

Status of the MUonE experiment

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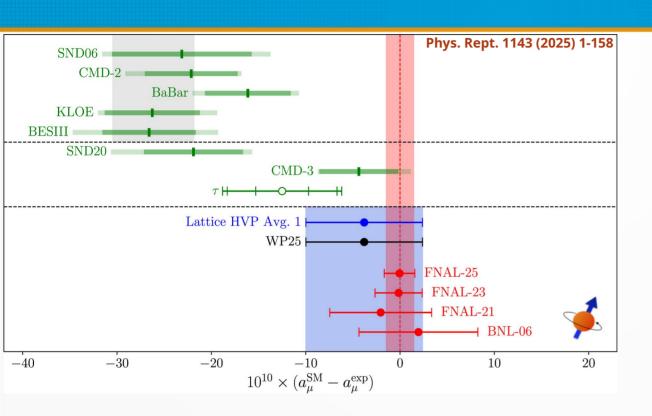


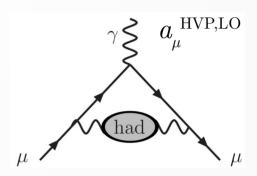
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Workshop on Flavour changing and conserving processes Anacapri, 30th September 2025

Muon g-2: current status







Main source of uncertainty of the theoretical prediction

Tensions in the evaluation of $a_{\mu}^{\rm HVP,LO}$ using lattice QCD (WP2025) or e^+e^- hadronic cross sections

A clarification of the theoretical prediction is needed

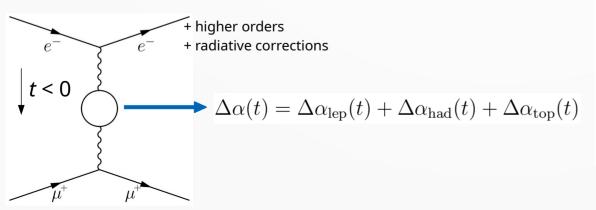


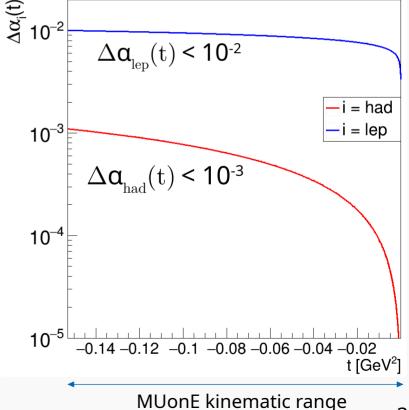
New independent evaluation of $a_{\mu}^{\text{HVP,LO}}$, based on the measurement of $\Delta a_{\text{had}}(t)$ in the space-like region

Phys. Lett. B 746 (2015), 325 Eur. Phys. J. C 77.3 (2017), 139 Letter of Intent CERN-SPSC-2019-026 Proposal for Phase 1 of the MUonE experiment

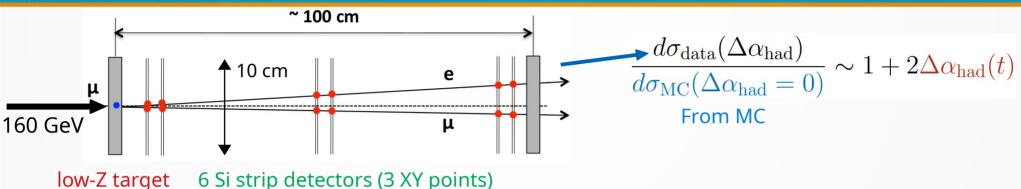
$$a_{\mu}^{HLO} = \frac{\alpha_0}{\pi} \int_0^1 \! dx (1-x) \Delta \alpha_{had}[t(x)]^{-t(x)} = \frac{x^2 m_{\mu}^2}{x-1} < 0 \qquad \stackrel{\text{\tiny $\frac{2}{5}$}}{\text{\tiny $\frac{2}{5}$}} \mathbf{10}^{-2}$$

Extract $\Delta \alpha_{had}(t)$ from the *shape* of $\mu e \rightarrow \mu e$ differential cross section





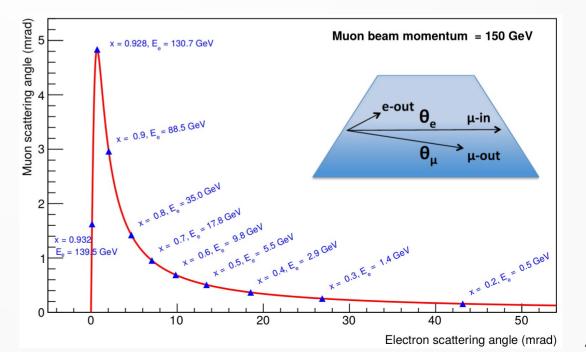




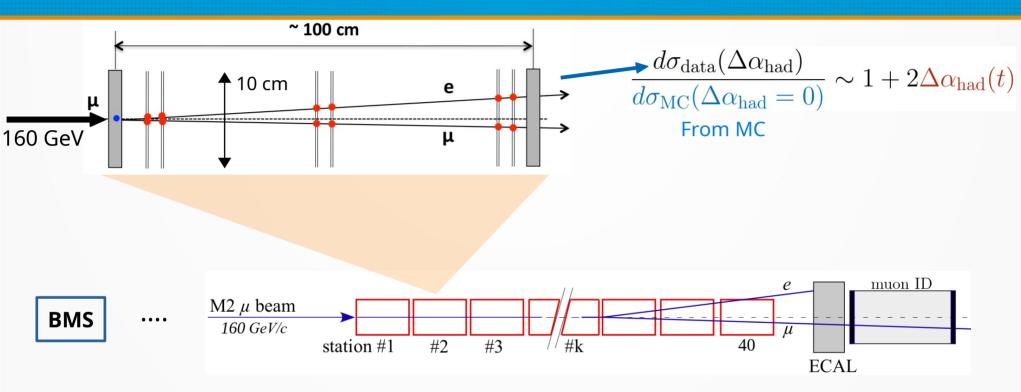
• Observables: (θ_e, θ_u)

~1.5 cm

- Exploit (θ_e, θ_μ) correlation to reject background (main source: μ N $\rightarrow \mu$ N e^+e^-)
- Boosted kinematics: θ_{μ} < 5 mrad, θ_{e} < 32 mrad







Modular layout:
 each station measures
 the incident muon direction
 for the following one

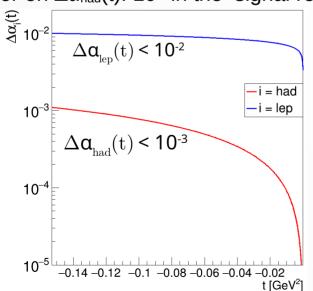
- ECAL: PID + e energy
- Muon ID: PID
- BMS: beam momentum spectrometer



MUonE final goal:

- ~3 years post LS3 (>2030)
- 40 stations
- 4×10¹² elastic events
- $a_{\mu}^{\text{HVP,LO}} < 0.5\%$
- ~1% precision on $\Delta \alpha_{had}(t)$

(error on $\Delta \alpha_{had}(t)$: 10⁻⁵ in the signal region)



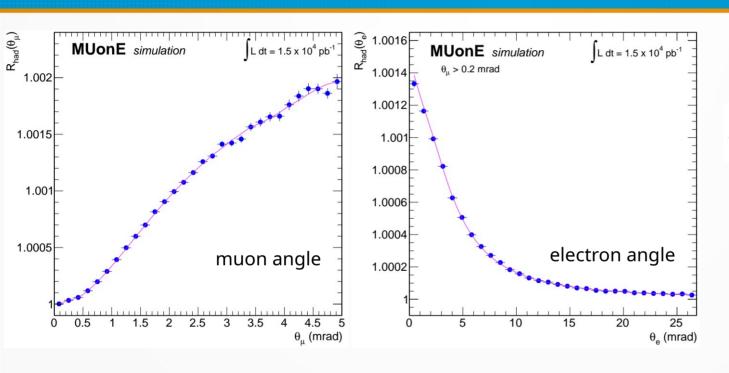
Systematic error goal:

10 ppm on the shape of the differential cross section

- 10 µm longitudinal alignment
- Beam energy measured to few MeV
- Multiple scattering 1%
- Angular intrinsic resolution
- Uniform detector response over full angular range
- Need of dedicated MC generators: signal (>NNLO), main backgrounds

Sensitivity to $\Delta \alpha_{had}(t)$





$$R_{had} = \frac{d\sigma(\Delta\alpha_{had}(t) \neq 0)}{d\sigma(\Delta\alpha_{had}(t) = 0)}$$

$$R_{had} \sim 1 + 2\Delta\alpha_{had}(t)$$

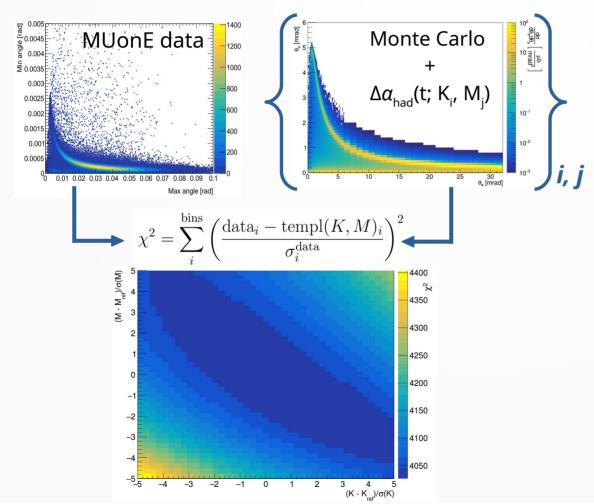
 $\Delta \alpha_{\rm had}(t)$ parameterization: inspired from the 1 loop QED contribution at t < 0:

$$\Delta\alpha_{had}(t) = KM \left\{ -\frac{5}{9} - \frac{4}{3}\frac{M}{t} + \left(\frac{4}{3}\frac{M^2}{t^2} + \frac{M}{3t} - \frac{1}{6}\right)\frac{2}{\sqrt{1 - \frac{4M}{t}}} \ln \left| \frac{1 - \sqrt{1 - \frac{4M}{t}}}{1 + \sqrt{1 - \frac{4M}{t}}} \right| \right\} \text{ 2 parameters: K, M}$$

