Using Adaptiveness and Causal Superpositions Against Noise in Quantum Metrology

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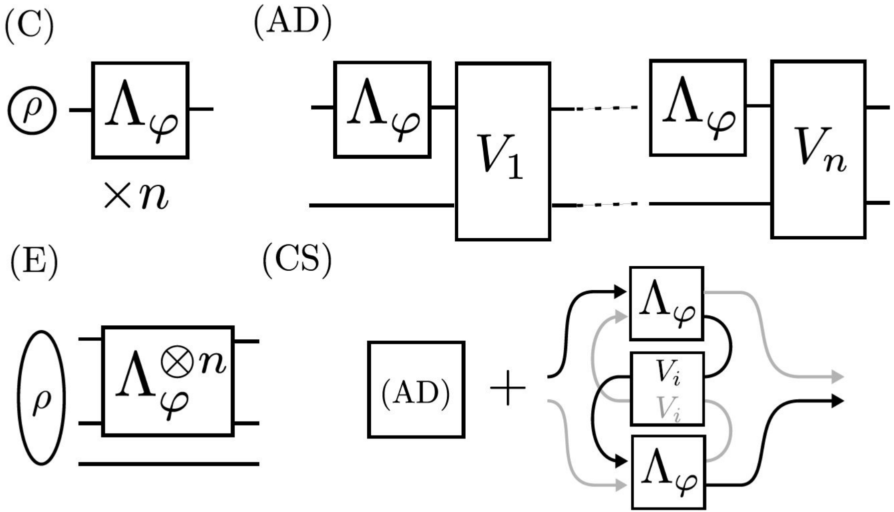

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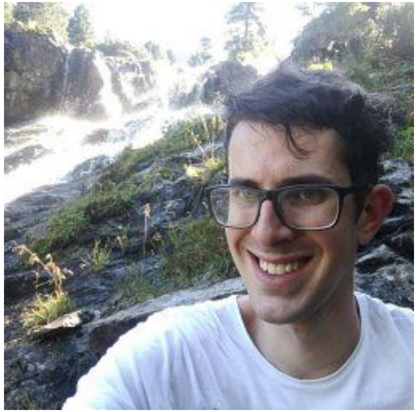
Phys. Rev. Lett. 131, 090801

Francesco Albarelli

Rafał Demkowicz-Dobrzański



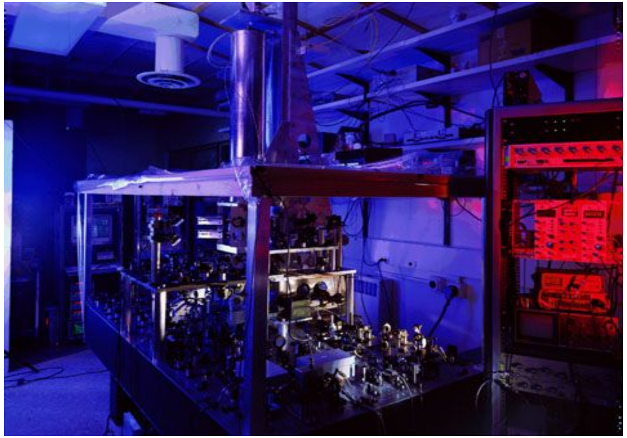
Wojciech Górecki

Quantum metrology

Gravitational wave detector (LIGO)

$\Delta L/L \approx 10^{-23}$

Cesium Fountain atomic clock

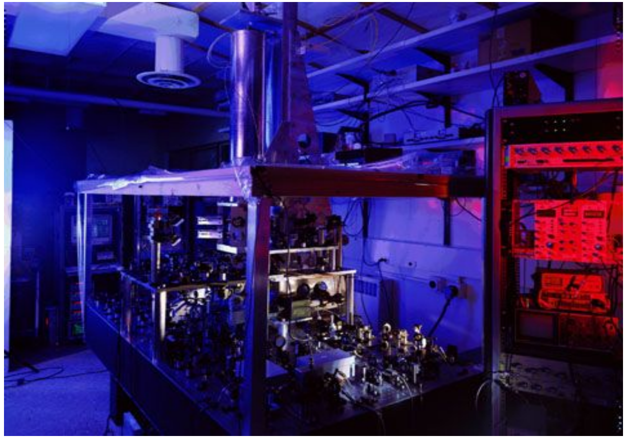


$\Delta t/t \approx 10^{-16}$

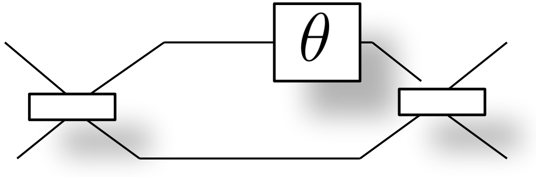
What are the precision limits?

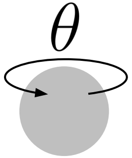
Quantum metrology (but for theorists)

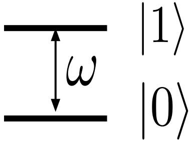
Michelson interferometer



Ramsey interferometry

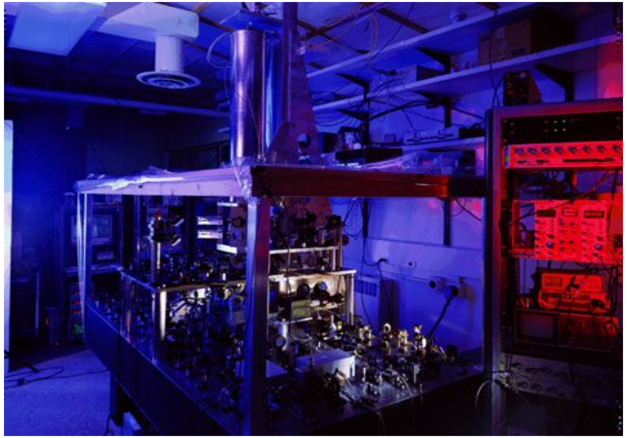






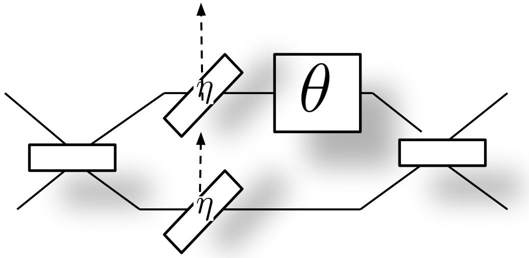
Quantum metrology (but for slightly less naive theorists)

