Quantum Effects in Early Universe Cosmology

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Outline

Plan

Motivation

- Quantum effects play a key role in early universe cosmology.
- Example: Quantum Theory of Cosmological Perturbations
- Current early Universe models are based on effective point particle quantum field theory (EFTs).
- Conceptual problems $\rightarrow$ breakdown of EFT.
- Require an approach beyond EFT consistent with a unified description of all forces of Nature.
- Superstring theory: best candidate for a unified quantum theory of all forces.
- Question: What emerges from a non-perturbative approach to superstring theory

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Standard Big Bang Cosmology: A Classical Theory

The SBB is based on:

- Cosmological principle: universe homogeneous and isotropic on large scales.
- General Relativity governing dynamics of space-time.
- Classical matter as source in the Einstein equations.
- Classical matter: cold (pressure-less) matter (describing the galaxies) + radiation(describing the CMB).

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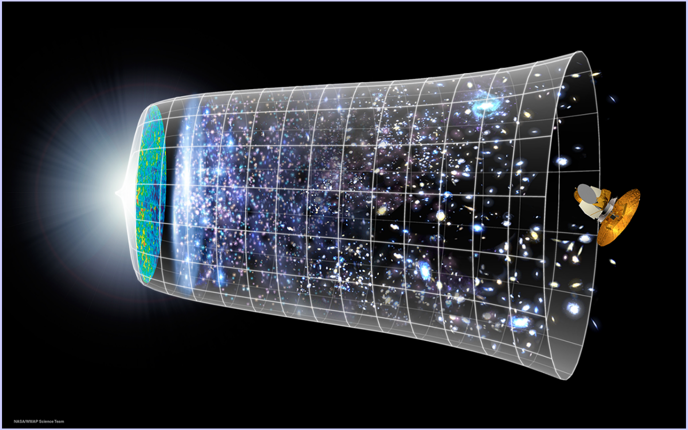
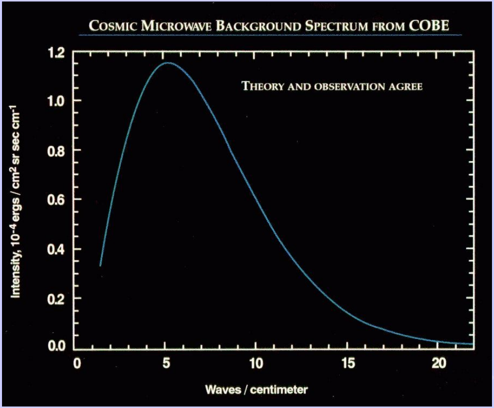


Credit: NASA/WMAP Science Team

Successes of the SBB Model

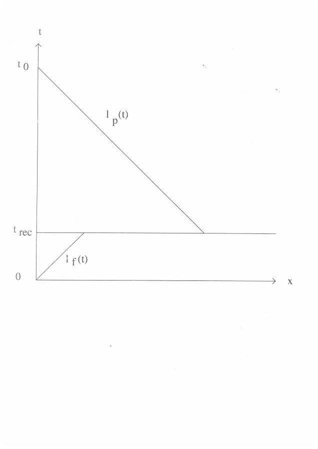
Key success: Existence and black body nature of the CMB.



Conceptual Problems of the SBB Model

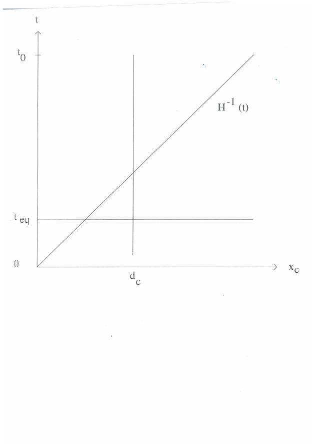
No explanation for the homogeneity, spatial flatness and large size and entropy of the universe.

Horizon problem of the SBB:



Conceptual Problems of the SBB Model II

No explanation of the observed inhomogeneities in the distribution of matter and anisotropies in the Cosmic Microwave Background possible!

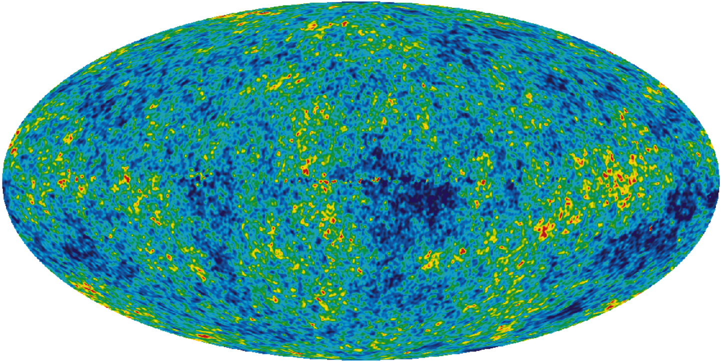


Message

A modified theory of the very early universe is required.

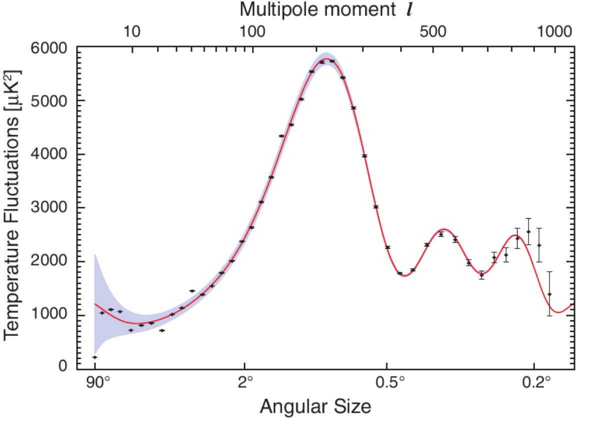
Plan

Anisotropies in the Cosmic Microwave Background (CMB)



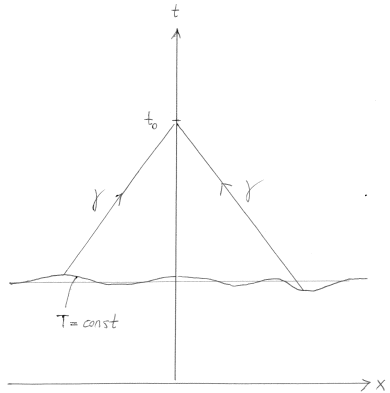
Credit: NASA/WMAP Science Team

Angular Power Spectrum of CMB Anisotropies



Credit: NASA/WMAP Science Team

Density Fluctuations lead to CMB Anisotropies



Early Work

1970 p.