Extraction of $a_{\mu}^{\rm \; HVP,LO}$: current workflow



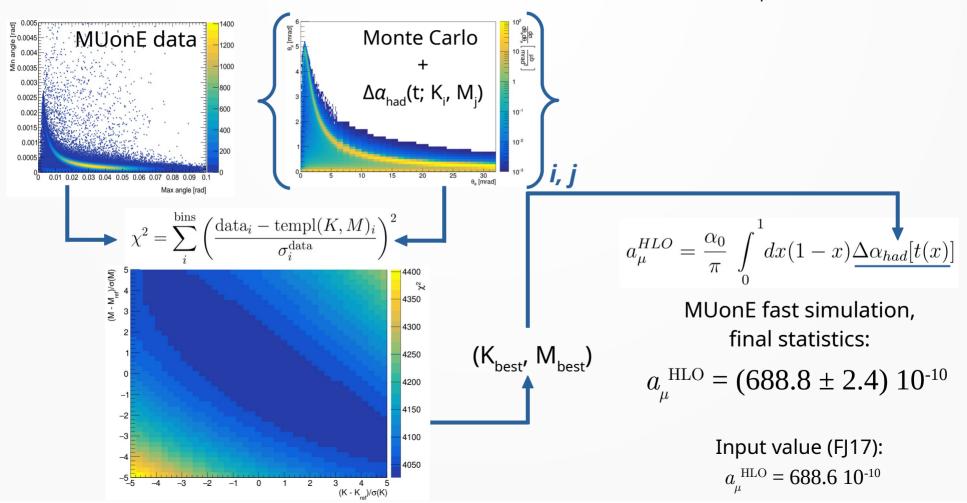
Extraction of $\Delta \alpha_{had}(t)$ through a template fit to the 2D (θ_e , θ_u) distribution:



Extraction of $a_{\mu}^{\rm \; HVP,LO}$: current workflow



Extraction of $\Delta \alpha_{had}(t)$ through a template fit to the 2D (θ_e , θ_{μ}) distribution:



Alternative method to compute $a_{\mu}^{\rm \; HVP,LO}$ from MUonE data



Based on Phys. Rev. D 85 (2012) Phys. Rev. D 96 (2017)

Phys. Lett. B 848 (2024)

Start from the dispersive integral

$$a_{\mu}^{\mathrm{HLO}} = rac{lpha^2}{3\pi^2} \int_{s_{\mathrm{th}}}^{\infty} rac{ds}{s} K(s) R(s)$$

Approximated kernel + Cauchy theorem to remove the problematic low energy part

$$a_{\mu}^{\mathrm{HLO}} = a_{\mu}^{\mathrm{HLO}\; \mathrm{(I)}} + a_{\mu}^{\mathrm{HLO}\; \mathrm{(II)}} + a_{\mu}^{\mathrm{HLO}\; \mathrm{(III)}} + a_{\mu}^{\mathrm{HLO}\; \mathrm{(IV)}}$$

$$a_{\mu}^{\mathrm{HLO}\;(\mathrm{I})} = -\frac{\alpha}{\pi} \sum_{n=1}^{3} \frac{c_{n}}{n!} \frac{d^{(n)}}{dt^{n}} \Delta \alpha_{had}(t) \bigg|_{t=0}$$

$$a_{\mu}^{\mathrm{HLO}\;(\mathrm{II})} = \frac{\alpha}{\pi} \frac{1}{2\pi i} \oint_{|s|=s_0} \frac{ds}{s} c_0 s \; \Pi_{had}(s) \Big|_{\mathrm{pQCD}}$$

$$a_{\mu}^{\mathrm{HLO (III)}} = \frac{\alpha^2}{3\pi^2} \int_{s_{\mathrm{th}}}^{s_0} \frac{ds}{s} [K(s) - K_1(s)] R(s)$$

$$a_{\mu}^{\mathrm{HLO}\;(\mathrm{IV})} = \frac{\alpha^2}{3\pi^2} \int_{s_s}^{\infty} \frac{ds}{s} [K(s) - \tilde{K}_1(s)] R(s)$$

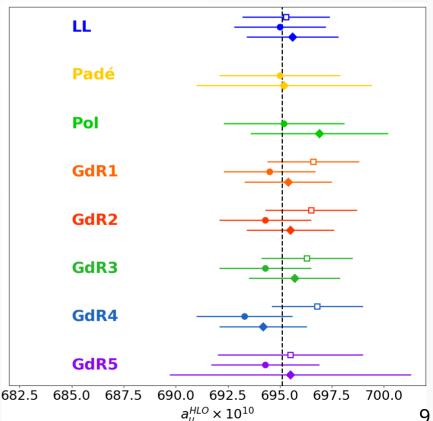
MUonE 99%

Time-like data

pQCD

1%

Competitive results independently of the $\Delta \alpha_{had}(t)$ parameterization

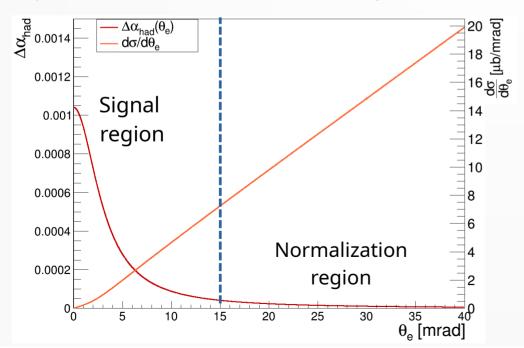


Strategy for the systematic effects

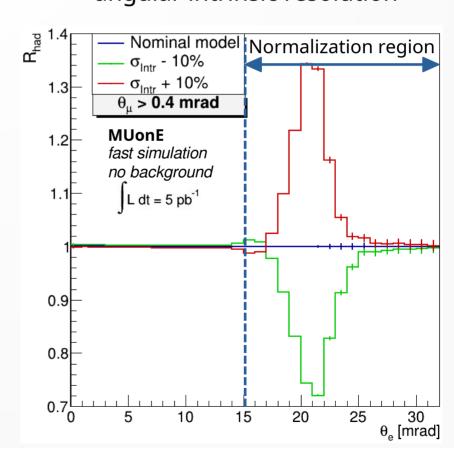


Promising strategy:

- Study the main systematics in the normalization region: large systematic effects but no sensitivity to $\Delta\alpha_{_{\rm had}}$
- Include residual systematics as nuisance parameters in a combined fit with signal



Example: ±10% systematic error on the angular intrinsic resolution



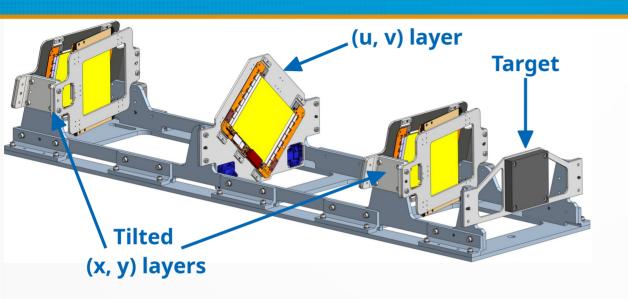
Staged approach towards the full experiment



- 2017: test beam, multiple scattering studies JINST 15 (2020) P01017
- 2018: test beam, elastic scattering properties and event selection studies
- 2021: first joint test CMS-MUonE with 4 2S modules prototypes (parasitic)
- 2022:
 - test 1 tracking station
 - test the calorimeter
- **2023**: test with 2 tracking stations + calorimeter
- 2024: 2 tracking stations (DAQ tests) + calorimeter (characterization)
- **2025**: run with a scaled version of the complete apparatus

Tracking system





INVAR (Fe/Ni alloy)

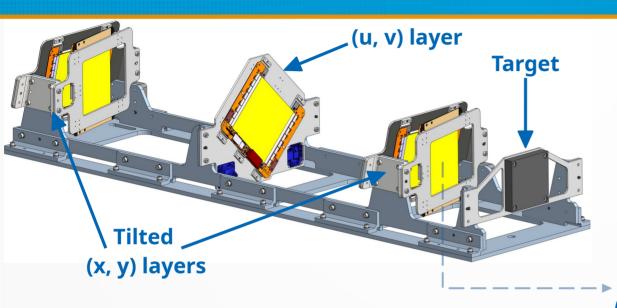
CTE ~ 1.2 ppm/K

Laser holographic system to monitor stability

- (x, y) layers: tilted by 233 mrad → ~2× hit resolution improvement
- (u, v) layers: solve reconstruction ambiguities

Tracking system





INVAR (Fe/Ni alloy)

CTE ~ 1.2 ppm/K

er holographic syste

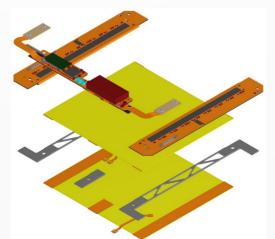
Laser holographic system to monitor stability

- (x, y) layers: tilted by 233 mrad → ~2× hit resolution improvement
- (u, v) layers: solve reconstruction ambiguities

2S modules (CMS Phase2 upgrade)

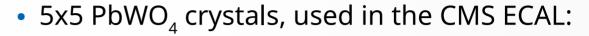
TDR CMS Tracker Phase2 Upgrade

- ~90 cm² active area
- $2 \times 320 \mu m$ thickness
- 40 MHz, binary readout
- 90 μm pitch
 (~26 μm hit resolution)



