The SM lifetime is slightly shorter

or “Revising the full one-loop gauge prefactor in electroweak vacuum stability”, soon to appear on PRL

[arXiv: 2406.05180] with M. Nemevšek, Y. Shoji, K. Trailović & L. Ubaldi

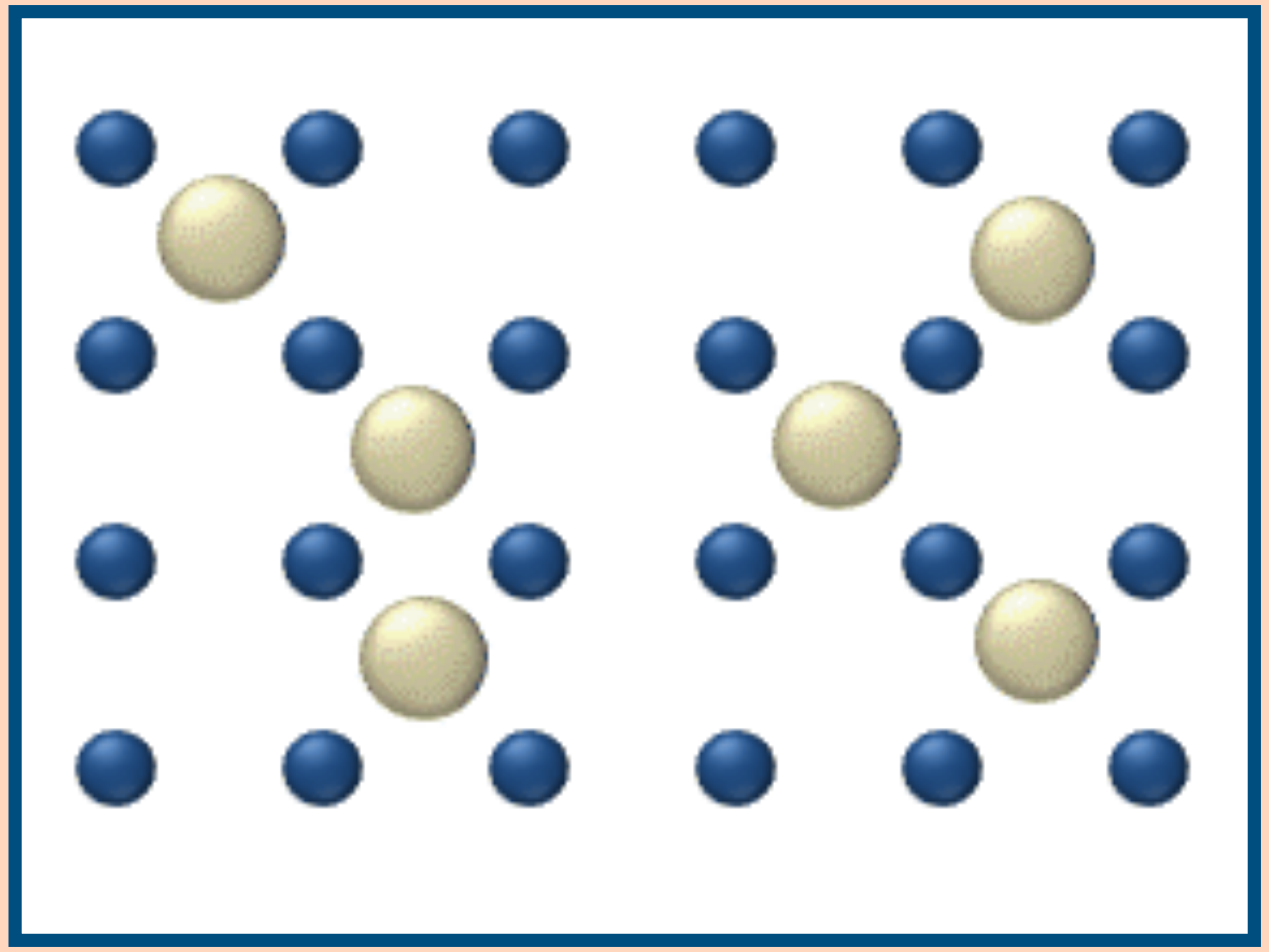
P. Baratella

Quantum vacuum

- Quantum fields Φ oscillate wildly
- In the vacuum state they typically average to zero, i.e. $\langle\Phi\rangle_{\mathrm{VAC}} = 0$ $\big($scalars can have $\langle\phi\rangle_{\mathrm{VAC}} = V$ $\big)$
- However $\langle\Phi^2\rangle_{\mathrm{VAC}} = \infty$

Quantum fluctuations

- Summing field modes randomly, it is possible to reach configurations that are very far from the average configuration
- Extremely unlikely
- Similar to thermal fluctuations



10 ordered; C(15,6) = 5005 total

Vacuum decay

- Usually they last for a little time
- Are there conditions for them to be amplified macroscopically?
- Focus on the real world (as we know it)