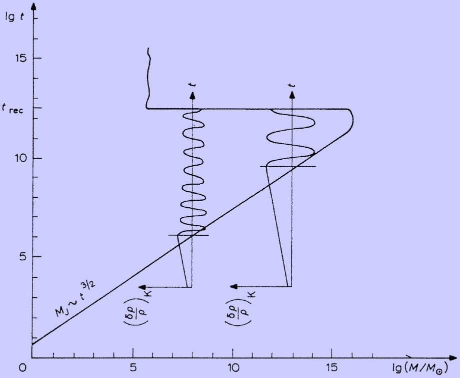


Fig. 1a. Diagram of gravitational instability in the 'big-bang' model. The region of instability is located to the right of the line $M_J(t)$; the region of stability to the left. The two additional lines of the graph demonstrate the temporal evolution of density perturbations of matter: growth until the moment when the considered mass is smaller than the Jeans mass and oscillations thereafter. It is apparent that at the moment of recombination perturbations corresponding to different masses correspond to different phases.

Predictions from 1970

R. Sunyaev and Y. Zel'dovich, Astrophys. and Space Science **7**, 3 (1970); P. Peebles and J. Yu, Ap. J. **162**, 815 (1970).

- Given a scale-invariant power spectrum of adiabatic fluctuations on "super-horizon" scales before t_{eq}, i.e. standing waves.
- $\rightarrow$ "correct" power spectrum of galaxies.
- $\rightarrow$ acoustic oscillations in CMB angular power spectrum.
- $\rightarrow$ baryon acoustic oscillations in matter power spectrum.

Hubble Radius vs. Horizon

- Horizon: forward light cone of a point on the initial Cauchy surface.
- Horizon: limit of causal influence.
- Hubble radius $l_H(t) \equiv H^{-1}(t)$
- Hubble radius relevant for the propagation of fluctuations.

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Criteria for a Successful Early Universe Scenario

- Horizon $\gg$ Hubble radius in order for the scenario to solve the “horizon problem” of Standard Big Bang Cosmology.
- Scales of cosmological interest today originate inside the Hubble radius at early times in order for a causal generation mechanism of fluctuations to be possible.
- Mechanism for producing a scale-invariant spectrum of curvature fluctuations on super-Hubble scales.

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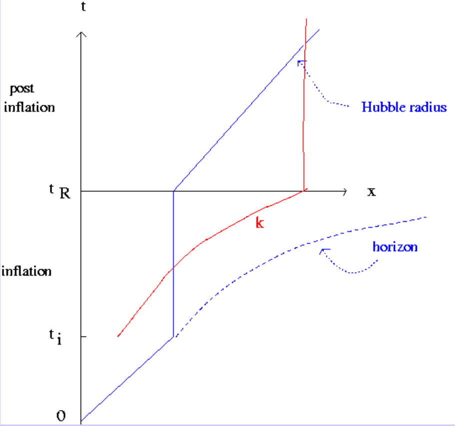
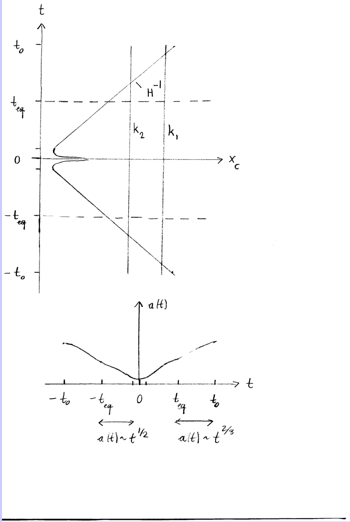
Inflation as a Solution



Origin of inhomogeneities: quantum vacuum fluctuations.

Bouncing Cosmology as a Solution

F. Finelli and R.B., *Phys. Rev. D65, 103522 (2002)*



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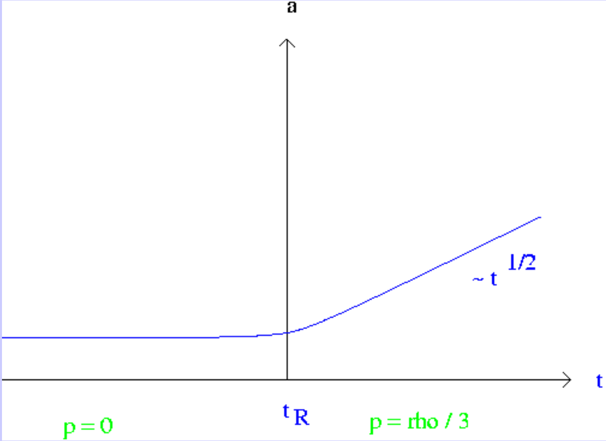
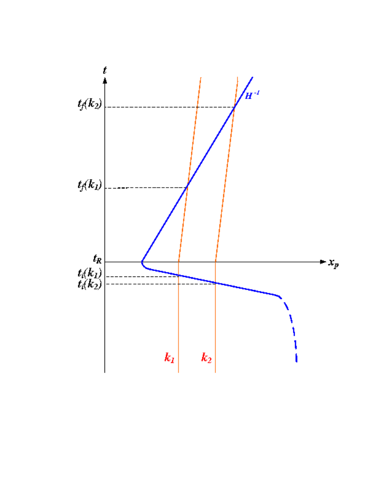
Emergent Universe

R.B. and C. Vafa, *Nucl. Phys. B316:391 (1989)*



Emergent Universe as a Solution

A. Nayeri, R.B. and C. Vafa, *Phys. Rev. Lett. 97:021302 (2006)*



Origin of inhomogeneities: thermal fluctuations.

Plan

Some Notation, Background

Background space-time metric:

$$ds^2 = dt^2 - a(t)^2 d\mathbf{x}^2 = a^2(\eta)[d\eta^2 - d\mathbf{x}^2]$$

Hubble scale:

$$l_H(t) = \frac{a}{\dot{a}}$$

Perfect fluid matter: energy density ρ, pressure p, equation of state parameter w

$$w = \frac{p}{\rho}$$

Note: $w = 1/3$ for radiation, $w = 0$ for cold matter, $w = -1$ for cosmological constant.

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Cosmological Perturbations

Cosmological fluctuations connect early universe theories with observations

- Fluctuations of matter $\rightarrow$ large-scale structure
- Fluctuations of metric $\rightarrow$ CMB anisotropies
- N.B.: Matter and metric fluctuations are coupled

Key facts:

- 1. Fluctuations are small today on large scales
- $\rightarrow$ fluctuations were very small in the early universe
- $\rightarrow$ can use linear perturbation theory
- 2. Sub-Hubble scales: matter fluctuations dominate
- Super-Hubble scales: metric fluctuations dominate

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- $\rightarrow$ linearization of the equations of motion about a cosmological background is self-consistent.
- $\rightarrow$ Fourier modes of the fluctuation variables evolve independently.

Note: On super-Hubble scales, the metric perturbations dominate over the matter fluctuations $\rightarrow$ QFT in curved background spacetime gives wrong results.

