A critical review of the Standard Model Prediction of the muon g-2

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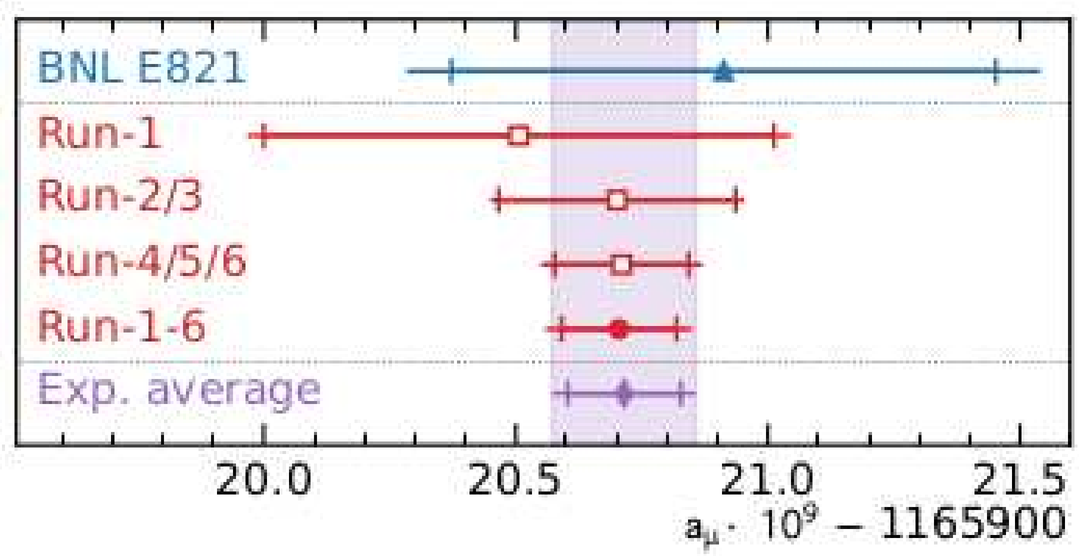
6th Workshop on flavour changing and conserving processes (FCCP2025) – Anacapri, Sept. 29 - Oct. 1, 2025

Introduction



$$a_\mu^{\text{exp}}(\text{FNAL E989}) = 116\,592\,0705(148) \cdot 10^{-12}\ [127\ \text{ppb}]$$

$$a_\mu^{\text{exp}}(\text{WA}) = 116\,592\,0715(145) \cdot 10^{-12}\ [124\ \text{ppb}]$$

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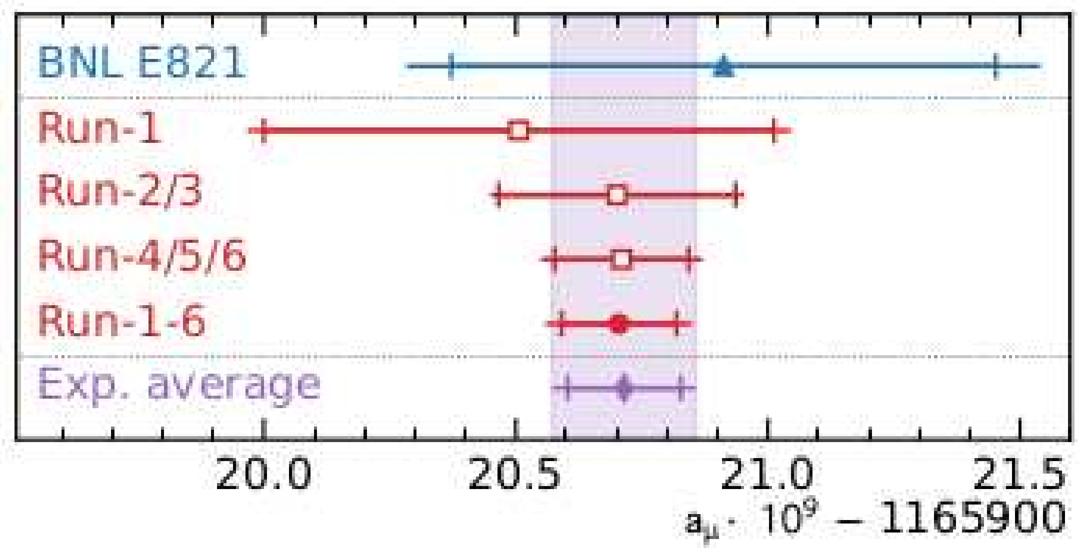

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$$a_\mu^{\text{SM}}(\text{WP25}) = 116\,592\,033(62)\cdot 10^{-11}\ [532\ \text{ppb}]$$

$$a_\mu^{\exp}(\text{WA}) - a_\mu^{\text{SM}}(\text{WP25}) = 38(63)\cdot 10^{-11}\ [532\ \text{ppb}]$$

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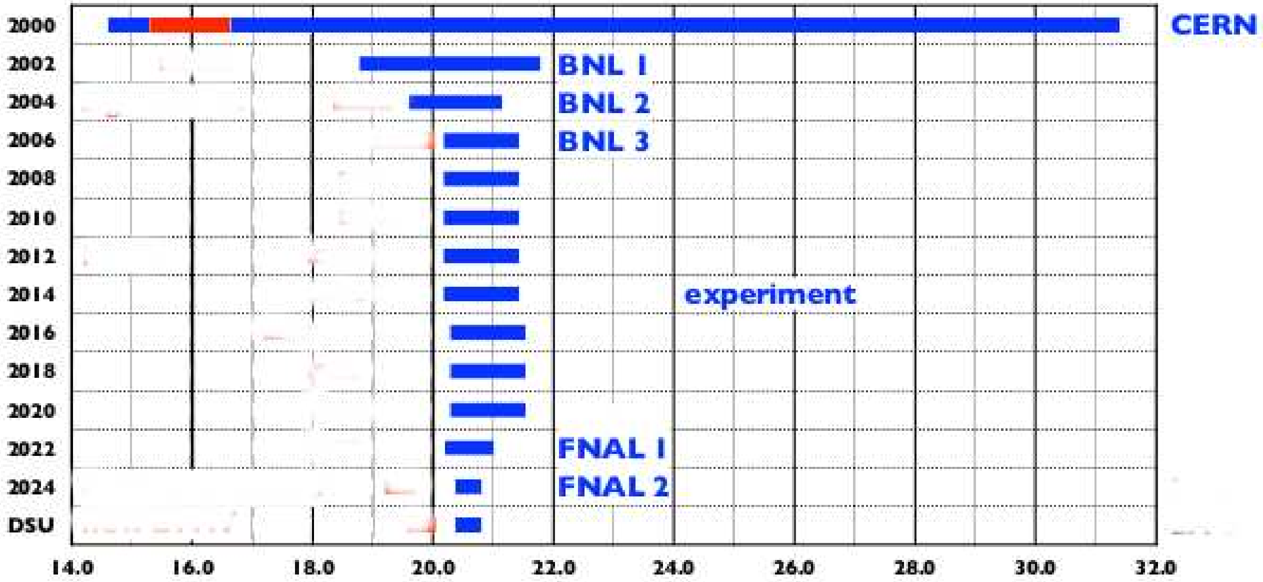

