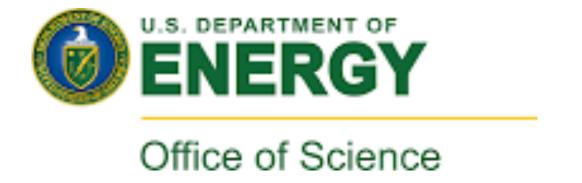
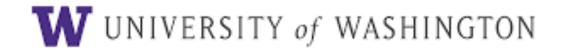
Workshop on Flavor Changing and Conserving Processes Villa Orlando, Anacapri September 29 - October I 2025

Status of the Cabibbo angle determination

Vincenzo Cirigliano

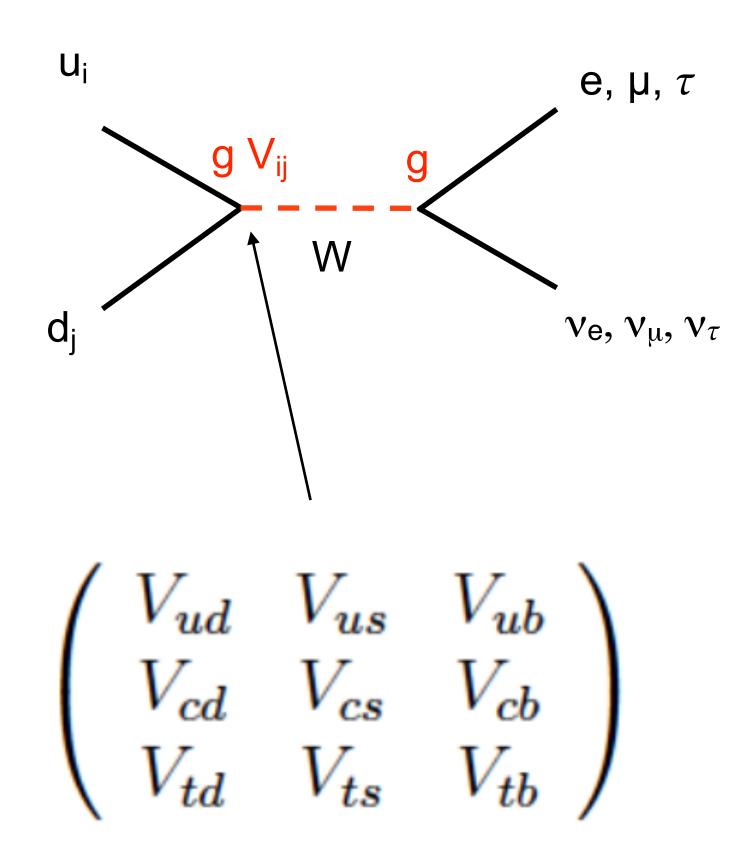






Semi-leptonic charged-current processes

• In the SM, W exchange between L-handed fermions \Rightarrow "V-A" currents & universality relations



Cabibbo-Kobayashi-Maskawa

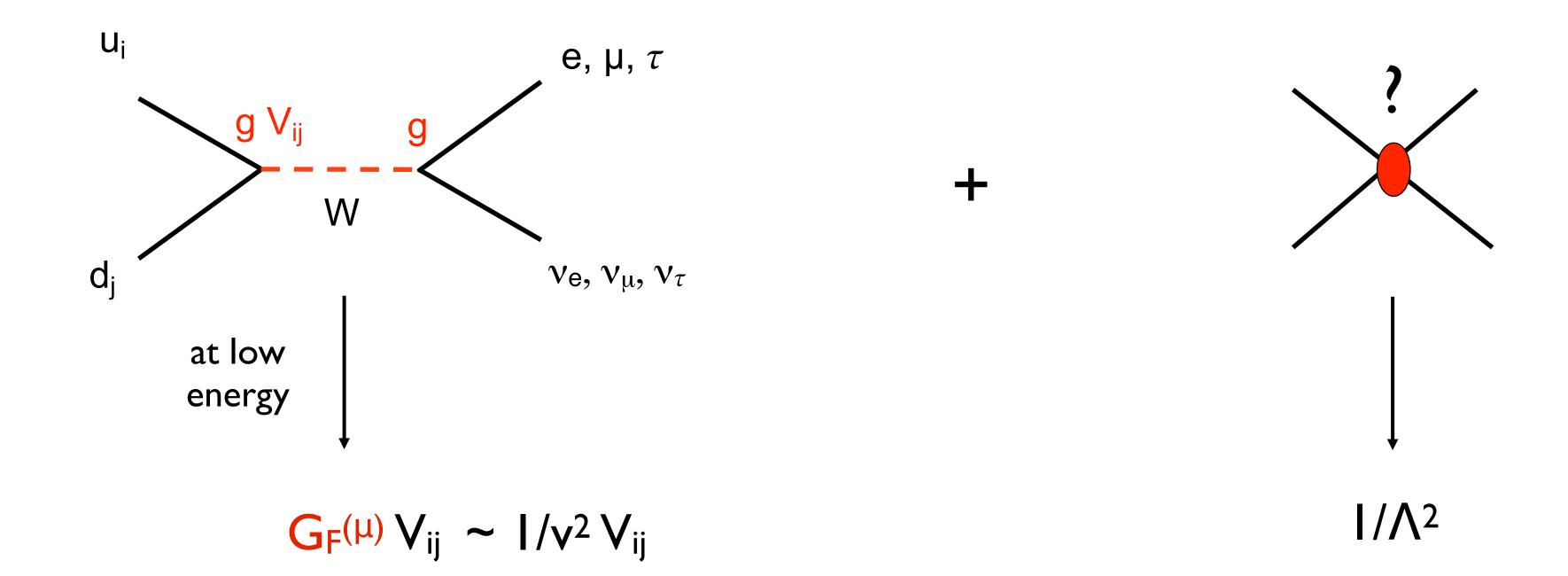
Cabibbo universality (CKM unitarity)

 V_{ud} and V_{us} are the most accurately known elements of the CKM matrix \Rightarrow

Ist row provides the most stringent test of universality & sensitivity to new physics

Semi-leptonic charged-current processes

• In the SM, W exchange between L-handed fermions \Rightarrow "V-A" currents & universality relations



New physics can spoil universality: $|V_{ud}|^2+|V_{us}|^2+|V_{us}|^2=1+O\left(\frac{v^2}{\Lambda^2}\right)$

Current precision \Rightarrow probe effective scale $\Lambda \sim 10 \text{ TeV}$

Compelling but challenging!

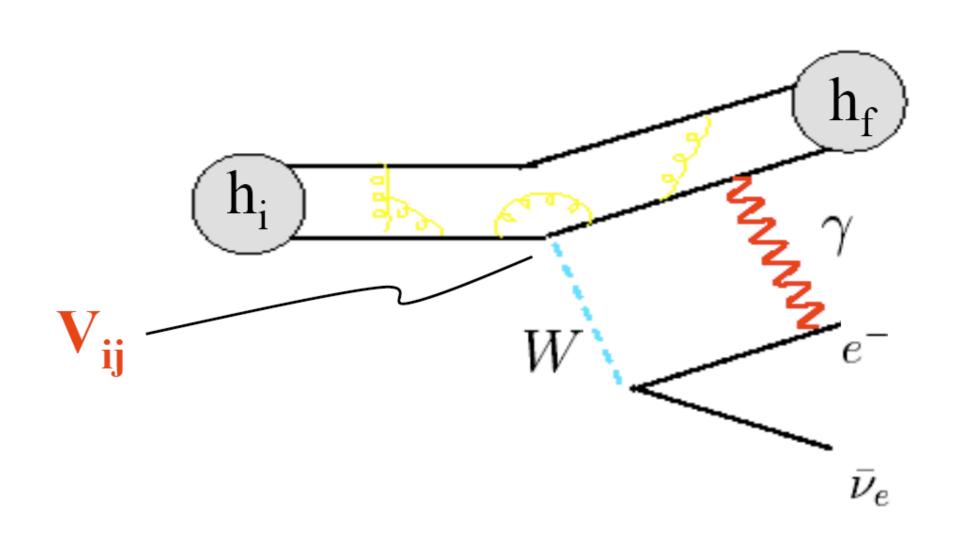
Outline

- Overview: paths to $V_{ud} \& V_{us}$ and current puzzles
- A closer look:
 - Status and prospects for selected channels
 - Radiative corrections to neutron and nuclear decays in EFT

Paths to V_{ud} & V_{us}: status and puzzles

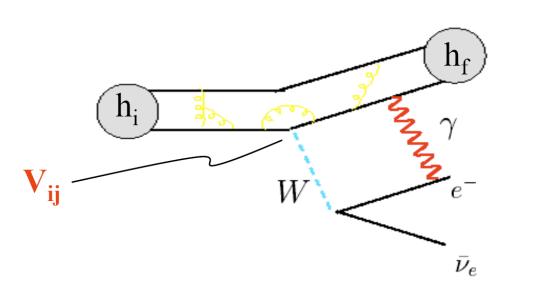
Paths to V_{ud} and V_{us}

	Hadron decays			Lepton decays
V _{ud}	$\pi^{\pm} \rightarrow \pi^{0} e \nu$ Nucl. $0^{+} \rightarrow 0^{+}$	$n \rightarrow pev$ Nucl. mirror decays	$\pi \to \mu \nu$	$ au o h_{NS} u$
V _{us}	$K \rightarrow \pi \mid \nu$	$\Lambda \rightarrow pev,$	$K \rightarrow \mu \nu$	$ au o h_S u$



The challenge of CKM precision tests

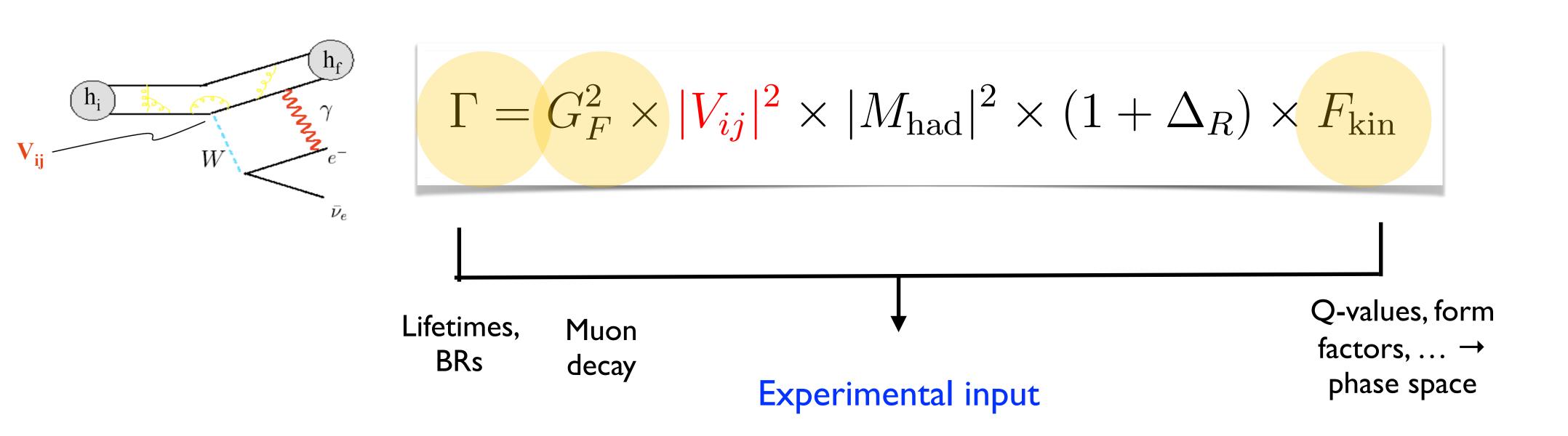
Extract $V_{us} = \sin\theta_C = \lambda$ and $V_{ud} = \cos\theta_C \simeq 1 - \lambda^2/2$ with sub-percent precision from decays involving hadrons (currently $\delta\lambda/\lambda \sim 0.2$ -0.5%)



$$\Gamma = G_F^2 \times |V_{ij}|^2 \times |M_{\text{had}}|^2 \times (1 + \Delta_R) \times F_{\text{kin}}$$

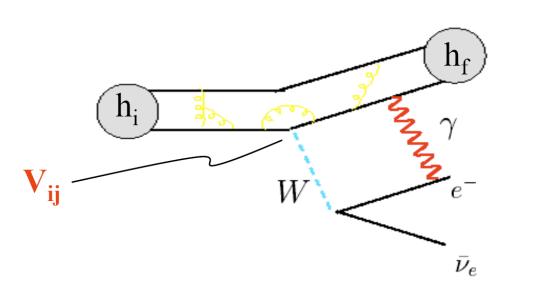
The challenge of CKM precision tests

Extract $V_{us} = \sin\theta_C = \lambda$ and $V_{ud} = \cos\theta_C \approx 1 - \lambda^2/2$ with sub-percent precision from decays involving hadrons (currently $\delta\lambda/\lambda \sim 0.2$ -0.5%)

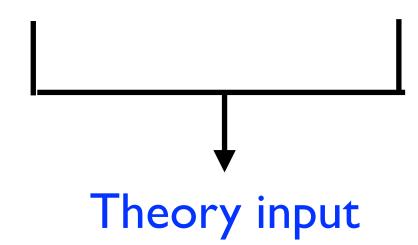


The challenge of CKM precision tests

Extract $V_{us}=\sin\theta_C=\lambda$ and $V_{ud}=\cos\theta_C\simeq I-\lambda^2/2$ with sub-percent precision from decays involving hadrons (currently $\delta\lambda/\lambda\sim0.2$ -0.5%)



$$\Gamma = G_F^2 \times |V_{ij}|^2 \times |M_{\text{had}}|^2 \times (1 + \Delta_R) \times F_{\text{kin}}$$



Hadronic / nuclear matrix elements of the weak V-A current, including small corrections such as those induced by electromagnetic radiative corrections $[(\alpha/\pi)\sim 2.\times 10^{-3}]$

Hadronic matrix elements

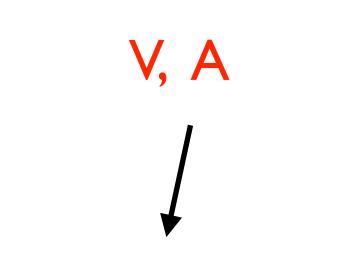
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V _{us}	$K \rightarrow \pi \mid \nu$	$\Lambda \rightarrow pev,$	$K \rightarrow \mu \nu$	$ au o h_S u$

Hadronic matrix elements: 'Vector - Axial' quark current



Traditionally "Golden modes": $< f |V_{\mu}| i > known in SU(2) [SU(3)] limit &$

corrections are 2nd order in SU(2) [SU(3)] breaking. Computed in lattice QCD for $K \rightarrow \pi$



Need experimental input on <f |A|i> / <f |V |i>

For neutron and hyperons, Lattice QCD catching up but not as precise as experiment



 $\begin{array}{c|c} <0 \mid A_{\mu} \mid M> \\ \text{(decay constants)} \\ \text{from Lattice QCD} \\ \text{[\sim0.2\%$]} \end{array}$

Use combination of data and theory (pQCD + lattice QCD)

Radiative corrections

	Hadron decays			Lepton decays
V _{ud}	$\pi^{\pm} \rightarrow \pi^{0} e \nu$ Nucl. $0^{+} \rightarrow 0^{+}$	$n \rightarrow pev$ Nucl. mirror decays	$\pi \to \mu \nu$	$ au o h_{NS} u$
V _{us}	$K \rightarrow \pi \mid \nu$	$\Lambda \rightarrow pev,$	$K \rightarrow \mu \nu$	$ au o h_S u$

Electroweak radiative corrections

Mesons and neutron:
well developed Effective Field
Theory (EFT) framework, with
non-perturbative input from lattice
QCD and / or dispersive methods
— systematically improvable

For leptonic meson decays: full lattice QCD+QED available

Recent activity to assess nuclear structure uncertainties:

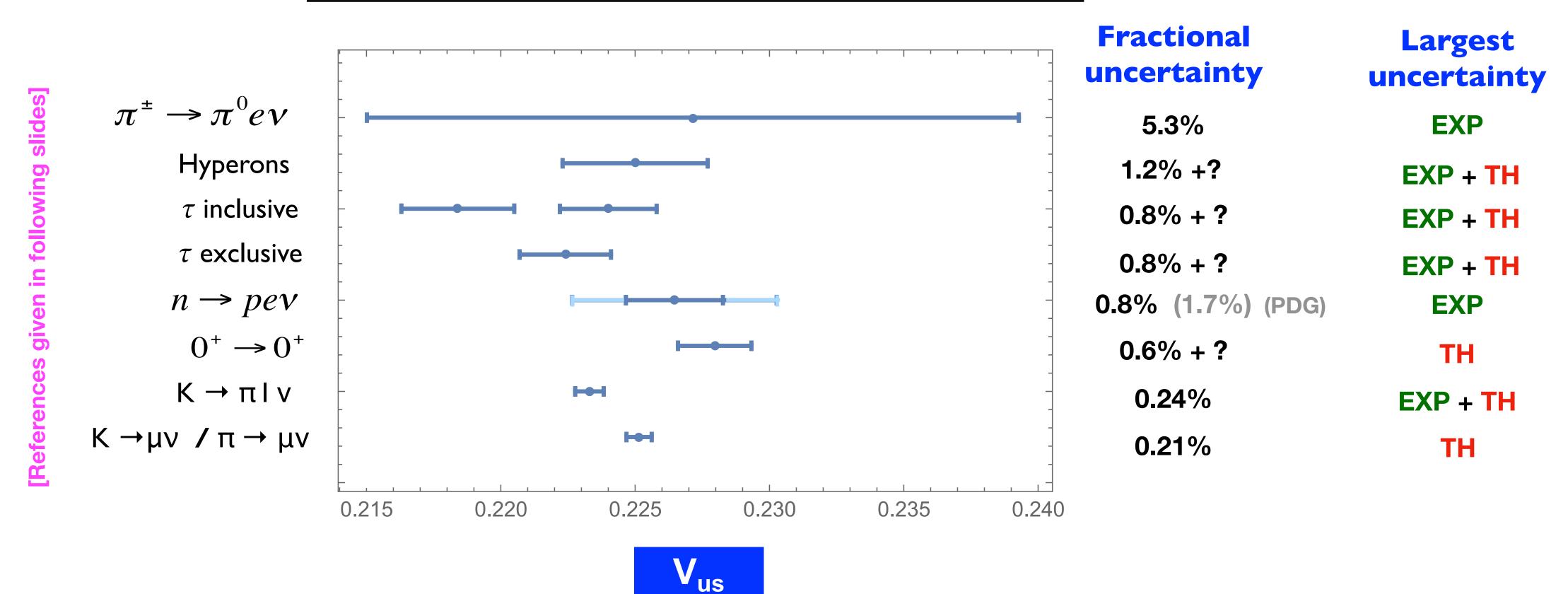
- Dispersive approach
- Chiral EFT

For exclusive channels, difficult to estimate the hadronic structure-dependent effects.

