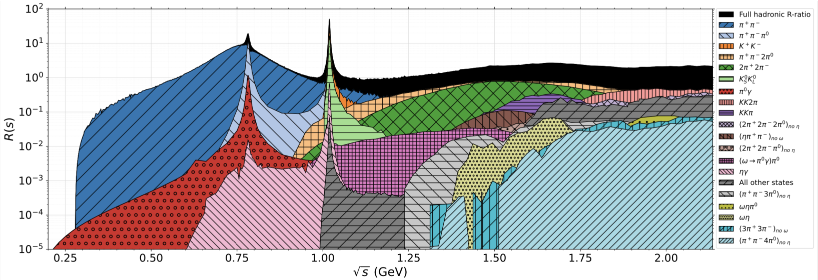
Dispersive Determination of the HVP Contributions to the Muon $g-2$

Aidan Wright

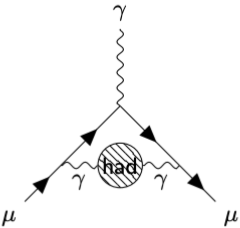
Science and Technology Facilities Council

LEVERHULME TRUST

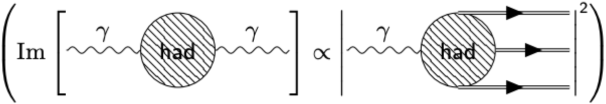


Dispersive Method

- Problem: QCD is non-perturbative at low $\sqrt{s}$.
- Implication: HVP of photon cannot be calculated in loop integrals etc.
- Solution: dispersion integral over the $e^+e^- \to hadrons$ cross section.



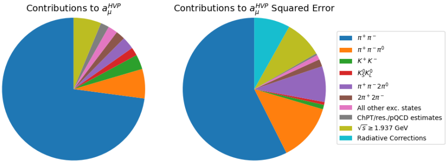
$$\begin{aligned}
&\propto \Pi^{had.}_{\mu\nu}(q^2) && \textit{by definition} \\
&\propto \Pi^{had.}(q^2) && \textit{due to gauge invariance} \\
&\propto \int \frac{\mathrm{Im}\left\{\Pi^{had.}(s)\right\}}{s\,(s-q^2-i\varepsilon)}ds && \textit{by analyticity and Cauchy's theorem} \\
&\propto \int \frac{\sigma^0_{had.}(s)}{s-q^2-i\varepsilon}ds && \textit{by unitarity} \implies \textit{the Optical Theorem}
\end{aligned}$$

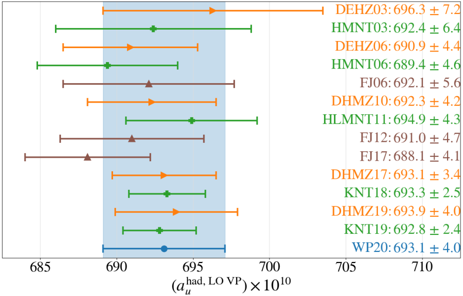


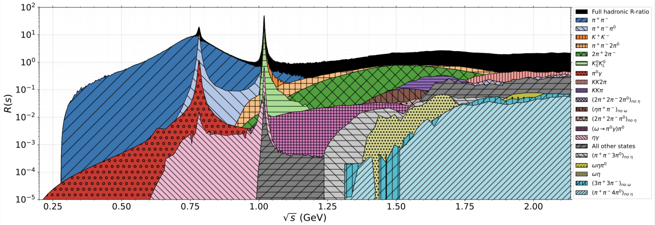
- For > 50 years, low energy $e^+e^- \to hadrons$ data have been collected...

Hadronic Data

- ~ 250 measurements in > 50 hadronic channels.
- Dominated ($> 70\%$) by $e^+e^- \to \pi^+\pi^-$.

