Victoria University of Wellington

Te Whare Wānanga o te Ūpoko o te Ika a Maui

— Dynamical horizons —
Vietri sul Mare, Amalfi, Italy

Matt Visser

17–21 February 2025
Thursday 20 Feb; 11:30–13:30;
Friday 21 Feb; 10:00–11:00;
Llyods Baia.

Outline:

Figure: Il buco nero di Monticello

Introduction

Introduction:

- Basic orientation...
  - Some examples...
  - Horizon penetrating coordinates?
    (*Versus* horizon non-penetrating coordinates?)
- What sort of horizon?
  - Killing horizon?
  - Isolated horizon?
  - Event horizon?
  - Apparent horizon?
  - Trapping horizon?
  - Dynamical horizon?

  All of these have drawbacks of one form or another...
- Carter–Penrose diagrams: “Nice” *versus* “not-so-nice” slices...
- Dynamical models *versus* kinematical models?
- Convergence conditions *versus* energy conditions?
- Raychaudhuri equation — which version?
- Semiclassical quantum physics *versus* full quantum physics?
- Summary

Reference materials

Books/Reviews:

- Hawking and Ellis — The large scale structure of spacetime.
- Padmanabhan — Gravitation
- Hayward — Black Holes : New Horizons
- Ashtekar and Krishnan —
  Isolated and dynamical horizons and their applications
  Living Rev.Rel. 7 (2004) 10 — e-Print: gr-qc/0407042 [gr-qc]
  (New review on arXiv this week.)
- Gourgoulhon —
  A 3+1 perspective on null hypersurfaces and isolated horizons
  Phys.Rept. 423 (2006) 159-294 — e-Print: gr-qc/0503113 [gr-qc]
- Booth — Black hole boundaries
  Can.J.Phys. 83 (2005) 1073-1099 — e-Print: gr-qc/0508107 [gr-qc]
- Hayward — Dynamics of black holes
  e-Print: 0810.0923 [gr-qc]

Selected articles:

- Hayward — Formation and evaporation of regular black holes
Phys.Rev.Lett. 96 (2006) 031103 — e-Print: gr-qc/0506126 [gr-qc]
- Hayward — General laws of black hole dynamics
Phys.Rev.D 49 (1994) 6467-6474
- Ashtekar and Krishnan — Dynamical horizons and their properties
Phys.Rev.D 68 (2003) 104030 — e-Print: gr-qc/0308033 [gr-qc]
- Andersson, Mars, and Simon —
Local existence of dynamical and trapping horizons
Phys.Rev.Lett. 95 (2005) 111102 — e-Print: gr-qc/0506013 [gr-qc]
- Nielsen and Visser — Production and decay of evolving horizons
CQG 23 (2006) 4637-4658 — e-Print: gr-qc/0510083 [gr-qc]
- Visser — Physical observability of horizons
Phys.Rev.D 90 (2014) 12, 127502 — e-Print: 1407.7295 [gr-qc]

Basic concepts

Basic concepts:

Definition

Black hole:

Any spacetime containing one or more horizons.

Standard idealized examples:

Schwarzschild, Reissner–Nordström, Kerr, Kerr–Newman.

But ultimately we need to consider more realistic astrophysical black holes.

Definition

Horizon:

Any effectively "one way" membrane in spacetime.

Typically associated with infinite gravitational redshift.

Need to go beyond idealized examples.

Ultimately we need to consider more realistic astrophysical horizons.

Basic concepts:

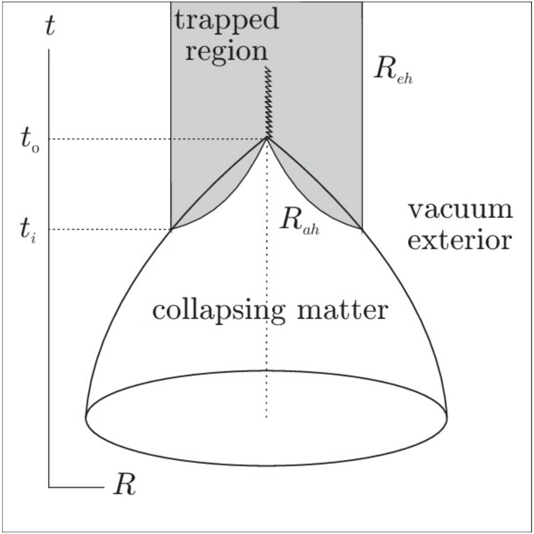


Figure: Schematic only — at least for now — many more details later...

Basic concepts:

- **Schwarzschild (Hilbert–Droste form, curvature coordinates)**:

$$\boxed{ds^2 = -\left(1 - \frac{2m}{r}\right) dt^2 + \frac{dr^2}{1 - 2m/r} + r^2 d\Omega^2.}$$

One horizon at the unique root of $(1 - 2m/r) = 0$:

$$\boxed{r_S = 2m.}$$

- The t coordinate is timelike only for $r > 2m$.
- The r coordinate is spacelike only for $r > 2m$.
- The t coordinate is spacelike for $r < 2m$.
- The r coordinate is timelike for $r < 2m$.
- The hypersurface $r = 2m$ is located at a coordinate singularity.
- Some of the nutters totally lose contact with reality at this point...

Basic concepts:

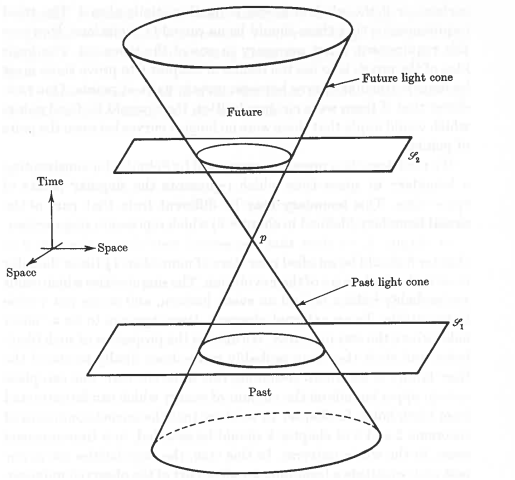


Figure: Locally the light cones always look like this...

Basic concepts:

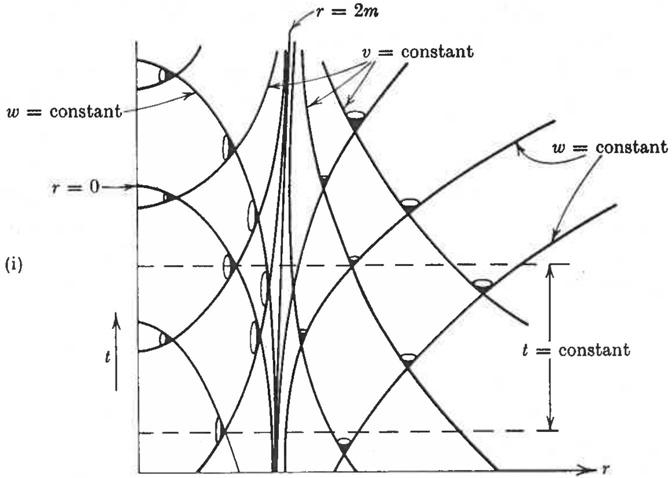


Figure: Schwarzschild light cones in Hilbert–Droste coordinates...
Note the cones flip by 90 degrees at $r = 2m$...
Note ingoing light rays pile up at $r = 2m$... (Horizon non-penetrating...)