Vacuum decay

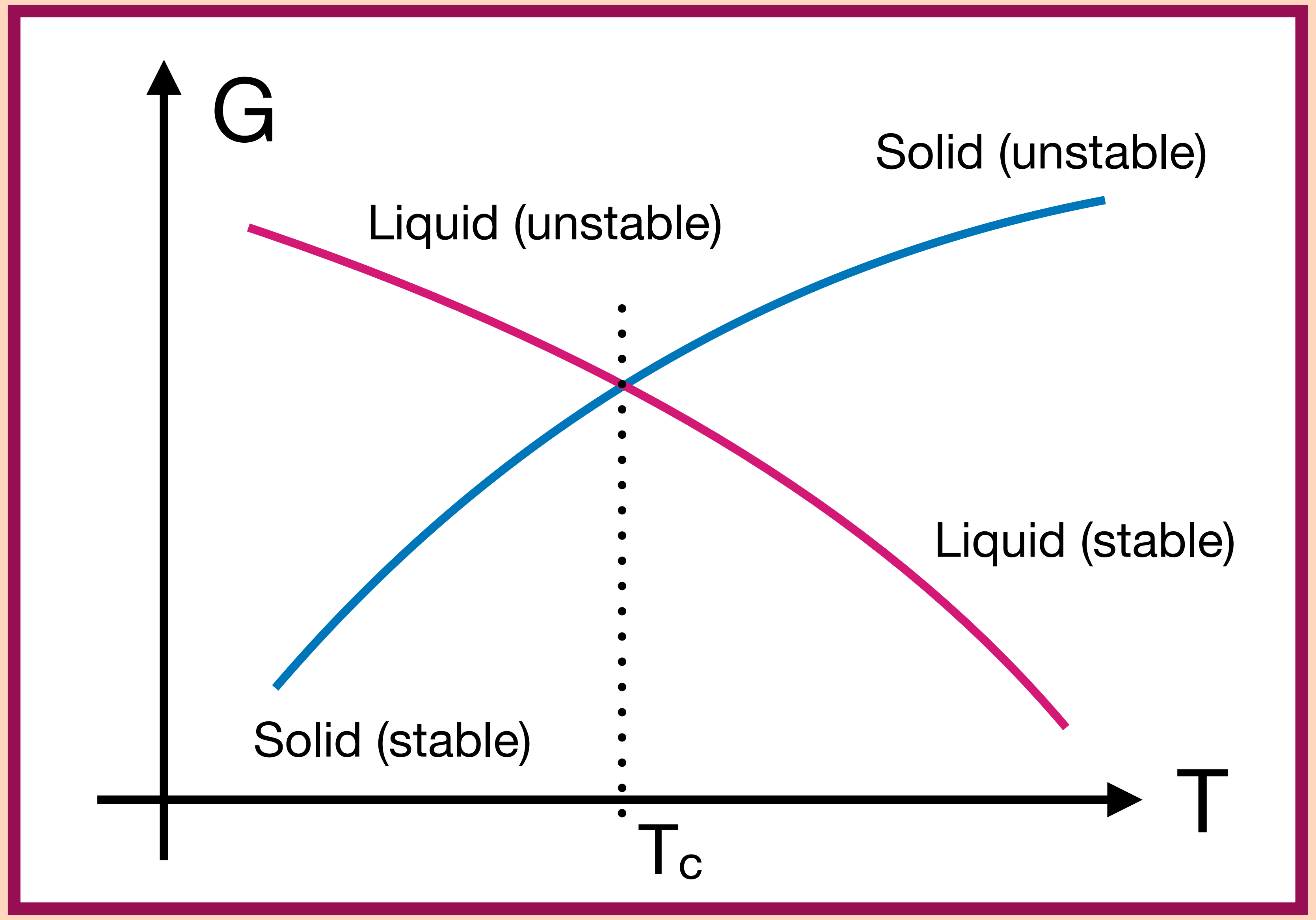
Conditions

- Look for constant field configurations whose energy density is smaller than zero (the energy density of the vacuum)
- If they exist, the vacuum is not the true vacuum
- It can decay by spontaneous formation of ``true vacuum`` bubbles
- Quantified by the rate of formation per unit volume γ_{bubble}

Thermal analogy

material with 1st order phase transition

- Analogy with a thermal system in the `wrong` phase



- For $T<T_c$ the liquid phase is thermodynamically disfavoured, but can be achieved by slow cooling
- It `decays` by induced or spontaneous formation of ice crystals

SM vacuum

Stable or not?

- The structure of our vacuum is governed by the Standard Model
- It has nonzero chiral (strong sector) and Higgs $\langle h \rangle = \mathrm{v}$ condensates
- The condition $\langle h \rangle = \mathrm{v}$ is obtained by minimising $V_{\mathrm{eff}}\left(h_{\mathrm{cl}}\right)$
- Are there other local minima?

Higgs (effective) potential

- At the electroweak scale $V = \lambda\left(h^2 - v^2\right)^2$
- For h $\gg$ v the EW v.e.v. can be neglected and $V = \lambda(h)\, h^4$
- $\lambda(h)$ can be understood as the coupling that governs $2 \to 2$ scattering at COM energy $\approx h$
- If λ crosses zero at $h*$, V becomes negative for $h > h*$

Higgs (effective) potential

Running of quartic λ

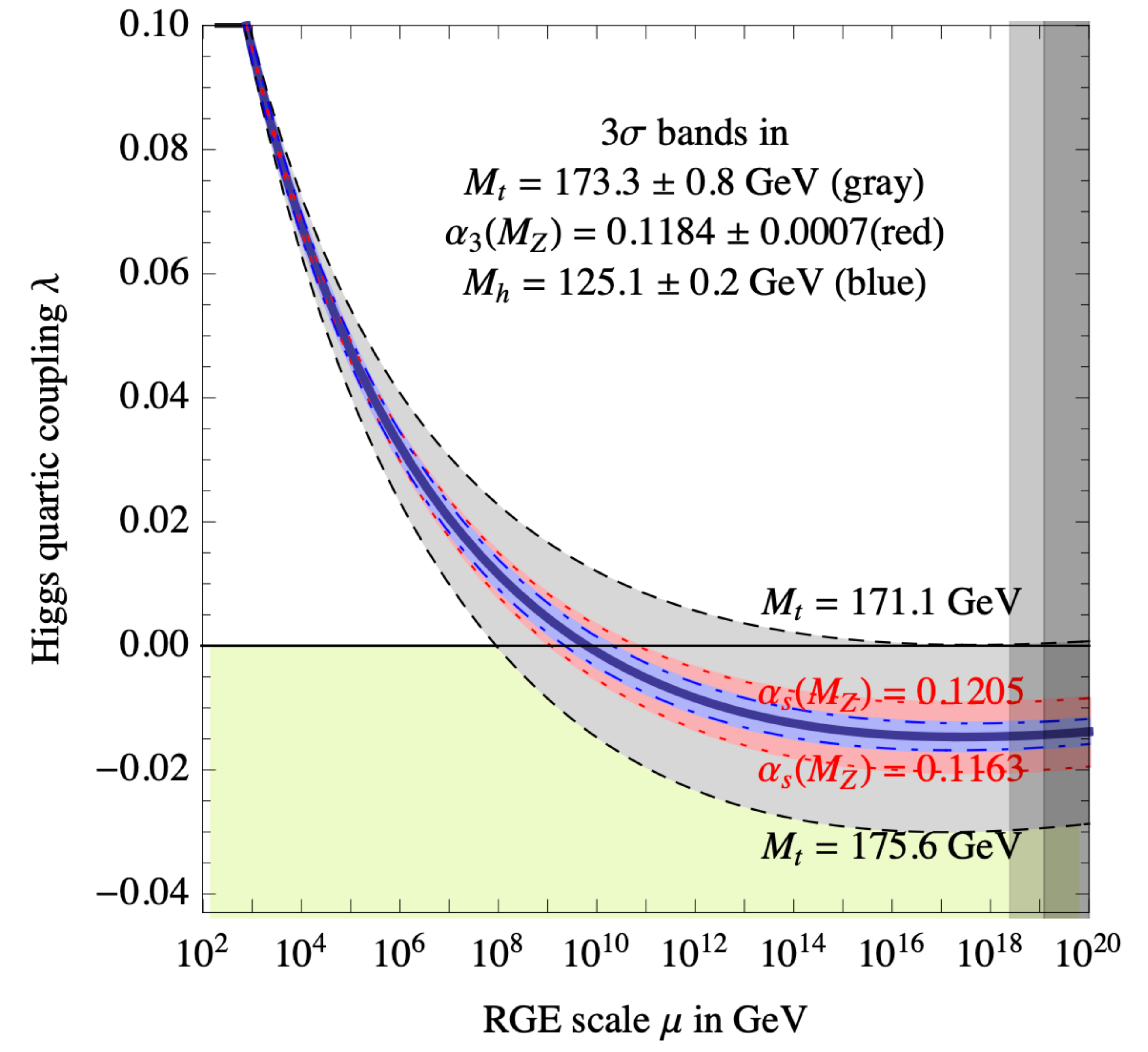
- $\lambda(h)$ is governed by β_λ
- Unlike all other couplings, β_λ does not need to be $\propto \lambda$
- Therefore λ can change sign in the SM