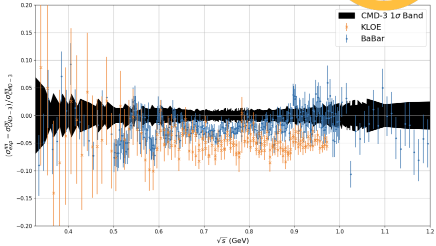


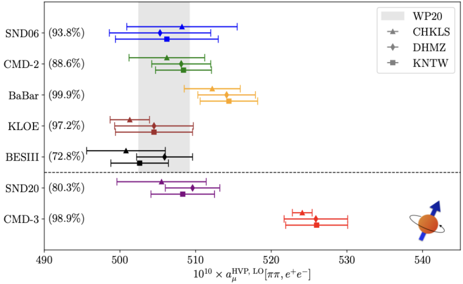




Tensions in $\pi^+\pi^-$

- **Historic problem:** $\sim 2.5\sigma$ KLOE/BaBar tension[a].
- **Historic Solution:** local error inflation; additional 'ad-hoc' systematic.
- **Current problem:** $> 5\sigma$ KLOE/CMD-3 tension, $\sim 2.5\sigma$ BaBar/CMD-3 tension!
  - CMD-3 'corroborated' by new SND preliminary.
  - BaBar confirm their earlier result.
- **Current Solution:** None as yet...
  - Nothing suggests earlier data is defective.
  - Dispersive method is robust.





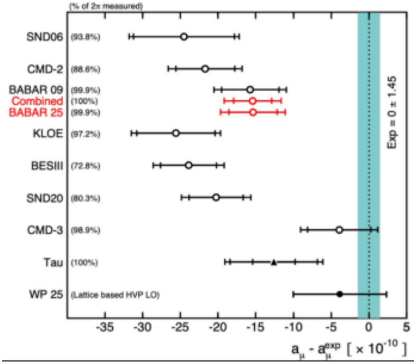
WP25, R. Aliberti et al - Phys. Rept. 1143 (2025)

[a] See penultimate slide...

New $\pi^+\pi^-$ Data

BaBar

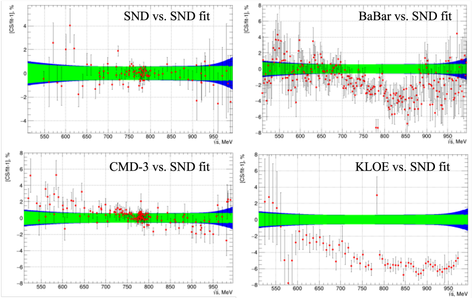
- Independent new method applied to all data.
- Reduced systematics in $0.5 \to 1.4$ GeV.
- **Excellent** agreement with 2009 data.



A. Pinto - 8th Plenary Workshop of the Muon $g-2$ TI

SND

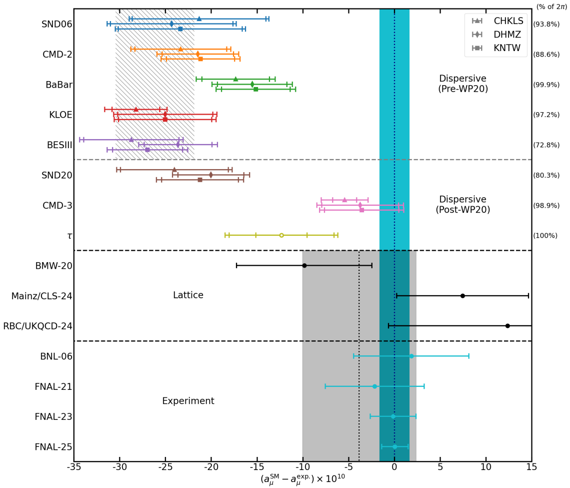
- Values considerably increased compared to SND20.
- Unaccounted systematic $\implies 2 \to 3\%$ higher value.
- Now $2\sigma >$ BaBar and comparable to CMD-3.



A. Kupich - 8th Plenary Workshop of the Muon $g-2$ TI

Implications...

