An analysis of entanglement harvesting beyond perturbation theory

Ireneo James Membrere[1,2,3,*]
(in collaboration with Adam Teixidó-Bonfill[1,2,3] and Eduardo Martín-Martínez[1,2,3])

[1]Department of Applied Mathematics, University of Waterloo, Waterloo, ON, N2L3G1, Canada
[2]Institute for Quantum Computing, University of Waterloo, Waterloo, ON, N2L3G1, Canada
[3]Perimeter Institute, Waterloo, ON, N2L 2Y5, Canada

[*]imembrere@uwaterloo.ca

Introduction

- QFT predicts the existence of correlations between different regions in a quantum field [1-2]
  - Entanglement harvesting protocol: entanglement between detectors is induced due to their interaction with a quantum field in its vacuum state [3]
  - Protocol has never been tested experimentally

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[4] C. Sabín, J. J. García-Ripoll, E. Solano, and J. León, “Dynamics of entanglement via propagating microwave photons”, Phys. Rev. B **81**, 184501 (2010).
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Introduction

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  - Entanglement harvesting protocol: entanglement between detectors is induced due to their interaction with a quantum field in its vacuum state [3]
  - Protocol has never been tested experimentally
- Some experimental proposals for realizing the entanglement harvesting protocol use superconducting circuits [4-5]
  - consist of superconducting qubits (detectors) coupled to a transmission line (electromagnetic field)

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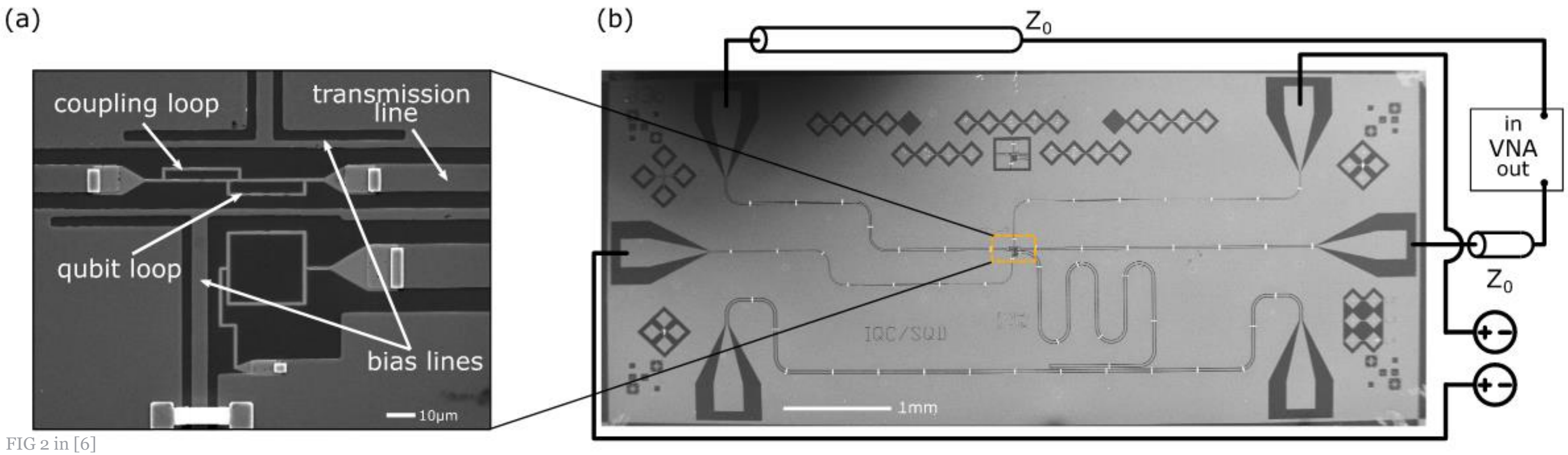

FIG 2 in [6]

- Ultrastrong coupling between superconducting qubit and transmission line has been demonstrated experimentally [6]
  - Regime outside validity of perturbation theory
  - Experiment in [5] utilizes ultrastrong coupling

[6] N. Janzen, X. Dai, S. Ren, J. Shi, and A. Lupascu, "Tunable coupler for mediating interactions between a two-level system and a waveguide from a decoupled state to the ultrastrong coupling regime", Phys. Rev. Res. **5**, 033155 (2023).

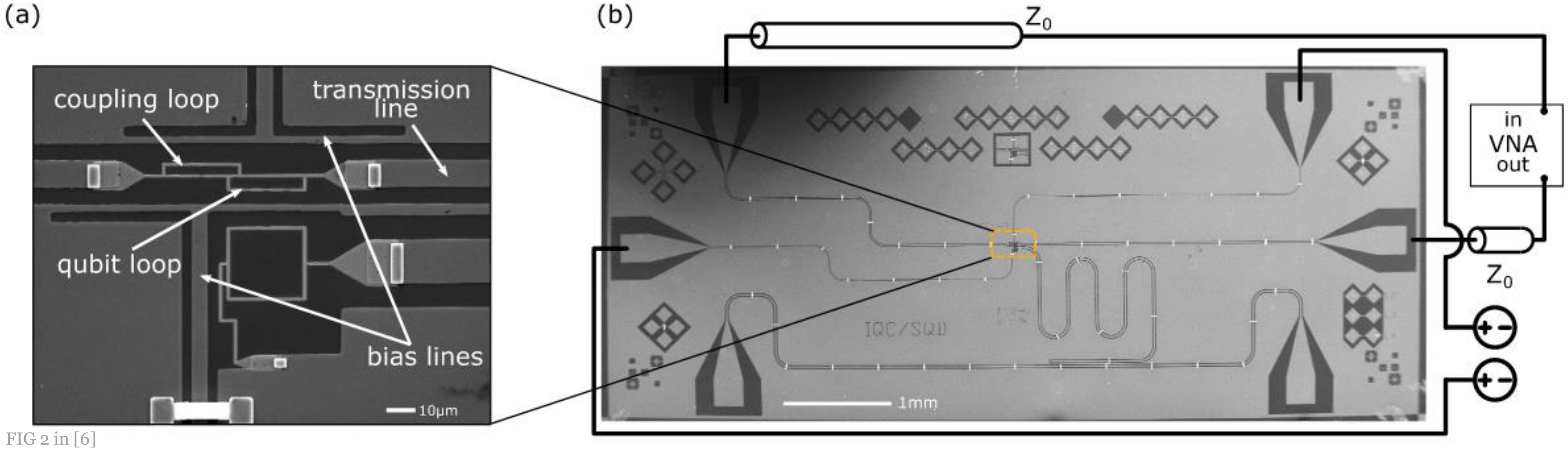

FIG 2 in [6]

- Ultrastrong coupling between superconducting qubit and transmission line has been demonstrated experimentally [6]
  - Regime outside validity of perturbation theory
  - Experiment in [5] utilizes ultrastrong coupling
- Goals of our study:
  1) Develop tools to model entanglement harvesting in regimes of current superconducting experiments: strong coupling, finite interaction time, field confined to a finite region
  2) Compare with perturbative model

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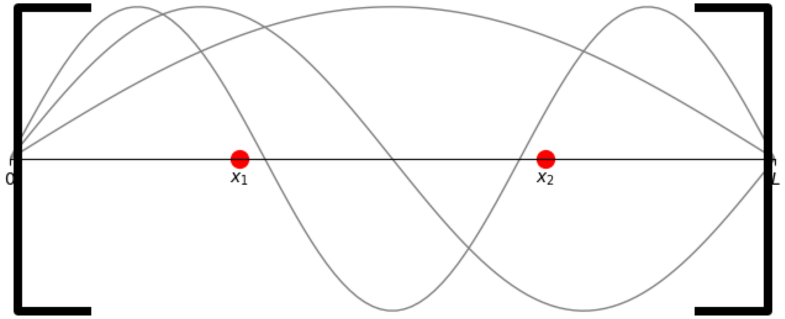
Setup

- Consider two stationary point-like detectors (bosonic modes e.g. harmonic oscillators) interacting with a scalar quantum field on 1+1-dimensional Minkowski spacetime confined to a cavity of length L



$$\hat{H}^H_{\text{free}} = \Omega_1 \hat{a}^\dagger_{d_1}\hat{a}_{d_1} + \Omega_2 \hat{a}^\dagger_{d_2}\hat{a}_{d_2} + \sum_k \omega_{n_k}\hat{a}^\dagger_{n_k}\hat{a}_{n_k}$$

$$\hat{H}^H_{\text{int}} = \sum_{j=1}^2 \sum_k \lambda_{jk}\chi_{jk}(t)(\hat{a}_{d_j} + \hat{a}^\dagger_{d_j})(\hat{a}_{n_k} v_{n_k}(x_j) + \hat{a}^\dagger_{n_k} v^*_{n_k}(x_j)) \ , u_n(t,x) = e^{-i\omega_n t} v_n(x)$$

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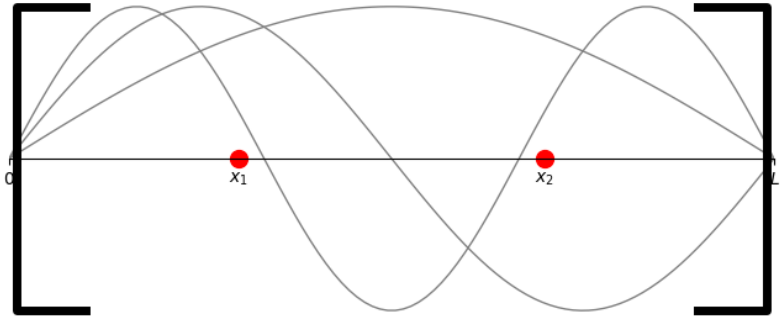

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- Dirichlet BCs: $u_n(t,x) = \frac{1}{\sqrt{\omega_n L}}e^{-i\omega_n t}\sin(k_n x), \omega_n = \sqrt{m^2 + k_n^2}, k_n = \frac{n\pi}{L}$
- Periodic BCs: $u_n(t,x) = \frac{1}{\sqrt{2\omega_n L}}e^{-i\omega_n t + ik_n x}, \omega_n = \sqrt{m^2 + k_n^2}, k_n = \frac{2n\pi}{L}$

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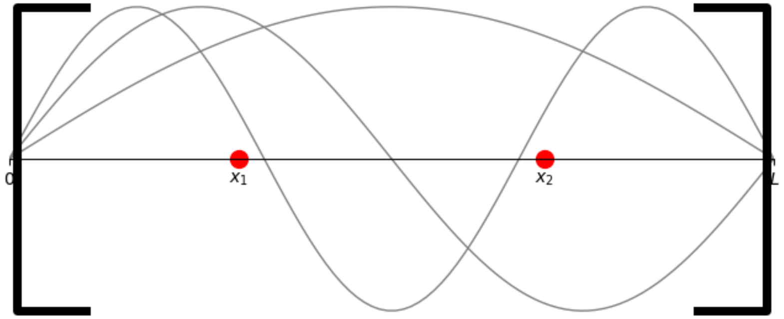

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- Simplifying assumptions:
  - Identical coupling constants and switching $\lambda_{jk} = \lambda, \chi_{jk}(t) = \chi(t), \forall j \forall k$
  - Initial state: $\hat{\rho}(t=0) = |0\rangle\langle 0|_{d_1} \otimes |0\rangle\langle 0|_{d_2} \otimes |0\rangle\langle 0|_{\hat{\phi}}$

Quantifying Entanglement

[7] E. Martín-Martínez, E. G. Brown, W. Donnelly, and A. Kempf, “Sustainable entanglement production from aquantum field”, Physical Review A **88**, 10.1103/physreva.88.052310 (2013).

Quantifying Entanglement

For a two-mode system AB, negativity is necessary and sufficient for the presence of entanglement [7]

$$\mathcal{N}(\hat{\rho}_{AB}) = \max\left(0, \sum_{\lambda_j \in \mathrm{Eig}(\hat{\rho}_{AB}^{\Gamma_A}), \lambda_j < 0} |\lambda_j|\right)$$

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Perturbative Methods

- At time $t > 0$, the evolution of the system can be calculated using the time evolution operator

$$\hat{\rho}(t) = \hat{U}(t)\hat{\rho}(t=0)\hat{U}^\dagger(t), \hat{U}(t) = \mathcal{T}\exp(-i\int_0^t dt' \hat{H}^I_{\text{int}}(t'))$$

[8] T. R. Perche, J. Polo-Gómez, B. d. S. L. Torres, and E. Martín-Martínez, "Fully relativistic entanglement harvesting", Physical Review D **109**, 10.1103/physrevd.109.045018 (2024).

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- As in [8], the time evolution operator can be expanded in a Dyson series giving the state of the detectors as

$$\hat{\rho}_{d_1 d_2}(t) = \mathrm{Tr}_{\hat{\phi}}(\hat{\rho}(t)) = \begin{bmatrix} M(t) & O_{7\times 2} \\ O_{2\times 7} & O_{2\times 2} \end{bmatrix} + \mathcal{O}(\lambda^4)$$

in the basis $\{|0\rangle_{d_1}, |1\rangle_{d_1}, |2\rangle_{d_1}\} \otimes \{|0\rangle_{d_2}, |1\rangle_{d_2}, |2\rangle_{d_2}\}$, where

$$M(t) = \begin{bmatrix} 1-\mathcal{L}_{11}(t)-\mathcal{L}_{22}(t) & 0 & \mathcal{K}_2(t)^* & 0 & \mathcal{M}(t)^* & 0 & \mathcal{K}_1(t)^* \\ 0 & \mathcal{L}_{22}(t) & 0 & \mathcal{L}_{12}(t)^* & 0 & 0 & 0 \\ \mathcal{K}_2(t) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \mathcal{L}_{12}(t) & 0 & \mathcal{L}_{11}(t) & 0 & 0 & 0 \\ \mathcal{M}(t) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \mathcal{K}_1(t) & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathcal{L}_{\mu\nu}(t) = \lambda^2 \int_0^t dt' \int_0^t dt'' W(t', x_\mu, t'', x_\nu) e^{-i\Omega_\mu t'}\chi(t') e^{i\Omega_\nu t''}\chi(t'')$$

$$\mathcal{M}(t) = -\lambda^2 \int_0^t dt' \int_0^t dt'' [\Theta(t'-t'')W(t', x_1, t'', x_2) + \Theta(t''-t')W(t'', x_2, t', x_1)] e^{i\Omega_1 t'}\chi(t') e^{i\Omega_2 t''}\chi(t'')$$

$$W(t, x, t', x') = \mathrm{Tr}(\hat{\rho}_{\hat{\phi}} \hat{\phi}(t,x)\hat{\phi}(t',x'))$$

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- Negativity $\mathcal{N}(t) = \max(0, \sqrt{|\mathcal{M}(t)|^2 + \frac{(\mathcal{L}_{11}(t)-\mathcal{L}_{22}(t))^2}{4}} - \frac{\mathcal{L}_{11}+\mathcal{L}_{22}}{2}) + \mathcal{O}(\lambda^4)$

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Non-Perturbative Methods

- For quadrature operators $\hat{q}_j = \frac{1}{\sqrt{2}}(\hat{a}_j + \hat{a}_j^\dagger), \hat{p}_j = \frac{i}{\sqrt{2}}(\hat{a}_j^\dagger - \hat{a}_j), j \in \{d_1, d_2, n_1, n_2, ..., n_N\}$, the Hamiltonian is quadratic

$$\hat{H}^H = \hat{\vec{x}}^T F_{\text{free}} \hat{\vec{x}} + \hat{\vec{x}}^T F_{\text{int}} \hat{\vec{x}} \qquad \hat{\vec{x}} = [\hat{q}_{d_1}, \hat{p}_{d_1}, \hat{q}_{d_2}, \hat{p}_{d_2}, \hat{q}_{n_1}, \hat{p}_{n_1}, ..., \hat{q}_{n_N}, \hat{p}_{n_N}]^T$$

where

$$F_{\text{free}} = \frac{1}{2}\text{diag}(\Omega_1, \Omega_1, \Omega_2, \Omega_2, \omega_{n_1}, \omega_{n_1}, ..., \omega_{n_N}, \omega_{n_N})$$

$$F_{\text{int}} = \begin{bmatrix} O_{4\times 4} & B^T \\ B & O_{2N\times 2N} \end{bmatrix} \qquad B = \begin{bmatrix} \vec{b}_1 & \vec{0} & \vec{b}_2 & \vec{0} \end{bmatrix}$$

$$\vec{b}_j = \lambda\chi(t)[\Re(v_{n_1}(x_j)), -\Im(v_{n_1}(x_j)), ..., \Re(v_{n_N}(x_j)), -\Im(v_{n_N}(x_j))]^T$$

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- At $t = 0$ the system is in a Gaussian state, therefore it will remain in a Gaussian state for $t > 0$
  - State of the system is completely characterized by first and second quadrature moments [9]

$$\xi_0^\mu(t) = \langle \hat{x}^\mu \rangle_{\hat{\rho}(t)}, \sigma^{\mu\nu} = \langle \hat{x}^\mu \hat{x}^\nu + \hat{x}^\nu \hat{x}^\mu \rangle_{\hat{\rho}(t)} - 2\langle \hat{x}^\mu \rangle_{\hat{\rho}(t)} \langle \hat{x}^\nu \rangle_{\hat{\rho}(t)}$$

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- Initial condition $\hat{\rho}(t=0) = |0\rangle\langle 0|_{d_1} \otimes |0\rangle\langle 0|_{d_2} \otimes |0\rangle\langle 0|_{\hat{\phi}}$ corresponds to

$$\sigma(t=0) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \oplus \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \oplus \bigoplus_{k=1}^{N} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \vec{\xi}_0(t) = \vec{0}\ \forall t$$

[10] W. G. Brenna, E. G. Brown, R. B. Mann, and E. Martín-Martínez, "Universality and thermalization in the unruh effect", Physical Review D **88**, 10.1103/physrevd.88.064031 (2013).
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- At time $t > 0$, covariance matrix evolves under symplectic transformation

$$\sigma(t) = S(t)\sigma(t=0)S(t)^T \qquad S(t) = S_0(t)S^I(t)$$

  - These symplectic matrices are generated by the Hamiltonian [10]

$$S_0(t) = \exp\left(\Omega(F_{\text{free}} + F_{\text{free}}^T)t\right)$$

$$\frac{dS^I(t)}{dt} = K_1^I(t)S^I(t), S^I(0) = I \qquad K_1^I(t) = (S_0(t))^{-1}K_1(t)S_0(t), K_1(t) = \Omega(F_{\text{int}} + F_{\text{int}}^T)$$

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- Negativity [11] $\mathcal{N}(t) = \frac{2^{E_N(t)}-1}{2}, E_N(t) = \max\left(0, -\log_2\sqrt{(\Delta - \sqrt{\Delta^2 - 4\det\sigma_d})/2}\right)$

$$\Delta = \det\sigma_1 + \det\sigma_2 - 2\det\sigma_{12}, \sigma_d = \begin{bmatrix} \sigma_1 & \sigma_{12} \\ \sigma_{12}^T & \sigma_2 \end{bmatrix}$$

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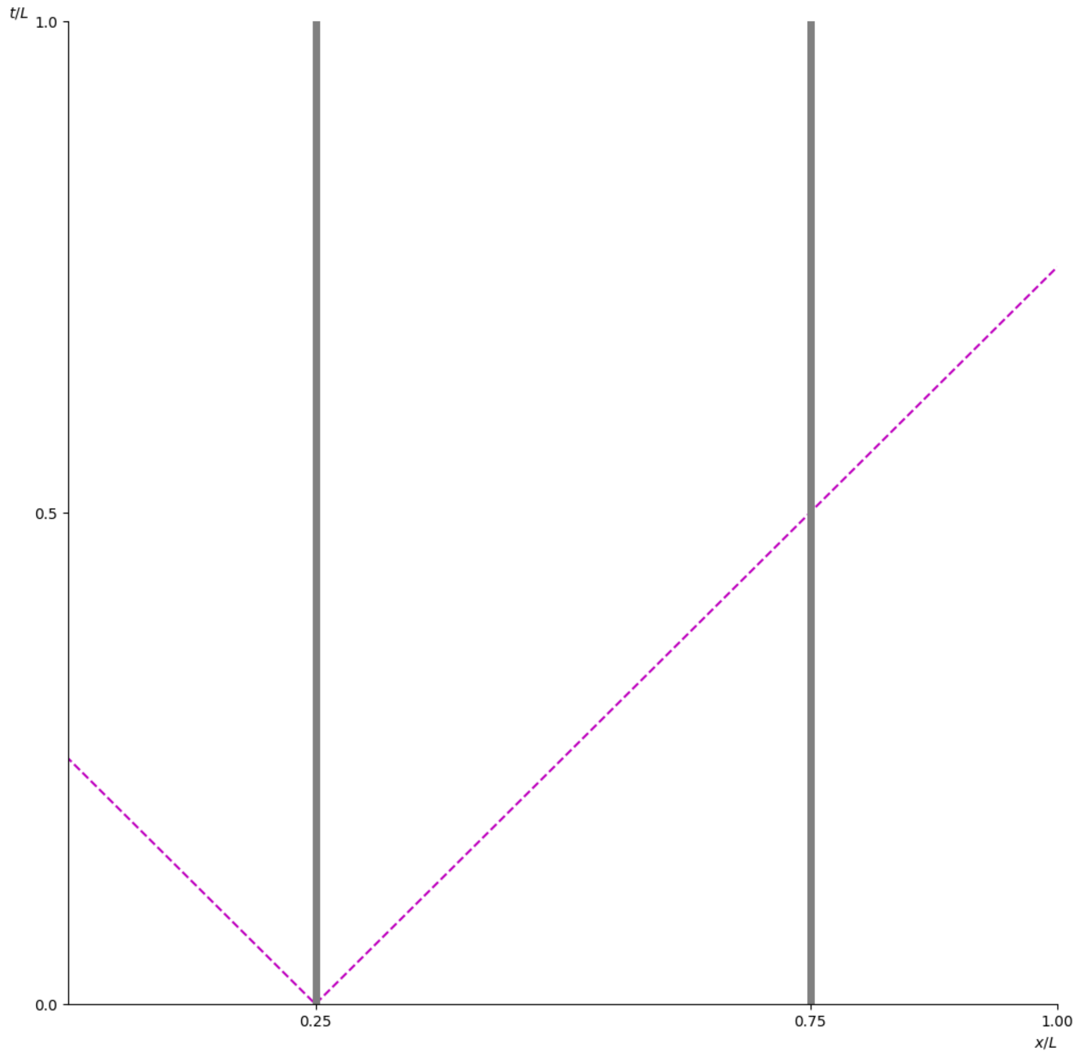
Results

- Detector parameters:

$\Omega_1 L$	25
$\Omega_2 L$	25
x_1/L	$1/4$
x_2/L	$3/4$



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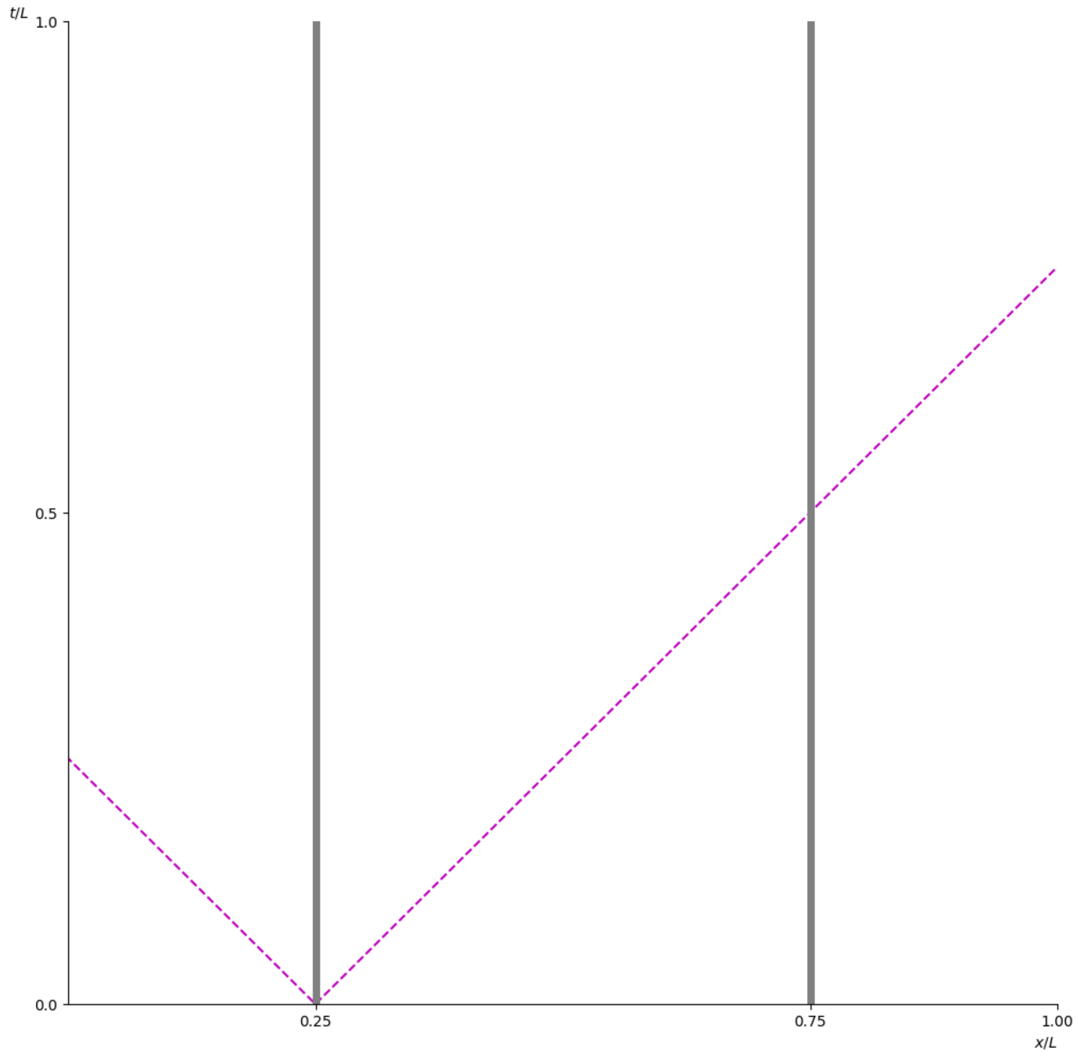
- Field parameters:

L	2π
m	0
Boundary Conditions	**Periodic**



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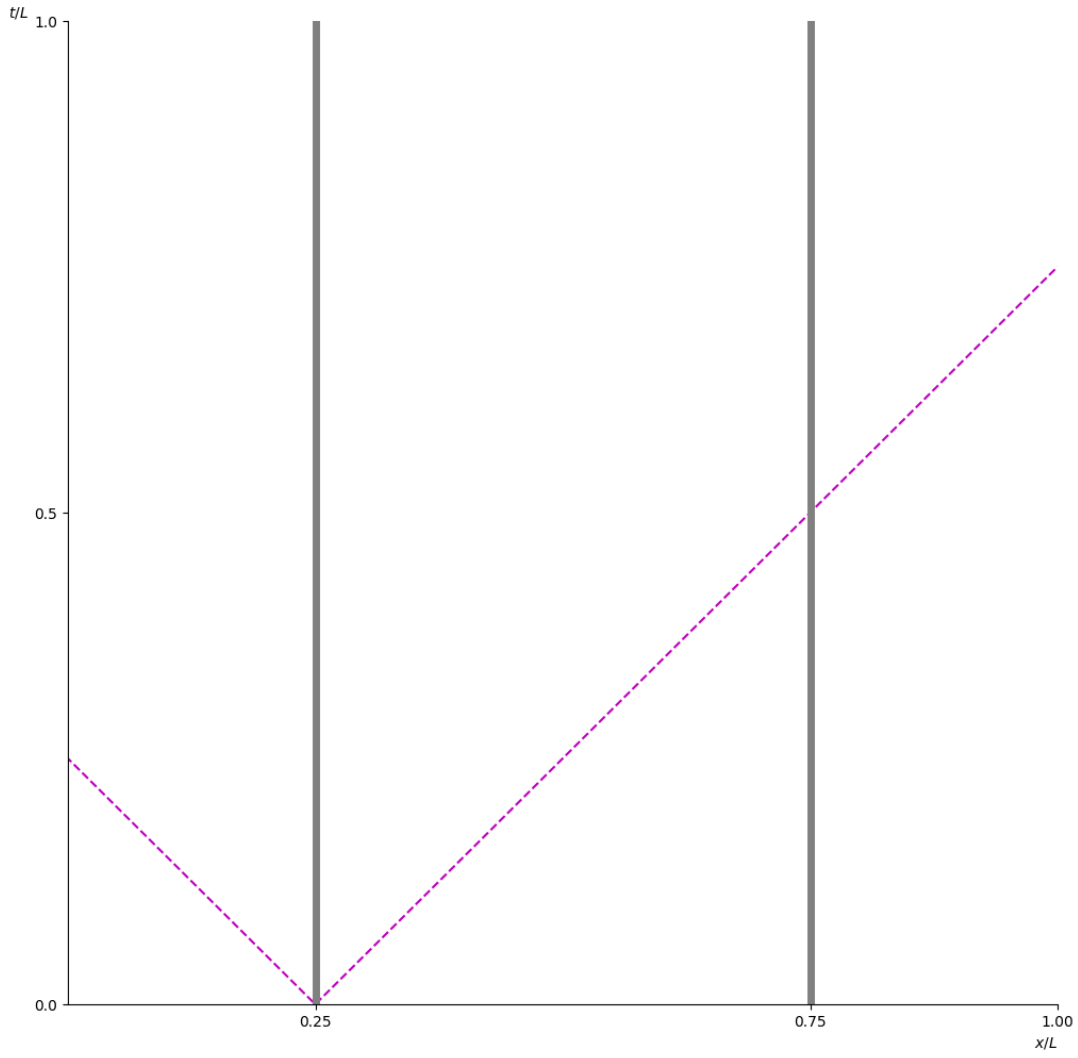
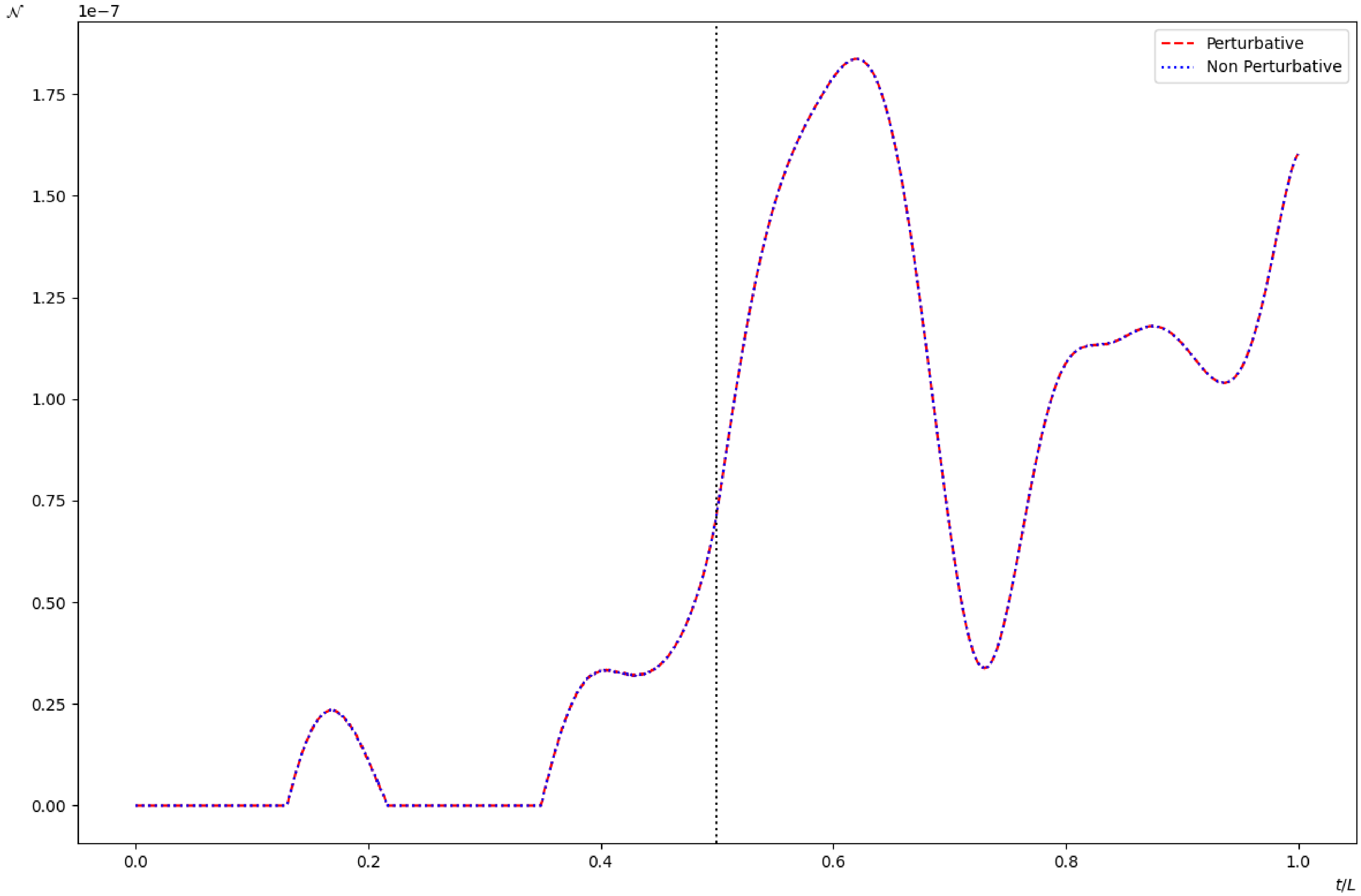
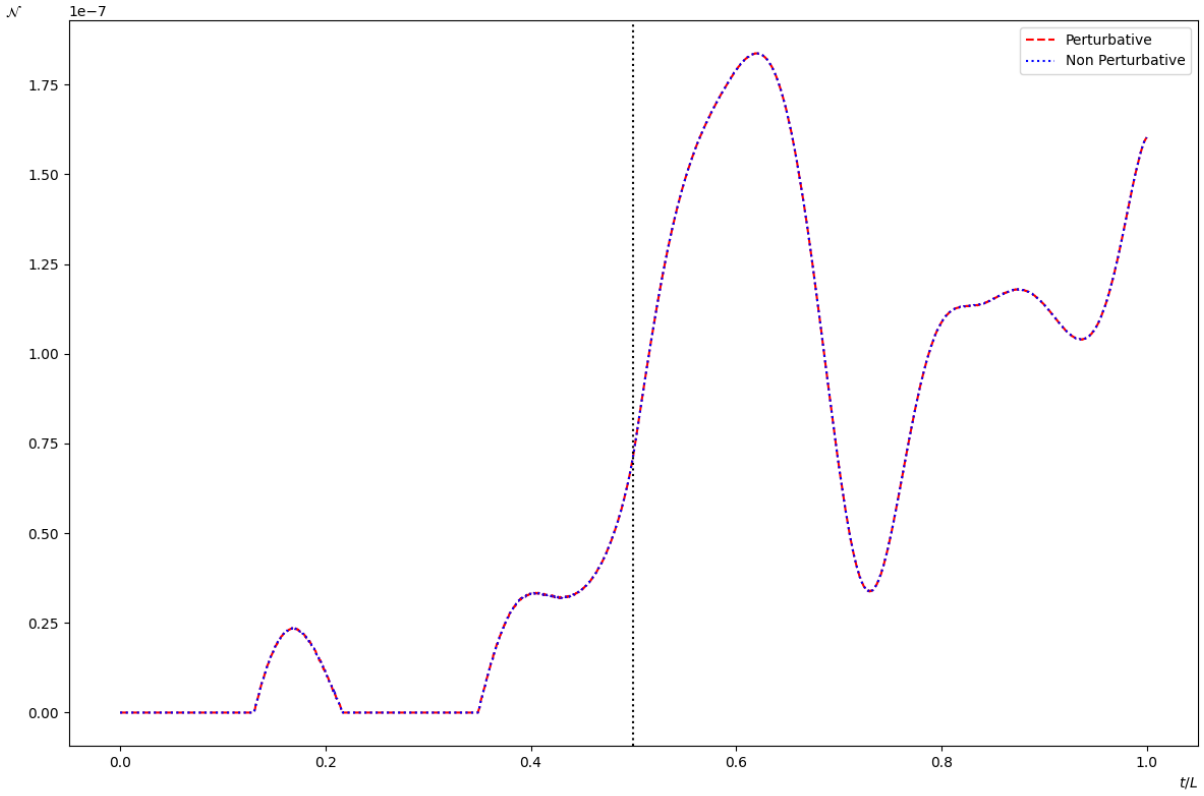
- Sharp switching $\chi(t) = \begin{cases} 1 & \text{if } t \in [0, L] \\ 0 & \text{otherwise} \end{cases}$

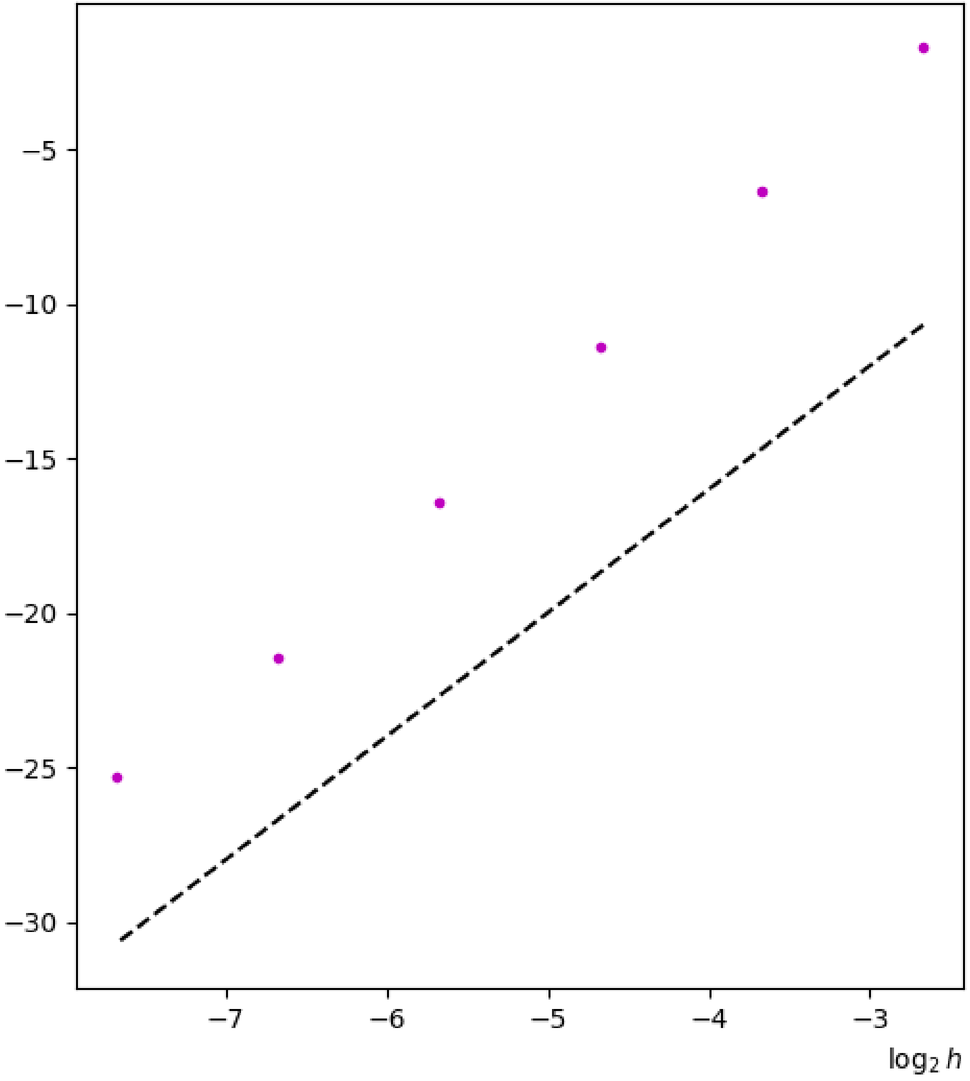




$$\lambda L = 1/100$$



$\log_2 \|\sigma_d^{(2h)}(t=0.18L) - \sigma_d^{(h)}(t=0.18L)\|_F$



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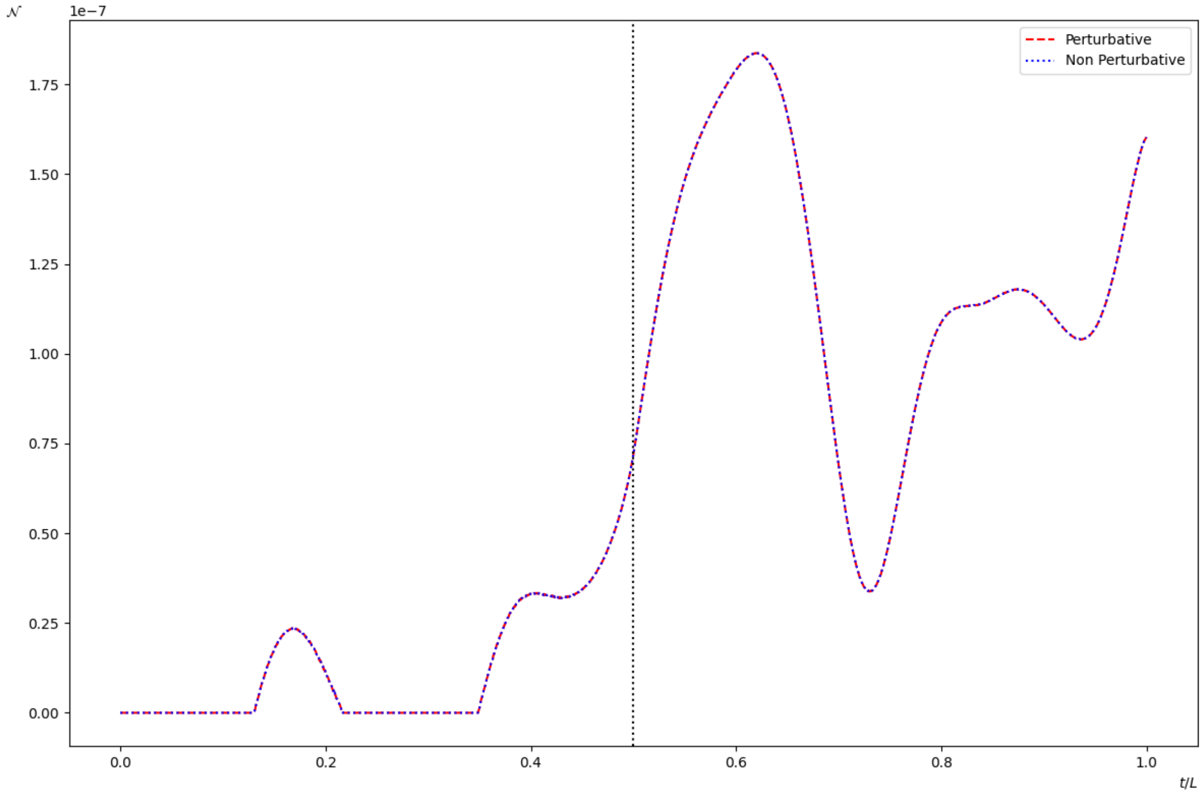
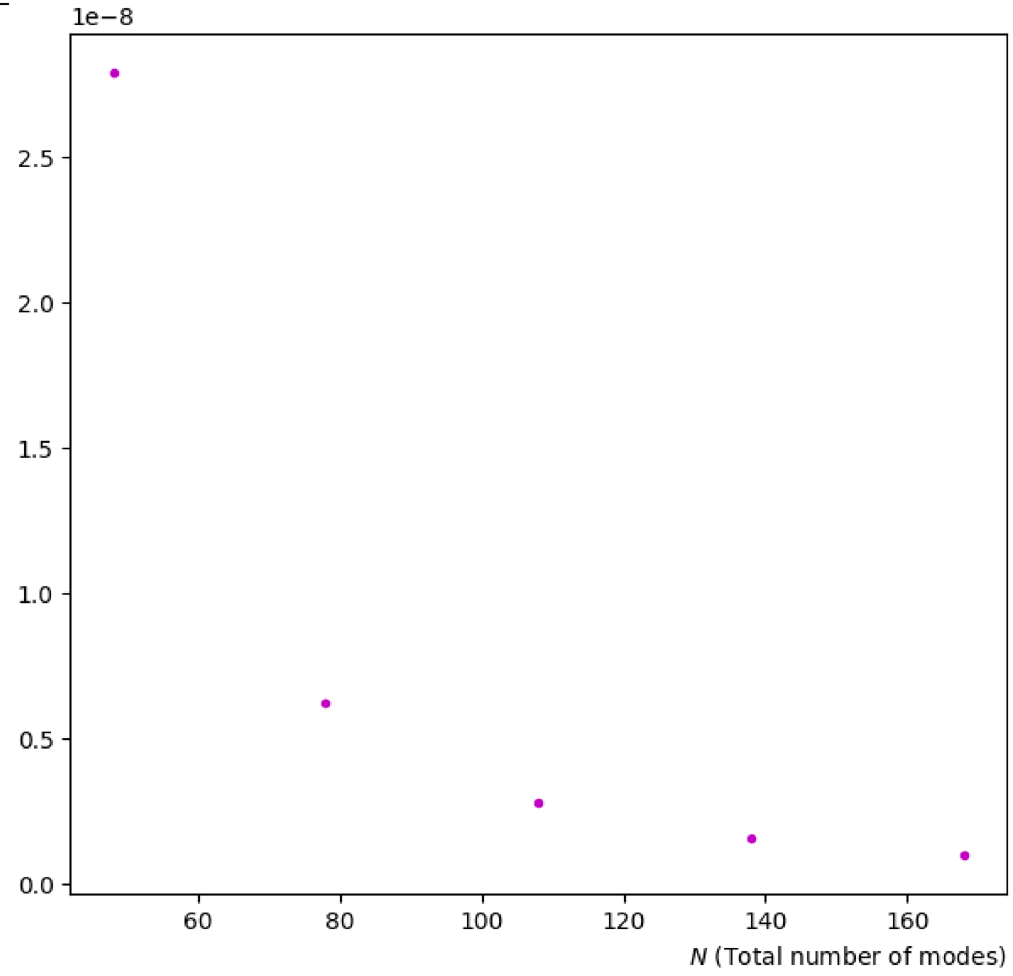


$$\frac{\|\sigma_d^{(N)}(t=0.18L) - \sigma_d^{(N-\Delta N)}(t=0.18L)\|_F}{\|\sigma_d^{(N-\Delta N)}(t=0.18L)\|_F}$$



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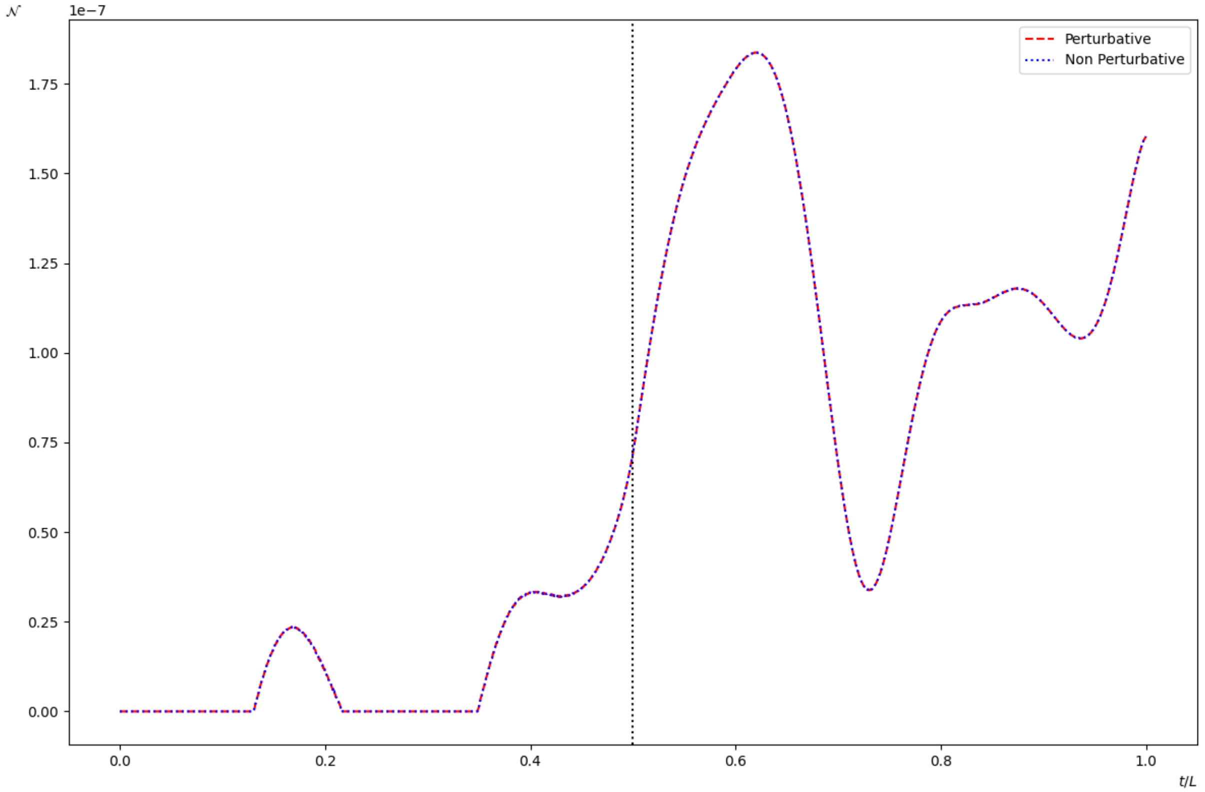
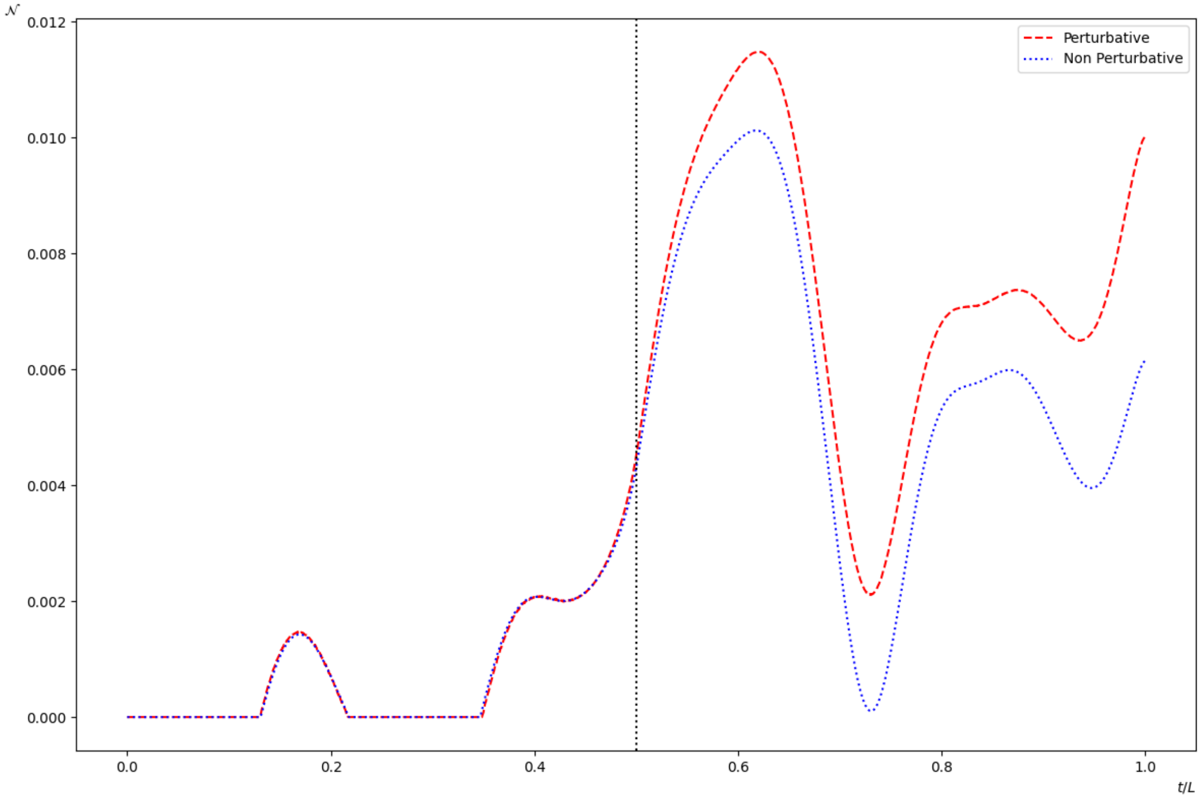
$\Omega_1 L = \Omega_2 L = 25$

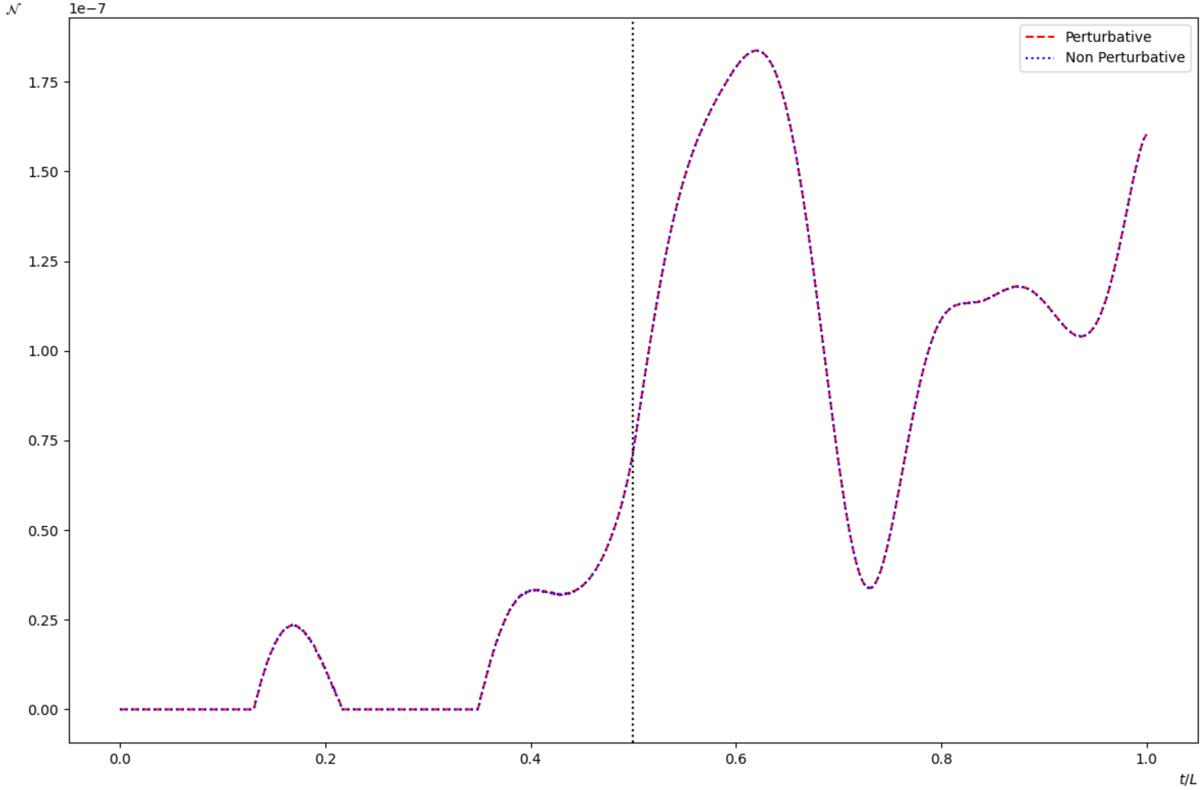


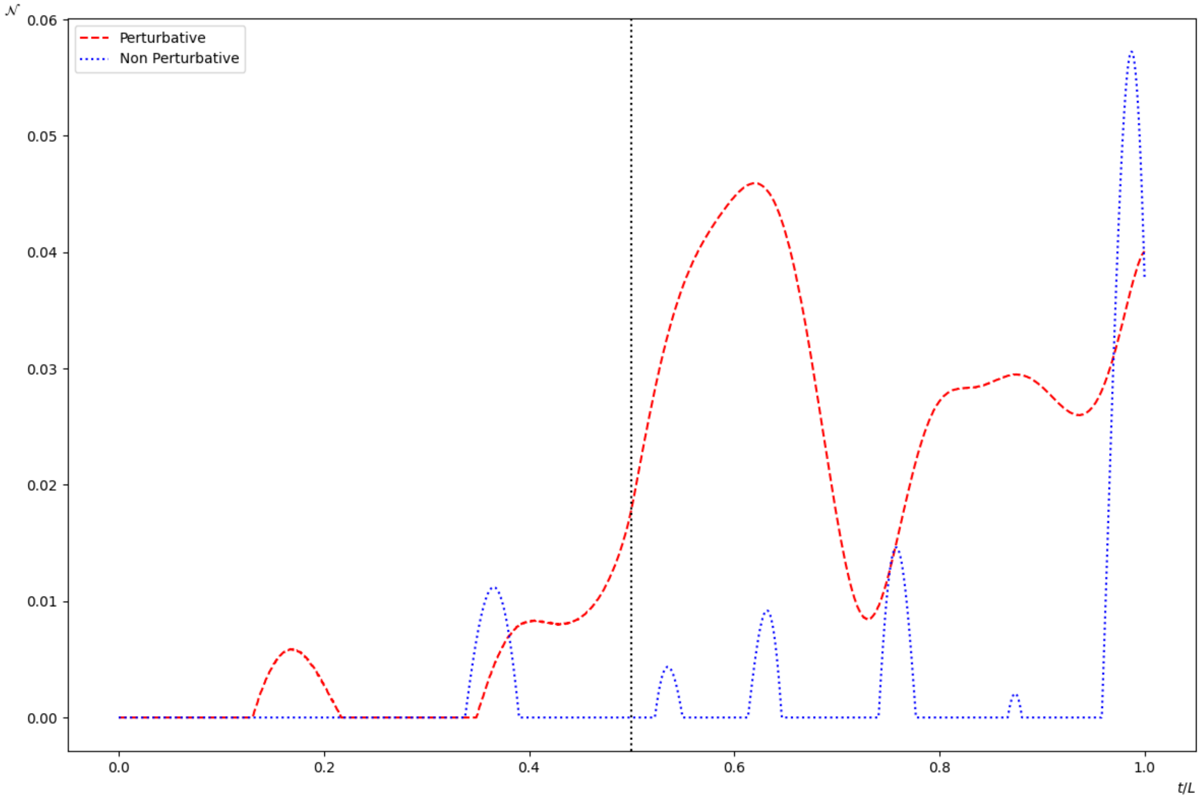


$\lambda L = 1/100$

$\lambda L = 2.5$

$$\Omega_1 L = \Omega_2 L = 25$$

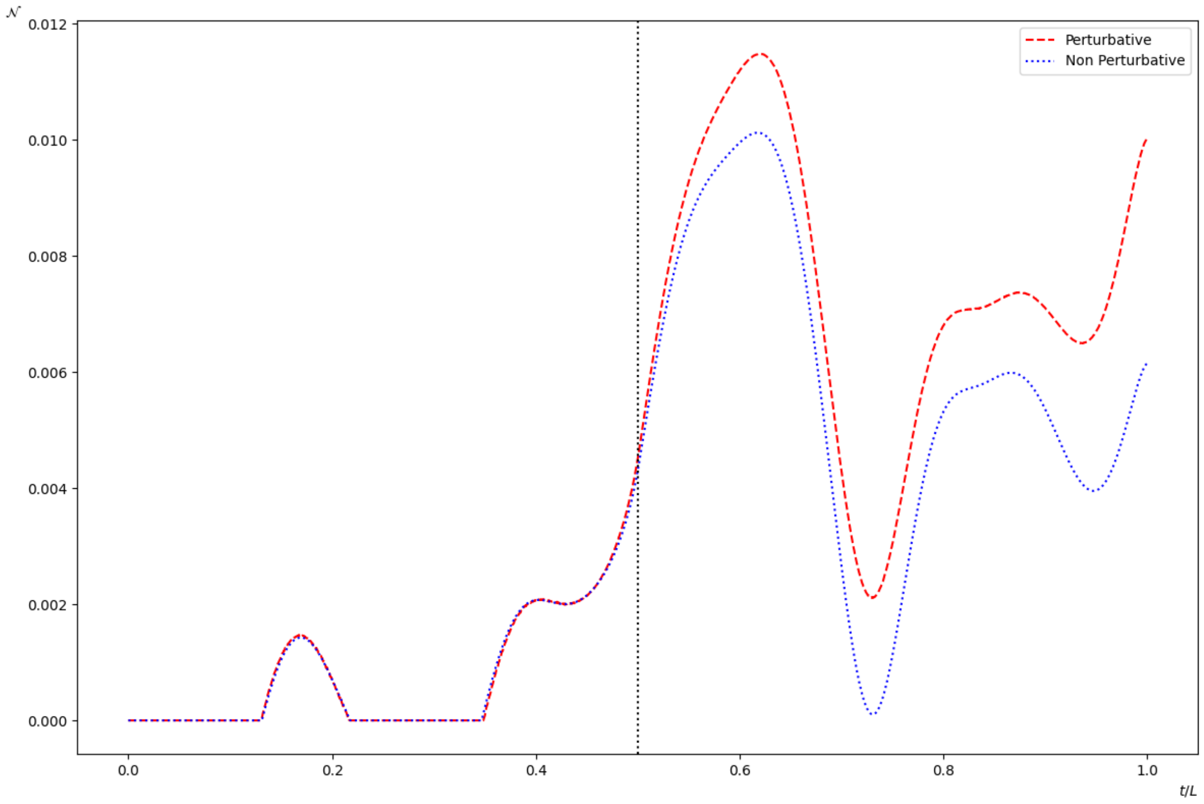


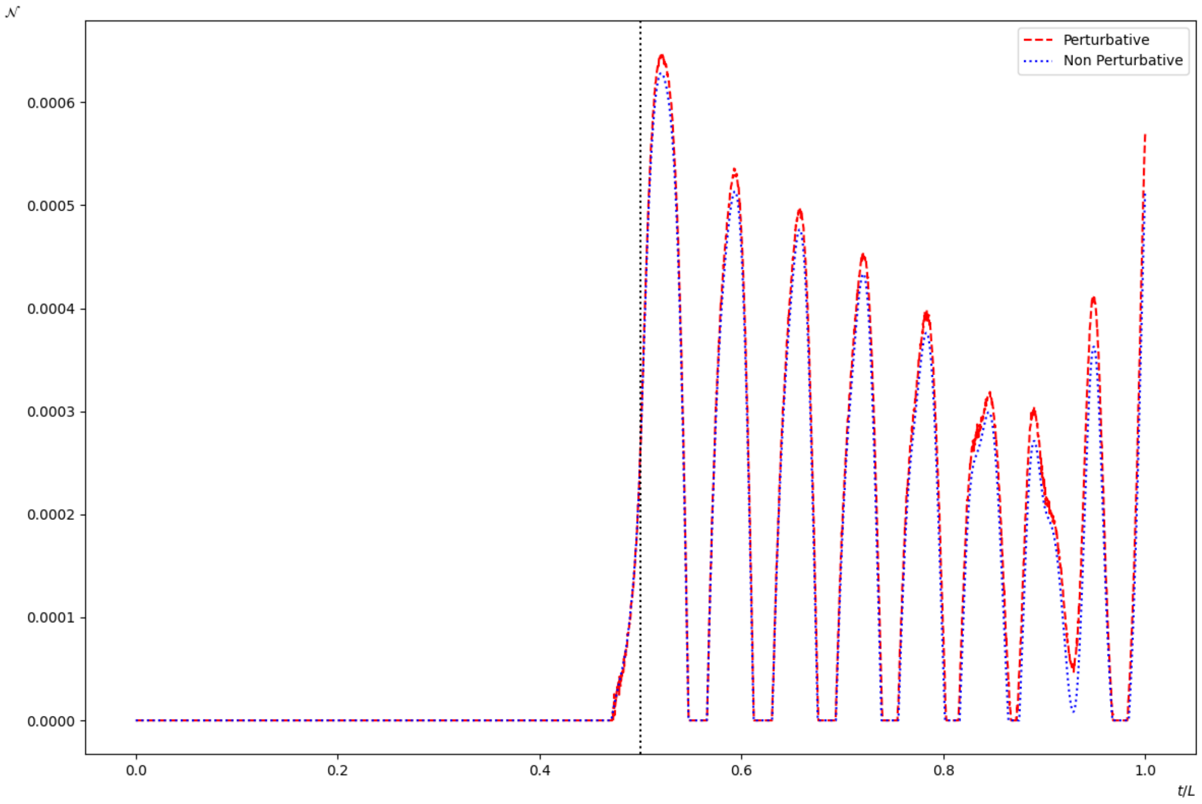


$$\lambda L = 1/100$$

$$\lambda L = 10$$

$\lambda L = 2.5$





$\Omega_1 L = \Omega_2 L = 25$

$\Omega_1 L = \Omega_2 L = 100$

Concluding Remarks

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