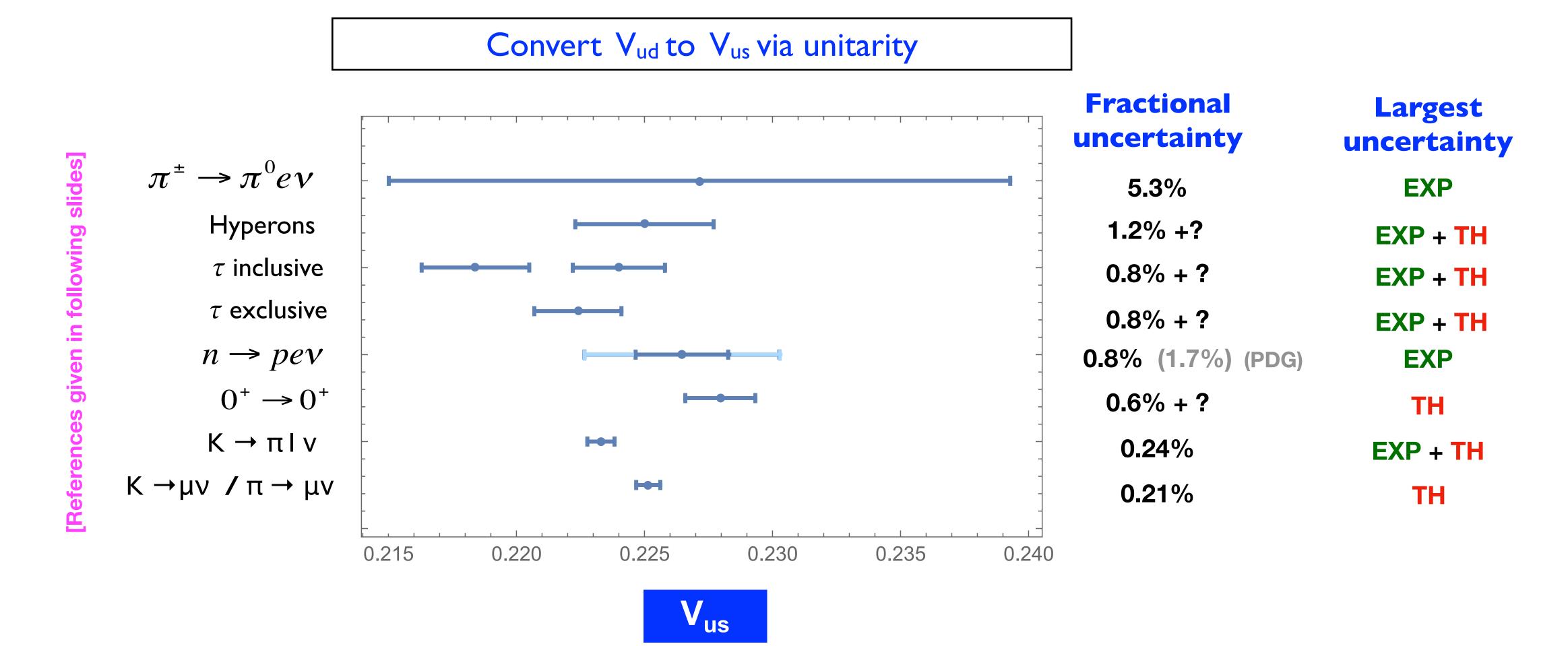
Lattice QCD+QED?

The Cabibbo angle — global view



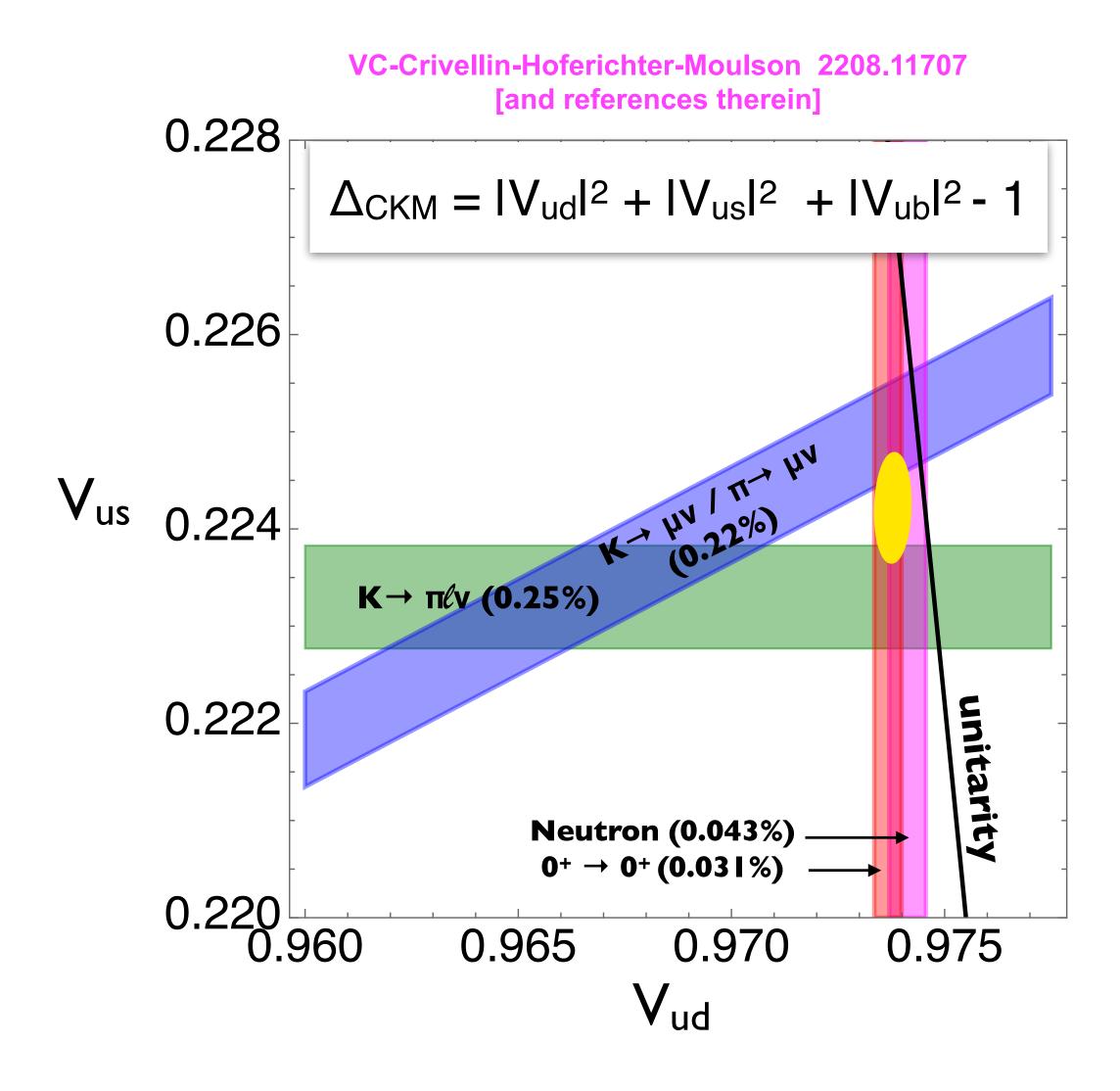


The Cabibbo angle — global view



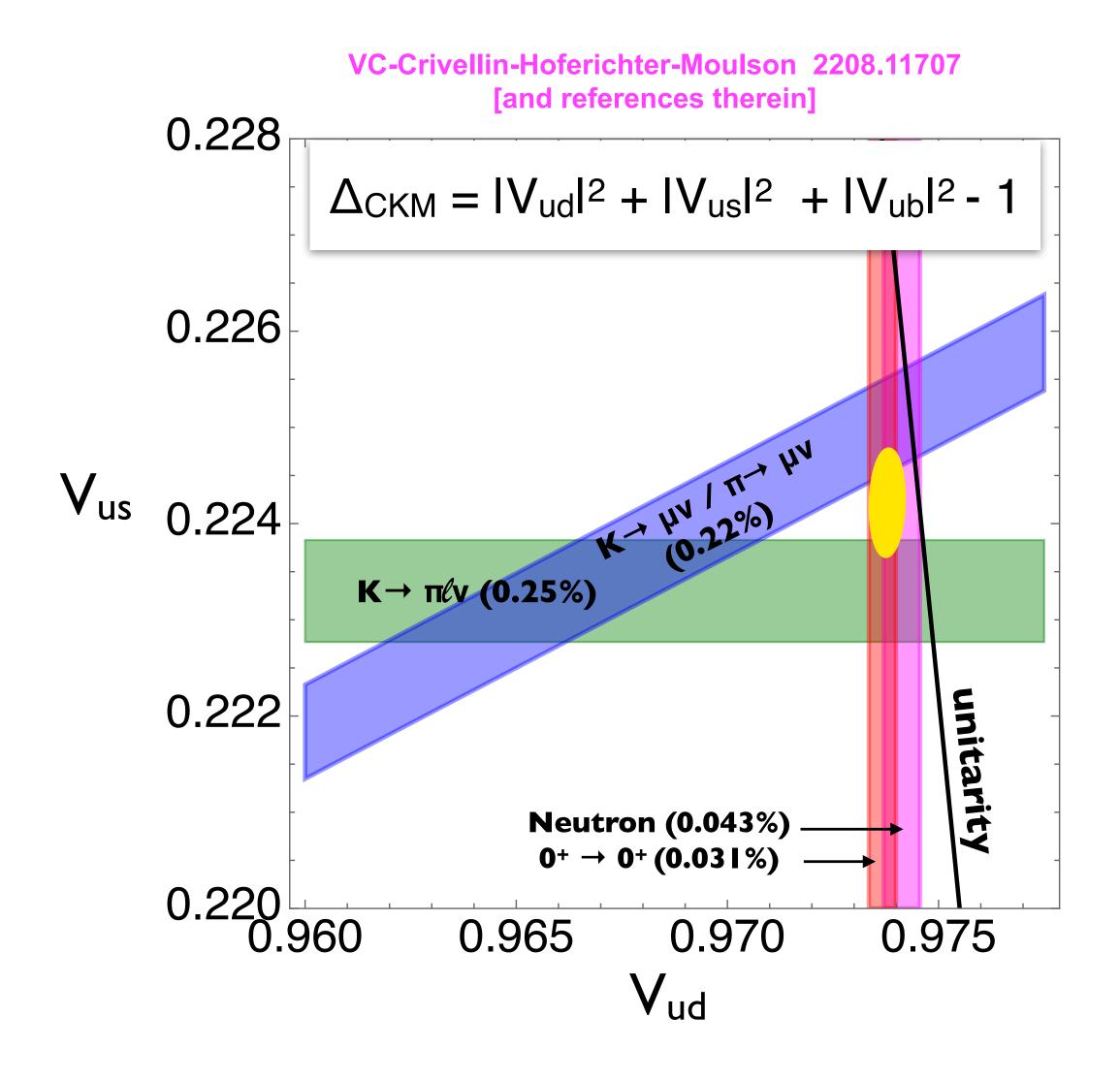
Tension among the most precise determinations

Tensions in the V_{ud}-V_{us} plane



- Bands don't intersect in the same region on the unitarity circle
- ~3 σ effect in global fit (Δ_{CKM} = -1.48(53) \times 10-3)

Tensions in the V_{ud}-V_{us} plane

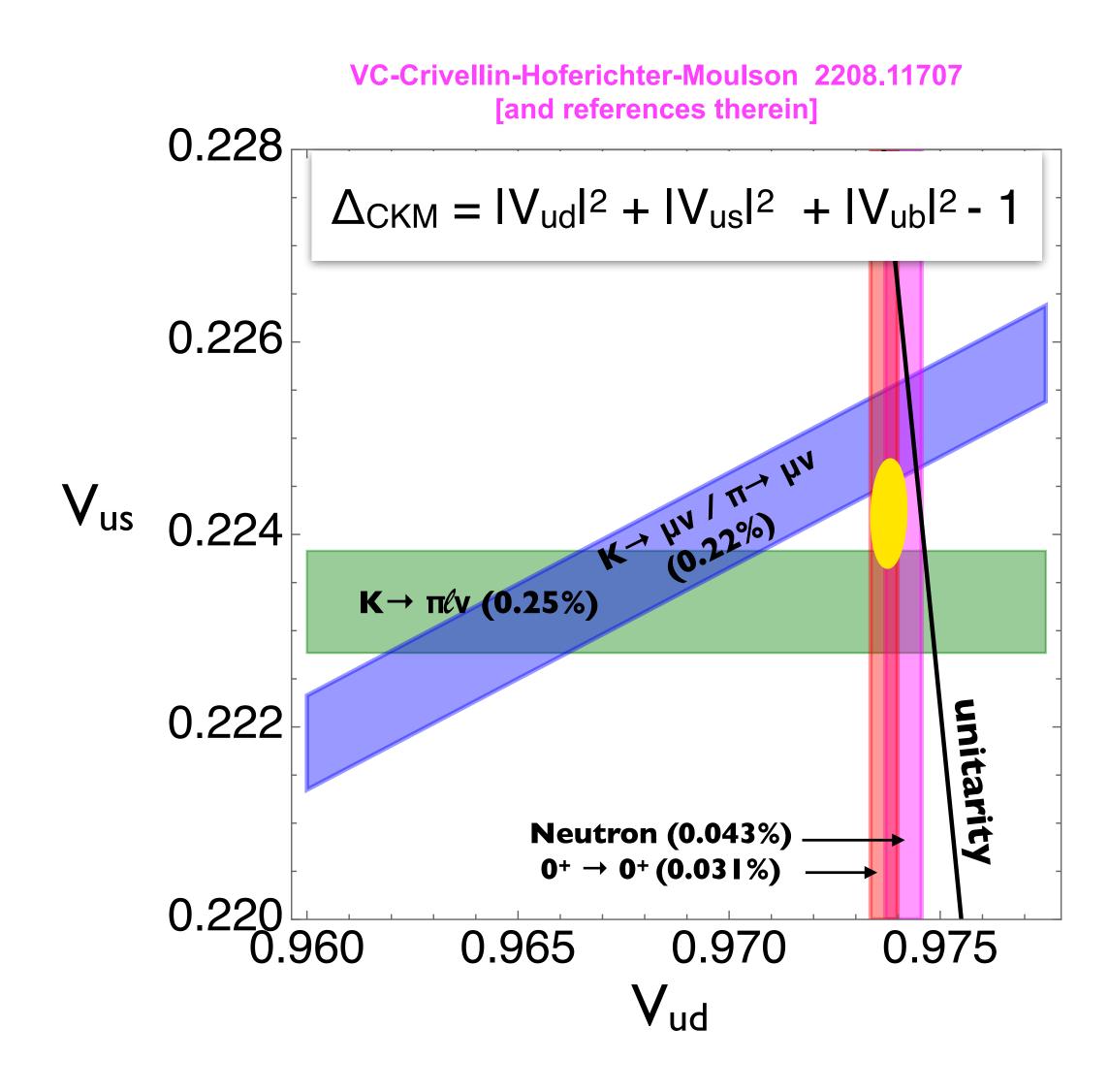


- Bands don't intersect in the same region on the unitarity circle
- ~3 σ effect in global fit (Δ_{CKM} = -1.48(53) \times 10-3)

