Status of radiative corrections and Monte Carlo tools for low-energy hadronic cross sections in e^+e^- collisions

Andrea Gurgone

on behalf of the RMCL2 Working Group

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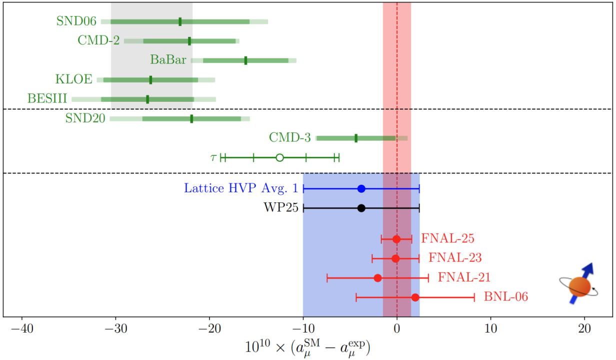
The RadioMonteCarLow 2 effort

- **RadioMonteCarLow 2** is a community effort focused on Monte Carlo tools and radiative corrections for e^+e^-collisions at low energies ($\sqrt{s} <$ few GeV)
- The goal is to assess the current state of MC codes, make them accessible, and further improve them where needed
  $\hookrightarrow$ A living repository of MC generators and benchmark results
- Close collaboration between theorists and experimental collaborations
  $\hookrightarrow$ BESIII, CMD-3, KLOE ...
- 7 codes: AfkQed, BabaYaga@NLO, KKMC, MCGPJ, McMule, Phokhara, Sherpa
- 3+3 processes (both for energy scan and radiative return):
  - $e^+e^- \to e^+e^-(\gamma)$
  - $e^+e^- \to \mu^+\mu^-(\gamma)$
  - $e^+e^- \to \pi^+\pi^-(\gamma)$

radiomontecarlow2.gitlab.io

The motivation: muon $g-2$ puzzle

- MC generators and radiative corrections have a crucial role in the measurement of $e^+e^- \to$ hadrons at flavour factories $\hookrightarrow$ HVP contribution to $(g-2)_\mu$
- Other precision measurements at flavour factories will join the game
- Large overlap (people+physics) with MUonE theory effort ($\mu^\pm e^- \to \mu^\pm e^-$)



The inspiration: original 2010 report

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THE EUROPEAN PHYSICAL JOURNAL C

Review

Quest for precision in hadronic cross sections at low energy: Monte Carlo tools vs. experimental data

Working Group on Radiative Corrections and Monte Carlo Generators for Low Energies

S. Actis[38], A. Arbuzov[9,e], G. Balossini[32,33], P. Beltrame[13], C. Bignamini[32,33], R. Bonciani[15], C.M. Carloni Calame[35], V. Cherepanov[25,26], M. Czakon[1], H. Czyż[19,a,f,i], A. Denig[22], S. Eidelman[25,26,g], G.V. Fedotovich[25,26,e], A. Ferroglia[23], J. Gluza[19], A. Grzelińska[8], M. Gunia[19], A. Hafner[22], F. Ignatov[25], S. Jadach[8], F. Jegerlehner[3,19,41], A. Kalinowski[29], W. Kluge[17], A. Korchin[20], J.H. Kühn[18], E.A. Kuraev[9], P. Lukin[25], P. Mastrolia[14], G. Montagna[32,33,b,d], S.E. Müller[22,f], F. Nguyen[34,d], O. Nicrosini[33], D. Nomura[36,h], G. Pakhlova[24], G. Pancheri[11], M. Passera[28], A. Penin[10], F. Piccinini[33], W. Płaczek[7], T. Przedzinski[6], E. Remiddi[4,5], T. Riemann[41], G. Rodrigo[37], P. Roig[27], O. Shekhovtsova[11], C.P. Shen[16], A.L. Sibidanov[25], T. Teubner[21,h], L. Trentadue[30,31], G. Venanzoni[11,c,i], J.J. van der Bij[12], P. Wang[2], B.F.L. Ward[39], Z. Was[8,g], M. Worek[40,19], C.Z. Yuan[2]

SciPost

SciPost Phys. Comm. Rep. 9 (2025)

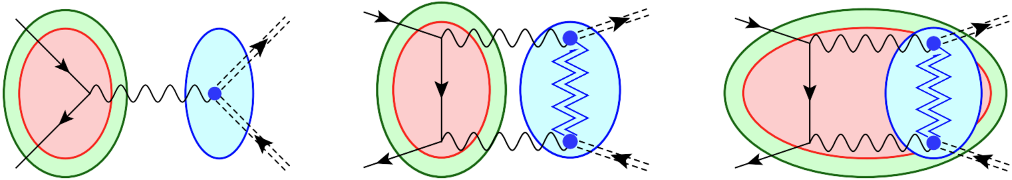
Radiative corrections and Monte Carlo tools for low-energy hadronic cross sections in e^+e^- collisions