Michelson interferometer

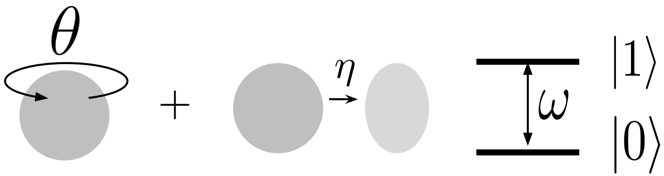


Ramsey interferometry

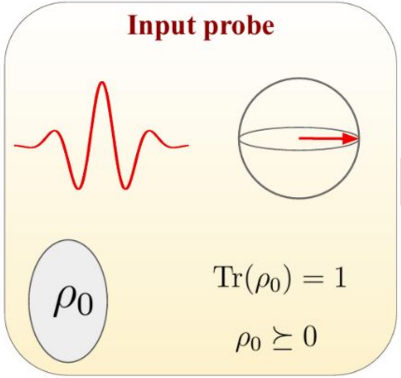
Quantum channel



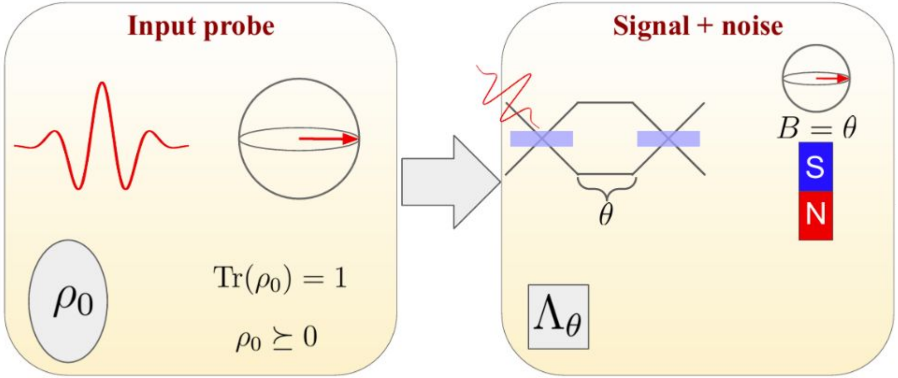
$$= \Lambda_\theta =$$



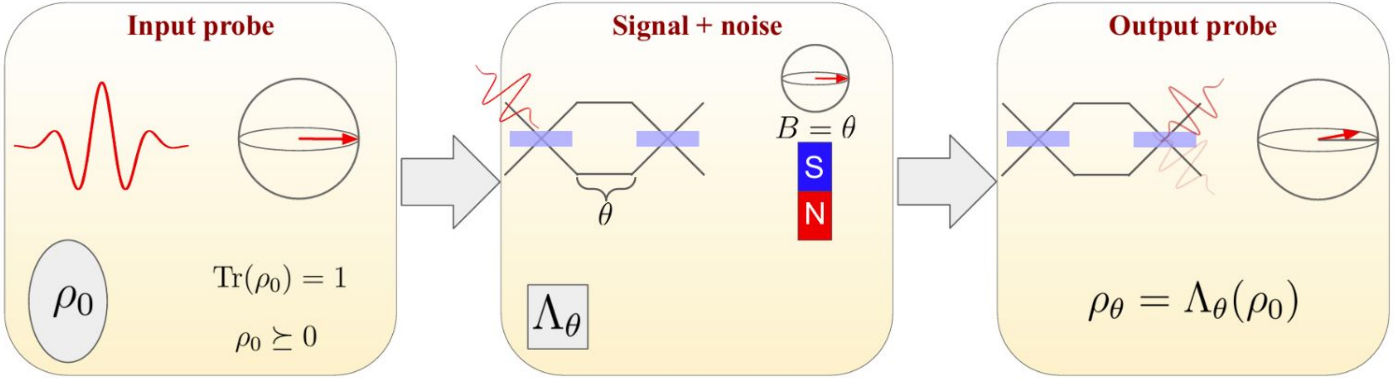
Quantum metrology: quantum channel estimation



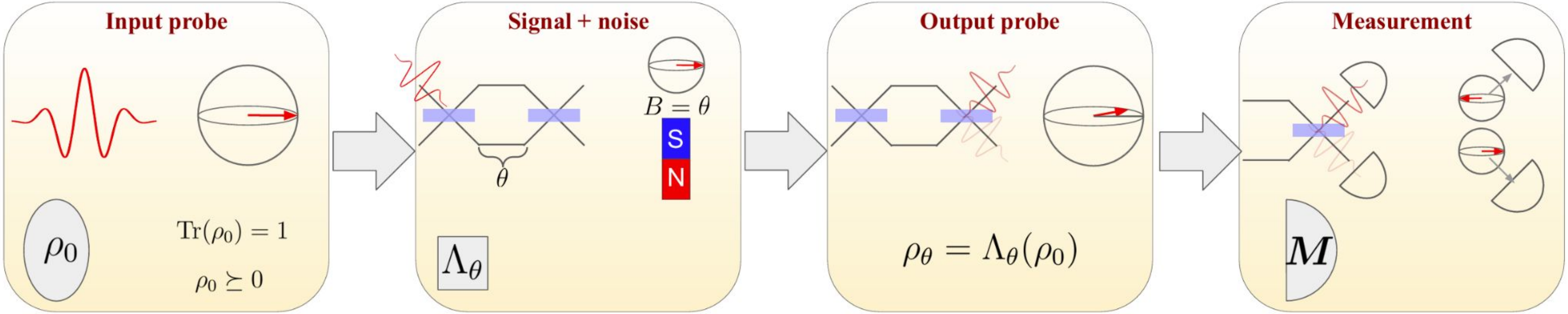
Quantum metrology: quantum channel estimation



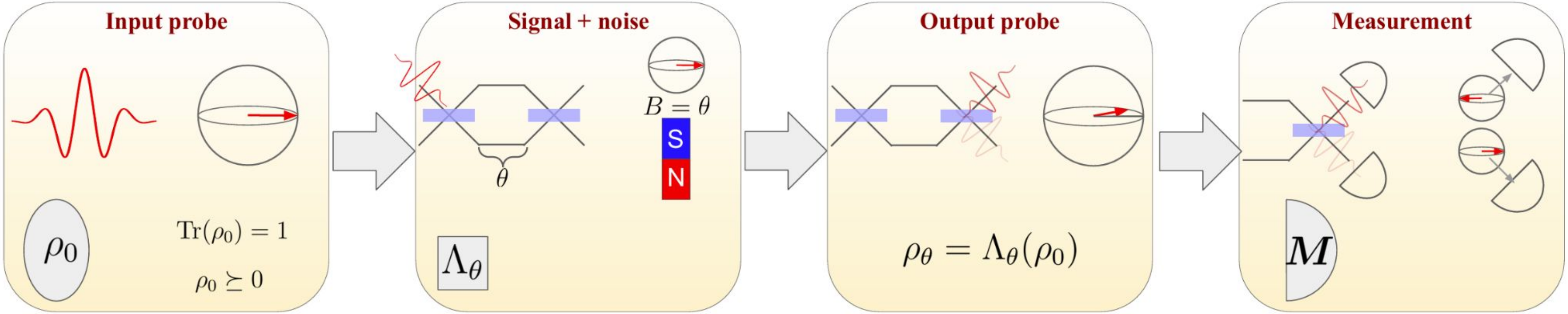
Quantum metrology: quantum channel estimation



Quantum metrology: quantum channel estimation



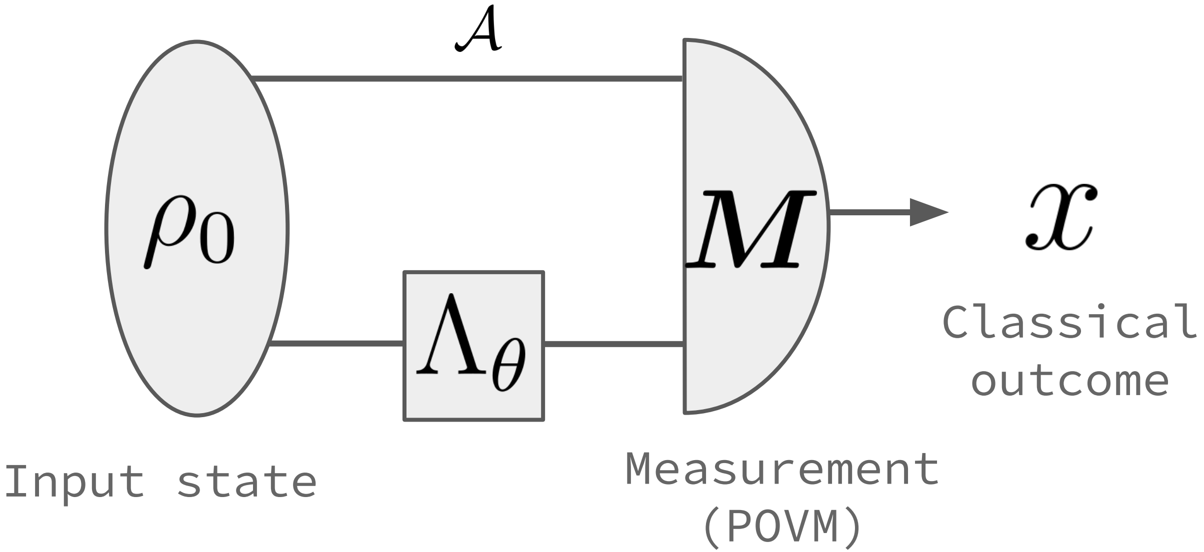
Quantum metrology: quantum channel estimation