For the enthusiasts

- Until ~2018, bands *did* intersect in the same region on the unitarity circle ($< 2\sigma$)
- Main changes since then:
 - V_{us} from KI3 decreased (<V> increased with smaller uncertainty, 2+I+I lattice QCD)
 - V_{ud} decreased (radiative corrections in nuclear & neutron increased with smaller uncertainty, dispersive)
 Seng et al., 1807.10197

Tensions in the V_{ud}-V_{us} plane



Next

- Closer look at selected channels
- Emphasis on recent developments in radiative corrections

A closer look at selected channels

	Hadron decays			Lepton decays
V _{ud}	$\pi^{\pm} \rightarrow \pi^{0} e \nu$ Nucl. $0^{+} \rightarrow 0^{+}$	$n \rightarrow pev$ Nucl. mirror decays	$\pi \to \mu \nu$	$ au o h_{NS} u$
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V_{ud} from pion β decay

$$\Gamma(\pi^+ \to \pi^0 e^+ \nu(\gamma)) = \frac{G_\mu^2 |V_{\rm ud}|^2 m_{\pi^+}^5 |f_+^{\pi}(0)|^2}{64\pi^3} (1 + RC_\pi) I_\pi,$$

• Vector form factor

$$f_{+}(0) = 1 - \frac{1}{(4\pi F_{\pi})^2} \frac{\left(M_{K^{+}}^2 - M_{K_0}^2\right)_{\text{QCD}}^2}{24M_{K}^2} = 1 + O\left(\frac{m_u - m_d}{\Lambda_{\text{QCD}}}\right)^2$$

Radiative corrections

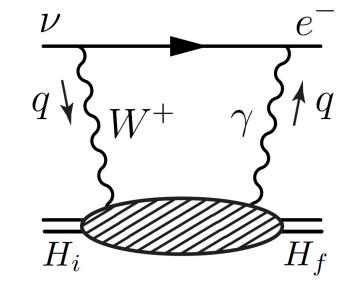
$$RC_{\pi} = 0.0342(10)$$
 (ChPT)

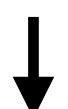
VC-Neufeld-Pichl 2002

Box diagram in Lattice QCD

$$RC_{\pi} = 0.0332(1)_{\gamma W}(3)_{HO}$$

Feng, Gorchtein, Jin, Ma, Seng, 2003.09798





Theory in good shape (cleanest channel)

0.3% total error on V_{ud} dominated by BR = $1.036(6)\times10^{-8}$ [PIBETA , hep-ex/ 0312030]

$$V_{ud}^{(\pi\beta)} = 0.97386 \, (281)_{BR} \, (9)_{\tau_{\pi}} \, (14)_{RC} \, (28)_{I_{\pi}} \, [283]_{\text{total}}$$

Uncertainty in π⁰ mass! [M. Hoferichter]

Experiment needs order-of-magnitude improvement in precision to be competitive → PIONEER @ PSI

2203.01908

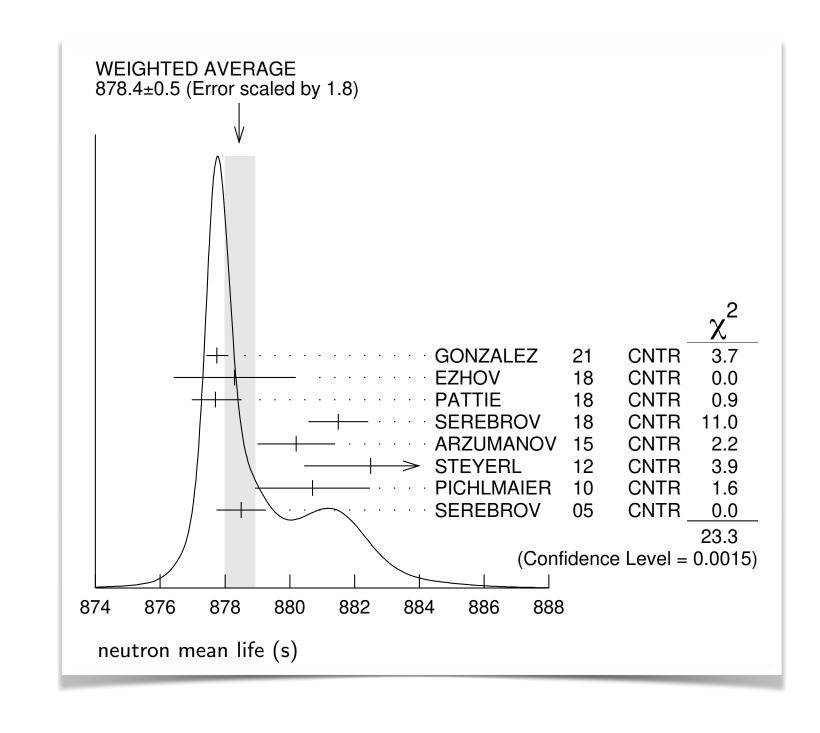
Vud from neutron decay

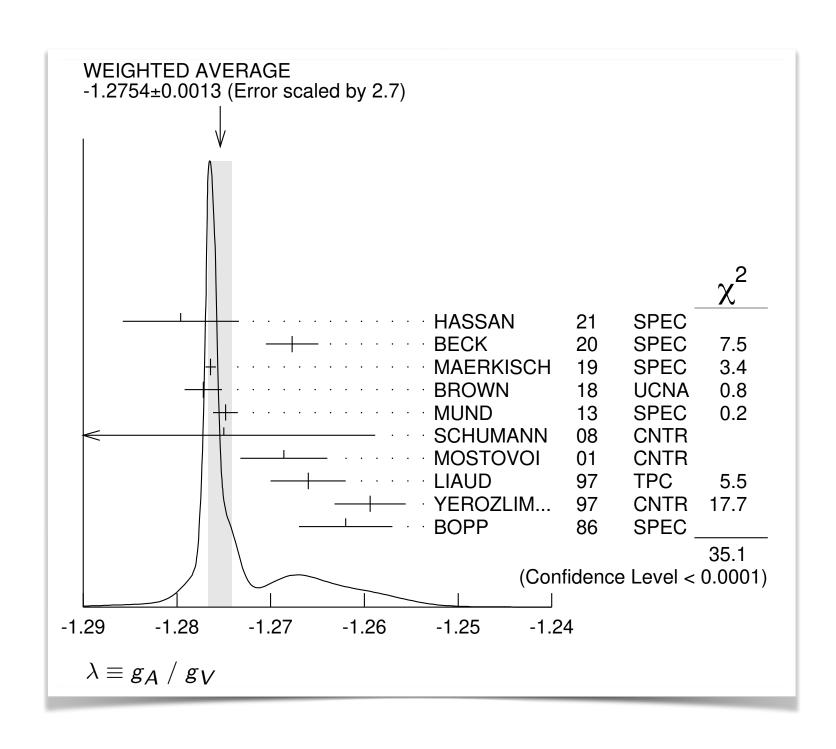
$$\lambda = g_A/g_V$$

$$\Gamma_n = \frac{G_F^2 |V_{ud}|^2 m_e^5}{2\pi^3} \left(1 + 3\lambda^2 \right) \cdot f_0 \cdot \left(1 + \Delta_f \right) \cdot \left(1 + \Delta_R \right),$$

Sirlin 1967-1978-1982
Seng et al. 1807.10197,
Czarnecki et al, 1907.06737,
Shiells et al. 2012.01580
Hayen 2010.07262,
Gorchtein-Seng 2106.09185

- Radiative corrections: radiative corrections in the Sirlin framework with dispersive input
- Experimental input: PDG averages include large scale factor, particularly for g_A / g_V





Vud from neutron decay

$$\lambda = g_A/g_V$$

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- Radiative corrections: radiative corrections in the Sirlin framework with dispersive input
- Experimental input: PDG averages include large scale factor, particularly for g_A / g_V

Single most precise measurements of lifetime and λ imply very competitive V_{ud} !

Maerkish et al, 1812.04666

Gonzalez et al, 2106.10375

$$V_{ud}^{\text{n,PDG}} = 0.97441(3)_{\Delta_f}(13)_{\Delta_R}(82)_{\lambda}(28)_{\tau_n}[88]_{\text{total}}$$

$$V_{ud}^{\text{n,best}} = 0.97413(3)_{\Delta_f}(13)_{\Delta_R}(35)_{\lambda}(20)_{\tau_n}[43]_{\text{total}}$$

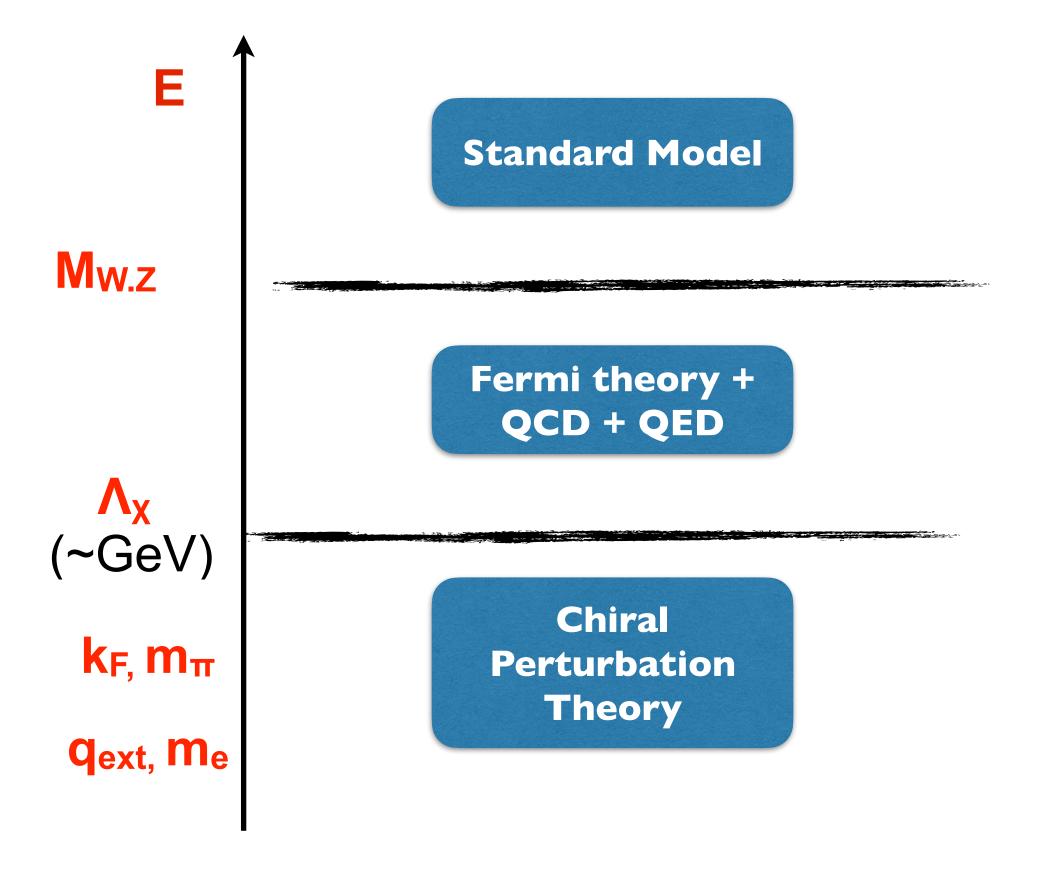
VC, Crivellin, Hoferichter. Moulson 2208.11707 and references therein

Need improvements in lifetime and g_A / g_V . Within reach in next 5 years

VC, W. Dekens, E. Mereghetti, O.Tomalak, 2306. 03138

• 'End-to-end' EFT approach for neutron decay, motivated by widely separated scales

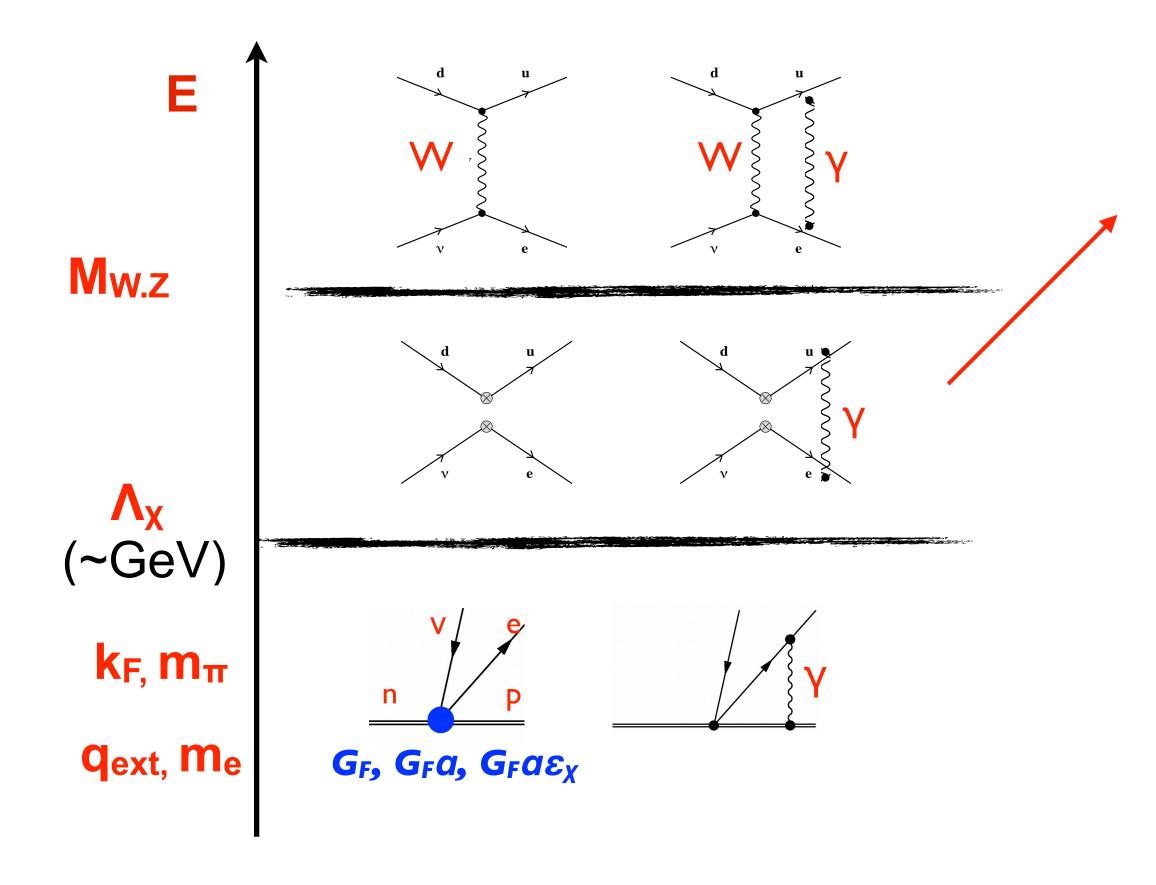
$$\Lambda_{BSM} >> M_W >> \Lambda_X >> Q \sim k_F \sim m_\pi >> m_e \sim q_{ext}$$



VC, W. Dekens, E. Mereghetti, O.Tomalak, 2306. 03138

• 'End-to-end' EFT approach for neutron decay, motivated by widely separated scales

$$\Lambda_{BSM} >> M_W >> \Lambda_X >> Q \sim k_F \sim m_\pi >> m_e \sim q_{ext}$$



$$\mathcal{L}_{\text{Fermi}} = -\frac{G_F}{\sqrt{2}} V_{ud} \, C_{\beta}(\mu) \, \bar{\ell} \gamma_{\alpha} (1 - \gamma_5) \nu_{\ell} \, \bar{u} \gamma^{\alpha} (1 - \gamma_5) d + \dots$$

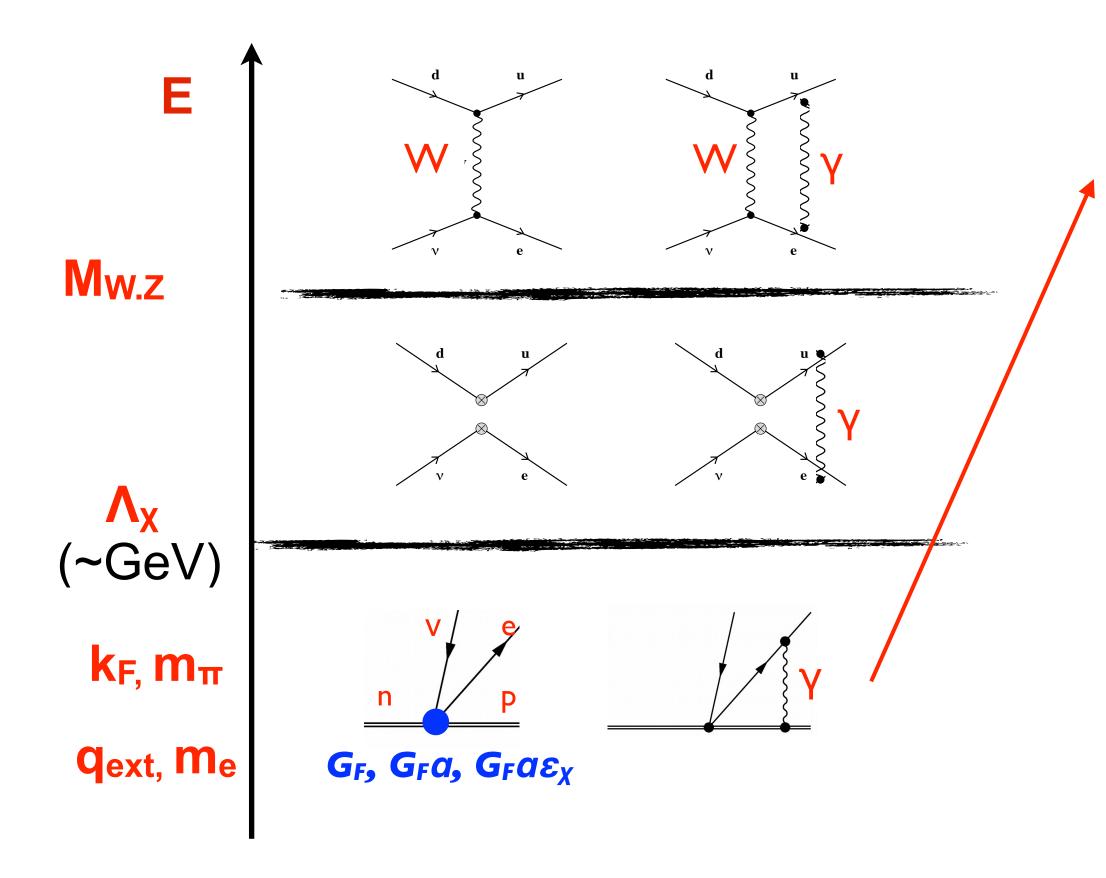
$$C_{\beta}(\mu) \sim 1 + \# (\alpha/\pi) \ln(M_{w}/\mu) + ...$$

