Status of the hadronic light-by-light contribution to the muon $g-2$ and holographic QCD predictions

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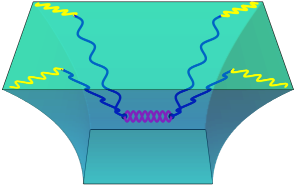
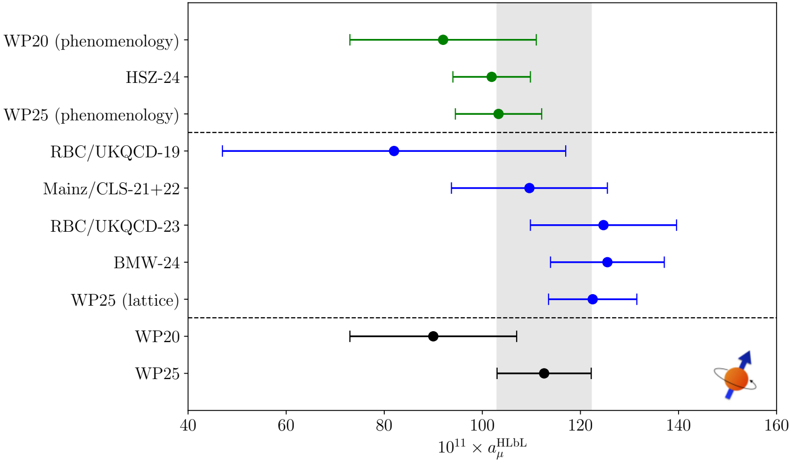


Status according to WP25



1.5σ tension between WP25 dispersive result and WP25 lattice average

HLbL contributions where hQCD might be of interest

Holographic QCD (hQCD) makes interesting(*) predictions
where WP error estimates from phenomenology are the largest:

(*) not too model-dependent in the class of models that match
m_ρ, f_π, and longitudinal short-distance constraints (LSDC)

Contribution	WP20-$a_\mu^{\rm HLbL} \times 10^{11}$	$\to$ WP25	hQCD [LMR22+CLMR25]
π^0, η, η'-poles	$93.8(4.0)$	$91.2^{+2.9}_{-2.4}$	$95.9(3.8)$
π, K-loops/boxes	$-16.4(0.2)$	$-16.4(0.2)$	
S-wave $\pi\pi$ rescattering	$-8(1)$	$-9.1(1.0)$	
scalars & tensors	$-1(3)$		
axial vectors	$6(6)$		see below
u, d, s-loops / short-distance	$15(10)$		
— of which LSDC: $13(6)$			
c-loop	$3(1)$	$3(1)$	
total	$92(19)$	$103.3(8.8)$	114^{+10}_{-4}

WP20 $\to$ WP25:
Subleading contributions from axials, scalars, tensors, and SDCs in new complete (up to tensors) dispersive analysis by Hoferichter, Stoffer & Zillinger [HSZ24, 2412.00178]

Chiral Lagrangians with resonances from holographic QCD

(Axial) vector mesons and pions are described by **5-d YM fields** $\mathcal{F}^{L,R}_{MN}$ for global $U(N_f)_L \times U(N_f)_R$ chiral symmetry of boundary theory

$$S_{\rm YM} \propto \frac{1}{g_5^2}\,\mathrm{tr}\int d^4x \int_0^{z_0} dz\, e^{-\Phi(z)}\sqrt{-g}\, g^{PR} g^{QS}\left(\mathcal{F}^{(L)}_{PQ}\mathcal{F}^{(L)}_{RS} + \mathcal{F}^{(R)}_{PQ}\mathcal{F}^{(R)}_{RS}\right),$$

where $P, Q, R, S = 0, \ldots, 3, z$ and $\mathcal{F}_{MN} = \partial_M \mathcal{B}_N - \partial_N \mathcal{B}_M - i[\mathcal{B}_M, \mathcal{B}_N]$

with conformal boundary at $z = 0$, and

either sharp cut-off of AdS$_5$ at z_0 (HW) or with nontrivial dilaton $z_0 = \infty$ (SW)
(SS: not asymptotically AdS$_5$, finite z_0 corresponding to point where D8 branes join)