- **Reissner–Nordstrom (curvature coordinates)**:

$$ds^2 = -(1 - 2m/r + Q^2/r^2)dt^2 + \frac{dr^2}{1 - 2m/r + Q^2/r^2} + r^2 d\Omega^2.$$

Two horizons at the roots of $(1 - 2m/r + Q^2/r^2) = 0$

$$r_\pm = m \pm \sqrt{m^2 - Q^2}.$$

- The t coordinate is timelike only for $r > r_+$ or $r < r_-$.
- The r coordinate is spacelike only for $r > r_+$ or $r < r_-$.
- The t coordinate is spacelike for $r \in (r_-, r_+)$.
- The r coordinate is timelike for $r \in (r_-, r_+)$.
- The hypersurfaces at $r = r_\pm$ are located at coordinate singularities.
- Some of the nutters totally lose contact with reality at this point...

- **Kottler (Schwarzschild–de Sitter; curvature coordinates)**:

$$ds^2 = -\left(1 - 2m/r - \frac{1}{3}\Lambda r^2\right) dt^2 + \frac{dr^2}{1 - 2m/r - \frac{1}{3}\Lambda r^2} + r^2 d\Omega^2.$$

Three (mathematical) horizons at roots of $(1 - 2m/r - \frac{1}{3}\Lambda r^2) = 0$.
At most two of these horizons are physical.
Write $\Lambda = 1/a^2$ and rewrite the cubic as $r^3 - 3ra^2 + 6ma^2$.
Then

$$r_\epsilon = 2a \sin\left(\frac{1}{3}\arcsin\left[\frac{3m}{a}\right] + \epsilon\frac{2\pi}{3}\right); \qquad \epsilon \in \{0, \pm 1\}.$$

- This process nicely generalizes...

Basic concepts:

- **Generic spherical symmetry (curvature coordinates)**:

$$ds^2 = -e^{-2\Phi(r)} \, f(r) \, dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2.$$

  Possibly many horizons r_{H_i} at roots of $f(r) = 0$.
  (Possibly zero horizons; a "compact object".)

- Eventually we will get around to adding time dependence:

$$ds^2 = -e^{-2\Phi(r,t)} \, f(r,t) \, dt^2 + \frac{dr^2}{f(r,t)} + r^2 d\Omega^2.$$

  Possibly many evolving horizons $r_{H_i}(t)$ at roots of $f(r,t) = 0$.
  (Possibly zero horizons; a "compact object".)

- Sometimes other coordinate systems are preferable...

- Pick a closed orientable spacelike 2-surface....
- Embedded in spacetime in a 2-sided fashion...
- Construct ingoing and outgoing null sheets...
- This is the usual case:

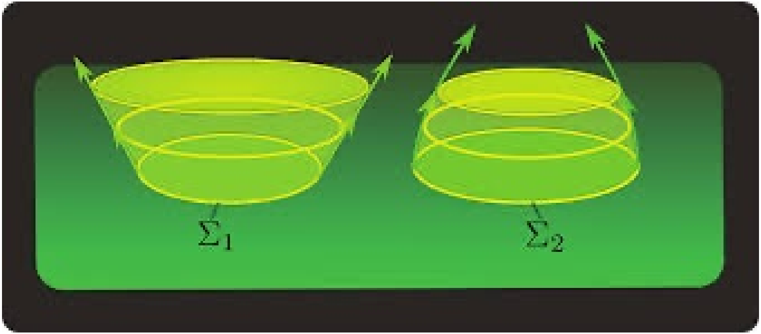


Figure: Non-trapped region — the usual case

Basic concepts:

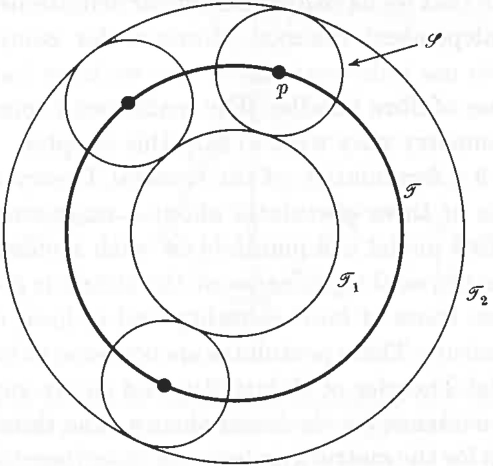


FIGURE 1. At some instant, the sphere $\mathscr{T}$ emits a flash of light. At a later time, the light from a point p forms a sphere $\mathscr{S}$ around p, and the envelopes $\mathscr{T}_1$ and $\mathscr{T}_2$ form the ingoing and outgoing wavefronts respectively. If the areas of *both* $\mathscr{T}_1$ and $\mathscr{T}_2$ are less than the area of $\mathscr{T}$, then $\mathscr{T}$ is a closed trapped surface.

- **Schwarzschild (Painlevé–Gullstrand form)**:

$$ds^2 = -dt^2 + \left(dr + \sqrt{2m/r}\,dt\right)^2 + r^2 d\Omega^2.$$

  - This is still Schwarzschild spacetime — the Ricci tensor is zero...
  - Note the spatial slices are flat...
  - Explicit coordinate transformation

$$dt_{PG} = dt_{HD} - \frac{\sqrt{2m/r}}{1-2m/r}\,dr.$$

  - Radial light rays: $ds^2 = 0$ implies $-dt^2 + (dr + \sqrt{2m/r}\,dt)^2 = 0$.
  - Thence

$$\frac{dr}{dt} = \pm 1 - \sqrt{2m/r}$$

  - "Outgoing" light rays dragged backwards ($dr/dt < 0$) once $r < 2m$.
  - Explicitly a trapped region...

- **Schwarzschild (Painlevé–Gullstrand form)**:

$$ds^2 = -dt^2 + \left(dr + \sqrt{2m/r}\,dt\right)^2 + r^2 d\Omega^2.$$

  - These Painlevé–Gullstrand coordinates are horizon-penetrating.
  - These Painlevé–Gullstrand coordinates (1922) pre-date Eddington's version (1924) of what are now called Kerr–Schild coordinates by at least 2 years.

$$ds^2 = -dt^2 + dr^2 + r^2 d\Omega^2 + \frac{2m}{r}(dr + dt)^2.$$

  - These Painlevé–Gullstrand coordinates (1922) pre-date what are now called Eddington–Finkelstein coordinates by some 36 years.

$$ds^2 = -(1 - 2m/r)dv^2 + 2\,dr\,dv + r^2 d\Omega^2.$$

  - The history is quite weird and twisty...