Known to LL~ $(\alpha \ln(M_w/\mu)^n$ and NLL ~ $\alpha (\alpha_S \ln(M_w/\mu))^n$, $\alpha (\alpha \ln(M_w/\mu))^n$

VC, W. Dekens, E. Mereghetti, O.Tomalak, 2306. 03138

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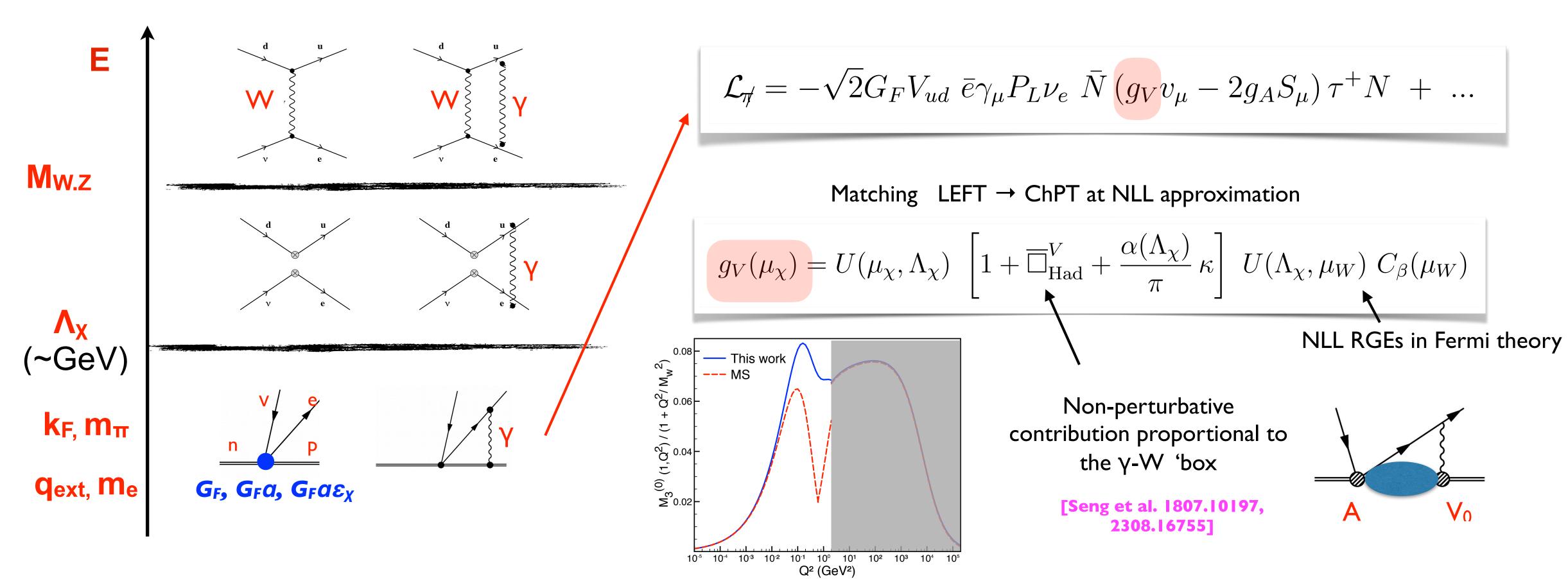


$$\mathcal{L}_{\pi} = -\sqrt{2}G_F V_{ud} \ \bar{e} \gamma_{\mu} P_L \nu_e \ \bar{N} \left(g_V v_{\mu} - 2g_A S_{\mu} \right) \tau^+ N + \dots$$

VC, W. Dekens, E. Mereghetti, O.Tomalak, 2306. 03138

• 'End-to-end' EFT approach for neutron decay, motivated by widely separated scales

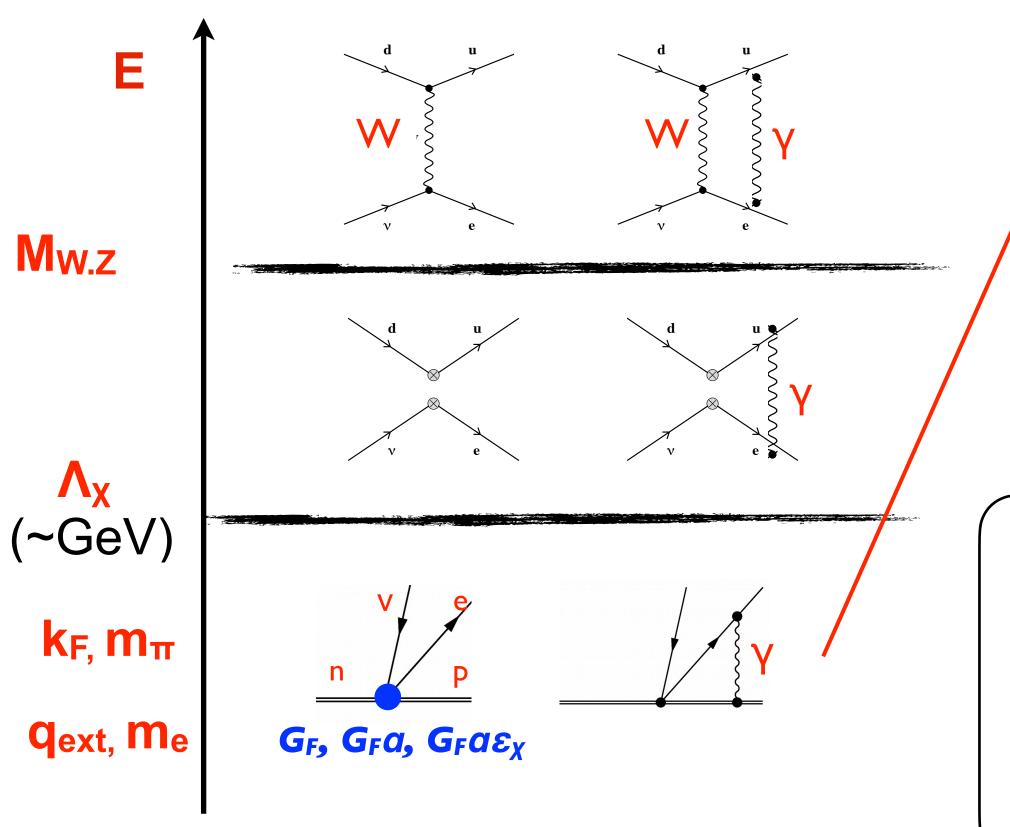
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VC, W. Dekens, E. Mereghetti, O. Tomalak, 2306. 03138

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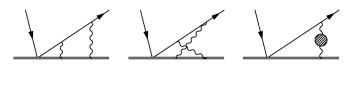
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Matching LEFT → ChPT at NLL approximation

$$g_{V}(\mu_{\chi}) = U(\mu_{\chi}, \Lambda_{\chi}) \left[1 + \overline{\square}_{\mathrm{Had}}^{V} + \frac{\alpha(\Lambda_{\chi})}{\pi} \kappa \right] U(\Lambda_{\chi}, \mu_{W}) C_{\beta}(\mu_{W})$$
NULL DOCE : CLOT

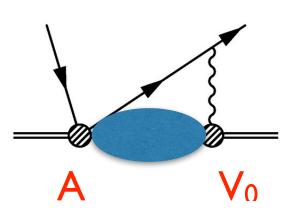
NLL RGE in ChPT

NLL ~ $\alpha (\alpha \ln(\Lambda_{\chi}/m_e))^n$



Non-perturbative contribution proportional to the γ-W 'box

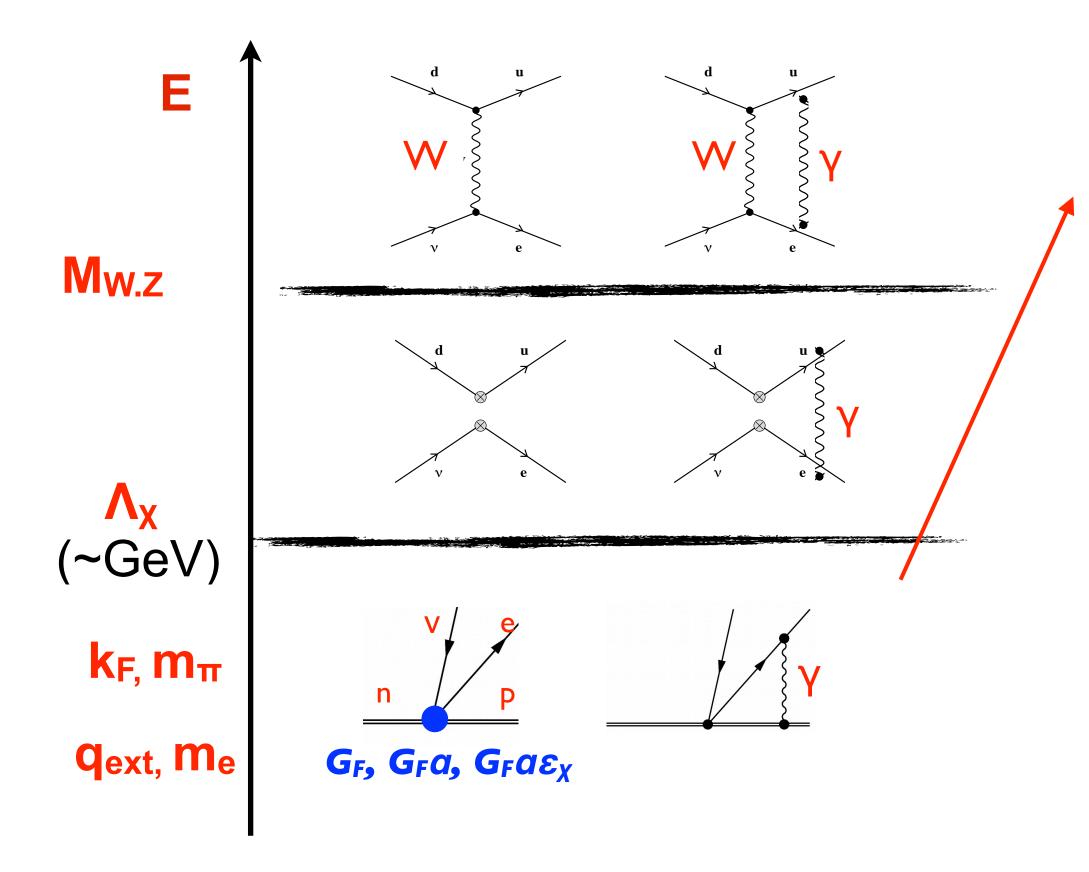
[Seng et al. 1807.10197, 2308.16755]



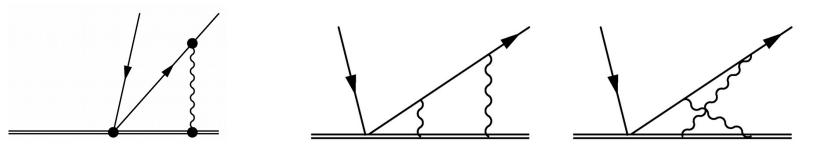
VC, W. Dekens, E. Mereghetti, O. Tomalak, 2306. 03138

• 'End-to-end' EFT approach for neutron decay, motivated by widely separated scales

$$\Lambda_{BSM} >> M_W >> \Lambda_X >> Q \sim k_F \sim m_\pi >> m_e \sim q_{ext}$$



 $O(\alpha, \alpha^2)$ matrix element in the low-energy EFT



No large logs but enhanced contributions $\sim (\pi \alpha/\beta)$, which we re-sum via the non-relativistic Fermi function *ansatz*

$$F_{NR}(\beta) = \frac{2\pi\alpha}{\beta} \frac{1}{1 - e^{-\frac{2\pi\alpha}{\beta}}} \approx 1 + \frac{\pi\alpha}{\beta} + \frac{\pi^2\alpha^2}{3\beta^2} + \dots \xrightarrow{m \to 0} 1 + \pi\alpha + \frac{\pi^2\alpha^2}{3} + \dots$$

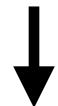
Enhanced terms are related to IR divergences in the loops, RG-based re-summation leads to (for $m_e \rightarrow 0$)

$$\exp\left[\frac{\pi\alpha}{\beta}\right] \xrightarrow[m\to 0]{} 1 + \pi\alpha + \frac{\pi^2\alpha^2}{2} + \dots \qquad \qquad \begin{array}{l} \text{Griend-Cao-Hill-} \\ \text{Plestid 2501.17916} \end{array}$$

Impact on V_{ud}

$$\lambda = g_A/g_V$$

$$\Gamma_n = \frac{G_F^2 |V_{ud}|^2 m_e^5}{2\pi^3} (1 + 3\lambda^2) \cdot f_0 \cdot (1 + \Delta_f) \cdot (1 + \Delta_R),$$



VC, W. Dekens, E. Mereghetti, O. Tomalak, 2306. 03138

Griend-Cao-Hill-Plestid 2501.17916

Griend-Cao-Hill-Plestid 2501.17916
$$\Delta_f = 3.573(5)_{\alpha \times \mathrm{recoil}} \% \quad \rightarrow \quad \Delta_f = 3.584(5)_{\alpha \times \mathrm{recoil}} \%$$

$$\Delta_R = 4.044(24)_{\text{Had}}(8)_{\alpha\alpha_s^2}(7)_{\alpha\epsilon_\chi^2}(5)_{\mu_\chi}[27]_{\text{total}}$$
 %

-0.024%

(Compared to pre-EFT analysis)

+0.061%

Impact on V_{ud}

$$\lambda = g_A/g_V$$

$$\Gamma_n = \frac{G_F^2 |V_{ud}|^2 m_e^5}{2\pi^3} (1 + 3\lambda^2) \cdot f_0 \cdot (1 + \Delta_f) \cdot (1 + \Delta_R),$$



