Detectors and moving boundaries

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Plan

Part A

- Historical remarks
- Localised quantum system interacting with a non-localised quantum field: detector!
- Time dependent systems: switching interaction on and off
- Six examples 1977–2025

Part B

At most one of:

- Vacua in locally de Sitter cosmologies, and how to distinguish them
- Temperature of acceleration, in spacetime and in the laboratory

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 - (e.g. moving boundary)
 - Theory: Moore 1970 Observation: Wilson et al 2011 Lähteenmäki et al 2013 (electrical simulation of motion)



Image: Johansson et al 2009

History

Cosmological 'particle creation'

Gravitational collapse

Late time outgoing thermal flux

 $T_H = \frac{1}{8\pi M}$

Parker 1968...

Hawking 1974, 1975



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Receding mirror in Minkowski

Tailored late time trajectory ⇒ Hawking-type radiation from the mirror Parker 1968...

Hawking 1974, 1975



Davies and Fulling 1977



Input

- "vacuum plus excitations" (old school)
- relies on notion of "positive frequency"
 - $\Rightarrow\,$ different positive frequency choices related by Bogoliubov transformations
- typically employs spread-out Fourier modes
 - \Rightarrow localising a "particle"?

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W. G. Unruh, public communication, 1976-present

Unruh(1976)-DeWitt(1979)

Quantum field

- D spacetime dimension
- ϕ real scalar field
- $|\Psi
 angle$ field initial state

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Two-state detector (atom)

- $\|0\rangle\!\rangle$ state with energy 0
- $\|1\rangle$ state with energy *E*
- $x(\tau)$ detector worldline,

 τ proper time

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Interaction

$$H_{\rm int}(\tau) = c \chi(\tau) \mu(\tau) \phi(\mathbf{x}(\tau))$$

- c coupling constant
- χ switching function, C_0^{∞} , real-valued
- μ detector's monopole moment operator

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$$F_{\chi}(E) = \int \mathrm{d}\tau' \mathrm{d}\tau'' \,\mathrm{e}^{-iE(\tau'-\tau'')} \,\chi(\tau') \chi(\tau'') \,W(\tau',\tau'')$$

 $W(\tau',\tau'') = \langle \Psi | \phi \big(\mathsf{x}(\tau') \big) \phi \big(\mathsf{x}(\tau'') \big) | \Psi \rangle \quad \text{Wightman function}$

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$$\begin{split} \mathcal{W}(\tau',\tau'') &= \langle \Psi | \phi \big(\mathsf{x}(\tau') \big) \phi \big(\mathsf{x}(\tau'') \big) | \Psi \rangle & \text{Wightman function} \\ & \text{(distribution)} \end{split}$$

Response function

$$F_{\chi}(E) = \int \mathrm{d}\tau' \mathrm{d}\tau'' \,\mathrm{e}^{-iE(\tau'-\tau'')} \,\chi(\tau')\chi(\tau'') \frac{W(\tau',\tau'')}{(\text{distribution})}$$

- $\mathit{W}:$ field initial state $|\Psi\rangle$ and detector motion
- χ : detector switching
- E: detector energy gap
 - E > 0: excitation
 - E < 0: de-excitation

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Theorem (Hörmander 1971)

If $|\Psi\rangle$ is Hadamard and $x(\tau)$ is smooth, then $W(\tau', \tau'')$ is a well-defined distribution on $\mathbb{R} \times \mathbb{R}$

Corollary

If χ is C_0^{∞} and $x(\tau)$ is smooth, $F_{\chi}(E)$ is well defined!

$$F_{\chi}(E) = \int d\tau' d\tau'' e^{-iE(\tau'-\tau'')} \chi(\tau') \chi(\tau'') \frac{W(\tau',\tau'')}{\text{(distribution)}}$$

Switching effects (artefacts?) intertwine with effects due to $|\Psi\rangle$, motion, energy gap

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Stationary motion: $W(\tau', \tau'') = W(\tau' - \tau'', 0)$

- uniform linear acceleration in Minkowski vacuum (Unruh effect)
- static observer outside Schwarzschild black hole,...