Basic concepts:

- You are just going to have to get used to dealing with multiple coordinate systems; they are just too useful.
- Changing coordinates will not change the physics — but **might** improve insight and understanding...
- Some of the nutters totally lose contact with reality at this point...
- Major dividing point: Horizon penetrating versus non-penetrating.
    - Coordinate systems with a diagonal metric (Hilbert-Droste, isotropic, tortoise) are not horizon penetrating and, (one way or another), are not great for dealing with horizons.
    - Horizon-penetrating coordinates (Painlevé–Gullstrand, Kerr–Schild, Eddington–Finkelstein) have a non-diagonal line element, (which sometimes makes computations messier), but one gets a much better intuition regarding horizons and trapped regions.

- **Schwarzschild (Painlevé–Gullstrand form)**:
  - **Rehash**:

$$ds^2 = -dt^2 + \left(dr + \sqrt{2m/r}\,dt\right)^2 + r^2 d\Omega^2.$$

  - Radial null curves: $ds^2 = 0$ implies $-dt^2 + \left(dr + \sqrt{2m/r}\,dt\right)^2$.
  - Thence

$$\frac{dr}{dt} = \pm 1 - \sqrt{2m/r}.$$

  - Ingoing light rays always have $dr/dt \leq -1 \leq 0$.
  - "Outgoing" light rays dragged backwards ($dr/dt < 0$) once $r < 2m$.
  - Explicitly a trapped region...
- Of course these notions generalize...

- **Schwarzschild (Kerr–Schild form)**:

$$ds^2 = -dt^2 + dr^2 + r^2 d\Omega^2 + \frac{2m}{r}(dr + dt)^2.$$

  - Radial null curves: $ds^2 = 0$ implies $-dt^2 + dr^2 + \frac{2m}{r}(dr + dt)^2 = 0$.
  - Thence

$$(dr + dt)\left(dr - dt + \frac{2m}{r}(dr + dt)\right) = 0.$$

  - Thence

$$\frac{dr}{dt} \in \left\{-1, \; \frac{1 - 2m/r}{1 + 2m/r}\right\}.$$

  - Ingoing light rays always have $dr/dt = -1$.
  - "Outgoing" light rays dragged backwards ($dr/dt < 0$) once $r < 2m$.
  - Explicitly a trapped region...
- Of course these notions generalize...

- **Schwarzschild (Eddington-Finkelstein form)**:

$$\boxed{ds^2 = -(1-2m/r)dv^2 + 2\,dr\,dv + r^2 d\Omega^2.}$$

  - Note v is a "null coordinate"...
  - Radial null curves: $ds^2 = 0$ implies $-(1-2m/r)dv^2 + 2\,dr\,dv = 0$.
  - Thence

$$\boxed{dv\,[2dr - (1-2m/r)dv] = 0.}$$

  - Thence

$$\boxed{dv = 0 \qquad \text{or} \qquad \frac{dr}{dv} = \frac{(1-2m/r)}{2}.}$$

  - Ingoing light rays always have $dr/dv = -\infty$.
  - "Outgoing" light rays dragged backwards ($dr/dv < 0$) once $r < 2m$.
  - Explicitly a trapped region...
- Of course these notions generalize...

Basic concepts:

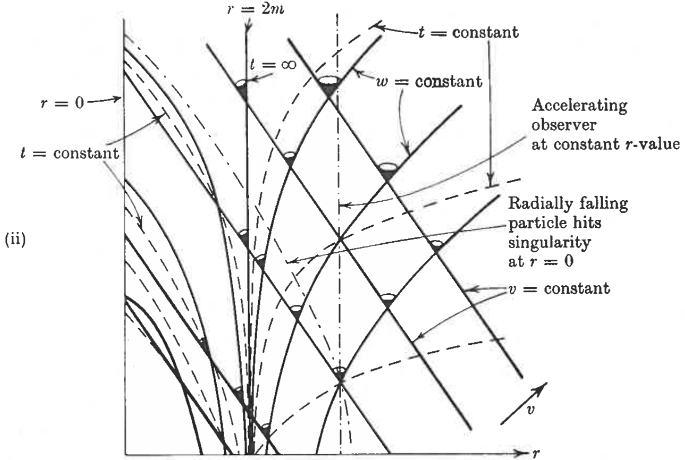


Figure: Schwarzschild light cones in Eddington–Finkelstein coordinates...
Note the cones tilt past the vertical at $r = 2m$...
Note ingoing light rays cheerfully cross $r = 2m$... (Horizon penetrating...)

Horizons

Killing Horizons:

- Essentially everything I have done so far refers to static spacetimes.
- Static spacetimes are particularly simple...
- (Stationary spacetimes are almost as good)...
- In static spacetimes all horizons are Killing horizons...
  - Time independent...
  - Specified by the asymptotically timelike time-translation Killing vector going null...

$$\boxed{g_{ab} K^a K^b = 0; \qquad K = K^a \partial_a = \partial_t; \qquad K^a = (1,0,0,0).}$$

  - The event/apparent/trapping/dynamical distinction is not yet important.
- What happens once we add time dependence?

- Consider these geometries (still spherically symmetric for simplicity):

$$\boxed{ds^2 = -e^{-2\Phi(r,t)} \left(1 - \frac{2m(r,t)}{r}\right) dt^2 + \frac{dr^2}{1-2m(r,t)/r} + r^2 d\Omega^2.}$$

$$\boxed{ds^2 = -c(r,t)^2 dt^2 + \left(dr + \sqrt{2m(r,t)/r}\; c(r,t)\; dt\right)^2 + r^2 d\Omega^2.}$$

$$\boxed{ds^2 = -(1-2m(r,v)/r)\; c(r,v)^2\; dv^2 + 2\,c(r,v)\, dr\, dv + r^2 d\Omega^2.}$$

- In all 3 cases:
  - To find the trapped regions just solve for $(1 - 2m(r,t)/r) < 0$, or $(1 - 2m(r,v)/r) < 0$.
  - **Bonus:** Check that $m(r,t)$, or $m(r,v)$, is just the Misner–Sharp quasi-local mass: $m(r,t) = \frac{r}{2}[1 - g^{ab}\,\partial_a r\,\partial_b r]$.

Apparent horizons:

- Trapped regions are relatively easy to find...
  (At least spherically symmetric trapped regions in spherical symmetry)
- Definition:
  Apparent horizons are just the boundaries of trapped regions.
- Apparent horizons can be determined by quasi-local measurements...
- (Not ultra-local measurements; you do need a finite-size laboratory...)
- **Warning**:
  - **Ultra-local**: Measurement at a point.
  - **Quasi-local**: Measurement using a finite-size laboratory.
- The word "local" is dangerously ambiguous.
- In spherical symmetry apparent horizons are easy to find

$$\boxed{\text{Solve:} \qquad 2m(r,t) = r \quad \text{to find} \quad r_H(t).}$$

- Why is this not the end of the story?

Apparent horizons:

- Even if the spacetime is spherically symmetric,
  you could go out of your way to cook up trapped surfaces
  that are not spherically symmetric.
- If you choose to do this, you can find lopsided trapped surfaces, leading to lop-sided appanent horizons, arbitrarily close to the origin, $r = 0$. (Where the radial null geodesics are otherwise well behaved.)
- Worse, or at least weirder, inner horizons are anti-trapped...
- Worse, if your spacetime is not spherically symmetric,
  all hell breaks loose...
- For these reasons apparent horizons are often deprecated....
- Still, in spherical symmetry with spherically symmetric trapped regions, they cover almost all the relevant physics.
- Even Stephen Hawking has been known to suppress technical quibbles and for pedagogy talk about "long-lived apparent horizons".