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Relativistic Theory of Cosmological Perturbations

V. Mukhanov, H. Feldman and RB, *Phys. Rep. 215, 203 (1992)*

Background:

$$ds^2 = a(\eta)^2 \left(d\eta^2 - d\mathbf{x}^2\right)$$

Linear fluctuations:

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + \delta g_{\mu\nu}$$

Linear Perturbation Equations

$$\delta G_{\mu\nu} = 8\pi G \delta T_{\mu\nu}$$

Too many degrees of freedom, coordinate artefacts

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Classification according to transformation under spatial rotations:

Classification

Scalar fluctuations ("cosmological perturbations")

$$\delta g_{\mu\nu} = a^2 \begin{pmatrix} 2\phi & -B_{,i} \\ -B_{,i} & 2(\psi\delta_{ij} - E_{,ij}) \end{pmatrix},$$

Vector perturbations:

$$\delta g_{\mu\nu} = a^2 \begin{pmatrix} 0 & -S_i \\ -S_i & F_{i,j} + F_{j,i} \end{pmatrix},$$

Tensor fluctuations (gravitational waves):

$$\delta g_{\mu\nu} = -a^2 \begin{pmatrix} 0 & 0 \\ 0 & h_{ij} \end{pmatrix},$$

$$\phi(x,\eta), \psi(x,\eta), E(x,\eta), B(x,\eta), \ldots S^i_{,i} = F^i_{,i} = h^i_{ij} = h^i_i = 0$$

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Note: at linear order there is no coupling between scalar, vector and tensor modes.

Reduction for Scalar Cosmological Perturbations

Coordinates are arbitrary in General Relativity

- 5 degrees of freedom (dof) for scalar cosmological perturbations (4 metric, 1 matter)
- 2 dof are coordinate artefacts $\rightarrow$ can set $E = B = 0$
- If matter has no anisotropic stress $\rightarrow$ one dof less ($\psi = \phi$)
- 1 constraint equation for cosmological perturbations (matter fluctuation induces gravitational potential)

$\rightarrow$ a single free function of space and time describes cosmological fluctuations.

Gravitational Waves

At linear order gravitational waves are gauge-independent.

$$h_{ij}(x,\eta) = h^{(}x,\eta)\epsilon_{ij}^{+} + h^{x}(x,\eta)\epsilon_{ij}^{x}$$

At linear order the two polarization modes evolve independently.

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Quantum Theory of Linearized Fluctuations

V. Mukhanov, H. Feldman and R.B., *Phys. Rep. 215:203 (1992)*

Step 1: Metric including fluctuations

$$ds^2 = a^2[(1+2\Phi)d\eta^2 - (1-2\Phi)d\mathbf{x}^2]$$

$$\varphi = \varphi_0 + \delta\varphi$$

Note: Φ and $\delta\varphi$ related by Einstein constraint equations

Step 2: Expand the action for matter and gravity to second order about the cosmological background:

$$S^{(2)} = \frac{1}{2}\int d^4x\left((v')^2 - v_{,i}v^{,i} + \frac{z''}{z}v^2\right)$$

$$v = a\left(\delta\varphi - \frac{z}{a}\Phi\right)$$

$$z = a\frac{\varphi_0'}{\mathcal{H}}$$

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Step 3: Resulting equation of motion (Fourier space)

$$v_k'' + (k^2 - \frac{z''}{z})v_k \,=\, 0$$

Features:

- oscillations on sub-Hubble scales
- squeezing on super-Hubble scales $v_k \sim z$

Quantum vacuum initial conditions:

$$v_k(\eta_i) \,=\, (\sqrt{2k})^{-1}$$

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Quantum Theory of Gravitational Waves

V. Mukhanov, H. Feldman and R.B., *Phys. Rep. 215:203 (1992)*

Steps:

- 1. Insert the metric ansatz for gravitational waves into the Einstein action.
- 2. Expand to quadratic order
- 3. Identify the canonical variable: $u_k = ah_k$

$$S^{(2)} = \frac{1}{2}\int d^4x((u')^2 - u_{,i}u^{,i} + \frac{a''}{a}u^2)$$

Resulting equation of motion:

$$u_k'' + (k^2 - \frac{a''}{a})u_k = 0\,.$$

Features:

- oscillations of u on sub-Hubble scales
- $\rightarrow$ damped oscillations of h on sub-Hubble scales
- squeezing of u on super-Hubble scales $u_k \sim a$
- $\rightarrow$ freezing out of h on super-Hubble scales.

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Free Scalar Matter Field on a Cosmological Background

- 1. Consider the Klein-Gordon action for φ in a cosmological background..
- 2. Identify the canonical variable: $u_k = a\varphi_k$

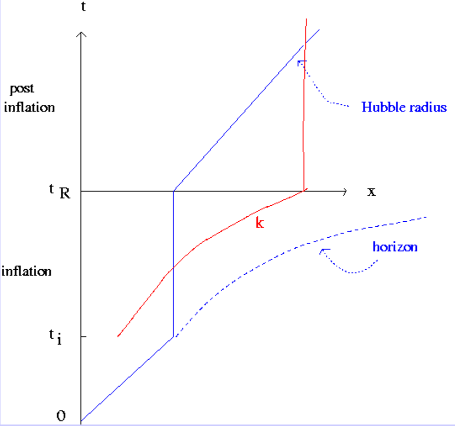
$$S(\varphi) = \frac{1}{2}\int d^4x\left((u')^2 - u_{,i}u^{,i} + \frac{a''}{a}u^2\right)$$

Resulting equation of motion:

$$u_k'' + \left(k^2 - \frac{a''}{a}\right)u_k = 0\,.$$

Warning: Very different evolution of φ_k compared to cosmological perturbations. QFT on a fixed background violates the Einstein constraint equations!.

Application to Inflation



Application to Inflation

$$v_k(t) \sim v_k(t_H(k)) \frac{z(t)}{z(t_H(k))} \sim v_k(t_i) \frac{a(t)}{a(t_H(k))}$$

$$v_k(t_i)) = \frac{1}{\sqrt{2k}}$$

$$a(t_H(k))^{-1} k = H$$

$$v_k(t) \sim a(t) H k^{-3/2}$$

Power spectrum of v:

$$P_v(k) = k^3 v_k(t)^2 \sim a(t)^2 H^2$$

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$$\mathcal{R} \equiv \frac{1}{z} v$$

Final result:

$$P_{\mathcal{R}}(k) = \left(\frac{a}{z}\right)^2 H^2 = \left(\frac{1}{\epsilon}\right)^2 H^2$$

Scale-invariant spectrum!

