fael et al., PRD 106 (2022) 034029
- Broggio et al., JHEP 01 (2023) 112
- Fael et al., PRD 107 (2023) 094017
- Engel, JHEP 07 (2023) 177
- Badger et al., JHEP 11 (2023) 041
- Fadin and Lee., JHEP 11 (2023) 148
- Ahmed et al., JHEP 01 (2024) 010
- Engel, JHEP 03 (2024) 004
- Abbiendi et al., PLB 854 (2024) 138720
- Plestid and Wise, arXiv:2403.12184;arXiv:2407.21752

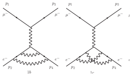
THANK YOU!

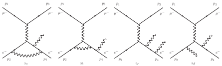
BACKUP

Details about radiative corrections for signal

First approximate *photonic* radiative corrections at **NNLO**

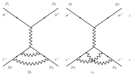
- **exact calculation of corrections along one lepton line with all finite mass effects**

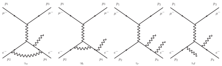




First approximate *photonic* radiative corrections at **NNLO**

- **exact calculation of corrections along one lepton line with all finite mass effects**





  - two independent calculations, with different IR singularities handling procedures (slicing and subtraction)

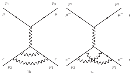
Carloni Calame et al., JHEP 11 (2020) 028,

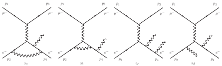
P. Banerjee, T. Engel, A. Signer, Y. Ulrich, SciPost Phys. 9 (2020) 027

  - implemented in **`Mesmer`** and **`McMule`**, perfect numerical agreement

First approximate *photonic* radiative corrections at **NNLO**

- **exact calculation of corrections along one lepton line with all finite mass effects**





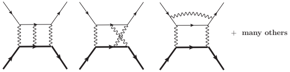
  - two independent calculations, with different IR singularities handling procedures (slicing and subtraction)

Carloni Calame et al., JHEP 11 (2020) 028,

P. Banerjee, T. Engel, A. Signer, Y. Ulrich, SciPost Phys. 9 (2020) 027

  - implemented in **`Mesmer`** and **`McMule`**, perfect numerical agreement

- **NNLO with finite mass effects and approximate up-down interference in `Mesmer`**
  - interference of LO $\mu e \to \mu e$ amplitude with



⇝ NNLO double-virtual amplitudes where at least 2 photons connect the e and μ lines are approximated according to the Yennie-Frautschi-Suura ('61) formalism to catch the IR divergent structure

Second step, *photonic* radiative corrections at **NNLO** (2023)

- **complete calculation of the amplitude $f^+f^- \to F^+F^-$ with $m_f = 0$, $m_F \neq 0$** R. Bonciani *et al.*, PRL 128 (2022)

Second step, *photonic* radiative corrections at **NNLO** (2023)

- **complete calculation of the amplitude $f^+f^- \to F^+F^-$ with $m_f = 0$, $m_F \neq 0$** R. Bonciani *et al.*, PRL 128 (2022)
- **"massification" to recover the leading m_e terms**, i.e. neglecting powers of m_e^2/Q^2

T. Engel, C. Gnendiger, A. Signer and Y. Ulrich, JHEP 02 (2019) 118

Y. Ulrich, *PoS* RADCOR2023 (2024) 077

Second step, *photonic* radiative corrections at **NNLO** (2023)

- **complete calculation of the amplitude $f^+f^- \to F^+F^-$ with $m_f = 0$, $m_F \neq 0$** R. Bonciani *et al.*, PRL 128 (2022)
- **"massification" to recover the leading m_e terms**, i.e. neglecting powers of m_e^2/Q^2

T. Engel, C. Gnendiger, A. Signer and Y. Ulrich, JHEP 02 (2019) 118

Y. Ulrich, *PoS* RADCOR2023 (2024) 077

- **FKS$^\ell$ subtraction scheme**

T. Engel, A. Signer, Y. Ulrich, JHEP 01 (2020) 085

Second step, *photonic* radiative corrections at **NNLO** (2023)

- **complete calculation of the amplitude $f^+f^- \to F^+F^-$ with $m_f = 0$, $m_F \neq 0$** R. Bonciani *et al.*, PRL 128 (2022)
- **"massification" to recover the leading m_e terms**, i.e. neglecting powers of m_e^2/Q^2