Event horizons:

- Event horizons in contrast require extremely strong global assumptions to even define them, let alone detect them...
- You need an asymptotically flat spacetime...

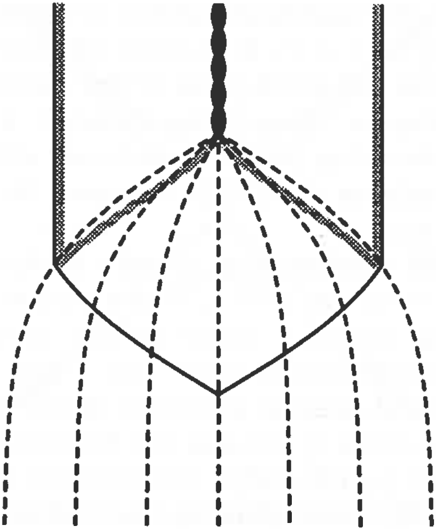
$$\boxed{\{i^-, \mathcal{J}^-, i^0, \mathcal{J}^+, i^+\}}$$

- At the very least you need to be able to define $\mathcal{J}^+$.
- The event horizon is then the boundary of the region from which future-directed null geodesics cannot reach $\mathcal{J}^+$.
- This implicitly requires teleology.
- This implicitly requires an immortal omniscient super observer.
- Event horizons are simply not physics.
- No finite-resource physicist or astronomer (finite time, not immortal; finite space, not omniscient) will ever be able to detect an event horizon, not even in principle.
- That's why we got rid of the luminiferous aether...
- Why bother? Great for proving mathematical theorems...

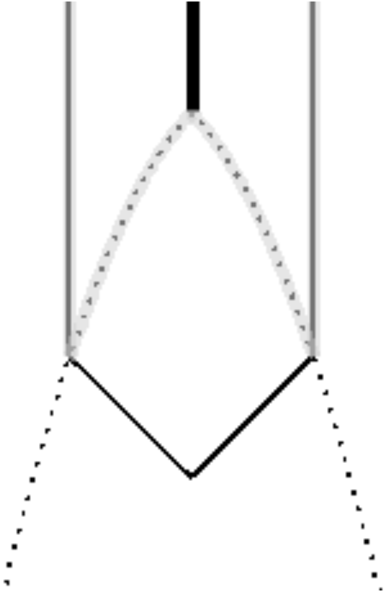
Apparent/Event horizons:

- Observation:
  - If accretion dominates over Hawking radiation,
    then the event horizon will lie slightly outside the apparent horizon.
  - If Hawking radiation dominates over accretion,
    then the event horizon will lie slightly inside the apparent horizon.
  - In the static limit the event/apparent horizons coincide.
- Remember:
  - Geodesics are an approximation applicable to the test particle limit...
  - If a finite-mass object falls into a black hole,
    then you *cannot* use the geodesic equations...
  - If a finite-mass object falls into a black hole, then once it is close enough, the event horizon will rise up to engulf it.

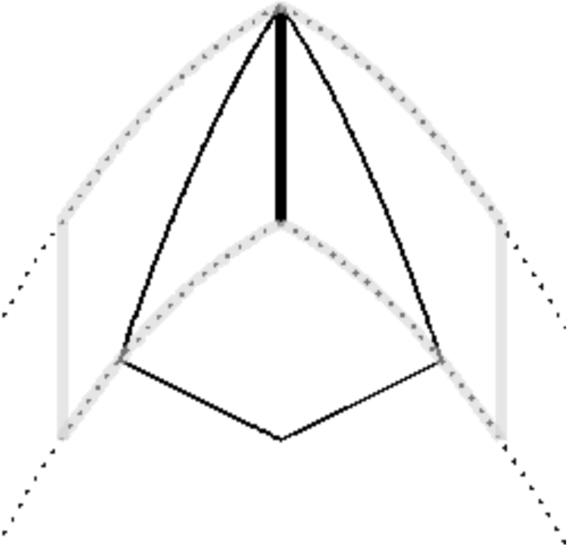
Apparent/Event horizons: Collapse and formation.



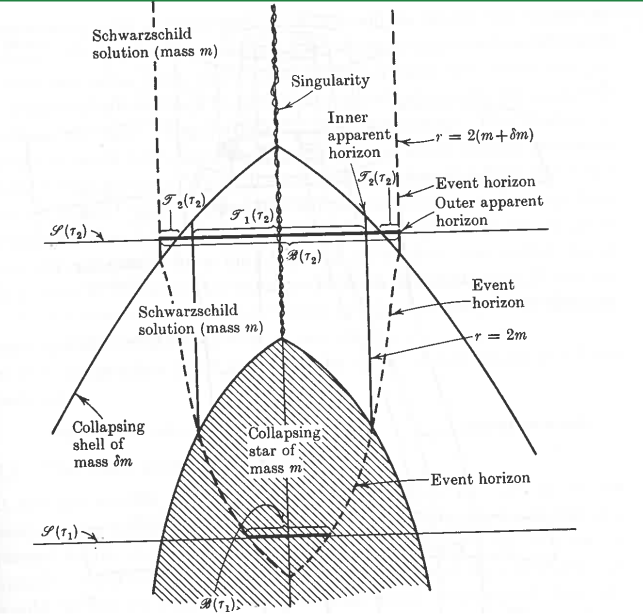
Apparent/Event horizons: Collapse and formation.



Apparent/Event horizons: Collapse and formation.



Apparent/Event horizons: Collapse and formation.



Apparent/Event horizons:

- You can see that both event and apparent horizons have their quirks...
- Event horizons can form in Riemann flat portions of spacetime...
  Zero gravity at formation; depends on something in the future...
- But at least the event horizons are future directed null surfaces...
- Apparent horizons can jump backwards and forwards in time...
- But at least the apparent horizons require strong gravitational "potential":

$$\boxed{(\text{potential}) := \frac{m(r,t)}{r} = \frac{1}{2}.}$$

- Neither event nor apparent horizons require strong gravitational fields:

$$\boxed{\nabla(\text{potential}) = (\text{arbitrarily small}).}$$

- Is there anything better we can do?

Isolated horizons:

Definition

Isolated horizon:
A "locally Killing" horizon — the black hole is taken (at least temporarily) to be in equilibrium with its environment.

- There is locally a hypersurface orthogonal Killing vector, defined on some finite region of the spacetime.
- This local Killing vector is timelike above the horizon, and spacelike below the horizon.
- No global Killing vector need exist.
- In particular defining surface gravity is rather indirect and tricky.
    - Pick a simple normalization for ingoing radial null curves: $\theta_k = 2/r$.
    - Enforce $g_{ab}\, k^a\, l^b = -1$.
    - Define $l^b\, \nabla_b l^a = \kappa\, l^a$.
- So the geometry is locally Schwarzschild/Reissner–Nordström, or some variant thereof.