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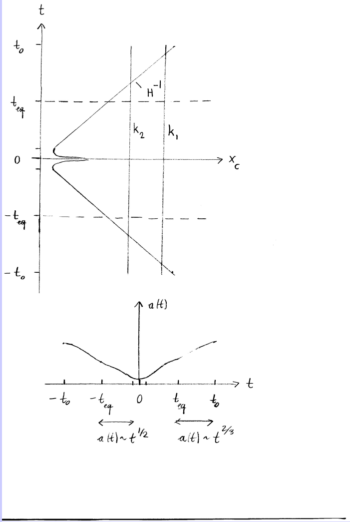
F. Finelli and RB, *Phys. Rev. D65, 103522 (2002))*



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Power spectrum of curvature fluctuations:

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Application to Ekpyrotic Scenario

J. Khoury et al, *Phys. Rev. D64*, 123522 (2001).

Ekpyrotic scenario: Einsteiin gravity coupled to a scalar field with negative exponential potential:

$$V(\phi) = -V_0 \exp(-\sqrt{2/p}\phi/m_{pl}) \quad p \ll 1 \, ,$$

Note: Since AdS is the favored ground state of perturbative string theory, such a potential is well justified.

Slow contraction:

$$a(t) \sim (-t)^p$$

$$w \simeq \frac{4}{3p} \gg 1 \, .$$

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- ρ increases during contraction.
- When $\rho = \rho_s$ EFT breaks down: new dof must be taken into account.
- $\rightarrow$ extra term in the action which appears at a fixed time: S-brane.
- $\rho_{S-brane} = 0$, $p_{S-brane} < 0 \rightarrow$ violation of the NEC
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Z. Wang and RB, *Phys. Rev. D102*, 023516 (2020).

- Begin with quantum vacuum fluctuations in the far past.
- **Deruelle-Mukhanov matching conditions** across the S-brane $\rightarrow \Phi$ continuous, v jumps (R. Durrer and F. Vernizzi, Phys. Rev. D66, 083503 (2002)).
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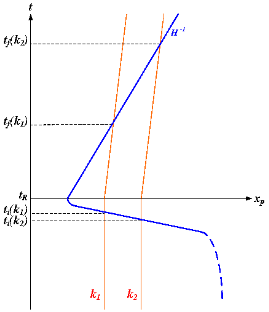

Application to Emergent Scenario: Formalism

A. Nayeri, RB and C. Vafa, *Phys. Rev. Lett. 97*, 021302 (2006).

$$ds^2 = a^2(\eta)\left((1+2\Phi)d\eta^2 - [(1-2\Phi)\delta_{ij} + h_{ij}]dx^i dx^j\right).$$

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$$\langle|\Phi(k)|^2\rangle = 16\pi^2 G^2 k^{-4} \langle \delta T^0{}_0(k) \delta T^0{}_0(k)\rangle,$$

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$$\langle \delta\rho^2(R) \rangle = \frac{T(k)^2}{R^6} C_V ,$$

For a gas of closed strings: holographic scaling of C_V:

$$C_V(R(k)) \approx 2 \frac{R^2/\ell_s^3}{T(k)(1 - T(k)/T_H)} ,$$

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Note: scale-invariant, and slight red tilt

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Application to Emergent Scenario: Prediction

RB, A. Nayeri, and S. Patil *Phys. Rev. D90*, 067301 (2014).

Quantitative prediction: consistency relation:

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Conventions:

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Back-Reaction of Cosmological Perturbations

R.B., hep-th/0210165

Beyond linear order:

- Coupling between scalar, vector and tensor modes.
- In particular: scalar modes induce gravitational waves at second order
- Coupling between different Fourier modes.
- In particular: linear fluctuations induce corrections to the background at 2nd order
- This is back-reaction of cosmological perturbations

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Formalism

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Starting point:

$$g_{\mu\nu}(x,t) = g^{(0)}_{\mu\nu}(t) + g^{(1)}_{\mu\nu}(x,t)$$

Does not satisfy Einstein equations at 2nd order.

- Insert metric ansatz into Einstein equations $G_{\mu\nu} = 8\pi G T_{\mu\nu}$
- Expand to 2nd order
- Move all terms quadratic in $g^{(1)}_{\mu\nu}$ to the matter side of the Einstein equations $\rightarrow T^{\text{eff}}_{\mu\nu}$.
- Average over space
- This yields the defining equation for a new background metric $g^{(0,br)}_{\mu\nu}$ which includes the effects of cosmological perturbations.

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$$G_{\mu\nu}(g^{(0,br)}_{\alpha\beta}) = 8\pi G < T^{\mathrm{eff}}_{\mu\nu} >$$

Applications to the Cosmological Constant Problem

- Setup: Einstein gravity with Λ and matter.
- Start with Λ dominating $\rightarrow$ inflation.
- Consider back-reaxtion of cosmological perturbations
- $< T^{\rm eff}_{\mu\nu} >$: contributions from all Fourier modes.
- Contributions from sub-Hubble modes constant
- Contributions from super-Hubble modes increasing
- Super-Hubble mode: negative gravitational energy overcomes positive matter energy.
- **equation of state** of infrared modes $p = -\rho$ with $\rho < 0$.
- $\rightarrow$ dynamical relaxation mechanism for Λ.

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- Setup: Einstein gravity with Λ and matter.
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- Need to focus on physically measurable quantities e.g. spatially averaged expansion rate.
- Require: physical clock
- Result: In the case of **purely adiabatic fluctuations**, there is **no physically measurable effect.**
- An observer at fixed value of the matter field φ will measure the same result with and without fluctuations (G. Geshnizjani and R.B., gr-qc/0204074).
- In the presence of entropy fluctuations, the back-reaction effect is real. (RB, Leila Graef, G. Marozzi and G. Vacca, Phys. Rev. D98, 103523 (2018)).

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Plan

Effective field theory (EFT) :

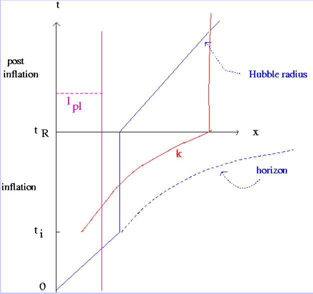
- Einstein gravity describing space and time.
- Matter obeying the usual energy conditions.
- Matter treated in terms of quantum fields
- Fields quantized according to canonical quantization.

Bouncing and Emergent Scenarios Require Going Beyond EFT

- Bouncing cosmology: $H = 0$ at the bounce point cannot be achieved using EFT.
- Emergent cosmology: $H = 0$ in the emergent phase cannot be achieved using EFT.

Trans-Planckian Problem

J. Martin and R.B., *Phys. Rev. D63, 123501 (2002)*



- **Success of inflation**: At early times scales are inside the Hubble radius $\rightarrow$ causal generation mechanism is possible.
- **Problem:** If time period of inflation is more than $70H^{-1}$, then $\lambda_p(t) < l_{pl}$ at the beginning of inflation.
- $\rightarrow$ breakdown of effective field theory; new physics MUST be taken into account when computing observables from inflation.