Calorimeter





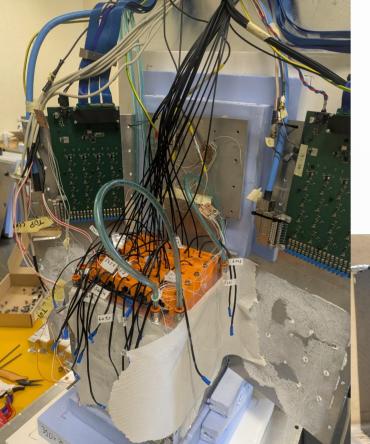
• area: 2.85×2.85 cm²

• length: 23 cm (~25 X₀)

Total ECAL area: ~14×14 cm²

Readout: 10x10 mm² APD

- End of TR 2023:
 ECAL data integrated in the main DAQ
 - TR 2025: tracker-ECAL time sync achieved



DAQ system

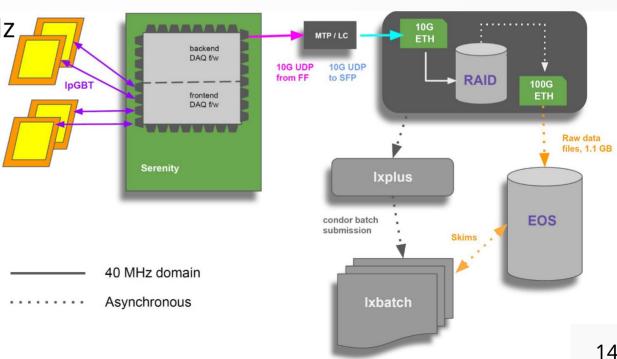


M2 beam line at CERN: unique environment

- High intensity: $\sim 2 \times 10^8 \,\mu^+ / 5s$ spill $\rightarrow \sim 40$ MHz
- Beam asynchronous to DAQ clock
- Serenity board (developed for CMS Phase2 Upgrade)

Triggerless readout @ 40 MHz

- Event aggregator on FPGA
- Data aggregation on 4 PCs
- Transmission to EOS for permanent storage

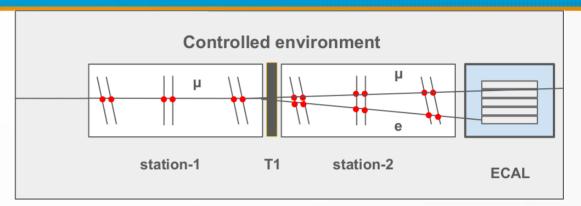


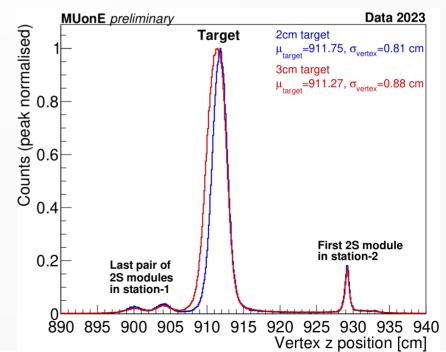
Test Run 2023



- 2 tracking stations
- C target
 (2 or 3 cm thickness used)
- ECAL

- Demonstrated continuous readout @40 MHz.
- Study detector performance, reconstruction algorithms, event selection.

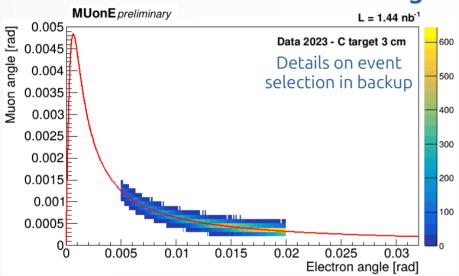




Test Run 2023 - Data/MC comparison



Select elastic events in a clear region

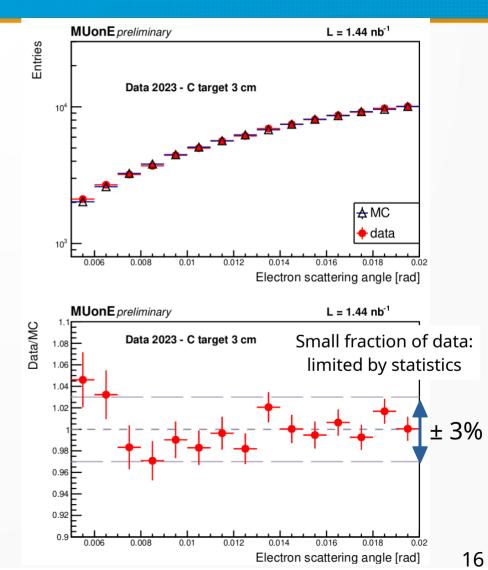


Count N_{μ} on target \rightarrow <u>luminosity estimate</u>

Data/MC comparison of the cross section within event selection:

$$\sigma_{\text{data}} = (75.1 \pm 3.1) \, \mu \text{b}$$

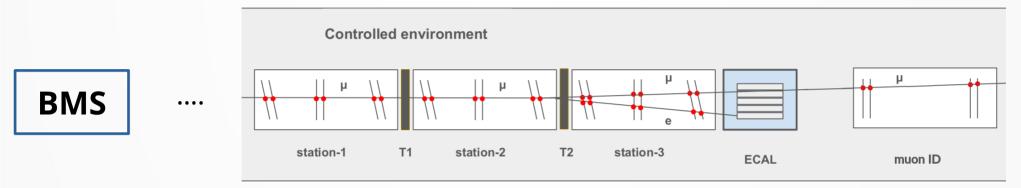
$$\sigma_{\rm MC} = (77.75 \pm 0.14) \,\mu b$$



MUonE Phase-1: Test Run 2025



Proposal for Phase 1 of the MUonE experiment

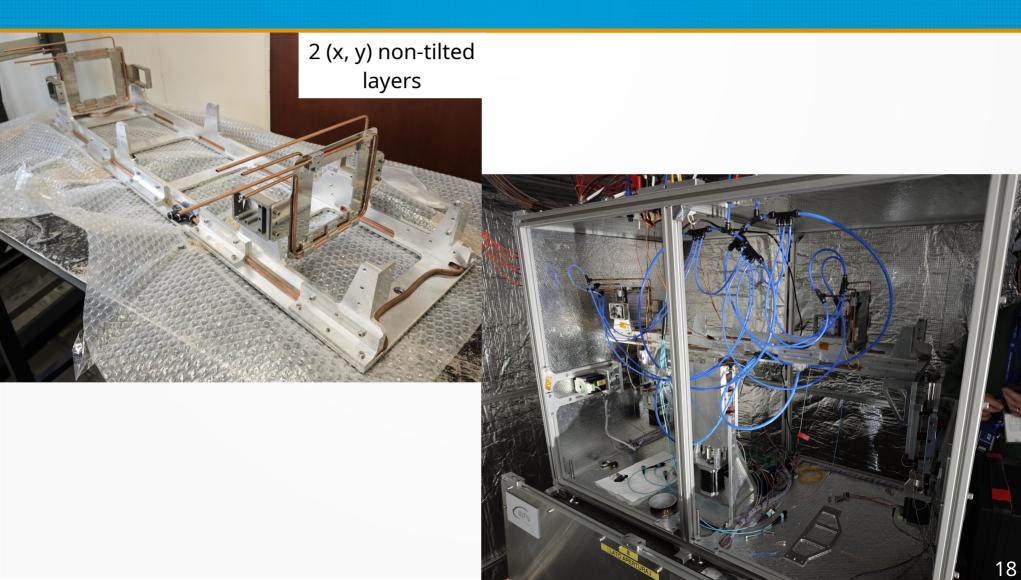


- 3 × tracking stations, each equipped with 6 pre-production 2S modules
- 2 × C targets (each 2 cm thick)
- ECAL: e^- PID + E_e measurement

- Timing detector: time of arrival of muons.
 2 plastic scintillators before and after the tracking stations
- Muon ID: μ PID. Equipped with 4 prototype 2S modules
- BMS: measure p_{μ} event by event. 2 × tracking stations, each equipped with 4 prototype 2S modules

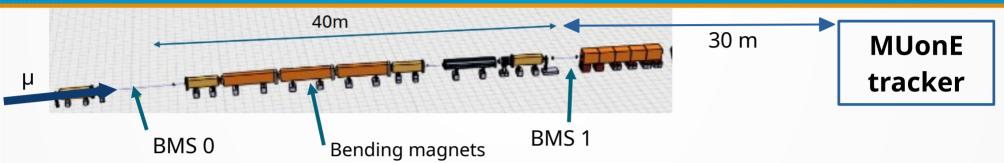
Muon ID





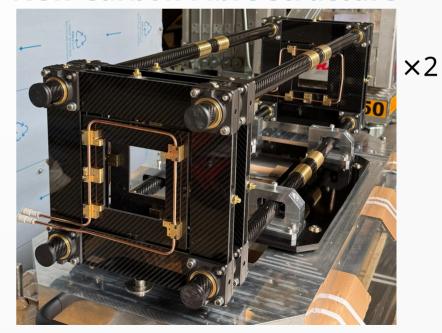
BMS (Beam Momentum Spectrometer)





- Bending power: 16 T*m
 (30 mrad @160 GeV)
- Proof of concept in 2025.
 Challenges:
 - Time synchronisation with the rest of the system
 - Alignment
 - B-field monitoring

New Carbon Fibre structure

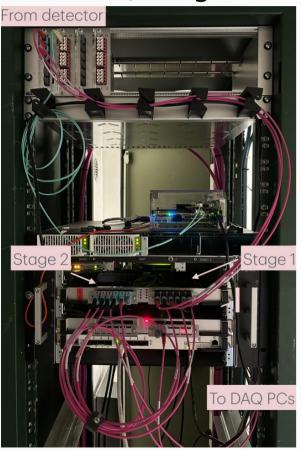


1m long, 2 (x, y) non-tilted layers

System increasing in complexity...