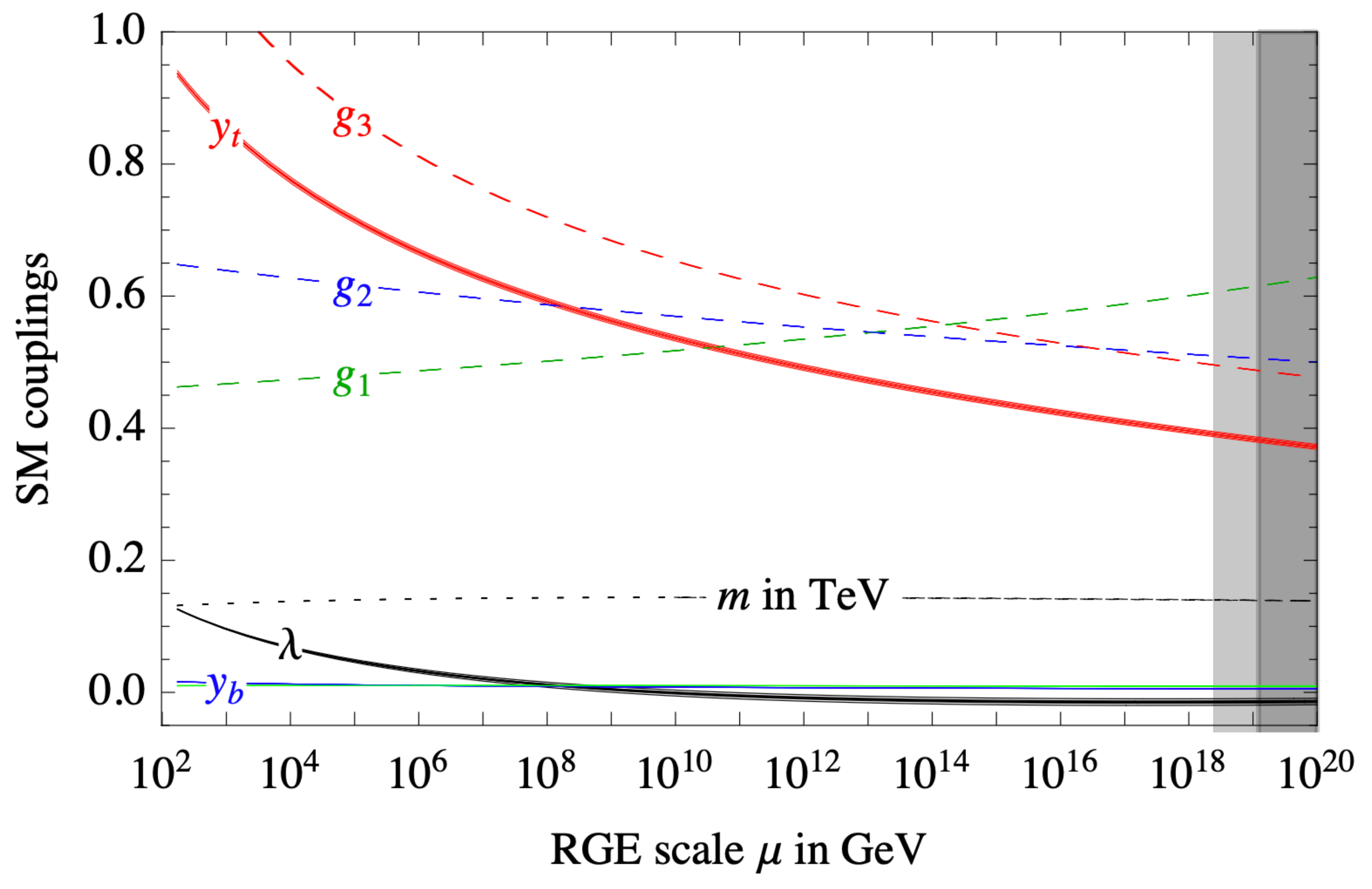
$$\beta_\lambda = -\frac{3y_t^4}{8\pi^2} + \ldots$$