- Analysis groups must manage tensions to estimate HVP.
- Tensions $\implies$ WP25 quoted only lattice - **but TI is not finished yet!**
- Unclear dispersive $\implies$ unclear $g-2$ interpretation.
- **Goal**: representative and accurate a_μ^{HVP} dispersive prediction to fully understand experiment implications.



Analysis Groups

KNTW

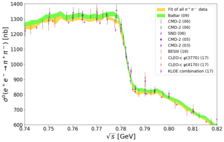
- Data dynamically clustered to prevent over-fitting.
- Utilises full given covariance matrices/ assumes full systematic correlation.
- Fit to avoid incurred d'Agostini bias.
- Full covariance matrices propagated to final result.

DHMZ

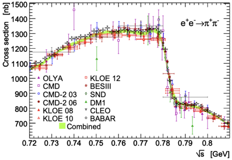
- Measurements quadratic spline interpolated and averaged on a fixed binning.
- Central value derived using uncertainties and local correlations.
- Uncertainties on channels generated from 'pseudo-experiments'.

CHKLS

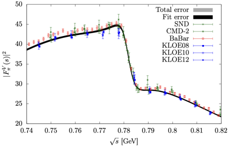
- Select channels' spectra constrained by analyticity and unitarity.
- $\implies$ Fit functions.
- Measurements fit and combined.
- Consistent w. DHMZ, KNTW despite method differing significantly.



KNT - arXiv:1911.00367v2



DHMZ - arXiv:1908.00921



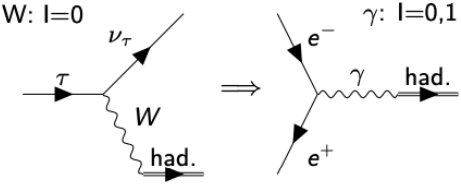
CHS - arXiv:1810.00007

Proposed Solutions - τ Data

The $e^+e^- \to \pi^+\pi^-$ data are in tension.
The $\tau \to \nu_\tau \pi^0 \pi$ data are not.
$\Longrightarrow$ **Supplement the former with the latter to get a more consistent average?**

- Requires calculation of **isospin breaking corrections**.
- Not included in WP20 average due to potentially large unknown uncertainties.
- Some progress with model-independent and lattice evaluations.
- **Concern:** interference between hadronic channels may not be negligible $\Longrightarrow$ large previously unaccounted terms.
- Full, assuredly accurate calculation not yet complete.

Conclusion: significant further study needed.



		Refs. [166, 194]	Ref. [209]	Refs. [237, 247]	Our estimate
Phase space		−7.88	−7.52	–	−7.7(2)
S_{EW}		−12.21(15)	−12.16(15)	–	−12.2(1.3)
G_{EM}		−1.92(90)	$(-1.67)^{+0.60}_{-1.39}$	–	−2.0(1.4)
FSR		4.67(47)	4.62(46)	4.42(4)	4.5(3)
ρ–ω mixing		4.0(4)	2.87(8)	3.79(19)	3.9(3)
$\frac{F_\pi^V}{f_+}$ (w/o ρ–ω)	ΔM_ρ	$0.20(^{+27}_{-19})(9)$	$1.95^{+1.56}_{-1.55}$	–	
	$\Delta\Gamma_\rho(\Delta M_\pi)$	4.09(0)(7)	3.37	–	
	$\Delta\Gamma_\rho(\pi\pi\gamma)$	−5.91(59)(48)	−6.66(73)	–	
	$\Delta\Gamma_\rho(g_{\rho\pi\pi})$	–	–	–	
	Total	−1.62(65)(63)	$(-1.34)^{+1.72}_{-1.71}$	–	−1.5(4.7)
Sum		−14.9(1.9)	$(-15.20)^{+2.26}_{-2.63}$	–	−15.0(5.1)

