Workshop on "Flavour changing and conserving processes" 2025 (FCCP2025)

Status of charged lepton flavor violation searches and lepton universality tests

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Flavour Changing and Conserving Processes (FCCP2025)
Villa Orlandi, Anacapri, Capri Island, Italy, 29 September 2025 to 1 October 2025

Lepton Flavour Violation

- Standard Model accidental symmetry conserves L_e, L_μ, L_τ
- νSM (including neutrino mixing) predicts extremely suppressed charged Lepton Flavour Violation, $\propto m^2_{\nu_i}/m^2_W$, e.g., $\mathcal{B}(\mu \to e\gamma) \sim 10^{-54}$
  $\Rightarrow$ can do clean searches for New Physics
- LFV processes
  - muon LFV decays
  - tau LFV decays
  - hadrons, W/Z, H decays to leptons

In the quest of New Physics, can be sensitive to very high scale:

- Kaon physics: $\frac{s\bar{d}s\bar{d}}{\Lambda^2} \Rightarrow \Lambda \gtrsim 10^5$ TeV
  [ε_K]
- Charged Lepton Flavour Violation: $\frac{\mu\bar{e}f\bar{f}}{\Lambda^2} \Rightarrow \Lambda \gtrsim 10^4$ TeV
  [$\mu \to e\gamma$]

[E.Passemar, FPCP 2022]

Experimental searches for LFV processes

Process	Current bound on BR	Future Sensitivity
$\mu \to e\gamma$	$< 4.2 \times 10^{-13}$ MEG	10^{-14} MEGII
$\mu \to \bar{e}ee$	$< 1.0 \times 10^{-12}$ SINDRUM	10^{-16} Mu3e
$\mu A \to eA$	$< 7 \times 10^{-13}$ SINDRUMII	$10^{-16} \to 10^{-18}$ COMET, Mu2e
$\tau \to l\gamma$	$< 3.3 \times 10^{-8}$	$3 \times 10^{-9}(e), 10^{-9}(\mu)$
$\tau \to e\bar{e}e$	$< 2.7 \times 10^{-8}$	5×10^{-9}
$\tau \to \mu\bar{\mu}\mu$	$< 2.1 \times 10^{-8}$	4×10^{-9}
$\tau \to \mu\bar{e}e, e\bar{\mu}\mu$	$< 1.8, 2.7 \times 10^{-8}$ Belle	$3, 5 \times 10^{-9}$ BelleII
...	...	...
$\tau \to l\pi^0$	$< 8.0 \times 10^{-8}$	4×10^{-9}
$\tau \to l\eta$	$< 6.5 \times 10^{-8}$	7×10^{-9}
$\tau \to l\rho$	$< 1.2 \times 10^{-8}$ Belle	10^{-9} BelleII
$K^0 \to \mu^\pm e^\mp$	$< 4.7 \times 10^{-12}$	
$B^0_d \to \tau^\pm \mu^\mp$	$< 1.2 \times 10^{-5}$ LHCb	$\sim 10^{-6}$?
...	...	...
$h \to e^\pm \mu^\mp$	$< 6.1 \times 10^{-5}$ Atlas	2.1×10^{-5}
$h \to e^\pm \tau^\mp$	$< 2.2 \times 10^{-3}$ CMS	2.4×10^{-4}
$h \to \tau^\pm \mu^\mp$	$< 1.5 \times 10^{-3}$ CMS	2.3×10^{-4} ILC
$Z \to e^\pm \mu^\mp$	$< 7.5 \times 10^{-7}$ Atlas	
$Z \to l^\pm \tau^\mp$	$< 10^{-7}$ Atlas	[M.Ardu, CLFV 2023]

- muon searches most sensitive
- tau searches probe many operators can distinguish models
- W, Z, H, hadrons decays complementary

Muon Lepton Flavour Violation

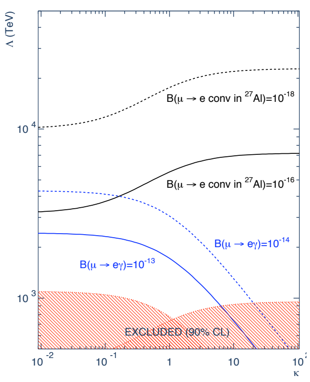
- effective theory for $\mu \to e\gamma$, $\mu \to eee$, $\mu N \to eN$ [Prog.Part.Nucl.Phys. 71 (2013) 75]

$$\mathcal{L}_{\text{CLFV}} = \begin{cases} \frac{1}{1+\kappa}\frac{m_\mu}{\Lambda^2}\bar{\mu}_R\sigma_{\mu\nu}e_L F^{\mu\nu} + h.c. \\ \frac{\kappa}{(1+\kappa)}\frac{1}{\Lambda^2}\bar{\mu}_L\gamma_\mu e_L\left(\bar{u}_L\gamma^\mu u_L + \bar{d}_L\gamma^\mu d_L\right) + h.c. \end{cases}$$

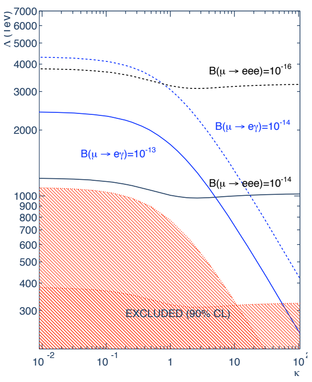
dipole term: SUSY, ...
$\mu \to e\gamma$, $\mu \to eee$

contact term, Z', leptoquark
$\mu N \to eN$, $\mu \to eee$
only via loops: $\mu \to e\gamma$

$\mu \to e\gamma$ vs. $\mu N \to eN$

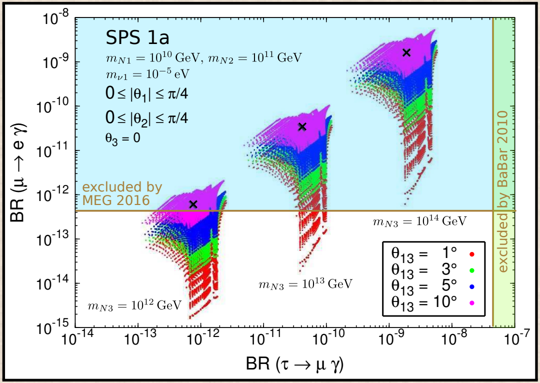


$\mu \to e\gamma$ vs. $\mu \to eee$

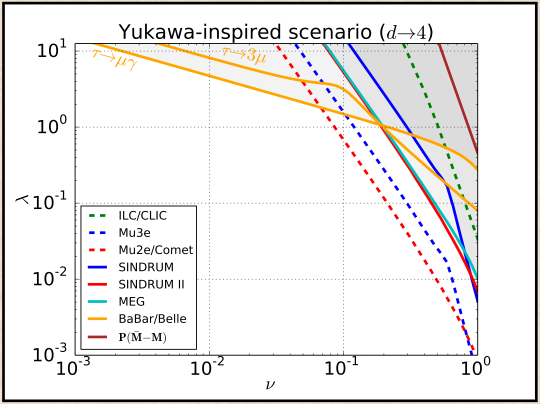


Tau LFV vs. Muon LFV searches

MSSM Seesaw
Antusch, Arganda, Herrero, Teixeira 2006



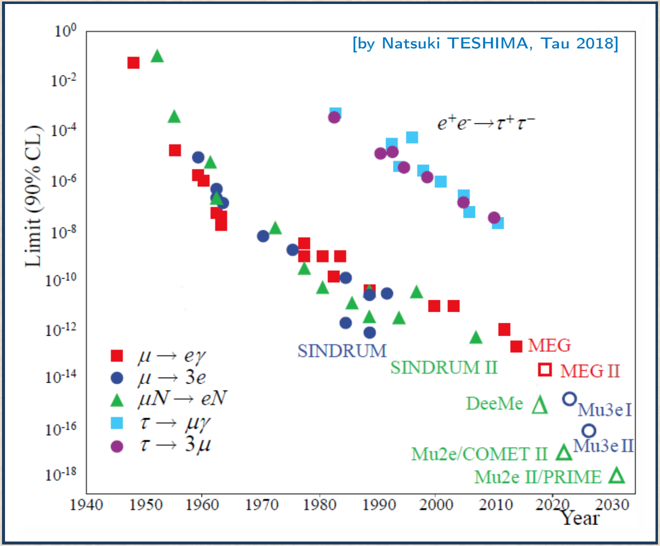
doubly charged scalar
Crivellin, Ghezzi, Panizzi, Pruna, Signer 2019