Riccardo Aliberti[1], Paolo Beltrame[2], Ettore Budassi[3,4], Carlo M. Carloni Calame[4], Gilberto Colangelo[5], Lorenzo Cotrozzi[2], Achim Denig[1], Anna Driutti[6,7], Tim Engel[8], Lois Flower[2,9], Andrea Gurgone[3,6,7], Martin Hoferichter[5], Fedor Ignatov[2], Sophie Kollatzsch[10,11], Bastian Kubis[12], Andrzej Kupść[13,14⋆], Fabian Lange[10,11], Alberto Lusiani[7,15], Stefan E. Müller[16], Jérémy Paltrinieri[2], Pau Petit Rosàs[2], Fulvio Piccinini[4], Alan Price[17], Lorenzo Punzi[7,15], Marco Rocco[10,18], Olga Shekhovtsova[19,20], Andrzej Siódmok[17], Adrian Signer[10,11⋆], Giovanni Stagnitto[21], Peter Stoffer[10,11], Thomas Teubner[2], William J. Torres Bobadilla[2], Francesco P. Ucci[3,4], Yannick Ulrich[2,5⋆] and Graziano Venanzoni[2,7⋆]

(RadioMonteCarLow 2 working group)

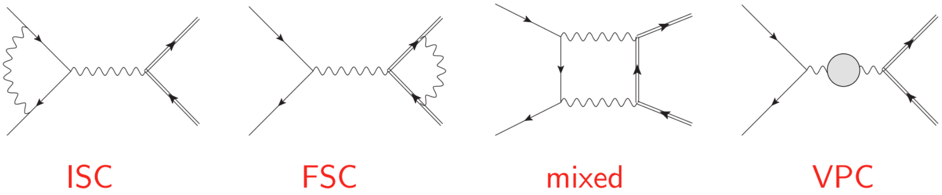
Organisation in Working Packages

- WP1 & WP2: fixed-order QED
- WP3: hadronic final states (mainly pions)
- WP3: all-order QED (resummation)
- WP5: experimental inputs



Terminology

- LO, NLO, NNLO: fixed-order contributions written in powers of α
  $\hookrightarrow$ Each order contains *all* diagrams, both real and virtual
- ISC, FSC: Initial-state and final-state contributions
- VPC: Vacuum polarisation contributions, both leptonic and hadronic
- CS: Collinear structures, resummation of collinear logs via structure functions
- PS: Parton Shower, exclusive resummation of collinear logs with angular effects
- YFS: Yennie-Frautschi-Suura exponentiation, exclusive resummation of soft logs



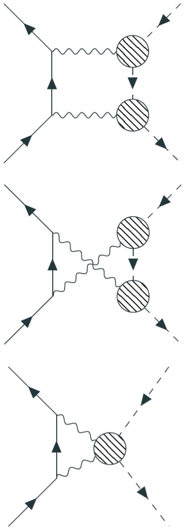
- **F×sQED**: Diagrams are computed in scalar QED and multiplied by a global form factor $F_\pi(q^2)$, where q^2 is chosen to ensure the cancellation of IR divergences
$\hookrightarrow$ $q^2 = m^2_{\pi\pi}$ for ISC, $q^2 = s$ for FSC and mixed

- **GVMD**: The form factor is written as a sum of Breit-Wigner functions. The propagator-like form allows one to solve the loop integral with standard techniques

$$F_\pi(q^2) = \sum_{v=0}^{N} c_v \frac{\Lambda_v^2}{\Lambda_v^2 - q^2} \qquad \text{with} \qquad \Lambda_v^2 = m_v^2 - i m_v \Gamma_v$$

- **FsQED**: Under the general assumptions unitarity and analyticity, the form factor is decomposed using the dispersion relation

$$\frac{F_\pi(q^2)}{q^2} = \frac{1}{q^2 - \lambda^2} - \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} \frac{ds'}{s'} \frac{\mathrm{Im} F_\pi(s')}{s'(q^2 - s')}$$



Phase 1 setup

- 3 channels $e^+e^- \to X^+X^-(\gamma)$
- 5 experimental scenarios
- 7 Monte Carlo codes