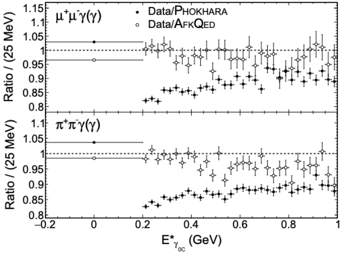
WP25, R. Aliberti et al - Phys. Rept. 1143 (2025), arXiv:2505.21476v3

Proposed Solutions - Radiative Correction Explanation

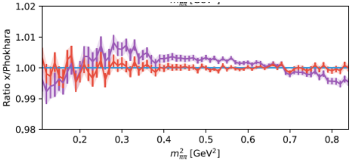
Processes $e^+e^- \to \pi^+\pi^- + n\gamma$ occur alongside $e^+e^- \to \pi^+\pi^-$.
Experiments often rely on Monte Carlo to handle additional photons.
$\Longrightarrow$ Issues with these Monte Carlos mean old data are defective?

- BaBar study of FSR in $\pi^+\pi^-$:
  - PHOKHARA generator potentially overestimates NLO.
  - Potentially significant NNLO contributions.
  - $\Longrightarrow$ Inaccuracy of BESIII and KLOE?
- KLOE and BESIII studies find much better agreement.
- Any deviation is $\lesssim 1\%$ and likely captured in quoted systematics.
- Detector effects studies ongoing.

Conclusion: Not the answer, but we should look forward to NNLO PHOKHARA.



J.P. Lees et al - arXiv:2308.05233



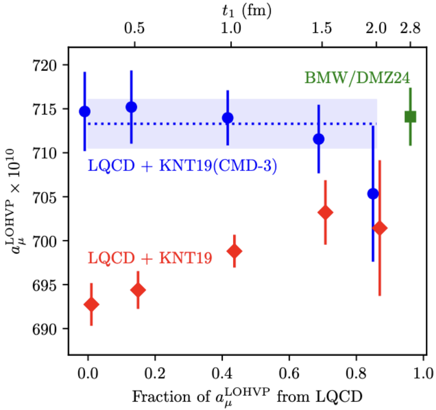
L. Cotrozzi - 8th Plenary Workshop of the Muon g − 2 TI

Proposed Solutions - Lattice Hybrids

Data are ∼consistent at small $\sqrt{s}$.
Lattice long distance windows have relatively large uncertainties.
$\Longrightarrow$ Supplement long distance lattice with low energy dispersive?

- Largely good idea - hybrid likely for next WP to maximise precision.
- Already exists: BMWc-DMZ hybrid with switch at 2.8 fm.
- **However:**
  - Does not touch root cause of tensions.
  - Value highly dependent on lattice/ dispersive switchover length.
- Latter - see effects of varying data input right.

Conclusion: not yet viable.



C.T.H. Davies et al - arXiv:2410.23882

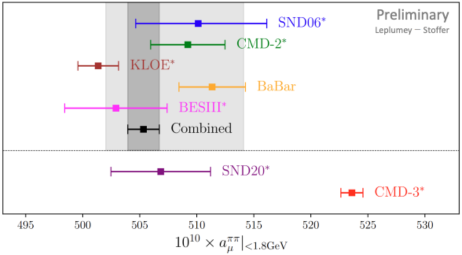
CHKLS Recent Work

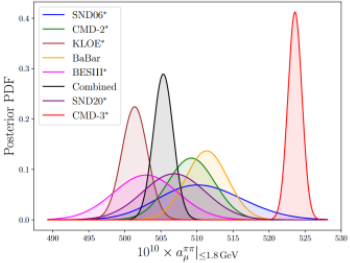
$$F_\pi^V(s) = \underbrace{\Omega_1^1(s)}_{\text{2 param.s}} \times \underbrace{G_{\omega(\phi)}(s)}_{\text{3(6) param.s}} \times G_{\text{in}}(s)$$

- Improved inelastic function:

$$G_{\text{in}}(z) = \frac{1}{\phi(z)} \frac{P_N(z)}{\prod_j (z - z_j)(z - z_j^*)}$$

  for OF $\phi(s)$, polynomial P_N and poles s_j.
- Bayesian parameter interference for improved fitting.
- **Exacerbated** CMD-3 tension.
- Strong correlation of lattice windows $\implies$ further issues with hybrid.