Chiral symmetry breaking either from

- extra bifundamental scalar field [Erlich-Katz-Son-Stephanov 2005] (HW1), or
- through different boundary conditions for vector/axial-vector fields at z_0 [Hirn-Sanz 2005] (HW2), [Sakai-Sugimoto 2004] (SS)

Vector meson dominance (VMD) naturally built in:
photons couple through *bulk-to-boundary propagators of vector gauge fields* whose normalizable modes give (infinite tower of!) vector mesons (ρ, ω, ϕ, ...)

Short distance constraints on TFFs

Crucially, hQCD models with asymptotic AdS$_5$ geometry reproduce
asymptotic momentum dependence of LCE [Brodsky-Lepage 1979-81]

- **Pseudoscalars** [Grigoryan & Radyushkin, PRD76,77,78 (2007-8)]:

$$F_{\pi^0\gamma^*\gamma^*}(Q_1^2,Q_2^2) \quad\to\quad \frac{2f_\pi}{Q^2}\sqrt{1-w^2}\int_0^\infty d\xi\,\xi^3 K_1(\xi\sqrt{1+w})K_1(\xi\sqrt{1-w})$$
$$= \frac{2f_\pi}{Q^2}\left[\frac{1}{w^2}-\frac{1-w^2}{2w^3}\ln\frac{1+w}{1-w}\right],$$

with $Q^2=\frac{1}{2}(Q_1^2+Q_2^2)\to\infty$, $w=(Q_1^2-Q_2^2)/(Q_1^2+Q_2^2)$,
corresponding to asymptotic behavior

$$F^\infty(Q^2,0)=\frac{2f_\pi}{Q^2}, \qquad F^\infty(Q^2,Q^2)=\frac{2f_\pi}{3Q^2} \quad (\Leftarrow \text{OPE}).$$

- **Axial vector mesons** [J. Leutgeb & AR, 1912.01596]
(confirmed by pQCD result of Hoferichter & Stoffer 2004.06127):

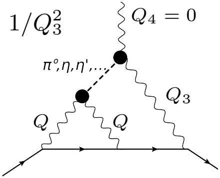
$$A_n(Q_1^2,Q_2^2)\to\frac{12\pi^2F_n^A}{N_cQ^4}\frac{1}{w^4}\left[w(3-2w)+\frac{1}{2}(w+3)(1-w)\ln\frac{1-w}{1+w}\right]$$

Melnikov-Vainshtein short-distance constraint

Melnikov and Vainshtein [hep-ph/0312226, PRD70(2004)]:
nonrenormalization theorem for axial anomaly implies
short-distance constraint for 4-photon-amplitude (in BTT basis w/ 54 structure functions):

$$\lim_{Q_3\to\infty}\lim_{Q\to\infty} Q^2 Q_3^2 \bar{\Pi}_1(Q,Q,Q_3) = -\frac{2}{3\pi^2}$$

each single meson exchange contribution gives 0
because propagator $\sim 1/Q_3^2$ and the two form factors $\sim 1/Q^2$ and $1/Q_3^2$



MV model: MV-SDC satisfied by replacing
external TFF by constant on-shell value, leading to
significant (almost +40%) increase of $a_\mu^{\pi^0,\eta,\eta'}$ by $\underline{38\times10^{-11}}$

WP estimate for MV-SDC based on Regge model of infinite tower of excited PS states
constructed to saturate MV-SDC with $\underline{\Delta a_\mu^{\rm PS} = 13(6)\times10^{-11}}$ [Colangelo et al., 1910.11881]

HW models: infinite tower of axials saturates MV-SDC to 100% in HW1 models,
with $a_\mu^{A(L)} = 23.2\times10^{-11}$ in chiral model;
no contribution to MV-SDC from excited PS, $a_\mu^{\pi^{0*}} = (0.8\ldots1.8)\times10^{-11}$