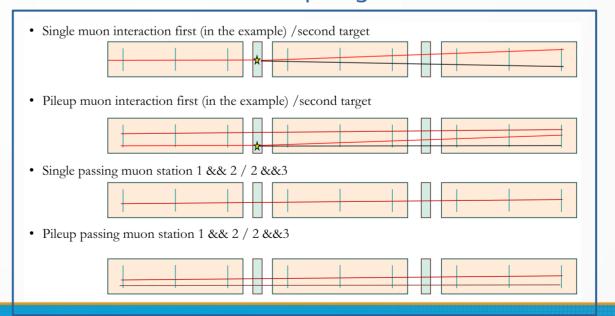
Move to a 2 stages
DAQ design



- **Stage 1:** 36 communication links with subdetectors
 - 30×2S modules
- 2×Timing Detector

- 4×ECAL
- Online selection based on tracker modules occupancy.
 ~×100 reduction of recorded events compared to 2023.

Event topologies



System increasing in complexity...



Move to a 2 stages
DAQ design



- **Stage 1:** 36 communication links with subdetectors
 - 30×2S modules
 - 2×Timing Detector

- 4×ECAL
- Online selection based on tracker modules occupancy.
 ~×100 reduction of recorded events compared to 2023.

- Stage 2: event building.
 - Group information from all subdetectors in a time-coherent packet of data.
 - Online decoding of data provides ready-to-use ntuples for DQM and prompt analysis.

Test Run 2025 - timeline

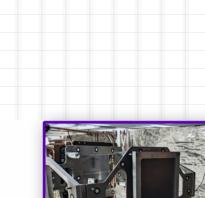


July

August

2S modules characterisation
Tracker, ECAL, Muon ID installation
DAQ and detector commissioning
Targets installed
First physics runs
Technical stop: LHC oxygen run
Resume physics runs
BMS installation

Overnight runs

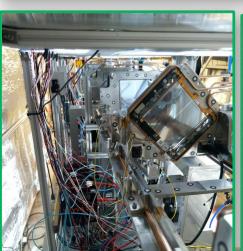


April

May

June

March

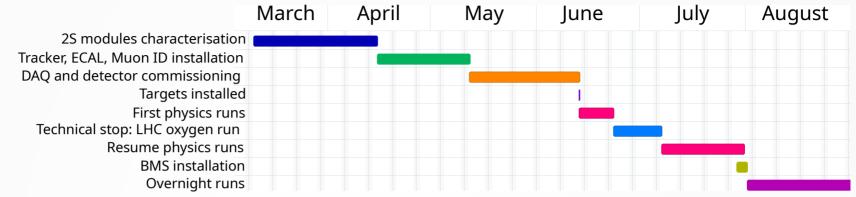


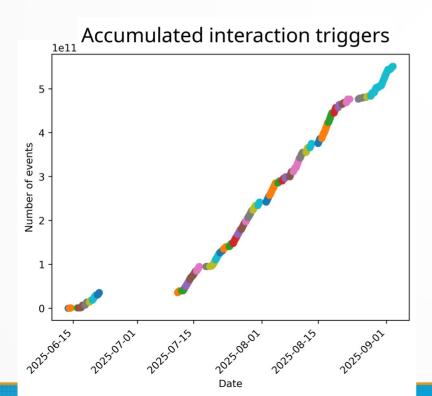




Test Run 2025 - timeline





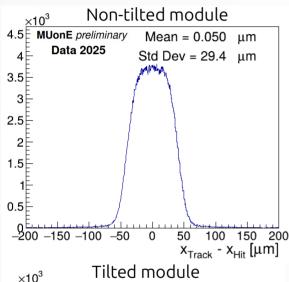


more than 5×10¹¹ interaction triggers recorded!

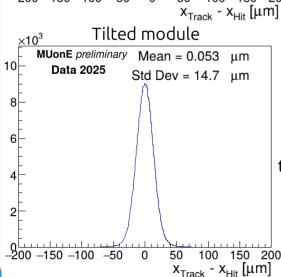
Tracker performance



Unbiased residuals

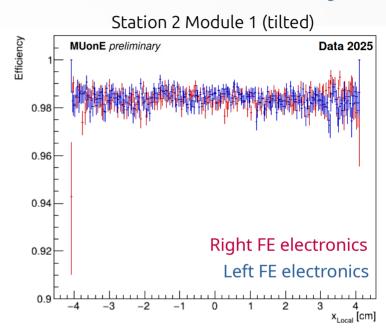


Clear difference in the resolution of tilted and non-tilted modules



*Track fit error to be subtracted from Std Dev to isolate the intrinsic resolution (work in progress)

Module efficiency



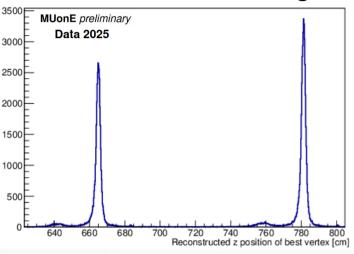
Uniform efficiency over the entire modules surface

Work in progress: efficiency time uniformity over the entire data taking

Elastic scattering events



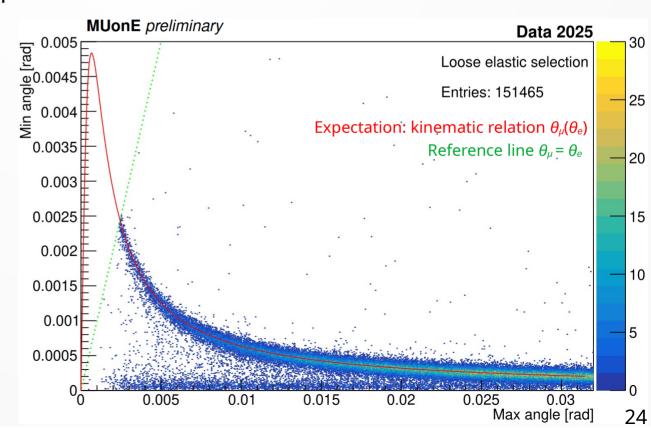




Can use ECAL or MuonID to resolve the ambiguity

Tracker-only analysis of elastic events

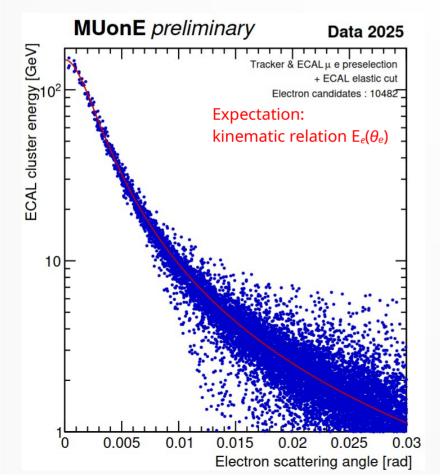
Not able to discriminate between μ and e: plot θ_{max} vs θ_{min} to avoid misidentification when θ_{e} < 5mrad

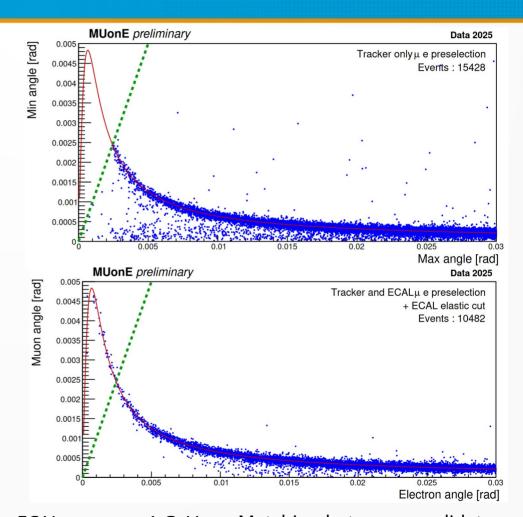


ECAL-based PID



Correlation between ECAL energy deposit and θ_e reconstructed in the tracker



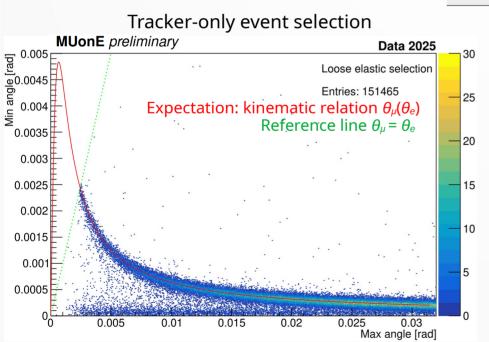


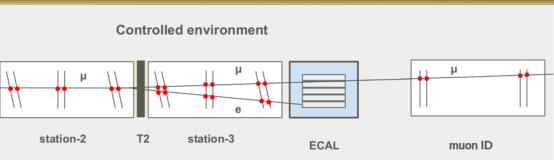
- ECAL energy > 1 GeV
 - Loose elastic selection
- Matching between candidate *e*-track and ECAL cluster centroid

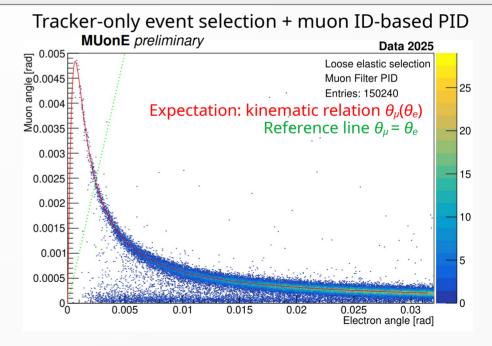
Muon ID-based PID



- Propagate tracks to the Muon ID
- Look for matching between a track and muon ID hits: select the muon track





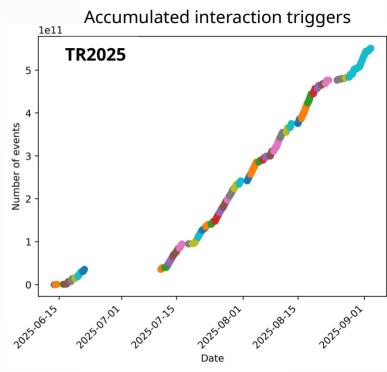


Work in progress: ECAL + muon ID combined PID

Conclusions



- MUonE aims to provide an independent evaluation of $a_{\mu}^{\rm HVP,LO}$ competitive with the latest results.
- 2025 Test Run:
 - Successful ~3 months data taking with 3 stations (2 targets), ECAL and muon ID.
 - Integration of the BMS in the last few days of run.
- Analysis campaign underway. Goals:
 - Proof of principle measurement of $\Delta \alpha_{lep}(t)$ (and comparison with 2023 data).
 - Preliminary measurement of $\Delta \alpha_{had}(t)$ (~20% statistical error + similar systematic).
 - Study systematic effects.
- 2025 data will serve as a basis for the full-scale experiment proposal (40 tracking stations + ancillary detectors) to be prepared during the CERN Long Shutdown 3.