Phase 1 setup

- 3 channels $e^+e^- \to X^+X^-(\gamma)$, both in Scan and Radiative-return mode:

  S: $e^+e^- \to e^+e^-$ $e^+e^- \to \mu^+\mu^-$ $e^+e^- \to \pi^+\pi^-$

  R: $e^+e^- \to e^+e^-\gamma$ $e^+e^- \to \mu^+\mu^-\gamma$ $e^+e^- \to \pi^+\pi^-\gamma$

- 5 experimental scenarios

- 7 Monte Carlo codes

Phase 1 setup

- 3 channels $e^+e^- \to X^+X^-(\gamma)$
- 5 experimental scenarios

  CMD: $e^+e^- \to X^+(p_+)X^-(p_-)$ at $\sqrt{s} = 0.7$ GeV
  cuts on $p_\pm$, $\theta_\pm$, $||\phi^+ - \phi^-| - \pi|$, $|\theta_+ + \theta_- - \pi|$

  KLOE-SA (untagged): $e^+e^- \to X^+(p_+)X^-(p_-)\gamma$ at $\sqrt{s} = 1.02$ GeV
  cuts on $p_\pm$, $\theta_\pm$, M_{XX}, if $\vec{p}_{\tilde{\gamma}} \equiv -(\vec{p}_+ + \vec{p}_-)$ then $\theta_{\tilde{\gamma}} \leq 15^\circ$ or $\theta_{\tilde{\gamma}} \geq 165^\circ$

  KLOE-LA (tagged): $e^+e^- \to X^+(p_+)X^-(p_-)\gamma$ at $\sqrt{s} = 1.02$ GeV
  cuts on $p_\pm$, $\theta_\pm$, $M_{XX\gamma}$ and the photon

  BESIII: $e^+e^- \to X^+(p_+)X^-(p_-)\gamma$ at $\sqrt{s} = 4$ GeV
  cuts on $p_\pm$, $\theta_\pm$, $M_{XX\gamma}$ and the photon

  B-factory: $e^+e^- \to X^+(p_+)X^-(p_-)\gamma$ at $\sqrt{s} = 10$ GeV
  cuts on $p_\pm$, $\theta_\pm$, $M_{XX\gamma}$ and the photon

- 7 Monte Carlo codes

Phase 1 setup

- 3 channels $e^+e^- \to X^+X^-(\gamma)$
- 5 experimental scenarios
- 7 Monte Carlo codes

AfkQed R: LO+ISC with CS, FSC with Photos, $X = \mu, \pi$

BabaYaga S: NLO+PS, R: LO + PS, F×sQED for $X = \pi$

KKMC S: LO+YFS, R: LO+YFS for $X = \mu$

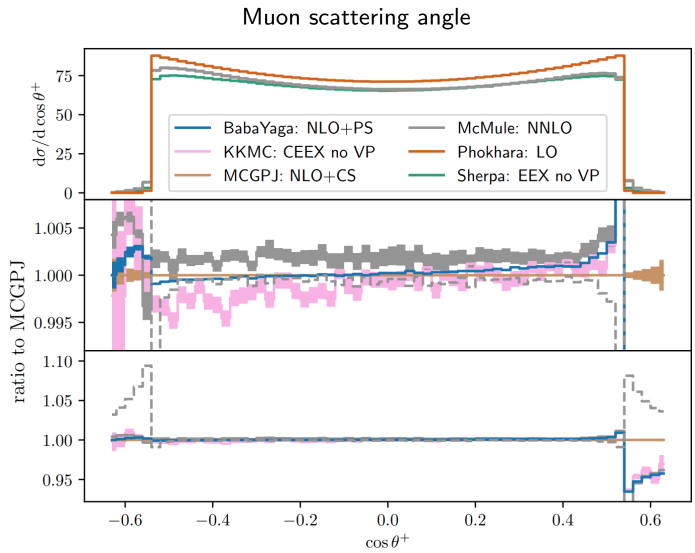
MCGPJ S: NLO+CS, R: LO+CS, GVMD for $X = \pi$