top Yukawa contribution at 1 loop

Higgs (effective) potential

Running of quartic λ



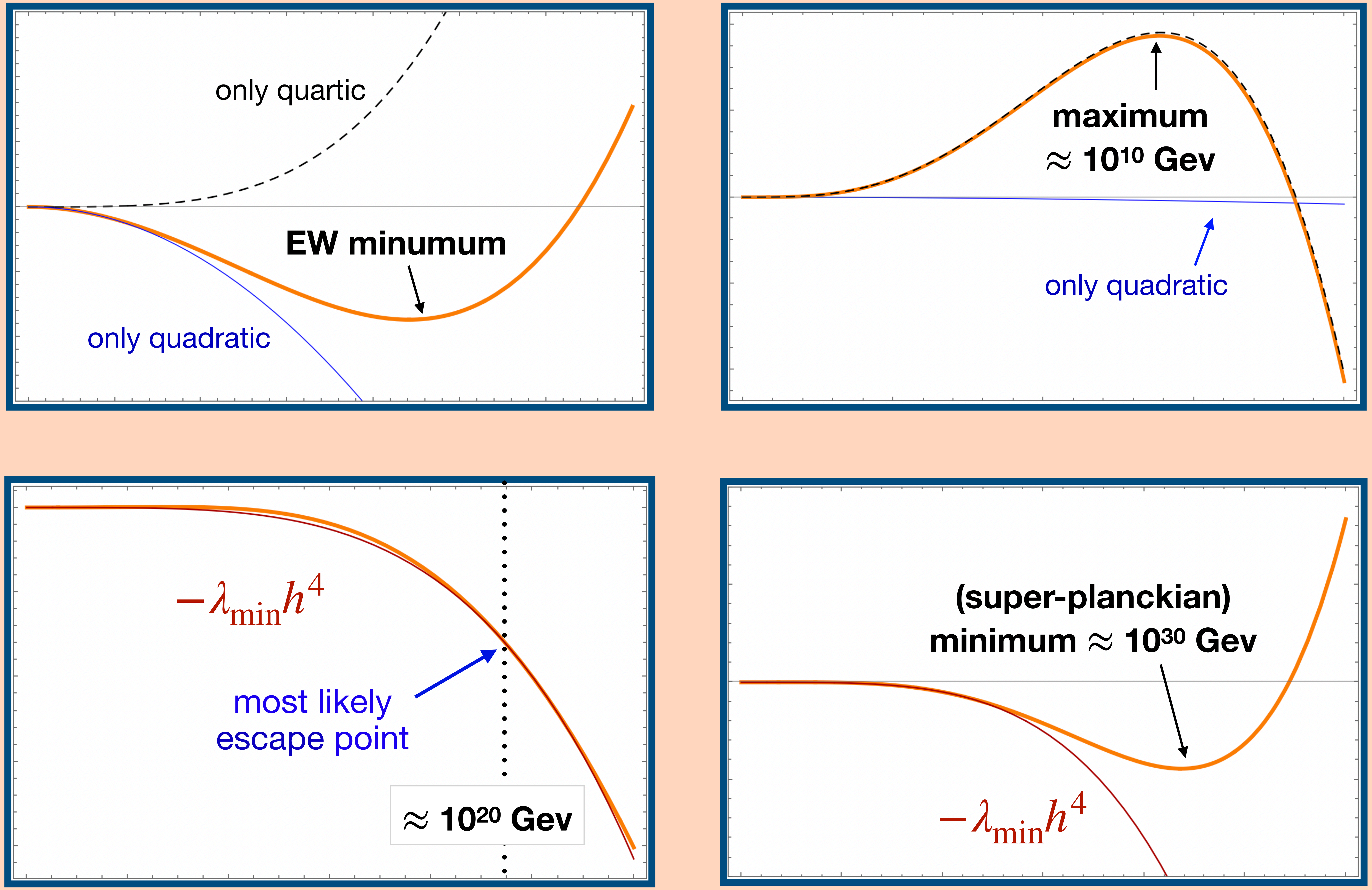


We use the NNLO formulae for SM parameters at $\mu = M_t$ from **Buttazzo et al (2013)**, with updated input parameters

From PDG 2024

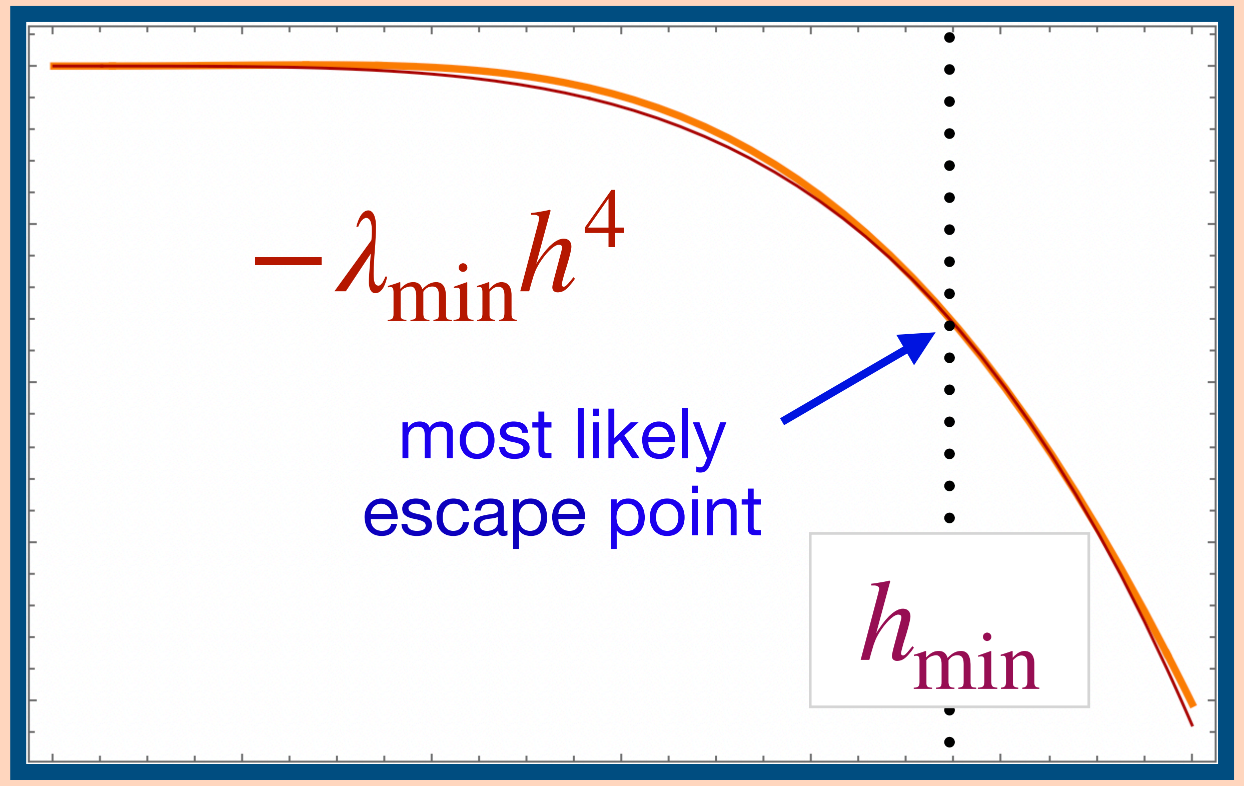
$$M_t = 172.57 \pm 0.29 \text{ GeV}$$
$$\alpha_3(M_Z) = 0.1180 \pm 0.0009$$
$$M_h = 125.20 \pm 0.11 \text{ GeV}$$

Higgs (effective) potential



Decay rate

Estimate



$h_{min} \approx 10^{20}\,\mathrm{GeV}$

$\lambda_{min} \approx 0.01$

- Decay rate mainly governed by the minimum of $\lambda(h)$
- Tunnelling exponent given by $B = 8\pi^2 / 3\lambda_{min}$

$$\gamma_{bubble} \approx h_{min}^4 e^{-B} \sim 10^{-900}\, T_U^{-4}$$

- Decay would happen mostly by nucleation of bubbles $\bar{h}(r) = \dfrac{\sqrt{\lambda_{min}^{-1}}\, R^{-1}}{1 + (\frac{r}{R})^2}$

$R \approx h_{min}^{-1}$

Decay rate

Towards a better precision

- Potential V(h) was extrapolated with RG equations at NNNLO precision
- However we only gave a rough estimate of the `prefactor`
- It can be obtained as a path integral over fluctuations about the most probable escape history, or Bounce, which is a saddle point of the action
- In a Gaussian approximation of the path integral, this is equivalent to account for one-loop effects
- Work in Euclidean signature…

Decay rate

The `prefactor`

S'' is a differential operator, e.g. for scalars $\Box + \lambda \bar{h}^2$

$$S(\bar{h}+\phi) = S(\bar{h}) + \frac{1}{2}\phi S'' \phi + \ldots$$

linear term absent as $\bar{h}$ solves the equations of motion

$$\boxed{S(\bar{h}) \equiv B}$$

$$\gamma_{\text{bubble}} = e^{-B}\frac{B^2}{4\pi^2}\left(\frac{\text{sDet}'S''}{\text{sDet}\,S_0''}\right)^{-\frac{1}{2}}$$

comes from expanding the action about the false vacuum solution

Computing the prefactor

$$\gamma_{\text{bubble}} = e^{-B}\frac{B^2}{4\pi^2}\left(\frac{s\text{Det}' S''}{s\text{Det}\, S_0''}\right)^{-\frac{1}{2}}$$

- Determinant of a differential operator is given by the product of eigenvalues
- On general grounds $[S'', J_{\mu\nu}]$ because of SO(4) invariance of S and $\bar{h}(\mathbf{x}, t)$
- S'' acts on scalars like h, spinors like the top, and gauge bosons Z_μ and $W^\pm_\mu$
- To diagonalise S'', we need to understand SO(4) rep. theory of these fields