Long time limit \rightarrow transition rate:



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Long time limit \rightarrow transition rate: yet within linear perturbation theory!





 $\delta \rightarrow 0$? Switching effects grow...

Hodgkinson and JL 2012



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$$D = 2, 3:$$
 $F_{\chi}(E)$ finite as $\delta \rightarrow 0$
 $D = 4, 5:$

D = 6:



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 $D = 2, 3: \quad F_{\chi}(E) \text{ finite } \text{ as } \delta \to 0$ $D = 4, 5: \quad F_{\chi}(E) \to \infty \quad \text{but } \quad \frac{d}{d\tau}F_{\chi}(E) \text{ finite } \text{ as } \delta \to 0$ **Instantaneous transition rate still well defined** (measurable in an ensemble of ensembles)

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Instantaneous transition rate divergent (for generic states/trajectories)

Remarks

- $W(\tau', \tau'')$ sometimes infrared divergent (eg D = 2 massless) Cure: $H_{int}(\tau) \propto \frac{d}{d\tau} \phi(\mathbf{x}(\tau))$ (but more singular at sharp switching)
- ► Fermions: $H_{int}(\tau) \propto \overline{\psi(x(\tau))}\psi(x(\tau))$ quadratic Takagi 1986 ⇒ more singular JL and Toussaint 2016
- Higher-order perturbation theory? More singular! Needed in entanglement harvesting by multiple detectors
- Spatially smeared detectors? Less singular!
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- Moving cavities Bruschi et al 2012, Lorek et al 2015,...
- Complementary track: open quantum systems approach Long time, weak coupling, Markovian approximation ⇒ Sharp switch-off well defined (after renormalisation) Benatti and Floreanini 2004,...
- ▶ Pointlike two-state detector \rightarrow pointlike SHO \Rightarrow Gaussian nonperturbative methods Lin and Hu 2007,...

Example: 1+1 receding mirror



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Example: 1+1 null shell collapse



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Juárez-Aubry and JL 2018



Massless field

Future:



Example: 1+1 null shell collapse

Late time state approaches the Unruh state on Kruskal (stationary outgoing flux) Juárez-Aubry and JL 2018



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Example: 1+1 Schwarzschild infall



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Vacua (stationary wrt Killing time)

- Boulware: Minkowski vacuum at infinity
- ► Hartle-Hawking-Israel: thermal equilibrium $T_{\infty} = 1/(8\pi M)$
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Outcomes (massless field)

- At infinity Boulware empty, HHI thermal, Unruh half-thermal
- ► HHI and Unruh:
 - (half-)thermality gradually lost during the fall
 - Horizon-crossing non-drastic
 - $\dot{F}_{\tau} \sim r^{-3/2}$ near the singularity

Conformal field, HHI state

Spinless Hodgkinson and JL 2012, Preciado-Rivas et al 2024



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 F_τ has discontinuous derivative ("glitch") when switch-off moment is on the lightcone of the switch-on moment

Happens already outside the horizon!

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Spinning Wang et al 2024

► HHI state has singularities beyond the inner horizon Steif 1994 ⇒ severe glitches there

Example: infall in a single-exterior BTZ hole Spadafora et al 2024



Example: infall in a single-exterior BTZ hole Spadafora et al 2024



Example: infall in a single-exterior BTZ hole Spadafora et al 2024



Infalling detector configurations on the geon:



New glitches after horizon-crossing



 \dot{F}_{τ} numerically difficult!







3+1 Schwarzschild infall (cont'd) Shallue and Carroll [arXiv:2501.06609]





3+1 Schwarzschild infall (cont'd) Shallue and Carroll [arXiv:2501.06609]





 $\Delta \lesssim M \text{ needed to resolve timescales near and beyond the horizon}$ $\Rightarrow F_{\chi} \text{ dominated by switching effects}$

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Notion of 'effective temperature' during the infall continues to be under debate!