McMule S: NNLO, R: NLO, only ISC for $X = \pi$

Phokhara S: LO, R: NLO, F×sQED for $X = \pi$

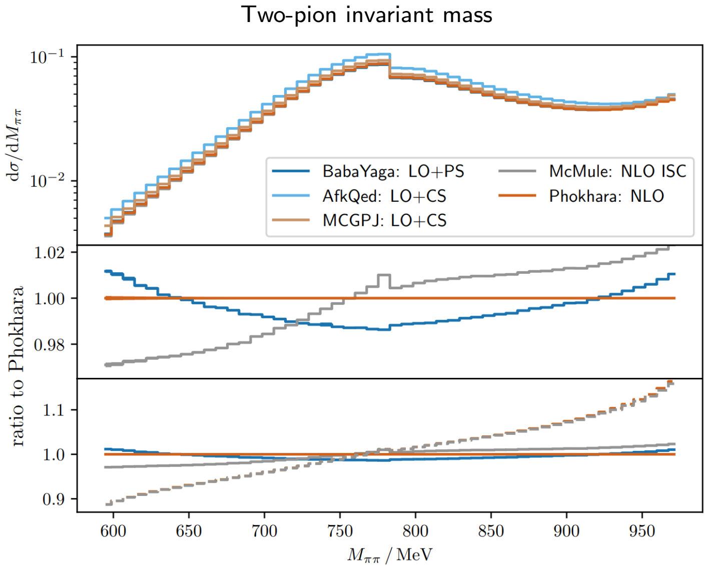
Sherpa S: NLO+YFS for $X = e, \pi$, LO+YFS for $X = \pi$

CMD-like scenario: $e^+e^- \to \mu^+\mu^-$



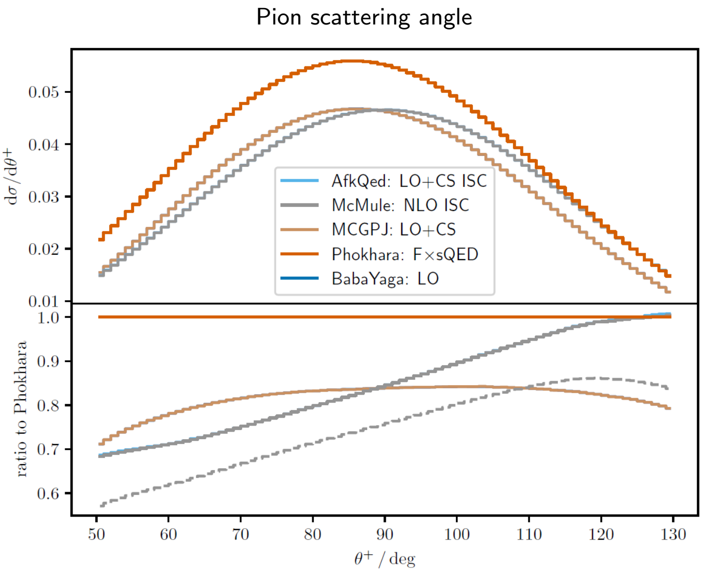
- No VPC for KKMC and SHERPA, but present for all other codes
- BABAYAGA close to MCGPJ despite two different resummation procedures (PS vs. CS)
- PHOKHARA is designed for radiative return, only LO for scan setups
- Dashed grey line in lower panels: MCMULE at NLO
- Agreement at $\sim 0.2\%$ at the center, larger deviations at edges

KLOE-like small-angle scenario: $e^+e^- \to \pi^+\pi^-\gamma$



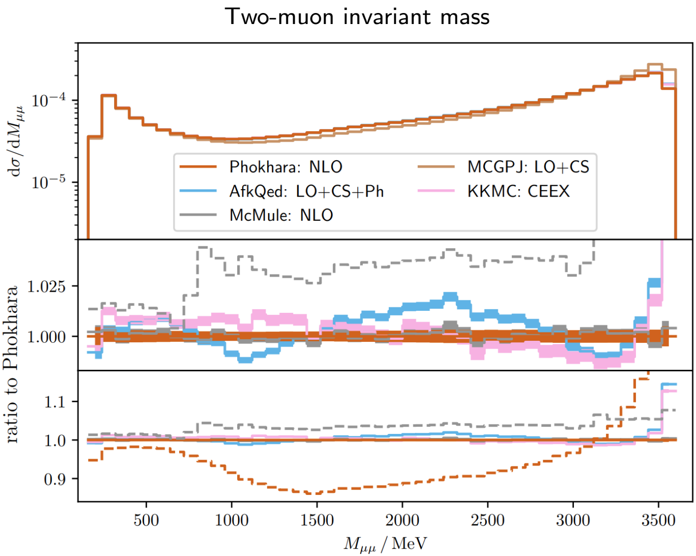
- PHOKHARA territory (full NLO)
- NLO corrections are large (10%)
- AFKQED and MCGPJ are not suitable for radiative return (LO + collinear photons)
- BABAYAGA is designed for $2 \to 2$ processes but the photon can be tagged from the exclusive PS
- McMule has ISC only (no radiation off pions). At LO, the effect is negligible, but is of $\sim 2\%$ at NLO