Spherically symmetric operators

- Similar (no, identical!) to the problem of diagonalising a hamiltonian for a point particle in a radial potential V(r)
- Basis of spherical harmonics Y_{lm} if the particle is a scalar, or spin-orbit effects can be neglected
- Different angular basis if the particle has spin $\frac{1}{2}$ and spin-orbit effects are important (Dirac equation)
- <u>In general only total</u> $J_{\mu\nu}$ <u>and not</u> $L_{\mu\nu}$ <u>commute with the hamiltonian</u> $(L^2$ ✅ $)$

Spherically symmetric operators

$$\partial^2 + \lambda\,\overline{h}^2(r)$$ ← **Higgs sector**

$$\gamma_\mu \partial_\mu + y_t\,\overline{h}(r)$$ ← **Top**

Gauge bosons coupled to would-be-NGB (ξ=1 background gauge)

$$\begin{pmatrix} \left(\partial^2 + g^2\overline{h}^2(r)\right)\delta_{\mu\nu} & 2g\hat{x}_\mu \overline{h}'(r) \\ 2g\hat{x}_\nu \overline{h}'(r) & \partial^2 + (g^2+\lambda)\overline{h}^2(r) \end{pmatrix}$$

Reduction to radial differential operators

3 angles in 4 Euclidean dimensions

$$\mathcal{O}\Big(c(r)Y_{j\sigma}(\vartheta)\Big) = Y_{j\sigma}(\vartheta)\ \mathcal{O}_j\Big(c(r)\Big)$$

Reduced radial operator depends only on j and not on polarisation σ

E.g. for scalars: $\left(\partial^2\right)_j \equiv \partial_r^2 + \frac{3}{r}\partial_r - \frac{j(j+2)}{r^2}$

Reduction to radial differential operators

$$\boxed{\mathcal{O}\Big(c(r)Y_{j\sigma}(\vartheta)\Big) = Y_{j\sigma}(\vartheta)\ \mathcal{O}_j\Big(c(r)\Big)}$$

True for any operator with the property $[\mathcal{O}, J_{\mu\nu}] = 0$
if one chooses the proper angular basis

Angular basis for all spins

- Focus on the angular part, i.e. consider fields on the three-sphere
- They transform under rotations as infinite dimensional representations

$$\phi_i(\theta) \to \rho_{ij}(R)\, \phi_j\left(R^{-1}\theta\right)$$

- Finite-dimensional rep. ρ depends on the `spin` of the field

Angular basis for all spins

Peter-Weyl theorem

- Unitary representations of compact groups admit a decomposition into finite dimensional irreducible representations
- Spherical harmonics for scalars in 3 dimensions $\phi(\theta, \phi) = \sum_{j,m} c_{jm} Y_{jm}(\theta, \phi)$
- More abstractly

$$V_\phi \simeq \bigoplus_{j=0}^{\infty} \mathbf{j}$$

Vector space of scalar fields on the 2-sphere

(2j+1)-dimensional rep. of SO(3)

spin 1/2

$$V_\psi \simeq \bigoplus_{j=0}^{\infty} \left(\mathbf{j} + \tfrac{1}{2}\right)$$

Angular basis for all spins

Peter-Weyl theorem for SO(4)

- Decompose S'' in (Euclidean) four-space with r=1
- Irreps. of SO(4) are labelled by two half-integers [SO(4) $\simeq$ SO(3) $\times$ SO(3)]

$$V_\phi \simeq \bigoplus_{j=0}^{\infty} \left(\frac{j}{2}, \frac{j}{2}\right)$$

Vector space of scalar fields on the 3-sphere

Corresponds to ``hyper-spherical harmonics`` basis

Angular basis for all spins

Construction

- Dirac spinors like the top are understood as the tensor product

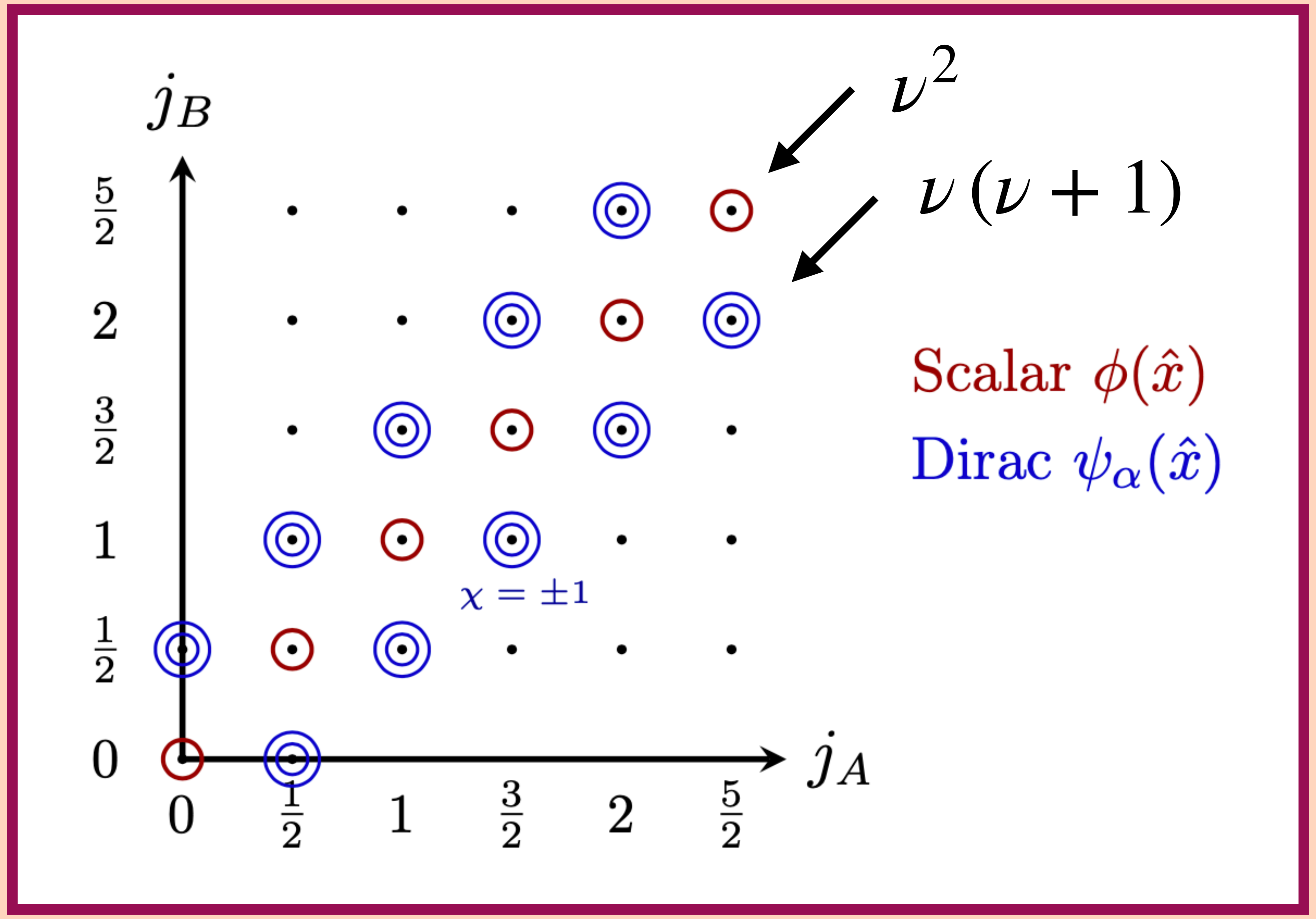
$$V_{\text{Dirac}} \simeq \left[\left(\frac{1}{2},0\right) \oplus \left(0,\frac{1}{2}\right)\right] \otimes V_\phi$$