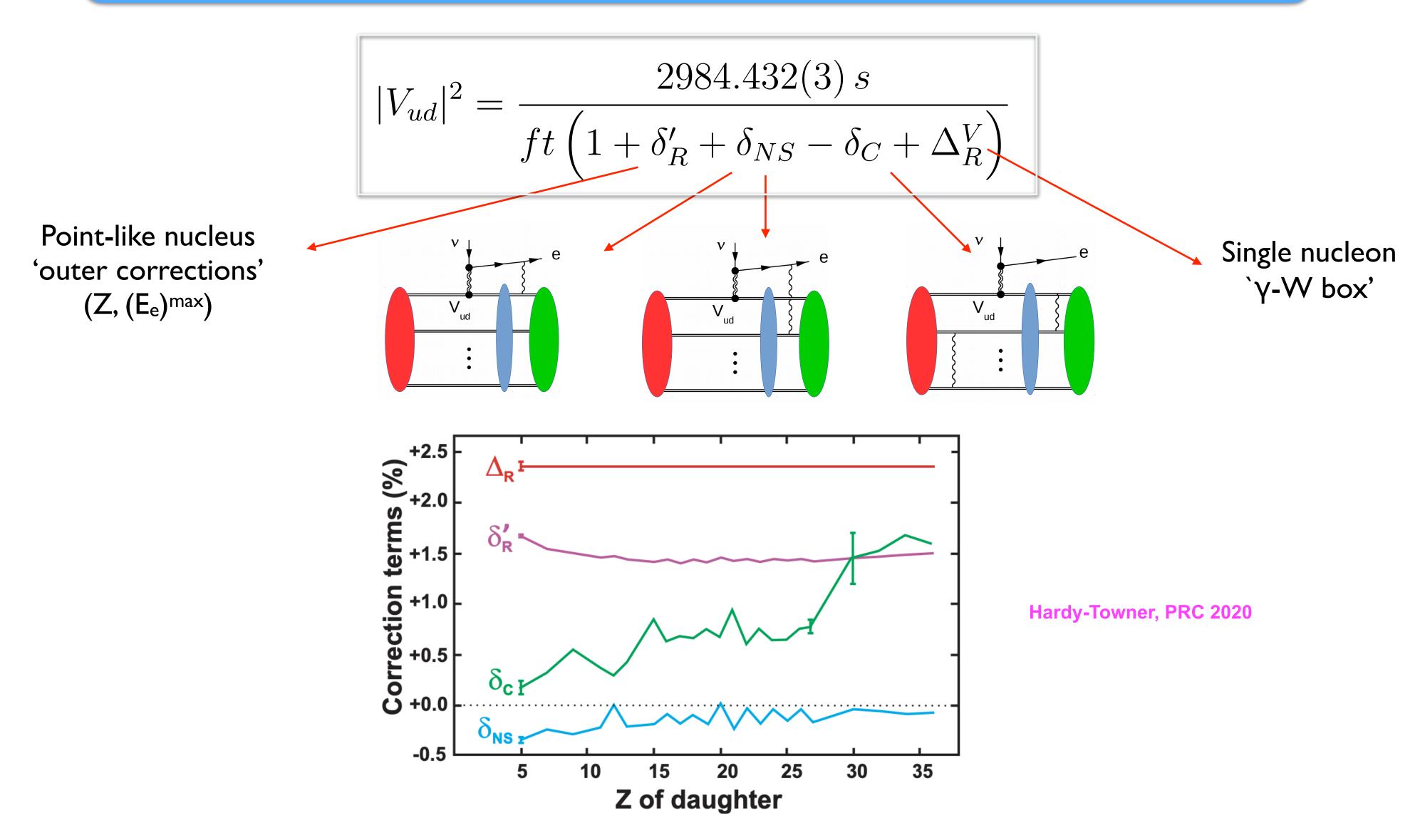
$$V_{ud}^{\text{n,PDG}} = 0.97424(2)_{\Delta_f}(13)_{\Delta_R}(82)_{\lambda}(28)_{\tau_n}[88]_{\text{total}}$$

$$V_{ud}^{\text{n,best}} = 0.97396(2)_{\Delta_f}(13)_{\Delta_R}(35)_{\lambda}(20)_{\tau_n}[42]_{\text{total}}$$

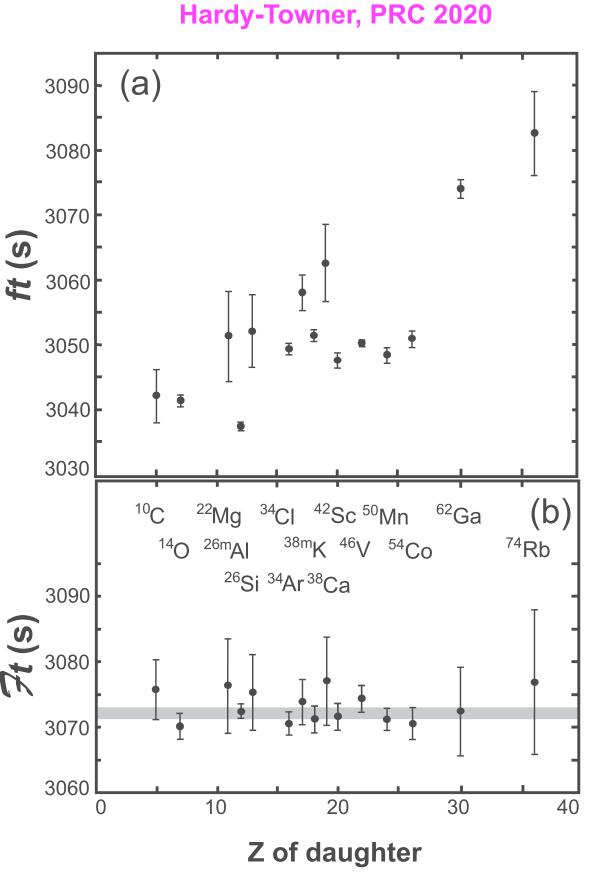
Overall shift of -0.0175% in V_{ud} (neutron) compared to pre-EFT literature —

larger than hadronic-structure uncertainty and relevant for target precision of 0.02%

V_{ud} from nuclear 0+ \rightarrow 0+ beta decays



V_{ud} from nuclear $0^+ \rightarrow 0^+$ beta decays



$$|V_{ud}|^2 = \underbrace{ft\left(1 + \delta_R' + \delta_{NS} - \delta_C + \Delta_R^V\right)}_{ft\left(1 + \delta_R' + \delta_{NS} - \delta_C + \Delta_R^V\right)}$$

$$V_{ud}^{0^+ \to 0^+} = 0.97367(11)_{\exp}(13)_{\Delta_R^V}(27)_{NS}[32]_{\text{total}}$$

Lots of activity

New analysis of nuclear weak form factors and phase space f

Gorchtein, Seng 2311.00044 and references therein

- New approaches towards structure dependent corrections $\delta_{C,NS}$
- Controlled uncertainties will be achieved for a range of A=10, 14, ...

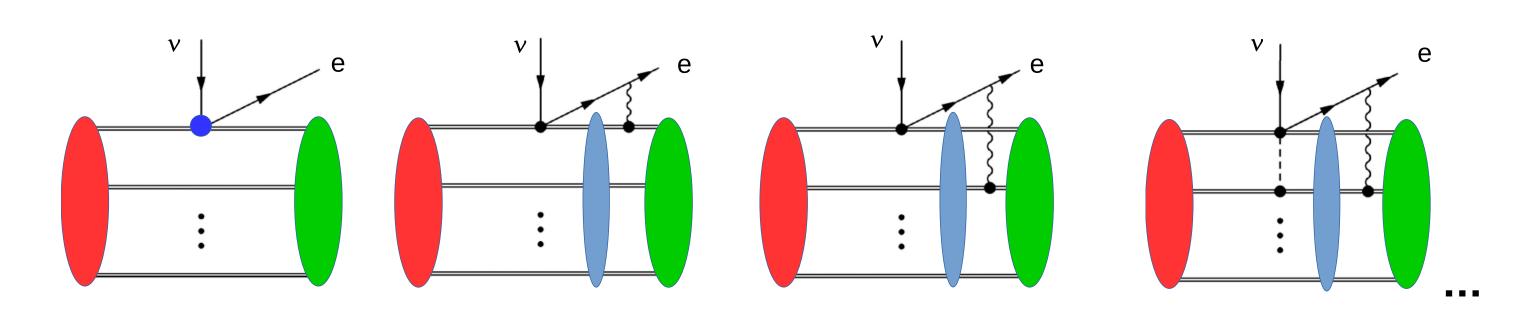
VC, W. Dekens,, J.de Vries, S. Gandolfi, M. Hoferichter, E, Mereghetti, 2405.18469, 2405.18464

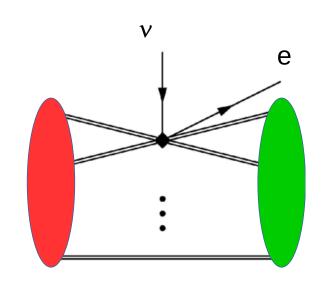
- Chiral EFT (NN, NNN, ...) with dynamical leptons and photons
- Hard photons leave behind local multi-nucleon electroweak operators (as in the one-nucleon case)

$$\mathcal{L}_W^{2b} = -\sqrt{2}e^2 G_F V_{ud} \ \bar{e}_L \gamma_0 \nu_L \times$$

$$N^{\dagger} \tau^+ N \ \left(e^2 g_{V1}^{NN} \ N^{\dagger} N + e^2 g_{V2}^{NN} \ N^{\dagger} \tau^3 N \right)$$

• Soft, potential, and ultra-soft photons contribute to multi-nucleon amplitudes





Soft: $(q^0, |\mathbf{q}|) \sim Q \sim k_F$ Potential: $(q^0, |\mathbf{q}|) \sim (Q^2/m_N, Q)$

Ultrasoft $(q^0, |\mathbf{q}|) \sim Q^2/m_N \ll k_F$

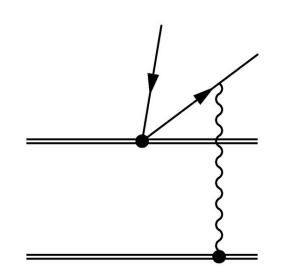
VC, W. Dekens,, J.de Vries, S. Gandolfi, M. Hoferichter, E, Mereghetti, 2405.18469, 2405.18464

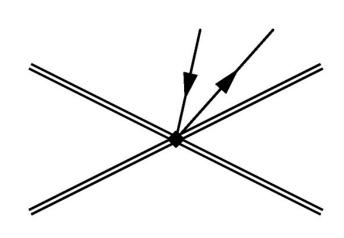
- Chiral EFT (NN, NNN, ...) with dynamical leptons and photons
- Hard photons leave behind local multi-nucleon electroweak operators (as in the one-nucleon case)

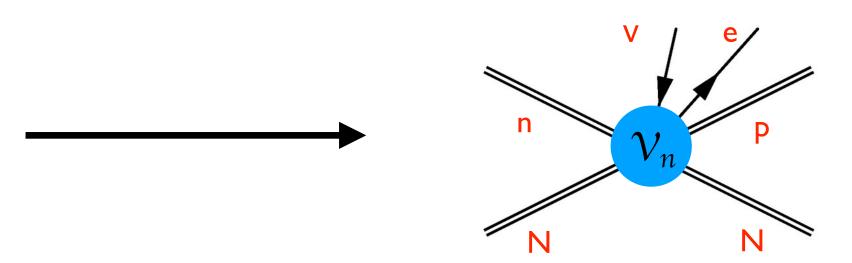
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$$N^{\dagger} \tau^+ N \ \left(e^2 g_{V1}^{NN} \ N^{\dagger} N + e^2 g_{V2}^{NN} \ N^{\dagger} \tau^3 N \right)$$

• 'Integrate out' soft & potential photons (and π 's) \rightarrow obtain EW n-body transition operators ('potentials')







$$\mathcal{V}_E \sim \frac{e^2 E_{e,\nu}}{\mathbf{q}^4} \quad \mathcal{V}_{\mathrm{mag}} \sim \frac{e^2}{m_N \mathbf{q}^2}$$

$$\mathcal{V}_{\text{contact}} \sim e^2 g_{V_1, V_2}^{NN} \sim e^2 \frac{1}{\Lambda_{\chi} F_{\pi}^2}$$

$$H_{EW} \supset \sqrt{2}G_F V_{ud} \ \bar{e}_L \gamma_0 \nu_L \times \sum_n c_n \mathcal{V}_n$$

 $G_F\alpha\epsilon_\pi$

GFae_X

$$\varepsilon_X = Q/\Lambda_X$$

G_Fae_X

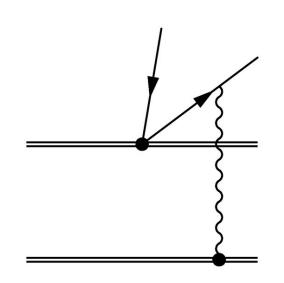
VC, W. Dekens,, J.de Vries, S. Gandolfi, M. Hoferichter, E, Mereghetti, 2405.18469, 2405.18464

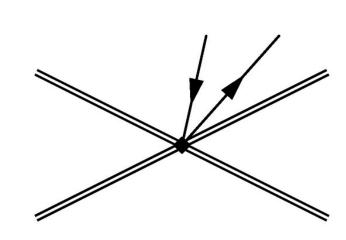
- Chiral EFT (NN, NNN, ...) with dynamical leptons and photons
- Hard photons leave behind local multi-nucleon electroweak operators (as in the one-nucleon case)

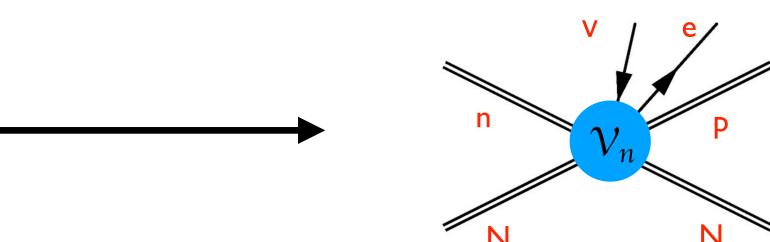
$$\mathcal{L}_W^{2b} = -\sqrt{2}e^2 G_F V_{ud} \ \bar{e}_L \gamma_0 \nu_L \times$$

$$N^{\dagger} \tau^+ N \ \left(e^2 g_{V1}^{NN} \ N^{\dagger} N + e^2 g_{V2}^{NN} \ N^{\dagger} \tau^3 N \right)$$

• 'Integrate out' soft & potential photons (and π 's) \rightarrow obtain EW n-body transition operators ('potentials')







Currently unknown LECs

$$\mathcal{V}_E \sim rac{e^2 E_{e,\nu}}{\mathbf{q}^4} \quad \mathcal{V}_{\mathrm{mag}} \sim rac{e^2}{m_N \mathbf{q}^2}$$

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 $G_F\alpha\epsilon_\pi$

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VC, W. Dekens,, J.de Vries, S. Gandolfi, M. Hoferichter, E, Mereghetti, 2405.18469, 2405.18464

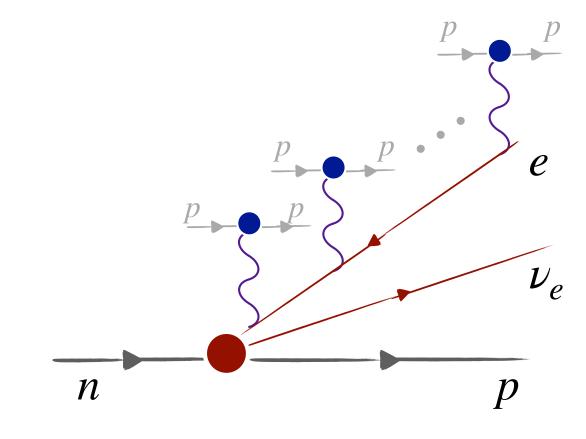
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• 'Integrate out' soft & potential photons (and π 's) \rightarrow obtain EW n-body transition operators ('potentials')

• Ultrasoft photons: Z-dependent running of effective couplings between m_π and m_e & matrix elements at μ ~ m_e



Courtesy of W. Dekens

Impact on V_{ud}: exploratory studies in QMC

140 \rightarrow 14N: δ_{NS} contributions in rough agreement with corresponding terms in Hardy-Towner 2020

VC, W. Dekens,, J.de Vries, S. Gandolfi, M. Hoferichter, E, Mereghetti, 2405.18469, 2405.18464

$$V_{ud}\big|_{14O} = 0.97411(10)_{\rm exp}(12)_{g_V}(22)_{\mu}(12)_{\delta_C}(43)_{g_V^{NN}}[55]_{\rm tot}$$
 Residual scale dependence due to missing terms of O(q27) in the Fermi function
$$g_{V1,V2}^{NN} = 1/(4m_N F_\pi^2)$$

 $O(\alpha^2 Z)$ in the Fermi function

$$V_{ud}^{\rm HT}|_{^{14}O} = 0.97405[37]_{\rm tot}$$
 (31) from $\delta_{\rm NS}$

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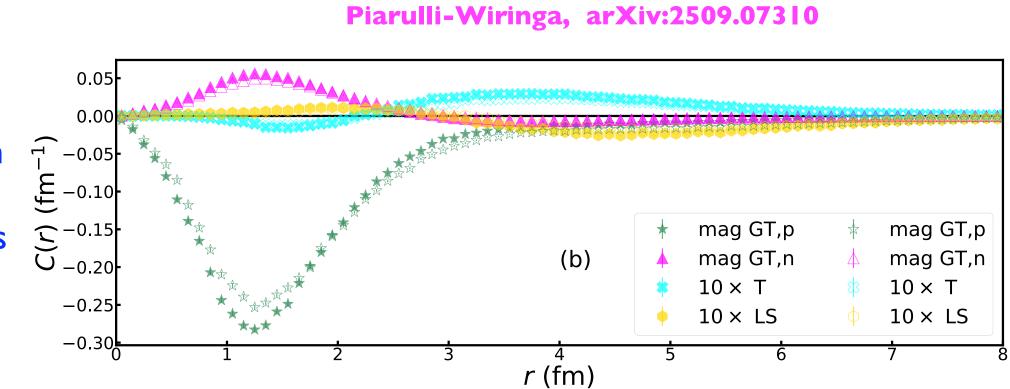
Largest uncertainty from unknown LECs. Assumes $g_{V1,V2}^{NN} = 1/(4m_N F_{\pi}^2)$

$$V_{ud}^{\rm HT}|_{^{14}O} = 0.97405[37]_{\rm tot}$$
 (31) from $\delta_{\rm NS}$

• $10C \rightarrow 10B$:

$$V_{ud}|_{^{10}\text{C}} = 0.97355(66)_{\text{exp}}(12)_{g_V}(17)_{\mu}(9)_{\delta_C}(38)_{g_V^{\text{NN}}}$$

0.02% spread from use of different chiral Hamiltonians



Empty and filled symbols correspond to two different chiral interactions

King-Carlson-Flores-Gandolfi-Mereghetti-Pastore-

Impact on V_{ud}: exploratory studies in QMC

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VC, W. Dekens,, J.de Vries, S. Gandolfi, M. Hoferichter, E, Mereghetti, 2405.18469, 2405.18464