BACKUP

Data-MC comparison of elastic events

Data sample: run $6 \rightarrow 97 \times 10^6$ events after skimming to be reconstructed

 $\underline{\text{MC}}$ sample: MESMER generated <u>signal elastic events</u> $\rightarrow 16.5 \times 10^6$ to be reconstructed with **realistic misalignment** scenario (simulated geometry from real metrology followed by track-based alignment as with real data)

Fiducial selection:

- $N_{hits_{S0}} = 6 \rightarrow 1 \text{ per module: } \underline{\text{golden muon}} \text{ (GM)};$
- GM impinges last 2 modules in S0 within ± 1.5 cm from centre in X and Y;
- Reconstructed GM with $\theta < 4 mrad$.

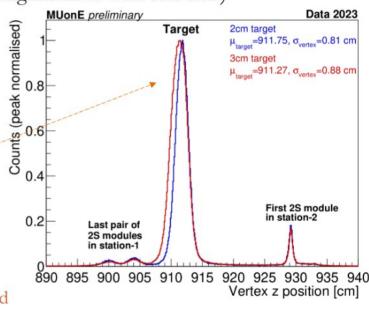
Elastic selection:

- $N_{hits_{s_1}} \le 15$;
- Reconstructed Z vertex > 906 *cm*;
- $\theta_{\mu} > 0.2 \, mrad$, $5 < \theta_{e} < 20 \, mrad$;
- Acoplanarity $|A_{\phi}| < 0.4 \ rad;$

- >0.2 mrad: main background removed
- >5 mrad: Avoid ambiguities in PID
- <20 mrad: region less affected by systematics

• Elasticity condition:
$$|\theta_{\mu} - \theta_{\mu}^{exp}(\theta_{e})| < 0.2 \, mrad$$

$$\theta_{\mu}^{exp}(\theta_e) = \arcsin \left\{ \sin \theta_e \sqrt{\frac{E_e^2(\theta_e) - m_e^2}{[E_{\mu} + m_e - E_e(\theta_e)]^2 - m_{\mu}^2}} \right\}$$



Absolute luminosity normalization

From the **knowledge of the number of golden muons** (passing the fiducial selection) that can potentially interact in the target, we can <u>estimate luminosity</u>:

Fiducial selection:

 $N_{hits_{S0}} = 6 \rightarrow 1 \text{ per module: } \underline{\text{golden muon}} \text{ (GM)};$

GM impinges last 2 modules in S0 within ± 1.5 cm from centre in X and Y;

Reconstructed GM with θ < 4 mrad.

Luminosity real data:

$$L = N_{\mu \text{oT}} \cdot d_{target} \cdot \rho_{target}^{e} =$$

Run 6 = $(1443.0 \pm 8.0) \mu b^{-1}$

Golden muons on target

Target thickness

Electron density target $\rho_{target}^e = \rho \cdot \frac{Z}{A} \cdot N_A$

$$\sigma = \frac{N_{elastic}}{\epsilon_{hw}L}$$

$$\epsilon_{hw} = 0.850 \pm 0.035$$
:

2 tracks reconstruction efficiency which depends on modules efficiency $(\epsilon_{mod} = 0.980 \pm 0.005)$ Main error on:

$$\rho = (1.83 \pm 0.01)g/cm^3$$

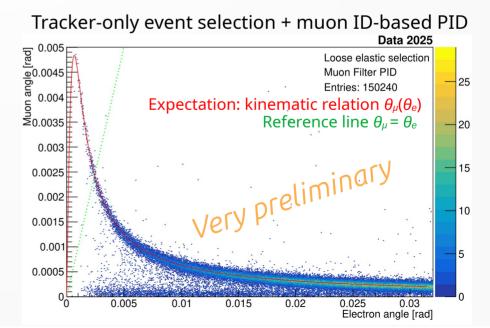
$$d_{target} = (3.000 \pm 0.001) cm$$

MC selection efficiency on σ_{MC} estimate: 76.5%

Muon ID-based PID event selection



- 1stub/module for each track
- 1 track in the station before target; 2 tracks in the station after target
- No stubs shared between different tracks
- $|z_{\text{vtx}} z_{\text{target}}| < 2 \text{ cm}$
- Reject acoplanar events (acoplanarity < 0.4 rad)



Summary of the main sources of systematic errors and corresponding uncertainties for the 2025 run



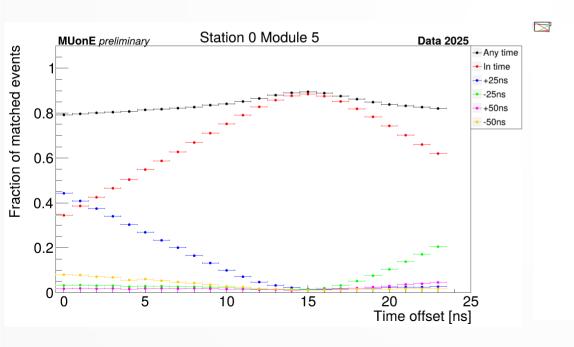
Source of systematic error	Uncertainty on the	Uncertainty	Uncertainty
	systematic source	on $\Delta lpha_{had}$	on $d\sigma$
Intrinsic angular resolution	0.5%	10%	200 ppm
Multiple scattering	1%	10%	200 ppm
Beam energy scale	25 MeV	10%	200 ppm
z coordinate	100 μm	10%	200 ppm
Beam energy spread (3.75%)			
current BMS	$4\% (\sigma_p/p \sim 1\%)$	1%	20 ppm
upgraded BMS	$1\% (\sigma_p/p < 0.5\%)$	< 0.5%	5ppm
Other contributions			
(i.e. tracking efficiency and		15%	300 ppm
reconstruction uniformity,			
residual background)			
Total		25%	500 ppm

TR 2025 - tracker time synchronisation

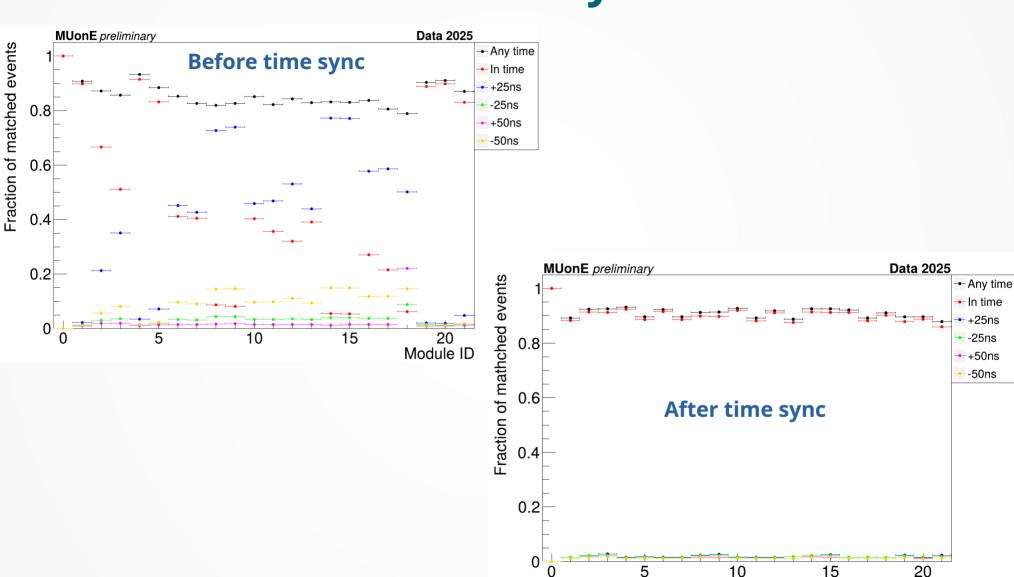


Delay:

Delay scanned [0, 24] ns



TR 2025 – tracker time synchronisation



Module ID

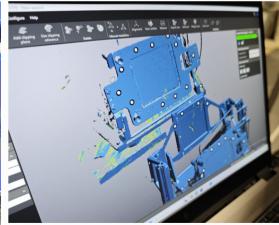
Alignment

- Hardware (stepper motors): center the beam profile on each module (< 500 μ m) align the longitudinal axis of each station to the beam axis (< 0.5mrad) align the 3 stations one relative to the other (< 200 μ m)
- Software: local χ^2 minimization on a sample of single passing muons.