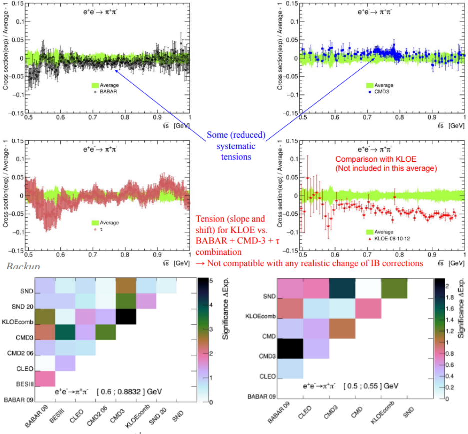




T. Leplumey - 8th Plenary Workshop of the Muon g − 2 TI

DHMZ Recent Work

- Significant effort to better understand tensions.
- Data from τ, BaBar and CMD-3:
  - Had proposed combinations based on these data.
  - Prior to interference concern, agreement was quite good.
- Local tensions assessment:
  - Measurements exhibit different levels of tension in different regions.
  - Most significant tensions are on and above ρ peak.
  - Datasets are compatible at low energies.



B. Malaescu - 8th Plenary Workshop of the Muon $g-2$ TI

KNTW - Blinding and New Analysis

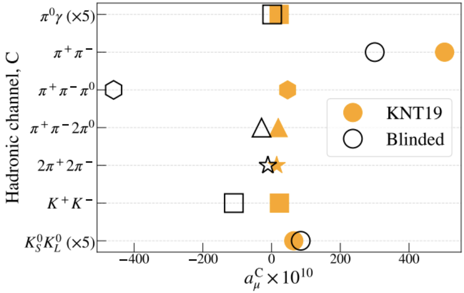
- Choices during combination:
  - Radiative correction routines;
  - Re-binning procedure;
  - Fitting procedure - correlations;
  - Additional constraints;
  - Error inflation;
  - Interpolation/integration;
  - Additional systematic uncertainties...
- KNTW-DHMZ difference - these clearly influence the central value!

	DHMZ19	KNT19	Difference
$\pi^+\pi^-$	507.85(0.83)(3.23)(0.55)	504.23(1.90)	3.62
$\pi^+\pi^-\pi^0$	46.21(0.40)(1.10)(0.86)	46.63(94)	−0.42
$\pi^+\pi^-\pi^+\pi^-$	13.68(0.03)(0.27)(0.14)	13.99(19)	−0.31
$\pi^+\pi^-\pi^0\pi^0$	18.03(0.06)(0.48)(0.26)	18.15(74)	−0.12
K^+K^-	23.08(0.20)(0.33)(0.21)	23.00(22)	0.08
$K_S K_L$	12.82(0.06)(0.18)(0.15)	13.04(19)	−0.22
$\pi^0\gamma$	4.41(0.06)(0.04)(0.07)	4.58(10)	−0.17
Sum of the above	626.08(0.95)(3.48)(1.47)	623.62(2.27)	2.46
[1.8, 3.7] GeV (without $c\bar{c}$)	33.45(71)	34.45(56)	−1.00
J/ψ, $\psi(2S)$	7.76(12)	7.84(19)	−0.08
$[3.7, \infty)$ GeV	17.15(31)	16.95(19)	0.20
Total $a_\mu^{\text{HVP, LO}}$	$694.0(1.0)(3.5)(1.6)(0.1)_\psi(0.7)_{\text{DV+QCD}}$	692.8(2.4)	1.2

Theory Initiative White Paper 2020

- Need to mitigate and/or quantify these effects $\implies$ KNTW new analysis.
- Want to avoid biases in updated procedure $\implies$ KNTW **blinded** analysis.