Trans-Planckian Censorship Conjecture (TCC)

A. Bedroya and C. Vafa., arXiv:1909.11063

No trans-Planckian modes exit the Hubble horizon.

$$ds^2 = dt^2 - a(t)^2 d\mathbf{x}^2$$

$$H(t) \equiv \frac{\dot{a}}{a}(t)$$

$$\boxed{\frac{a(t_R)}{a(t_i)} l_{pl} < H(t_R)^{-1}}$$

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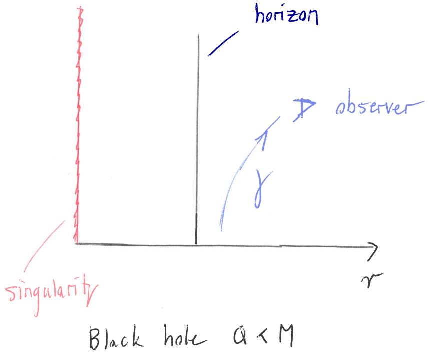
Justification

R.B. arXiv:1911.06056

Analogy with Penrose's Cosmic Censorship Hypothesis:



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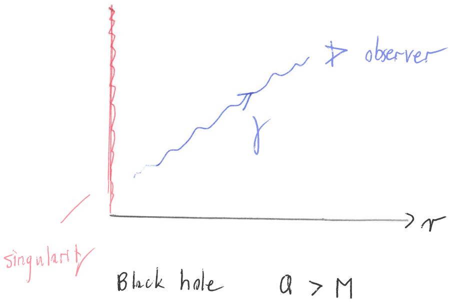

Justification

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- Effective field theory of General Relatiivty allows for solutions with timelike singularities: super-extremal black holes.
- $\rightarrow$ Cauchy problem not well defined for observer external to black holes.
- Evolution non-unitary for external observer.
- Conjecture: ultraviolet physics $\rightarrow$ **external observer** shielded from the singularity and non-unitarity by horizon.

Cosmological Version of the Censorship Conjecture

Translation

- Position space $\rightarrow$ momentum space.
- Singularity $\rightarrow$ trans-Planckian modes.
- Black Hole horizon $\rightarrow$ Hubble horizon.

Observer measuring super-Hubble horizon modes must be shielded from trans-Planckian modes.

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Why Hubble Horizon?

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- Recall: Fluctuations only oscillate on sub-Hubble scales.
- Recall: Fluctuations freeze out, become **squeezed states** and **classicalize** on super-Hubble scales.
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Unitarity Problem

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- Recall: non-unitarity of **effective field theory** in an expanding universe (N. Weiss, Phys. Rev. D32, 3228 (1985); J. Cotler and A. Strominger, arXiv:2201.11658).
- $\mathcal{H}$ is the product Hilbert space of harmonic oscillator Hilbert spaces for all **comoving** wave numbers k
- UV cutoff: time dependent k_{max} : $k_{max}(t)a(t)^{-1} = m_{pl}$
- Continuous mode creation $\rightarrow$ non-unitarity.
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Effective Field Theory (EFT) and the CC Problem

- EFT: expand **fields** in comoving Fourier space.
- Quantize each Fourier mode like a harmonic oscillator $\rightarrow$ ground state energy.
- Add up ground state energies $\rightarrow$ CC problem.
- The usual quantum view of the CC problem is an artefact of an EFT analysis!

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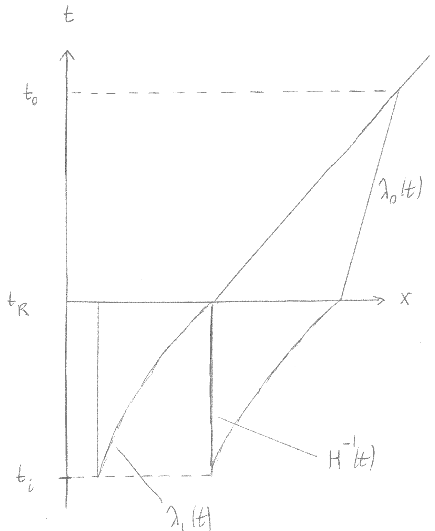
Application of the Second Law of Thermodynamics

S. Brahma, O. Alaryani and RB, arXiv:2005.09688

- Consider entanglement entropy density $s_E(t)$ between sub- and super-Hubble modes.
- Consider an **phase of inflationary expansion.**
- $s_E(t)$ increases in time since the phase space of super-Hubble modes grows.
- **Demand**: $s_E(t)$ remain smaller than the post-inflationary thermal entropy.
- $\rightarrow$ Duration of inflation is bounded from above, consistent with the TCC.

Application to EFT Description of Inflation

A. Bedroya, R.B., M. Loverde and C. Vafa., arXiv:1909.11106



Application to EFT Descriptions of Inflation

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TCC implies:

$$\frac{a(t_R)}{a(t_*)} l_{pl} < H(t_R)^{-1}$$

Demanding that inflation yields a causal mechanism for generating CMB anisotropies implies:

$$H_0^{-1} \frac{a(t_0)}{a(t_R)} \frac{a(t_R)}{a(t_*)} < H^{-1}(t_*)$$

Implications

Upper bound on the **energy scale of inflation**:

$$V^{1/4} < 3 \times 10^9 \mathrm{GeV}$$

$\rightarrow$ upper bound on the **primordial tensor to scalar ratio** r:

$$r < 10^{-30}$$

Note: Secondary tensors will be larger than the primary ones.

Implications for Dark Energy

- Dark Energy cannot be a bare cosmological constant.
- Quintessence models of Dark Energy are constrained (L. Heisenberg et al. arXiv:2003.13283]

Trans-Planckian Censorship and Cosmological Scenarios

- **Bouncing cosmologies** are consistent with the TCC provided that the energy scale at the bounce is lower than the Planck scale.
- **Emergent cosmologies** are consistent with the TCC provided that the energy scale of the emergence phase is lower than the Planck scale.
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Requirements

Requirements for early universe cosmology:

- Quantum theory
- Unified theory of all four forces
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Challenge for Superstring Cosmology

Challenge for superstring cosmology: non-perturbative superstring theory does not yet exist.