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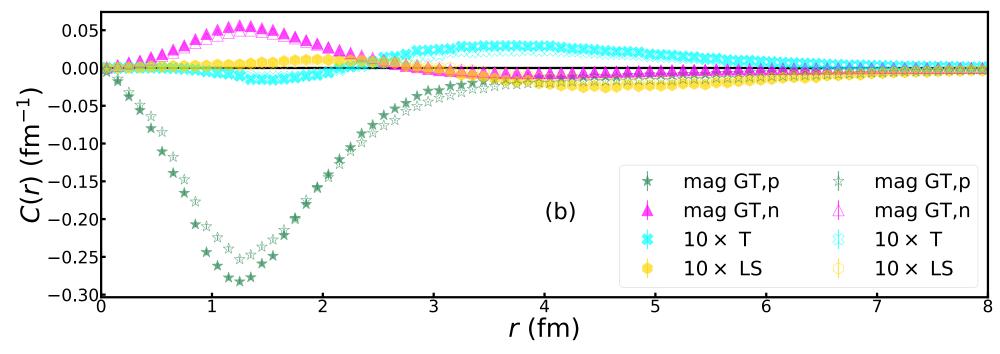
$$V_{ud}|_{^{10}\text{C}}^{\text{HT}} = 0.97318(66)_{\text{exp}}(9)_{\Delta_R^V}(24)_{\delta_{NS}}(9)_{\delta_C}$$

$$V_{ud}|_{^{10}\text{C}}^{\text{NCSM}} = 0.97317(66)_{\exp}(9)_{\Delta_R^V} (16)_{\delta_{NS}} (9)_{\delta_C}$$

0.02% spread from use of different chiral Hamiltonians

Gennari, Drissi, Gorchtein, Navratil,, Seng, Phys. Rev. Lett. 134, 012501 (2025)



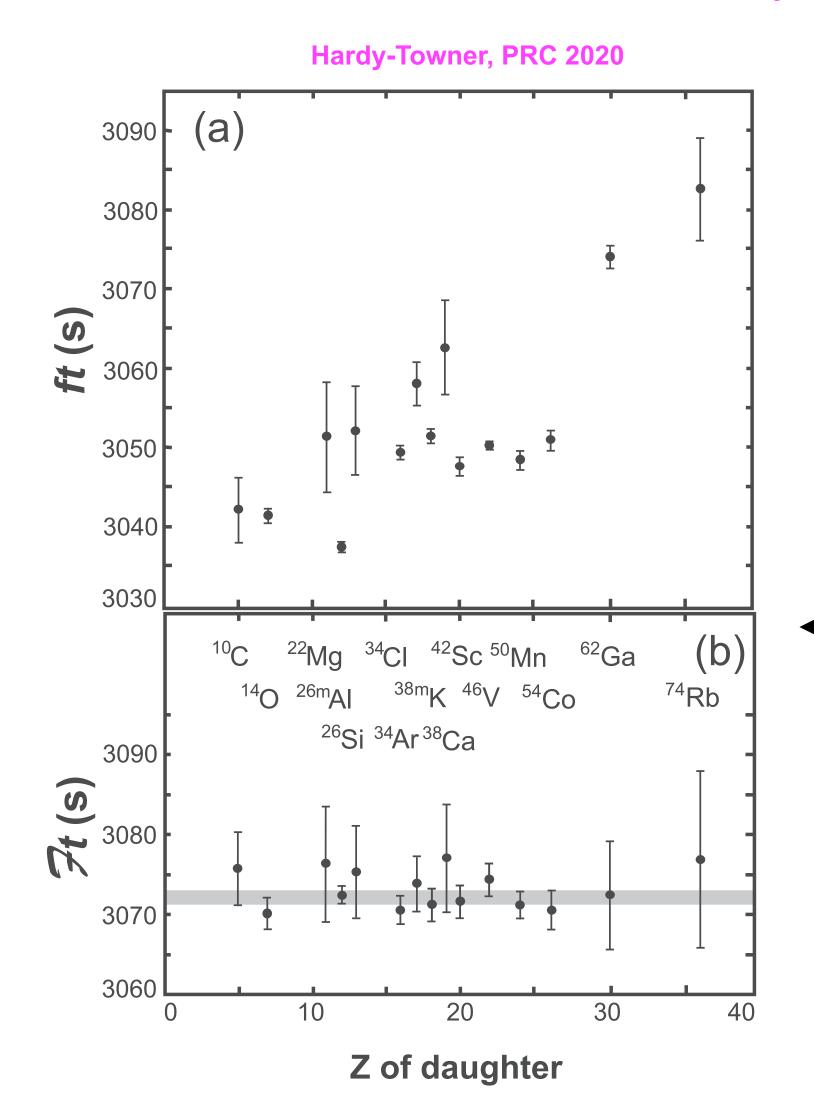


Empty and filled symbols correspond to two different chiral interactions

Reasonable agreement with HT & dispersive + NCSM

Path forward in the EFT approach

VC, W. Dekens,, J.de Vries, S. Gandolfi, M. Hoferichter, E, Mereghetti, 2405.18469, 2405.18464



- EFT has identified new method to compute structuredependent corrections and (temporarily) increased the uncertainty. But in the long run it will allow for robust uncertainty quantification
- LECs can be obtained by
 - Fitting data (along with V_{ud} and possibly BSM effective couplings) once NME calculations for several isotopes become available
 - Theory: dispersive analysis, Lattice QCD

V_{us} from $K \rightarrow \pi I V$ decays

$$\Gamma_{K \to \pi \ell \nu(\gamma)} = \frac{C_K^2 G_F^2 S_{EW} |V_{us}|^2 M_K^5}{192\pi^3} |f_+^{K\pi}(0)|^2 I_{K\ell} \left(1 + 2\Delta_{K\ell}^{EM} + 2\Delta_K^{IB}\right)$$

• Lattice calculations of $\langle \pi | V | K \rangle @ 0.2\%$:

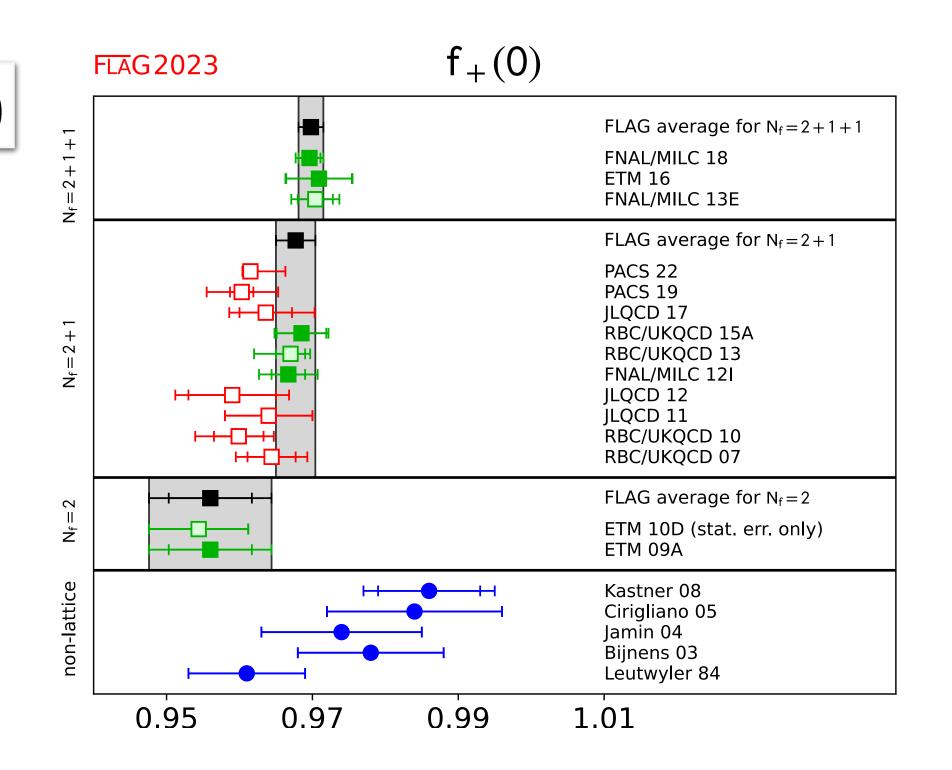
$$f_+^{K\pi}(0) = 0.9698(17)$$

• New radiative corrections based on current algebra + lattice QCD + ChPT. Consistent with old ChPT, with reduced uncertainties

	Cirigliano et al. '08	Seng et al. '21
$\Delta^{EM}(K^0_{e3})$ [%]	0.50 ± 0.11	0.580 ± 0.016
$\Delta^{EM}(K^+_{e3})$ [%]	0.05 ± 0.12	0.105 ± 0.023
$\Delta^{EM}(K^+_{\ \mu 3})$ [%]	0.70 ± 0.11	0.770 ± 0.019
$\Delta^{EM}(K^0_{\mu 3})$ [%]	0.01 ± 0.12	0.025 ± 0.027

NEW: Seng et al, 1910.13209, 2103.00975. 2103.4843. 2107.14708. 2203.05217. Ma et al. 2102.12048

OLD: VC, Giannotti, Neufeld 0807.4607



V_{us} from $K \rightarrow \pi I V$ decays

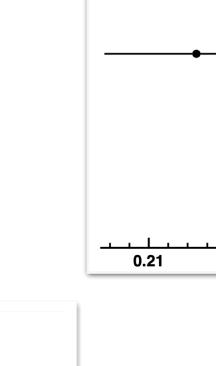
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Lattice calculations of $<\pi |V|K> @ 0.2\%$: $f_{\perp}^{K\pi}(0) = 0.9698(17)$

$$f_+^{K\pi}(0) = 0.9698(17)$$

- New radiative corrections based on current algebra + lattice QCD + ChPT. Consistent with old ChPT, with reduced uncertainties
- Experimental input has received only small updates since 2010

Flavianet WG, 1005.2323 Moulson 1704.04104



 $|V_{us}|f_+(0)$

 $K_I \mu 3$

 $K_{S}e3$

 $K^{\pm}e3$

 $K^{\pm}\mu 3$

$$V_{us}^{K_{\ell 3}} = 0.22330(35)_{\exp}(39)_{f_{+}}(8)_{RC+IB}[53]_{total}$$

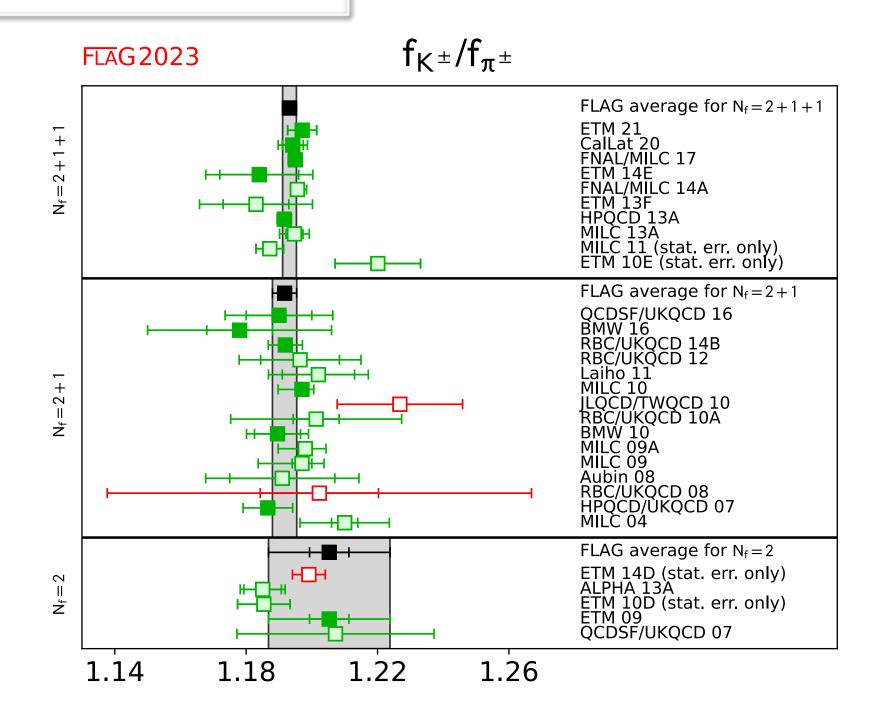
Potential issue: definition of 'isosymmetric QCD' in lattice (f₊(0)) vs calculations of $\Delta^{\text{EM, IB}}$

V_{us} from $K \rightarrow \mu \nu$ decays

$$\frac{|V_{us}|}{|V_{ud}|} \frac{f_K}{f_{\pi}} = \left(\frac{\Gamma_{K \to \mu\nu(\gamma)} \ m_{\pi^{\pm}}}{\Gamma_{\pi \to \mu\nu(\gamma)} \ m_{K^{\pm}}}\right)^{1/2} \frac{1 - m_{\mu}^2 / m_{\pi^{\pm}}^2}{1 - m_{\mu}^2 / m_{K^{\pm}}^2} \left(1 - \frac{\Delta_{\text{RC+IB}}^{K\pi}}{2}\right)$$

- Lattice QCD calculations of F_K/F_{π} are at the 0.2% level
- First calculations of radiative and isospin-breaking corrections in LQCD.
 Compatible with ChPT, factor of ~2 more precise

ChPT:	LQCDI: Di Carlo et al.,	LQCD2: Boyle et al.,
VC-Neufeld, 1102.0563	1904.08731	2211.12865
$\Delta_{\text{RC+IB}}^{K\pi} = -1.12(21)\%$	$\Delta_{\text{RC+IB}}^{K\pi} = -1.26(14)\%$	$\Delta_{\text{RC+IB}}^{K\pi} = -0.86(40)\%$

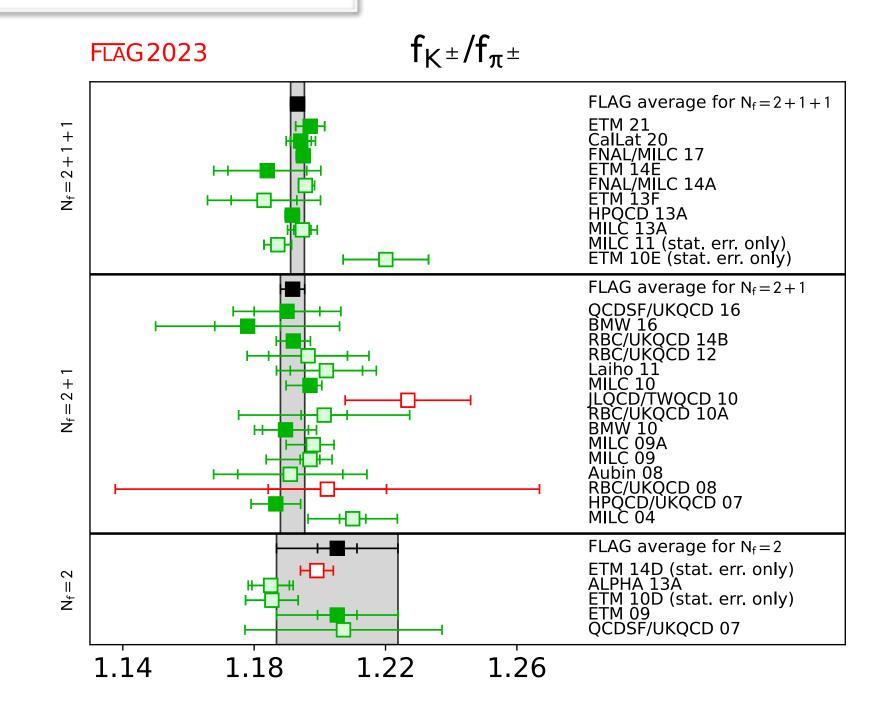


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Potential issue:

 $K_{\mu 2}$ BR dominated by one measurement (KLOE)

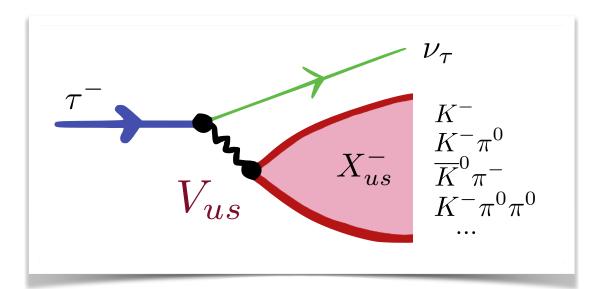
 $K_{\mu 3}/K_{\mu 2}$ BR measurement at 0.2-0.5% would have significant impact (NA62)

$$\frac{V_{us}}{V_{ud}}\Big|_{K_{\ell 2}/\pi_{\ell 2}} = 0.23108(23)_{\exp}(42)_{F_K/F_{\pi}}(16)_{RC+IB}[51]_{total}$$

Vus from tau decays

• Inclusive $(\tau \rightarrow X_s v)$: need integrated spectral functions (exp) + theory (pQCD (OPE) \rightarrow Lattice QCD)