T. Engel, C. Gnendiger, A. Signer and Y. Ulrich, JHEP 02 (2019) 118

Y. Ulrich, *PoS* RADCOR2023 (2024) 077

- **FKS$^\ell$ subtraction scheme**

T. Engel, A. Signer, Y. Ulrich, JHEP 01 (2020) 085

- **Next-to-soft stabilisation**, to obtain numerical stability in real-virtual corrections with soft and/or collinear photon configurations

T. Engel, A. Signer, Y. Ulrich, JHEP 04 (2022) 097; T. Engel, JHEP 07 (2023) 177

Second step, *photonic* radiative corrections at **NNLO** (2023)

- **complete calculation of the amplitude $f^+ f^- \to F^+ F^-$ with $m_f = 0$, $m_F \neq 0$** R. Bonciani *et al.*, PRL 128 (2022)
- **"massification" to recover the leading m_e terms**, i.e. neglecting powers of m_e^2/Q^2

  T. Engel, C. Gnendiger, A. Signer and Y. Ulrich, JHEP 02 (2019) 118

  Y. Ulrich, *PoS* RADCOR2023 (2024) 077

- **FKS$^\ell$ subtraction scheme**

  T. Engel, A. Signer, Y. Ulrich, JHEP 01 (2020) 085

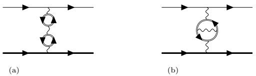
- **Next-to-soft stabilisation**, to obtain numerical stability in real-virtual corrections with soft and/or collinear photon configurations

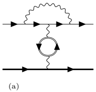
  T. Engel, A. Signer, Y. Ulrich, JHEP 04 (2022) 097; T. Engel, JHEP 07 (2023) 177

- with the above ingredients
  - **NNLO calculation neglecting terms of $\mathcal{O}(m_e^2/Q^2)$ in `McMule`**

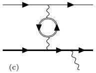
  A. Broggio et al., JHEP 01 (2023) 112

- any lepton (and hadron) in the VP blobs
- interfered with $\mu e \to \mu e$ or $\mu e \to \mu e \gamma$ amplitudes



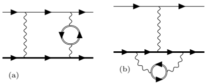


$+ \cdots$



$+ \cdots$

- interfered with $\mu e \to \mu e$ amplitude



$+ \cdots$

- 2-loop integral evaluated with dispersion relation techniques in **`Mesmer`**

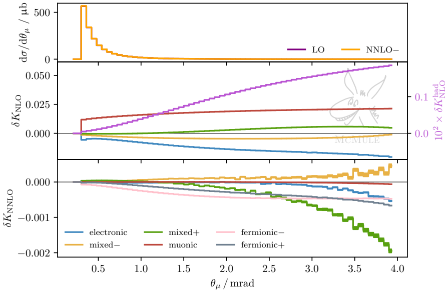
used e.g. in the past for Bhabha: Actis et al., Phys. Rev. Lett. 100 (2008) 131602; Carloni Calame et al., JHEP 07 (2011) 126

$$\frac{g_{\mu\nu}}{q^2 + i\epsilon} \to g_{\mu\nu} \frac{\alpha}{3\pi} \int_{4m_\ell^2}^{\infty} \frac{dz}{z} \frac{R_\ell(z)}{q^2 - z + i\epsilon} = g_{\mu\nu} \frac{\alpha}{3\pi} \int_{4m_\ell^2}^{\infty} \frac{dz}{z} \frac{1}{q^2 - z + i\epsilon} \left(1 + \frac{4m_\ell^2}{2z}\right) \sqrt{1 - \frac{4m_\ell^2}{z}}$$

- 2-loop integral evaluated (also) with hyperspherical method in **`McMule`** — M. Fael, JHEP02 (2019) 027