Axial vectors and SDCs contributions updated

WP 2020 estimate: $a_\mu^{\text{axials+SDC+}P^*} = 21(16) \times 10^{-11}$

- **hQCD model [LMR22] prediction** (with $m_s > m_{u,d}$ and $\mathrm{U(1)}_A$ anomaly) considerably larger:

$$a_\mu^{\text{axials+MV-SDC+}P^*} = 32.7^{+3.0}_{-0.0} \times 10^{-11}$$

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- confirmed by **new dispersive results** of Hoferichter, Stoffer & Zillinger [HSZ 2412.00178 w/o $\{S,T\}_{IR}$]:

$$a_{\mu,\text{dispersive}}^{\text{axials+SDC+}P^*} = 36.4(4.6) \times 10^{-11}$$

[1] Range of models considered in LMR24 around "best-guess" model LMR22(F_ρ-fit)

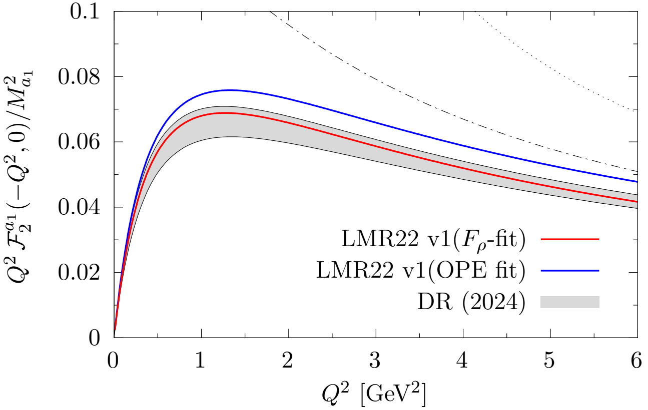
Axials and SDCs contributions in detail

Region	$a_\mu^{\cdots} \times 10^{11}$	hQCD[LMR22(F_ρ-fit)]	dispersive[HSZ24]
IR (all $Q_i < 1.5$ GeV)	a_1	4.2	3.8(7)
	$f_1 + f_1'$	8.9	8.4(1.4)
	AV^*	0.7	
	PS^*	1.7	
	eff.poles		2.0
	Sum	15.4	14.2(1.6)
Mixed	a_1	2.4	
	$f_1 + f_1'$	7.1	
	AV^*	1.9	
	PS^*	-0.04	
	Sum	11.4†	15.9(1.7)
UV (all $Q_i > 1.5$ GeV)		5.7†	$6.2^{+0.2}_{-0.3}$

†: $11.4 \to 13.5$, $5.7 \to 6.3$ once tensors are included (see below)

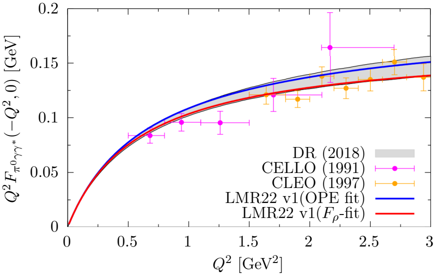
Axial vector TFF (singly virtual)

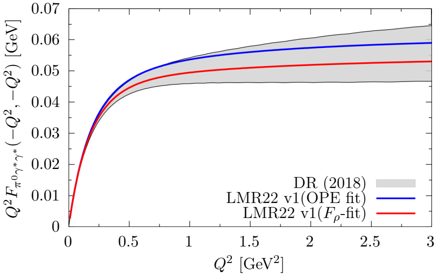
Surprisingly good agreement of a_1 TFF comparison of best guess hQCD model with dispersive results from Lüdtke et al.[2410.11946]



Pion TFFs (singly and doubly virtual)

Similarly good agreement for leading HLbL contribution of π^0:





π^0 TFF comparison of hQCD model [LMR22] with dispersive results (DR) from Hoferichter et al.[1808.04823] (Fig. 53 in WP2).