- Similarly for gauge bosons, which are vector fields

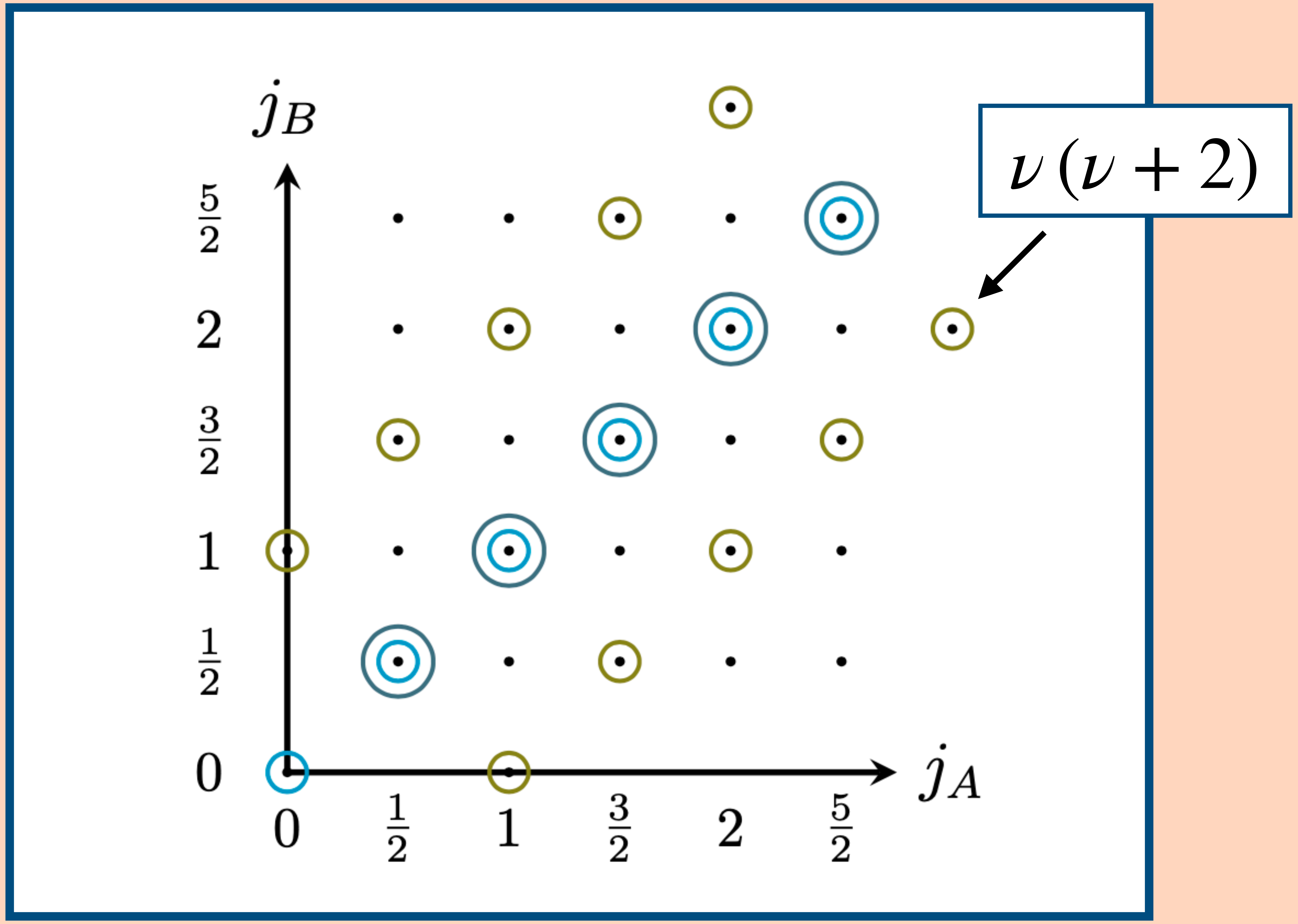
$$V_{A_\mu} \simeq \left(\frac{1}{2},\frac{1}{2}\right) \otimes V_\phi$$