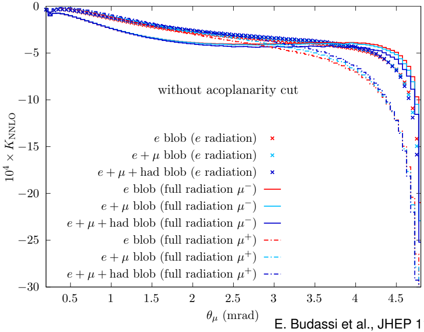
NNLO order of magnitude

McMule



A. Broggio et al., JHEP 01 (2023) 112

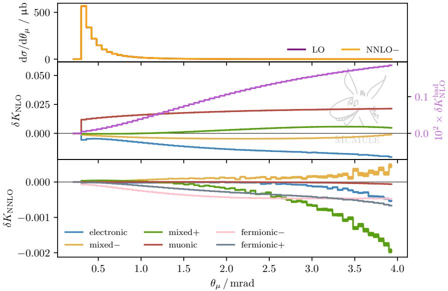
Mesmer



E. Budassi et al., JHEP 11 (2021) 098

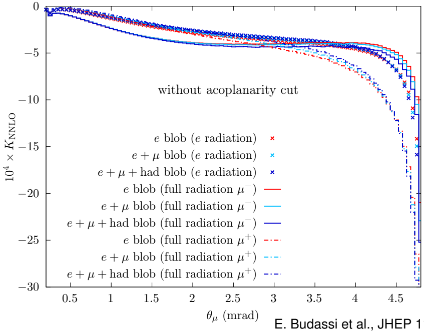
NNLO order of magnitude

McMule



A. Broggio et al., JHEP 01 (2023) 112

Mesmer

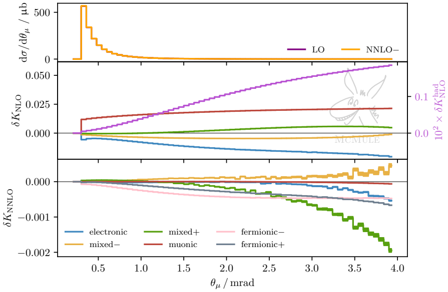


E. Budassi et al., JHEP 11 (2021) 098

- **NNLO corrections at the $10^{-4} - 10^{-3}$ level**

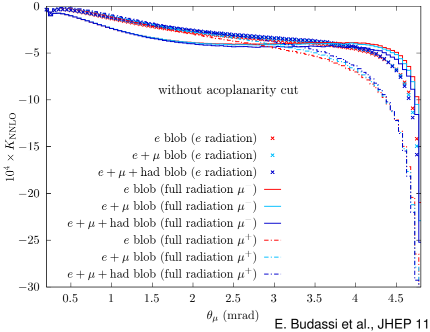
NNLO order of magnitude

McMule



A. Broggio et al., JHEP 01 (2023) 112

Mesmer

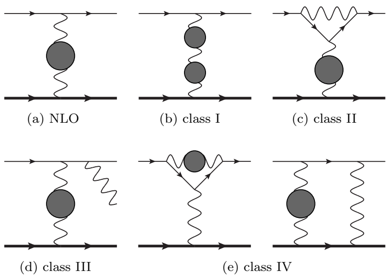


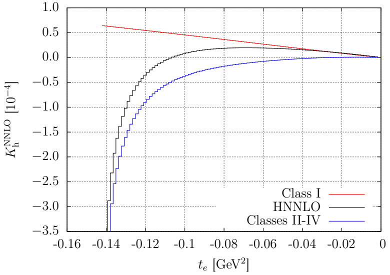
E. Budassi et al., JHEP 11 (2021) 098

- **NNLO corrections at the $10^{-4} - 10^{-3}$ level**
- **eventually fixed order calculations need to be matched to resummation of higher order corrections, through PS techniques (e.g. BaBayaga) or YFS techniques (e.g. KKMC/SHERPA)**