η and η' TFFs (singly and doubly virtual)

hQCD model [LMR22] predicts somewhat larger values for TFFs and a_μ (in units of 10^{-11}):

	a_μ dispersive [WP2]	a_μ hQCD
π^0	$63.0^{+2.7}_{-2.1}$	$63.4(2.7)$
η	$14.7(9)$	$17.6(1.7)$
η'	$13.5(7)$	$14.9(2.0)$
Sum	$91.2^{+2.9}_{-2.4}$	$95.9(3.8)$

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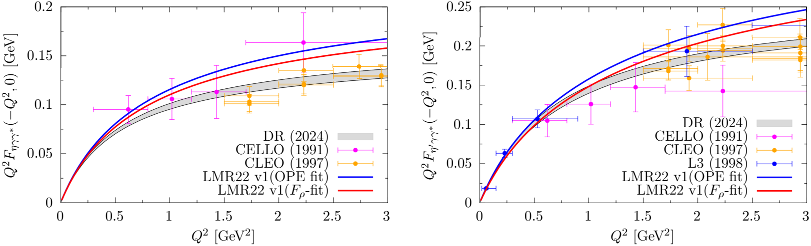

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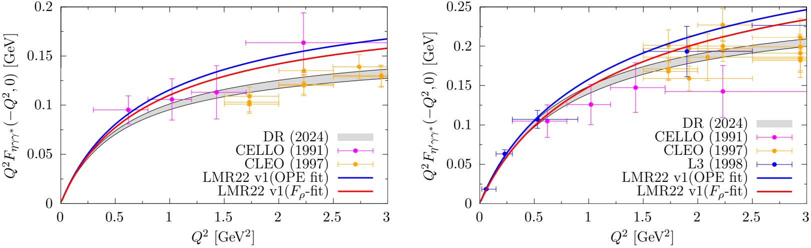

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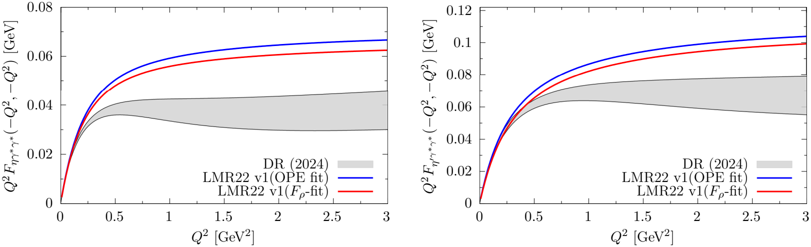

Doubly virtual η, η' TFF comparison of hQCD model with dispersive results [2411.08098]

New puzzle: Tensor mesons

WP20: Despite their prominence in photon collisions,
contribution from tensor mesons was considered to be almost negligibly small

With simple Quark Model (QM) ansatz for TFFs, Danilkin & Vanderhaeghen [1611.04646] got $\boxed{a_\mu^{\mathrm{T}} = +0.64(13) \times 10^{-11}}$ for ground-state tensor multiplet

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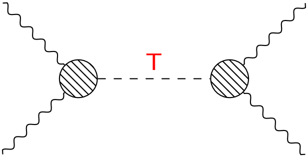
CLMR25: hQCD gives instead $\boxed{a_\mu^{\mathrm{T}} = +2.9 \times 10^{-11} \text{ and } a_\mu^{\mathrm{T}^*} = +5.6 \times 10^{-11}}$!!

Tensor mesons

T= $f_2(1270), a_2(1320), f_2'(1430), ..$



TFF decomposition:

$$\mathcal{M}^{\mu\nu\alpha\beta}(q_1^2, q_2^2) = \sum_{i=1}^{5} \frac{\mathcal{F}_i^T(q_1^2, q_2^2)}{m_T^{n_i}} T_i^{\mu\nu\alpha\beta},$$

QM: only $\mathcal{F}_1^T$ nonvanishing with scale Λ_T either $\sim M_T$ (DV) or M_ρ (HSZ24)
and match to real-photon decay rate

$$\mathcal{F}_1^T(-Q_1^2, -Q_2^2) = \frac{\mathcal{F}_1^T(0,0)}{(1 + (Q_1^2 + Q_2^2)/\Lambda_T^2)^2}, \quad \mathcal{F}_{2,3,4,5}^T = 0$$