Isolated horizon:

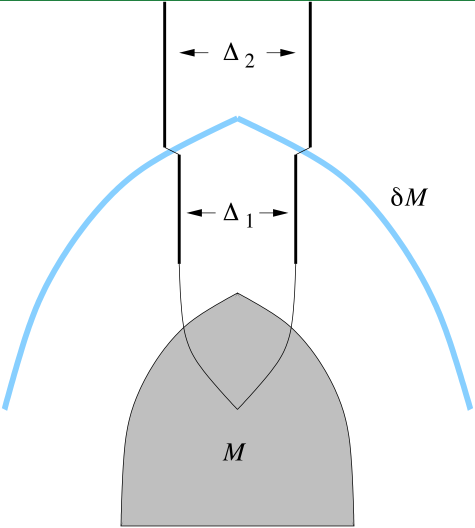


Figure: Two isolated horizons... (not really to scale)... not entirely accurate...

Trapping horizons:

Definition

Trapping horizon (outer):
Smooth 3 dimensional submanifold of spacetime,
that can be foliated by a family of spacelike 2-surfaces:

- Expansion θ_l of the outgoing null normal l^a vanishes.
- Expansion θ_k of the ingoing null normal k^a is negative.
- Derivative of outgoing expansion along ingoing null vector is negative.

- No real need to specify ahead of time whether the 3-surface is timelike/null/spacelike.
    - Spacelike $\Longleftrightarrow$ accretion dominated...
    - Null $\Longleftrightarrow$ equilibrium (isolated horizon)...
    - Timelike $\Longleftrightarrow$ Hawking evaporation dominated...

Trapping horizons:

Technical details:

- Projection tensor:

$$h_{ab} = g_{ab} + \frac{(k_a l_b + l_a k_b)}{(g_{cd} k^c l^d)}$$

$$h_{ab}\, g^{cd}\, h_{de} = h_{ae}$$

- Expansions:

$$\theta_k = h_{ab}\, \nabla^a k^b; \qquad \theta_l = h_{ab}\, \nabla^a l^b;$$

$$\theta_k < 0; \qquad \theta_l = 0.$$

- Outer trapping horizons have

$$k^a \nabla_a \theta_l < 0.$$

- Inner trapping horizons flip this sign

$$k^a \nabla_a \theta_l > 0.$$

- No *a priori* need to restrict attention to spherical symmetry.

- But in spherical symmetry it does simplify:
- Find the outgoing and ingoing radial null trajectories,

$$\left(\frac{dr_\pm}{dt}\right)_{(r,t)}.$$

- Demand:

$$\left.\frac{dr_+}{dt}\right|_H = 0; \qquad \left.\frac{dr_-}{dt}\right|_H < 0; \qquad \left[\frac{d}{dr}\left(\frac{dr_+}{dt}\right)\right]_H > 0.$$

- This can be reduced to a simple grad-student algorithm...

Dynamical horizons:

- Dynamical horizons are closely related to trapping horizons.

Definition

Dynamical horizon:
Smooth 3 dimensional **spacelike** submanifold of spacetime,
that can be foliated by a family of spacelike 2-surfaces:

- Expansion θ_l of the outgoing null normal l^a vanishes.
- Expansion θ_k of the ingoing null normal k^a is negative.

- This definition presupposes that accretion dominates...
- This definition excludes isolated or Killing horizons...
- This definition excludes event horizons...
- No condition on the derivative of the outgoing expansion along the ingoing null vector...
- Cannot (without more structure) distinguish inner versus outer...

Horizons — Summary:

- There are many subtly different notions of black hole horizon floating around in the literature...
- (Non-expanding horizons, marginal horizons, chronology horizons...)
- My personal favourites:
  - When full generality is required, use trapping horizons....
  - When simplicity and clarity is required, restrict attention to spherically symmetric spacetimes with spherically symmetric foliations, then trapping horizons simplify tremendously ... (You can then largely get away with using apparent horizons...)
- My personal unfavourite:
  - Event horizon — teleological — not physics — requires immortal omniscient super observers...

Carter–Penrose diagrams

- Carter introduced the notion of conformal causal diagrams, and Penrose popularized their use.
- Conformal transformations $g_{ab} \to \Omega^2(x)\, g_{ab}$ preserve the light-cone stucture.
- Conformal transformations $g_{ab} \to \Omega^2(x)\, g_{ab}$ do not preserve the volume, and can squeeze an entire (1+1) slice of spacetime onto a finite sheet of paper.
- Convention: Light rays travel at $\pm 45^\circ$...
  (otherwise it's not called a Carter–Penrose diagram...)
- Some examples (and cautions) below...

Carter–Penrose diagrams:

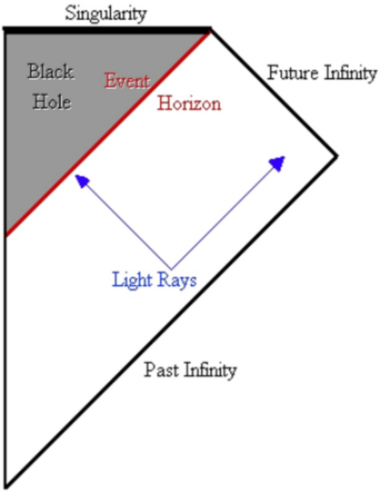


Figure: Classical collapsed object in asymptotically flat spacetime...

Carter–Penrose diagrams:

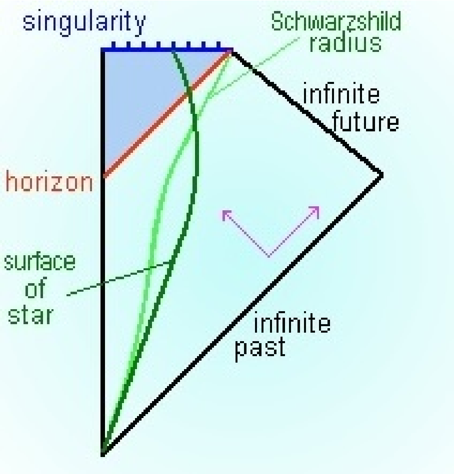


Figure: Classical collapsing object in asymptotically flat spacetime...
Where is the apparent horizon?

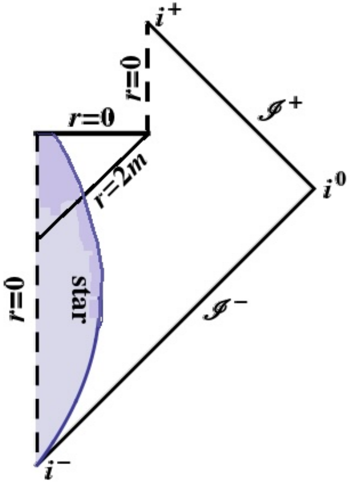


Figure: Collapse followed by Hawking evaporation...
Where is the apparent horizon? What happens at the endpoint?

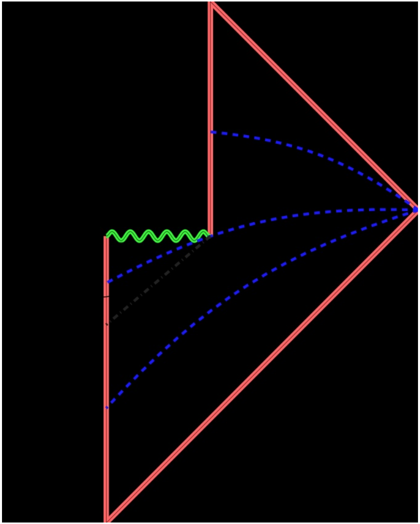


Figure: Collapse followed by Hawking evaporation...
Where are the horizons? What happens at the endpoint?