Options:

- A. Consider a toy model of superstring theory.
- A model based on new degrees of freedom and new symmetries which distinguish string theory from point particle theories.
- Example: String gas cosmology
- B: work with a proposed non-perturbative definition of superstring theory.
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Plan

Idea: make use of the new symmetries and new degrees of freedom which string theory provides to construct a new theory of the very early universe.
Assumption: Matter is a gas of fundamental strings
Assumption: Space is compact, e.g. a torus.
Key points:

- New degrees of freedom: string oscillatory modes
- Leads to a maximal temperature for a gas of strings, the Hagedorn temperature
- New degrees of freedom: string winding modes
- Leads to a new symmetry: physics at large R is equivalent to physics at small R

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R.B. and C. Vafa, *Nucl. Phys. B316:391 (1989)*

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T-Duality

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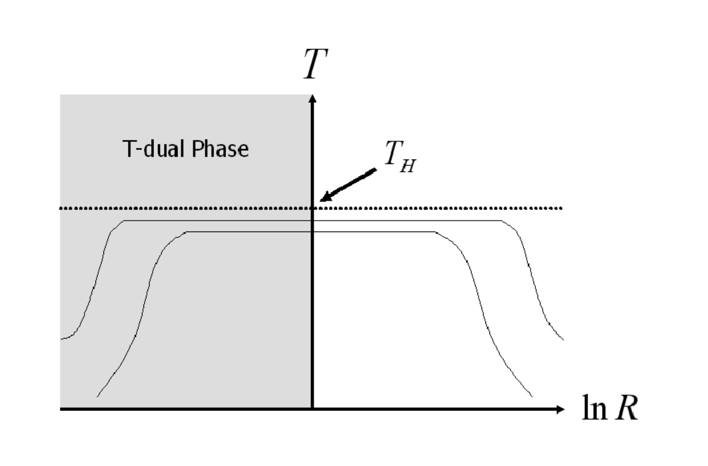
- Momentum modes: $E_n = n/R$
- Winding modes: $E_m = mR$
- Duality: $R \rightarrow 1/R \quad (n, m) \rightarrow (m, n)$
- Mass spectrum of string states unchanged
- Symmetry of vertex operators
- Symmetry at non-perturbative level $\rightarrow$ existence of D-branes

Note: usual EFTs have lost the T-duality symmetry.

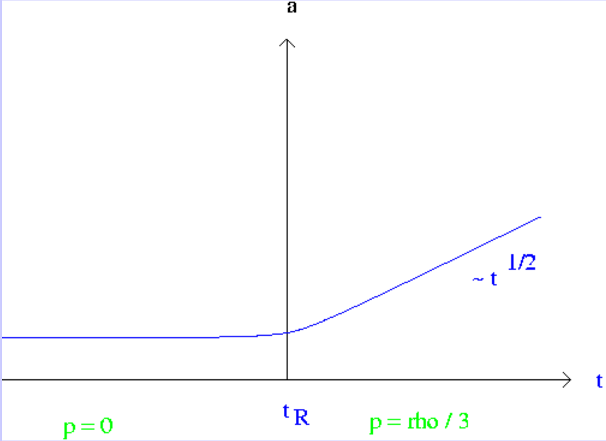
Adiabatic Considerations

R.B. and C. Vafa, *Nucl. Phys. B316:391 (1989)*

Temperature-size relation in string gas cosmology



Background for string gas cosmology



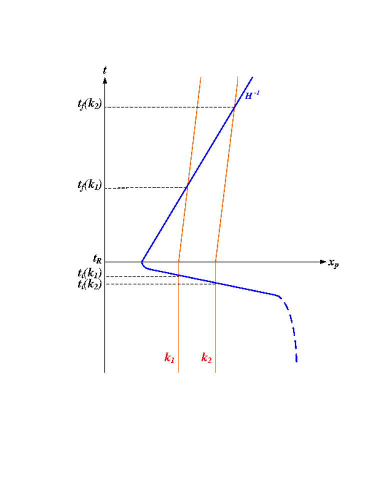
Origin of 3-d Space

R.B. and C. Vafa, *Nucl. Phys. B316:391 (1989)*

- A spatial dimension can only expand it string winding modes can annihilate into loops.
- This process only allows 3 spatial dimensions to become large.
- $SO(9) \to SO(6) \times SO(3)$ symmetry breaking.

Structure formation in string gas cosmology

A. Nayeri, R.B. and C. Vafa, *Phys. Rev. Lett. 97:021302 (2006)*



N.B. Perturbations originate as thermal string gas fluctuations.

Method

- Calculate matter correlation functions in the Hagedorn phase (neglecting the metric fluctuations)
- For fixed k, convert the matter fluctuations to metric fluctuations at Hubble radius crossing $t = t_i(k)$
- Evolve the metric fluctuations for $t > t_i(k)$ using the usual theory of cosmological perturbations

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Extracting the Metric Fluctuations

Ansatz for the metric including cosmological perturbations and gravitational waves:

$$ds^2 = a^2(\eta)\big((1+2\Phi)d\eta^2 - [(1-2\Phi)\delta_{ij} + h_{ij}]dx^i dx^j\big) .$$

Inserting into the perturbed Einstein equations yields

$$\langle|\Phi(k)|^2\rangle = 16\pi^2 G^2 k^{-4} \langle \delta T^0{}_0(k) \delta T^0{}_0(k) \rangle ,$$

$$\langle|h(k)|^2\rangle = 16\pi^2 G^2 k^{-4} \langle \delta T^i{}_j(k) \delta T^i{}_j(k) \rangle .$$

Power Spectrum of Cosmological Perturbations

Key ingredient: For thermal fluctuations:

$$\langle \delta\rho^2 \rangle \, = \, \frac{T^2}{R^6} C_V \, .$$

Key ingredient: For string thermodynamics in a compact space

$$C_V \, \approx \, 2 \frac{R^2/\ell_s^3}{T(1 - T/T_H)} \, .$$

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Power spectrum of cosmological fluctuations

$$\begin{aligned}
P_\Phi(k) &= 8G^2 k^{-1} < |\delta\rho(k)|^2 > \\
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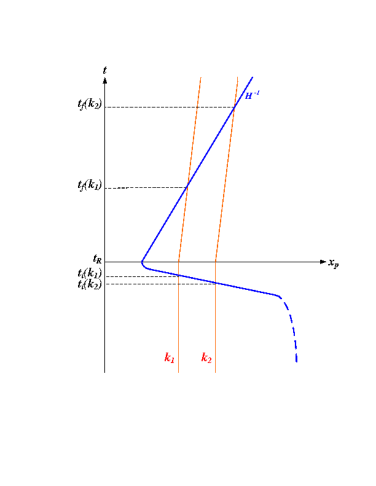

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Key features:

- scale-invariant like for inflation
- slight red tilt like for inflation

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R.B., A. Nayeri, S. Patil and C. Vafa, *Phys. Rev. Lett. (2007)*

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Achilles Heel of String Gas Cosmology

No dynamics for the Hagedorn phase!

Plan

Particle theory vs. Strting Theory

Particle theory:

- Classical particle motion: well understood
- Quantum scattering of particles: well understood (Feynman rules)
- **Non-perturbative analysis**: theory of an arbitrary number of particles: replace particles by fields.
- **Particles** emerge as point-like excitations of fields.

