The HVP contribution to the muon g-2 from the lattice

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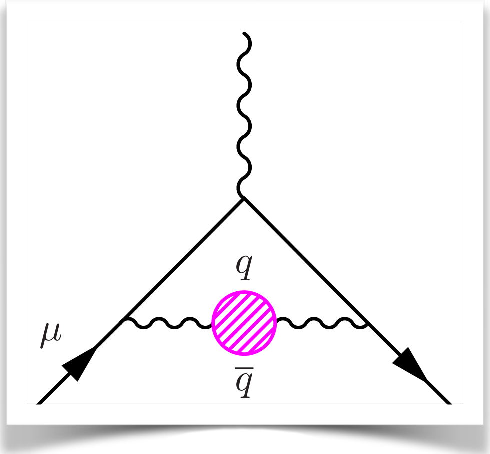
Workshop on Flavour Changing and Conserving Processes

Anacapri

29th September 2025

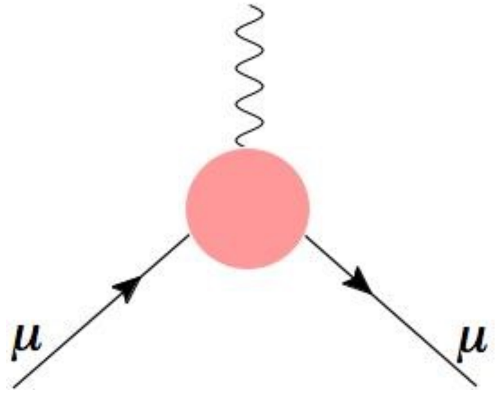
OUTLINE



- Introduction
- HVP from the lattice & window obs.
- The BMW/DMZ-24 calculation

Introduction

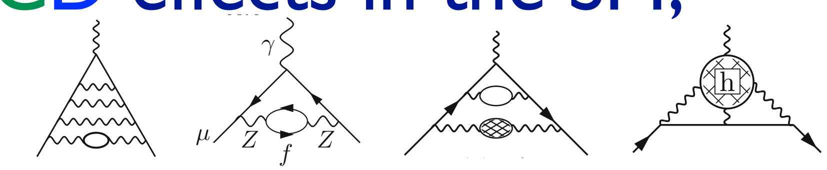
Muon magnetic anomaly



$$= (-i\,e)\,\bar{u}(p')\left[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m}F_2(q^2)\right]u(p)$$

muon anomalous magnetic moment: $a_\mu \equiv \dfrac{g_\mu - 2}{2} = F_2(0)$

- is generated by quantum loops;
- receives contribution from QED, EW and QCD effects in the SM;
- is a sensitive probe of new physics

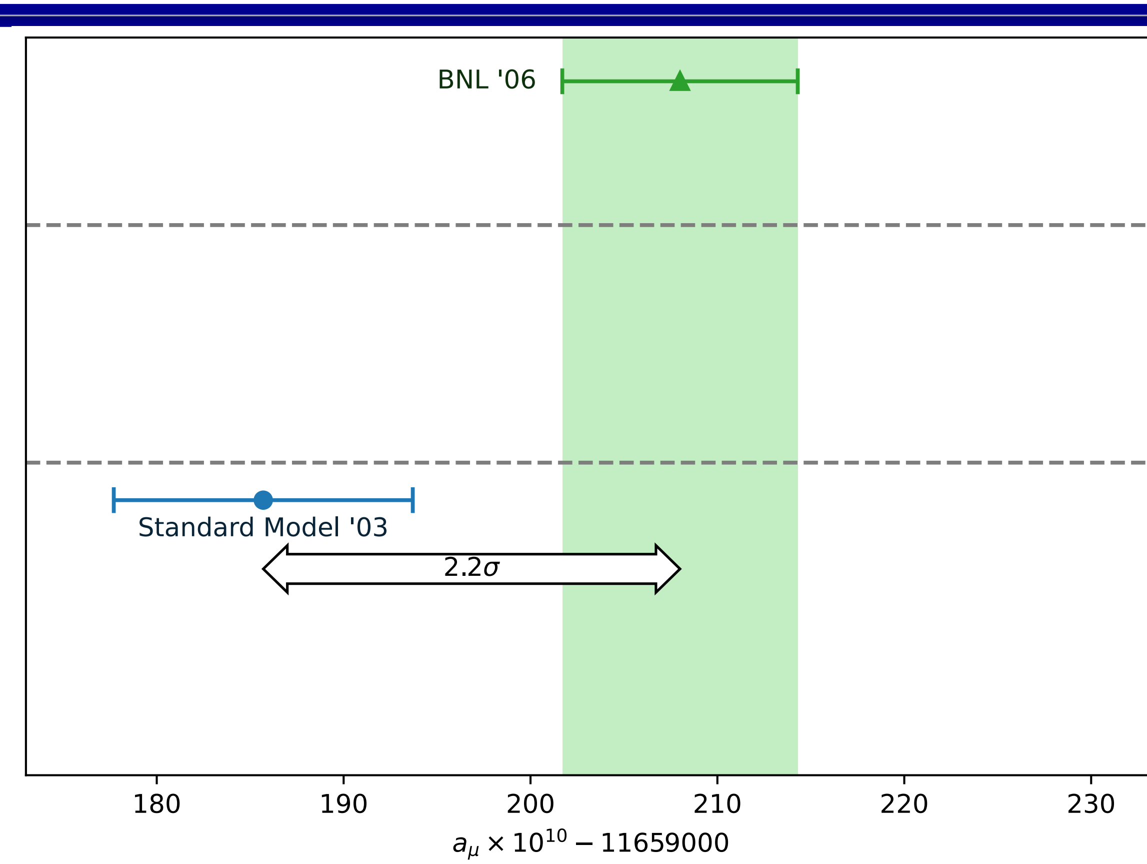


SM contributions to $a_\mu [\times 10^{10}]$

5-loop QED	11 658 471.8931(104)
2-loop EW	15.36(10)
HVP LO	693.1(4.0)
HVP NLO	-9.83(7)
HVP NNLO	1.24(1)
HLbL	9.2(1.8)

Aoyama *et al.* [WP] 2020

Theory error dominated by hadronic physics



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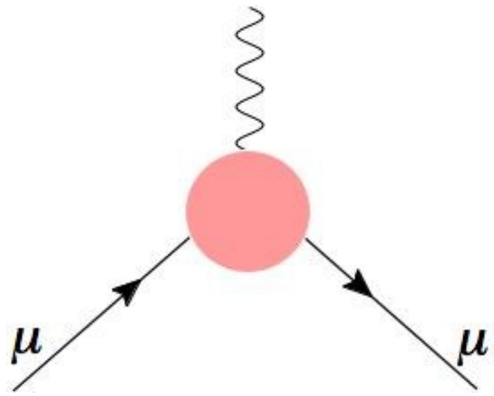

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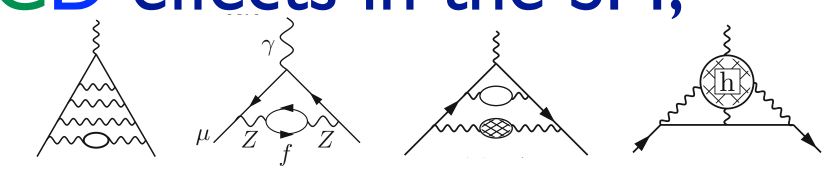

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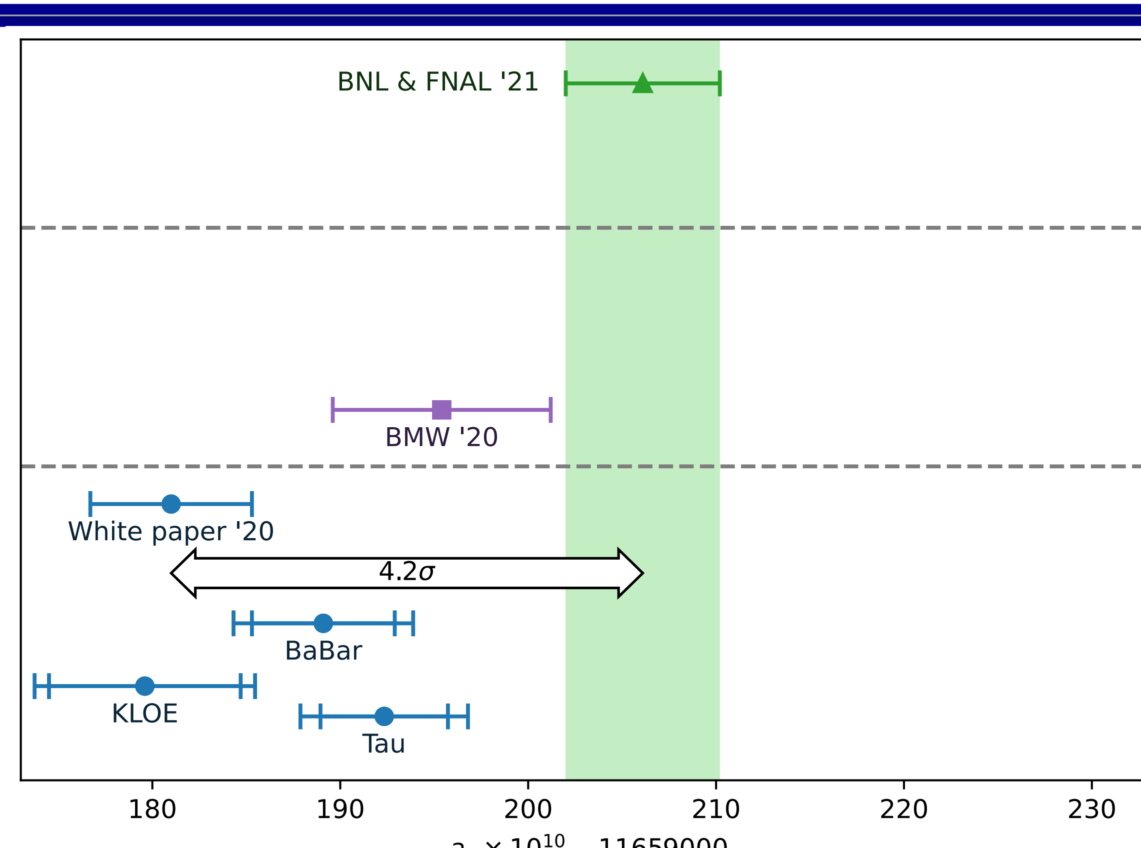

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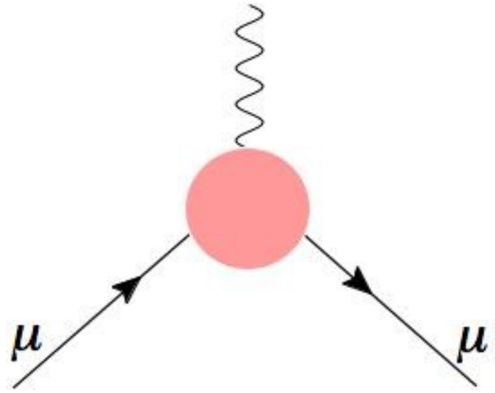

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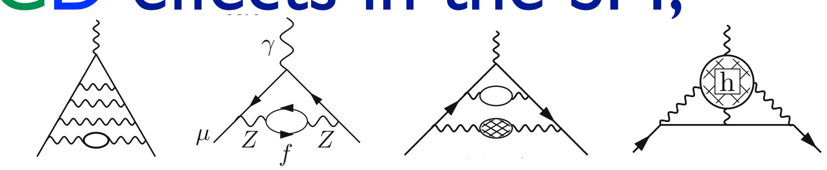

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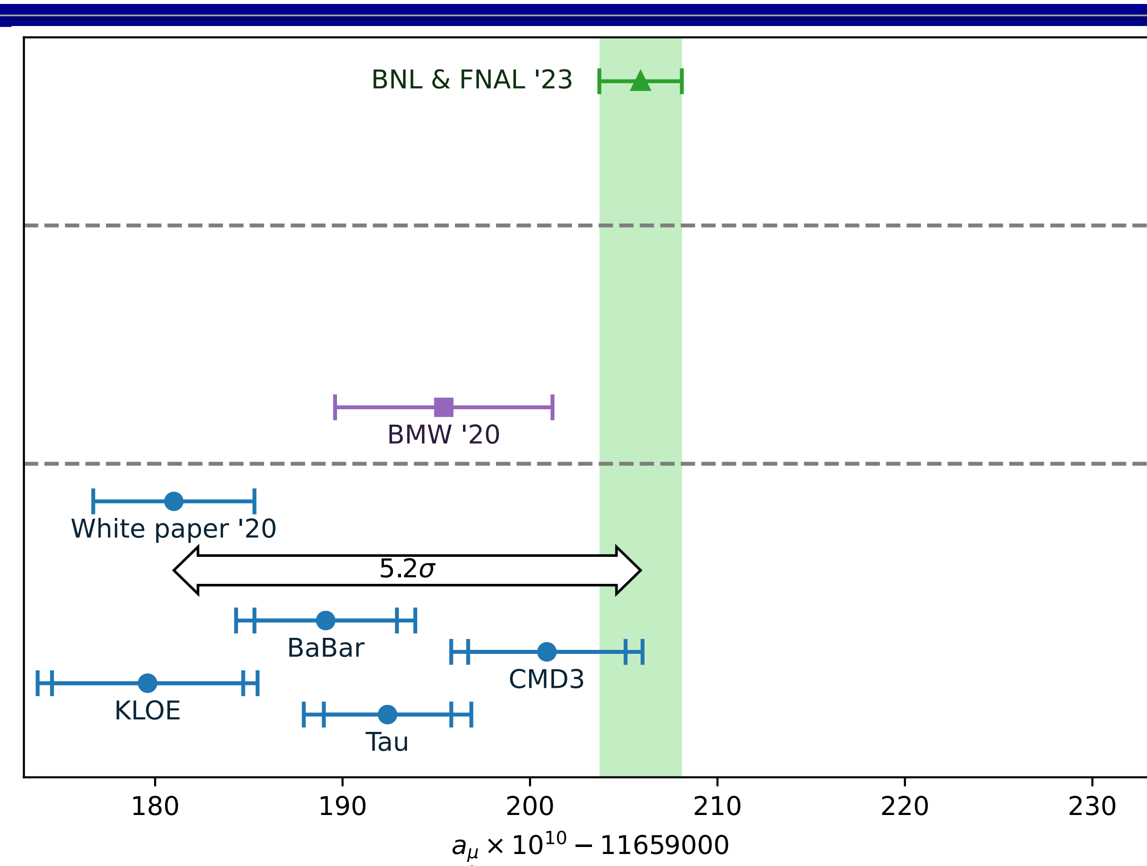

g_μ-2 Theory Initiative

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Review article

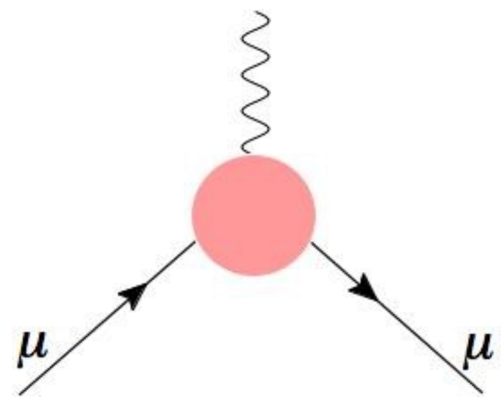
The anomalous magnetic moment of the muon in the Standard Model: an update

R. Aliberti [1,2], T. Aoyama [3], E. Balzani [4,5], A. Bashir [6,7], G. Benton [8,9], J. Bijnens [10], V. Biloshytskyi [1,2], T. Blum [11,12], D. Boito [13], M. Bruno [14,15], E. Budassi [16,17], S. Burri [18], L. Cappiello [19], C.M. Carloni Calame [17], M. Cè [14,15], V. Cirigliano [20,21], D.A. Clarke [22], G. Colangelo [18,*], L. Cotrozzi [23], M. Cottini [18], I. Danilkin [1,2], M. Davier [24,*], M. Della Morte [25], A. Denig [1,2,26,27], C. DeTar [22], V. Druzhinin [28], G. Eichmann [29], A.X. El-Khadra [8,9,*], E. Estrada [30], X. Feng [31,32,33], C.S. Fischer [34,35], R. Frezzotti [36], G. Gagliardi [37], A. Gérardin [38], M. Ghilardi [16,17], D. Giusti [39,40], M. Golterman [41], S. Gonzàlez-Solís [42,43], S. Gottlieb [44], R. Gruber [45], A. Guevara [46], V. Gülpers [47], A. Gurgone [48,49], F. Hagelstein [1,2], M. Hayakawa [50,51], N. Hermansson-Truedsson [10,47], A. Hoecker [52], M. Hoferichter [18,*], B.-L. Hoid [1,2], S. Holz [18], R.J. Hudspith [53], F. Ignatov [23], L. Jin [11], N. Kalntis [18], G. Kanwar [47], A. Keshavarzi [54], J. Komijani [45], J. Koponen [1,2], S. Kuberski [55], B. Kubis [56], A. Kupich [28], A. Kupść [57,58], S. Lahert [22], S. Laporta [4,5], C. Lehner [40,*], M. Lellmann [1], L. Lellouch [38,*], T. Leplumey [59,60], J. Leutgeb [61], T. Lin [31], Q. Liu [62], I. Logashenko [28], C.Y. London [13], G. López Castro [30], J. Lüdtke [63], A. Lusiani [49,64], A. Lutz [24], J. Mager [61], B. Malaescu [65], K. Maltman [66,67], M.K. Marinković [45], J. Márquez [30], P. Masjuan [68,69], H.B. Meyer [1,2,26,27], T. Mibe [70,*], N. Miller [26,27], A. Miramontes [71,72], A. Miranda [68], G. Montagna [16,17], S.E. Müller [73], E.T. Neil [74], A.V. Nesterenko [28], O. Nicrosini [17], M. Nio [51,75], D. Nomura [76], J. Paltrinieri [23], L. Parato [45], J. Parrino [40], V. Pascalutsa [1,2], M. Passera [5,77], S. Peris [68,69], P. Petit Rosàs [23], F. Piccinini [17,78], R.N. Pilato [23], L. Polat [65,24], A. Portelli [47], D. Portillo-Sánchez [30], M. Procura [63], L. Punzi [49,64], K. Raya [7], A. Rebhan [61], C.F. Redmer [1,2], B.L. Roberts [79,*], A. Rodríguez-Sánchez [72], P. Roig [30,72], J. Ruiz de Elvira [80], P. Sánchez-Puertas [81], A. Signer [82,60], J.W. Sitison [74], D. Stamen [56], D. Stöckinger [83], H. Stöckinger-Kim [83], P. Stoffer [60,82], Y. Sue [70], P. Tavella [45], T. Teubner [23,*], J.-N. Toelstede [60,82], G. Toledo [84], W.J. Torres Bobadilla [23], J.T. Tsang [55], F.P. Ucci [16,17], Y. Ulrich [23], R.S. Van de Water [85], G. Venanzoni [23,49], S. Volkov [86], G. von Hippel [1,2], G. Wang [38], U. Wenger [18], H. Wittig [1,2,26,27,*], A. Wright [23], E. Zaid [23], M. Zanke [56], Z. Zhang [24], M. Zillinger [18],

A B S T R A C T

We present the current Standard Model (SM) prediction for the muon anomalous magnetic moment, a_μ, updating the first White Paper (WP20) [1]. The pure QED and electroweak contributions have been further consolidated, while hadronic contributions continue to be responsible for the bulk of the uncertainty of the SM prediction. Significant progress has been achieved in the hadronic light-by-light scattering contribution using both the data-driven dispersive approach as well as lattice-QCD calculations, leading to a reduction of the uncertainty by almost a factor of two. The most important development since WP20 is the change in the estimate of the leading-order hadronic-vacuum-polarization (LO HVP) contribution. A new measurement of the $e^+e^- \to \pi^+\pi^-$ cross section by CMD-3 has increased the tensions among data-driven dispersive evaluations of the LO HVP contribution to a level that makes it impossible to combine the results in a meaningful way. At the same time, the attainable precision of lattice-QCD calculations has increased substantially and allows for a consolidated lattice-QCD average of the LO HVP contribution with a precision of about 0.9%. Adopting the latter in this update has resulted in a major upward shift of the total SM prediction, which now reads $a_\mu^{\rm SM} = 116\,592\,033(62) \times 10^{-11}$ (530 ppb). When compared against the current experimental average based on the E821 experiment and runs 1–6 of E989 at Fermilab, one finds $a_\mu^{\rm exp} - a_\mu^{\rm SM} = 38(63) \times 10^{-11}$, which implies that there is no tension between the SM and experiment at the current level of precision. The final precision of E989 (127 ppb) is the target of future efforts by the Theory Initiative. The resolution of the tensions among data-driven dispersive evaluations of the LO HVP contribution will be a key element in this endeavor.