- typical "dipole-dominance" models
  $\mathcal{B}(\tau \to \mu\gamma)$ 10-10000 larger than $\mathcal{B}(\mu \to e\gamma)$
  but μ searches sensitivity $>$ 10000 larger than τ
  $\Rightarrow$ experimental muon LFV searches more effective

- tau LFV upper limits may be most constraining
  - for specific New Physics models
  - and / or for specific parameters space regions

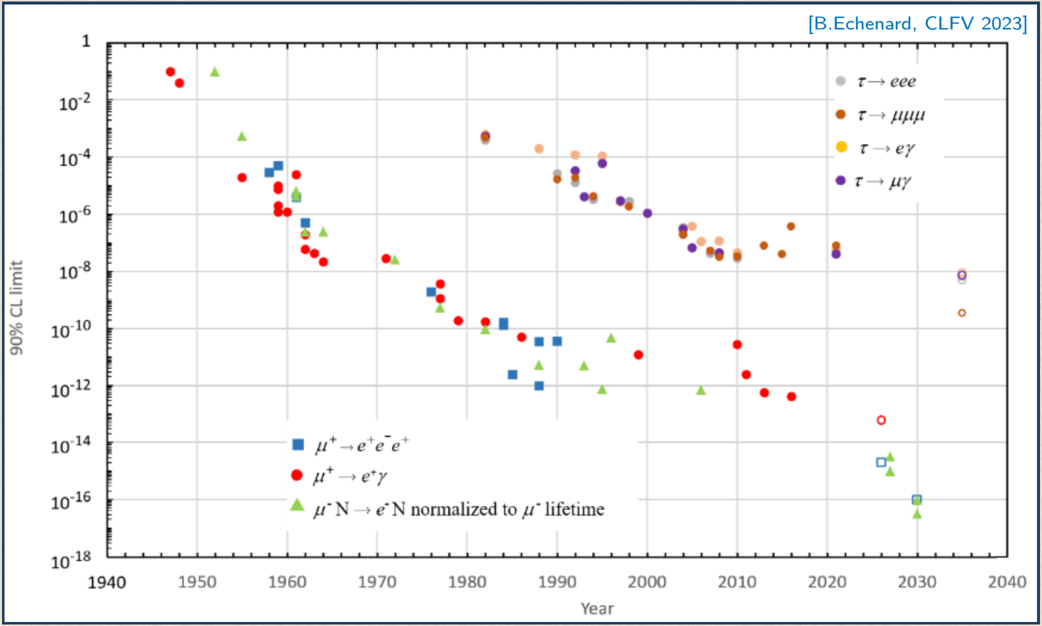
Muon LFV searches results and prospects



MEG, MEG-II searches

- MEG 2016 final limit: $[\mathcal{B}(\mu^+ \to e^+\gamma) < 4.2{\cdot}10^{-13}$ (90% CL)]
- MEG-II Run 1 (2021) search Euro.Phys.J.C 84 (2024) 3, 216 $\mathcal{B}(\mu^+ \to e^+\gamma) < 7.5{\cdot}10^{-13}$ (90% CL)
- combined MEG 2016 + MEG-II 2024: $\mathcal{B}(\mu^+ \to e^+\gamma) < 3.1{\cdot}10^{-13}$ (90% CL)
- MEG-II EPS-2025: $\mathcal{B}(\mu^+ \to e^+\gamma) < 1.5{\cdot}10^{-13}$ (90% CL)
- MEG-II EPS-2025 prospects (2026): $\mathcal{B}(\mu^+ \to e^+\gamma) < 0.6{\cdot}10^{-13}$ (90% CL)

LFV searches status and prospects



Searches for LFV in tau decays

- present upper limits mainly from CLEO, *BABAR*, Belle
- LHCb, CMS and ATLAS performed searches on $\tau^- \to \mu^-\mu^+\mu^-$, $\tau^- \to \bar{p}\mu^+\mu^-$
- Belle II will significantly improve B-factories limits
- possible future players FCC-ee(Z), CEPC, STCF

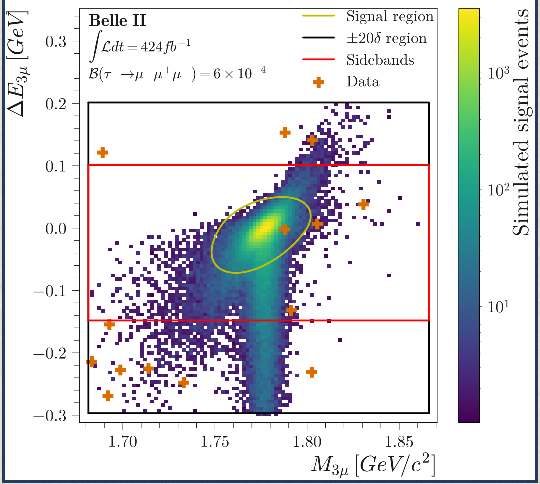
new Belle II limit on $\tau^- \to \mu^-\mu^+\mu^-$

- $\mathcal{B}(\tau^- \to \mu^-\mu^+\mu^-) < 1.9{\cdot}10^{-8}$ JHEP 09 (2024) 062, 424 fb^{-1}
- $>2\times$ larger selection efficiency of previous Belle analysis!
- previous $\mathcal{B}(\tau^- \to \mu^-\mu^+\mu^-) < 2.1{\cdot}10^{-8}$ 90% CL, Phys.Lett.B687 (2010) 139-143, 782 fb^{-1}

Belle II 2024 search for $\tau^- \to \mu^-\mu^+\mu^-$

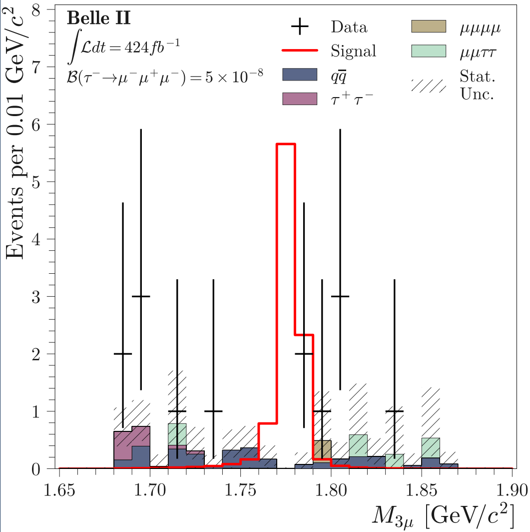
- select $\tau^- \to \mu^-\mu^+\mu^-$ candidates
- compute $M_{3\mu}$, $\Delta E_{3\mu} = E_{3\mu} - E_{\text{CM}}/2$
- estimate expected background events in signal region with simulation calibrated on sidebands
- search for excess over background
- compute upper limnit if no evidence of signal