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- Yields an emergent early universe cosmology.

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S. Brahma, R.B. and S. Laliberte, arXiv:2108.1152

Starting point: BFSS matrix model at high temperatures.

- BFSS model is a quantum mechanical model of 10 $N \times N$ bosonic Hermitean matrices and 16 Fermionic $N \times N$ matrices.
- Supersymmetric
- Note: no space!
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T. Banks, W. Fischler, S. Shenker and L. Susskind, Phys. Rev. D **55**, 5112 (1997), B. de Wit, J. Hoppe and H. Nicolai, Nucl. Phys. **B305** (1988), 545.

$$L = \frac{1}{2g^2}\left[\mathrm{Tr}\left(\frac{1}{2}(D_t X_i)^2 - \frac{1}{4}[X_i, X_j]^2\right)\right] + L_f$$

- X_i, $i = 1, \ldots d$ are $N \times N$ Hermitean matrices.
- D_t: gauge covariant derivative (contains a matrix A_0)

't Hooft limit: $N \to \infty$ with $\lambda \equiv g^2 N = g_s l_s^{-3} N$ fixed.

Note: It is precisely in $d = 9$ that both a supersymmetric extension is possible and a normalizable zero energy state exists (J. Froehlich, G-M. Graf, D. Hasler, J. Hoppe and S-T. Yau, hep-th/9904182).

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Thermal Initial State

N. Kawahara, J. Nishimura and S. Takeuchi, JHEP **12**, 103 (2007)

- Consider a high temperature state.
- Matsubara expansion:

$$X_i(t) = \sum_n X_i^n e^{2\pi i T t}$$

- Note: no Matsubara zero modes for the Fermionic matrices.
- $S_{BFSS} = S_0 + \mathcal{O}(1/T)$
- $S_0 = S_{IKKT}^{\mathrm{bosonic}}$: contribution of the $n = 0$ modes.
- At high temperatures, the bosonic sector of the (Euclidean) BFSS model is well approximated by the bosonic sector of the (Euclidean) IKKT matrix model.

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IKKT Matrix Model

N. Ishibashi, H. Kawai, Y. Kitazawa and A. Tsuchiya, Nucl. Phys. B **498**, 467 (1997).

Proposed as a non-perturbative definition of the IIB Superstring theory.

Action:

$$S_{IKKT} = -\frac{1}{g^2}\mathrm{Tr}\left(\frac{1}{4}[A^a, A^b][A_a, A_b] + \frac{i}{2}\bar{\psi}_\alpha(\mathcal{C}\Gamma^a)_{\alpha\beta}[A_a, \psi_\beta]\right),$$

Partition function:

$$Z = \int dA d\psi e^{iS}$$

Emergent Time from Matrix Theory

Y. Ito, J. Nishimura and A. Tsuchiya, arXiv:1506.04795

- Eigenvalues of A_0 become emergent time.
- Work in the basis in which A_0 is diagonal.
- Numerical studies: $\frac{1}{N}\langle \mathrm{Tr} A_0^2 \rangle \sim \kappa N$
- $\rightarrow t_{max} \sim \sqrt{N}$
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- $\sum_i \langle |A_i|^2_{ab} \rangle$ decays when $|a - b| > n_c$
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- $\sum_i \langle |A_i|^2_{ab} \rangle \sim$ constant when $|a - b| < n_c$

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Emergent Space from Matrix Theory

S. Kim, J. Nishimura and A. Tsuchiya, arXiv:1108.1540

- Eigenvalues of A_0 become emergent time, continuous in $N \rightarrow \infty$ limit.
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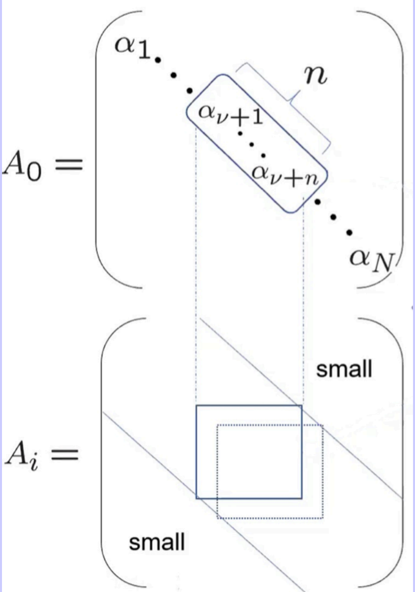
- Pick $n \times n$ blocks $\tilde{A}_i(t)$ about the diagonal ($n < n_c$)


$A_0 =$
α_1
n
$\alpha_{\nu+1}$
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α_N
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Spontaneous Symmetry Breaking in Matrix Theory

J. Nishimura, PoS CORFU **2019**, 178 (2020) [arXiv:2006.00768 [hep-lat]].

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- **Extent of space** in direction i

$$x_i(t)^2 \equiv \left\langle \frac{1}{n}\mathrm{Tr}(\bar{A}_i)(t))^2 \right\rangle ,$$

- In a thermal state there is spontaneous symmetry breaking: $SO(9) \to SO(6) \times SO(3)$: three dimensions of space become larger, the others are confined. [J. Nishimura and G. Vernizzi, JHEP **0004**, 015 (2000);]S.-W. Kim, J. Nishimura and A. Tsuchiya, Phys. Rev. Lett. **109**, 011601 (2012)]

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Emergent Metric from Matrix Theory

S. Brahma, R.B. and S. Laliberte, arXiv:2206.12468

- Eigenvalues of A_0 become emergent time, continuous in $N \to \infty$ limit.
- Work in the basis in which A_0 is diagonal: pick n (comoving spatial coordinate) and consider the block matrix $\tilde{A}_i(t)$.
- Physical distance between $n_i = 0$ and n_i (emergent space):

$$l^2_{phys,i}(n,_i\, t) \equiv \left\langle \mathrm{Tr}(\bar{A}_i)(t))^2 \right\rangle ,$$

- $l_{phys,i}(n_i) \sim n_i$ (for $n_i < n_c$)
- Emergent infinite and continuous space in $N \to \infty$ limit.
- Emergent metric (S. Brahma, R.B. and S. Laliberte, arXiv:2206.12468).

$$g_{ii}(n_i)^{1/2} = \frac{d}{dn_i} l_{phys,i}(n_i)$$

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No Flatness Problem in Matrix Theory Cosmology

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Result:

$$g_{ii}(n_i, t) = \mathcal{A}(t)\delta_{ii} \quad i = 1, 2, 3$$

$SO(3)$ symmetry $\rightarrow$

$$g_{ij}(\mathbf{n}, t) = \mathcal{A}(t)\delta_{ij} \quad i = 1, 2, 3$$

$\rightarrow$ spatially flat.

Note: Local Lorentz invariance emerges in $N \rightarrow \infty$ limit.