Tensor mesons in hQCD

hQCD describes tensors via metric fluctuations h_{MN} [Katz et al. 0510388]

$$S = -2k_T \int d^5x \sqrt{g} \left(\mathcal{R} + 2\Lambda\right) + \frac{1}{2g_5^2} \mathrm{tr} \int d^5x \sqrt{g} F_{MN} F^{MN}$$

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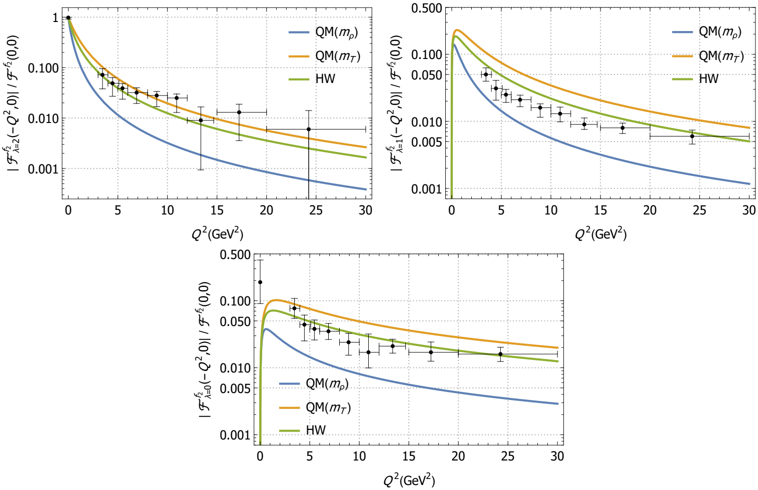
Non-zero $\mathcal{F}_i^T$: [CLMR 2501.09699]

$$\mathcal{F}_1^T(-Q_1^2, -Q_2^2) = -m_T \frac{1}{g_5^2}\mathrm{tr}\mathcal{Q}^2 \int \frac{dz}{z} h_n(z) \mathcal{J}(z, Q_1)\mathcal{J}(z, Q_2),$$
$$\mathcal{F}_3^T(-Q_1^2, -Q_2^2) = -m_T^3 \frac{1}{g_5^2}\mathrm{tr}\mathcal{Q}^2 \int \frac{dz}{z} h_n(z) \frac{\partial_z \mathcal{J}(z, Q_1)}{Q_1^2} \frac{\partial_z \mathcal{J}(z, Q_2)}{Q_2^2}.$$

$\mathcal{F}_1^T$ and $\mathcal{F}_3^T$ originate both from $5d$ covariant $F_{MN}F^{MN}$.
Quark model ansatz (QM) only has nonzero $\mathcal{F}_1^T$.

Neither QM nor hQCD tensor TFFs reproduce the LC asymptotics, which involves *all* $\mathcal{F}_i^T$ simultaneously.
Comparison with resonance chiral theory suggests that $\mathcal{F}_1^T$ and $\mathcal{F}_3^T$ may be equally important at low energies.

- $m_T = 1235$ MeV (experimentally: $1275.4(8)$ MeV)
- $\Gamma_{\gamma\gamma} = 2.3$ keV (experimentally: $2.65(45)$ keV)



Nice agreement with singly virtual data but no sensitivity to $\mathcal{F}_3^T$

(Without $\mathcal{F}_3^T$, hQCD result for $\mathcal{F}_1^T$ would give similar results as QM!!)