Carter–Penrose diagrams:

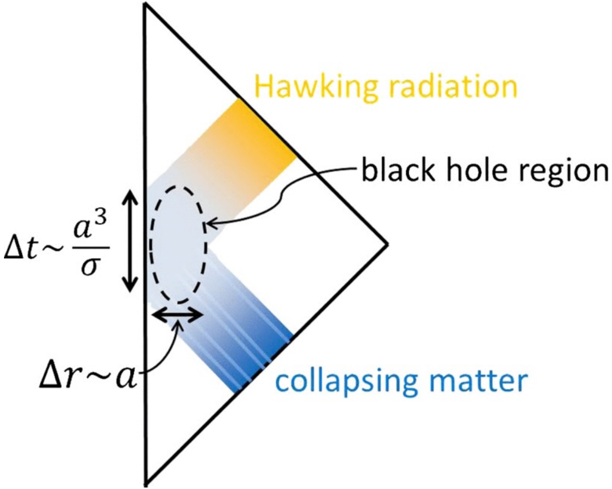


Figure: Collapse and evaporation without strict event horizons...
Where are the trapping horizons?

Carter–Penrose diagrams:

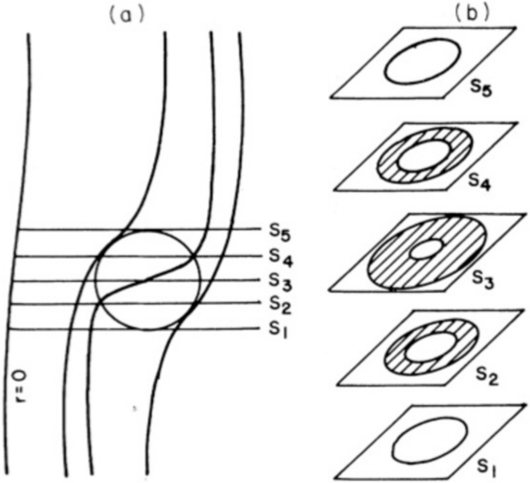


Figure: Collapse without singularities... Where are the horizons?
(Bergmann–Roman 1983)

Carter–Penrose diagrams:

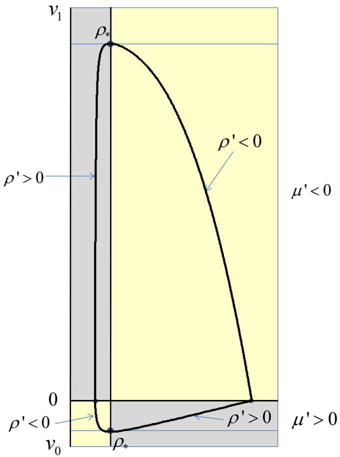


Figure 1. Apparent horizon structure.

Figure: Collapse and evaporation without singularities? (Frolov 2017)

Nice slice argument:

- The "nice slice argument" is quite common but is particularly tricky.
- Precise statement of the "nice slice argument" is impossible to find.
- Explicit reference for the "nice slice argument" seems non-existent? (Undocumented personal communication?)
- **Warning:**
  As currently used in the literature:
  - To the extent that "nice slices" exist,
    they are not as nice as advertised...
  - To the extent that "nice slices" are as nice as advertised,
    they do not exist...
- Carter–Penrose diagrams are good for understanding causal structure, and causal topology.
- Carter–Penrose diagrams can be grossly misleading when it comes to metrical properties.

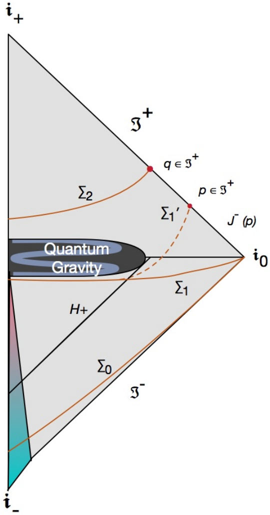


Figure: Collapse and evaporation with quantum gravity (Planck slop) region...
Where are the horizons? What type of horizon?

- Let

$$\boxed{\mathcal{P} = \text{(Planck slop region)}.}$$

- Define

$$\boxed{\mathcal{H}_- = \partial(\mathcal{J}_-(\mathcal{P})).}$$

- Past Planck slop horizon?
- Define

$$\boxed{\mathcal{H}_+ = \partial(\mathcal{J}_+(\mathcal{P})).}$$

- Future Planck slop horizon?
- A while ago I tried introdcing the notion of a "reliability horizon", but no-one took up the idea...
- Maybe the phrase "Planck slop horizon" will have more impact?
- In counterpoint, you might also want to consider the "domain of dependence" of the Planck slop...

Carter–Penrose diagrams:

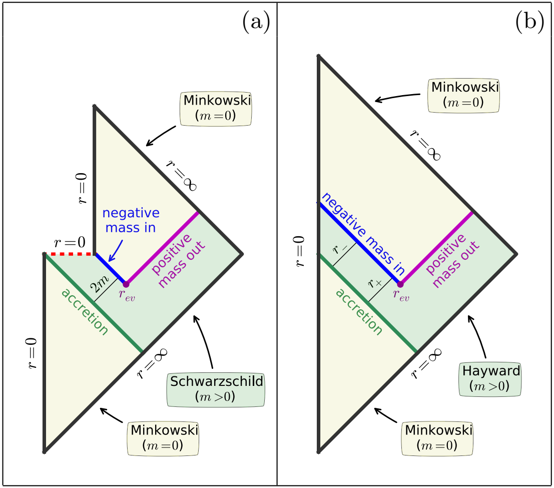


Figure: Collapse and evaporation with double null shell...
Where are the horizons? What type of horizon?

Dynamical/kinematical models

Dynamical/kinematical models:

- Should you use a dynamical model or a kinematical model?
- Depends:
  How much you actually know about the system in question?
- Can you find an actual solution to a reasonably well defined set of equations of motion?
- If so, great...
- Otherwise, build a model, use symmetries, general principles...
- Sometimes, once you have an interesting toy model,
  you can reverse engineer a suitable Lagrangian...
- EG: Simpson-Visser spacetime, originally just a model, was subsequently reverse engineered into a solution of a system involving a minimally coupled phantom scalar and nonlinear electrodynamics...
- Reverse engineering is often fine-tuned and fragile...

Example:

- Morris–Thorne wormhole model (1988):

$$ds^2 = -dt^2 + dr^2 + (r^2 + a^2)\, d\Omega^2.$$

- Morris–Thorne wormhole solution (1988):

Introduce a negative kinetic energy scalar (phantom scalar).

- The transition from model to solution can be almost trivial...
- **Warning:**
  Please do not go down the "reversed polarity coupling to gravity" rabbit hole...