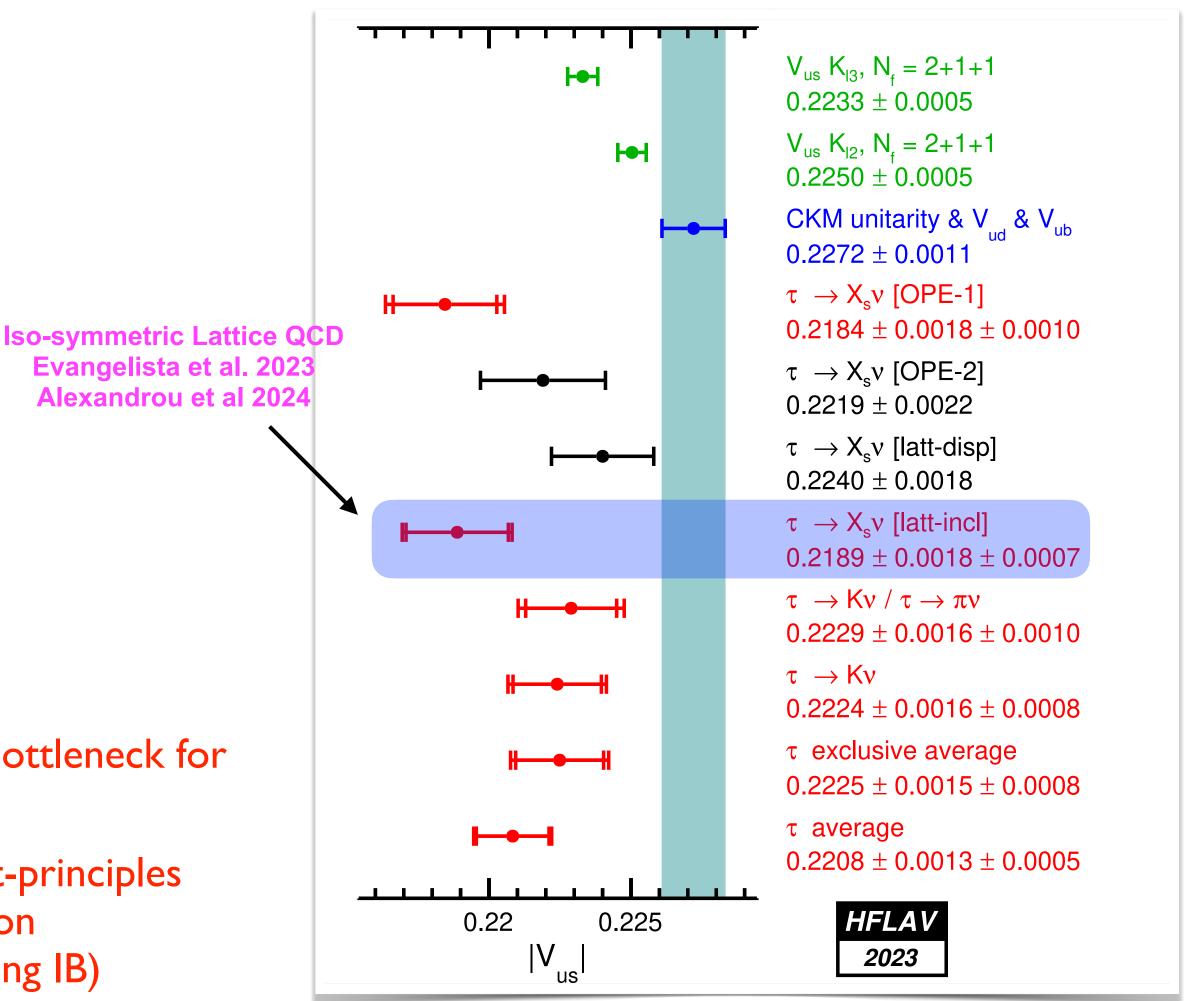
M. Di Carlo's talk at CKM 2025



• Exclusive $(\tau \rightarrow KV / \tau \rightarrow \pi \nu)$: need partial widths, decay constants (LQCD) & radiative corrections

Theory prospects:

- (I) Radiative corrections are a bottleneck for exclusive modes;
 - (2) lattice QCD provides first-principles inclusive determination (in the future also including IB)



Experimental prospects:

Belle-II and possibly tau-charm factory & FCC-ee

Summary of expected / desired developments

• Experiment:

- Neutron decay: aim for $\delta \tau_n \sim 0.1s$ [UCN τ +] and $\delta g_A/g_A \sim 0.01\%$ [PERC] to get $\delta V_{ud} \sim 1.5 \cdot 10^{-4}$
- Pion beta decay BR: 6x to 10x at PIONEER phases II, III [~10 years]
- New $K_{\mu 3}/K_{\mu 2}$ BR measurement @0.2% at NA62 will shed light on KI3 vs KI2 tension
- τ decays: Belle-II will reduce experimental uncertainties by > 2x

• Theory:

- Radiative corrections in lattice QCD+QED or hybrid: $K \to \pi l \nu$, $\pi^+ \to \pi^0 e^+ \nu$, $n \to pe\nu$, $\tau \to K \nu$, τ inclusive
- Nuclear decays: EFT for radiative corrections coupled to first-principles nuclear calculations for δ_{NS} , δ_{C}

Summary and outlook

The Cabibbo angle is the cornerstone of the CKM matrix and the Cabibbo universality test is a precision tool to explore what may lie beyond the Standard Model

• Current tensions in Cabibbo universality test could point to new physics at $\Lambda \sim$ few TeV

For a detailed analysis in the SM-EFT see VC, W. Dekens, J. De Vries, E. Mereghetti, T. Tong, 2311.00021

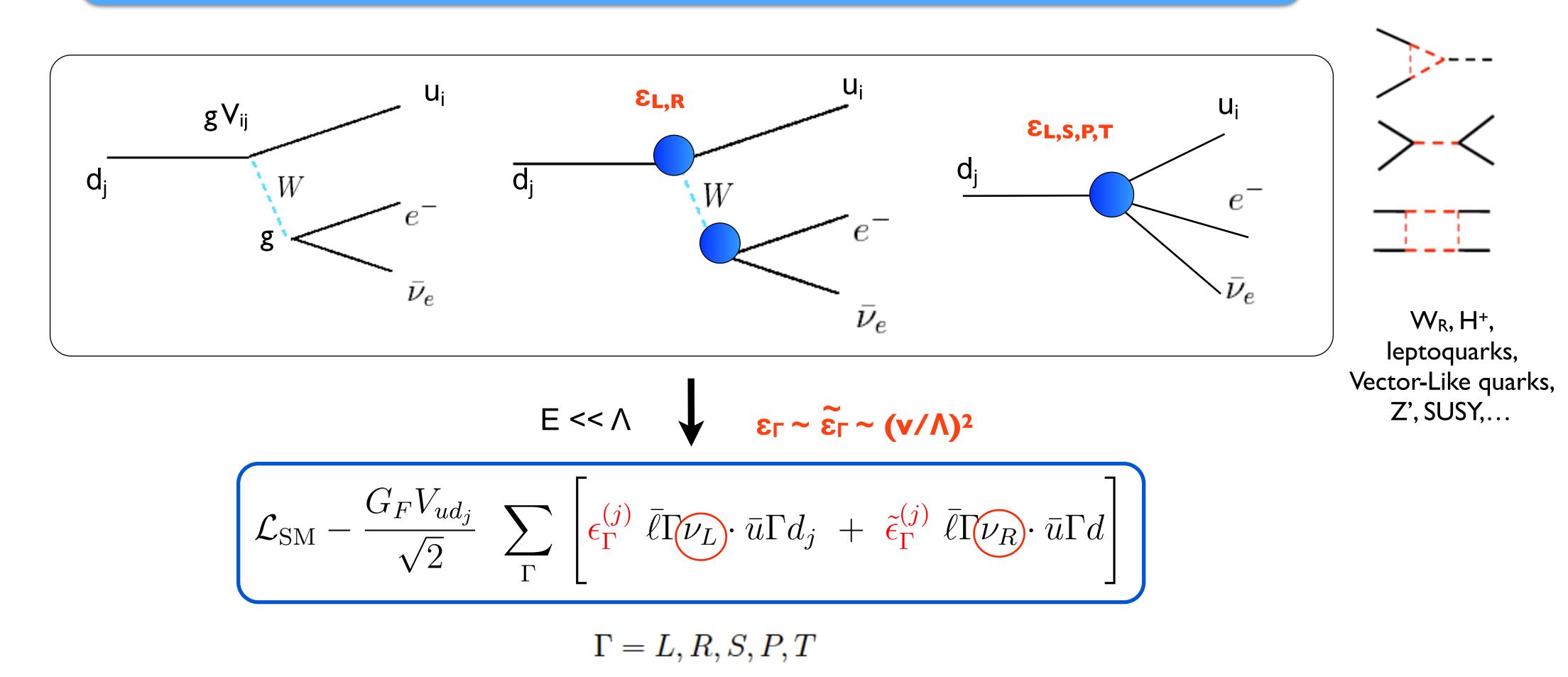
- However, further scrutiny is needed
 - Experiment: neutron, K, π , τ
 - Theory: lattice QCD+QED for neutron, K, π , τ ; EFT+ 'ab-initio' methods for nuclei

Expect decisive improvements in the 5-10 year frame

Backup

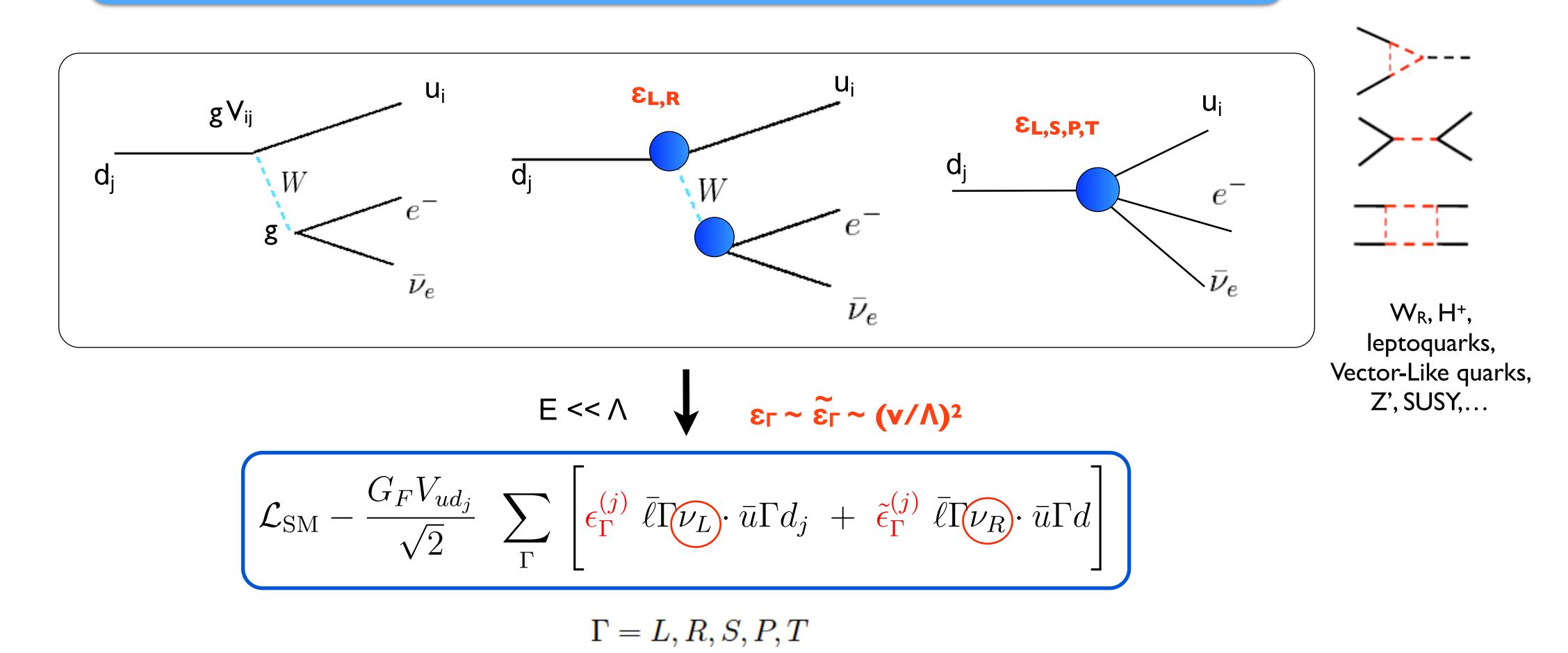
Cabibbo universality and physics beyond the Standard Model

Semileptonic processes beyond the SM



BSM effects parameterized by 10(ud) + 10(us) effective couplings at E ~ GeV They map into vertex corrections and 4-Fermion interactions above the EW scale

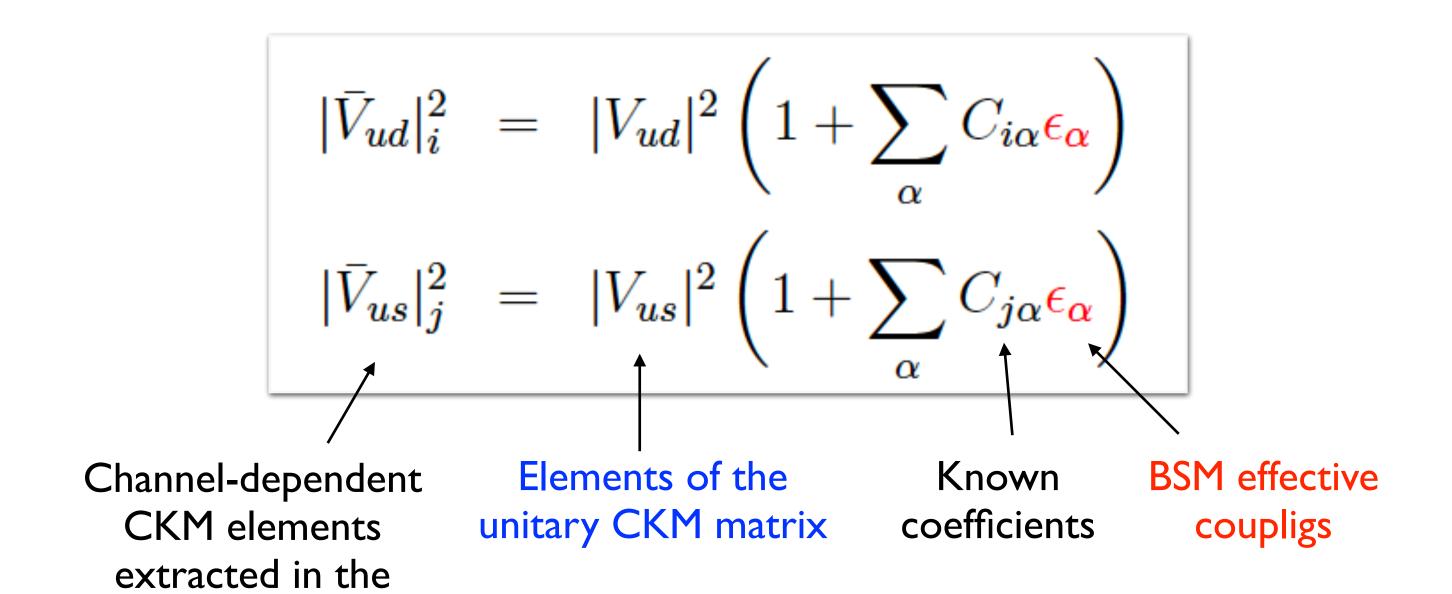
Semileptonic processes beyond the SM



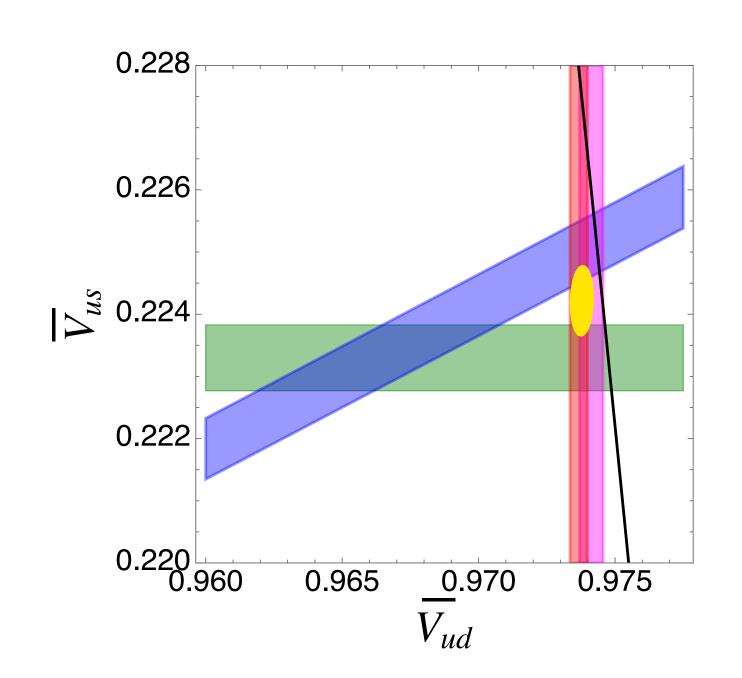
 Δ_{CKM} tension confirmed: points to specific new physics

 Δ_{CKM} tension removed: strong constraints, complementary to traditional 'precision electroweak observables'

Corrections to V_{ud} and V_{us}



'SM-like analysis'