Angular basis for all spins

Construction



$A_\mu(\theta)$

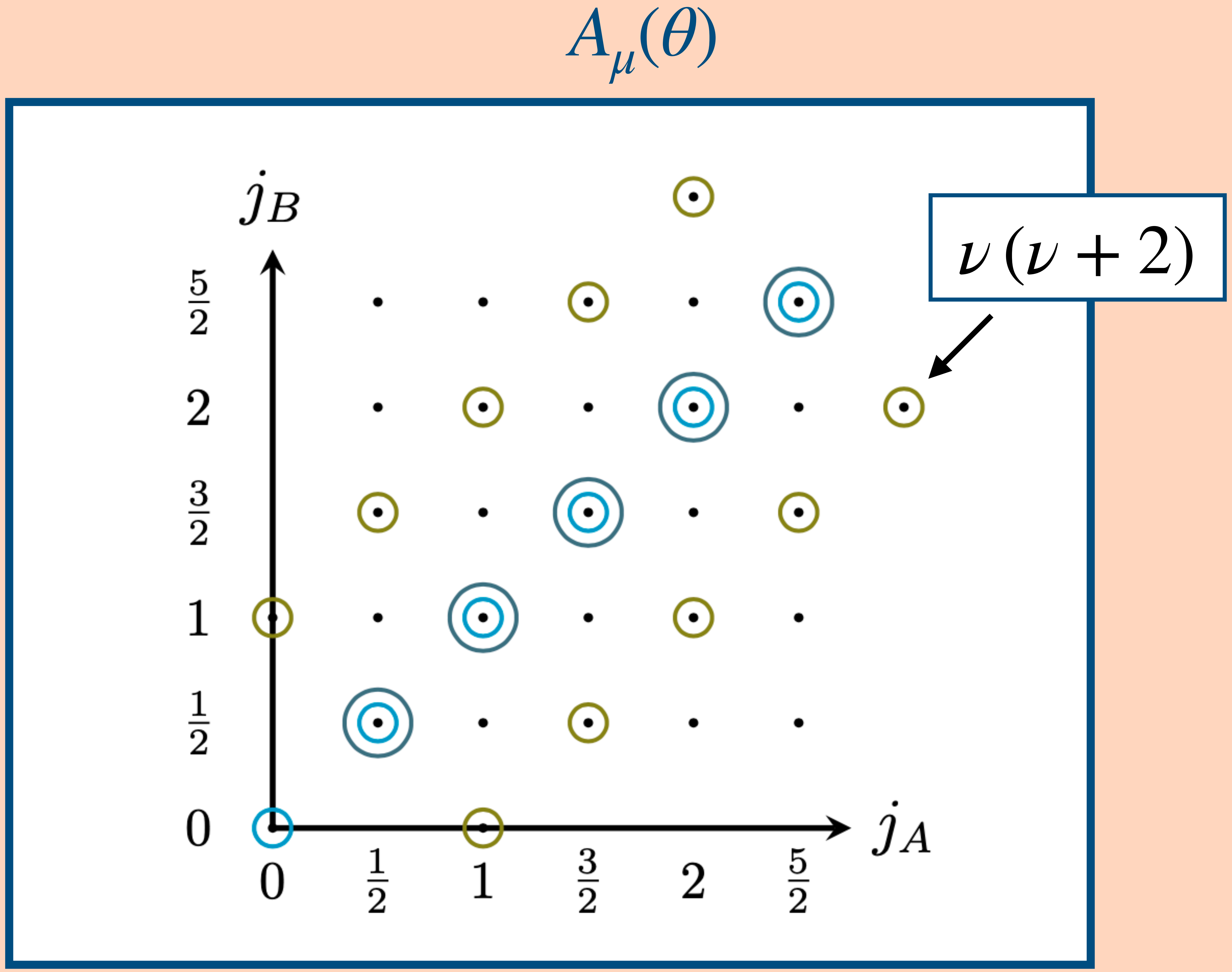


Easy to count number of independent polarisations ≡ dimension of SO(4) multiplet = $(2j_A+1)(2j_B+1)$

Transverse modes

- Number of independent transverse polarisations was wrong in previous work on the topic
- It enters in γ_{bubble} because of the prefactor

$$\det S'' = \prod_{\text{sectors}} \left[\prod_{\nu} \left(\det S''_{\nu} \right)^{d_\nu} \right]$$



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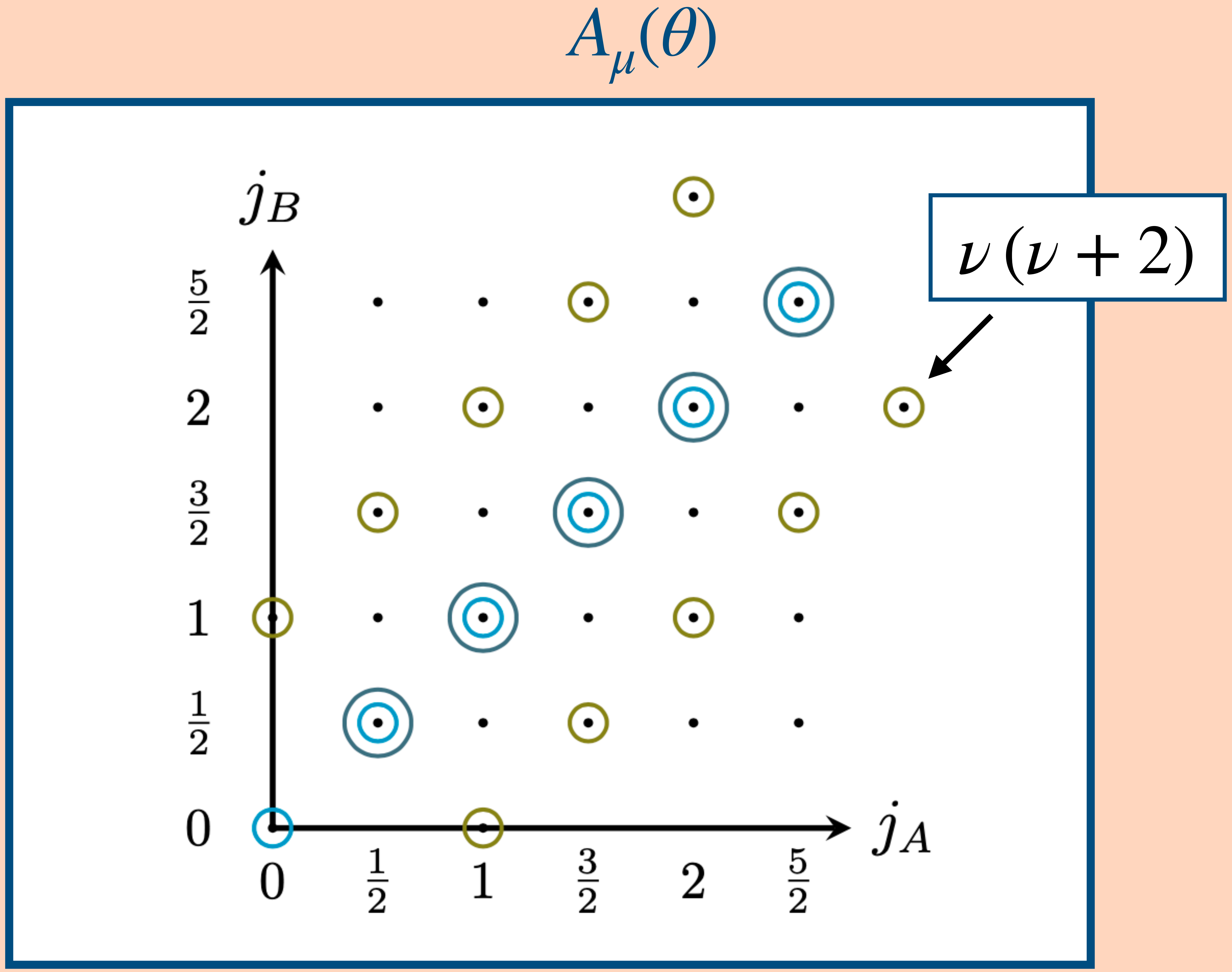
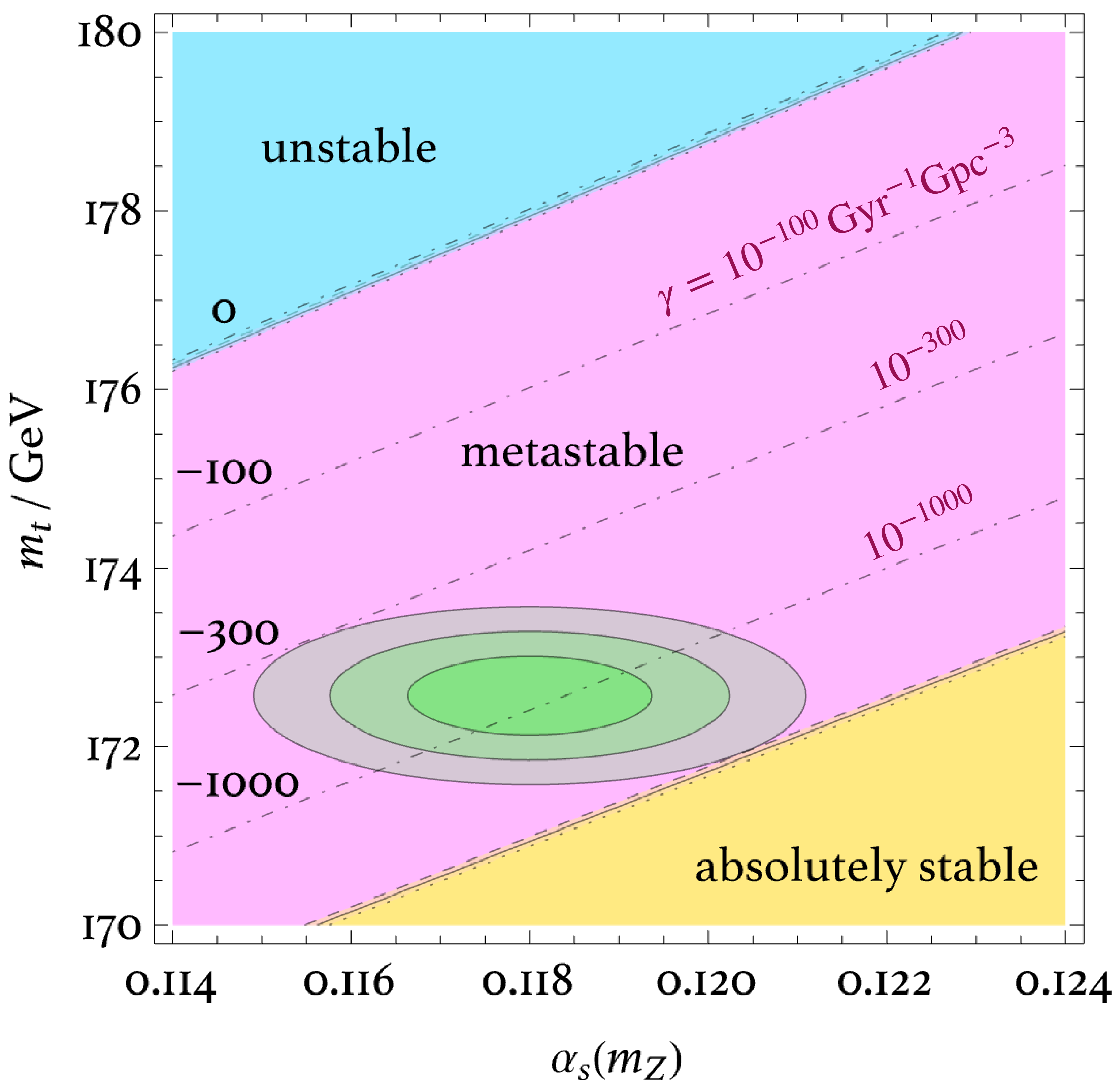