Dynamical/kinematical models:

Example:

- Simpson–Visser wormhole model (2018):

$$ds^2 = -\left(1 - \frac{2m}{\sqrt{r^2+a^2}}\right) dt^2 + \frac{dr^2}{1 - \frac{2m}{\sqrt{r^2+a^2}}} + (r^2+a^2)\, d\Omega^2.$$

- We cooked this up as a simple 2-parameter model interpolating Schwarzschild $\longleftrightarrow$ Morris–Thorne.
- Simpson–Visser wormhole solution (Bronnikov–Walia 2021):

Introduce both a negative kinetic energy scalar and nonlinear EM.

- The transition from model to solution can be almost trivial...
- There is now an entire micro-industry of reverse engineering various models to make them solutions of some sort of toy dynamical model (Lagrangian). I recommend care and discretion...

Convergence/energy conditions

Convergence/energy conditions:

- Should one use energy conditions
  (constraints on the stress-energy tensor)
  or purely geometrical convergence conditions
  (constraints in the Einstein/Ricci tensor)?
- This is ultimately a sociology of physics question,
  not really a scientific question.
- In standard Einstein gravity

$$\boxed{G_{ab} = 8\pi G_N \; T_{ab}.}$$

- So then (energy condition) $\Longleftrightarrow$ (convergence condition).
- In modified gravity?

Convergence/energy conditions:

- In modified gravity, quite generally

$$\boxed{\text{some-function}\left(\textit{Riemann}, \textit{metric}, T_{ab}, \text{other-stuff}\right) = 0.}$$

- Quite often this can be rearranged into the form

$$\boxed{G_{ab} = \text{some-other-function}\left(\textit{metric}, \textit{Weyl}, T_{ab}, \text{other-stuff}\right) = 0.}$$

- If so, then

$$\boxed{G_{ab} = 8\pi G_N \; T_{ab}^{\text{effective}}.}$$

- If so, then even in very many (almost all) modified gravities

$$\boxed{(\text{energy condition on } \; T_{ab}^{\text{effective}}) \iff (\text{convergence condition}).}$$

- This has been extremely well known for decades...

Convergence/energy conditions:

- But this is one place where the sociology of physics trumps the actual science...
- There is now a truly vast industry churning out endless repetitive and superficial papers on "energy condition violations in modified gravity"...
- Enter this part of the literature at your peril...

- In the meantime, merely to short circuit some of the chatter, it might be a good idea (sociologically, not scientifically) to focus on convergence conditions...
- Expect to see me talking more about convergence conditions in the future...
- It is important to usefully communicate with the widest possible cross section of the broader community...

Hawking-Ellis classification:

To hopefully short-circuit a whole extra level of (potential) future nonsense:

- The Hawking-Ellis (Segre-Plebanski) classification of stress-energy tensors really has very little to do with stress-energy tensors...
- The Hawking-Ellis (Segre-Plebanski) classification is really a classification of $T^1{}_1$ tensors...
- Look for invariant eigenvalues and eigenvectors...
  - Either solve
  $$\det(T_{ab} - \lambda\, g_{ab}) = 0$$
  - Or solve
  $$\det(T^a{}_b - \lambda\, \delta^a{}_b) = 0$$
- Note $T^a{}_b$ need not be symmetric...
- (Math) Need the whole Jordan normal form decomposition...
- (Physics) Need type I–II-III-IV.

Raychaudhuri equation

Raychaudhuri equation:

- Which version of the Raychaudhuri equation should we use?
  - There is the obvious (and physically important) dichotomy between timelike and null versions of the Raychaudhuri equation.
  - The spacelike version of the Raychaudhuri equation is rarely used. (Classical tachyons exhibit a long list of dubious properties...)
  - More subtle is the dichotomy between null affine and null non-affine paramterizations.
    - For some issues, null affine paramterization is best.
    - For other issues, null non-affine paramterization is best.
  - There is also another rat's nest that can be opened up by looking at non-geodesic versions of the Raychaudhuri equation. (You could try to consider bounding the 4-acceleration.)

Timelike Raychaudhuri equation:

- Consider a 4-velocity field V^a, with 4-acceleration field A^a.
- Define $B_a{}^b = \nabla_a V^b$.
- Compute

$$\frac{dB_a{}^b}{ds} = V^c \nabla_c B_a{}^b = \nabla_a A^b - B_a{}^c B_c{}^b - R_{ac}{}^b{}_d V^c V^d$$

- Contract

$$\frac{d\theta}{ds} = V^c \nabla_c B_a{}^a = \nabla_a A^a - B_a{}^c B_c{}^a - R_{cd} V^c V^d$$

- Geodesic plus hypersurface orthogonal

$$\frac{d\theta}{ds} \leq -\frac{\theta^2}{3} - R_{cd} V^c V^d$$

- Add TCC (timelike convergence condition) $R_{cd} V^c V^d \geq 0$

$$\frac{d\theta}{ds} \leq -\frac{\theta^2}{3}$$

Timelike Raychaudhuri equation:

- Integrate the inequality
$$\frac{d\theta}{ds} \leq -\frac{\theta^2}{3}$$
to get
$$\theta(s) \leq \frac{\theta_0}{1+\frac{\theta_0}{3}(s-s_0)}.$$
- If you start out at s_0 with $\theta_0 < 0$ then you encounter a crushing singularity $\theta(s) \to -\infty$ at some finite proper time, at or before
$$s = s_0 + \frac{3}{|\theta_0|}$$
provided, of course, you do not hit the edge of spacetime first...
- A timelike observer hitting the edge of spacetime at finite proper time is at least as bad as encountering a crushing singularity...
- The situation for null geodesics is more subtle...

Null Raychaudhuri equation:

- Consider a null vector field ℓ^a.
- Null geodesic plus affine parameter plus hypersurface orthogonal

$$\frac{d\theta}{d\lambda} \leq -\frac{\theta^2}{2} - R_{cd}\ell^c\ell^d$$

- Add NCC (null convergence condition) $R_{cd}\ell^c\ell^d \geq 0$

$$\frac{d\theta}{d\lambda} \leq -\frac{\theta^2}{2}$$

- Integrate this inequality to get

$$\theta(\lambda) \leq \frac{\theta_0}{1 + \frac{\theta_0}{2}(\lambda - \lambda_0)}.$$

Null Raychaudhuri equation:

- If you start out at λ_0 with $\theta_0 < 0$ then you encounter a crushing singularity $\theta(\lambda) \to -\infty$ at some finite affine parameter, at or before

$$\lambda = \lambda_0 + \frac{2}{|\theta_0|},$$

  provided, of course, you do not run out of affine parameter first...
- A null observer hitting running out of affine parameter is nowhere near as bad as a timelike observer hitting the edge of spacetime...
- Example: static inner horizons (Cauchy horizons).
- Discussion:
  Violations of the null convergence condition in kinematical transitions between singular and regular black holes, horizonless compact objects, and bounces
  Borissova, Liberati, Visser. 2502.00548 [gr-qc]

Non-affine null Raychaudhuri equation:

- From from the affine parameterized equation

$$\frac{d\theta}{d\lambda} \leq -\frac{\theta^2}{2}$$

use $d\lambda = F(\hat\lambda) d\hat\lambda$ and $\theta = \hat\theta / F(\hat\lambda)$ to get

$$\frac{d\hat\theta}{d\hat\lambda} \leq +\frac{F'}{F}\,\hat\theta - \frac{\hat\theta^2}{2} = +\kappa\,\hat\theta - \frac{\hat\theta^2}{2}$$

- Integrate this inequality to get

$$\hat\theta(\lambda) \leq \frac{\hat\theta_0 \exp(\int \kappa d\hat\lambda)}{1 + \frac{\theta_0}{2}\int \exp(\int \kappa d\hat\lambda) d\hat\lambda}.$$

- Still get crushing singularity, now at or before

$$\int \exp\left(\int \kappa d\hat\lambda\right) d\hat\lambda = \frac{2}{|\hat\theta_0|}.$$

- Just some annoying details to sort out...