NNLO hadronic contributions

- using the dispersion relation approach





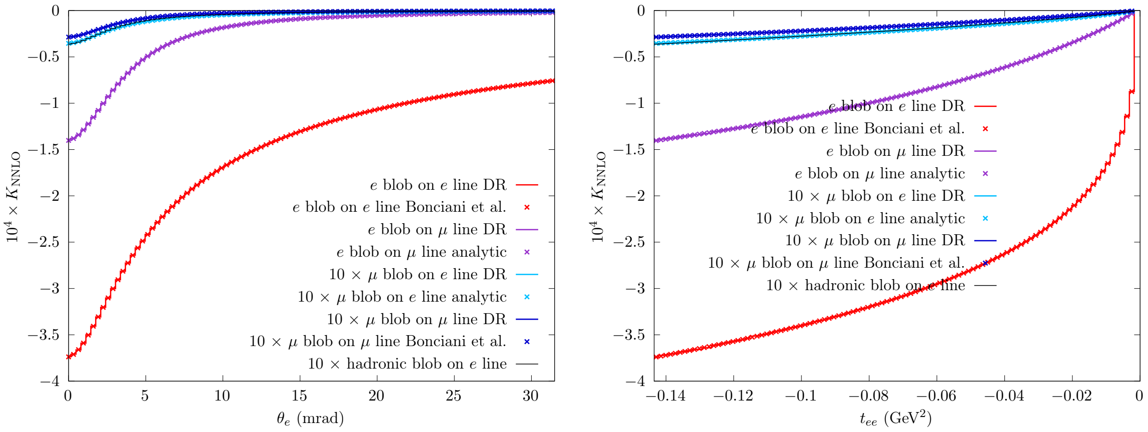
Fael, Passera, Phys. Rev. Lett. 122 (2019) 192001

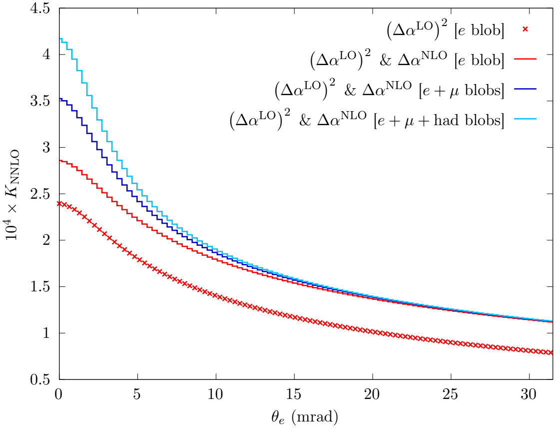
- corrections of the order of 10^{-4}
- hyperspherical integration method to calculate hadronic NNLO corrections, where the hadronic vacuum polarization is employed in the space-like region (used in **`McMule`**)

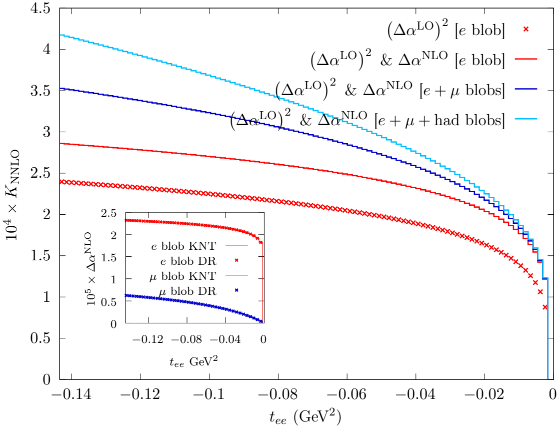
M. Fael, JHEP02 (2019) 027

Virtual leptonic (and hadronic NNLO) vertex corrections

(Budassi et al., 2021)




$(\Delta\alpha^{\rm LO})^2$ [e blob]
$(\Delta\alpha^{\rm LO})^2$ & $\Delta\alpha^{\rm NLO}$ [e blob]
$(\Delta\alpha^{\rm LO})^2$ & $\Delta\alpha^{\rm NLO}$ [$e+\mu$ blobs]
$(\Delta\alpha^{\rm LO})^2$ & $\Delta\alpha^{\rm NLO}$ [$e+\mu+$had blobs]
$10^4 \times K_{\rm NNLO}$
θ_e (mrad)


$(\Delta\alpha^{\rm LO})^2$ [e blob]
$(\Delta\alpha^{\rm LO})^2$ & $\Delta\alpha^{\rm NLO}$ [e blob]
$(\Delta\alpha^{\rm LO})^2$ & $\Delta\alpha^{\rm NLO}$ [$e+\mu$ blobs]
$(\Delta\alpha^{\rm LO})^2$ & $\Delta\alpha^{\rm NLO}$ [$e+\mu+$had blobs]
$10^4 \times K_{\rm NNLO}$
t_{ee} (GeV^2)
e blob KNT
e blob DR
μ blob KNT
μ blob DR
$10^5 \times \Delta\alpha^{\rm NLO}$
t_{ee} GeV^2

E. Budassi et al., JHEP 11 (2021) 098