Muon magnetic anomaly



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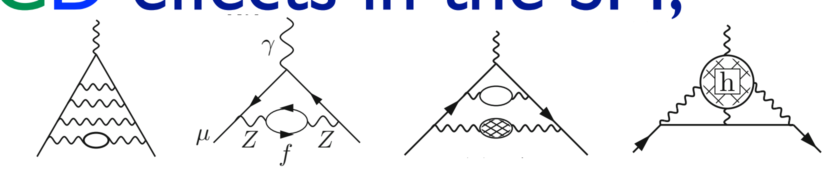

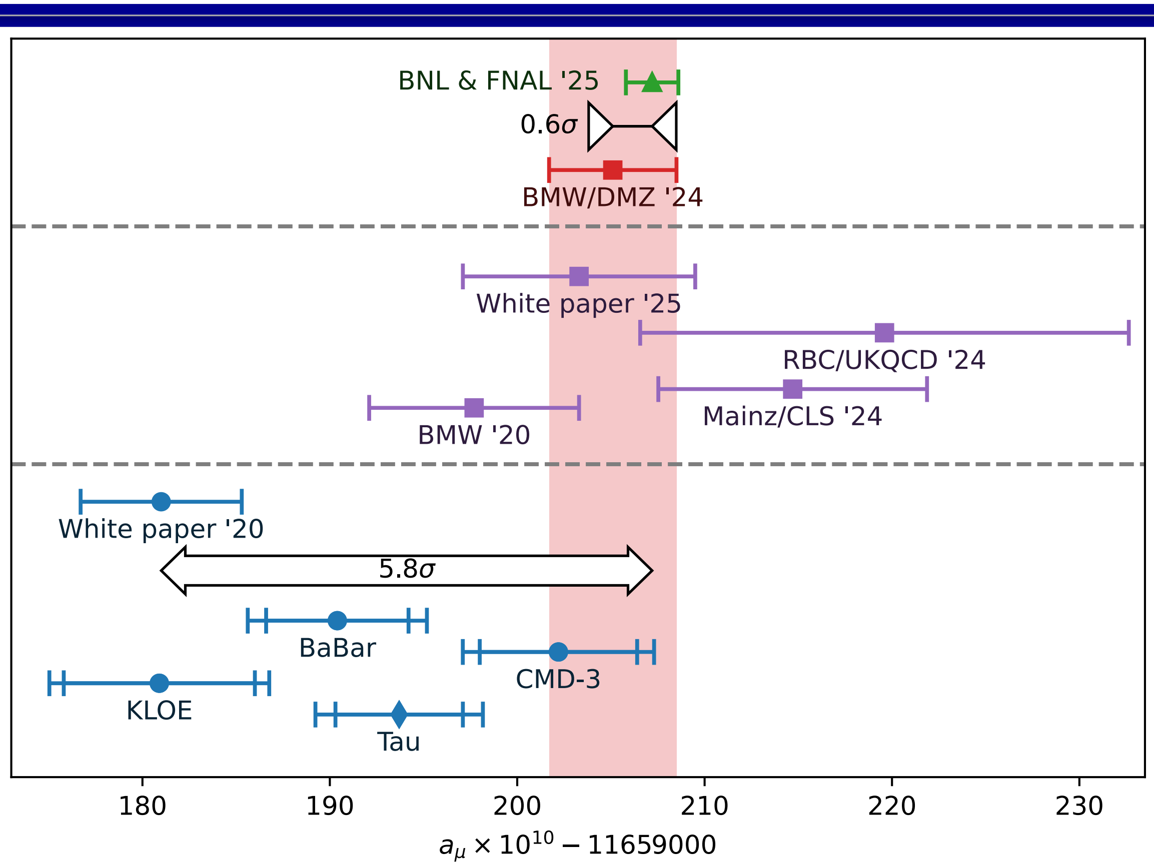

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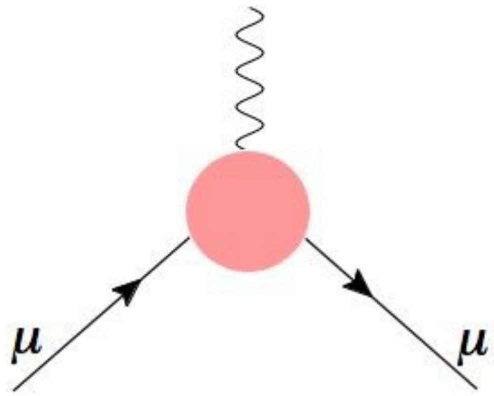
HLbL	11.55(99)

Aliberti *et al.* [WP] 2025

Muon magnetic anomaly



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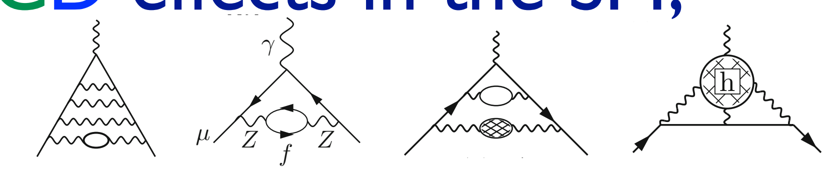
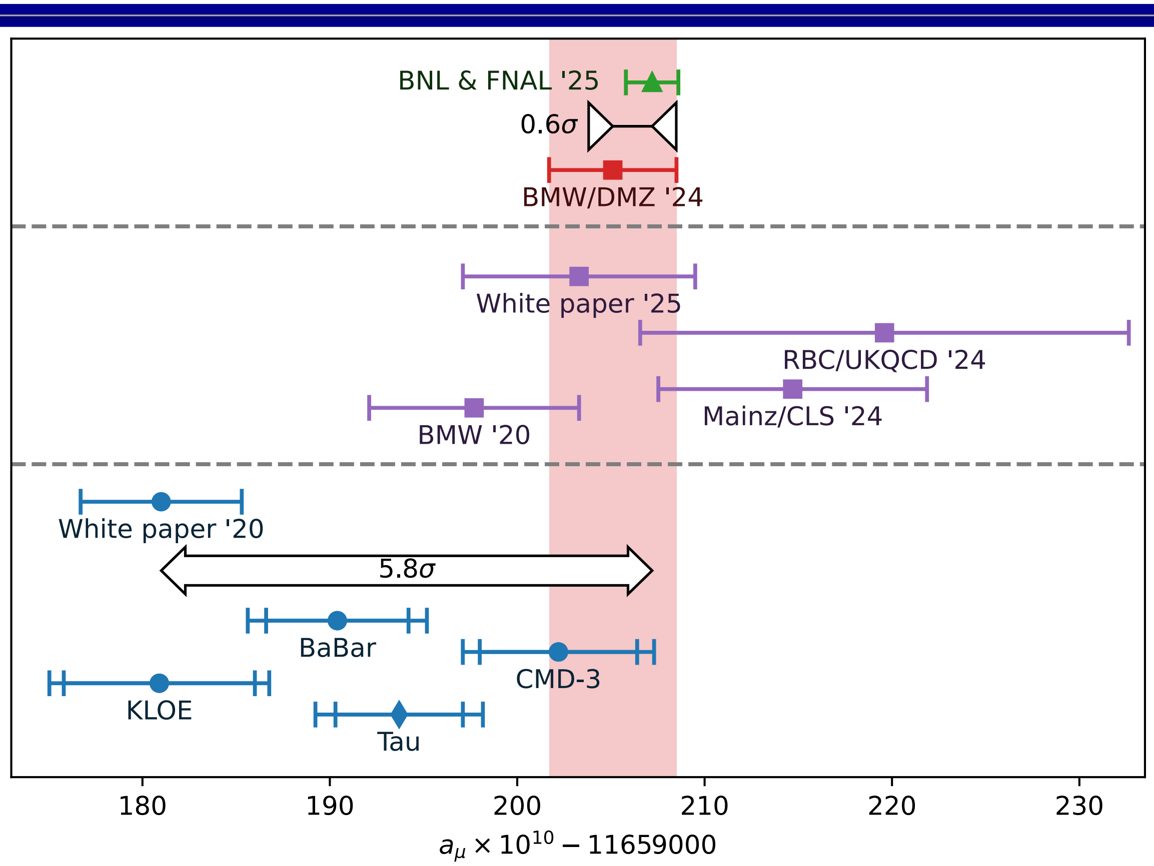
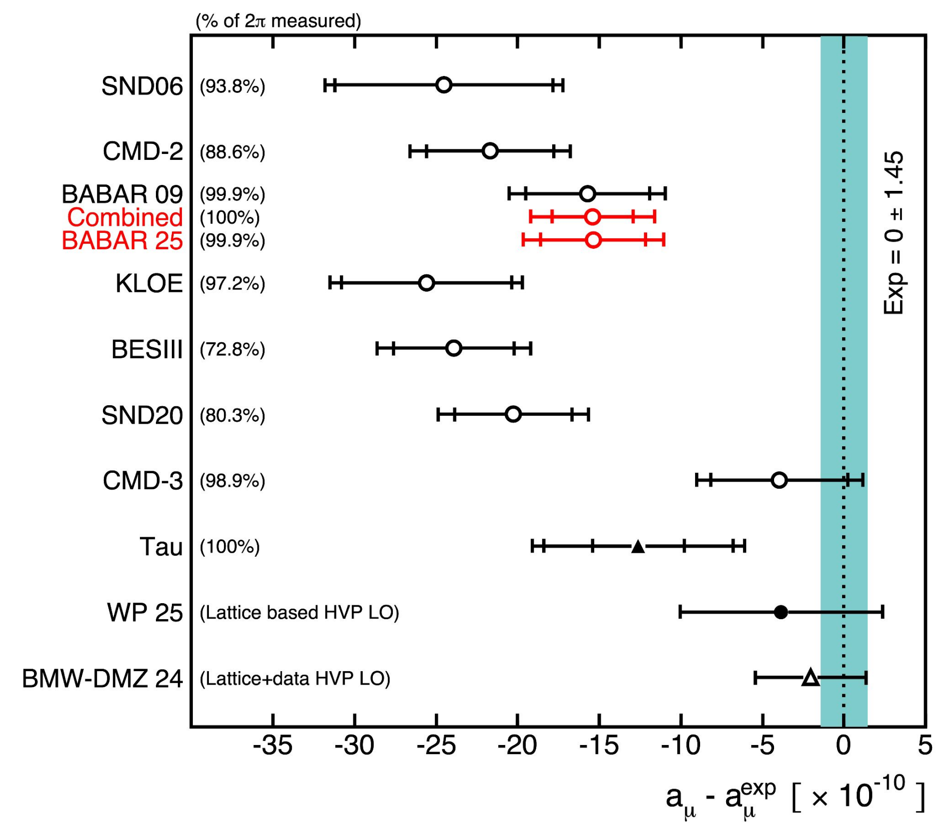






Hadronic contributions

$$a_\mu^{\text{exp}} - a_\mu^{\text{QED}} - a_\mu^{\text{EW}} = 718.9(4.1) \times 10^{-10} \stackrel{?}{=} a_\mu^{\text{had}}$$

Clearly right order of magnitude:

$$a_\mu^{\text{had}} = O\left(\left(\frac{\alpha}{\pi}\right)^2 \left(\frac{m_\mu}{M_\rho}\right)^2\right) = O\left(10^{-7}\right)$$

(already Gourdin & de Rafael '69 found $a_\mu^{\text{had}} = 650(50) \times 10^{-10}$)

Huge challenge: theory of strong interaction between quarks and gluons, QCD, hugely nonlinear at energies relevant for a_μ

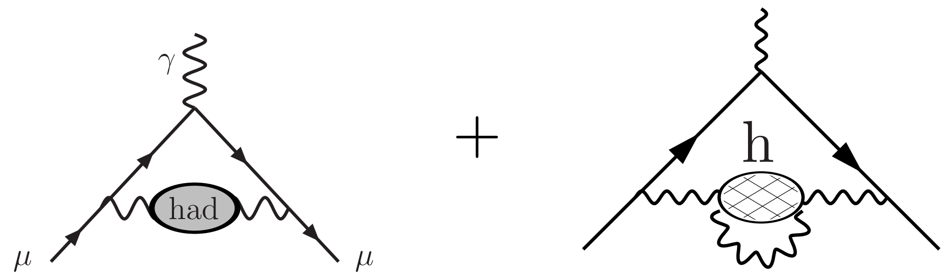
$\rightarrow$ perturbative methods used for electromagnetic and weak interactions do not work

$\rightarrow$ need nonperturbative approaches

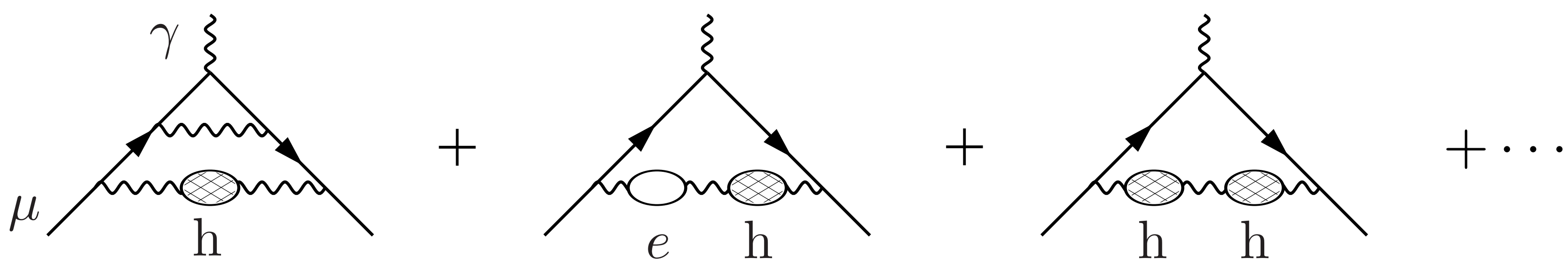
Write

$$a_\mu^{\text{had}} = a_\mu^{\text{LO-HVP}} + a_\mu^{\text{HO-HVP}} + a_\mu^{\text{HLbyL}} + O\left(\left(\frac{\alpha}{\pi}\right)^4\right)$$

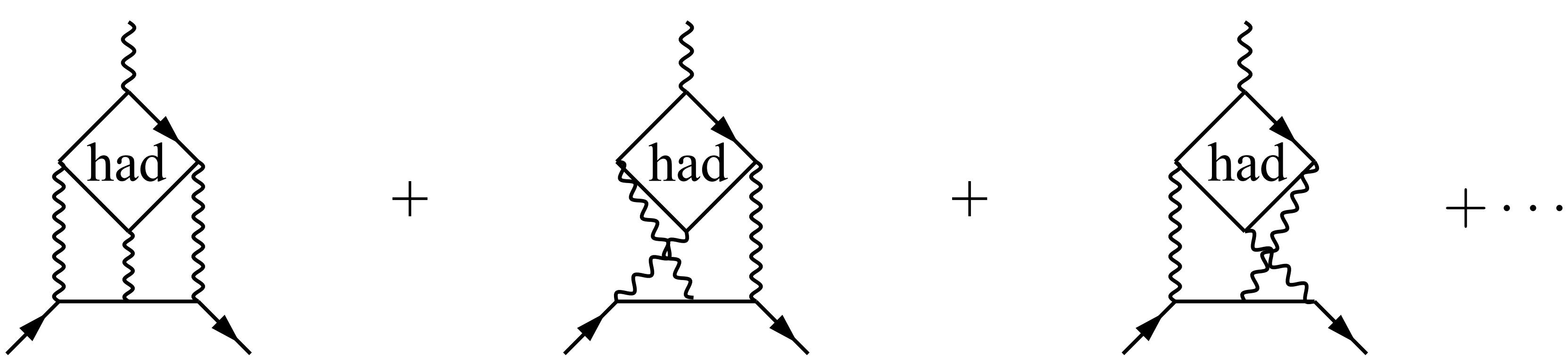
Hadronic contributions: diagrams



$\rightarrow a_\mu^{\text{LO-HVP}} = O\left(\left(\frac{\alpha}{\pi}\right)^2\right)$