$$a_\mu^{\text{blind}}[i] = \frac{1}{4\pi^3} \int_{s_{th}}^{\infty} ds \left\{ \sigma_i^0(s) K_\mu(s)\, B_i(s) \right\}$$

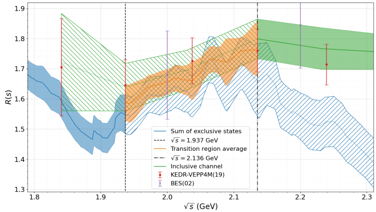


KNTW - arXiv:2409.02827, Phys. Rev. D 111, L011901

KNTW New Analysis Progress

"Re-Baselining"

- (*Minor*) Corrections of KNT19 analysis:
  - Checks of database against literature.
  - More detailed systematic covariance matrix construction.
- Completions of KNT19 analysis features:
  - Lagrange polynomial interpolation of all resonances.
  - Exclusive/inclusive transition region.
- Estimates of KNT19 method systematics:
  - Two unfixed aspects of procedure.
  - Systematics would be $\sim 4.3\%$ of KNT19 squared error budget.

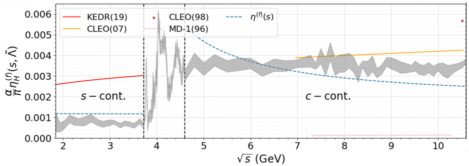


FSR Studies

- Revisited $K\bar{K}$ - confident in KNT19 conclusions for scan experiments.
- Looking at 3π with input of MH.
- Inclusive channel (grey band):
  - Improve 1% syst. with $q\bar{q}$ treatment.

$$R_{(\gamma)} = \left(1 + \frac{\alpha}{\pi} \sum_{\text{q=uds(c)}} Q_q^2 \eta^{(f)}(s, m_q^2)\right) R$$

  - Datasets often FSR inclusive, hard correction needed for four datasets.
  - Estimated 20% drop in $\Delta\alpha^{(5)}_{\text{had.}}(M_Z^2)$ uncertainty!



KNTW Correlations Study

- KNTW/DHMZ $\iff$ different correlation handling?
- Assess 'uncertainties on uncertainties' with decorrelation procedure for **systematics**:

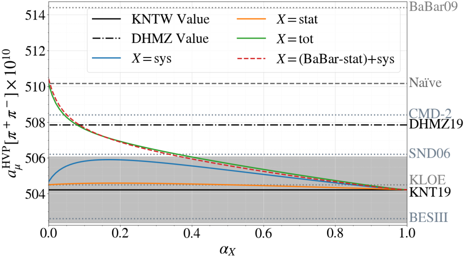
$$\tilde{C}_{ij} = \alpha\, C_{ij} + (1-\alpha)\,\mathrm{diag}\left[C_{ij}\right].$$

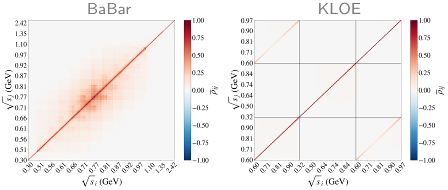
- Blue line - does not replicate DHMZ etc.
- Use to estimate systematic uncertainty:

$$d^{\rho} a_\mu^{\pi^+\pi^-} = \pm 1.68 << d^{\mathrm{KLOE/BaBar}} a_\mu^{\pi^+\pi^-};$$

extension to all channels $= \pm 1.95$.

- Implication (green line) - need to vary stat. and syst. or 'KLOE favoured'.
- Difference driven by BaBar statistics.
- More advanced decorr.s possible but this provides an ∼upper bound uncertainty.





arXiv2510.XXXXX

Conclusions

- Significant tensions remain within dispersive HVP evaluations.
- No (complete) explanation has yet been provided.
- The KNTW new analysis will attempt to accommodate these tensions but is blinded and ongoing - so no real results yet!
- Historic (but not current) tensions understood using new procedure.
- **The full implications of the result of the $g-2$ experiment cannot be known until the dispersive result is known.**