Metrology measurements of the detector (< 100 μm precision)

- 3D scanner photogrammetry: position and orientation of each module within a station
- Laser survey:
 relative position of the different
 subdetectors; absolute position
 with respect to beam elements
- To be used as starting point of software alignment

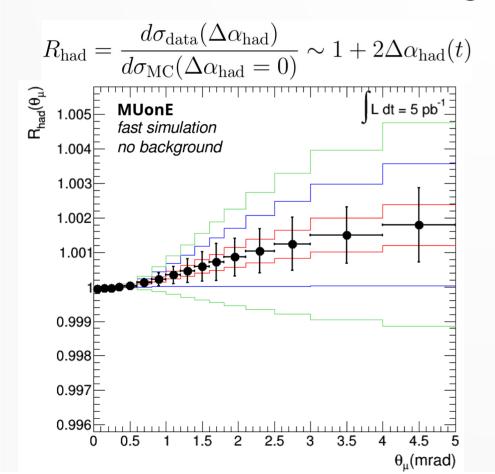




First measurement of $\Delta\alpha_{had}(t)$



Expected event yield: ~10° elastic events within acceptance (one order of magnitude larger than 2023)



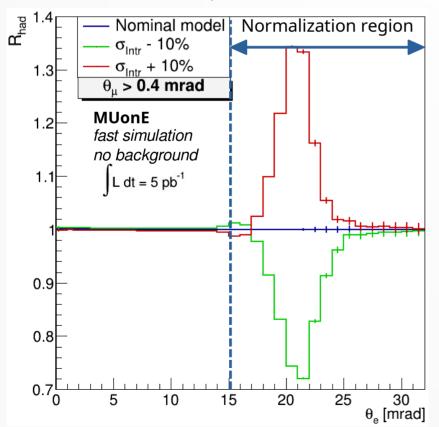
Template fit procedure to extract $\Delta \alpha_{had}(t)$ $\Delta \alpha_{had}(t) \simeq -\frac{1}{15}Kt$ 50 $K = 0.136 \pm 0.026$ 40 (20% stat error) 30 20 10 **MUonE** fast simulation L dt = 5 pb^{-1} no background 0.2 0.1 0.3

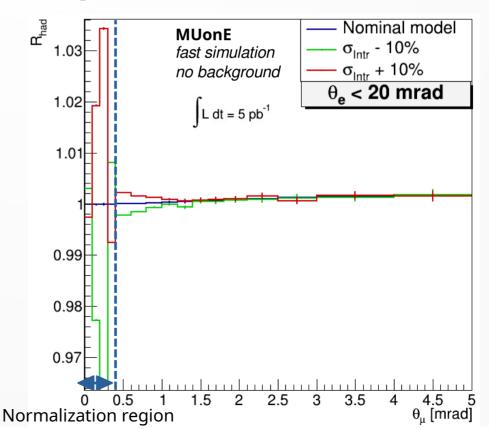
The need of including systematic effects in the analysis



Some systematic effects can produce huge distortions in the shape of the elastic scattering cross section

Example: ±10% error on the angular intrinsic resolution



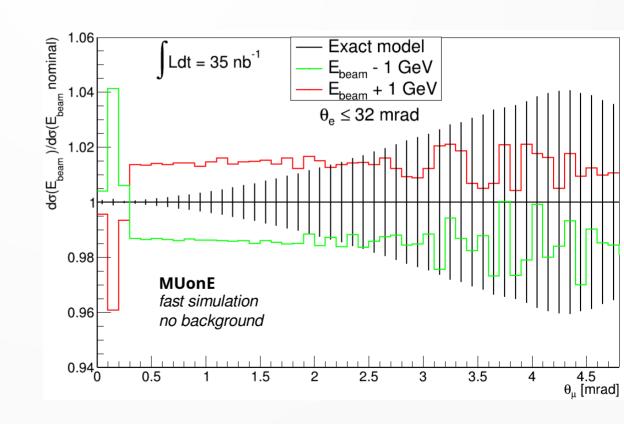


Systematic error on the muon beam energy



Accelerator division provides E_{beam} with O(1%) precision (~ 1 GeV)

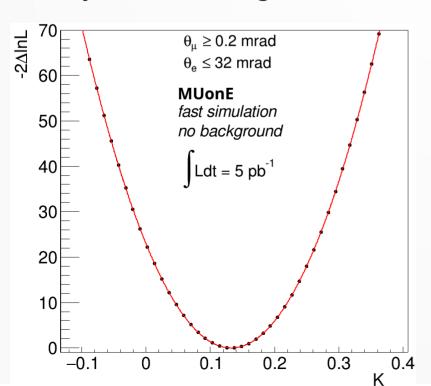
This effect can be seen from our data in 1h of data taking per station



Combined fit signal + systematics



- Include residual systematics as nuisance parameters in the fit.
- Simultaneous likelihood fit to K and systematics using the Combine tool.



- $K_{ref} = 0.137$
- shift MS: +0.5%
- shift intr. res: +5%
- shift E_{beam}: +6 MeV

Selection cuts	Fit results	
$\theta_e \le 32 \text{mrad}$ $\theta_\mu \ge 0.2 \text{mrad}$	$K = 0.133 \pm 0.028$	
	$\mu_{\rm MS} = (0.47 \pm 0.03)\%$	
	$\mu_{\text{Intr}} = (5.02 \pm 0.02)\%$	
	$\mu_{\rm E_{Beam}} = (6.5 \pm 0.5) \; {\rm MeV}$	
	$\nu = -0.001 \pm 0.003$	

Similar results also for different selection cuts

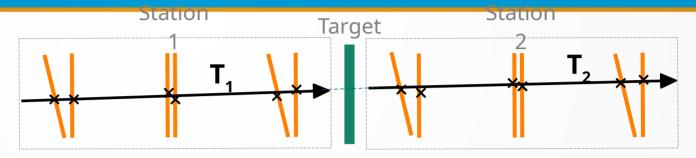
Input shifts identified correctly.

No degradation on the signal parameter

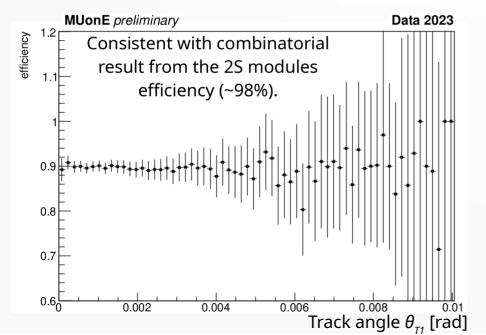


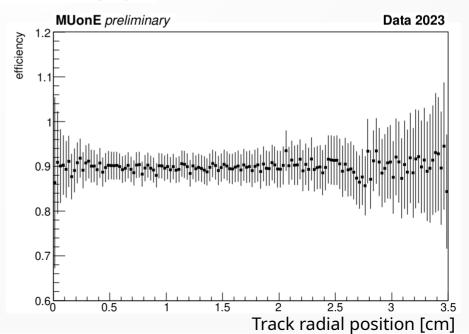
TR 2023 – tracking efficiency

Select events with single passing muons.



Tracking efficiency =
$$\frac{N(T_2 \cdot T_1)}{N(T_1)}$$

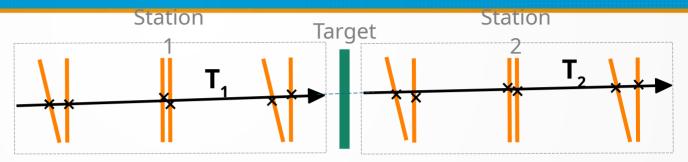


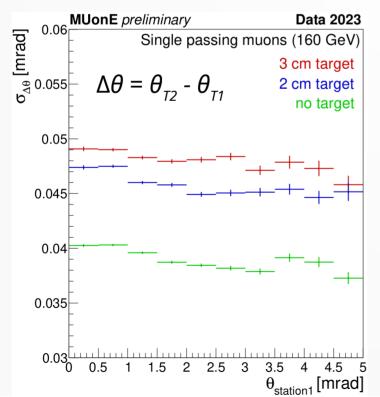


TR 2023 – angular resolution and MS effects



Select events with single passing muons.



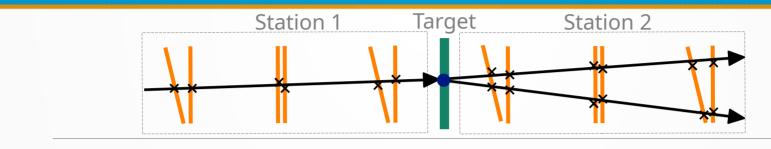


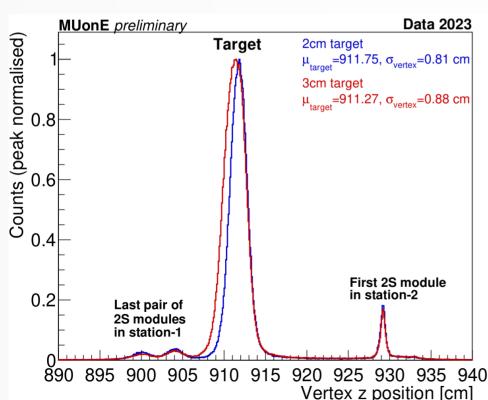
Target	$\sigma_{\Delta\theta}$ [μ rad]	$\sigma_{MS}(\text{target}) [\mu \text{rad}]$	σ_{MS}^{PDG} (target) [μ rad]
3 cm C	48.9 ± 2.1	28.1 ± 0.6	28.2
2 cm C	46.8 ± 2.1	24.3 ± 1.4	22.6
No Target	40.0 ± 2.2		

- Angular resolution of a station: ~28 µrad
- Target MS effects: good agreement with the expectations

TR 2023 - vertexing







- Simple selection: events with 2 outgoing tracks within geometrical acceptance (0.2 – 32 mrad)
- The target center is shifted by 0.5 cm by changing between 3 cm and 2 cm target
- Interactions in the Si sensors are visible
- Vertex resolution: ~8 mm

TR 2023 μ -e elastic scattering event selection



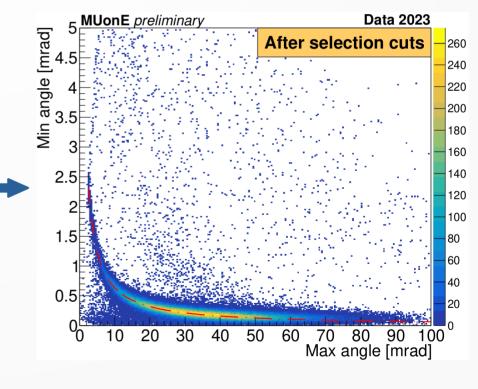
Pre-selection

- Single μ_{in} candidate
- μ_{out} , e_{out} pair candidate

Data 2023 **MUonE** preliminary Min angle [mrad] 2 2 3 2 3 3 3 **Before selection cuts** 3000 2500 2000 1500 1.5 1000 500 0.5 Max angle [mrad]

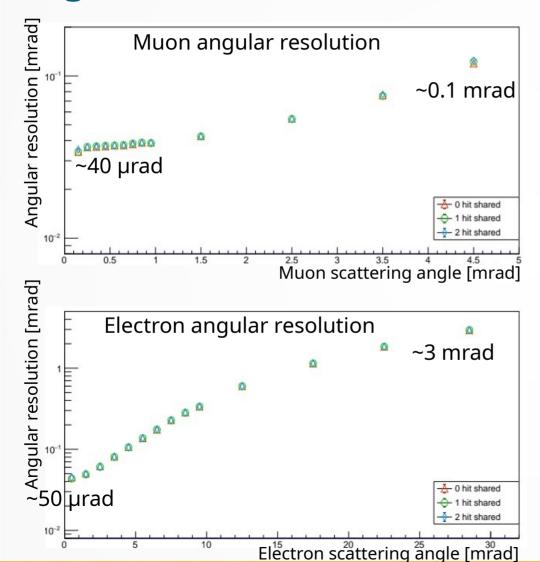
Initial event selection

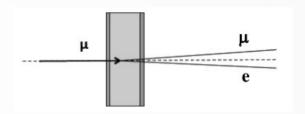
- ≥ 1 hit/module
- Cut on N_{hits} (station2)
- $|z_{vtx} z_{target}| < 3 \text{ cm}$
- Acoplanarity cut



TR 2023 - MC performance: angular resolution of scattered particles



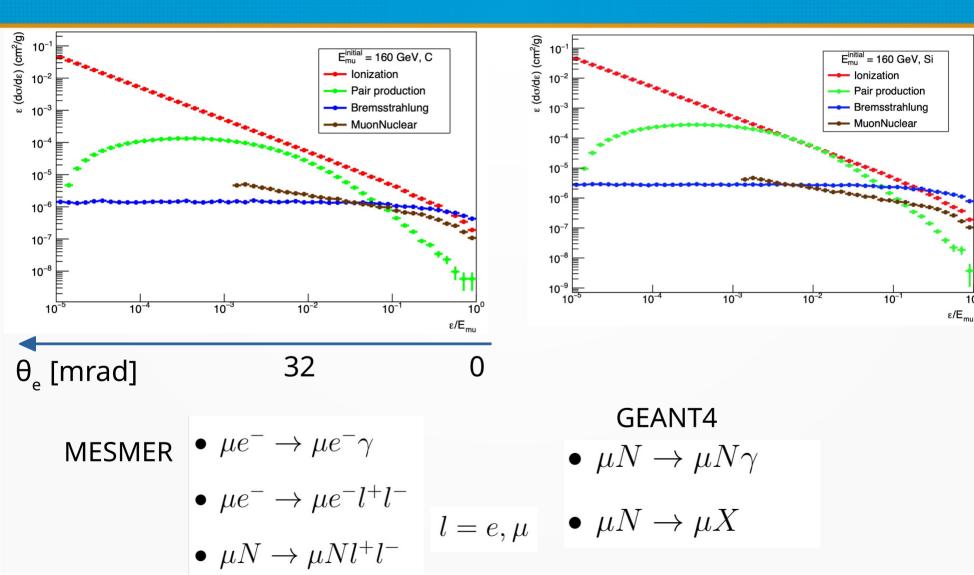




- Compare track reconstruction with MC truth.
- Muon angle: ~40 µrad resolution for small scattering angles.
- Electron angle: stronger impact of MS.
 Resolution is ~3 mrad for large scattering angles (E_e ~1–2 GeV).

Backgrounds

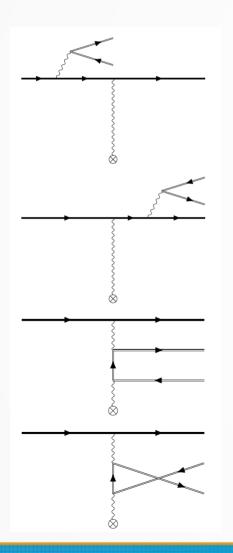




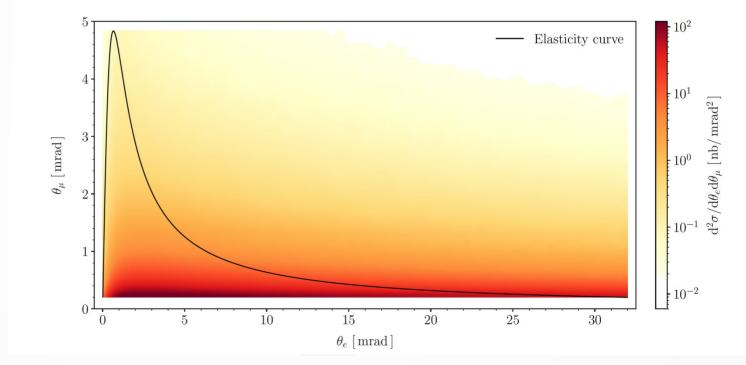
 $l = e, \mu \quad \bullet \quad \mu N \to \mu X$

New Background MC generator

Main background: e+e- pair production
Implemented in MESMER
and interfaced with the MUonE detector simulation

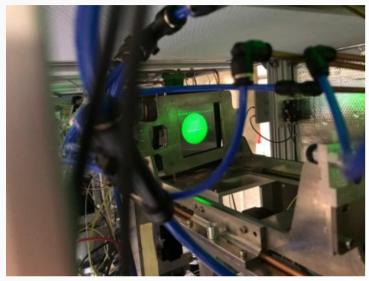


Numerical results for $\mu^+ C \rightarrow \mu^+ C e^+ e^-$ (3)

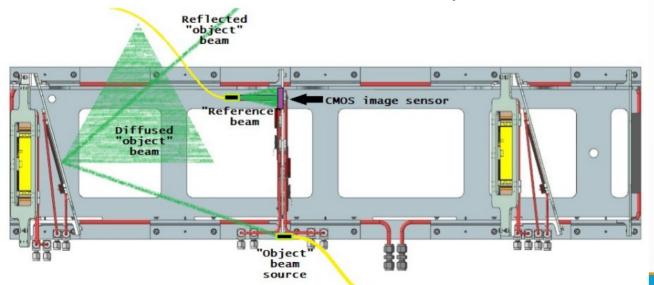


Laser holographic system



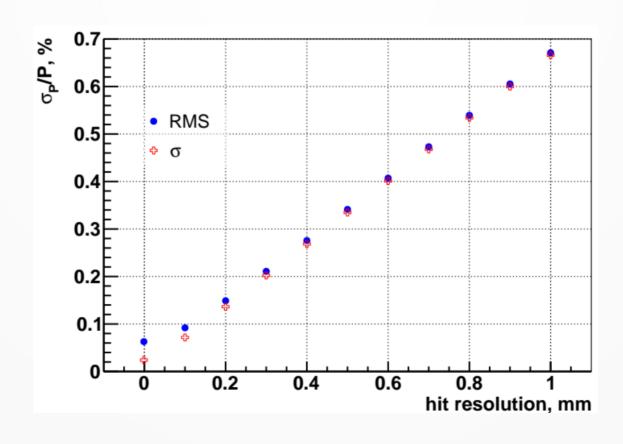


- Compare holographic images of the same object at different time
- Fringe pattern is related to deformations of the mechanical structure
- 532 nm fiber-coupled laser.
 Resolution: ~0.25 μm (half wavelength)
- Current limitation: Si sensors are sensitive to visible light → continuous monitoring is not possible
- Improvement: use >1500 nm laser (IR)



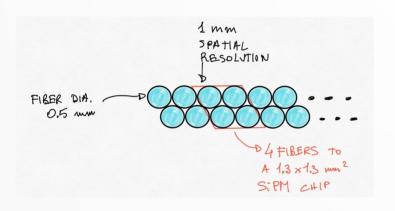
BMS (Beam Momentum Spectrometer)





Muon ID - SciFi

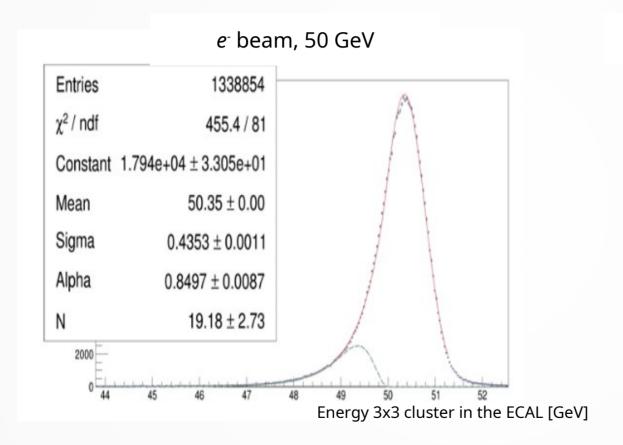




prototype

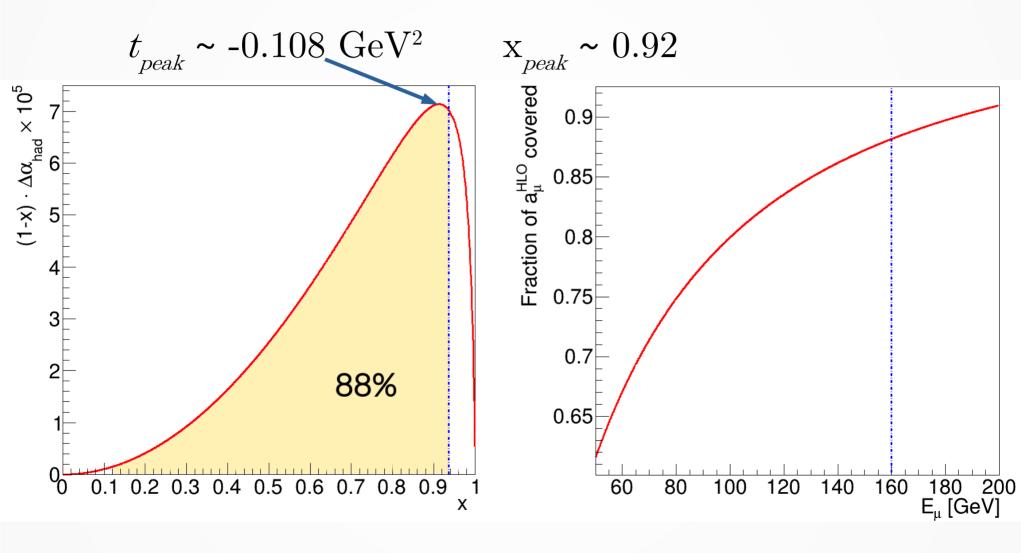


- Polistyrene round fibres. 4 fibres coupled to 1 SiPM.
- <0.5 ns timing resolution.
- Pitch: 1.25mm. Expected resolution: ~360 μm
- Same technology could be used as timing detector between BMS and main tracker.



TB 2024, M2 beamline

x < 0.936

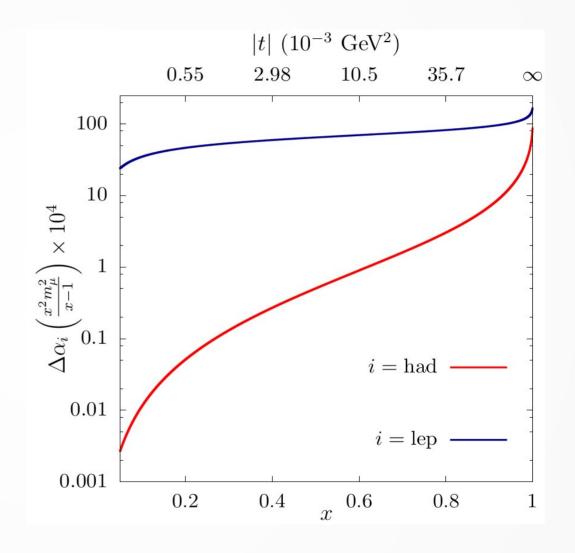


 160 GeV muon beam on atomic electrons.

$$\sqrt{s} \sim 420 \, \mathrm{MeV}$$

$$-0.153 \,\mathrm{GeV}^2 < t < 0 \,\mathrm{GeV}^2$$

$$\Delta \alpha_{had}(t) \lesssim 10^{-3}$$



$\Delta\alpha_{had}$ parameterization



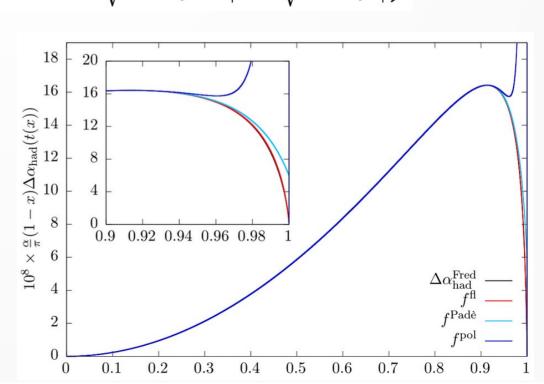
Inspired from the 1 loop QED contribution of lepton pairs and top quark at t < 0

$$\Delta\alpha_{had}(t) = KM \left\{ -\frac{5}{9} - \frac{4}{3} \frac{M}{t} + \left(\frac{4}{3} \frac{M^2}{t^2} + \frac{M}{3t} - \frac{1}{6} \right) \frac{2}{\sqrt{1 - \frac{4M}{t}}} \ln \left| \frac{1 - \sqrt{1 - \frac{4M}{t}}}{1 + \sqrt{1 - \frac{4M}{t}}} \right| \right\} \text{ 2 parameters: } \text{K, M}$$

Allows to calculate the full value of $a_{\mu}^{\;\;\mathrm{HLO}}$

Dominant behaviour in the MUonE kinematic region:

$$\Delta \alpha_{had}(t) \simeq -\frac{1}{15}Kt$$



Alternative method to compute $a_{"}^{ m HVP,LO}$ from MUonE data



$$a_{\mu}^{\rm HLO} = \frac{\alpha^2}{3\pi^2} \int_{s_{\rm th}}^{\infty} \frac{ds}{s} K(s) R(s)$$

$$K(s) = \int_{0}^{1} dx \frac{x^{2}(1-x)}{x^{2} + (1-x)s/m_{\mu}^{2}}$$

$$s_{\rm th} = m_{\pi^0}^2 \qquad \qquad s_0 \gtrsim (2 \, \mathrm{GeV})^2$$

$$s_0 \gtrsim (2 \, {\rm GeV})$$

$$R(s) \propto \sigma(e^+e^- \to \text{hadrons})$$

$$\frac{\alpha^2}{3\pi^2} \int_{s_{\rm th}}^{s_0} \frac{ds}{s} K(s) R(s) + \frac{\alpha^2}{3\pi^2} \int_{s_0}^{\infty} \frac{ds}{s} K(s) R(s)$$
 pQCD

$$-\mathrm{Im}\Pi_{had}(s) = \frac{\alpha}{3}R(s)$$

Alternative method to compute $a_{\mu}^{\ \ \mathrm{HVP,LO}}$ from MUonE data



Low energy integral

$$\int_{s_{\text{th}}}^{s_0} \frac{ds}{s} K(s) \frac{\text{Im}\Pi_{had}(s)}{\pi} =$$

$$\int_{s_{\text{th}}}^{s_0} \frac{ds}{s} [K(s) - K_1(s)] \frac{\text{Im}\Pi_{had}(s)}{\pi} + \int_{s_{\text{th}}}^{s_0} \frac{ds}{s} K_1(s) \frac{\text{Im}\Pi_{had}(s)}{\pi}$$

$$K_1(s) = a_0 s + \sum_{n=1}^{3} \frac{a_n}{s^n}$$

 $K_1(s)$ approximates K(s) for $s < s_0$. Meromorphic function: no cuts, poles in s = 0.

Two different techniques to get $K_1(s)$:

- 1) Least squares minimization
- 2) Minimize $\int_{s_{\text{th}}}^{s_0} \frac{ds}{s} |K(s) K_1(s)| R(s)$

Alternative method to compute $a_{\mu}^{\ \ \mathrm{HVP,LO}}$ from MUonE data



Im s

pole

Low energy integral

Use Cauchy's theorem

$$\int_{s_{\rm th}}^{s_0} \frac{ds}{s} K_1(s) \frac{{\rm Im} \Pi_{had}(s)}{\pi} =$$

$$\operatorname{Res}\left[\Pi_{had}(s)\frac{K_1(s)}{s}\right]_{s=0} - \frac{1}{2\pi i} \oint_{|s|=s_0} \frac{ds}{s} K_1(s) \Pi_{had}(s) \Big|_{pQ}$$

$$\operatorname{Res} \left[\Pi_{had}(s) \frac{K_1(s)}{s} \right]_{s=0} = \sum_{n=1}^{3} \frac{c_n}{n!} \frac{d^{(n)}}{ds^n} \Pi_{had}(s) \bigg|_{s=0} = \sum_{n=1}^{3} \frac{c_n}{n!} \frac{d^{(n)}}{dt^n} \Delta \alpha_{had}(t) \bigg|_{t=0}$$

From MUonE

Alternative method to compute $a_{"}^{ m HVP,LO}$ from MUonE data



High energy integral

$$\int_{s_0}^{\infty} \frac{ds}{s} K(s) \frac{\operatorname{Im}\Pi_{had}(s)}{\pi} =$$

$$\int_{s_0}^{\infty} \frac{ds}{s} [K(s) - \tilde{K}_1(s)] \frac{\operatorname{Im}\Pi_{had}(s)}{\pi} + \int_{s_0}^{\infty} \frac{ds}{s} \tilde{K}_1(s) \frac{\operatorname{Im}\Pi_{had}(s)}{\pi}$$

$$\tilde{K}_1(s) = K_1(s) - c_0 s$$

Similar strategy for the high energy part

$$-\int_{s_0}^{\infty} \frac{ds}{s} \tilde{K}_1(s) \frac{\operatorname{Im}\Pi_{had}(s)}{\pi}$$

$$\frac{ds}{s} \tilde{K}_1(s) \Pi_{had}(s)$$

$$\left| \int_{s_0}^{\infty} \frac{ds}{s} \tilde{K}_1(s) \frac{\operatorname{Im}\Pi_{had}(s)}{\pi} \right| = \left| \frac{1}{2\pi i} \oint_{|s|=s_0} \frac{ds}{s} \tilde{K}_1(s) \Pi_{had}(s) \right|_{pQC}$$

$$\oint_{|s|=s_0} \frac{ds}{s} \tilde{K}_1(s) \Pi_{had}(s)$$