Experiment: steady path towards reduction of uncertainty

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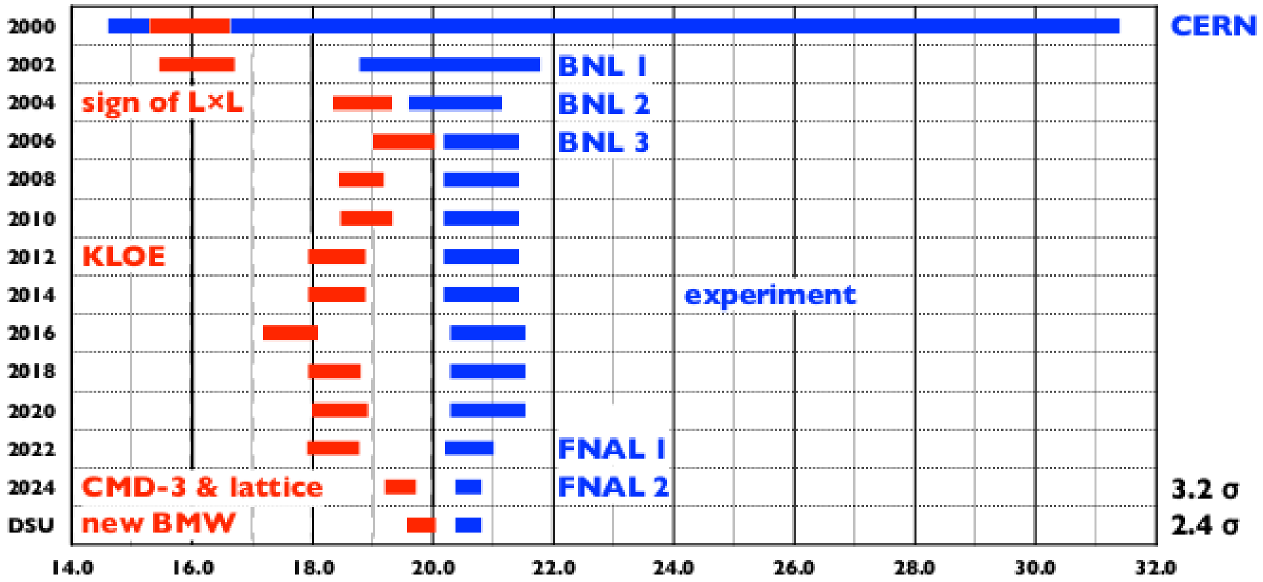

J. Erler 2505.03457

Experiment: steady path towards reduction of uncertainty

Theory: a much more chaotic journey

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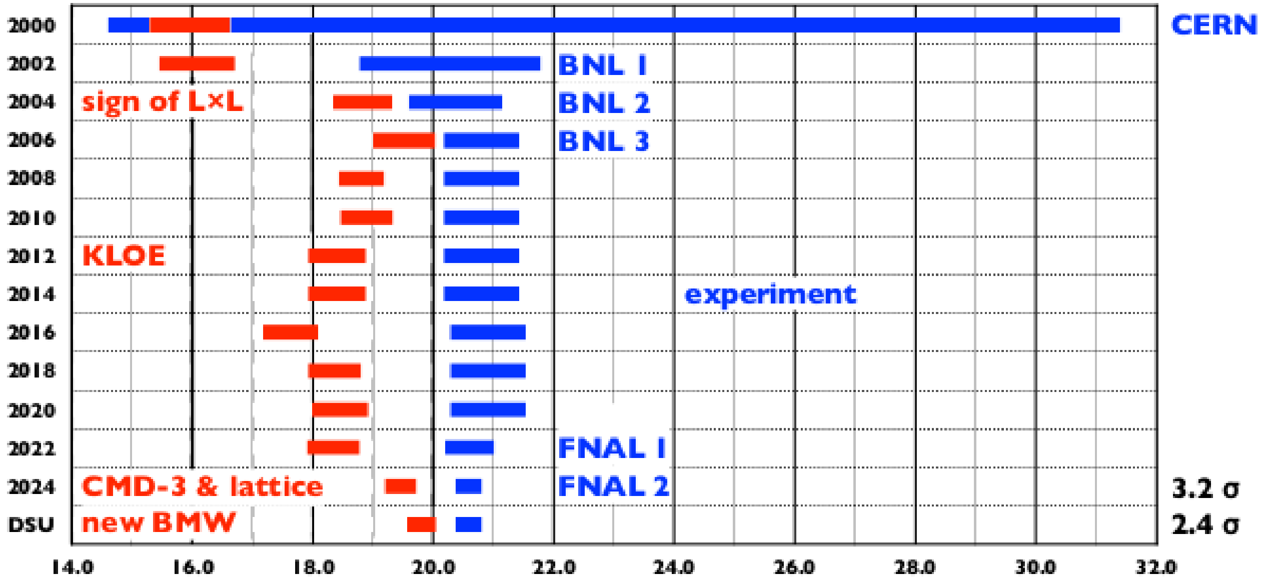

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Experiment: steady path towards reduction of uncertainty

Theory: a much more chaotic journey

Present status of SM theory prediction Phys. Rep. 887, 1 (2020) Phys. Rep. 1143, 1 (2025)

SM prediction (the White Paper(s) in a nutshell)

Considering SM contributions only, one has, by order of importance

$$a_\ell = a_\ell^{\mathsf{QED}} + a_\ell^{\rm had} + a_\ell^{\rm weak}$$

a_ℓ^{QED}: loops with only photons and leptons

$a_\ell^{\rm had}$: loops with photons and leptons and at least one quark loop dressed with gluons

$a_\ell^{\rm weak}$: loops with also contributions from the weak sector

QED contribution :

$\longrightarrow$ loops with only photons and leptons

$\longrightarrow$ can be computed in perturbation theory (conceptually clear)

$$a_\ell^{\text{QED}} = C_\ell^{(2)}\left(\frac{\alpha}{\pi}\right) + C_\ell^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + C_\ell^{(6)}\left(\frac{\alpha}{\pi}\right)^3 + C_\ell^{(8)}\left(\frac{\alpha}{\pi}\right)^4 + C_\ell^{(10)}\left(\frac{\alpha}{\pi}\right)^5 + \ldots$$

$\longrightarrow$ need to go to high orders $\quad \Delta a_\mu^{\text{exp}} \sim 14 \cdot 10^{-11} \quad [\Delta a_e^{\text{exp}} \sim 13 \cdot 10^{-14}]$

$$(\alpha/\pi)^4 = 2.91\ldots \cdot 10^{-11} \qquad (\alpha/\pi)^5 = 6.76\ldots \cdot 10^{-14}$$

$\longrightarrow$ becomes technically challenging $(1, 6, 72, 891, 12\,672, \ldots)$

$\longrightarrow$ mass effects are very crucial! $[C_\mu^{(10)} = 750.72(93)$ vs. $C_e^{(10)} = 4.952(55)]$

$\longrightarrow$ requires an input for α

QED contribution :

$$a_\mu^{\rm QED}(Cs) = 116\,584\,718.932(23)_\alpha(7)_{\rm mass}(17)_{\alpha^4}(6)_{\alpha^5}(100)_{\alpha^6}[104] \cdot 10^{-11}$$

$$a_\mu^{\rm QED}(a_e) = 116\,584\,718.833(13)_\alpha(7)_{\rm mass}(17)_{\alpha^4}(6)_{\alpha^5}(100)_{\alpha^6}[103] \cdot 10^{-11}$$

$$a_\mu^{\rm QED}(Rb) = 116\,584\,718.795(8)_\alpha(7)_{\rm mass}(17)_{\alpha^4}(6)_{\alpha^5}(100)_{\alpha^6}[102] \cdot 10^{-11}$$

$\longrightarrow$

$$a_\mu^{\rm QED} = 116\,584\,718.8(2) \cdot 10^{-11}$$

$$a_\mu^{\rm exp}({\rm WA}) = 116\,592\,071.5(14.5) \cdot 10^{-11}$$

$$a_\mu^{\rm exp}({\rm WA}) - a_\mu^{\rm QED} = 735.3(1.5) \cdot 10^{-10}$$

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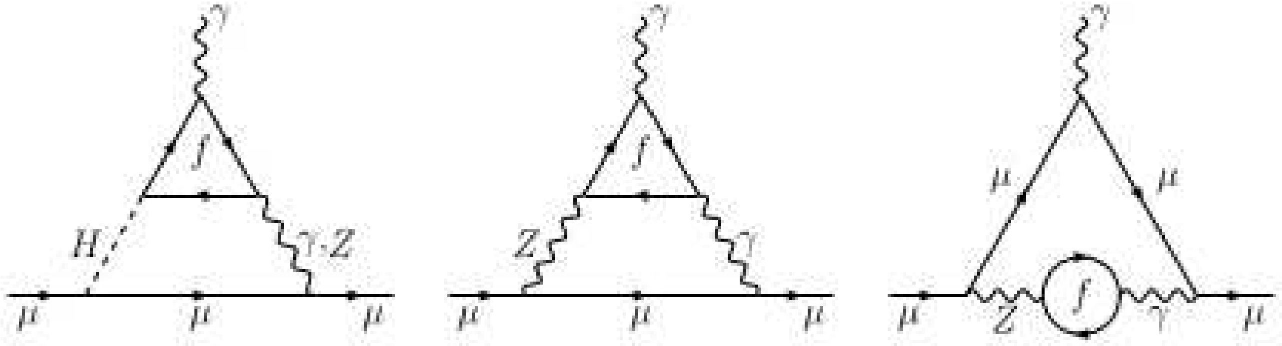
- QED provides more than 99.99% of the total experimental value
- The missing part has to be provided by weak and strong interactions (or else, new physics...)

Weak contribution:

$\longrightarrow$ loops with Z^0, H, ν_ℓ,...

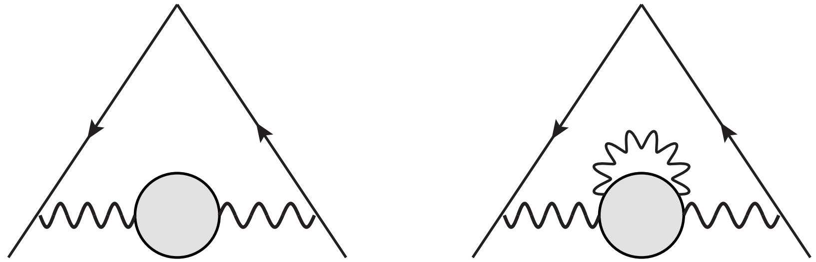
$\longrightarrow$ can (almost) be computed in perturbation theory



$$a_\mu^{\text{weak}} = 15.44(4) \cdot 10^{-10}$$

$$a_\mu^{\text{exp}} - a_\mu^{\text{QED}} - a_\mu^{\text{weak}} = 719.86(1.50) \cdot 10^{-10}$$

HVP contribution:



$\longrightarrow$ starts at $\mathcal{O}(\alpha^2)$

$\longrightarrow$ can be expressed as

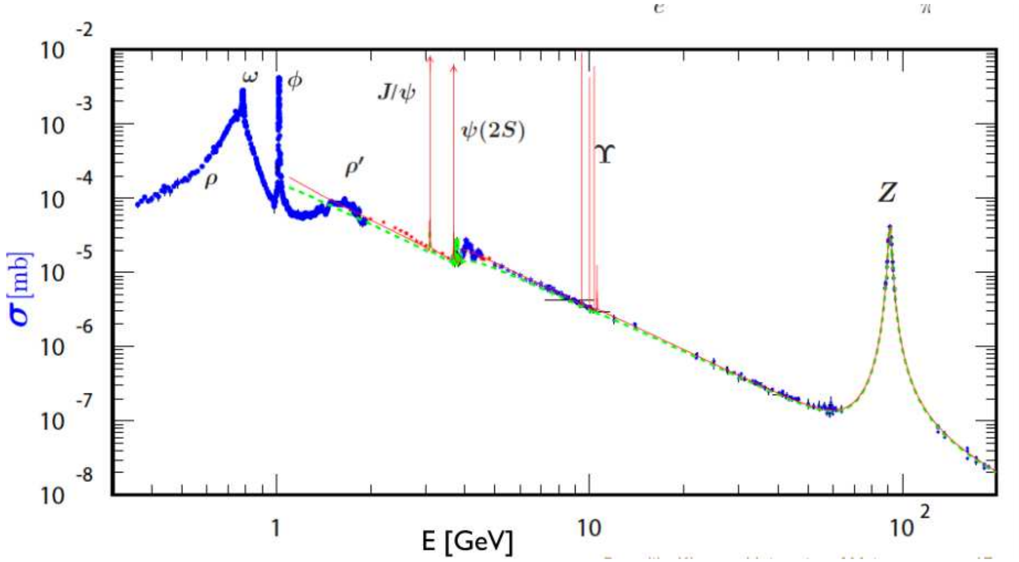
$$a_\mu^{\text{HVP-LO}} = \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^2 \int_{M_\pi^2}^{\infty} \frac{dt}{t}\, K(t) R^{\text{had}}(t) \qquad K(t) = \int_0^1 dx\, \frac{x^2(1-x)}{x^2 + (1-x)\,\frac{t}{m_\mu^2}} \sim m_\mu^2/(3t)$$