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Physical Origin of the Emergence of Three Spatial Dimensions in IKKT Model

R.B. and J. Pasiecznik, arXiv:2409.00254

- Hypothesis: The BV mechanism can be embedded into matrix cosmology: winding D1-strings allow only three dimensions of space to become large.
- What are D1-strings in the IKKT matrix theory?
- D1-strings are solitonic matrix excitations.
- **Analogy**: particles are point-like excitations of fields in QFT.
- Thermal initial state contains all excitations of Type IIB string theory..

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- **Free energy** consideration: a spatial dimension can only expand if winding D1 strings can annihilate.
- **In the supersymmetric IKKT model**, the force between parallel D1-strings vanishes (to leading order).
- $\rightarrow$ decay of winding strings requires crossing of world sheets.
- **Vanishing probability** in more than 3 large spatial dimensions.
- $\rightarrow$ precisely three spatial dimensions become large.

Some details

Consider a **matrix background** (Note: breaks supersymmetry)

$$A^b_\mu = A^q_\mu + A^{D1}_\mu ,$$

where A^{D1}_μ is the classical matrix configuration for a pair of parallel D1-strings along the 1 axis separated by a distance b along the 2 axis.

Consider **fluctuations** about this background, insert into the expression for the **one loop effective action** W_{eff}.

Result: W_{eff} independent of $b \rightarrow$ vanishing force between the parallel D1-strings.

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with eigenvalue distribution

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Plan

Late Time Dynamics

- Consider the expansion of the 3-d large space.
- Late time evolution given by the classical matrix equations of motion.
- Result:

$$\mathcal{A}(t) \sim t^{1/2}$$

- Note: no sign of a cosmological constant.

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Matrix Theory Cosmology

S. Brahma, R.B. and S. Laliberte, arXiv:2107.11512

- We assume that the spontaneous symmetry breaking $SO(9) \rightarrow SO(3) \times SO(6)$ observed in the IKKT model also holds in the BFSS model.
- Using the Gaussian approximation method we have shown the existence of a symmetry breaking phase transition in the BFSS model (S. Brahma, RB and S. Laliberte, arXiv:2209.01255).
- **Thermal correlation functions** in the three large spatial dimensions calculated in the high temperature state of the BFSS model (following the formalism developed in String Gas Cosmology).
- $\rightarrow$ curvature fluctuations and gravitational waves.

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Matrix Theory Cosmology: Thermal Fluctuations

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- Start with the BFSS partition function .
- Note: $\frac{1}{T}$ correction terms in the BFSS action are crucial!
- Calculate matter correlation functions in the emergent phase.
- For fixed k, convert the matter fluctuations to metric fluctuations at Hubble radius crossing $t = t_i(k)$.
- Evolve the metric fluctuations for $t > t_i(k)$ using the usual theory of cosmological perturbations.

Note: the matter correlation functions are given by partial derivatives of the **finite temperature partition function** with respect to T (density fluctuations) or R (pressure perturbations).

Extracting the Metric Fluctuations

Ansatz for the metric including cosmological perturbations and gravitational waves:

$$ds^2 = a^2(\eta)\big((1+2\Phi)d\eta^2 - [(1-2\Phi)\delta_{ij} + h_{ij}]dx^i dx^j\big) .$$

Inserting into the perturbed Einstein equations yields

$$\langle|\Phi(k)|^2\rangle = 16\pi^2 G^2 k^{-4} \langle \delta T^0{}_0(k) \delta T^0{}_0(k) \rangle ,$$

$$\langle|h(k)|^2\rangle = 16\pi^2 G^2 k^{-4} \langle \delta T^i{}_j(k) \delta T^i{}_j(k) \rangle .$$

Note: We assume the validity of the semi-classical Einstein equations in the far IR.

Extracting the Metric Fluctuations

Ansatz for the metric including cosmological perturbations and gravitational waves:

$$ds^2 = a^2(\eta)\big((1+2\Phi)d\eta^2 - [(1-2\Phi)\delta_{ij} + h_{ij}]dx^i dx^j\big) .$$

Inserting into the perturbed Einstein equations yields

$$\langle|\Phi(k)|^2\rangle = 16\pi^2 G^2 k^{-4} \langle\delta T^0{}_0(k)\delta T^0{}_0(k)\rangle ,$$

$$\langle|h(k)|^2\rangle = 16\pi^2 G^2 k^{-4} \langle\delta T^i{}_j(k)\delta T^i{}_j(k)\rangle .$$

Note: We assume the validity of the semi-classical Einstein equations in the far IR.

Computation of Fluctuations

S. Brahma, R.B. and S. Laliberte, arXiv:2107.11512

$$P(k) = k^3(\delta\Phi(k))^2 = 16\pi^2 G^2 k^2 T^2 C_V(R)$$

$$C_V(R) = \frac{\partial}{\partial T} E(R)$$

$$E = -\frac{\partial}{\partial \beta} \ln Z(\beta)$$

Matrix Theory Cosmology: Results

S. Brahma, R.B. and S. Laliberte, arXiv:2107.11512

Thermal fluctuations in the emergent phase $\rightarrow$

- Scale-invariant spectrum of curvature fluctuations
- With a Poisson contribution for UV scales.
- Scale-invariant spectrum of gravitational waves.

$\rightarrow$ BFSS matrix model yields emergent infinite space, emergent infinite time, emergent spatially flat metric and an emergent early universe phase with thermal fluctuations leading to scale-invariant curvature fluctuations and gravitational waves.

Note: Horizon problem automatically solved.

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Open Problems

- Include the effects of the fermionic sector.
- Understand **phase transition** to the expanding phase of Big Bang Cosmology.
- Understand the **emergence of GR** in the IR.
- Spectral indices?
- What about Dark Energy?
- Emergent low energy effective field theory for localized excitations.

Plan

Conclusions

- BFSS matrix model is a proposal for a non-perturbative definition of superstring theory. Consider a high temperature state of the BFSS model.
- $\rightarrow$ emergent time, 3D space and metric. Emergent space is spatially flat and infinite.
- **Thermal fluctuations** of the BFSS model $\rightarrow$ scale-invariant spectra of cosmological perturbations and gravitational waves.
- **Horizon problem, flatness problem** and **formation of structure problem** of Standard Big Bang Cosmology resolved without requiring inflation.
- Transition from an emergent phase to the radiation phase of expansion. No cosmological constant.

Overall Conclusions

- Inflation is not the only viable early universe scenario.
- Cosmological perturbation theory is the key tool to connect early universe physics with observations.
- EFT inevitably breaks down in the early universe.
- Starting from the BFSS matrix model in a finite temperature state, we can obtain emergent space-time, an emergent metric and an emergent early universe cosmology.
- Key prediction: blue tilt of the gravitational wave spectrum.