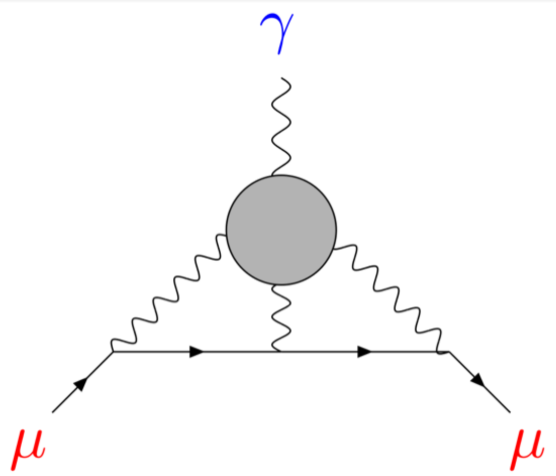


$\rightarrow a_\mu^{\text{NLO-HVP}} = O\left(\left(\frac{\alpha}{\pi}\right)^3\right)$

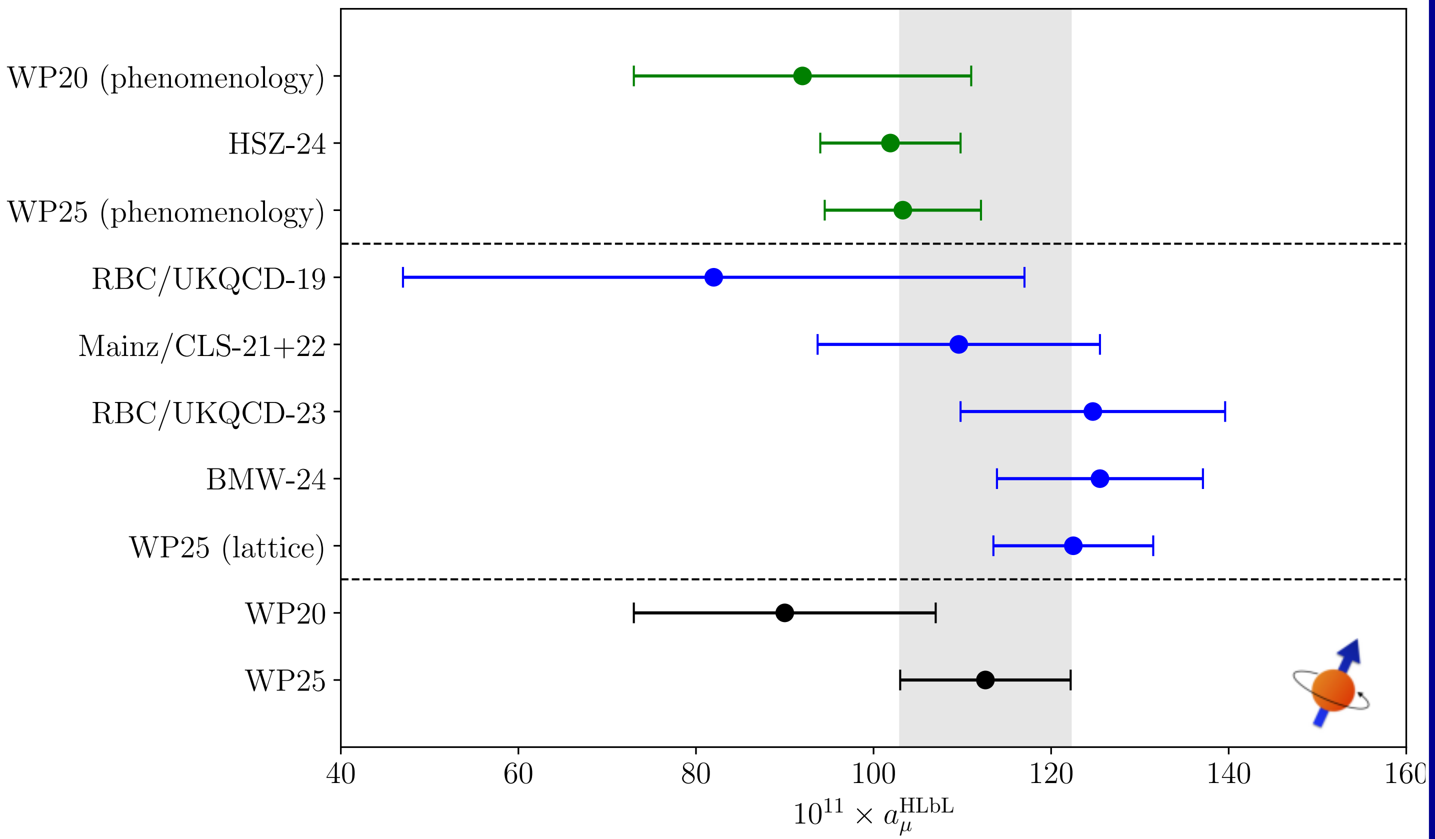


$\rightarrow a_\mu^{\text{HLbL}} = O\left(\left(\frac{\alpha}{\pi}\right)^3\right)$

Hadronic light-by-light



- HLbL much more complicated than HVP, but ultimate precision needed is $\simeq 10\%$ instead of $\simeq 0.2\%$
- For many years, only accessible to models of QCD w/ difficult to estimate systematics (Prades et al '09): $a_\mu^{\text{HLbL}} = 10.5(2.6) \times 10^{-10}$
- Also, lattice QCD calculations were exploratory and incomplete
- Tremendous progress in past 5 years:
  - $\rightarrow$ Phenomenology: rigorous data driven approach [Colangelo, Hoferichter, Kubis, Procura, Stoffer,… '15-'20]
  - $\rightarrow$ Lattice: three solid lattice calculations
- All agree w/ older model results but error estimate much more solid and will improv
- Agreed upon average w/ NLO HLbL and conservative error estimates
- $a_\mu^{\text{exp}} - a_\mu^{\text{QED}} - a_\mu^{\text{EW}} - a_\mu^{\text{HLbL}} = 709.7(4.5) \times 10^{-10} \stackrel{?}{=} a_\mu^{\text{HVP}}$



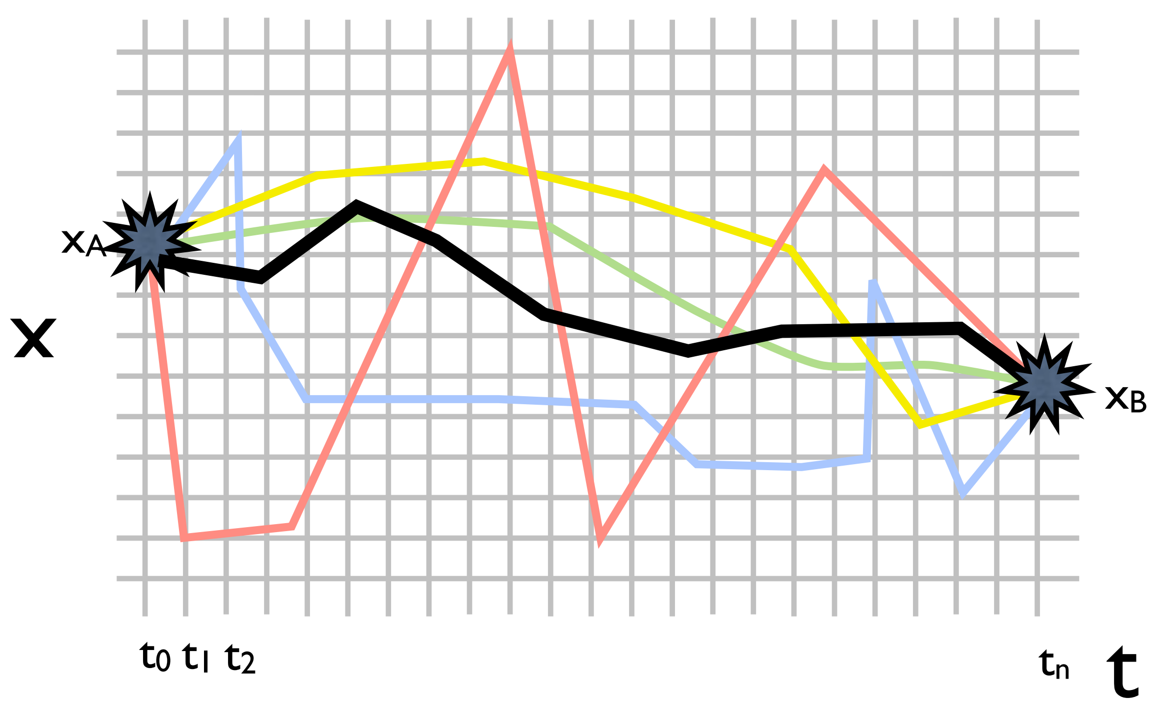
Aliberti *et al.* [WP] 2025

Small interlude:
Lattice QCD

Lattice QCD

Numerical first-principles approach to non-perturbative QCD

- Discretise QCD onto 4D space-time lattice
- QCD equations ⟷ integrals over the values of quark and gluon fields on each site/link (QCD path integral)
- ~10^{12} variables (for state-of-the-art)

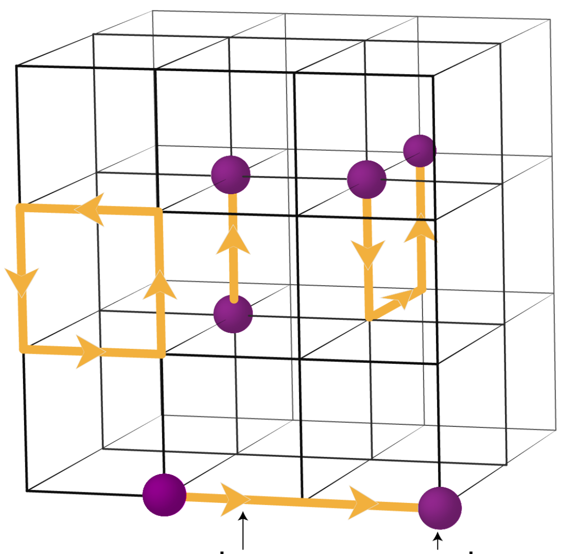


- Evaluate by importance sampling
- Paths near classical action dominate
- Calculate physics on a set (ensemble) of samples of the quark and gluon fields

Lattice QCD

Numerical first-principles approach to non-perturbative QCD

- Euclidean space-time $t \to i\tau$
- Finite lattice spacing a
- Volume $L^3 \times T = 64^3 \times 128$
- Boundary conditions



Approximate the QCD path integral by **Monte Carlo**

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}A \mathcal{D}\overline{\psi} \mathcal{D}\psi \mathcal{O}[A, \overline{\psi}\psi] e^{-S[A,\overline{\psi}\psi]} \quad \rightarrow \quad \langle \mathcal{O} \rangle \simeq \frac{1}{N_{\mathrm{conf}}} \sum_{i}^{N_{\mathrm{conf}}} \mathcal{O}([U^i])$$

with field configurations U^i distributed according to $e^{-S[U]}$

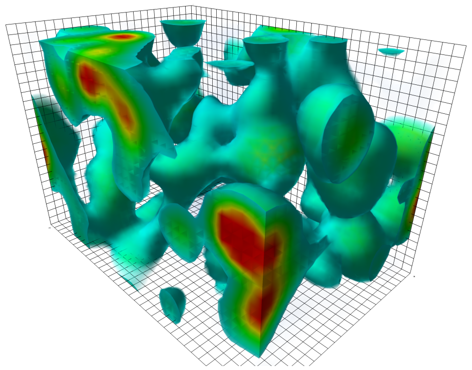
Lattice QCD

Workflow of a lattice QCD calculation

1. Generate field configurations via Hybrid Monte Carlo
   - Leadership-class computing
   - ~100K cores or 1000GPUs, 10's of TF-years
   - O(100-1000) configurations, each ~10-100GB

2. Compute propagators
   - Large sparse matrix inversion
   - ~few 100s GPUs
   - 10x field config in size, many per config

3. Contract into correlation functions
   - ~few GPUs
   - O(100k-1M) copies



Lattice QCD action density, Leinweber, CSSM, Adelaide, 2003

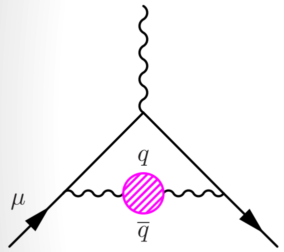
Hadrons are emergent phenomena of statistical average over background gluon configurations

- 1 year on supercomputer ~ 100k years on laptop

HVP from the lattice

&

Window observables

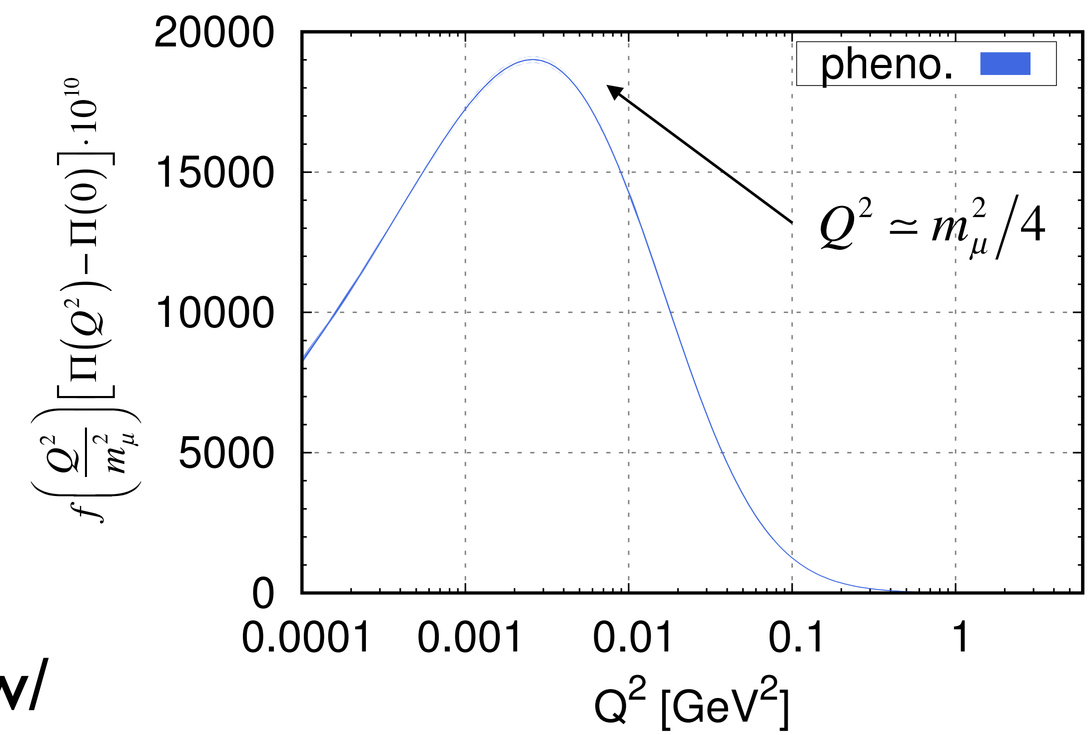


HVP from LQCD

$$\Pi_{\mu\nu}(Q) = \int d^4x\, e^{iQ\cdot x} \langle J_\mu(x) J_\nu(0) \rangle = \left[\delta_{\mu\nu} Q^2 - Q_\mu Q_\nu\right] \Pi(Q^2)$$

$$a_\mu^{\text{HVP,LO}} = 4\alpha_{em}^2 \int_0^\infty dQ^2 \,\frac{1}{m_\mu^2}\, f\left(\frac{Q^2}{m_\mu^2}\right)\left[\Pi(Q^2) - \Pi(0)\right]$$

B. E. Lautrup *et al.*, 1972



F. Jegerlehner, "alphaQEDc17"

FV & $a \neq 0$: A. discrete momenta ($Q_{\min} = 2\pi/T > m_\mu/2$); B. $\Pi_{\mu\nu}(0) \neq 0$ in FV contaminates $\Pi(Q^2) \sim \Pi_{\mu\nu}(Q)/Q^2$ for $Q^2 \to 0$ w/ very large FV effects; C. $\Pi(0) \sim \ln(a)$

↓

Time-Momentum Representation

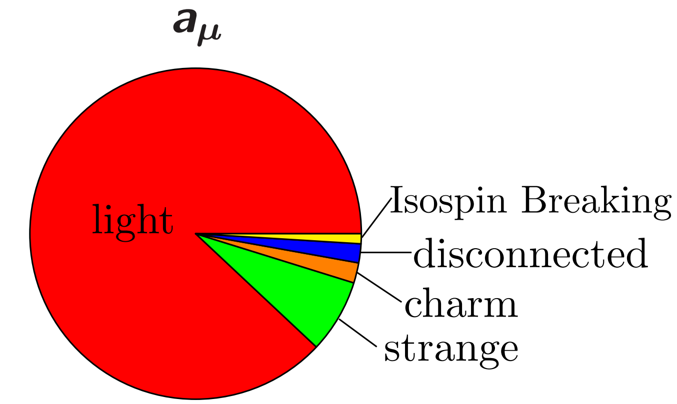
$$a_\mu^{\text{HVP,LO}} = 4\alpha_{em}^2 \int_0^\infty dt\, \tilde{f}(t)\, V(t)$$

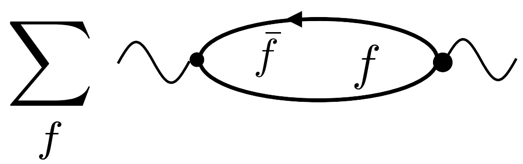
$$V(t) \equiv \frac{1}{3} \sum_{i=1,2,3} \int d\vec{x}\, \langle J_i(\vec{x},t) J_i(0) \rangle$$