$\longrightarrow$ the low-energy region dominates

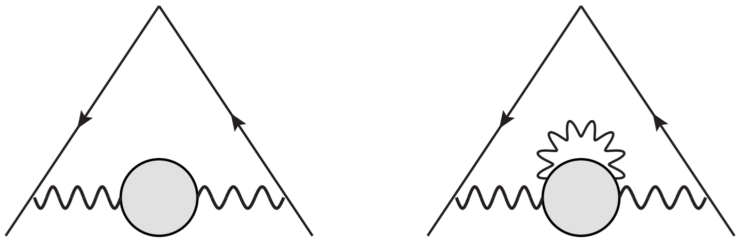
$\longrightarrow$ cannot be handled in perturbation theory

$\longrightarrow$ non-perturbative approaches have been developed:
data-driven, lattice QCD

HVP contribution: $\sigma(e^+e^- \to \mathrm{had})$



$\longrightarrow$



$\longrightarrow$ $e^+e^- \to \mathrm{had}$ cross-section measurements

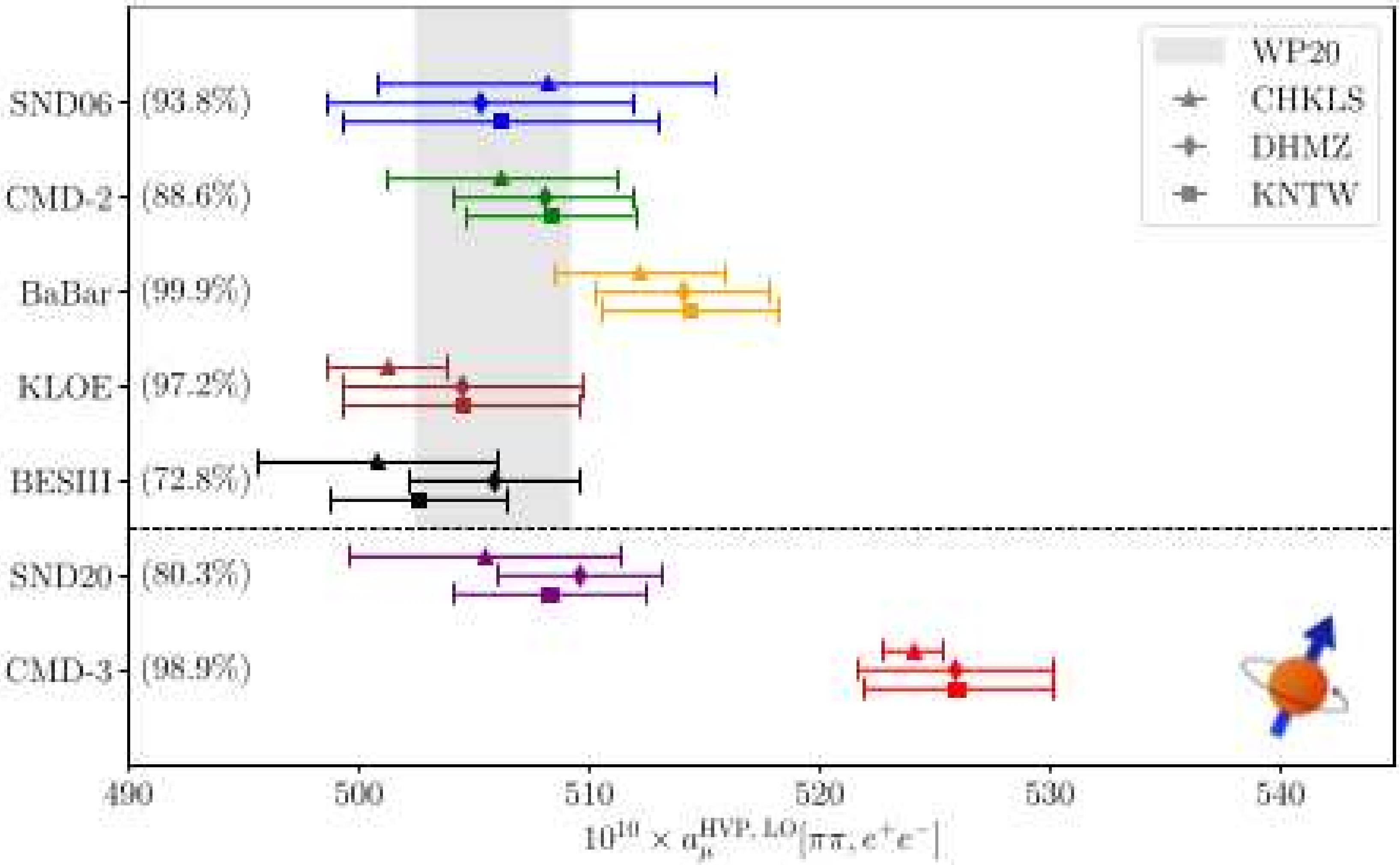
$\longrightarrow$ more than 40 exclusive channels have been measured

$\longrightarrow$ $\pi\pi$ channel dominates

$\longrightarrow$ many experiments have contributed:

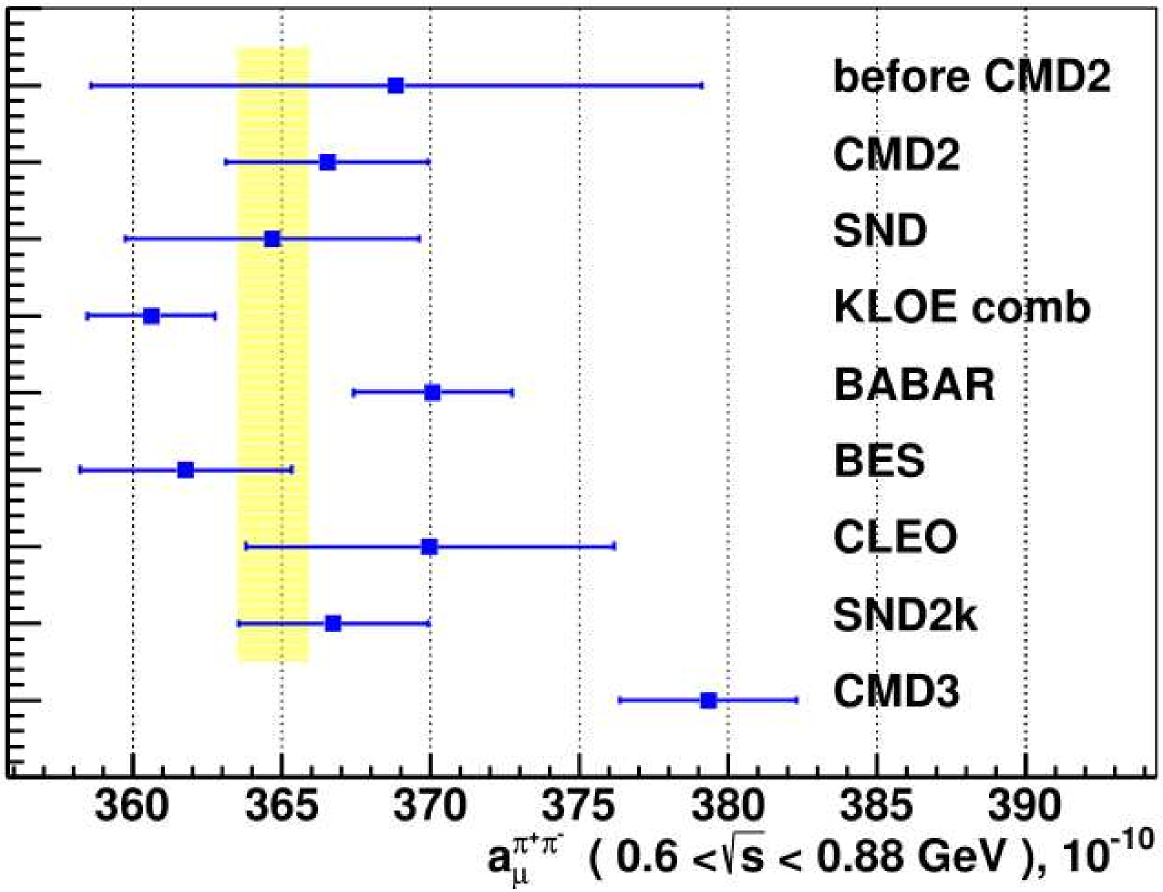
ISR: BaBar, KLOE, BESIII, Belle II,...
scan: CMD-2, SND, CMD-3 (VEPP 2000)