Revising γ_{EW}

- We recomputed γ_{EW} with updated SM parameters and revised d_ν^T
- No room for absolute stability within 3σ
- Gauge sector changes by 6%
- Main change in γ_{EW} comes from experimental side

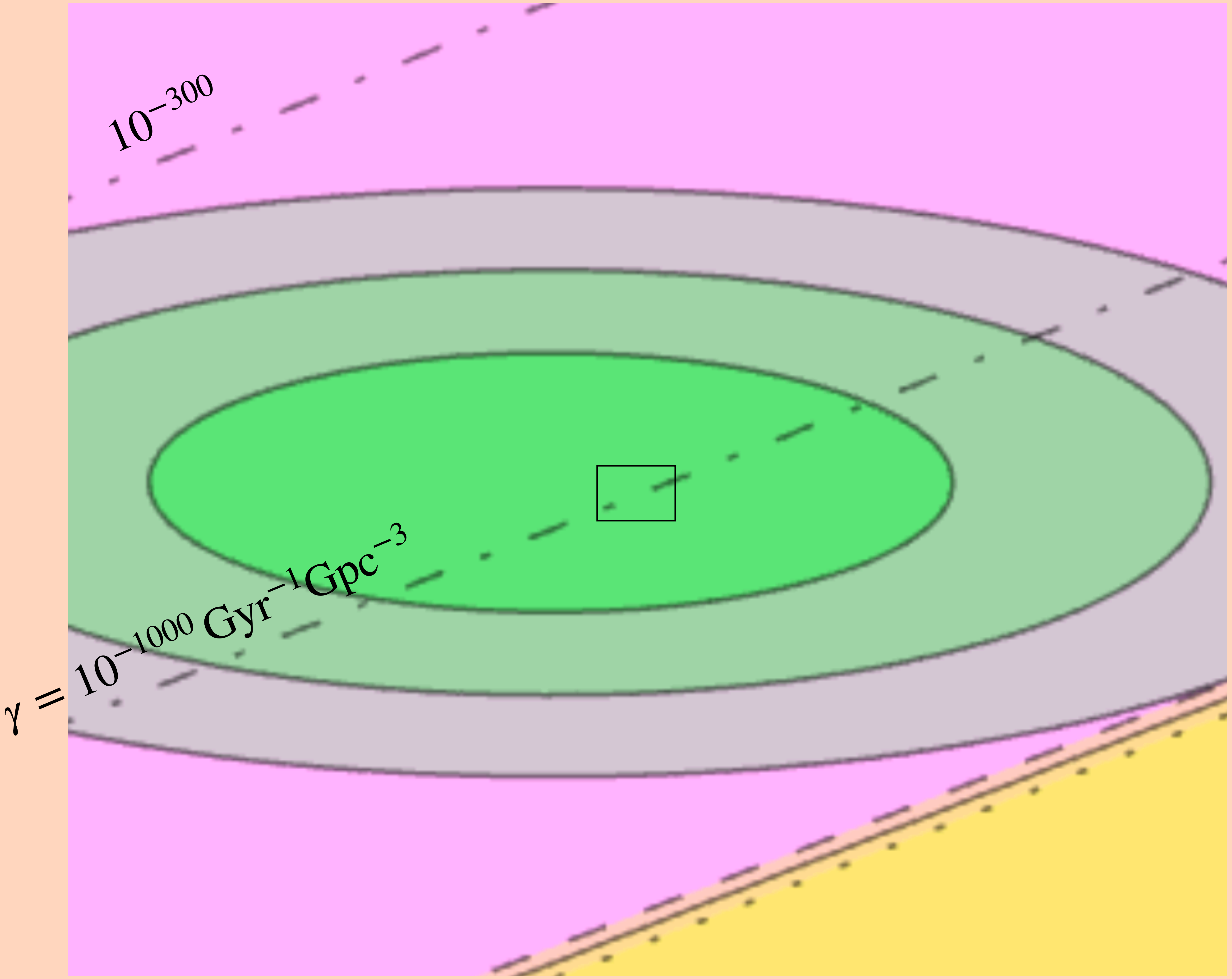
$$\log_{10}\left(\gamma_{\mathrm{EW}}\,\mathrm{Gyr}\,\mathrm{Gpc}^3\right) = -871^{+35+175+209}_{-37-253-330}$$

m_{Higgs} m_{top} α_s



+ 6 from corrected multiplicity

Revising γ_{EW}



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