D. Bernecker and H. B. Meyer, 2011

Time-Momentum Representation

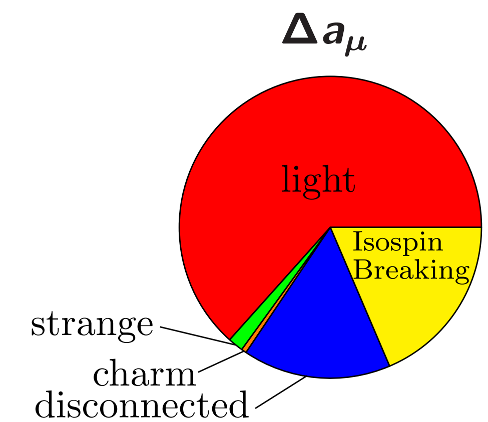
- No reliance on exp. data, except for hadronic quantities used to calibrate the simulation ($M_\pi, M_K, M_{nucl}, \ldots$)
- Can perform an explicit quark flavor separation of $a_\mu^{\mathrm{HVP,LO}}$

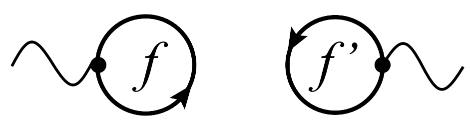




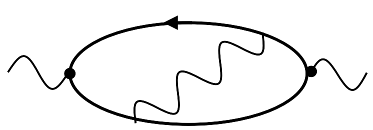
light-quark connected $a_\mu^{\mathrm{HVP,LO}}(\mathrm{ud}) \sim 90\,\%$ of total

s,c-quark connected $a_\mu^{\mathrm{HVP,LO}}(\mathrm{s,c}) \sim 8\,\%\,, 2\,\%$ of total



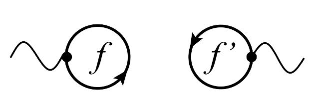
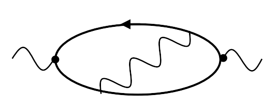


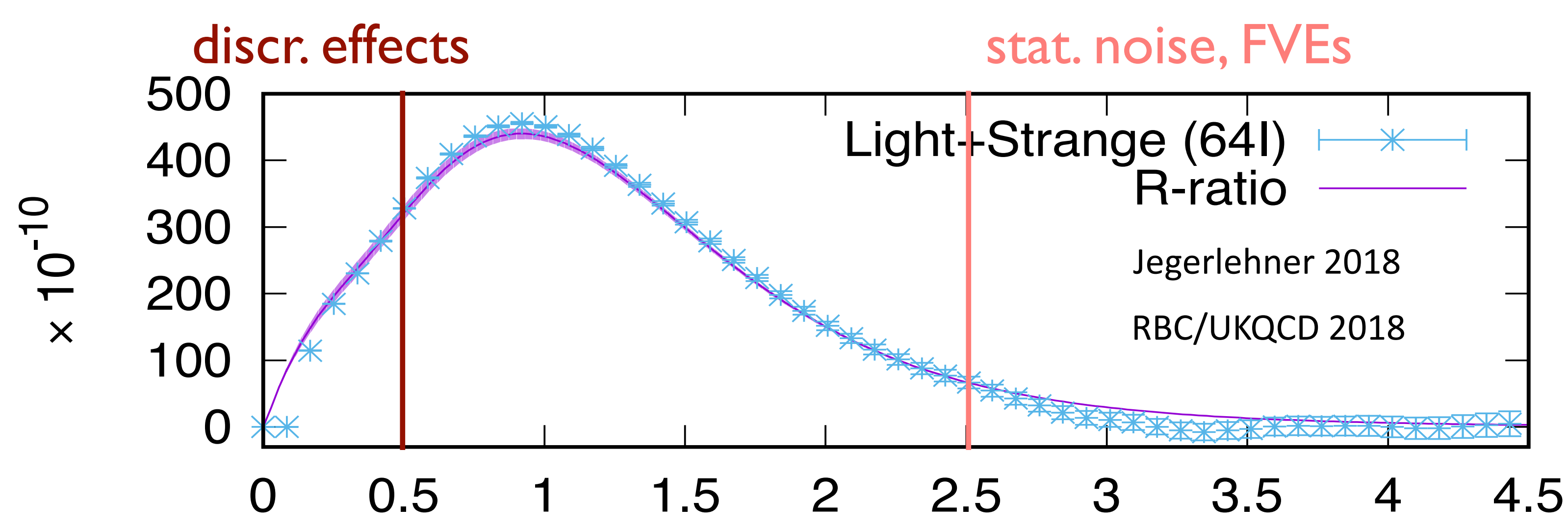
disconnected $a_{\mu,disc}^{\mathrm{HVP,LO}} \sim 2\,\%$ of total



IB ($m_u \neq m_d + \mathrm{QED}$) $\delta a_\mu^{\mathrm{HVP,LO}} \sim 1\,\%$ of total

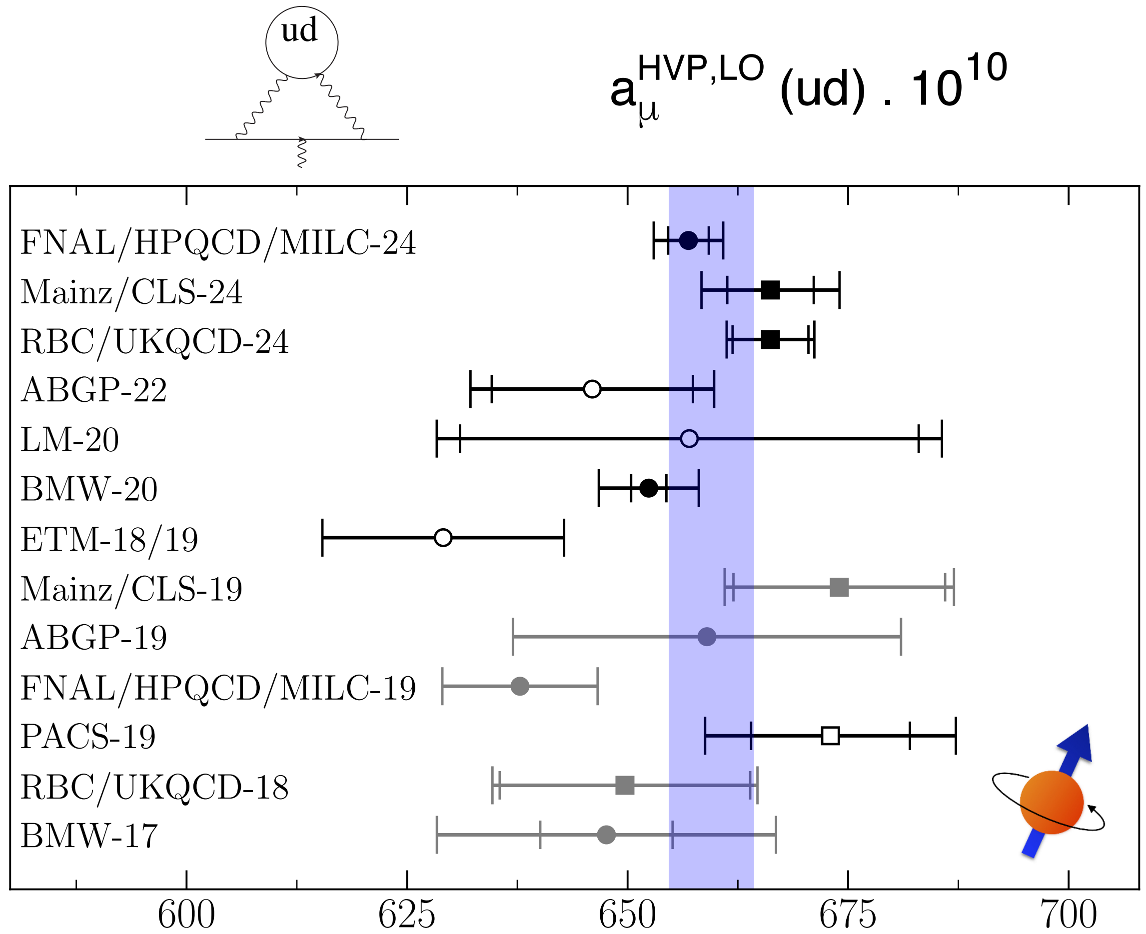
Challenges:

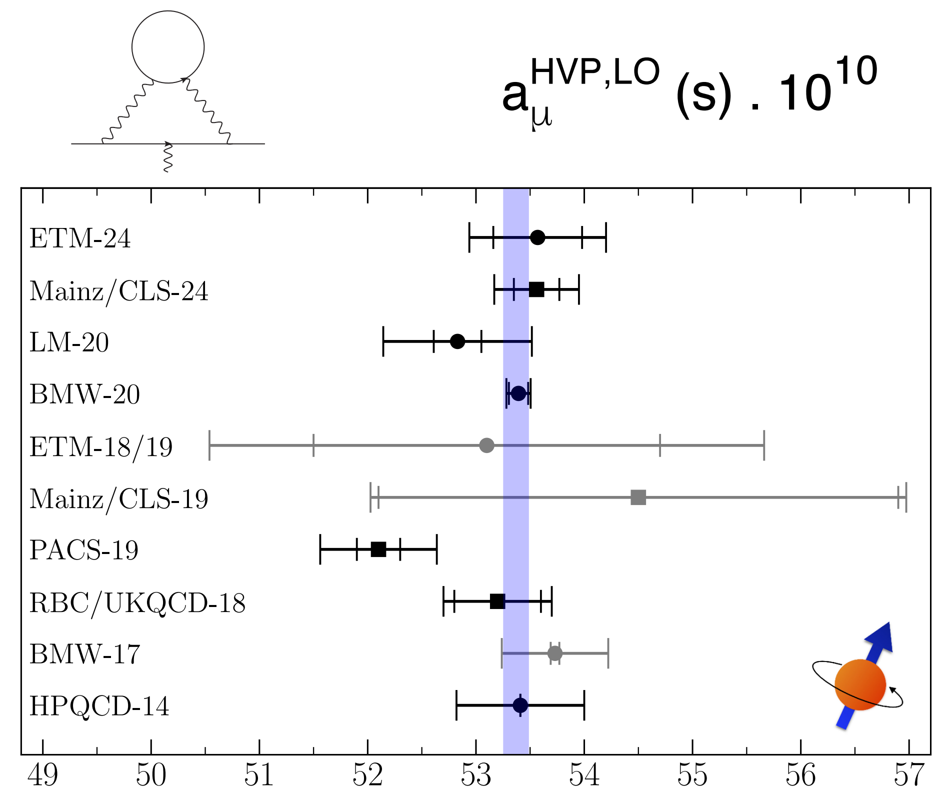
- sub-percent stat. precision
  exp. growing StN ratio in $V(t)$ as $t \to \infty$
- correct for FVEs, control discr. effects (scale setting and continuum extrap.)
- quark-disconn. diagrams 
  control stat. & stochastic noise
- isospin-breaking: $m_u \neq m_d$, $\alpha_{em} \neq 0$ 

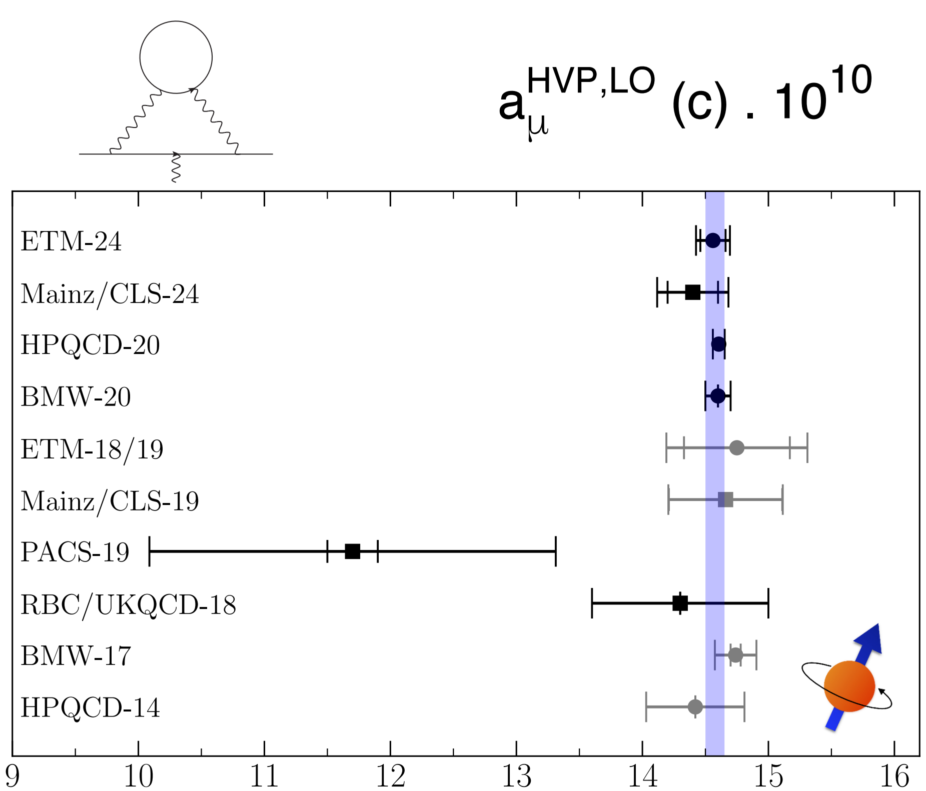


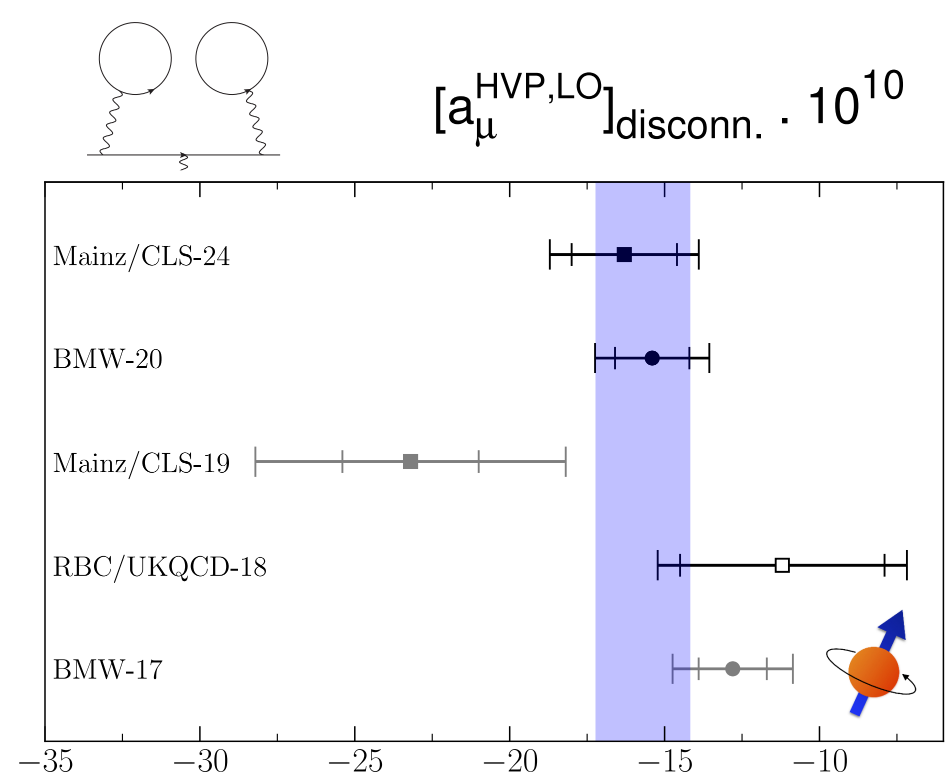
Results for each contribution

WP '25









Windows "on the g-2 mystery"

Restrict integration over Euclidean time to sub-intervals

→ reduce/enhance sensitivity to systematic effects

$$a_\mu^{\text{HVP,LO}} = a_\mu^{SD} + a_\mu^{W} + a_\mu^{LD}$$

$$a_\mu^{SD}(f; t_0, \Delta) \equiv 4\alpha_{em}^2 \int_0^\infty dt\, \tilde{f}(t) V^f(t) \left[1 - \Theta\left(t, t_0, \Delta\right)\right]$$

$$a_\mu^{W}(f; t_0, t_1, \Delta) \equiv 4\alpha_{em}^2 \int_0^\infty dt\, \tilde{f}(t) V^f(t) \left[\Theta\left(t, t_0, \Delta\right) - \Theta\left(t, t_1, \Delta\right)\right]$$

$$a_\mu^{LD}(f; t_1, \Delta) \equiv 4\alpha_{em}^2 \int_0^\infty dt\, \tilde{f}(t) V^f(t)\, \Theta\left(t, t_1, \Delta\right)$$

$$\Theta\left(t, t', \Delta\right) = \frac{1}{1 + e^{-2(t-t')/\Delta}}$$

"Standard" choice:

$$t_0 = 0.4\ \text{fm} \quad t_1 = 1.0\ \text{fm}$$

$$\Delta = 0.15\ \text{fm}$$

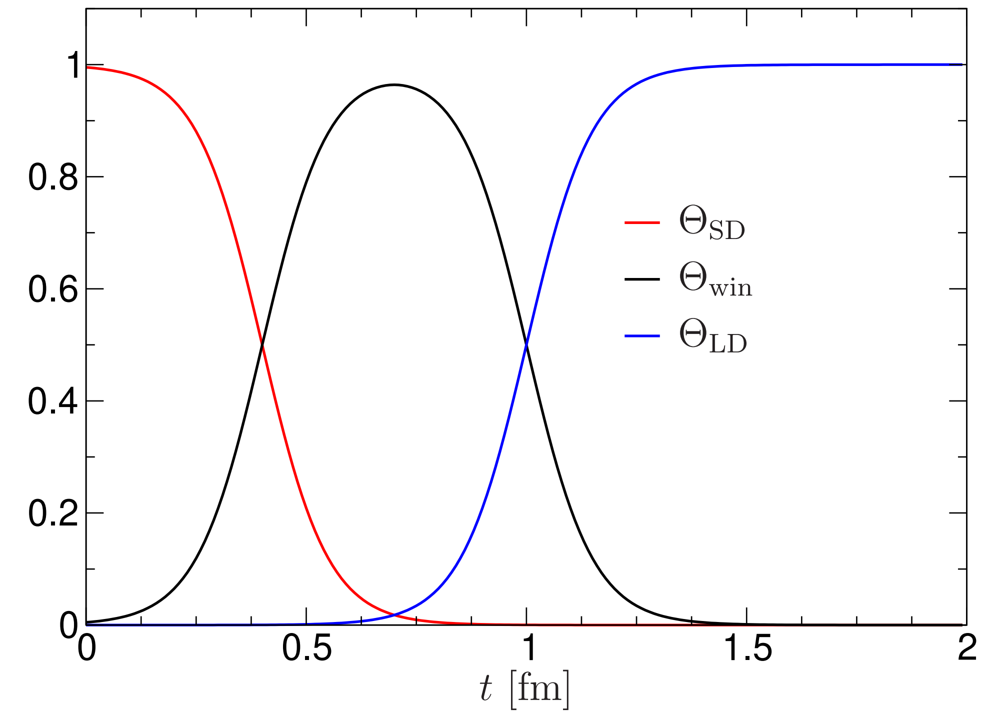
RBC/UKQCD 2018

Intermediate window

- Reduced FVEs
- Much better StN ratio

→ Precision test of different lattice calculations

→ Commensurate uncertainties compared to dispersive evaluations

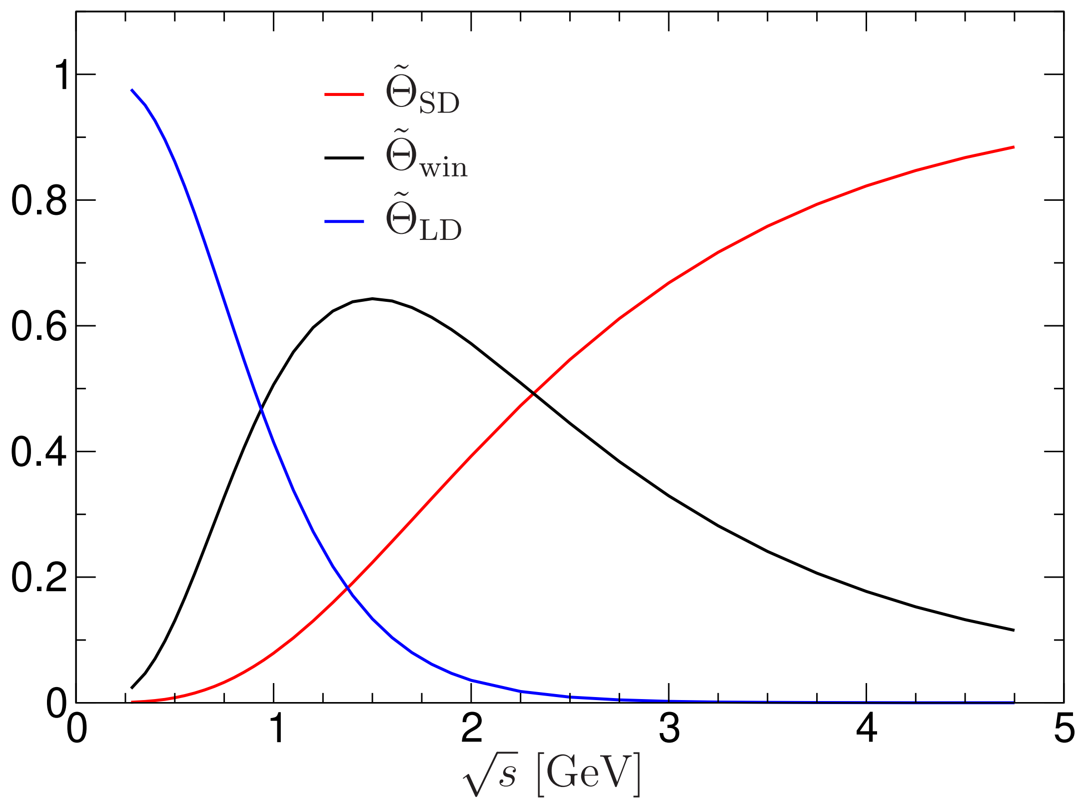


Comparison with R-ratio

$$V(t) = \frac{1}{12\pi^2}\int_{M_{\pi^0}}^{\infty} d(\sqrt{s})\, R(s)\, s\, e^{-\sqrt{s}t} \qquad R(s) = \frac{3s}{4\pi\alpha_{em}^2}\sigma(s, e^+e^- \to hadrons)$$

Insert $V(t)$ into the expression for TMR

$$a_{\mu,win}^{\mathrm{HVP,LO}} = 4\alpha_{em}^2 \int_{M_{\pi^0}}^{\infty} d(\sqrt{s}) R(s) \frac{1}{12\pi^2}\, s \int_0^{\infty} dt\, \tilde{f}(t)\, \Theta_{win}(t)\, e^{-\sqrt{s}t}$$



Colangelo *et al.* 2022

	a_{SD}^{HVP}	a_{int}^{HVP}	a_{LD}^{HVP}	a_{total}^{HVP}
All channels	68.4(5)	229.4(1.4)	395.1(2.4)	693.0(3.9)
	[9.9%]	[33.1%]	[57.0%]	[100%]
2π below 1.0 GeV	13.7(1)	138.3(1.2)	342.3(2.3)	494.3(3.6)
	[2.8%]	[28.0%]	[69.2%]	[100%]
3π below 1.8 GeV	2.5(1)	18.5(4)	25.3(6)	46.4(1.0)
	[5.5%]	[39.9%]	[54.6%]	[100%]
White Paper [1]	–	–	–	693.1(4.0)
RBC/UKQCD [24]	–	231.9(1.5)	–	715.4(18.7)
BMWc [36]	–	236.7(1.4)	–	707.5(5.5)
BMWc/KNT [7, 36]	–	229.7(1.3)	–	–
Mainz/CLS [99]	–	237.30(1.46)	–	–
ETMC [100]	69.33(29)	235.0(1.1)	–	–

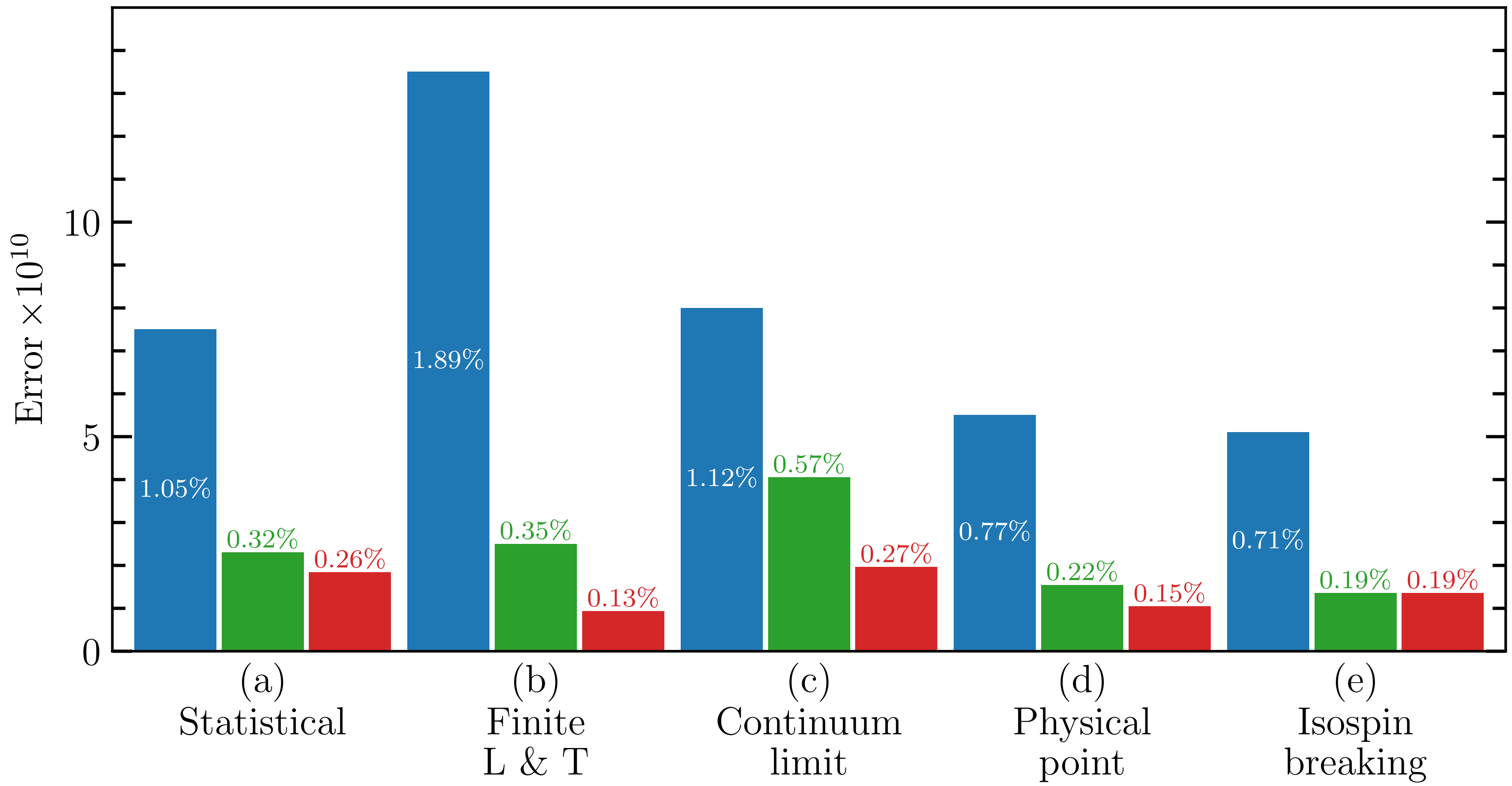
The BMW/DMZ-24 calculation

<u>including an update</u>

BMW/DMZ-24 calculation

High precision calculation of the hadronic vacuum polarisation contribution to the muon anomaly

A. Boccaletti[1,2], Sz. Borsanyi[1], M. Davier[3], Z. Fodor[1,4,5,2,6,7,*], F. Frech[1], A. Gérardin[8], D. Giusti[2,9], A.Yu. Kotov[2], L. Lellouch[8], Th. Lippert[2], A. Lupo[8], B. Malaescu[10], S. Mutzel[8,11], A. Portelli[12,13], A. Risch[1], M. Sjö[8], F. Stokes[2,14], K.K. Szabo[1,2], B.C. Toth[1], G. Wang[8], Z. Zhang[3]



- New lattice spacing $a = 0.048$ fm (same cost as all of BMWc '20) $\longrightarrow$ divides a^2 effects by 2
- Over 30,000 gauge configurations, 10's of millions of measurements

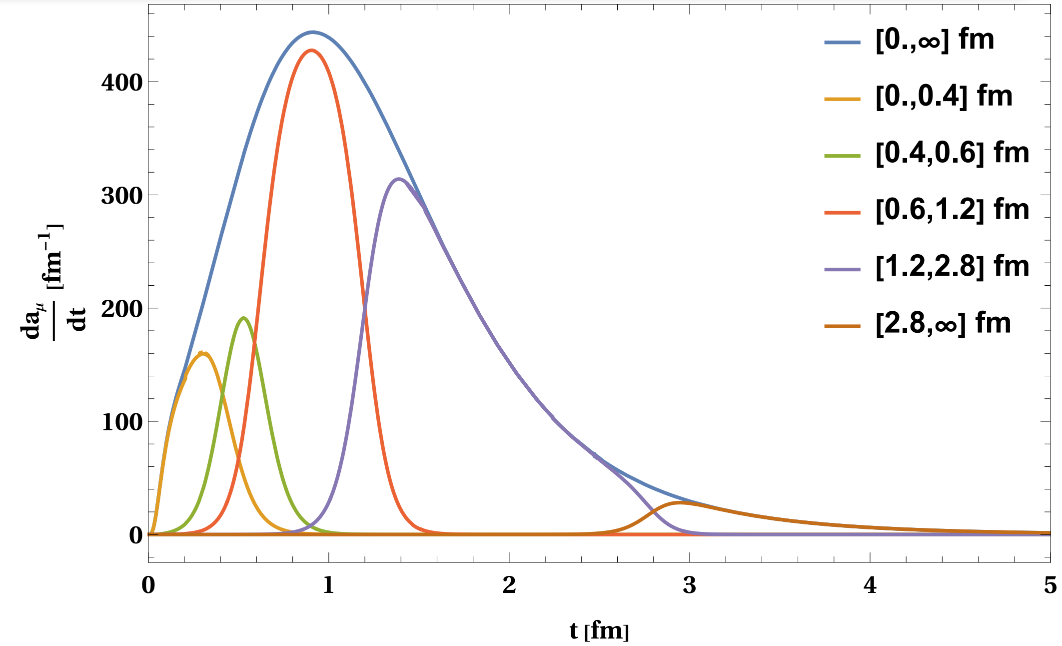
Strategy for improvement

- New simulations on finer lattice spacing: $128^3 \times 192$ w/ $a = 0.048\,\mathrm{fm}$
- Completely revamped analysis vs BMWc '20
- Break up analysis into optimized set of windows: $0-0.4$, $0.4-0.6$, $0.6-1.2$, $1.2-2.8\,\mathrm{fm}$
- Combined fit to $a_{\mu,\text{win,04-06}}^{\text{LO-HVP}}$, $a_{\mu,\text{win,06-12}}^{\text{LO-HVP}}$, $a_{\mu,\text{win,12-28}}^{\text{LO-HVP}}$
- Continuum extrapolate $I = 0$ instead of disconnected

$\to$ reduces statistical uncertainty
$\to$ reduces $a \to 0$ error

- Data-driven evaluation of tail: $a_{\mu,28\text{-}\infty}^{\text{LO-HVP}}$ (proposed and used w/ $1\,\mathrm{fm} \to \infty$ [RBC/UKQCD '18])

$\to$ reduces FV effect $18.5(2.5) \to 9.3(9)$, i.e. cv $\div 2$ & err $\div 3$
$\to$ reduces LD noise
$\to$ reduces LD taste breaking and $a \to 0$ error



[plot made w/ KNT '18 data set]

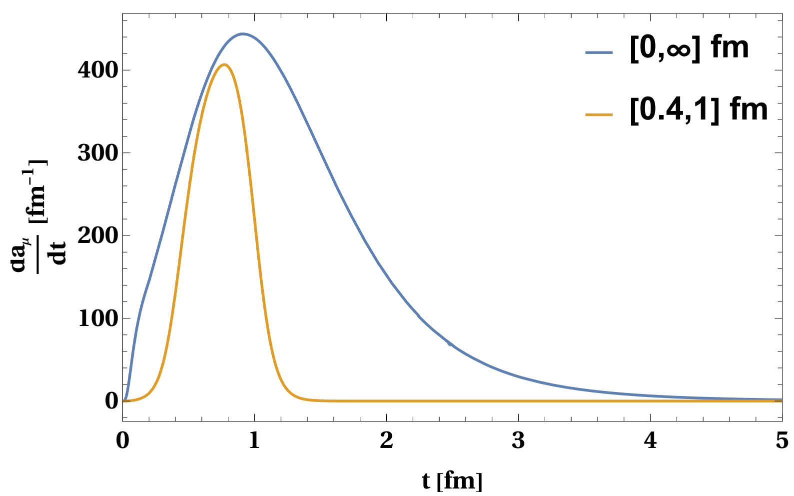
Fully blinded analysis:

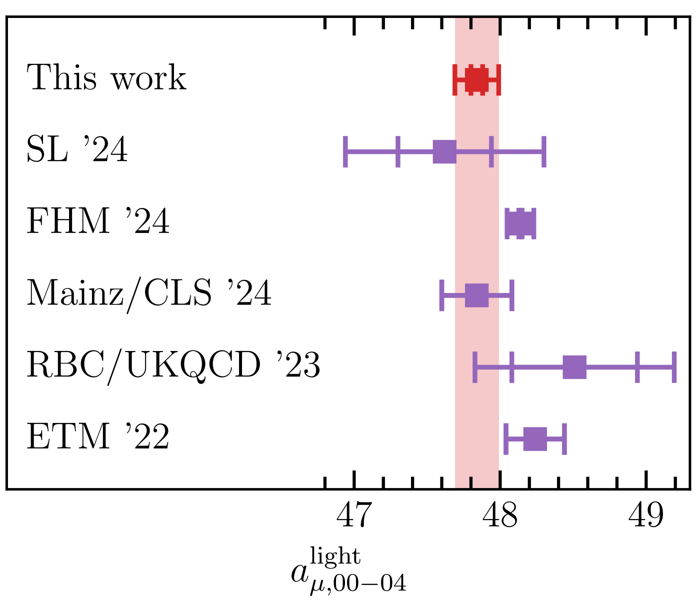
- Independent blinding by factor $\pm 3\%$ on correlator for each window and component, including data-driven tail
- $\gtrsim 2$ independent analyses of all blinded $a_\mu^{\text{LO-HVP}}$ contributions (and of other aspects)
- Once agreement reached, partial unblinding to allow sum of contributions
- Full unblinding on July 12, 2024, w/ automatic script that made appropriate changes in all figures and text
- Paper submitted to arXiv on July 15, 2024

July 12, 2024: unblinding

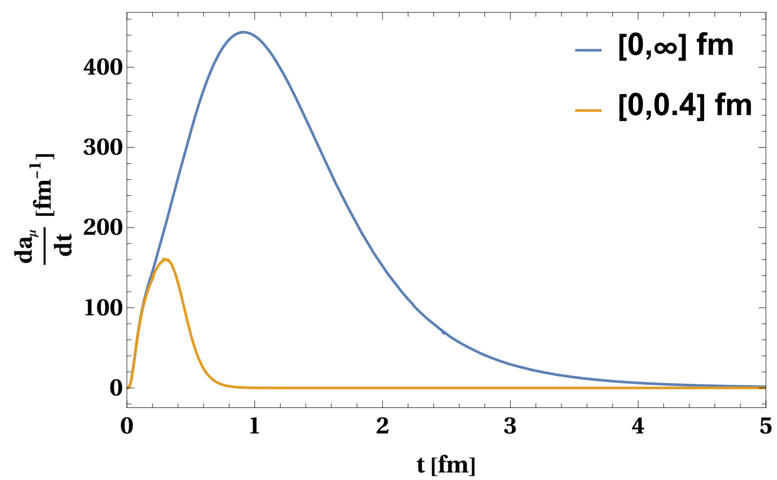
Benchmarking of lattice calculations: windows