HVP contribution: $e^+e^- \to \pi\pi$



$\sqrt{s} \leq 1.8$ GeV

HVP contribution: $e^+e^- \to \pi\pi$



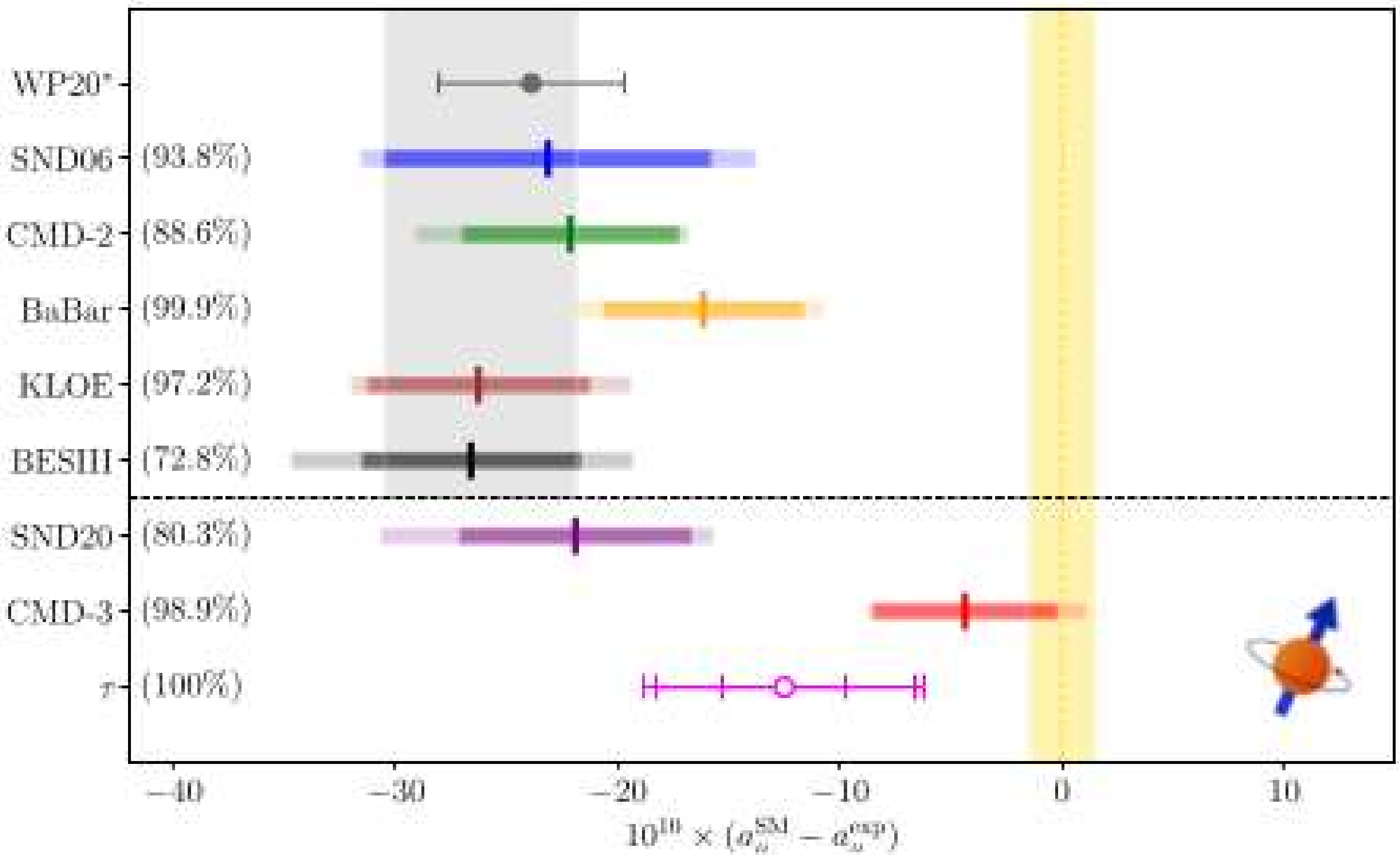
HVP contribution: $\tau^{\pm} \to \pi^{\pm}\pi^{0}\nu_{\tau}$

$\longrightarrow$ data from several experiments:

Belle, CLEO, ALEPH, OPAL

$\longrightarrow$ need to control isospin rotation from $\pi^{\pm}\pi^{0}$ channel to $\pi^{+}\pi^{-}$!