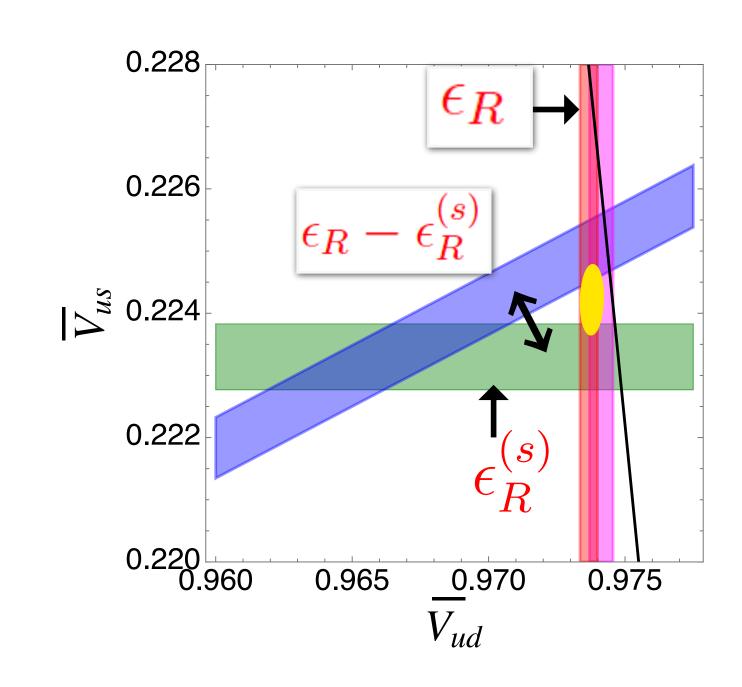


Find set of ε 's so that V_{ud} and V_{us} bands meet on the unitarity circle

Corrections to V_{ud} and V_{us}

$$|\bar{V}_{ud}|_i^2 = |V_{ud}|^2 \left(1 + \sum_{\alpha} C_{i\alpha} \epsilon_{\alpha}\right)$$

$$|\bar{V}_{us}|_j^2 = |V_{us}|^2 \left(1 + \sum_{\alpha} C_{j\alpha} \epsilon_{\alpha}\right)$$
 Channel-dependent CKM elements of the unitary CKM matrix coefficients extracted in the



Alioli et al 1703.04751 Grossman-Passemar-Schacht 1911.07821 VC-Crivellin-Hoferichter-Moulson 2208.11707 VC, W. Dekens, J. De Vries, E. Mereghetti, T. Tong, 2311.00021

'SM-like analysis'

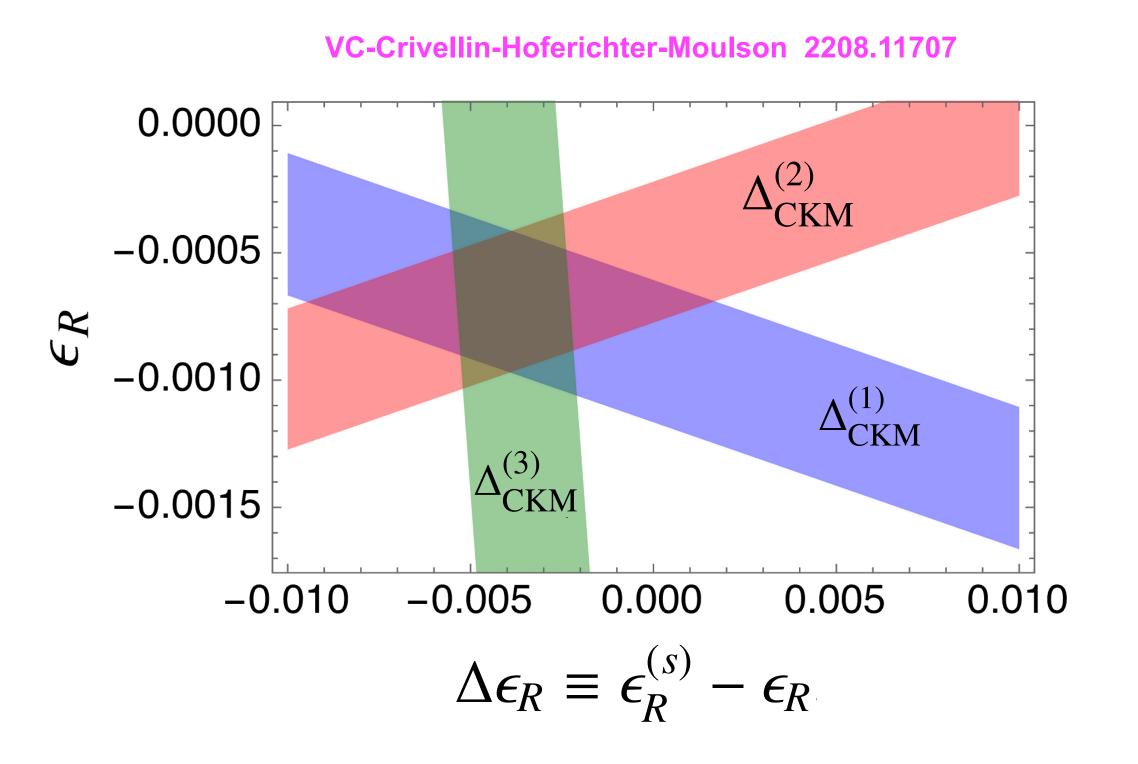
Find set of ε 's so that V_{ud} and V_{us} bands meet on the unitarity circle

Simplest 'solution': right-handed (V+A) quark currents

CKM elements from vector (axial) channels are shifted by $1+\epsilon_R$ (1- ϵ_R).

 V_{us}/V_{ud} , V_{ud} and V_{us} shift in correlated way, can resolve all tensions!

Unveiling R-handed quark currents?



$$\Delta_{CKM}^{(1)} = |V_{ud}^{\beta}|^2 + |V_{us}^{K_{\ell 3}}|^2 - 1$$

$$= -1.76(56) \times 10^{-3}$$

$$\Delta_{CKM}^{(2)} = |V_{ud}^{\beta}|^2 + |V_{us}^{K_{\ell 2}/\pi_{\ell 2},\beta}|^2 - 1$$

$$= -0.98(58) \times 10^{-3}$$

$$\Delta_{CKM}^{(3)} = |V_{ud}^{K_{\ell 2}/\pi_{\ell 2},K_{\ell 3}}|^2 + |V_{us}^{K_{\ell 3}}|^2 - 1$$

$$= -1.64(63) \times 10^{-2}$$

$$\bullet$$

$$\bullet$$

$$\Delta \epsilon_R = -0.69(27) \times 10^{-3}$$

$$\Delta \epsilon_R = -3.9(1.6) \times 10^{-3}$$

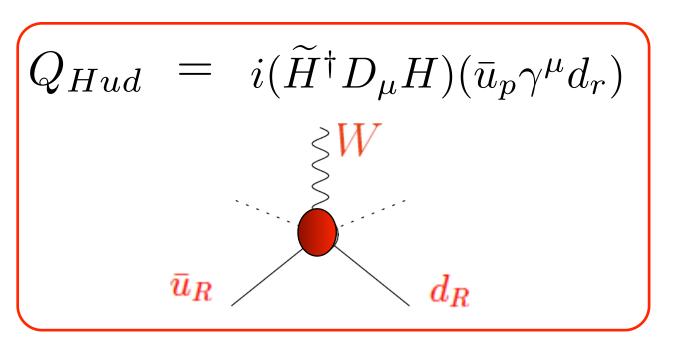
$$\Lambda_R \sim 5-10 \text{ TeV}$$

- Preferred ranges are not in conflict with other constraints from β decays, nor from $K \to (\pi\pi)_{l=2}$
- Does the R-handed current explanation survive after taking into account high energy data?

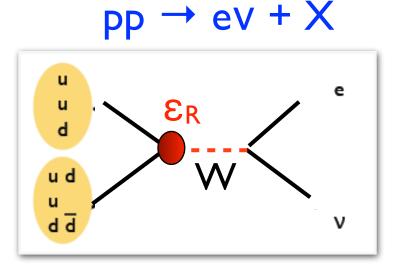
ε_R: high scale origin and constraints

For a detailed analysis in the SM-EFT see VC, W. Dekens, J. De Vries, E. Mereghetti, T. Tong, 2311.00021

- ϵ_R originates from SU(2)xU(1) invariant vertex corrections
- ER only weakly constrained by LHC processes

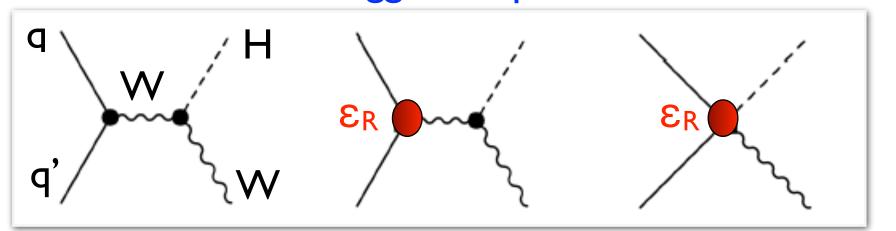


Same shape as the SMW exchange → weak sensitivity



VC, Graesser, Gonzalez-Alonso 1210.4553

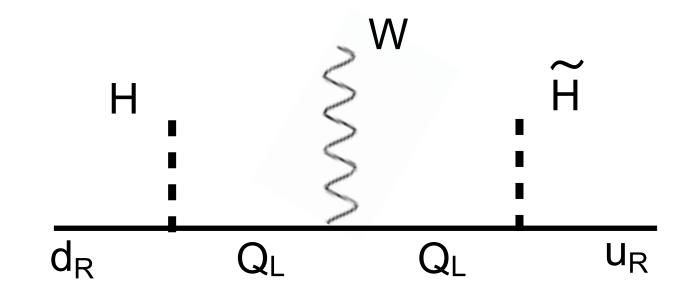
Associated Higgs +W production



S. Alioli, VC, W. Dekens, J. de Vries, E. Mereghetti 1703.04751

Current LHC results allow for to $\varepsilon_R \sim 5\%$

• ϵ_R can be generated at tree level by W_L - W_R mixing in LRSM or by exchange of vector-like quarks**

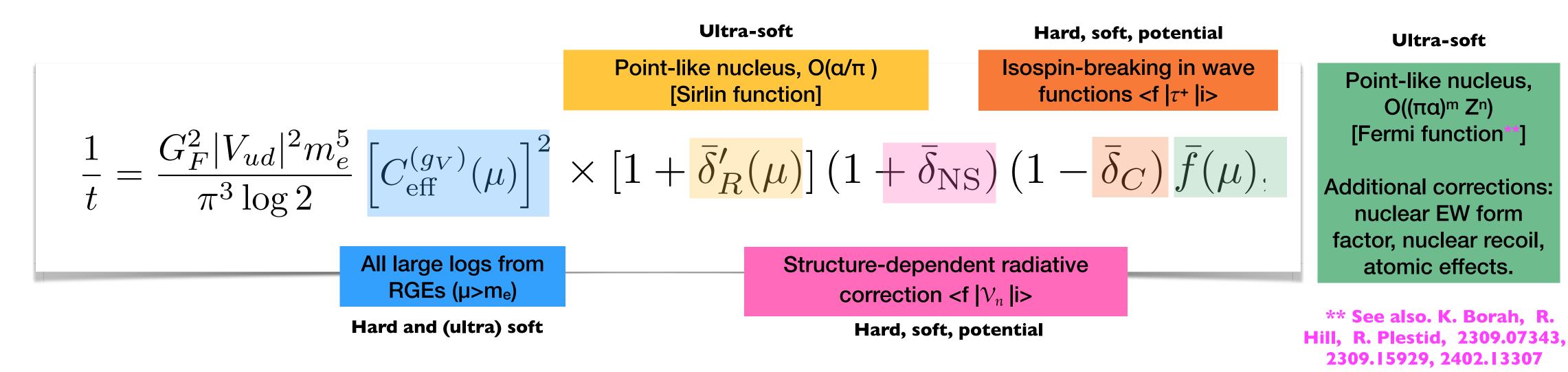


Additional backup

Nuclear decay rate in EFT

VC, W. Dekens,, J.de Vries, S. Gandolfi, M. Hoferichter, E, Mereghetti, 2405.18469, 2405.18464

EFT-based decay rate formula reorganizes 'traditional' corrections using EFT principles



- Need for improvement
 - Two currently unknown LECs contributing to δ_{NS} to $O(G_F \alpha \epsilon_X)$
 - Two- and three- body potentials to $O(G_F\alpha(\epsilon_X)^2)$: may be relevant at 0.01%, needed to check EFT convergence
 - Non-logarithmic terms of $O(\alpha^2 Z)$ in the Fermi function (finite parts of two-loop diagrams)

Vus from hyperon decays

$$\Gamma = \frac{G_F^2}{60\pi^3} (M_B - M_b)^5 (1 - 3\delta) |V_{us}|^2 |f_1^{B \to b}(0)|^2 (1 + \Delta_{RC}) \left[1 + 3 \left| \frac{g_1^{B \to b}(0)}{f_1^{B \to b}(0)} \right|^2 + \cdots \right] \qquad \delta = \frac{M_B - M_B}{M_B + M_B}$$

- Use SU(3) limit for vector form factor f₁(0)
- Extract g₁/f₁ from data

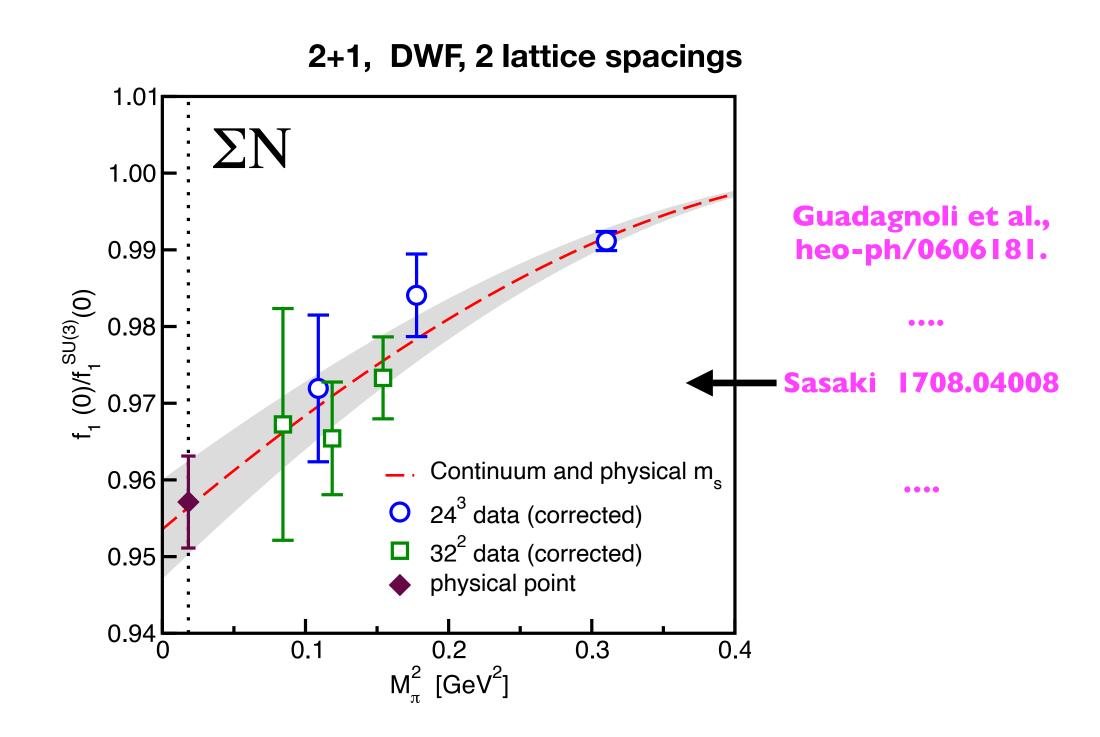
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Gasisso Girai		Hop pinodoi Edd

Decay	Rate	g_1/f_1	V_{us}	
Process	(μsec^{-1})			
$\Lambda \to p e^- \overline{\nu}$	3.161(58)	0.718(15)	0.2224 ± 0.0034	
$\Sigma^- \to ne^- \overline{\nu}$	6.88(24)	-0.340(17)	0.2282 ± 0.0049	
$\Xi^- \to \Lambda e^- \overline{\nu}$	3.44(19)	0.25(5)	0.2367 ± 0.0099	
$\Xi^0 \to \Sigma^+ e^- \overline{\nu}$	0.876(71)	1.32(+.22/18)	0.209 ± 0.027	
Combined		_ (0.2250 ± 0.0027	

V_{us} @ %-level in best channels.

No theoretical uncertainty included

- SU(3) in $f_1(0)$: quark model, I/N_c , $ChPT \rightarrow LQCD$
- Negative shift of few percent with uncertainty ~1%



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• Negative shift of few percent with uncertainty ~1%

Cabibbo-Swallow-Winston. hep-ph/0307298

2+1, DWF, 2 lattice spacings

1.01

Competitive extraction of V_{us} will require improved theory input (LQCD) and experimental progress (LHCb?)

V_{us} @ %-level in best channels.
No theoretical uncertainty included