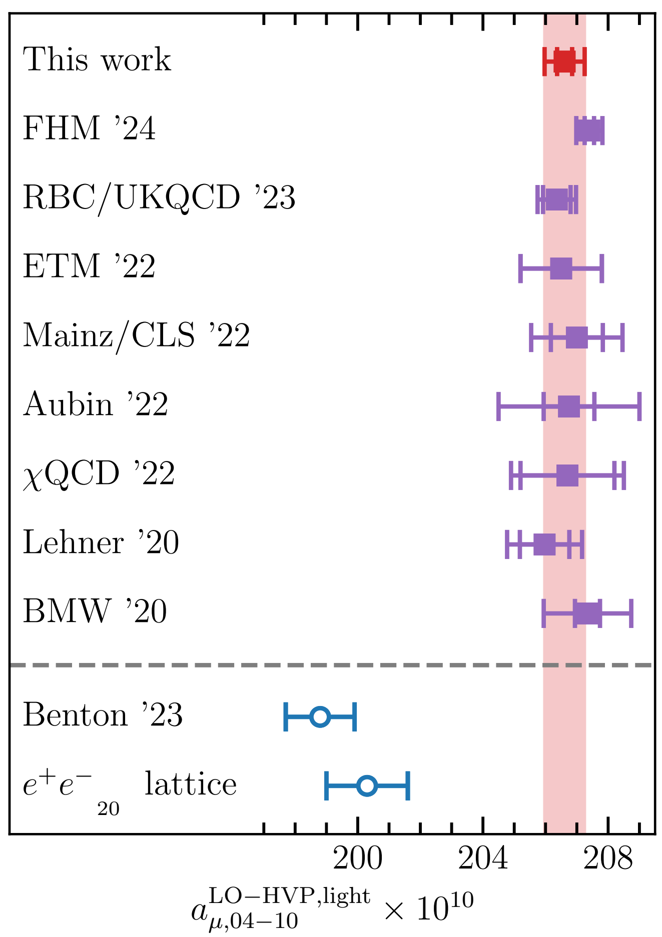
$0 \to 0.4$ fm



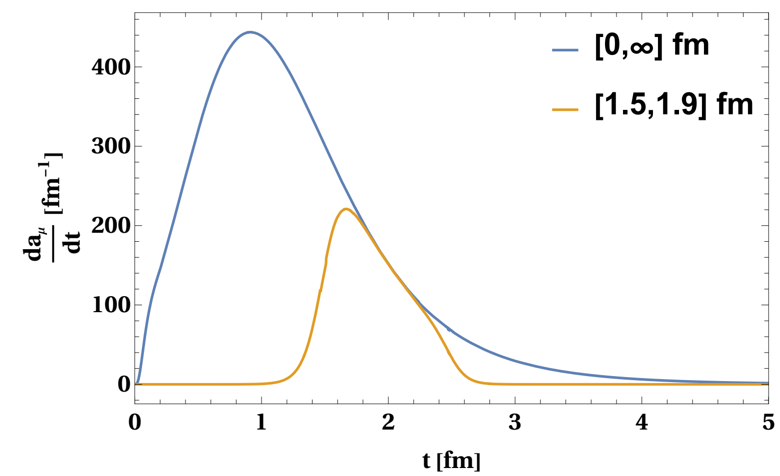


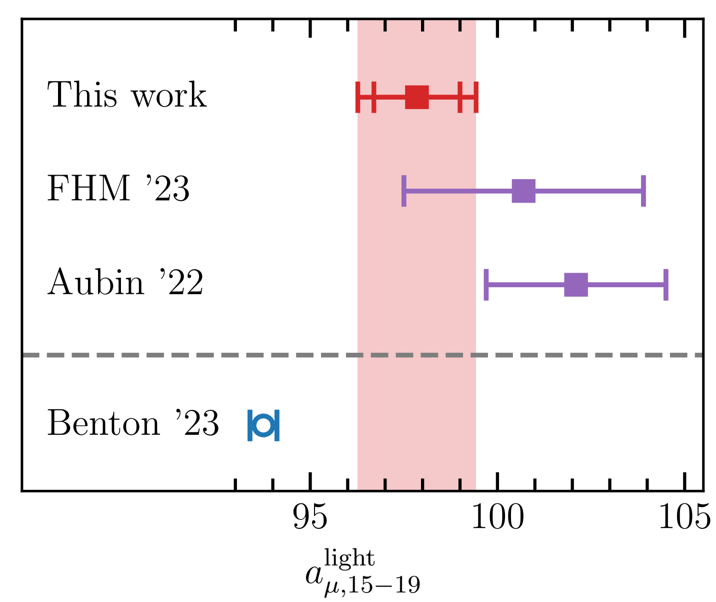
$0.4 \to 1$ fm



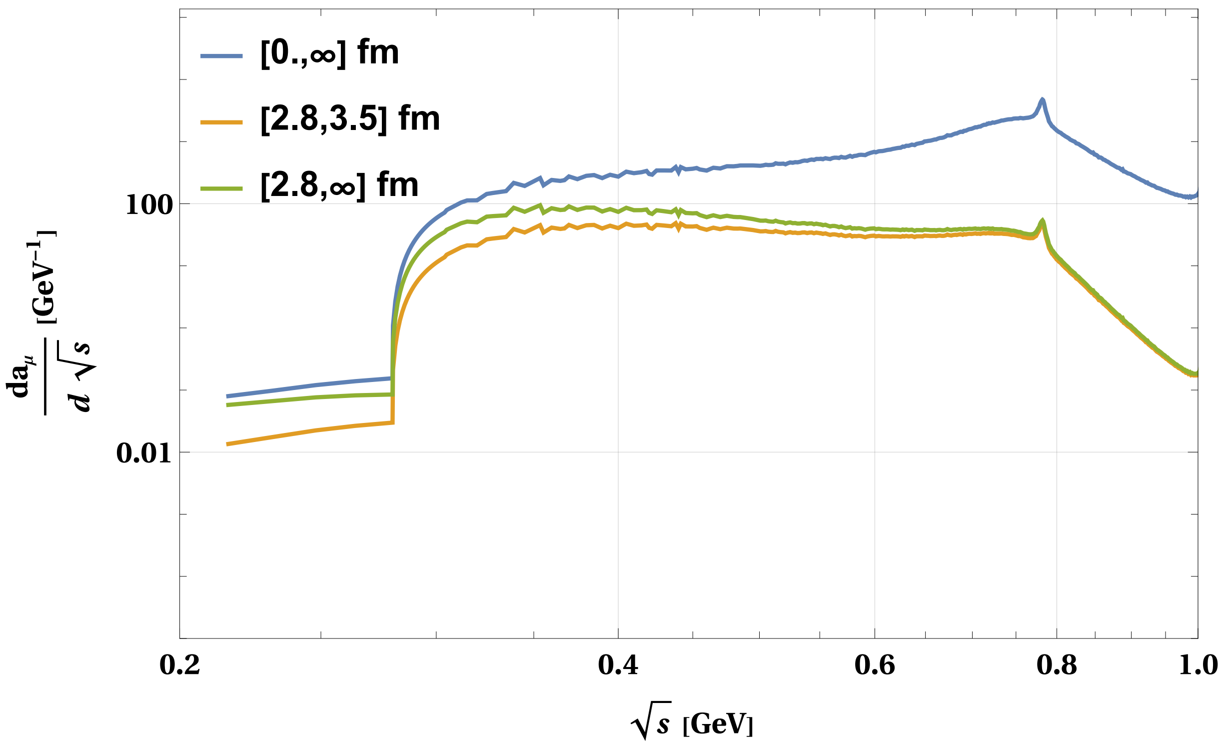


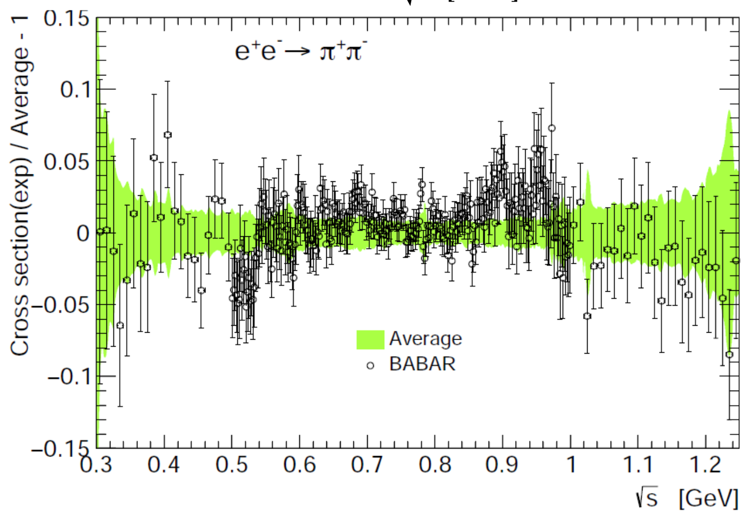
$1.5 \to 1.9$ fm

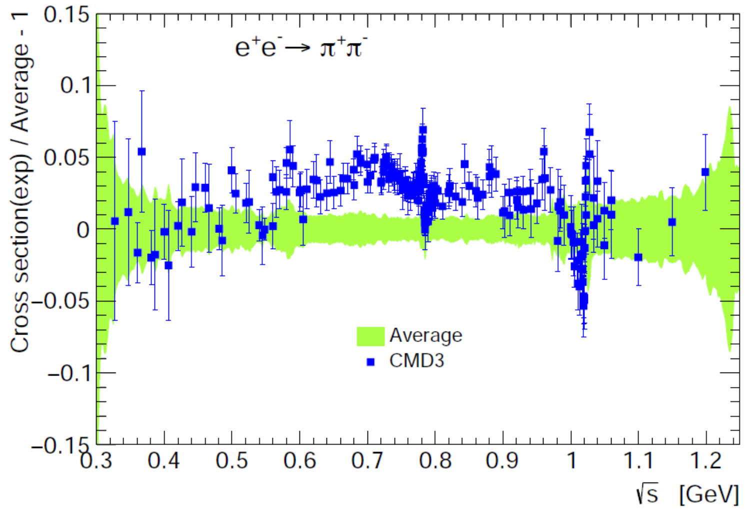




Tail contribution

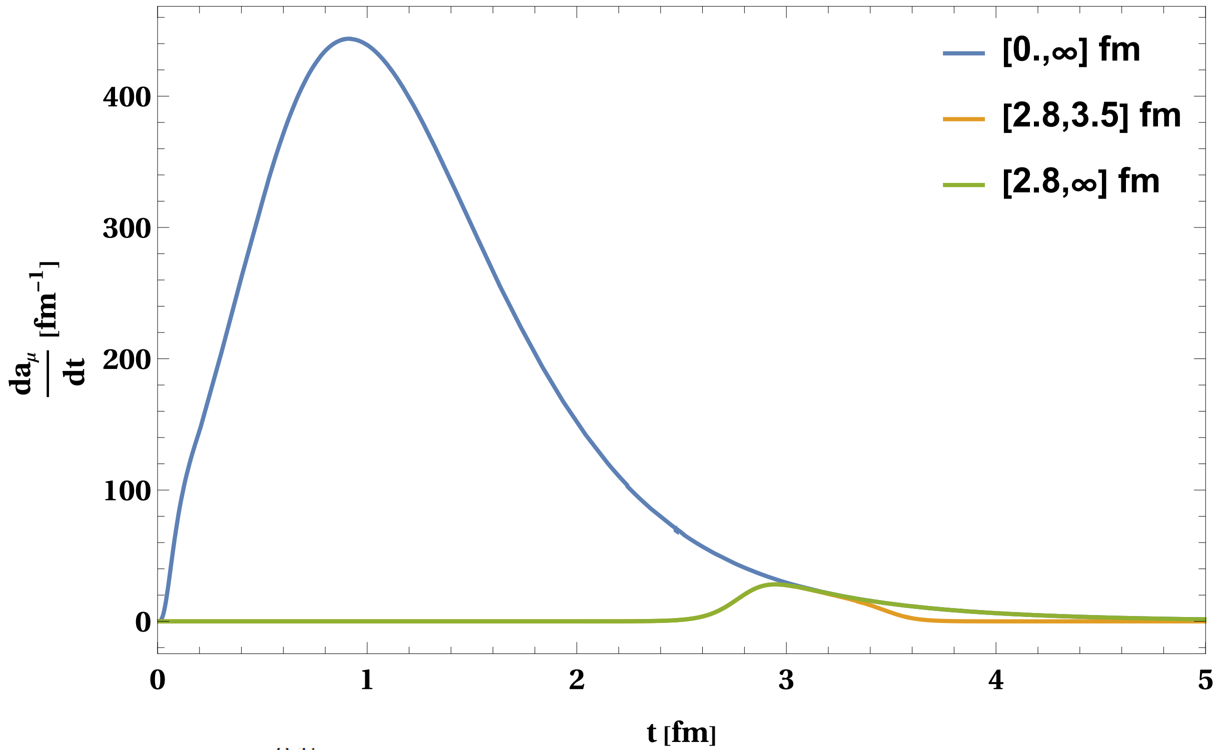


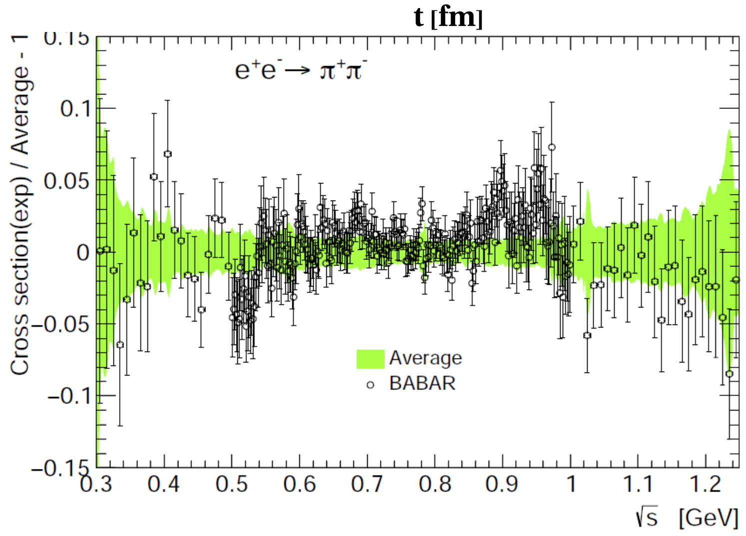


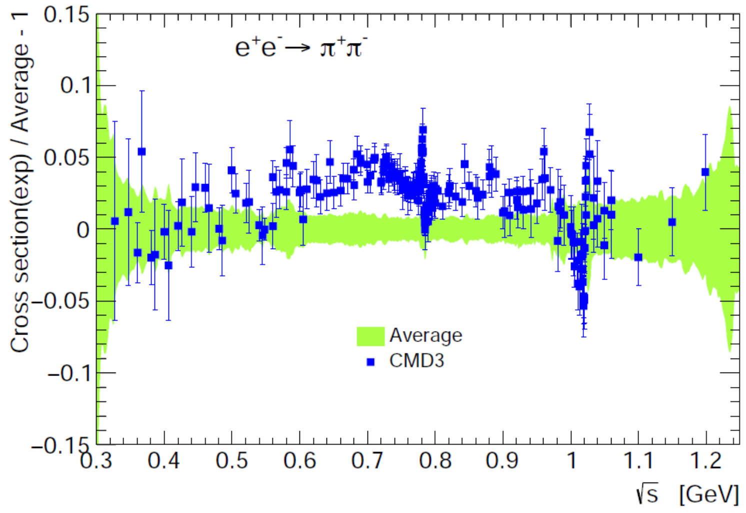


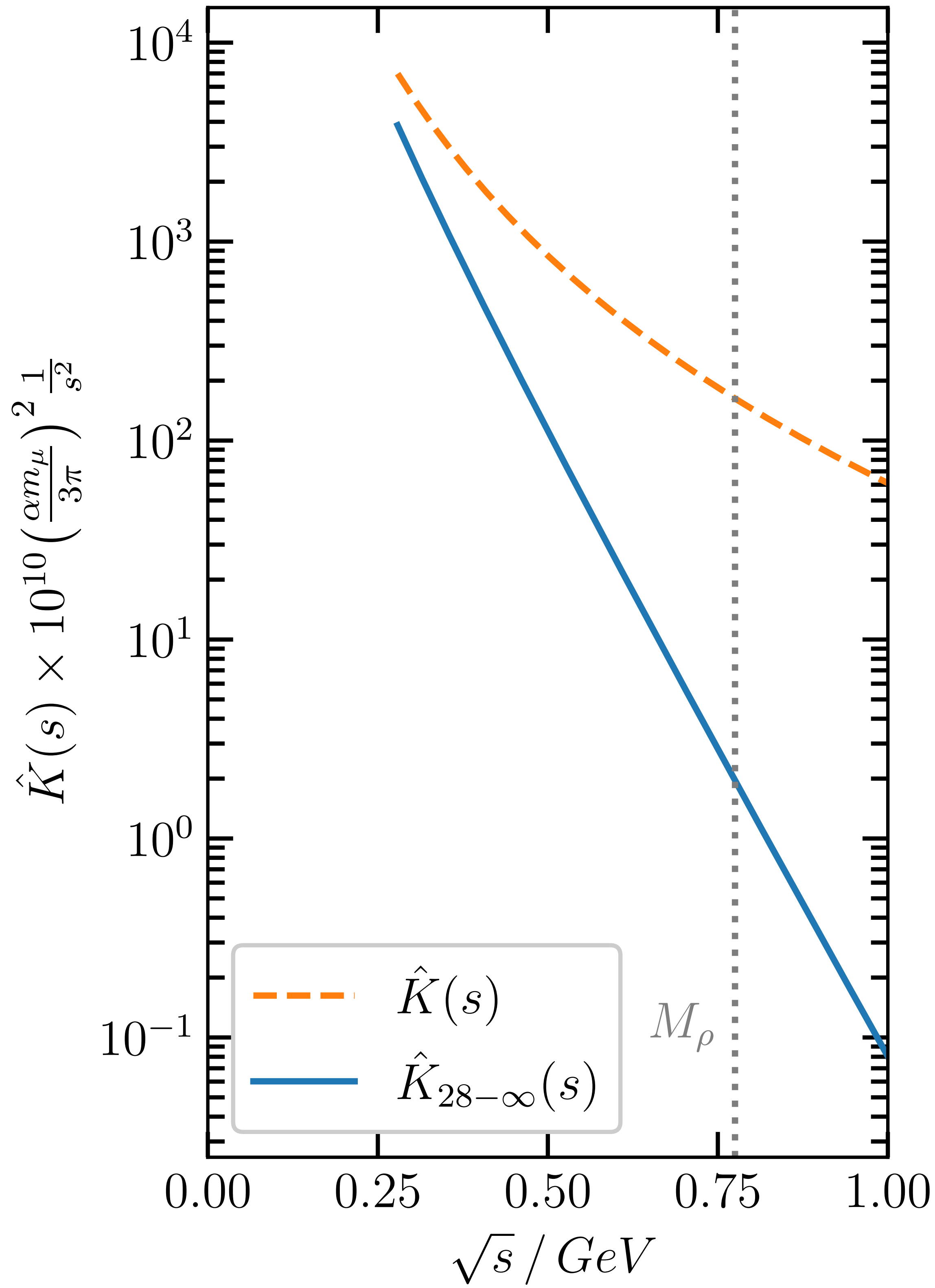
- Lattice computation up to $t = 2.8$ fm : $> 95\,\%$ of final result for $a_\mu^{\text{LO-HVP}}$
- Tail $a_{\mu,28-\infty}^{\text{LO-HVP}}$ computed using $e^+e^- \to \text{hadrons}$ for $t > 2.8$ fm : $\lesssim 5\,\%$ of final result for $a_\mu^{\text{LO-HVP}}$
- Tail dominated by cross section below ρ peak: $\sim 75\,\%$ for $\sqrt{s} \leq 0.63$ GeV
- All measurements agree to within 1.4σ for $\sqrt{s} \lesssim 0.55$ GeV. Tensions that plague $a_\mu^{\text{LO-HVP}}$ & $a_{\mu,\text{win}}^{\text{LO-HVP}}$ not present here
- Partial tail $a_{\mu,28-35}^{\text{LO-HVP}}$ for comparison with lattice; dominated by cross section below ρ peak: $\sim 70\,\%$ for $\sqrt{s} \leq 0.63$ GeV

Tail contribution

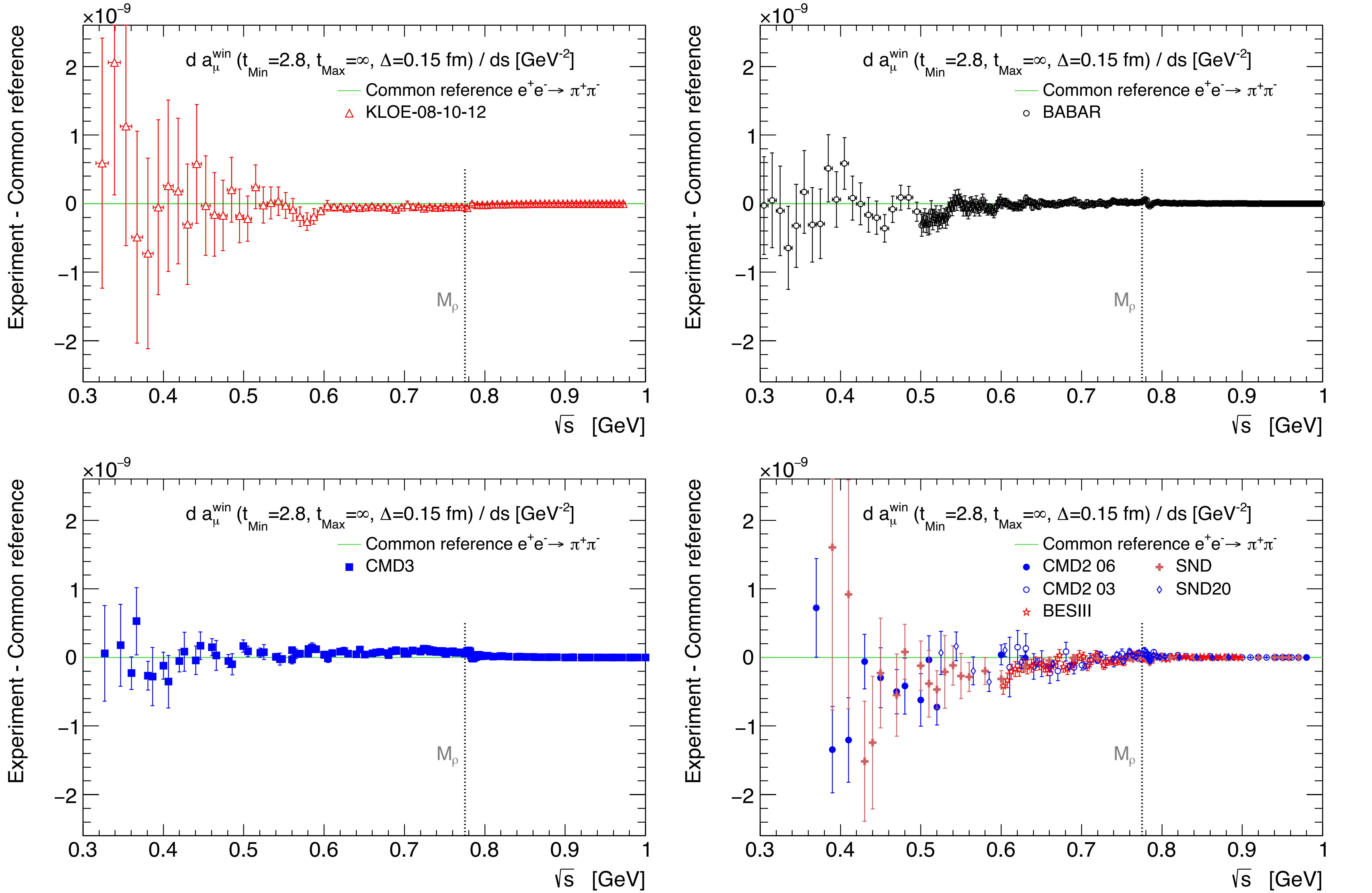




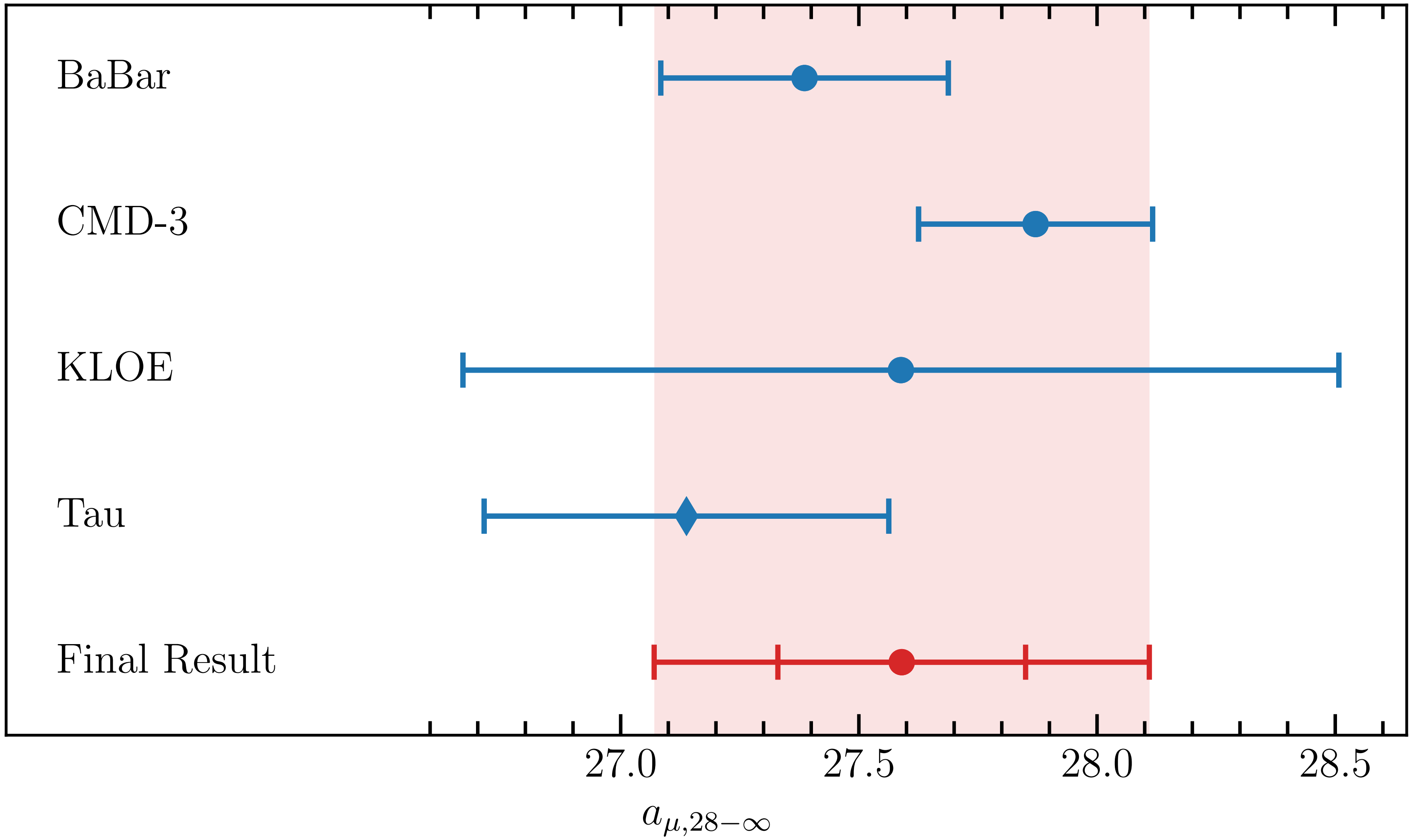




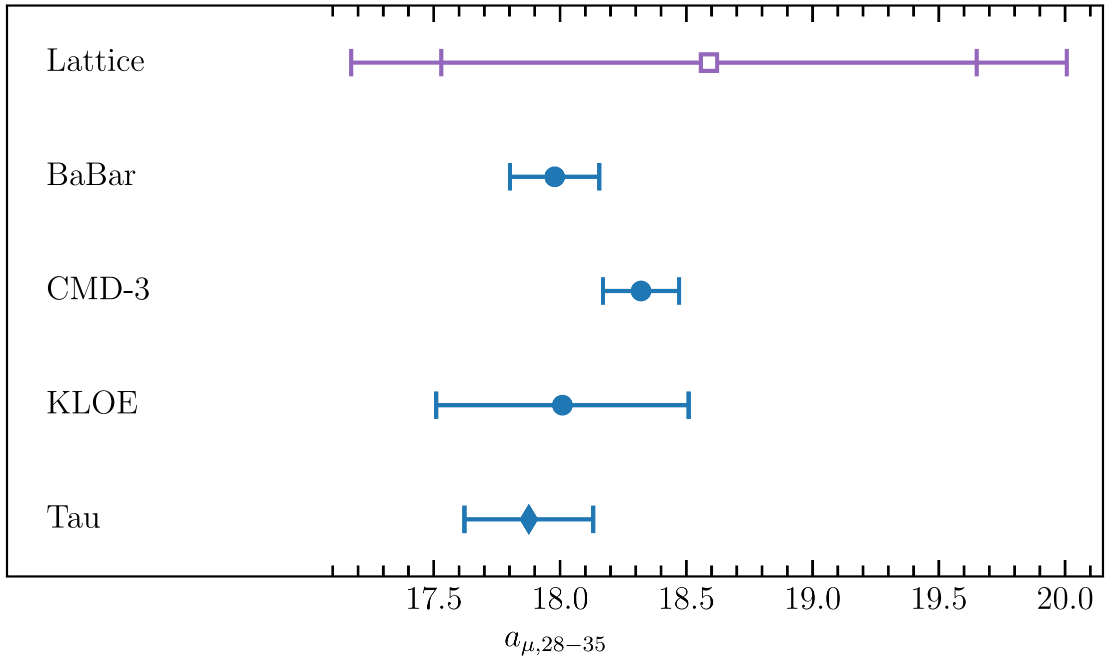
Cross section and the tail



Data-driven tail



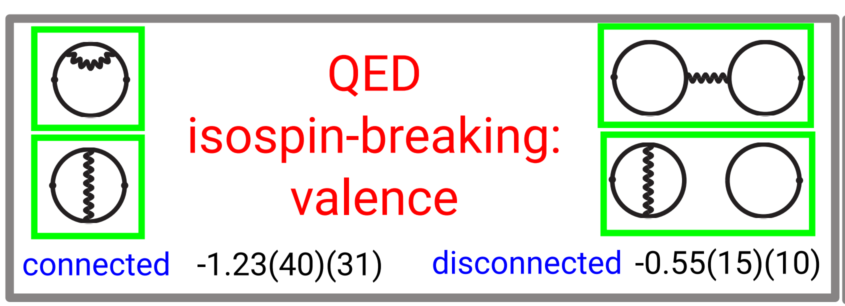
- Only $\lesssim$ 5% of final result for a_μ
- Contributes $\sim$ 65% to total squared uncertainty improvement: 5.5 $\to$ 3.3
- Even if the error was arbitrarily doubled, the effect on total uncertainty would be insignificant

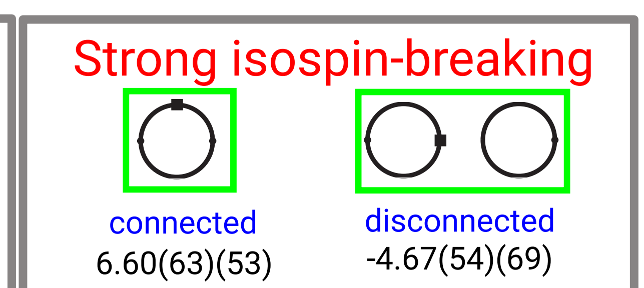


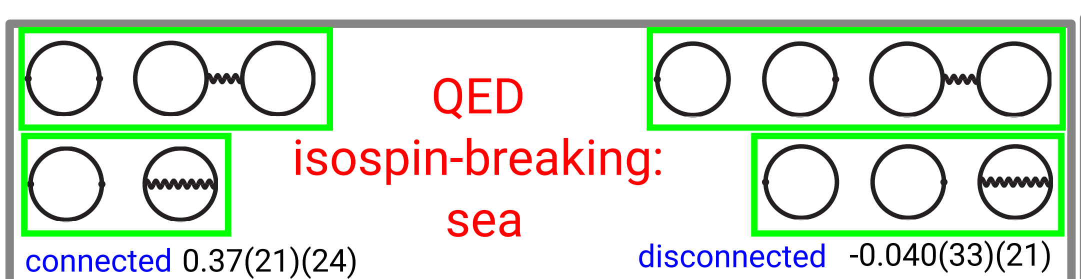
Summary of all contributions [BMW/DMZ-24]

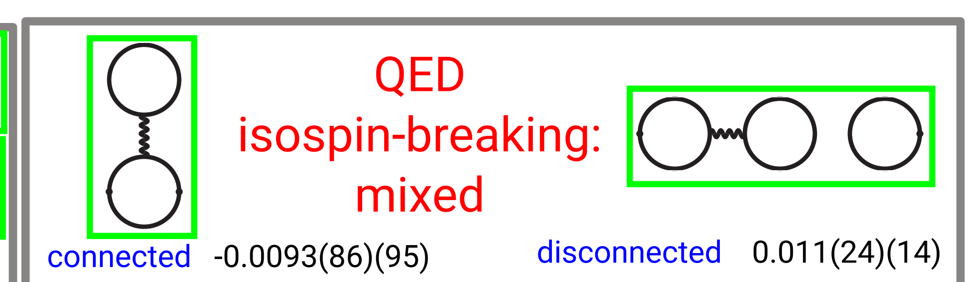
light and disconnected $00-28$	$618.6(1.9)(2.3)[3.0]$	this work, Equation (34)
strange $00-28$	$53.19(13)(16)[21]$	this work, Equation (37)
charm $00-28$	$14.64(24)(28)[37]$	this work, Equation (40)
light qed	$-1.57(42)(35)$	[5], Table 15 corrected in Equation (45)
light sib	$6.60(63)(53)$	[5], Table 15
disconnected qed	$-0.58(14)(10)$	[5], Table 15
disconnected sib	$-4.67(54)(69)$	[5], Table 15
disconnected charm	$0.0(1)$	[31], Section 4 in Supp. Mat.
strange qed	$-0.0136(86)(76)$	[5], Table 15
charm qed	$0.0182(36)$	[43]
bottom	$0.271(37)$	[44]
tail from data-driven $28-\infty$	$27.59(17)(9)[26]$	this work, Equation (50)
total	$714.1(2.2)(2.5)[3.3]$	