HVP contribution: $e^+e^- \to \mathrm{had}$



Improvements in many channels (other than $\pi\pi$) since WP20

HVP contribution: lattice QCD

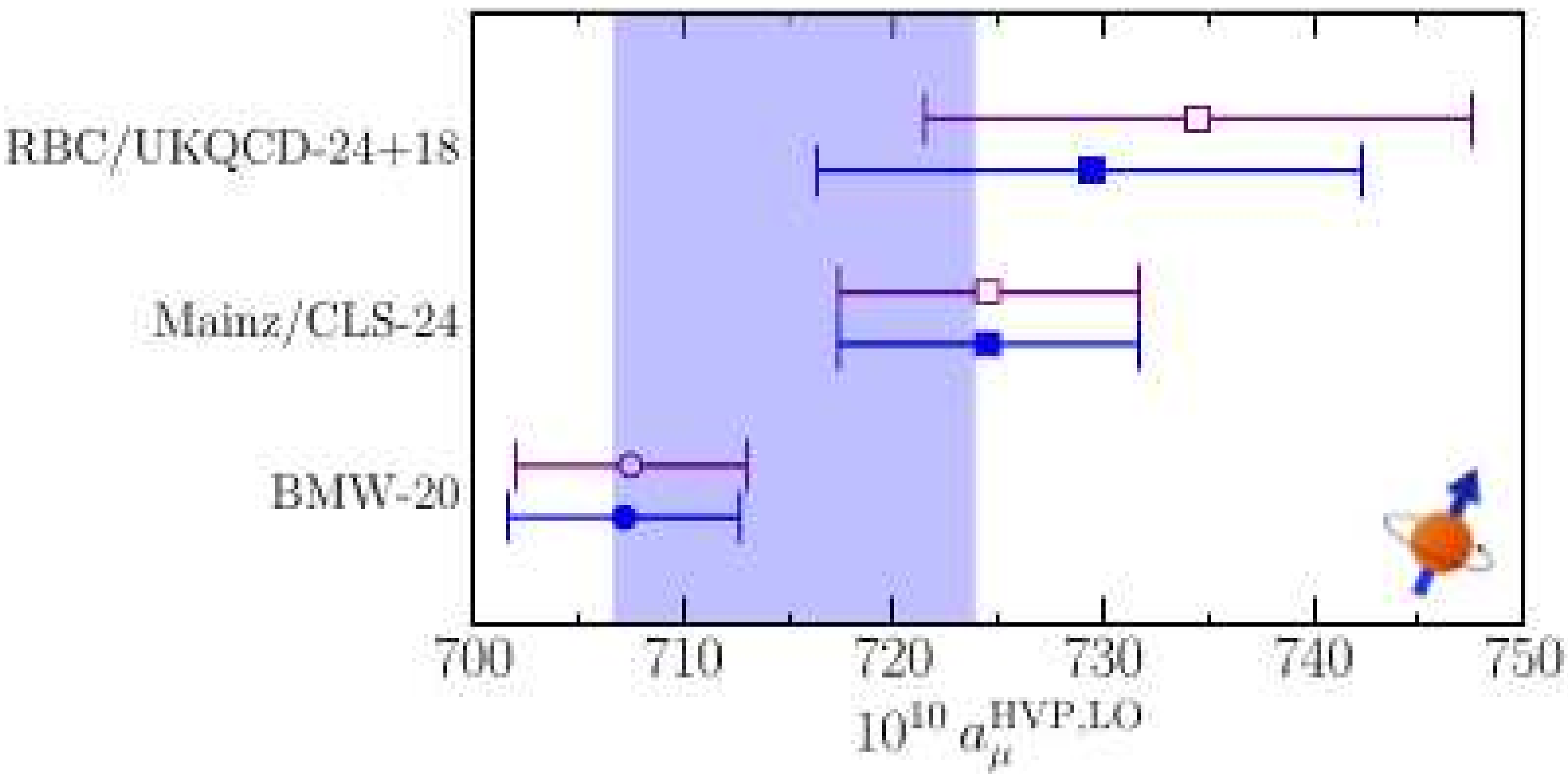
$\longrightarrow$ discretized and finite-volume space-time

$\longrightarrow$ dispersive integral becomes integral over the Euclidian time axis

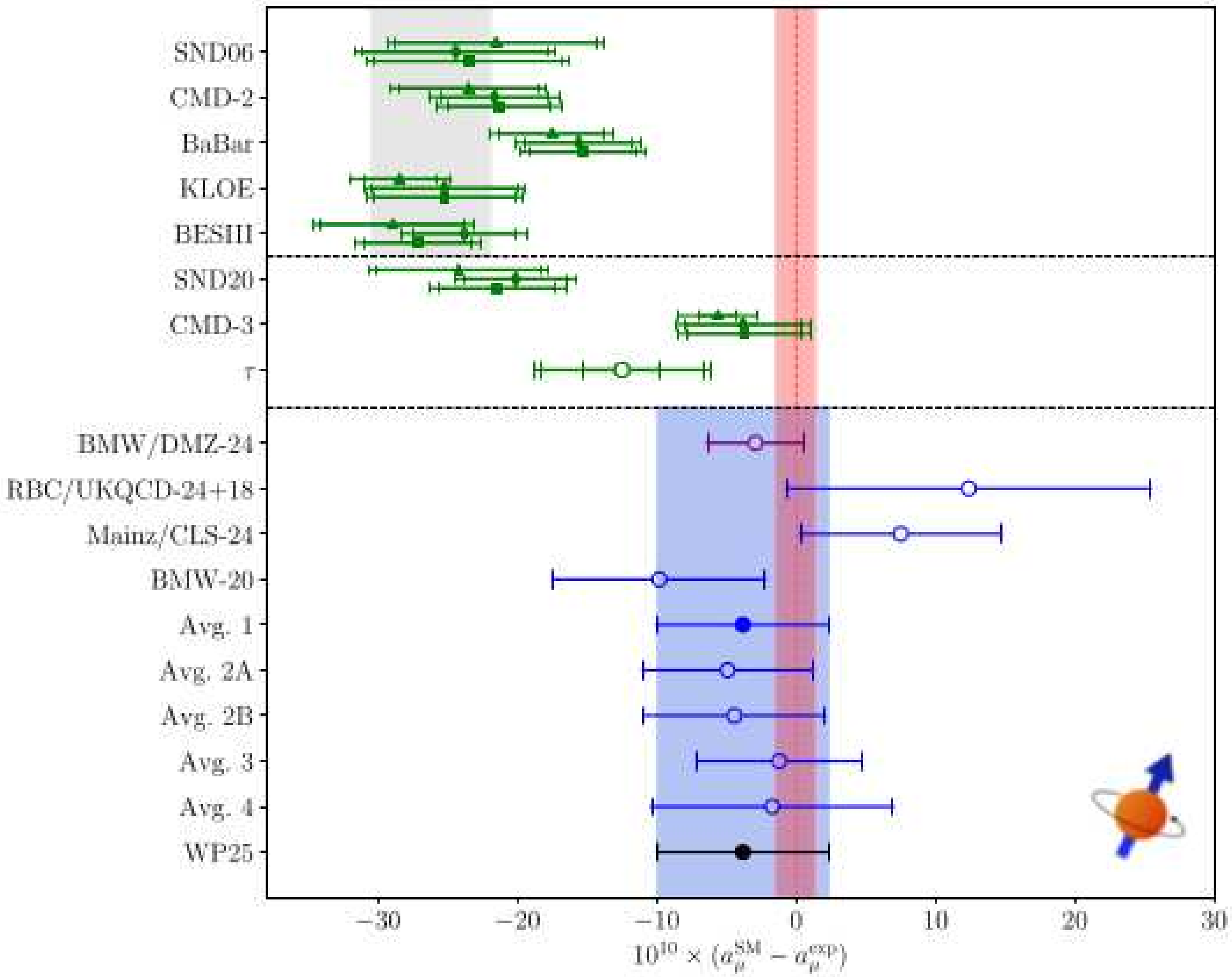
$\longrightarrow$ independent ensembles of gauge-field configurations

$\longrightarrow$ different fermion discretizations

HVP contribution: lattice QCD

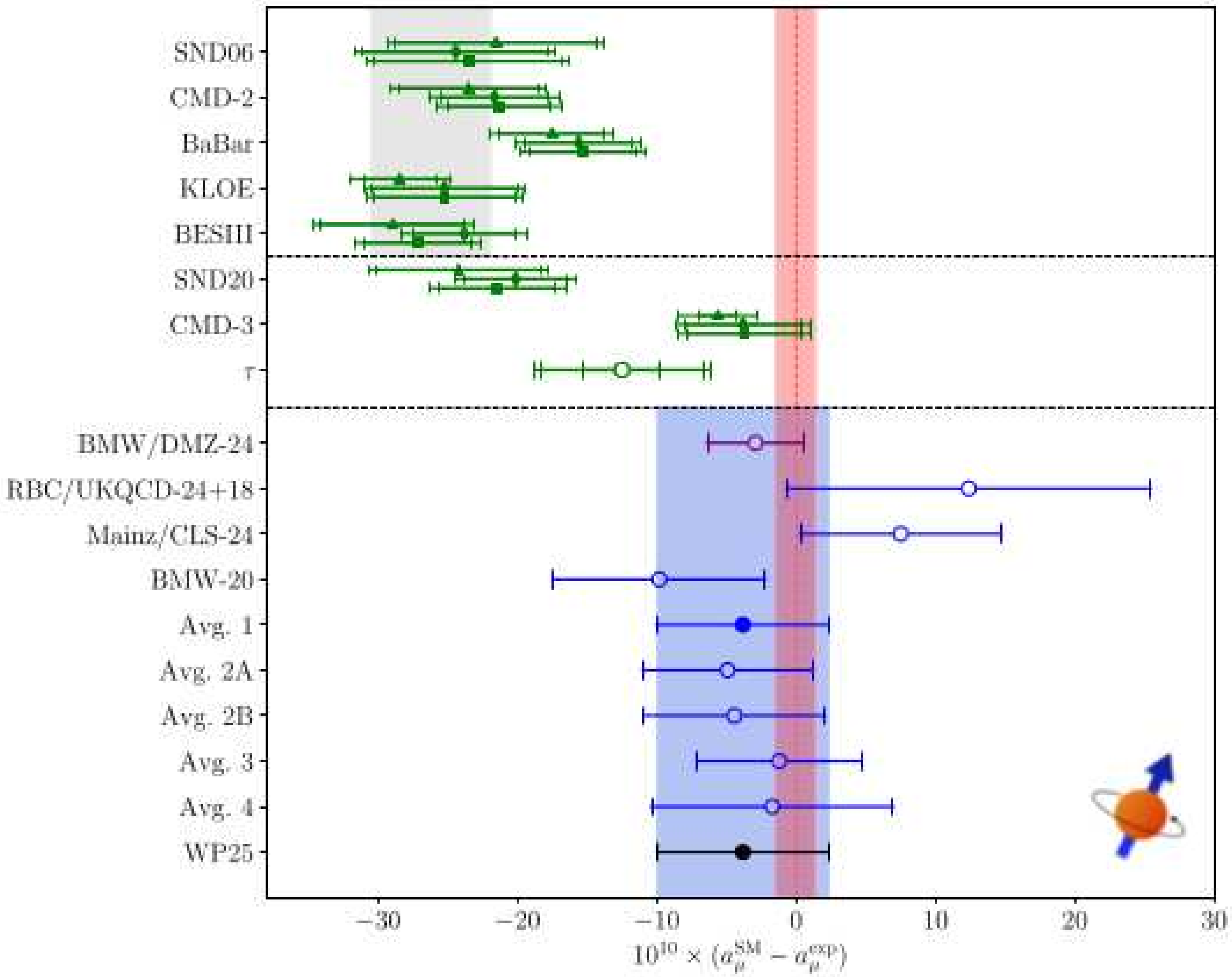


HVP contribution: summary



clear tensions among data-driven determinations but also
between lattice and data-driven determinations (except CMD-3 and τ)!

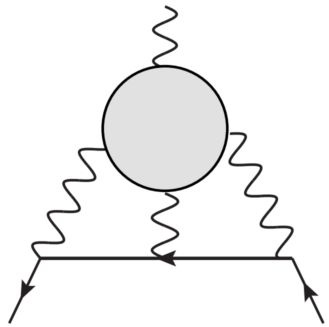
HVP contribution: summary



lattice QCD: from $711.6(18.4)$ [2.6%] (WP20) to $713.2(6.1)$ [0.9%] (WP25)

$$a_\mu^{\rm HVP;tot}[\rm WP25] = 704.5(6.1) \cdot 10^{-10}$$

HLxL contribution:



$\longrightarrow$ starts at $\mathcal{O}(\alpha^3)$

$\longrightarrow$ non-perturbative aspects important

$\longrightarrow$ several approaches have been considered:

dispersion relations, lattice QCD

analytical approaches (holographic QCD, rational approximants, resonance models, SDE,...)

$\longrightarrow$ short-distance constraints have been established

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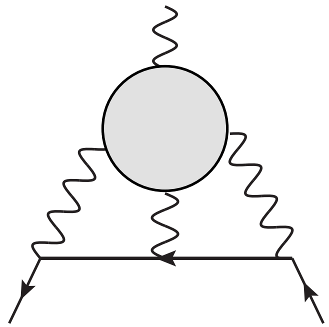

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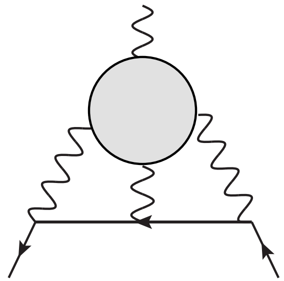
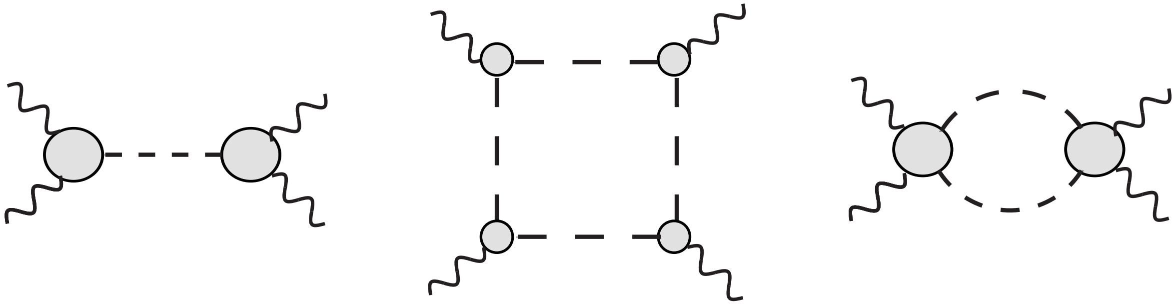
HLxL contribution: dispersion relations

⟶ not described by a single experimental observable

? ⟶ 

⟶ several contributions identified through pole and cut singularities



⟶ require input for form factors (experiment, lattice,...)

⟶ match to QCD short-distance constraints

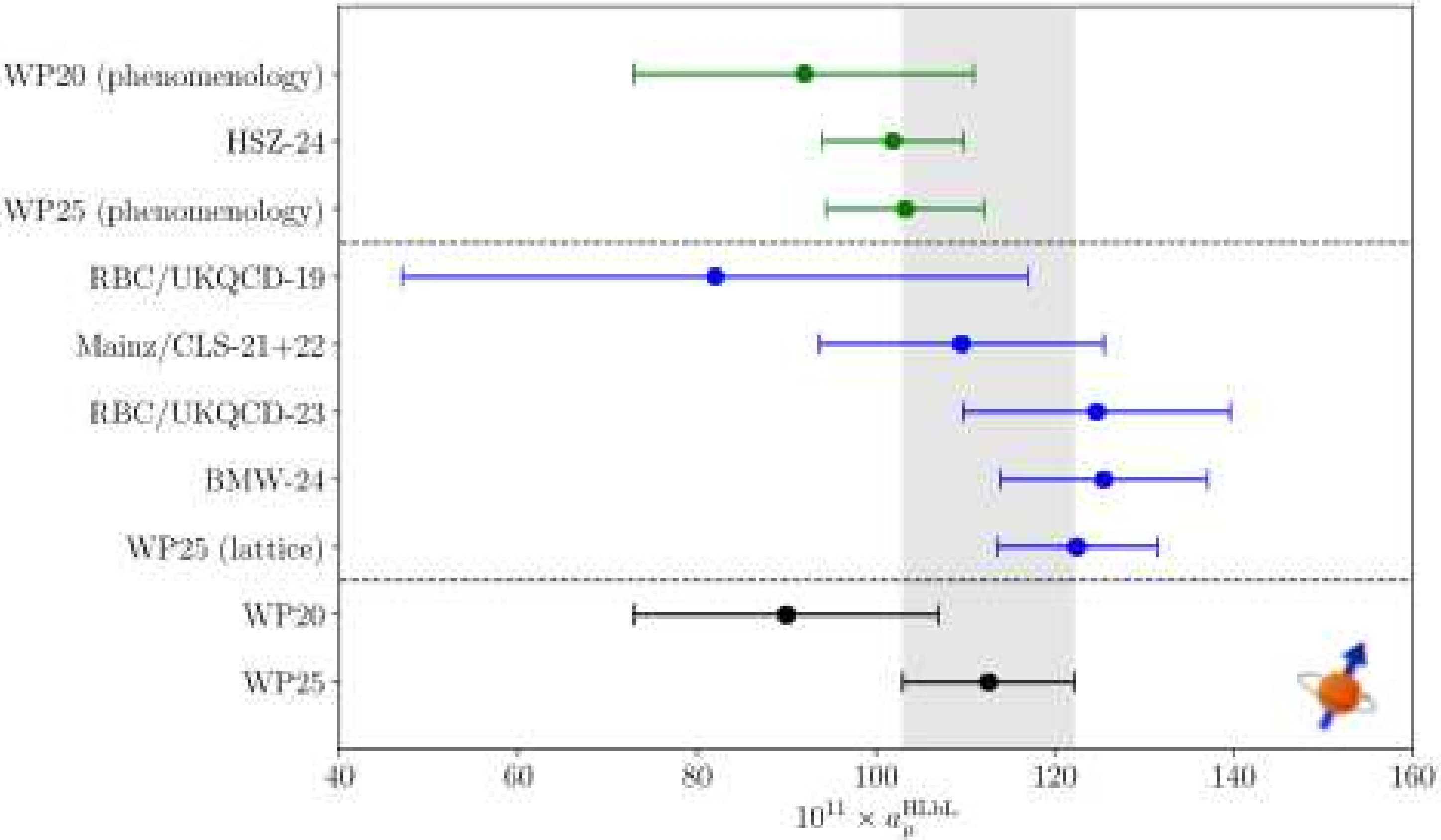
HLxL contribution: lattice QCD

$\longrightarrow$ several independent determinations

$\longrightarrow$ with different fermionic actions

$\longrightarrow$ quark-disconnected contributions, isospin-breaking effects

HLxL contribution: summary



$$a_\mu^{\rm HLxL}[\text{pheno}] = 103.3(8.8) \cdot 10^{-11} \quad [a_\mu^{\rm HLxL,NLO}[\text{pheno}] = 2.6(6) \cdot 10^{-11}]$$

$$a_\mu^{\rm HLxL}[\text{lattice}] = 112.5(9.0) \cdot 10^{-11} \quad [(7.1)_{\rm stat}(5.6)_{\rm syst}]$$

$$a_\mu^{\rm HLxL}[\text{WP25}] = 112.6(9.6) \cdot 10^{-11} \quad [\text{was } 90(17) \cdot 10^{-11}]$$

Summary:

Contribution	WP25	WP20
HVP LO (lattice)	7132(61)	7116(184)
HVP LO (e^+e^-, τ)	Table 5	6931(40)*
HVP NLO (e^+e^-)	−99.6(1.3)	−98.3(7)
HVP NNLO (e^+e^-)	12.4(1)	12.4(1)
HLbL (phenomenology)	103.3(8.8)	92(19)
HLbL NLO (phenomenology)	2.6(6)	2(1)
HLbL (lattice)	122.5(9.0)	82(35)
HLbL (phenomenology + lattice)	112.6(9.6)	90(17)
QED	116 584 718.8(2)	116 584 718.931(104)
EW	154.4(4)	153.6(1.0)
HVP (LO + NLO + NNLO)	7045(61)	6845(40)
HLbL (phenomenology + lattice + NLO)	115.5(9.9)	92(18)
Total SM Value	116 592 033(62)	116 591 810(43)

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Strong implication on the theoretical side, coordinated within the g-2 Theory Initiative (WP20 and WP25)

Collateral effect: breakthrough in the performances of lattice calculations

$\longrightarrow$ unprecedented achievement in precision and control over the main systematic effects (continuum and infinite-volume limits)

$\longrightarrow$ diversity of approaches (fermion types, window observables,...)

The lattice community will undoubtedly benefit from this impetus/momentum to deal with other challenges in the future

Concluding remarks

Quest to understand serious tensions between different hadronic cross-section measurements will continue (e.g. radiative corrections, KLOE re-analysis, new experimental data,...)

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The result on a_μ, combined with constraints coming from other observables, will probably recompose somewhat the landscape of BSM models

$\rightarrow$ we'll see in the future (and possibly in other talks at this meeting)

Plenty of good reasons and material

to continue this series of meetings

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Thanks for your attention!