$M_{3\mu}$ vs. $\Delta E_{3\mu}$

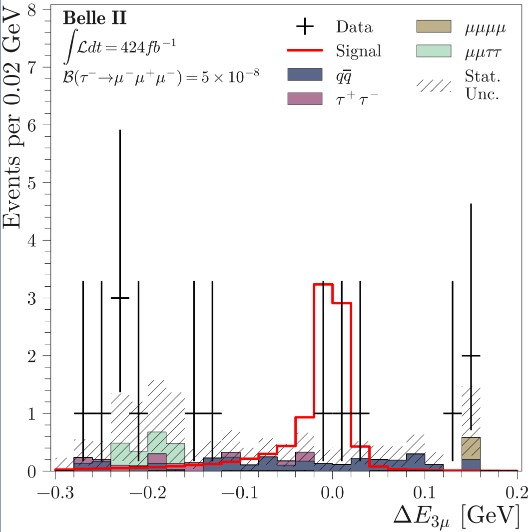


Belle II 2024 search for $\tau^- \to \mu^-\mu^+\mu^-$

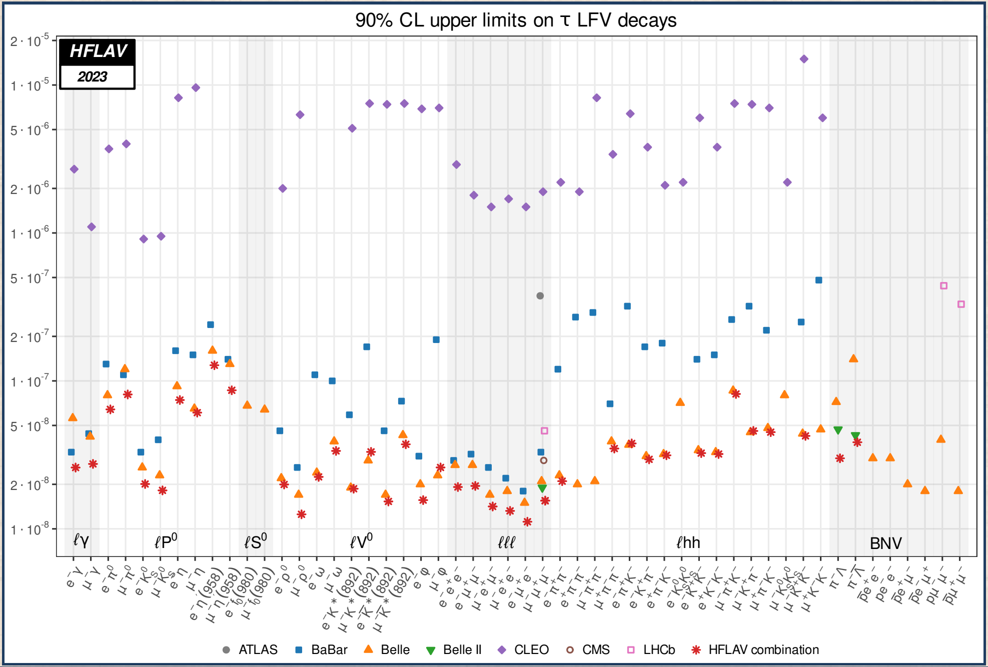
$M_{3\mu}$ projection



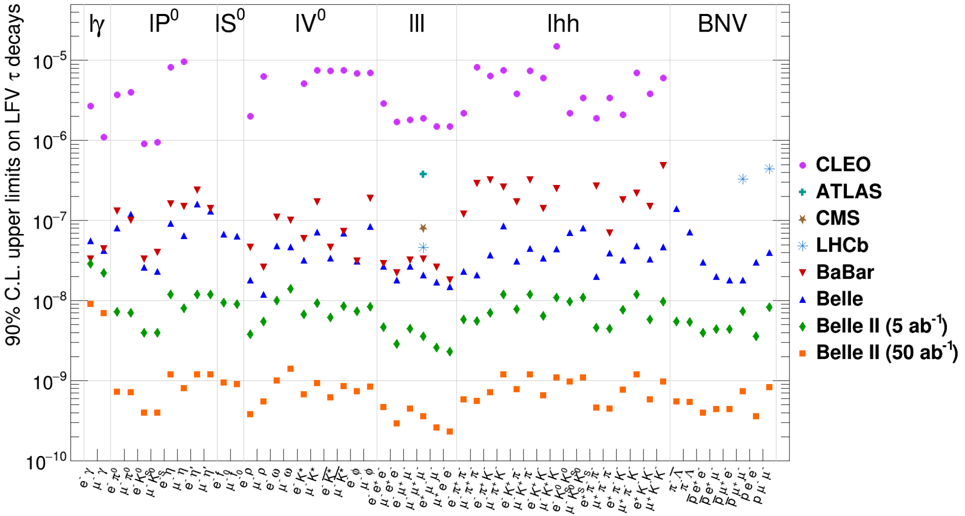
$\Delta E_{3\mu}$ projection



Tau LFV decay upper limits

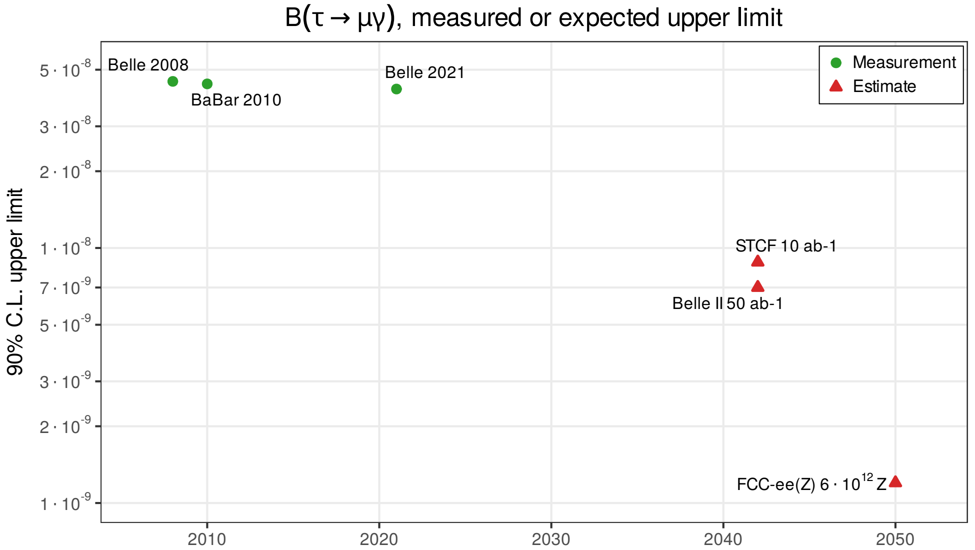


Tau LFV decay upper limits, Belle II expected



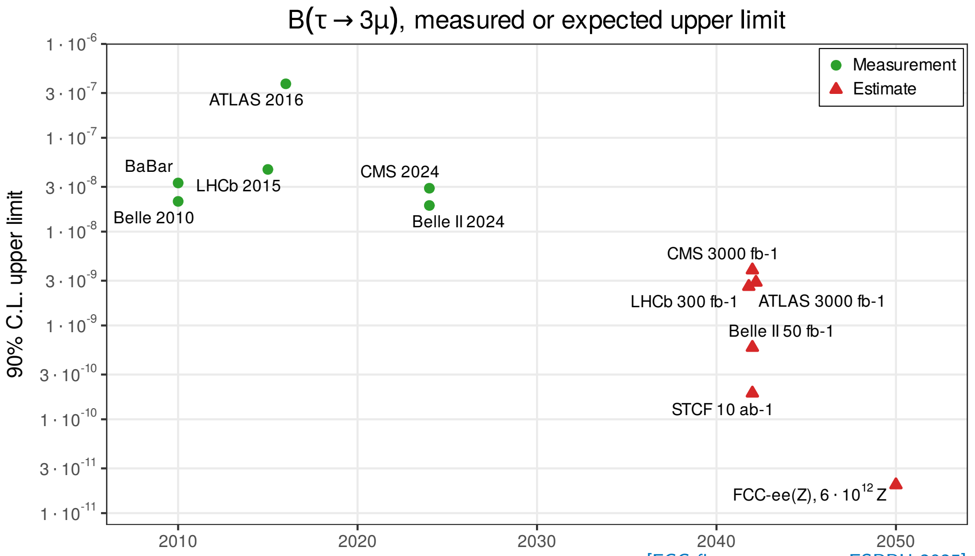
[Snowmass 2021 White Paper: Charged lepton flavor violation in the tau sector]

LFV search for $\tau \to \mu\gamma$



[FCC flavour prospects ESPPU 2025]

LFV search for $\tau \to 3\mu$



[FCC flavour prospects ESPPU 2025]

Experimental LFV searches with tau in final state

$B^0 \to \mu^\pm e^\mp$	1.0×10^{-9}	90%	LHCb
$B_s^0 \to \mu^\pm e^\mp$	5.4×10^{-9}	90%	LHCb
$B^0 \to K^{*0}\mu^+ e^-$	5.7×10^{-9}	90%	LHCb
$B^0 \to K^{*0}\mu^- e^+$	6.8×10^{-9}	90%	LHCb
$B_s^0 \to \phi\mu^\pm e^\mp$	1.6×10^{-8}	90%	LHCb
$B^+ \to K^+\mu^+ e^-$	6.4×10^{-9}	90%	LHCb
$B^+ \to K^+\mu^- e^+$	7.0×10^{-9}	90%	LHCb
$B^0 \to \tau^\pm \mu^\mp$	1.5×10^{-5}	90%	LHCb
$B_s^0 \to \tau^\pm \mu^\mp$	4.2×10^{-5}	90%	LHCb
$B^0 \to \tau^\pm e^\mp$	1.6×10^{-5}	90%	Belle
$B^0 \to K^{*0}\tau^+ \mu^-$	1.0×10^{-5}	90%	LHCb
$B^0 \to K^{*0}\tau^- \mu^+$	8.2×10^{-6}	90%	LHCb
$D^0 \to \mu^\pm e^\mp$	1.3×10^{-8}	90%	LHCb
$D^+ \to \pi^+\mu^+ e^-$	2.2×10^{-7}	90%	LHCb
$D^+ \to \pi^+\mu^- e^+$	2.1×10^{-7}	90%	LHCb
$D^+ \to K^+\mu^+ e^-$	1.0×10^{-7}	90%	LHCb
$D^+ \to K^+\mu^+ e^-$	7.5×10^{-8}	90%	LHCb

$K_L^0 \to \mu^\pm e^\mp$	4.7×10^{-12}	90%	BNL
$K_L^0 \to e^\pm e^\pm \mu^\mp \mu^\mp$	4.12×10^{-11}	90%	KTeV
$K_L^0 \to \pi^0 \mu^\pm e^\mp$	7.56×10^{-11}	90%	KTeV
$K_L^0 \to \pi^0\pi^0 \mu^\pm e^\mp$	1.64×10^{-10}	90%	KTeV
$K^+ \to \mu^- \nu e^+ e^+$	8.1×10^{-11}	90%	NA62
$Z \to e^\pm \mu^\mp$	7.5×10^{-7}	95%	ATLAS
$Z \to e^\pm \tau^\mp$	5.0×10^{-6}	95%	ATLAS
$Z \to \mu^\pm \tau^\mp$	6.5×10^{-6}	95%	ATLAS
$H \to e^\pm \mu^\mp$	6.2×10^{-5}	95%	ATLAS
$H \to e^\pm \tau^\mp$	2.0×10^{-3}	95%	ATLAS
$H \to \mu^\pm \tau^\mp$	1.5×10^{-3}	95%	CMS

- information taken on Nov 2024

Lepton Flavour Universality (LFU) tests

charged weak current coupling ratios for leptons' decay widths

- Standard Model predicts that all interactions are lepton-flavour-invariant (except Higgs couplings)
- in the following, mainly consider charged weak interaction couplings

$$\frac{G_F}{(\hbar c)^3} = \frac{\sqrt{2}}{8}\frac{g^2}{M_W^2 c^4}$$ adimensional coupling constant

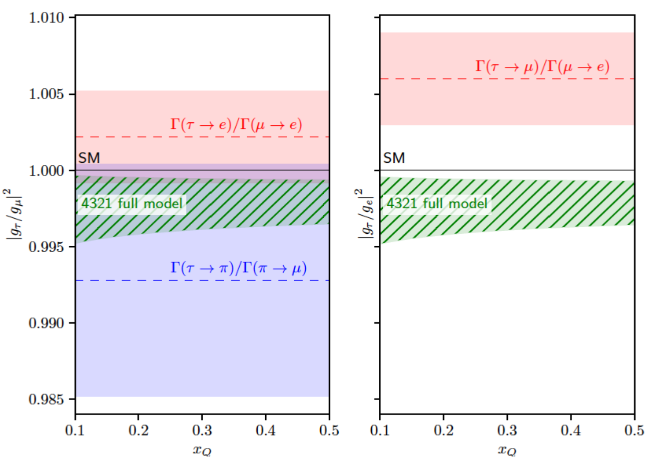
$$\left|\frac{g_\tau}{g_\mu}\right|^2_{\text{exp}} = \frac{\Gamma_{\text{exp}}(\tau^- \to e^- \bar{\nu}_e \nu_\tau)}{\Gamma_{\text{SM}}(\tau^- \to e^- \bar{\nu}_e \nu_\tau)} \Big/ \frac{\Gamma_{\text{exp}}(\mu^- \to e^- \bar{\nu}_e \nu_\mu)}{\Gamma_{\text{SM}}(\mu^- \to e^- \bar{\nu}_e \nu_\mu)}$$ coupling constant ratio $$\left|\frac{g_\tau}{g_\mu}\right|^2_{\text{SM}} = 1$$

- lepton universality tests related to
  - B anomalies (R_D, R_{D^*}, R_K, R_K^*, ...)
    - [Feruglio, Paradisi, Pattori JHEP 09 (2017) 061], [Allwicher, Isidori, Selimovic, 2021], [Allwicher, Isidori, Lizana, Selimovic, Stefanek, 2023]
  - Cabibbo angle anomaly
    - [Coutinho, Crivellin, Manzari, PRL 125 (2020) 071802], [Crivellin, Hoferichter, PRL 125 (2020) 111801]

Precise tau LFU measurements can confirm or exclude B anomalies models

- constraints on 4321 vector lepto-quark model for B anomalies
  [Allwicher, Isidori, Selimovic, 2021], [Allwicher, Isidori, Lizana, Selimovic, Stefanek, 2023]

today g_τ/g_μ, g_e/g_μ bounds



[G.Isidori, Jul 2023]

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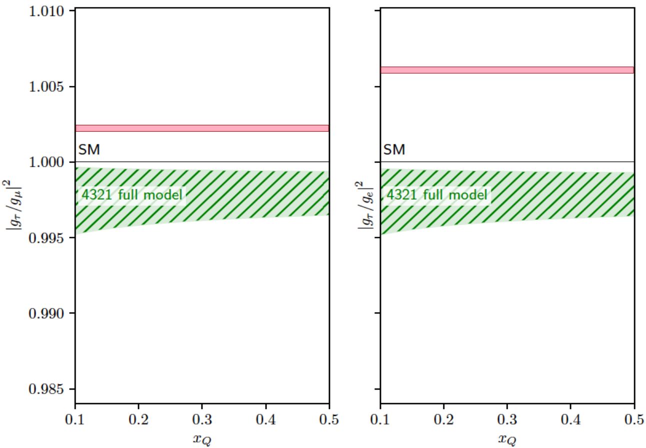
future FCC-ee g_τ/g_μ, g_e/g_μ may exclude model



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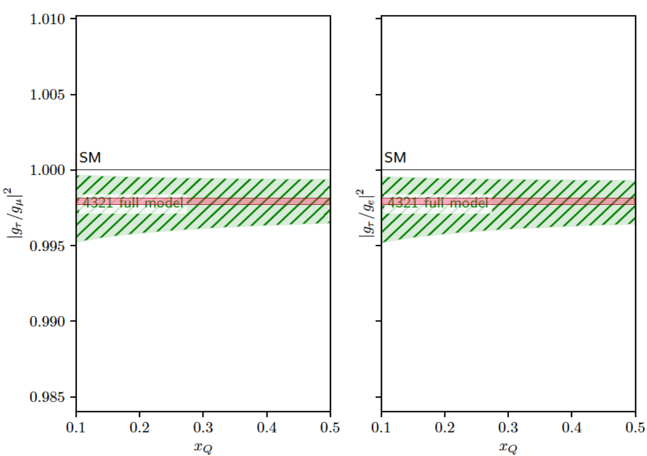


[G.Isidori, Jul 2023]

Lepton universality tests

2013 [A.Pich, Precision Tau Physics (2014)]

	$\Gamma_{\tau\to\mu}/\Gamma_{\tau\to e}$	$\Gamma_{\pi\to\mu}/\Gamma_{\pi\to e}$	$\Gamma_{K\to\mu}/\Gamma_{K\to e}$	$\Gamma_{K\to\pi\mu}/\Gamma_{K\to\pi e}$	$\Gamma_{W\to\mu}/\Gamma_{W\to e}$
$\lvert g_\mu/g_e\rvert$	1.0018(14)	1.0021(16)	0.9978(20)	1.0010(25)	0.996(10)
	$\Gamma_{\tau\to e}/\Gamma_{\mu\to e}$	$\Gamma_{\tau\to\pi}/\Gamma_{\pi\to\mu}$	$\Gamma_{\tau\to K}/\Gamma_{K\to\mu}$		$\Gamma_{W\to\tau}/\Gamma_{W\to\mu}$
$\lvert g_\tau/g_\mu\rvert$	1.0011(15)	0.9962(27)	0.9858(70)		1.034(13)
	$\Gamma_{\tau\to\mu}/\Gamma_{\mu\to e}$				$\Gamma_{W\to\tau}/\Gamma_{W\to e}$
$\lvert g_\tau/g_e\rvert$	1.0030(15)				1.031(13)

2024 [V.Cirigliano *et al.*, 2022] [HFLAV 2023 report] [PDG 2024]

	$\Gamma_{\tau\to\mu}/\Gamma_{\tau\to e}$	$\Gamma_{\pi\to\mu}/\Gamma_{\pi\to e}$	$\Gamma_{K\to\mu}/\Gamma_{K\to e}$	$\Gamma_{K\to\pi\mu}/\Gamma_{K\to\pi e}$	$\Gamma_{W\to\mu}/\Gamma_{W\to e}$
$\lvert g_\mu/g_e\rvert$	1.002(11)	1.0010(9)	0.9978(18)	1.0009(18)	1.001(3)
	$\Gamma_{\tau\to e}/\Gamma_{\mu\to e}$	$\Gamma_{\tau\to\pi}/\Gamma_{\pi\to\mu}$	$\Gamma_{\tau\to K}/\Gamma_{K\to\mu}$		$\Gamma_{W\to\tau}/\Gamma_{W\to\mu}$
$\lvert g_\tau/g_\mu\rvert$	1.0016(14)	0.9958(38)	0.9856(75)		1.007(10)
	$\Gamma_{\tau\to\mu}/\Gamma_{\mu\to e}$				$\Gamma_{W\to\tau}/\Gamma_{W\to e}$
$\lvert g_\tau/g_e\rvert$	1.0018(14)				1.008(10)

- in most cases, tau measurements produce most precise LFU tests
- most precise LFU test using $\Gamma_{\pi\to\mu}/\Gamma_{\pi\to e}$, limited by $\mathcal{B}(\pi\to e\nu)$, best by [PIENU(TRIUMF) 2015]

μ/e universality from $\mathcal{B}(\pi \to e\nu(\gamma))/\mathcal{B}(\pi \to \mu\nu(\gamma))$

- $R^{\pi}_{e/\mu,\,\text{SM}} = 1.23524(15)\cdot 10^{-4}$
- $R^{\pi}_{e/\mu,\,\text{PIENU(2015)}} = 1.2344(23)\text{stat}(19)\text{syst}\cdot 10^{-4}$ PRL 115 (2015) 071801
- $R^{\pi}_{e/\mu,\,\text{EXP}} = 1.2327(23)\cdot 10^{-4}$ PDG 2024
- on-going PEN experiment at PSI also aims to measure $R^{\pi}_{e/\mu}$

prospects

- PIENU(TRIUMF) on-going
- PEN(PSI) on-going
- PIONEER(PSI) in preparation, factor 15× improvement, ∼theory precision

Lepton universality tests with tau leptonic branching fractions

$$\Gamma_{\text{SM}}[\mathcal{L} \to \nu_{\mathcal{L}} \ell \bar{\nu}_\ell (\gamma)] = \Gamma_{\mathcal{L}\ell} = \Gamma_{\mathcal{L}} \mathcal{B}_{\mathcal{L}\ell} = \frac{\mathcal{B}_{\mathcal{L}\ell}}{\tau_{\mathcal{L}}} = \frac{G_{\mathcal{L}} G_\ell m_{\mathcal{L}}^5}{192\pi^3} f^{\mathcal{L}\ell} \left(1 + \delta R_\gamma^{\mathcal{L}\ell}\right) \left(1 + \delta R_W^{\mathcal{L}\ell}\right)$$

$$G_{\mathcal{L}} = \frac{g_{\mathcal{L}}^2}{4\sqrt{2} M_W^2} \; ; \qquad f^{\mathcal{L}\ell} = 1 - 8\rho + 8\rho^3 - \rho^4 - 12\rho^2 \ln\rho \quad \text{with} \quad \rho = \rho^{\mathcal{L}\ell} = m_\ell^2 / m_{\mathcal{L}}^2$$

$$\delta R_\gamma^{\mathcal{L}\ell} = \frac{\alpha(m_{\mathcal{L}})}{2\pi} \left(\frac{25}{4} - \pi^2 \right)$$ QED radiative correction, 1st order

$$\delta R_W^{\mathcal{L}\ell} = \frac{3}{5} \frac{m_{\mathcal{L}}^2}{M_W^2} + \frac{9}{5} \frac{m_\ell^2}{M_W^2}$$ EW radiative correction

[Marciano, 1988], [Pich, Precision Tau Physics, 2014] (LO QED radiative corrections)

Lepton universality tests with tau leptonic branching fractions

$$\left(\frac{g_\tau}{g_\mu}\right) = \sqrt{\frac{\mathcal{B}_{\tau e}}{\mathcal{B}_{\mu e}} \frac{\tau_\mu m_\mu^5 f^{\mu e}(1+\delta R_\gamma^{\mu e})(1+\delta R_W^{\mu e})}{\tau_\tau m_\tau^5 f^{\tau e}(1+\delta R_\gamma^{\tau e})(1+\delta R_W^{\tau e})}} \quad 1.0016 \pm 0.0014$$

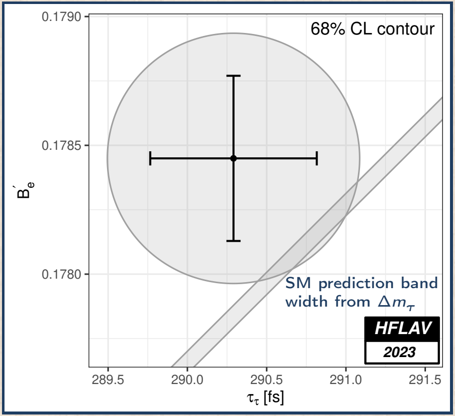
$$\left(\frac{g_\tau}{g_e}\right) = \sqrt{\frac{\mathcal{B}_{\tau \mu}}{\mathcal{B}_{\mu e}} \frac{\tau_\mu m_\mu^5 f^{\mu e}(1+\delta R_\gamma^{\mu e})(1+\delta R_W^{\mu e})}{\tau_\tau m_\tau^5 f^{\tau \mu}(1+\delta R_\gamma^{\tau \mu})(1+\delta R_W^{\tau \mu})}} \quad 1.0018 \pm 0.0014$$

$$\left(\frac{g_\mu}{g_e}\right) = \sqrt{\frac{\mathcal{B}_{\tau \mu}}{\mathcal{B}_{\tau e}} \frac{f^{\tau e}(1+\delta R_\gamma^{\tau e})(1+\delta R_W^{\tau e})}{f^{\tau \mu}(1+\delta R_\gamma^{\tau \mu})(1+\delta R_W^{\tau \mu})}} \quad 1.0002 \pm 0.0011$$

[HFLAV 2023 report]

- tau LFU tests limited to 1100–1400 ppm from branching fractions and tau lifetime measurements
- Belle II may improve precision on lifetime, and with hard work perhaps on BRs
- FCC-ee can probably improve by a factor of 10 (statistically 1000)

Canonical tau lepton universality test plot



[HFLAV 2023]

$(g_\tau/g_{e\mu}) = 1.0017 \pm 0.0013$
$[g_{e\mu} = g_e = g_\mu$ assuming $g_e = g_\mu]$

$\Delta(g_\tau/g_{e\mu})$ contributions

input	Δinput	$\Delta(g_\tau/g_{e\mu})$
$\mathcal{B}'_{\tau\to e}$	0.180%	0.090%
τ_τ	0.181%	0.090%
m_τ	0.005%	0.012%
total		0.128%

best measurements

$\mathcal{B}'_{\tau\to e}$	ALEPH
τ_τ	Belle
m_τ	Belle II

- $\mathcal{B}'(\tau \to e\bar{\nu}\nu) =$ average of $\begin{cases} \mathcal{B}(\tau \to e\bar{\nu}\nu) \\ \mathcal{B}(\tau \to \mu\bar{\nu}\nu) \cdot \dfrac{f^{\tau e} R_\gamma^{\tau e} R_W^{\tau e}}{f^{\tau\mu} R_\gamma^{\tau\mu} R_W^{\tau\mu}} \end{cases}$
- $\dfrac{\mathcal{B}'(\tau \to e\bar{\nu}\nu)\tau_\mu}{\mathcal{B}(\mu \to e\bar{\nu}\nu)\tau_\tau} = \dfrac{g_\tau^2}{g_{e\mu}^2} \dfrac{m_\tau^5 f^{\tau e} R_\gamma^{\tau e} R_W^{\tau e}}{m_\mu^5 f^{\mu e} R_\gamma^{\mu e} R_W^{\mu e}}$
- $\left(\dfrac{g_\tau}{g_{e\mu}}\right)^2 = \dfrac{\mathcal{B}'(\tau \to e\bar{\nu}\nu)}{\mathcal{B}(\mu \to e\bar{\nu}\nu)} \dfrac{\tau_\mu}{\tau_\tau} \dfrac{m_\mu^5}{m_\tau^5} \dfrac{f^{\mu e} R_\gamma^{\mu e} R_W^{\mu e}}{f^{\tau e} R_\gamma^{\tau e} R_W^{\tau e}}$

HFLAV Tau branching fraction fit

	inputs	examples
171	measurements of τ branching fractions & branching ratios	$\mathcal{B}(\tau^- \to \pi^- \nu_\tau)$, $\dfrac{\mathcal{B}(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau)}{\mathcal{B}(\tau^- \to e^- \bar{\nu}_e \nu_\tau)}$
1	nuisance fit parameter measurement (new feature)	$\mathcal{B}(a_1^- \to \pi^- \gamma) = 0.0021 \pm 0.0008$ [Phys.Rept. 421 (2005) 191]
91	constraints	$\mathcal{B}_{\tau\to\mu\nu\bar{\nu}/\tau\to e\nu\bar{\nu}} = \mathcal{B}_{\tau\to\mu\nu\bar{\nu}}/\mathcal{B}_{\tau\to e\nu\bar{\nu}}$ tau-decay final states related through different η, ω decay modes
1	uncertainty scale factor	5.44 scale factor for inconsistent BABAR/ Belle $\mathcal{B}(\tau^- \to K^- K^- K^+ \nu_\tau)$

external uncertain parameters: correct for updated values, account from induced uncertainties' correlations

$$\chi^2 = \sum_{ijkl} (m_i - M_{ik} q_k) \left(V^{-1}\right)_{ij} (m_j - M_{jl} q_l) + \sum_r \frac{(n_r - p_r)^2}{\sigma^2_{n_r}} ,$$

m_i, n_i measurements
q_i, p_i fit parameters
V_{ij} covariance of m_i, M_{ij} model matrix

137 fit parameters 1 nuisance fit parameter (new feature)
covariance matrix of fit parameters and nuisance fit parameter
χ^2/d.o.f. = 138/125 $P(\chi^2) = 20.2\%$
unitarity residual $\mathcal{B}_{\text{ur}} = 1 - \mathcal{B}_{\text{all}} = 0.0007 \pm 0.0011$

- HFLAV unitarity-constrained tau BR fit used for PDG tau BR “fit” averages
  - use of nuisance fit parameter removes some modifications of HFLAV tau BR fit for PDG “fit” averages

NLO QED radiative corrections for LFU tests using tau leptonic BRs

$$\Gamma[\mathcal{L} \to \nu_\mathcal{L} \ell \bar{\nu}_\ell (\gamma)] = \Gamma_{\mathcal{L}\ell} = \Gamma_\mathcal{L} \mathcal{B}_{\mathcal{L}\ell} = \frac{\mathcal{B}_{\mathcal{L}\ell}}{\tau_\mathcal{L}} = \frac{G_\mathcal{L} G_\ell m_\mathcal{L}^5}{192\pi^3} f^{\mathcal{L}\ell} \left(1 + \delta R_\gamma^{\mathcal{L}\ell}\right) \left(1 + \delta R_W^{\mathcal{L}\ell}\right)$$

$$G_\mathcal{L} = \frac{g_\mathcal{L}^2}{4\sqrt{2} M_W^2} \; ; \qquad f^{\mathcal{L}\ell} = 1 - 8\rho + 8\rho^3 - \rho^4 - 12\rho^2 \ln\rho \quad \text{with} \quad \rho = \rho^{\mathcal{L}\ell} = m_\ell^2 / m_\mathcal{L}^2$$

$$\delta R_\gamma^{\mathcal{L}\ell} = \frac{\alpha(m_\mathcal{L})}{2\pi} \left(\frac{25}{4} - \pi^2\right)$$

NLO 3rd order $\rightarrow$ $\delta R_\gamma^{\mathcal{L}\ell} = H_1^{\mathcal{L}\ell} \frac{\alpha(m_\mathcal{L})}{\pi} + H_2^{\mathcal{L}\ell} \frac{\alpha^2(m_\mathcal{L})}{\pi^3} + H_3^{\mathcal{L}\ell} \frac{\alpha^3(m_\mathcal{L})}{\pi^3}$

$$\delta R_W^{\mathcal{L}\ell} = \frac{3}{5} \frac{m_\mathcal{L}^2}{M_W^2} + \frac{9}{5} \frac{m_\ell^2}{M_W^2}$$

EW radiative correction

$$H_1^{\mathcal{L}\ell} = \frac{25}{8} - \frac{\pi^2}{2} - \left(9 + 4\pi^2 + 12 \ln \rho\right) \rho + 16\pi^2 \rho^{3/2} \quad \text{with} \quad \rho = \rho^{\mathcal{L}\ell} = m_\ell^2 / m_\mathcal{L}^2$$

$$H_2^{\mathcal{L}\ell} = \frac{156815}{5184} - \frac{518}{81}\pi^2 - \frac{895}{36}\zeta(3) + \frac{67}{720}\pi^4 + \frac{53}{6}\pi^2 \ln 2 + H_{2,\text{had}}^{\mathcal{L}\ell} - \frac{5}{4}\pi^2 \sqrt{\rho} \quad \text{with} \quad \rho = \rho^{\mathcal{L}\ell} = m_\ell^2 / m_\mathcal{L}^2$$

$H_{2,\text{had}}^{\mu e} = -0.042 \pm 0.002$ $\quad H_{2,\text{had}}^{\tau\mu} = ?$ $\quad H_{2,\text{had}}^{\tau e} = ?$ $\quad$ missing values set to zero in the following

$H_3^{\mu e} = -15.3 \pm 2.3$ $\quad H_3^{\tau\mu} = ?$ $\quad H_3^{\tau e} = ?$ $\quad$ missing values set to zero in the following

[J.Erler & A.Freitas, PDG EW review, 2022] $\quad$ [using modified minimal subtraction scheme ($\overline{\text{MS}}$)]

NLO QED radiative corrections for LFU tests using tau leptonic BRs / 2

- for tau leptonic partial widths use up to 2nd order QED radiative correction formula without $H_{2,\text{had}}^{\tau(\mu,e)}$, $H_3^{\tau(\mu,e)}$
- tau LFU tests uncertainties up to 2.4 ppm if missing terms $H_{2,\text{had}}^{\tau(\mu,e)} \sim 20 \times H_{2,\text{had}}^{\mu e}$, $H_3^{\tau(\mu,e)} \sim 20 \times H_3^{\mu e}$
  - desirable calculations for tau (even order-of-magnitude useful)
- tau mass uncertainty on phase space factors contributes uncertainty up to 1.3 ppm
- $\alpha(m_\tau)$ computed using [F.Jegerlehner 2023 code alphaQEDr23.f], uncertainty up to 0.3 ppm on tau LFU tests

Effect of NLO QED corrections on tau LFU tests

	QED rad. corr. 1st order	QED rad. corr. 2nd order
$\left(\frac{g_\tau}{g_\mu}\right) = \sqrt{\frac{\mathcal{B}_{\tau e}}{\mathcal{B}_{\mu e}} \frac{\tau_\mu m_\mu^5 f^{\mu e}(1+\delta R_\gamma^{\mu e})(1+\delta R_W^{\mu e})}{\tau_\tau m_\tau^5 f^{\tau e}(1+\delta R_\gamma^{\tau e})(1+\delta R_W^{\tau e})}}$	1.0016 ± 0.0014	1.0016 ± 0.0014
$\left(\frac{g_\tau}{g_e}\right) = \sqrt{\frac{\mathcal{B}_{\tau \mu}}{\mathcal{B}_{\mu e}} \frac{\tau_\mu m_\mu^5 f^{\mu e}(1+\delta R_\gamma^{\mu e})(1+\delta R_W^{\mu e})}{\tau_\tau m_\tau^5 f^{\tau \mu}(1+\delta R_\gamma^{\tau \mu})(1+\delta R_W^{\tau \mu})}}$	1.0018 ± 0.0014	1.0017 ± 0.0014
$\left(\frac{g_\mu}{g_e}\right) = \sqrt{\frac{\mathcal{B}_{\tau \mu}}{\mathcal{B}_{\tau e}} \frac{f^{\tau e}(1+\delta R_\gamma^{\tau e})(1+\delta R_W^{\tau e})}{f^{\tau \mu}(1+\delta R_\gamma^{\tau \mu})(1+\delta R_W^{\tau \mu})}}$	1.0002 ± 0.0011	1.0001 ± 0.0011

[A.L., EPS-HEP 2025]

- NLO QED radiative corrections ~negligible with current experimental precision
- importance will increse as Belle II and FCC / CEPT / STCF will improve experimental precision

Tau-muon universality using tau hadronic branching fractions

[HFLAV 2023]

$$\left(\frac{g_\tau}{g_\mu}\right)_h^2 = \frac{\mathcal{B}(\tau \to h\nu_\tau)}{\mathcal{B}(h \to \mu\bar{\nu}_\mu)} \frac{2 m_h m_\mu^2 \tau_h}{(1+\delta R_{\tau/h}) m_\tau^3 \tau_\tau} \left(\frac{1 - m_\mu^2/m_h^2}{1 - m_h^2/m_\tau^2}\right)^2 ,$$

- $h = \pi,\ K$
- $\delta R_{\tau/\pi} = (0.18 \pm 0.57)\%$ [PhysRevD.104.L091502 (2021)]
- $\delta R_{\tau/K} = (0.97 \pm 0.58)\%$ [PhysRevD.104.L091502 (2021)]
- $\mathcal{B}(\tau \to \pi\nu_\tau)$ $\mathcal{B}(\tau \to K\nu_\tau)$ [HFLAV 2023]
- $\mathcal{B}(\pi \to \mu\bar{\nu}_\mu)$, $\mathcal{B}(K \to \mu\bar{\nu}_\mu)$ [PDG 2023]

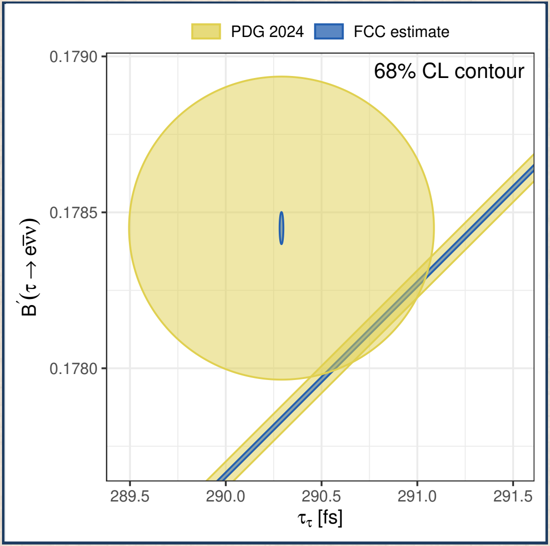
$$\left(\frac{g_\tau}{g_\mu}\right)_\pi = 0.996 \pm 0.004 , \qquad \left(\frac{g_\tau}{g_\mu}\right)_K = 0.986 \pm 0.008 .$$

- precision limited to ~0.29% from $\delta R_{\tau/\pi}$, $\delta R_{\tau/K}$

$$\left(\frac{g_\tau}{g_\mu}\right)_{\tau+\pi+K} = 1.0011 \pm 0.0014 , \quad \text{average of} \quad \left(\frac{g_\tau}{g_\mu}\right)_\tau , \left(\frac{g_\tau}{g_\mu}\right)_\pi , \left(\frac{g_\tau}{g_\mu}\right)_K$$

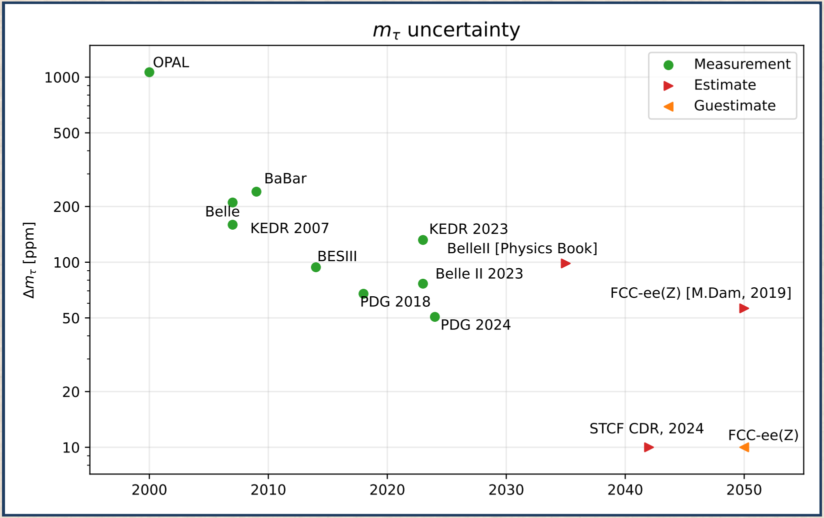
- assuming uncorrelated $\delta R_{\tau/\pi}$ and $\delta R_{\tau/K}$

Canonical tau lepton universality plot extrapolation at FCC-ee



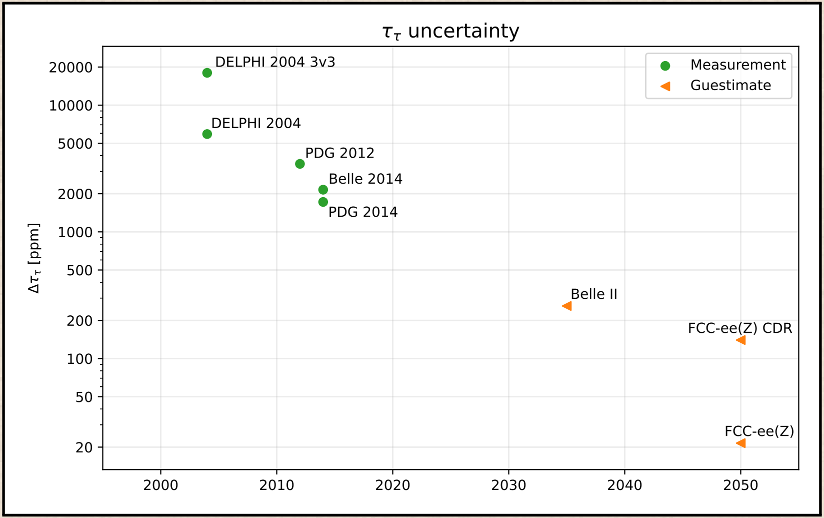
FCC-ee(Z) with $6{\cdot}10^{12}$ Z

Tau mass measurement prospects



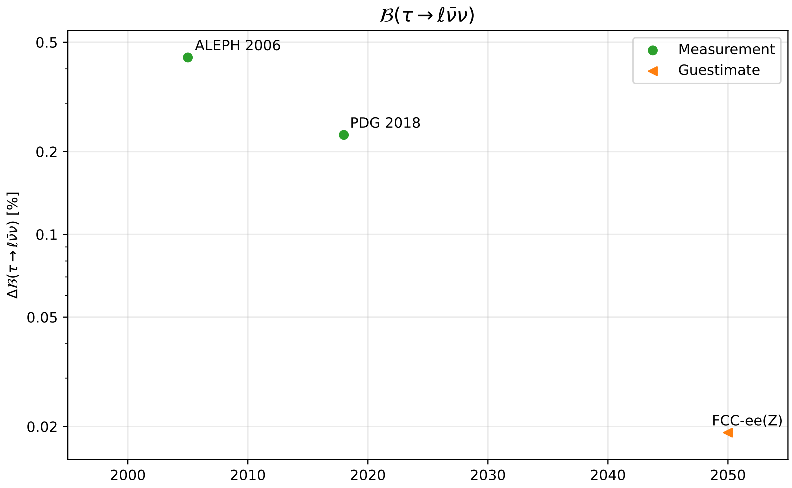
FCC-ee(Z) with $6{\cdot}10^{12}$ Z

Tau Lifetime measurement prospects



- FCC-ee(Z) CDR estimate [M. Dam, 1999]
- FCC-ee(Z) estimate [A.L., Tau Physics Prospects at FCC-ee, doi:10.17181/57pxj-6xd43]
- Belle II estimate [Belle II Physics Book]

Tau leptonic Branching fractions prospects at FCC-ee and other facilities



- FCC-ee(Z) with $6{\cdot}10^{12}$ Z
- Belle II is working on these measurements

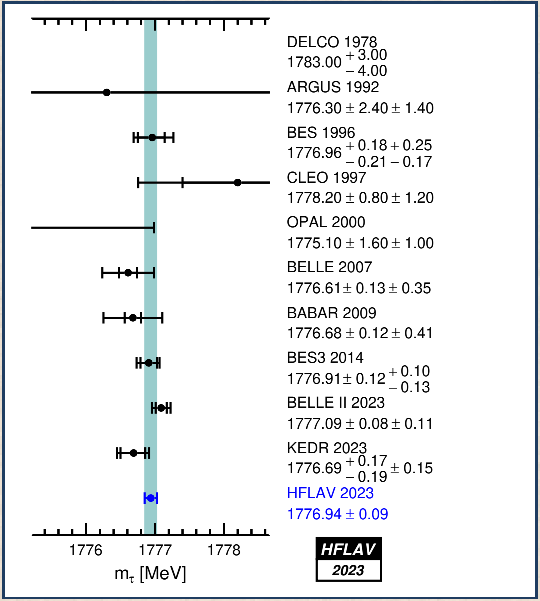
Summary

- LFV status and prospects
- LFU status and prospects
  - improved HFLAV tau BR fit for HFLAV 2023 report (accepted by PRD, arXiv:2411.18639 [hep-ex])
  - first assessment of NLO QED radiative corrections

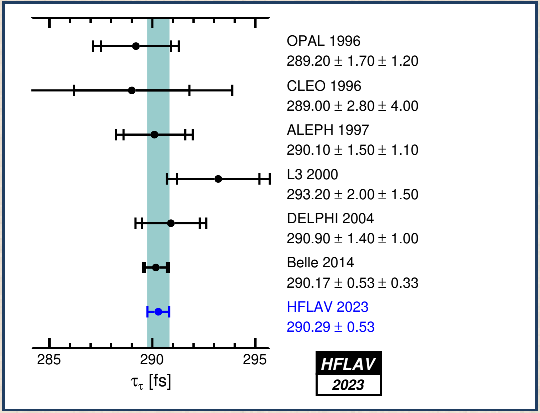
End

Backup slides

HFLAV Tau mass measurements



HFLAV Tau lifetime measurements



HFLAV Tau branching fractions measurements