KLOE-like large-angle scenario: $e^+e^- \to \pi^+\pi^-\gamma$



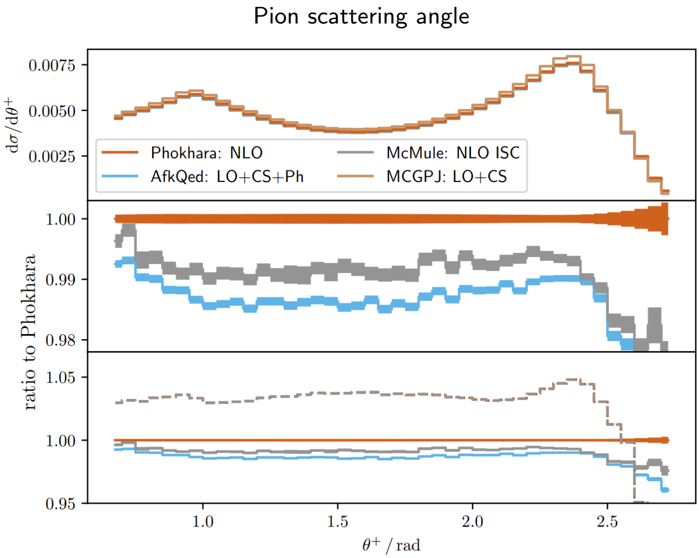
- PHOKHARA territory (full NLO)
- MCGPJ $\simeq$ PHOKHARA at LO
- AFKQED $\simeq$ MCMULE at NLO
- FSR huge at LO (10 – 20%), see MCMULE at LO (dashed grey line)
- Even larger at NLO (20 - 40%)
- A reliable implementation of FSC is crucial for this scenario

BESIII-like scenario: $e^+e^- \to \mu^+\mu^-\gamma$



- No VPC, technical comparison
- VPC $> 3\%$ (see McMule at NLO with VPC, dashed grey line)
- NLO corrections are large ($\sim 10\%$)
- Deviations between codes for the non-VP part are within 2% except at the end of the distribution

B-factory scenario: $e^+e^- \to \pi^+\pi^-\gamma$



- LO: FSC $\sim 0.01\%$ (suppressed by form factor at $\sqrt{s} = 10$ GeV)
- NLO: FSC 1 - 2%
- 1% precision is a long way ahead
- At least NLOPS and NNLO needed

- The various MC groups are working to further improve the QED accuracy of their codes:
  - $\hookrightarrow$ Complete NNLO and NLOPS computations for $2 \to 3$ processes
  - $\hookrightarrow$ Towards NNLOPS and N^{3}LO for ISC for $2 \to 2$ processes
- Systematic implementation of $e^+e^- \to \pi^+\pi^-(\gamma)$ beyond F$\times$sQED
  - $\hookrightarrow$ Already done in BabaYaga@NLO and McMule
- Expand the processes under investigation ($e^-e^+ \to \gamma\gamma, \pi^+\pi^-\pi^0 ...$)
- Evaluate the theory error (e.g. comparison between NNLO and NLOPS)
- Strengthen collaboration with experiments to better understand the impact of radiative corrections on data analysis $\to$ No longer just a *theory exercise* but actual physics

RadioMonteCarLow 2 members for Phase 1

Riccardo Aliberti, Paolo Beltrame, Ettore Budassi, Carlo M. Carloni Calame, Gilberto Colangelo, Lorenzo Cotrozzi, Achim Denig, Anna Driutti, Tim Engel, Lois Flower, Andrea Gurgone, Martin Hoferichter, Fedor Ignatov, Sophie Kollatzsch, Bastian Kubis, Andrzej Kupsc, Fabian Lange, Alberto Lusiani, Stefan E. Muller, Jeremy Paltrinieri, Pau Petit Rosas, Fulvio Piccinini, Alan Price, Lorenzo Punzi, Marco Rocco, Olga Shekhovtsova, Andrzej Siodmok, Adrian Signer, Giovanni Stagnitto, Peter Stoffer, Thomas Teubner, William J. Torres Bobadilla, Francesco P. Ucci, Yannick Ulrich, Graziano Venanzoni

RMCL2 Workshop in Pisa, May 2025

Contact Andrzej Kupsc, Adrian Signer, Yannick Ulrich, or Graziano Venanzoni if you are interested in joining the RadioMonteCarLow2 effort, have feedback or want to discuss