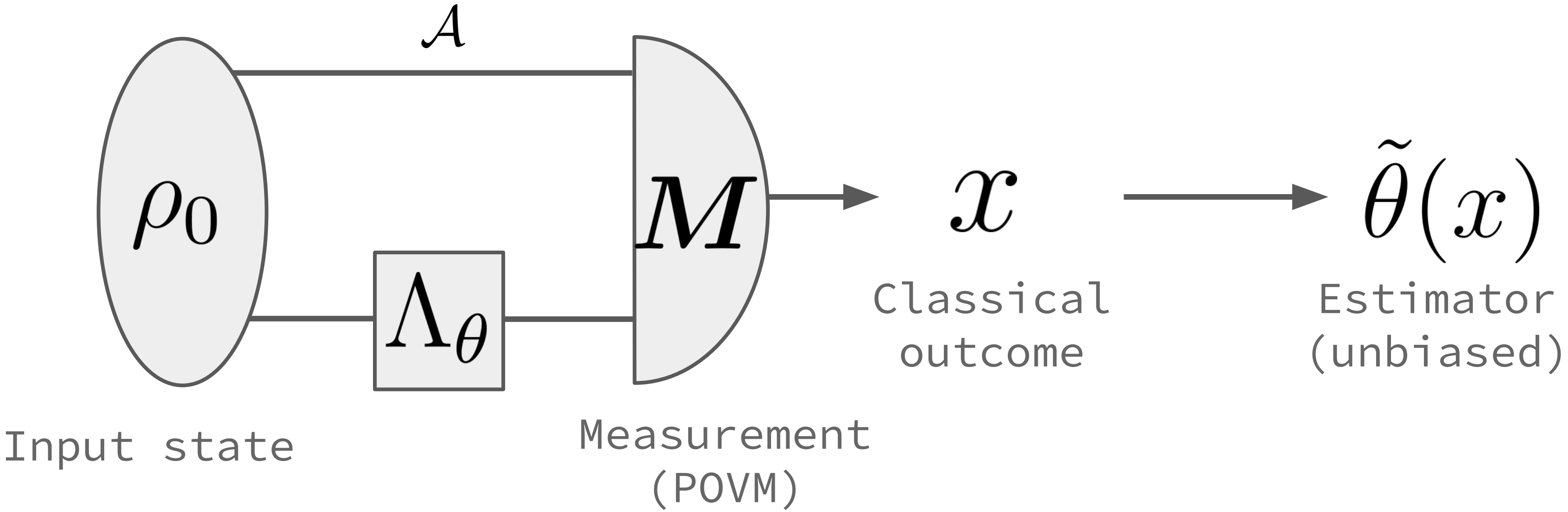
What is the best precision of θ estimation?

$$\min_{\rho_0, \mathbf{M}, \tilde{\theta}} \Delta\tilde{\theta} = ?$$

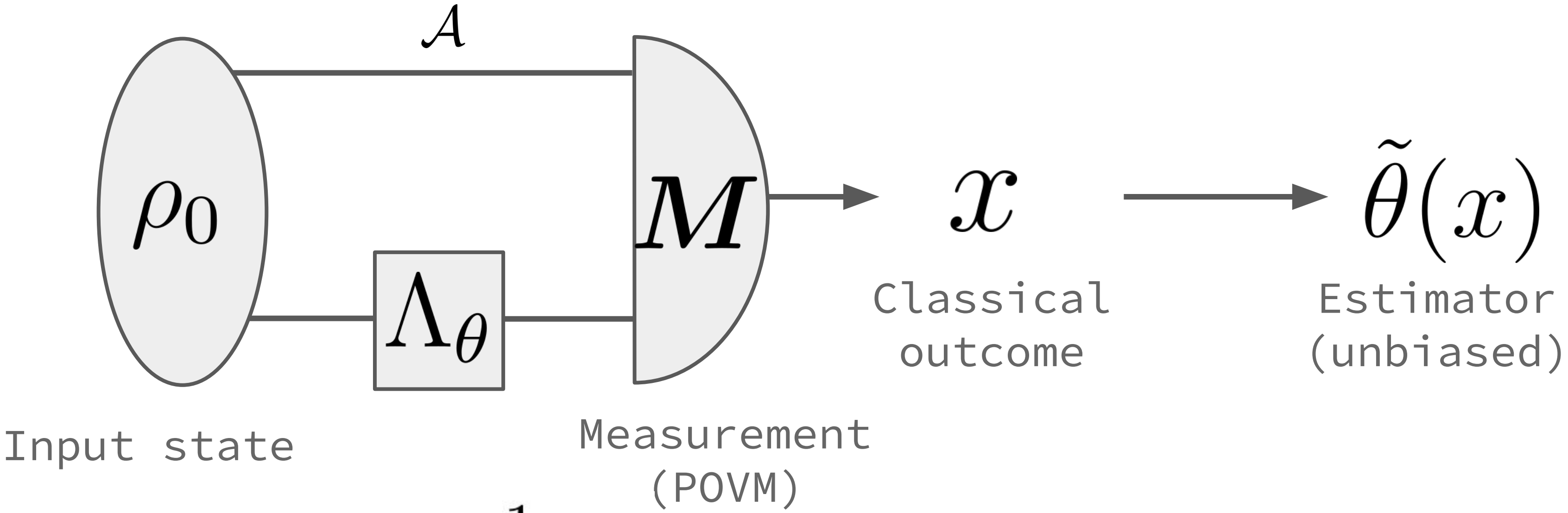
Quantum metrology = quantum channel estimation



Quantum metrology = quantum channel estimation

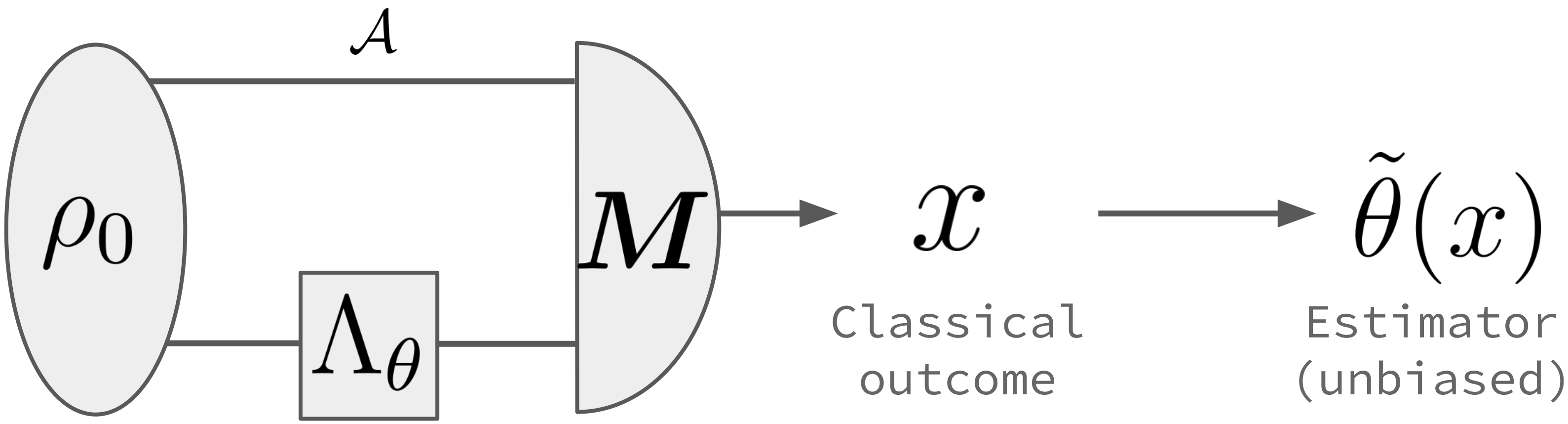


Quantum metrology = quantum channel estimation



$$\min_{\rho_0, \mathbf{M}, \tilde{\theta}} \Delta^2 \tilde{\theta} = \frac{1}{F(\Lambda_\theta)}$$

Quantum metrology = quantum channel estimation

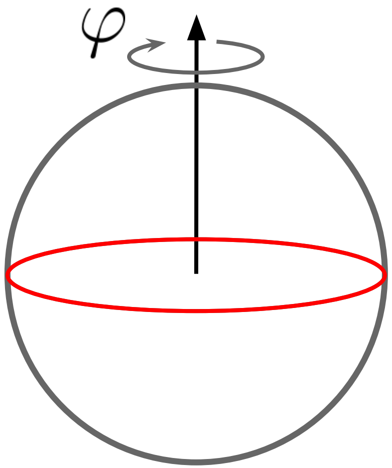


$$\min_{\rho_0, \mathbf{M}, \tilde{\theta}} \Delta^2 \tilde{\theta} = \frac{1}{F(\Lambda_\theta)}$$

$$F(\Lambda_\theta) = 4 \min_{\{K_k\}} \| \sum_k \dot{K}_k^\dagger \dot{K}_k \|$$

Kraus representations

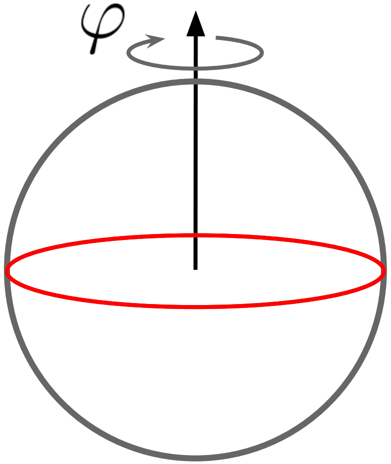
Example: Phase estimation



$U_\varphi = e^{-i\varphi\sigma_z/2}$

$F_Q^{(1)} = 1$

Example: Phase estimation



$$U_\varphi = e^{-i\varphi\sigma_z/2}$$

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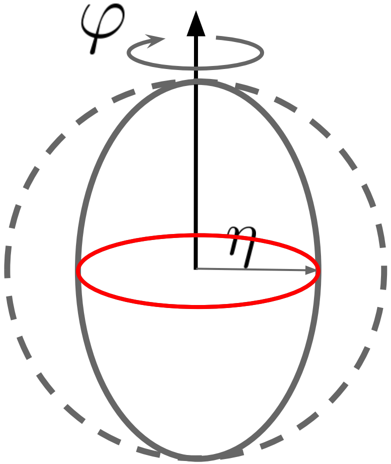
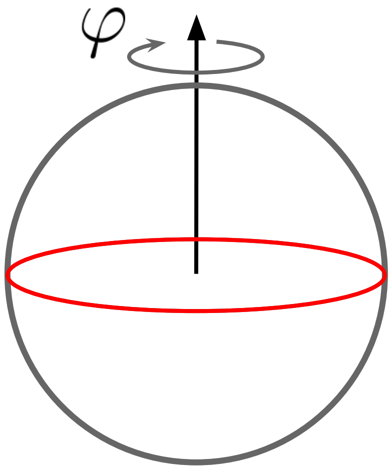


$$K_0 = U_\varphi \sqrt{\frac{1+\eta}{2}}\mathbb{I}$$

$$K_1 = U_\varphi \sqrt{\frac{1-\eta}{2}}\sigma_z$$

$$F_Q^{(1)} = \eta^2$$

Example: Phase estimation



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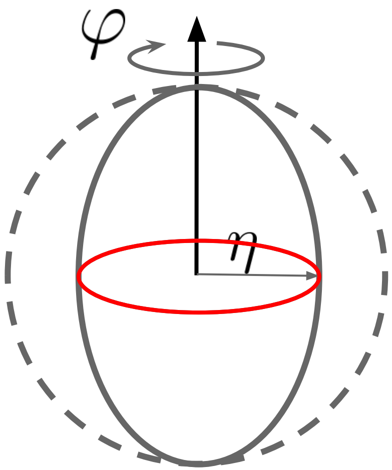

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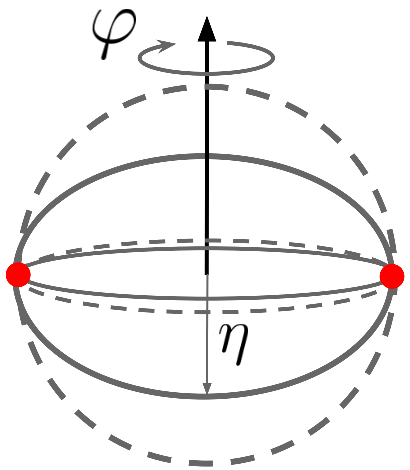
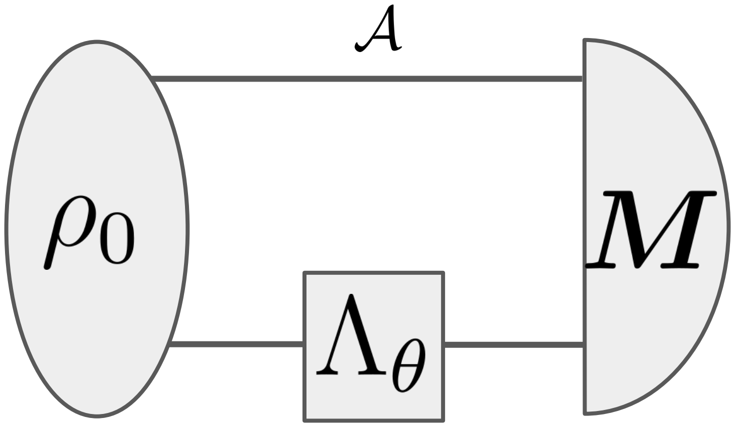


$$K_0 = U_\varphi \sqrt{\frac{1+\eta}{2}}\mathbb{I}$$

$$K_1 = U_\varphi \sqrt{\frac{1-\eta}{2}}\sigma_x$$

$$F_Q^{(1)} = 1$$

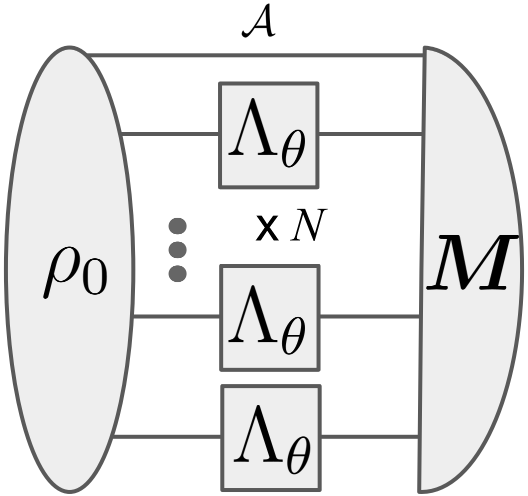
Quantum metrology = quantum channels estimation



x N

$$\min \Delta^2 \tilde{\theta} = \frac{1}{N F(\Lambda_\theta)}$$

Quantum metrology: quantum channels estimation

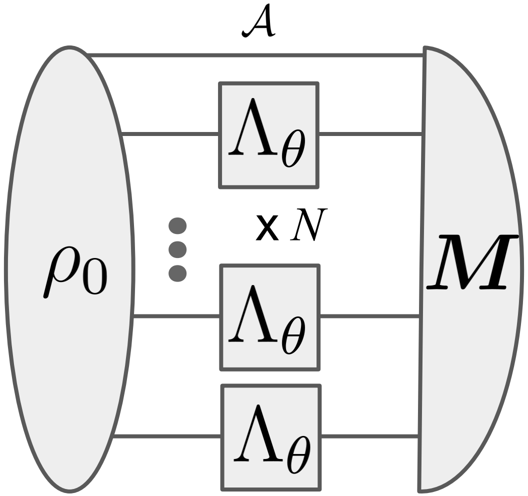


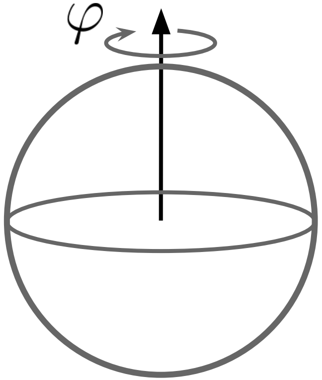
$$\mathcal{F}_{\mathrm{E}}^{(N)}(\Lambda_\theta) \equiv \mathcal{F}(\Lambda_\theta^{\otimes N}) \geq N\mathcal{F}(\Lambda_\theta)$$

(usually >)

R. Demkowicz-Dobrzański, J. Kołodyński, M. Guta, Nat. Comm. 2012
S. Zhou, L. Jiang, PRX Quantum 2021

Quantum metrology: quantum channels estimation





$$|\psi_0\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle^{\otimes N} + |1\rangle^{\otimes N}\right)$$

$$F_Q^{(N)} = N^2$$

$$\mathcal{F}_{\mathrm{E}}^{(N)}(\Lambda_\theta) \equiv \mathcal{F}(\Lambda_\theta^{\otimes N}) \geq N\mathcal{F}(\Lambda_\theta)$$

(usually >)

$\mathcal{F}_{\mathrm{E}}^{(N)}(\Lambda_\theta) \sim N^2$: Heisenberg scaling

R. Demkowicz-Dobrzański, J. Kołodyński, M. Guta, Nat. Comm. 2012

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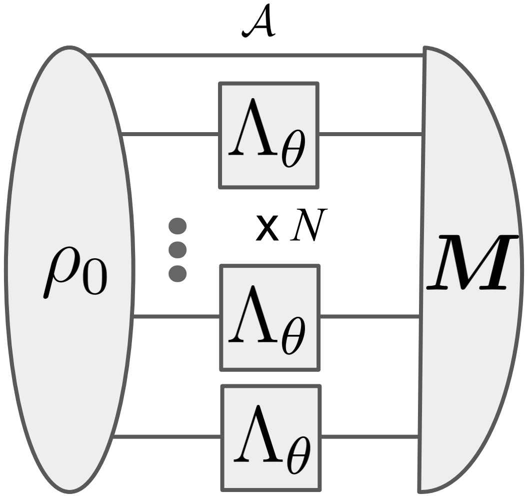
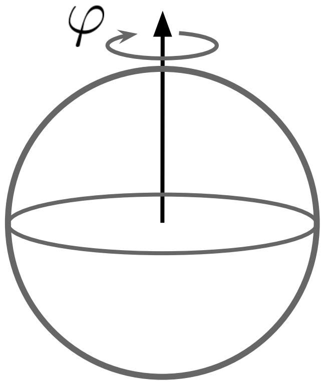
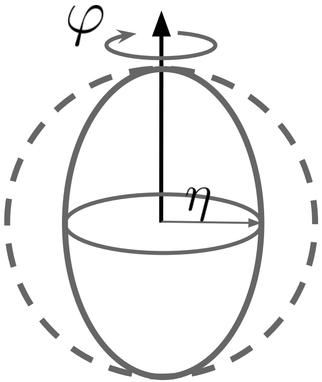
Quantum metrology: quantum channels estimation







$$\mathcal{F}_{\mathrm{E}}^{(N)}(\Lambda_\theta) \equiv \mathcal{F}(\Lambda_\theta^{\otimes N}) \geq N\mathcal{F}(\Lambda_\theta)$$

(usually >)

$\mathcal{F}_{\mathrm{E}}^{(N)}(\Lambda_\theta) \sim N^2$: Heisenberg scaling

$\mathcal{F}_{\mathrm{E}}^{(N)}(\Lambda_\theta) \sim N$: standard scaling

$$|\psi_0\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle^{\otimes N} + |1\rangle^{\otimes N}\right)$$

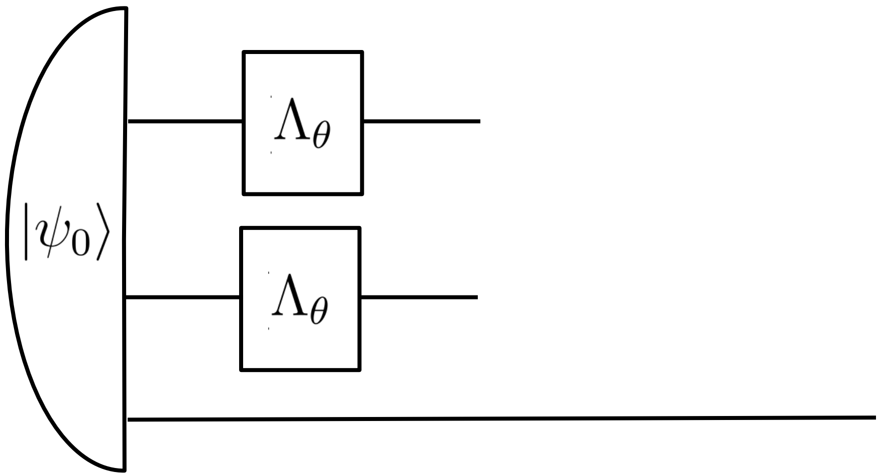
$$F_Q^{(N)} = N^2$$

$|\psi_0\rangle$:squeezed state

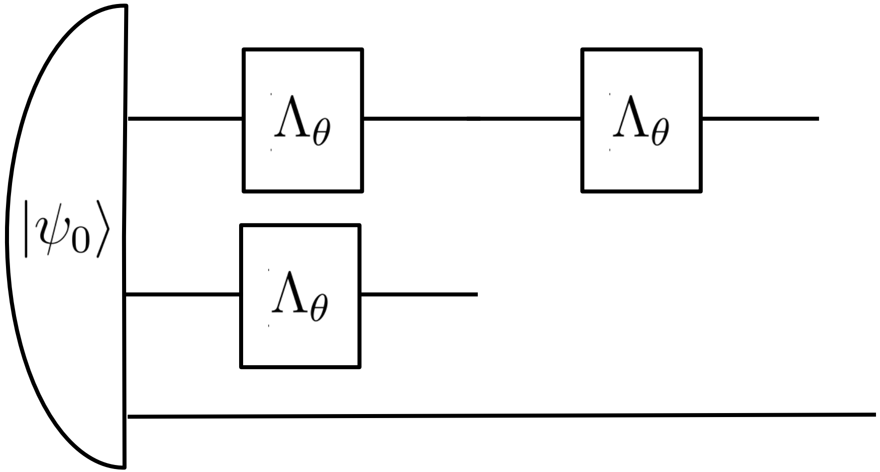
$$F_Q^{(N)}/N \to \frac{\eta^2}{1-\eta^2}$$

R. Demkowicz-Dobrzański, J. Kołodyński, M. Guta, Nat. Comm. 2012
S. Zhou, L. Jiang, PRX Quantum 2021

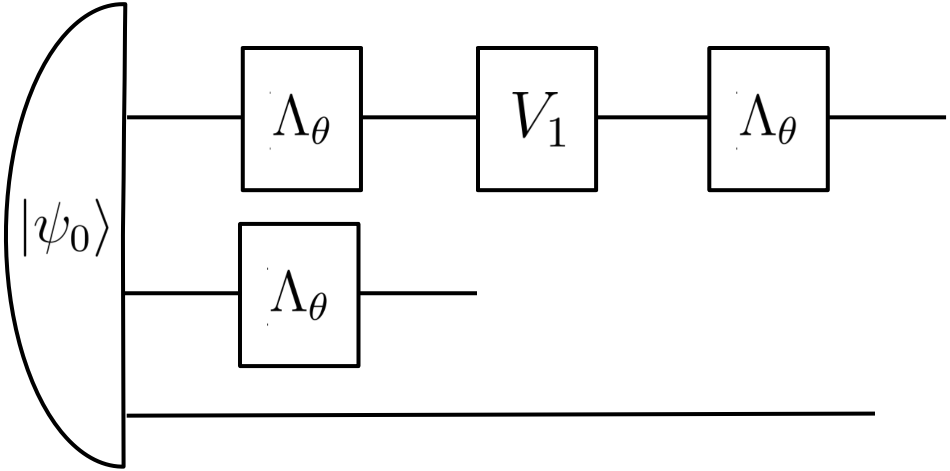
General many channel schemes



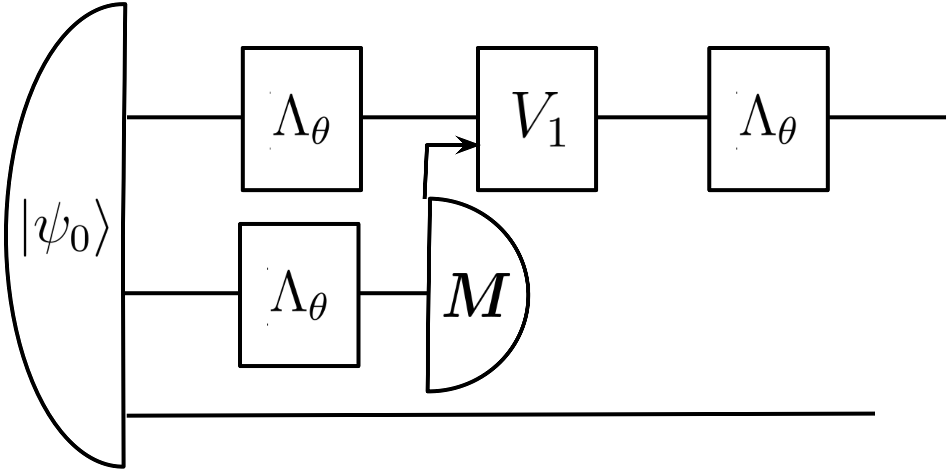
General many channel schemes



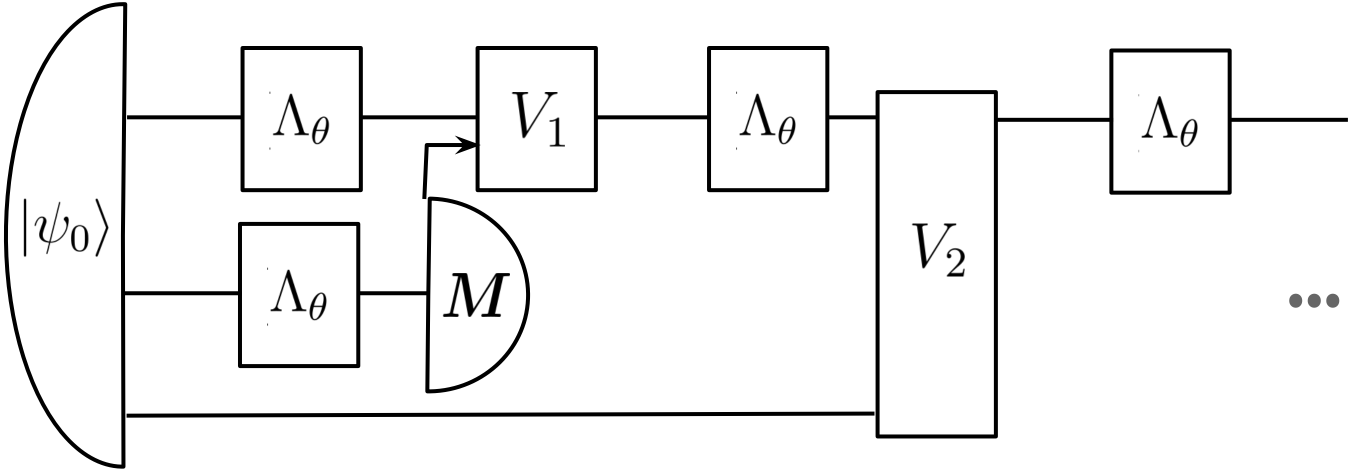
General many channel schemes



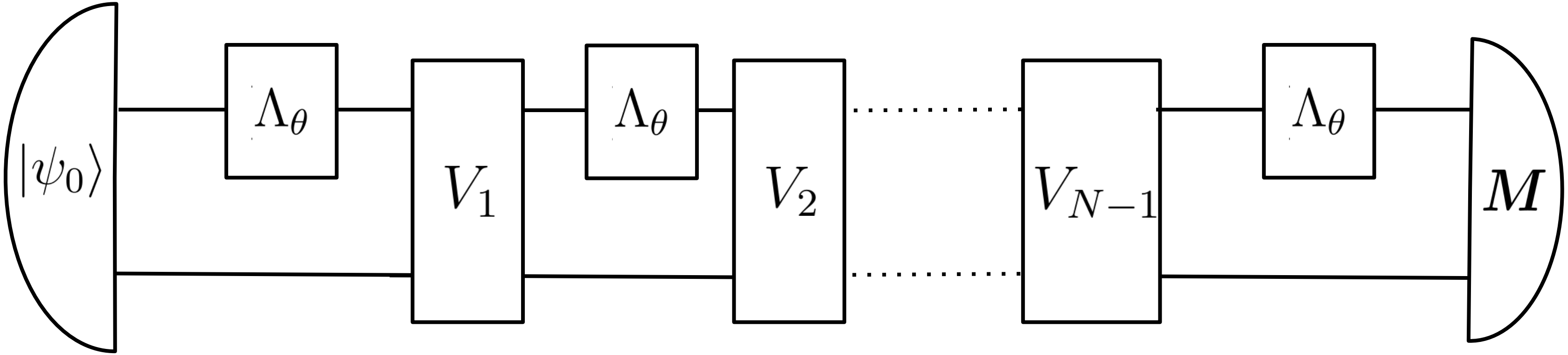
General many channel schemes



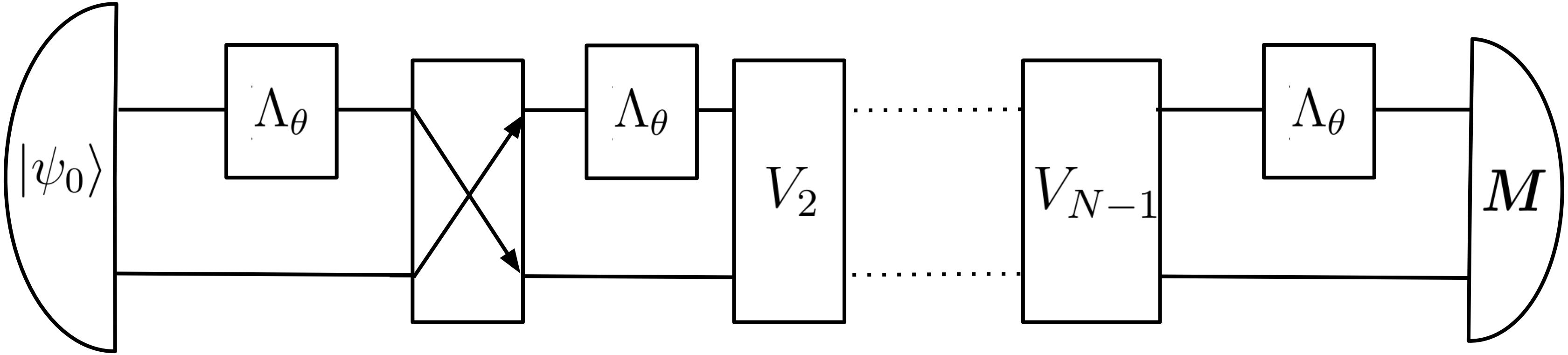
General many channel schemes



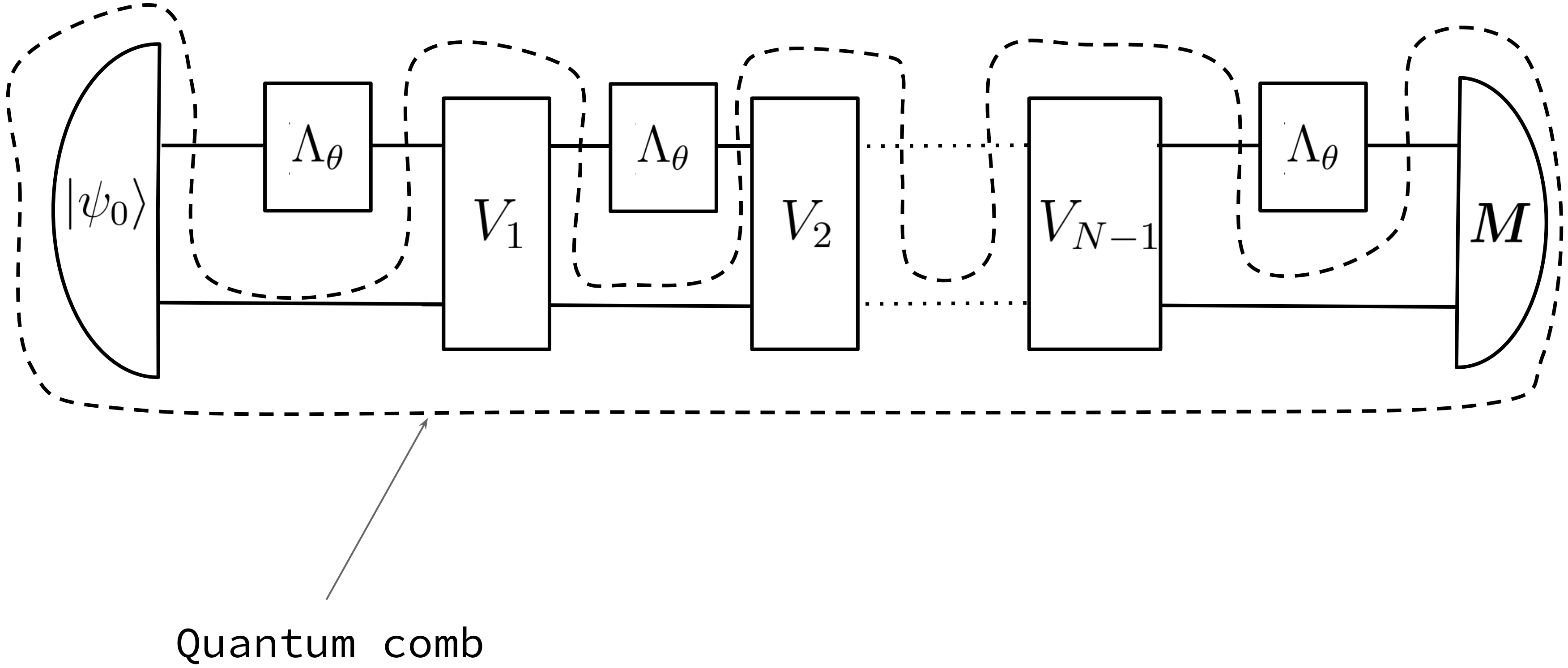
General many channel schemes



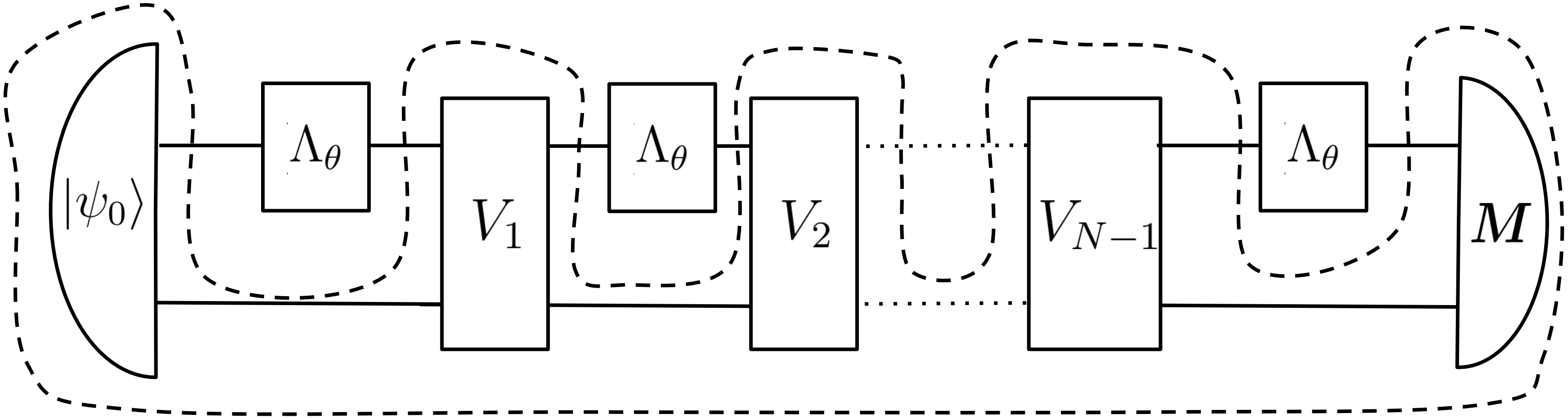
General many channel schemes



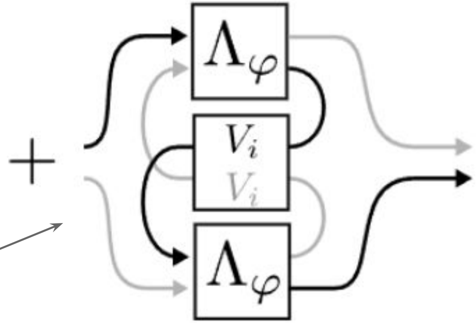
General many channel schemes



Even more general many channel schemes

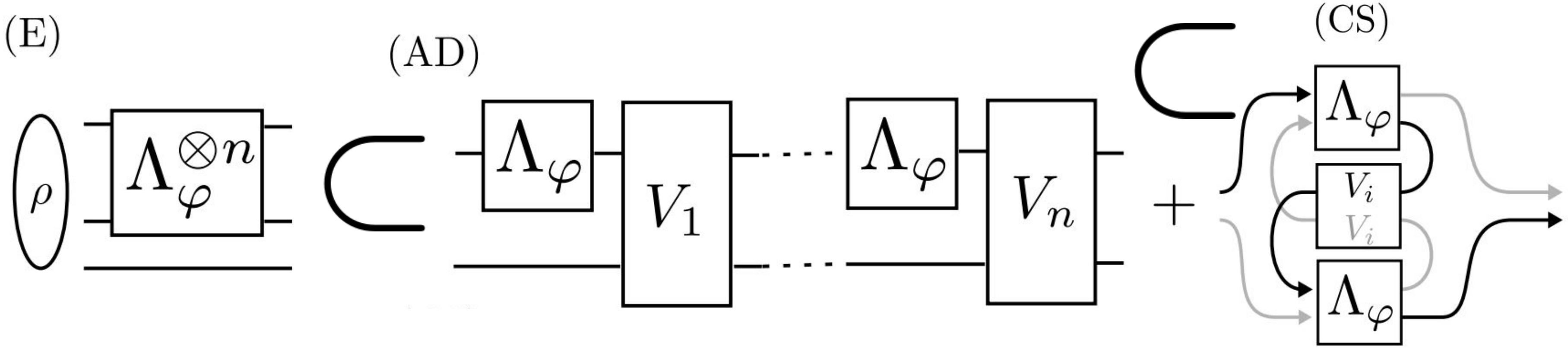


Quantum comb



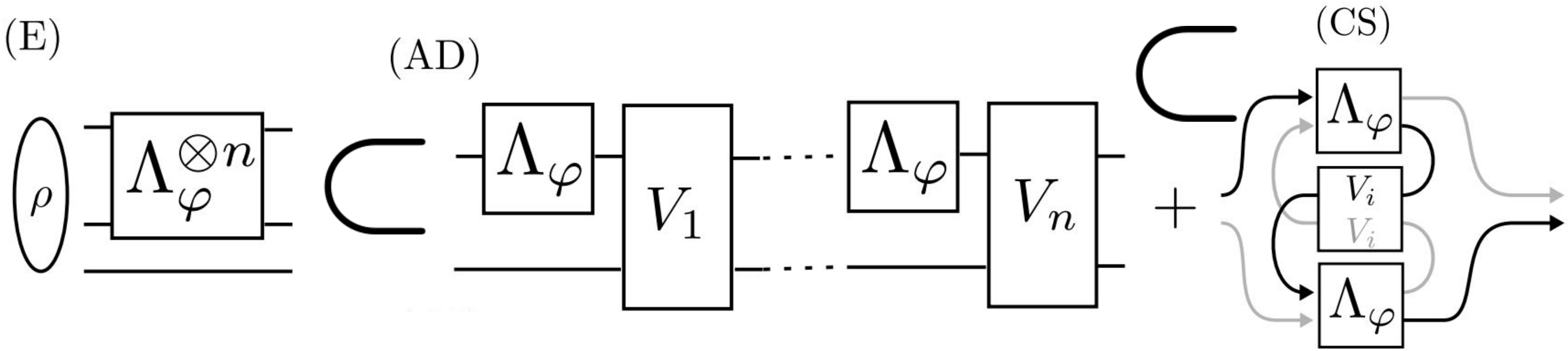
Quantum switch

Hierarchy of metrological strategies



Liu, Qiushi, et al. "Optimal strategies of quantum metrology with a strict hierarchy." *Physical Review Letters* 130.7 (2023): 070803.

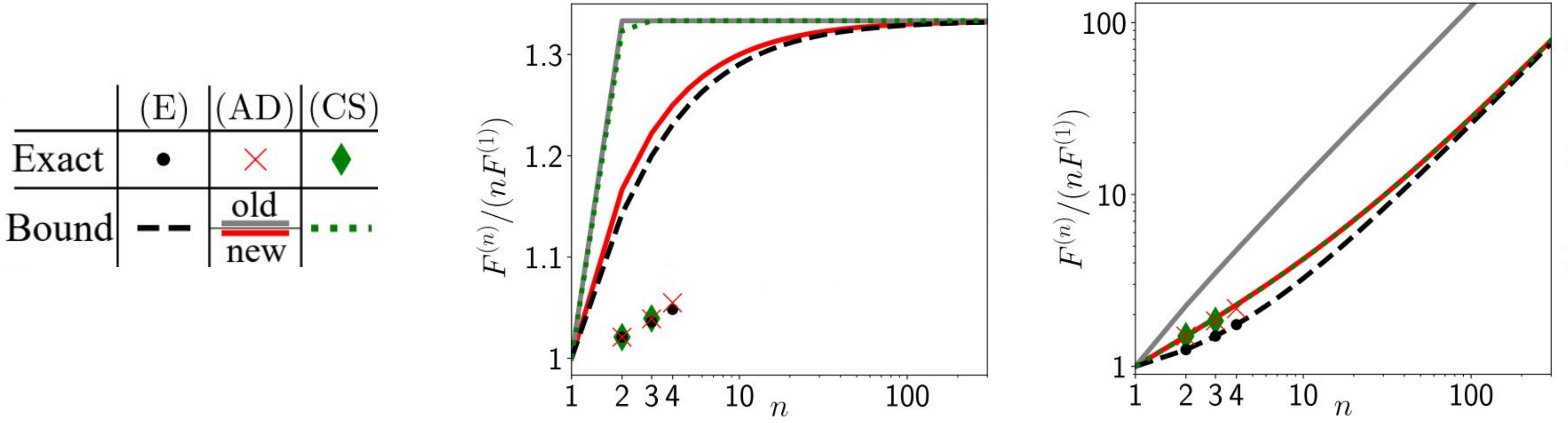
Asymptotic equivalence of metrological strategies



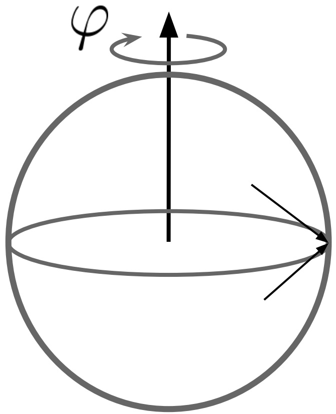
These three schemes are equally good asymptotically!

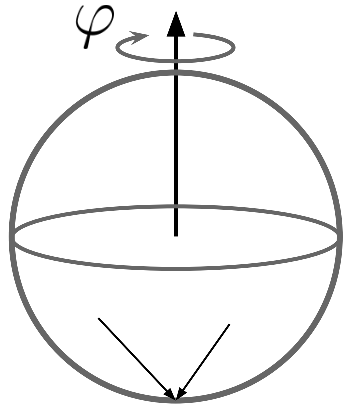
SK, W. Górecki, F. Albarelli, R. Demkowicz-Dobrzański, Using Adaptiveness and Causal Superpositions Against Noise in Quantum Metrology, Phys. Rev. Lett. 131, 090801

Examples (dephasing)

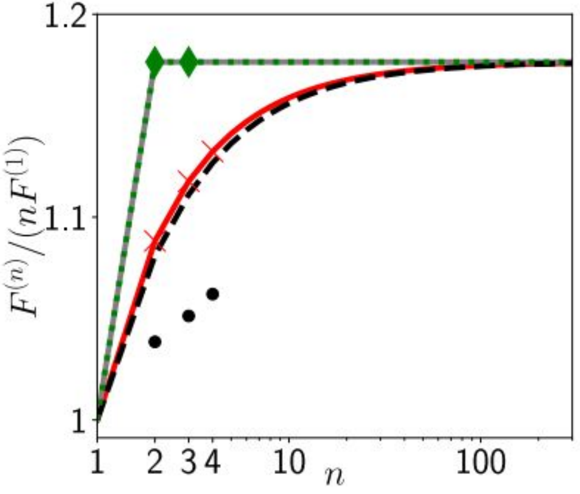


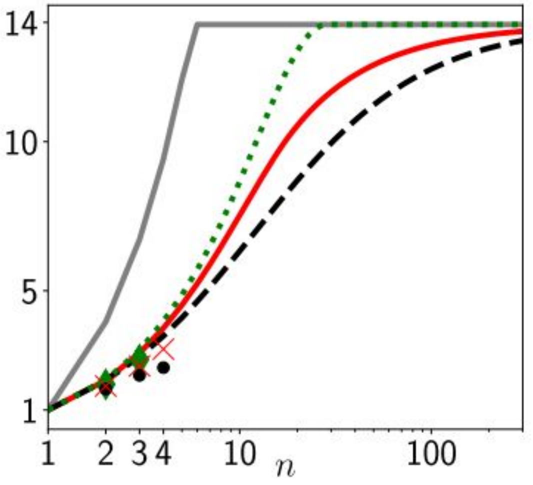
Examples (damping)





	(E)	(AD)	(CS)
Exact	●	×	◆
Bound	– –	old / new	···



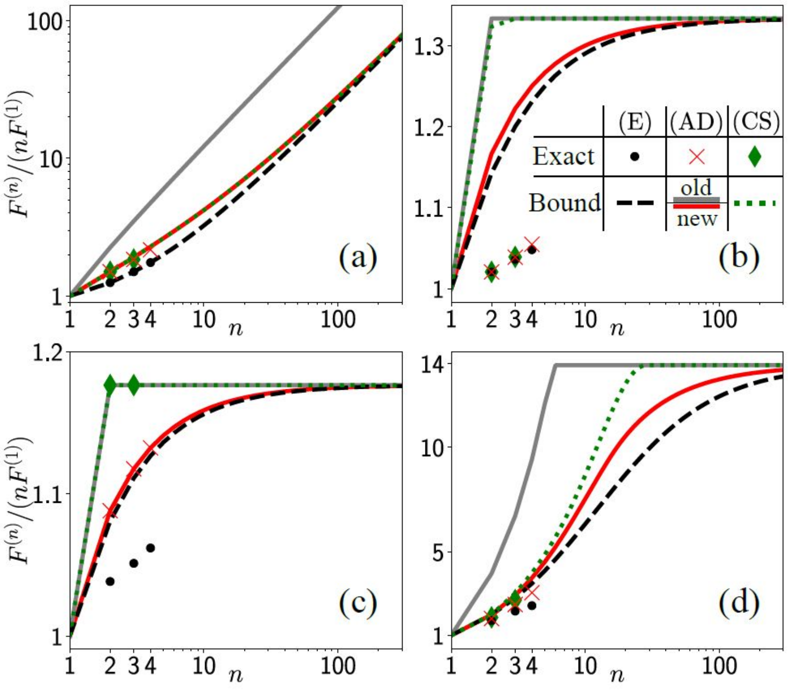


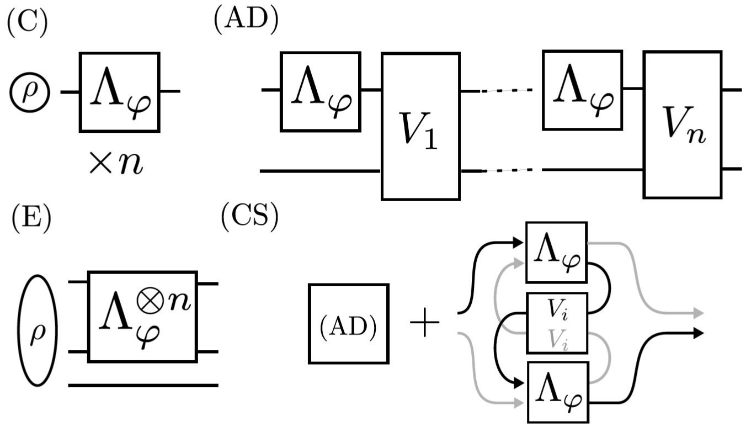
CONCLUSIONS

- Adaptiveness and causal superpositions may help, but not asymptotically

- Entanglement is a crucial resource in a quantum metrology

Thanks for your attention!





Phys. Rev. Lett. 131, 090801 (2023)