$$a_\mu^{\text{LO-HVP}} \times 10^{10} = 714.1(2.2)(2.5)[3.3] \quad [0.46\%]$$

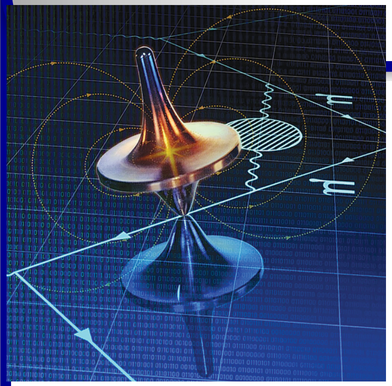


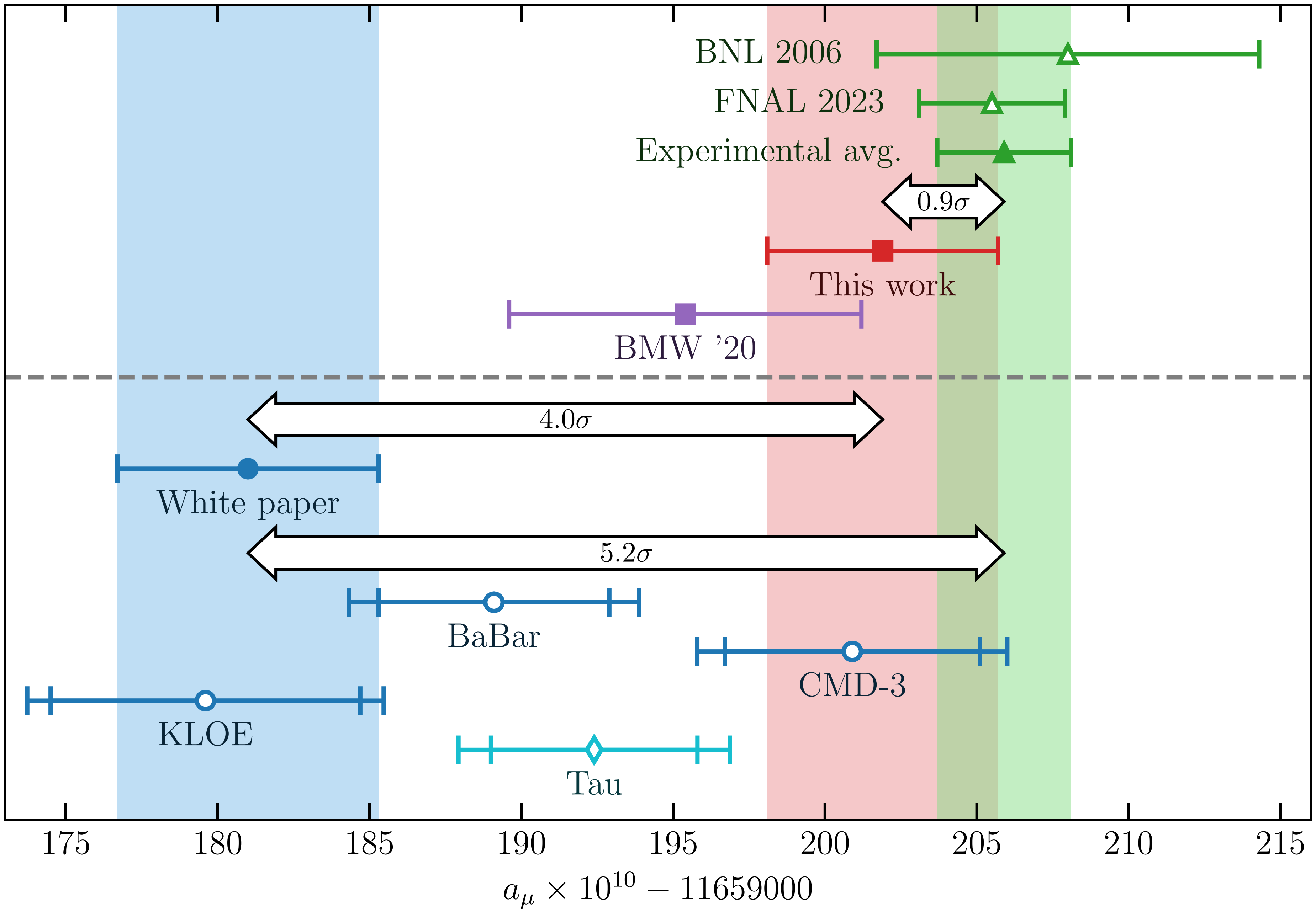






BMW-DMZ ’24 vs g-2 experiment



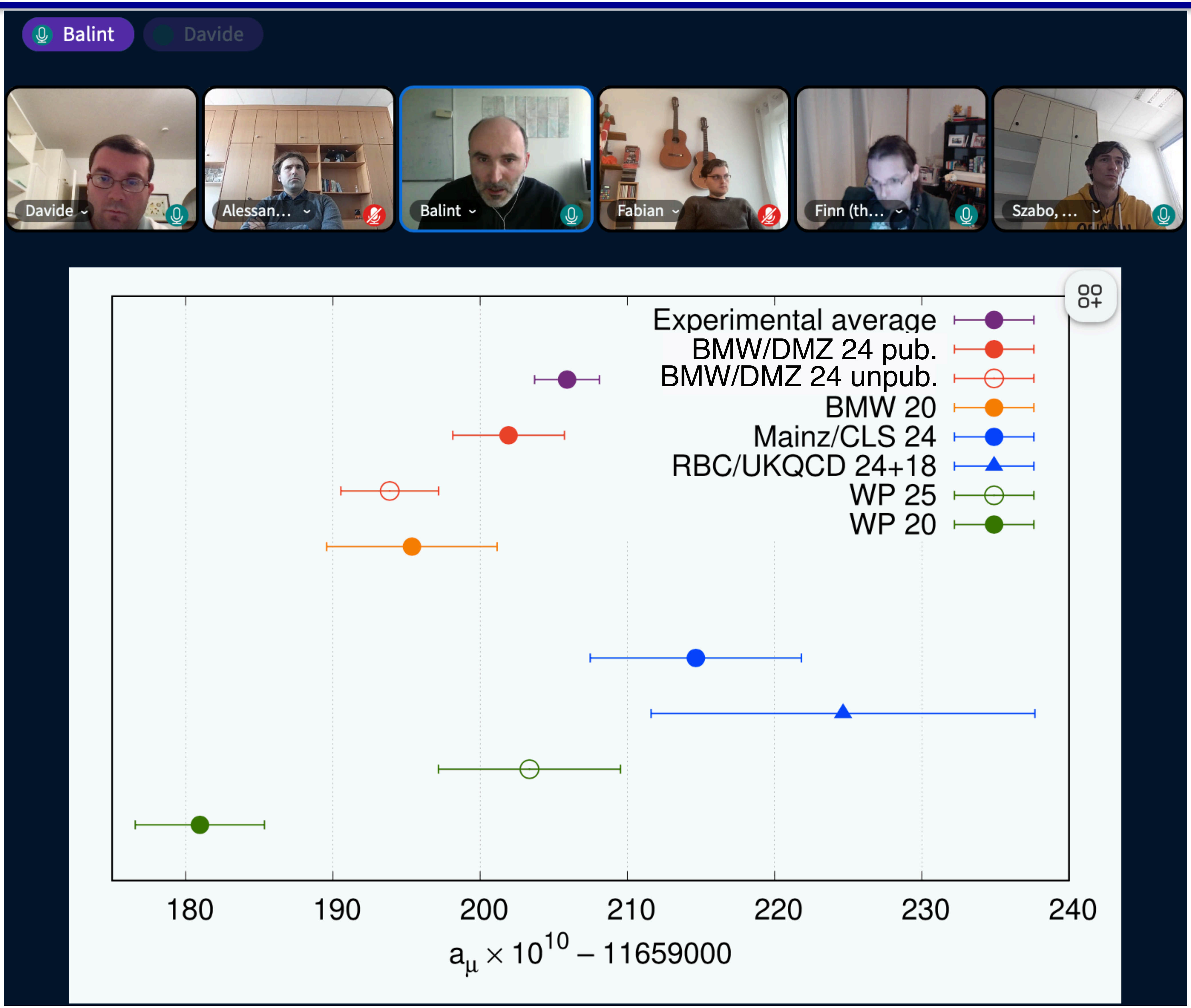


Indicates Standard Model confirmed to 0.32 ppm!

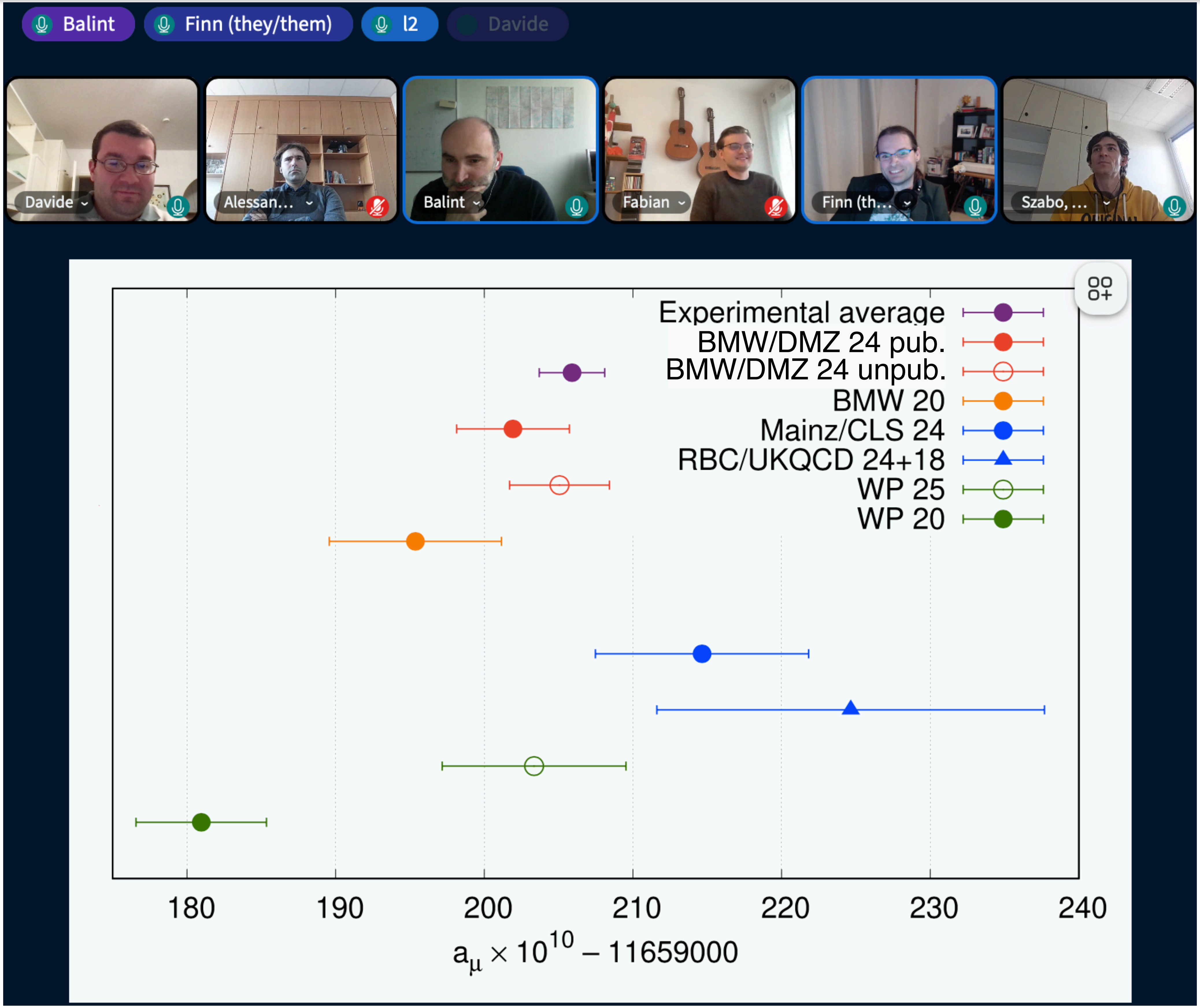
Podcast (generated by AI) on the current status of muon g-2:

https://drive.google.com/file/d/1aAi9CWSPVEYv2SMMxuGQT3l3KmEKGwKu/view?usp=d

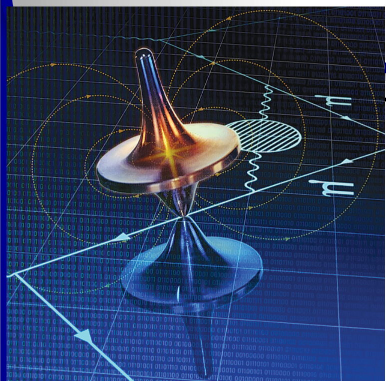
May 28, 2025: unblinding for new scale setting (before)

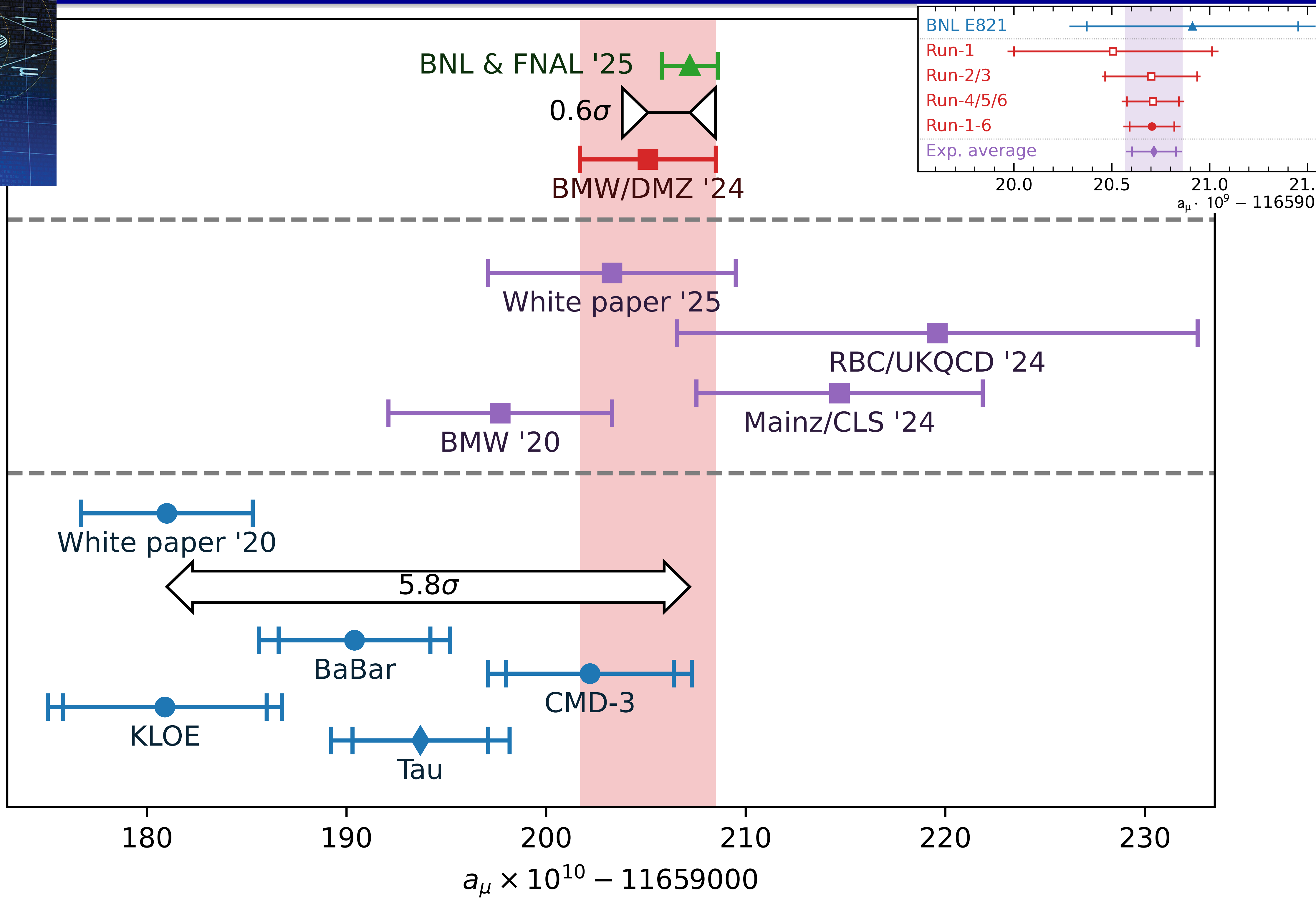


May 28, 2025: unblinding for new scale setting (after)



BMW-DMZ '24 vs g-2 experiment

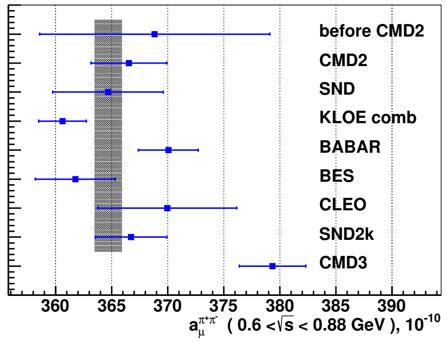




Indicates Standard Model confirmed to ~~0.32~~ 0.29 ppm!

Summary and Outlook

- Tremendous progress in lattice calculations of HVP (and HLbL!) contributions
- New BMW-DMZ calculation to 0.44% w/ fully blinded analysis, confirming the SM to 0.29 ppm
- Good agreement between lattice calculations for various windows
- Compared to WP ’20, in WP ’25 the SM prediction is dominated by lattice calculations, w/ consolidated averages from many independent groups
- Awaiting J-PARC entirely new method measurement
- Awaiting new KLOE, BESIII, Belle II, CMD3, SND2 data/analysis to clarify tensions in $\pi^+\pi^-$



- $\mu e \to \mu e$ experiment MUonE very important for experimental cross-check and complementarity w/ LQCD

Challenges of a full lattice calculation

To make contact with experiment need:

- **A valid approximation to the SM**
  - $\to$ at least u, d, s in the sea w/ $m_u = m_d \ll m_s$ (N_f=2+1) $\Rightarrow \sigma \sim 1\%$
  - $\to$ better also include c (N_f=2+1+1) & $m_u \leq m_d$ & EM $\Rightarrow \sigma \sim 0.1\%$
- **u & d w/ masses well w/in $SU(2)$ chiral regime** : $\sigma_\chi \sim (M_\pi/4\pi F_\pi)^2$
  - $\to$ $M_\pi \sim 135\,\mathrm{MeV}$ or many $M_\pi \leq 400\,\mathrm{MeV}$ w/ $M_\pi^{\mathrm{min}} < 200\,\mathrm{MeV}$ for $M_\pi \to 135\,\mathrm{MeV}$
- **a $\to$ 0** : $\sigma_a \sim (a\Lambda_{\mathrm{QCD}})^n,\ (am_q)^n,\ (a|\vec{p}|)^n$ w/ $a^{-1} \sim 2 \div 4\,\mathrm{fm}$
  - $\to$ at least 3 a's $\leq 0.1\,\mathrm{fm}$ for $a \to 0$
- **L $\to \infty$** : $\sigma_L \sim (M_\pi/4\pi F_\pi)^2 \times e^{-LM_\pi}$ for stable hadrons, $\sim 1/L^n$ for resonances, QED, ...
  - $\to$ many L w/ $(LM_\pi)^{\mathrm{max}} \gtrsim 4$ for stable hadrons & better otherwise to allow for $L \to \infty$
- These requirements $\Rightarrow O(10^{12})$ **dofs** that have to be integrated over
- **Renormalization** : best done nonperturbatively
- **A signal** : $\sigma_{\mathrm{stat}} \sim 1/\sqrt{N_{\mathrm{meas}}}$, reduce w/ $N_{\mathrm{meas}} \to \infty$