Tensors and $g-2$ in 4pt. kinematics

k_T	M_T [GeV]	$\Gamma_{\gamma\gamma}$ [keV]	IR	Mixed	a_μ $[10^{-11}]$
F_ρ fit	1.235	2.3+0.8+0.2	2.93	0.23	3.17
OPE fit	1.235	2.6+0.9+0.2	3.28	0.25	3.55
by $\Gamma_{\gamma\gamma}$	1.2754(8)	2.65(45)	2.28	0.16	2.4(4)
	1.3182(6)	1.01(9)	0.85	0.05	0.9(1)
	1.5173(24)	0.08(2)	0.06	0.003	0.06(2)
	$f_2+a_2+f_2'$		3.19	0.21	3.4(4)

For hQCD model at low energies (IR: $Q_i < 1.5$ GeV):

$$a_{\mu\ \mathrm{IR}}^{f_2+a_2+f_2'} = +2.9(4) \times 10^{-11}$$

Quark model (only $\mathcal{F}_1^T \neq 0$) as in HSZ:

$$a_{\mu\ \mathrm{IR}}^{f_2+a_2+f_2'}[\mathrm{QM}(m_\rho)] = -2.5(8) \times 10^{-11}$$

Excited states and SDCs

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Excited states crucial in understanding MV and symmetric SDCs:

[Melnikov-Vainshtein 2003], [Bijnens et al. 2019]

$$\mathcal{C}_{\mathrm{MV}} = \lim_{Q_3\to\infty} \lim_{Q\to\infty} Q^2 Q_3^2 \bar{\Pi}_1(Q,Q,Q_3) = -\frac{2}{3\pi^2},$$
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hQCD axials contribution after resummation (in OPE fit):

$$\mathcal{C}^A_{\mathrm{MV}} = \mathcal{C}_{\mathrm{MV}}$$
$$\mathcal{C}^A_{\mathrm{sym}} = 0.81\, \mathcal{C}_{\mathrm{sym}}$$

Excited tensor states

Tensor contributions can be resummed similarly

$$\bar{\Pi}_1(Q,Q,Q_3) = -\frac{4}{k_T}\left(\frac{\mathrm{tr}\mathcal{Q}^2}{g_5^2}\right)^2 \iint_0^{z_0} \frac{dzdz'}{zz'} \times \mathcal{J}(z,Q)\mathcal{J}(z',Q)\frac{\partial_{z'}\mathcal{J}(z',Q_3)}{Q_3^2}\partial_{z'}G(z,z';0).$$

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Excited tensors contribute to symmetric SDC but not to MV SDC!

Fitting k_T s.t. tensors supply missing 19% also yields $\Gamma_{\gamma\gamma}$ in superb agreement with data. In F_ρ fit (best guess), both constraints are satsified at $\sim 90\%$ level in accordance with α_s corrections at large but finite Q.

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	$f_2 + a_2 + f_2'$		3.4(4)

Transverse SDCs (preliminary)

Symmetric limit of $\hat{\Pi}_i$ compared to the quark loop (QL):[2]

$$\lim_{Q\to\infty} Q^{a_i}\hat{\Pi}_i(Q,Q,Q):$$

$\hat{i}$	A	T	A/QL	T/QL	sum
1	-0.03657	-0.00846	81.22	18.78	100 %
4	-0.0061	-0.01144	37.50	70.40	108 %
7	-0.0061	-0.00256	75.00	31.55	107 %
17	$+0.0061$	$+0.00034$	59.04	3.33	62 %
39	$+0.0183$	$+0.00769$	68.80	28.94	98 %

Tensors improve all transverse SDCs significantly (except $\hat{\Pi}_{17}$).

[2] $a_i = 4$ for $i = 1, 4$ and $a_i = 6$ else.

Effective tensor pole contributions to the $g-2$

Low-energy contributions of whole tower:

$$a_\mu^{\text{eff.tensor poles}}|_{\text{IR}} = 5.6(7) \times 10^{-11}$$

Large compared to contribution from ground state multiplet

$$a_\mu^{f_2+a_2+f_2'}|_{\text{IR}} = 2.9(4) \times 10^{-11}$$

- Excited f_2 in IR $a_\mu \sim 2.26 \times 10^{-11}$.
- In hQCD model: $m = 2.2$ GeV and $\Gamma_{\gamma\gamma} = 0.56$ keV.
- In nature: even lighter $f_2(1565)$ with larger $\Gamma_{\gamma\gamma} = 0.7 \pm 0.14$ keV!

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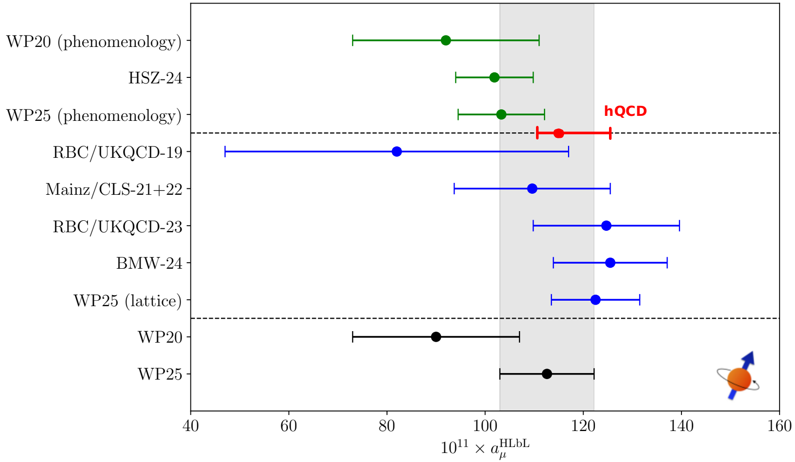
large excited state contributions within 4pt. kinematics dispersive approach plausible!

Status according to WP2025 vs. hQCD



hQCD: completed by negative contribution from pseudoscalar boxes (dispersive)

[Eq. (5.52) in WP2]

Conclusions

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- Axials used to be the big remaining unknown, but now nearly perfect concordance of hQCD and dispersive results
- Sizeable contribution from tensors as predicted by hQCD would reconcile WP25(phenomenology) with WP25(lattice)
- To be confirmed/falsified by full-fledged dispersive approach
- Urgent need for more experimental data on tensor mesons, in particular for doubly virtual TFFs given the game-changing role of $\mathcal{F}_3^T$

Appendix

Massive HW1+U(1)$_A$-Anomaly Model [LMR, 2211.16562]

$N_f = 2+1$ with $m_s \approx 24.3 m_{u,d}$ and Witten-Veneziano mechanism for η' mass
Two version of UV fits:

a) $g_5 = 2\pi$ such that UV constraints on TFF satisfied to 100%
b) $g_5 = 5.94$ such that f_ρ is fitted ($\approx 90\%$ of asymptotic SDCs)

Tuning of gluon condensate Ξ (neglected by KS) $\to$ virtually exact fit of m_η and $m_{\eta'}$:

Version a) (OPE fit)

	m [MeV]	m–m^{exp} [%]	f^8	f^0	f_G	$\lvert F(0,0)\rvert$	$F-F^{\text{exp}}$
π^0	135	(input)	0	0	0	0.277	
η	557	+1.7%	0.101	0.027	-0.030	0.275	+1(2)% (!)
η'	950	-0.8%	-0.0385	0.113	-0.077	0.340	-0(2)% (!)
G/η''	1992	?	-0.027	0.005	0.053	0.116	
	m [MeV]	m–m^{exp} [%]	F_A^8/m_A	F_A^0/m_A	$A^8(0,0)$	$A^{0\vee 3}(0,0)$	
a_1	1363	+11%	0	0	0	20.96	
f_1	1481	+15%	0.176	0.0365	20.77	3.857	
f_1'	1810	+27%	-0.030	0.201	-3.842	20.07	

gluon condensate parameter $\lvert\Xi\rvert = 0.01051\ \text{GeV}^4$

Massive HW1+U(1)$_A$-Anomaly Model [LMR, 2211.16562]

$N_f = 2+1$ with $m_s \approx 24.3 m_{u,d}$ and Witten-Veneziano mechanism for η' mass
Two version of UV fits:

a) $g_5 = 2\pi$ such that UV constraints on TFF satisfied to 100%
b) $g_5 = 5.94$ such that f_ρ is fitted ($\approx 90\%$ of asymptotic SDCs)

Tuning of gluon condensate Ξ (neglected by KS) $\to$ virtually exact fit of m_η and $m_{\eta'}$:

Version b) (our current "best guess" regarding a_μ)

	m [MeV]	m–$m^{\rm exp}$ [%]	f^8	f^0	f_G	$\|F(0,0)\|$	$F - F^{\rm exp}$
π^0	135	(input)	0	0	0	0.276	
η	561	+2.4%	0.103	0.030	-0.031	0.268	+2(2)%
η'	947	-1.1%	-0.039	0.121	-0.082	0.313	-8(2)%
G/η''	1943	?	-0.030	0.0076	0.048	0.111	
	m [MeV]	m–$m^{\rm exp}$ [%]	F_A^8/m_A	F_A^0/m_A	$A^8(0,0)$	$A^{0\vee 3}(0,0)$	
a_1	1278	+4%	0	0	0	19.46	
f_1	1410	+10%	0.176	0.029	19.58	2.69	
f_1'	1820	+28%	-0.017	0.219	-2.56	19.00	

gluon condensate parameter $|\Xi| = 0.01416\ {\rm GeV}^4$

PS: $f^{8,0}$'s within a few % of χPT values

AV: f_1-f_1' mixing angle $\phi_f - \phi_f^{\rm ideal}$ about twice as large as indicated by L3 data

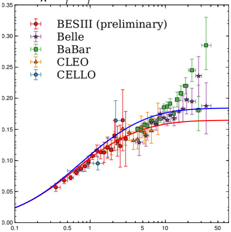
(ϕ_f strongly dependent on Ξ; but sum $a_\mu^{f_1} + a_\mu^{f_1'}$ rather insensitive)

a_μ in HW1+U(1)$_A$-Anomaly Model [LMR, 2211.16562]

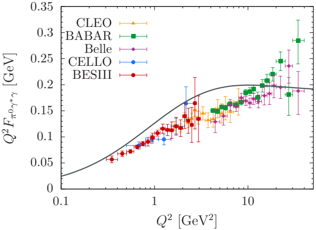
comparing also to **Soft-Wall model of P. Colangelo, F. Giannuzzi, S. Nicotri**
with $m_s > m_{u,d}$, accurate η, η' masses, good $F(0,0)$, and correct U(1)$_A$ anomaly
(CGN 2301.06456: scalar sector with $m_s > m_{u,d}$ and U(1)$_A$ anomaly;
CGN 2402.07579: axial vector contributions, but only in a simpler, flavor symmetric set-up!)

$a_\mu^{\cdots} \times 10^{11}$	LMR(OPE fit)	LMR(F_ρ-fit)	CGN(OPE fit)	WP2020
π^0	66.1	63.4	75.2	$63.0^{+2.7}_{-2.1}$
η	19.3	17.6	21.2	16.3(1.4)
η'	16.9	14.9	12.3	14.5(1.9)
$PSGB/\eta''$	0.2	0.2	5.1	
$\sum_{PS^*}$	1.6	1.4	$\gg$1.7	
PS poles total	104	97.5	>115.5	93.8(4.0)

$Q^2 F_{\pi^0\gamma^*\gamma}(Q^2, 0)$ [GeV] in LMR 2211.16562



CGN 2301.06456:



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$PSGB/\eta''$	0.2	0.2	5.1	
$\sum_{PS^*}$	1.6	1.4	$\gg$1.7	
PS poles total	104	97.5	>115.5	93.8(4.0)
a_1	7.8	7.1	9.0	
$f_1 + f_1'$	20.0	17.9=2.5×7.1	$3^\star \times 9.0$	
$\sum_{a_1^*}$	2.2	2.4	$1.3^\dagger$	
$\sum_{f_1^{(\prime)*}}$	3.6	3.0	$3^\star \times 1.3^\dagger$	
AV+LSDC total	34	30.5	41.3	19(12)
total	138	128	>157	113(16)

*: due to U(3) flavor symmetry

†: We (LMR) can reproduce results for first few resonances, but not infinite sum

SW model actually has a fundamental problem [Kwee & Lebed, 0712.1811], to be checked!