Non-affine null Raychaudhuri equation:

- Example: Consider this metric...

$$ds^2 = -f(r,v)dv^2 + 2dvdr + r^2 d\Omega^2 \quad \text{with} \quad f(r,v) = 1 - \frac{2m(r,v)}{r}\;.$$

(Not the most general, but sufficient for illustrative purposes)

- Introduce radial null vectors

$$k^\mu = -\partial_r = (0,-1,0,0) \qquad \text{and} \qquad l^\mu = \partial_v + \frac{f}{2}\partial_r = \left(1, \frac{f}{2}, 0, 0\right)$$

- Normalization $g(k,k) = 0 = g(l,l)$ and $g(k,l) = -1$.
- The ingoing null geodesics are affine parameterized, $k^\nu\,\nabla_\nu k^\mu = 0$.
- The "outgoing" null geodesics are not affine parameterized, $l^\nu\,\nabla_\nu l^\mu = \frac{1}{2} f'(r,v)\; l^\mu \neq 0$.
- NCC:

$$R_{\mu\nu}\; k^\mu k^\nu = 0; \qquad R_{\mu\nu}\; l^\mu l^\nu = -\frac{\dot{f}}{r} = \frac{2\dot{m}}{r^2}\;.$$

- NCC always OK for ingoing null geodesics, but depends on $\dot{m}$ for outgoing geodesics...

Affine null Raychaudhuri equation:

- Example: Consider the same metric...

$$ds^2 = -f(r,v)dv^2 + 2dvdr + r^2 d\Omega^2 \quad \text{with} \quad f(r,v) = 1 - \frac{2m(r,v)}{r} \;.$$

- Introduce (different) radial null vectors

$$k^\mu = -\partial_r = (0,-1,0,0) \qquad \text{and} \qquad l^\mu = \frac{2}{f}\partial_v + \partial_r = \left(\frac{2}{f}, 1, 0, 0\right)$$

- Normalization $g(k,k) = 0 = g(l,l)$ and $g(k,l) = -\frac{2}{f}$.
- The ingoing and outgoing null geodesics are now both affine parameterized, $k^\nu \; \nabla_\nu k^\mu = 0$ and $l^\nu \; \nabla_\nu l^\mu = 0$.
- NCC:

$$R_{\mu\nu} \; k^\mu k^\nu = 0; \qquad R_{\mu\nu} \; l^\mu l^\nu = -\frac{4\dot{f}}{f^2 r} = \frac{8\dot{m}}{f^2 r^2} \;.$$

- NCC always OK for ingoing null geodesics, but depends on $\dot{m}$ for outgoing geodesics...
- Nasty divide by zero at inner and outer apparent horizons...

Affine/non-affine null Raychaudhuri equation:

- There is a trade off...
- Note in general

$$\theta = \frac{\ell^a \nabla_a A}{A} = \frac{\ell^a \nabla_a r^2}{r^2} = \frac{2\ell^r}{r}$$

- Non affine:

$$l^a = \left(1, \frac{f}{2}, 0, 0\right); \qquad \theta_l = \frac{f}{r}$$

  With this normalization $\theta_l \to 0$ at inner and outer apparent horizons.
- Affine:

$$l^a = \left(\frac{2}{f}, 1, 0, 0\right); \qquad \theta_l = \frac{2}{r}.$$

  With this normalization θ_l finite at inner and outer apparent horizons.
- Expansions at inner/outer apparent horizons might be "unexpected".
- Learn to live with it...

Evolving inner horizon:

- Regardless of whether you choose affine or non affine parameterization, the "outgoing" null geodesics are dragged backwards and accumulate at the inner apparent horizon.
- So is the inner apparent horizon growing or shrinking?
  - If the inner horizon shrinks down to zero, then eventually it hits $r = 0$, and eventually you have a one-apparent-horizon object, qualitatively similar to Schwarzschild.
  - If the inner horizon expands, then (barring miracles) it eventually hits the outer horizon, and you have an (instantaneously) extremal object, which might in turn evaporate completely — cf Bergmann-Roman, Frolov, "topologically toroidal trapping horizon"...
- Either way, the r coordinate remains a useful null affine parameter....
- Test particle limit implied; geodesic equations $\Longrightarrow$ test particle limit...

Efficient Computation of Null Affine Parameters, Matt Visser
Universe 9 (2023) 521   e-Print: 2211.07835 [gr-qc]

Quantum physics

Quantum physics:

- I have said very little so far that is explicitly quantum...
- Quantum physics has been implicit in the discussion...
- Most of the "quantum" calculations we can do are actually semiclassical...
  (Classical spacetime background, idealized quantum "matter")
- For example:
  - Hawking radiation...
  - Cosmological particle production...
  - Vacuum expectation values (VEVs), of renormalized stress-energy tensors (RSETs)...
  - Quantum modified spacetimes...
- Trying to even define a horizon in a fully quantum spacetime setting is, ah, not yet really a viable proposition...
- (You really need the semi-classical background to even get started)...

Quantum physics:

Attempts at full quantum gravity include:

- Canonical quantum gravity:
  (Coherent formalism, but almost impossible to calculate anything)
- Wheeler-DeWitt equation:
  (Coherent formalism, but almost impossible to calculate anything)
- Minisuperspace:
  (Very limited toy models.)
- Loop quantum gravity:
  (Technically challenging)
- Causal dynamical triangulations:
  (Technically challenging)
- Causal sets:
  (Technically challenging)
- String based models:
  (50 years of promises.)

- At least for the time being, the best way forward seems to be classical and semi-classical GR.
- There are more than enough tractable classical and semi-classical GR problems to keep generations of grad students (and senior professors for that matter) usefully occupied.
- Keep at least one eye out for what the astronomers/astrophysicists actually need, practical and efficient ways for interpreting observational results...
- Keep at least one eye out for checking that what you are working on is (at least in principle) observationally testable...
- **Planck energy accelerator?**

Summary

Summary:

- Trapping horizons good...
  (Ditto dynamical, isolated horizons and their variants)...
- Apparent horizons tolerable...
  (At least in spherical symmetry with spherical foliations)...
- Event horizons bad...
  (Unless you are an external super-observer)...

- There is still an awful lot of interesting research to be done...

Crimes against Italian cuisine

Crimes against Italian cuisine:

Crimes against Italian cuisine:

Crimes against Italian cuisine:

Crimes against Italian cuisine:

Crimes against Italian cuisine:

Crimes against Italian cuisine:

End: