The Localization Problem

An antinomy between measurability and causal dynamics

Evan P. G. Gale

The University of Queensland

15th Annual Conference on Relativistic Quantum Information (North)

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Outline

Introduction

What is **localization**? It is about where things are found in space and time

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Problem: we do not understand localization in relativistic quantum theory

Why should we care?

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Digital Object Identifier (DOI) https://doi.org/10.1007/s00220-020-03800-6

Communications in
Mathematical Physics

Check for updates

Quantum Fields and Local Measurements

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Quantum Reference Frames for Lorentz Symmetry

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[3]Institute for Theoretical Physics, ETH Zurich, Switzerland

Since their first introduction, Quantum Reference Frame (QRF) transformations have been extensively discussed, generalising the covariance of physical laws to the quantum domain. Despite important progress, a formulation of QRF transformations for Lorentz symmetry is still lacking. The present work aims to fill this gap. We first introduce a reformulation of relativistic quantum mechanics independent of any notion of preferred temporal slicing. Based on this, we define transformations that switch between the perspectives of different relativistic QRFs. We introduce a notion of "quantum Lorentz transformations" and "superposition of Lorentz boosts", acting on the external degrees of freedom of a quantum particle. We analyse two effects, superposition of time dilations and superposition of length contractions, that arise only if the reference frames exhibit both relativistic and quantum-mechanical features. Finally, we discuss how the effects could be observed by measuring the wave-packet extensions from relativistic QRFs.

Several works have reported extensions of QRFs to relativistic systems [17, 28, 31]. Ref. [17] introduces a "quantum Wigner rotation" that allows moving to the rest frame of a particle even if it is moving in a superposition of relativistic velocities with respect to the laboratory frame. This was used as a tool to solve the problem of an operational definition of spin in the relativistic regime. However, it focused only on the transformation of the internal degrees of freedom. Ref. [31] introduces QRF transformations for spacetime translation symmetry and applies them to describe a quantum superposition of special-relativistic time dilation to second order in $(p/mc)^2$. Despite recent progress, a relativistic extension of QRF transformations, in the sense of Lorentz symmetry, is still missing.

In the present work we extend the quantum reference frame formalism to relativistic quantum systems carrying Lorentz symmetry. This is challenging because Lorentz transformations mix space and time, which calls for a framework that treats both space and time on the same footing. Hence, we use a spacetime representation of

Why should we care?

PHYSICAL REVIEW D **111,** 046026 (2025)

Gravitational entanglement witness through Einstein ring image

Youka Kaku[*] and Yasusada Nambu[†]
Department of Physics, Graduate School of Science, Nagoya University, Chikusa, Nagoya 464-8602, Japan

(Received 21 November 2024; accepted 30 January 2025; published 24 February 2025)

We investigate the interplay between quantum theory and gravity by exploring gravitational lensing and Einstein ring images in a weak gravitational field induced by a mass source in spatial quantum superposition. We analyze a quantum massless scalar field propagating in two distinct models of gravity: the first quantized Newtonian gravity (QG) model, which generates quantum entanglement between the mass source and other systems, and the Schrödinger-Newton (SN) gravity model, which does not produce entanglement. Visualizing the two-point correlation function of the scalar field, we find that the QG model produces a composition of multiple Einstein rings, reflecting the spatial superposition of the mass source. By contrast, the SN model yields a single deformed ring image, representing a classical spacetime configuration. Furthermore, we introduce a specific quantity named the which-path information indicator and visualize its image. The QG model again reveals multiple Einstein rings, while the image intensity in the SN model notably vanishes. Our findings provide a visual approach to witness gravity-induced entanglement through distinct features in Einstein ring images. This study advances our understanding of quantum effects in general relativistic contexts and establishes a foundation for future studies of other relativistic phenomena.

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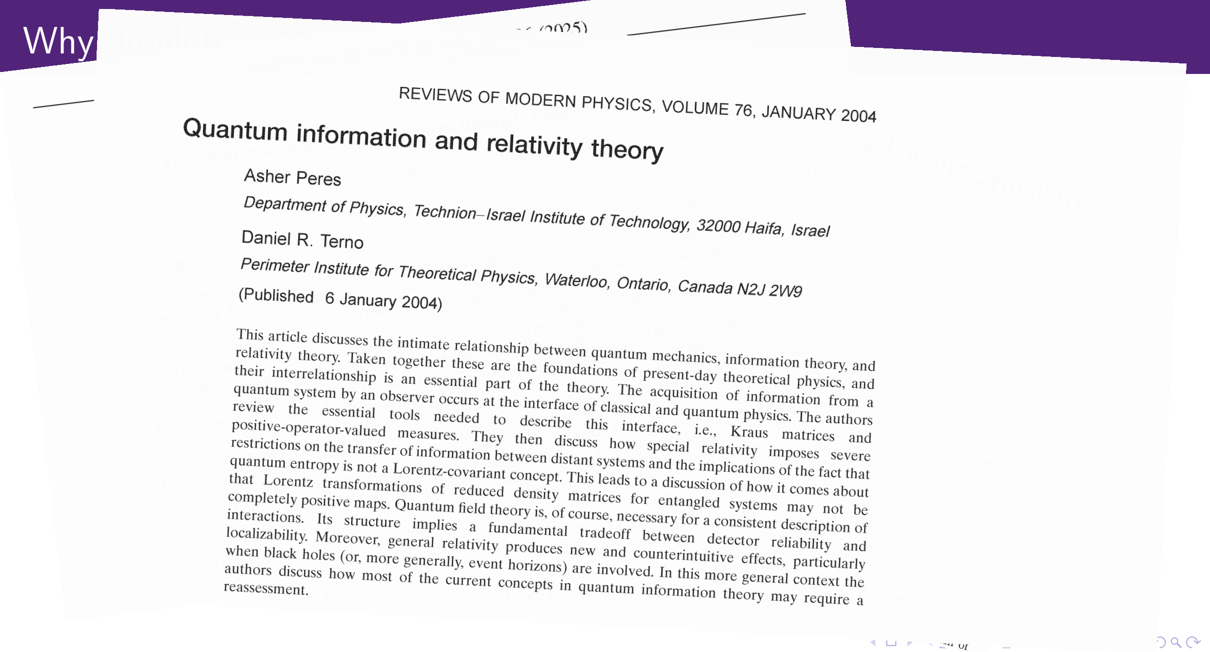

REVIEWS OF MODERN PHYSICS, VOLUME 76, JANUARY 2004

Quantum information and relativity theory

Asher Peres

Department of Physics, Technion–Israel Institute of Technology, 32000 Haifa, Israel

Daniel R. Terno

Perimeter Institute for Theoretical Physics, Waterloo, Ontario, Canada N2J 2W9

(Published 6 January 2004)

This article discusses the intimate relationship between quantum mechanics, information theory, and relativity theory. Taken together these are the foundations of present-day theoretical physics, and their interrelationship is an essential part of the theory. The acquisition of information from a quantum system by an observer occurs at the interface of classical and quantum physics. The authors review the essential tools needed to describe this interface, i.e., Kraus matrices and positive-operator-valued measures. They then discuss how special relativity imposes severe restrictions on the transfer of information between distant systems and the implications of the fact that quantum entropy is not a Lorentz-covariant concept. This leads to a discussion of how it comes about that Lorentz transformations of reduced density matrices for entangled systems may not be completely positive maps. Quantum field theory is, of course, necessary for a consistent description of interactions. Its structure implies a fundamental tradeoff between detector reliability and localizability. Moreover, general relativity produces new and counterintuitive effects, particularly when black holes (or, more generally, event horizons) are involved. In this more general context the authors discuss how most of the current concepts in quantum information theory may require a reassessment.

Why should we care?

The real physical problem is how localized detectors can be. The idealization of "one detector per spacetime point" is obviously impossible. How can we manage to ensure that two detectors have zero probability of overlapping? There appears to be a fundamental tradeoff between detector reliability and localizability. The bottom line is how to formulate a relativistic interaction between a detector and the detected system. [...] This problem seems to be very far from a solution. Completely new notions may have to be invented.[1]

[1] A. Peres and D. R. Terno, Rev. Mod. Phys. **76**, 93–123 (2004).

Two conceptions of localized measurements

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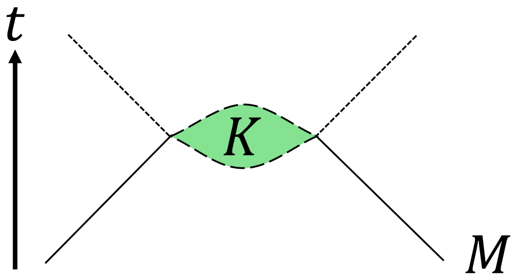
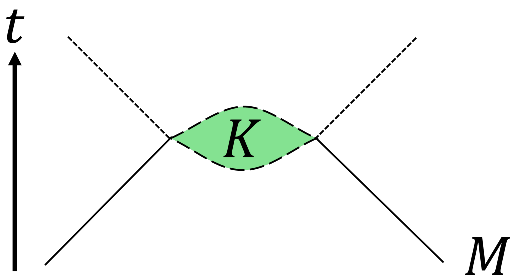
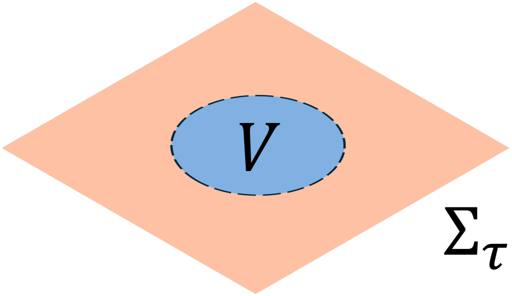


4D (Lorentz-invariant localization):
Localized system or local scattering operation in coupling region K of spacetime manifold M

Two conceptions of localized measurements





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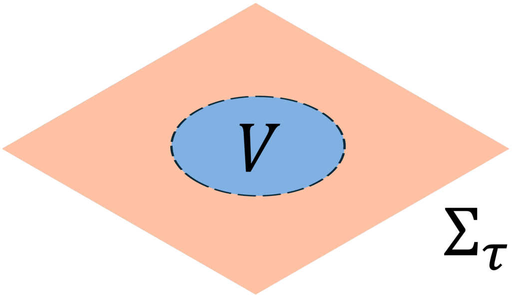

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Newton-Wigner localization problem I

Let S_0 be the set of localized states at the origin.

Newton and Wigner[2] took the following postulates:

[2] T. D. Newton and E. P. Wigner, Rev. Mod. Phys. **21**, 400–406 (1949).

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(d) **Regularity**: Generators of the Lorentz group are applicable to the localized states.

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Newton-Wigner localization problem II

Adopting standard L^2-inner product

$$\langle\psi|\phi\rangle := \int d^3\boldsymbol{x}\,\psi^*(\boldsymbol{x})\phi(\boldsymbol{x}) = \int d^3\boldsymbol{p}\,\psi^*(\boldsymbol{p})\phi(\boldsymbol{p})\,,$$

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(i) NW wavefunction given by Fourier transform

$$\psi_{\mathrm{NW}}(\boldsymbol{x}) = \frac{1}{(2\pi)^{3/2}}\int d^3\boldsymbol{p}\,\widetilde{\psi}_{\mathrm{NW}}(\boldsymbol{p})e^{-i\boldsymbol{p}\cdot\boldsymbol{x}}\,,$$

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(ii) NW position operator has standard representations $[\boldsymbol{x}_{\mathrm{NW}}]_x = \boldsymbol{x}$ or $[\boldsymbol{x}_{\mathrm{NW}}]_p = i\boldsymbol{\nabla}_p$.

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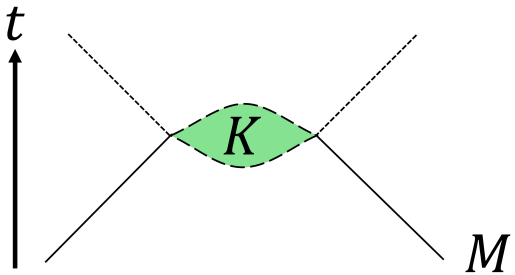
NW wavefunction not microcausal

$$[\psi_{\mathrm{NW}}(x), \psi_{\mathrm{NW}}(x')] \neq 0$$

for spacelike separation at different times

Two conceptions of localized measurements



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NW incompatible with Lorentz boosts—Philips dropped orthogonality postulate

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Lorentz-invariant localization problem II

Philips only considered spin-0 case. Adopting Klein-Gordon inner product

$$(\psi,\phi) := i\int_t d^3\boldsymbol{x}\,\psi^*\partial_t\phi - \phi\partial_t\psi^* = \int \frac{d^3\boldsymbol{p}}{2p_0}\,\psi^*(\boldsymbol{p})\phi(\boldsymbol{p})\,,$$

and Lorentz-invariant normalisation $\langle\boldsymbol{p}|\boldsymbol{p}'\rangle = 2p_0\delta^{(3)}(\boldsymbol{p}-\boldsymbol{p}')$ with $p_0 = \sqrt{p^2+m^2}$, then:

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But Lorentz-invariant localization scheme also has serious issues!

[4] I. R. de Oliveira, PhD thesis (Universidade de São Paulo, São Paulo, Brazil, 2024).

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Same problems in contemporary AQFT approach, known as modular localization[4] (see brief review by Yngvason[5])

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Malament's theorem[6]

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$$P(V_t)P(V_t') = P(V_t')P(V_t) = 0$$

(d) Microcausality between spacelike regions

$$[P(V_t), P(V'_{t'})] = 0$$

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Hegerfeldt's theorem[7]

Stronger no-go theorem

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> *If at $t = 0$ a particle is strictly localized in a bounded region V_0 then, unless it remains in V_0 for all times, it cannot be strictly localized in a bounded region V, however large, for any finite time interval thereafter, and the particle localization immediately develops infinite "tails".*

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- observables without explicit position dependence (e.g., number operator, stress-energy density, etc.)

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Summary

	Three-dimensional	**Four-dimensional**
Wave equation	Schrödinger eq.	Hyperbolic eq.
Inner product	Probability measure	Lorentz invariant
Position operator	Lorentz covariant ✗ Self-adjoint ✓ Orthogonal states ✓	Lorentz covariant ✓ Self-adjoint ✗ Orthogonal states ✗
Causality	Energy bounded below, superluminal propagation	Microcausal dynamics, but no measurement framework

Outline

Antinomy between localized observables and causal dynamics

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The localization problem

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On instantaneous states in relativistic quantum theory

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The state is not abolished, it withers away: How quantum field theory became a theory of scattering

CrossMark

Alexander S. Blum

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On instantaneous states in relativistic quantum theory

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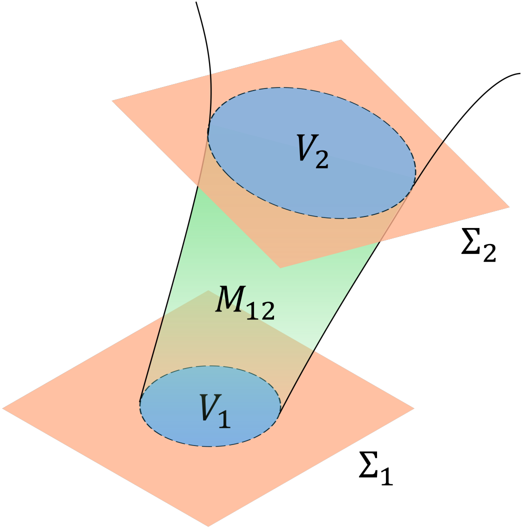
- have worldine or worldtube describing evolution of system to asymptotic in- and out-states



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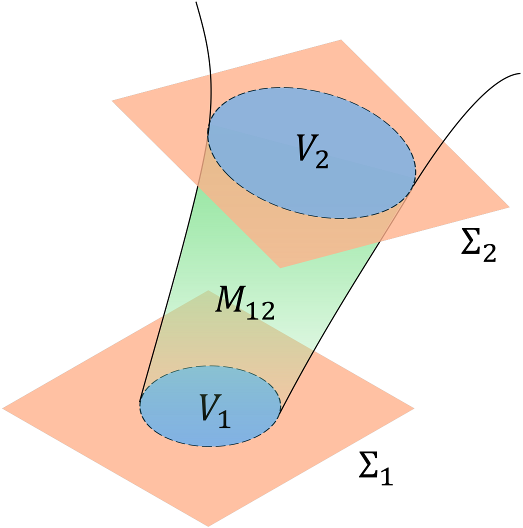

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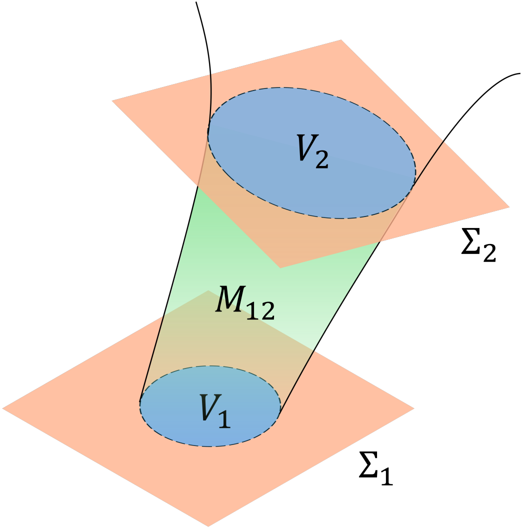

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- want von Neumann measurement scheme with localized observables in volumes V_1 and V_2
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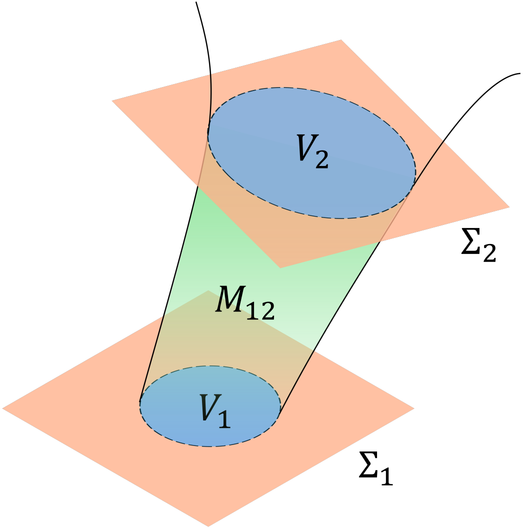
- **must interchange between three- and four-dimensional descriptions**



Outline

Conclusion and outlook

More than just localization—implications for causality, dynamics and measurability

- Essential feature missing from current description of relativistic quantum theory—instantaneous, localized states
- Dual structures—three- versus four-dimensional description—measurement framework versus causal dynamics
- Nature of duality crucial to further understanding of present foundational problems

arXiv:2503.15254 [quant-ph]

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Why is this Hamiltonian not $H \sim \sqrt{\boldsymbol{p}^2 + m^2}$?

Relation between unitary representations

Foldy and Wouthuysen[9] derived the following unitary transformation

$$U_{\mathrm{FW}} = \exp\left(\frac{\beta\boldsymbol{\alpha}\cdot\boldsymbol{p}}{2|\boldsymbol{p}|}\tan^{-1}\left(\frac{|\boldsymbol{p}|}{m}\right)\right) ,$$

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Foldy[10] obtained ‘canonical form’ for arbitrary spin

$$i\partial_t \psi_{2s+1}(\boldsymbol{x}, t) = \beta\sqrt{-\boldsymbol{\nabla}^2 + m^2}\,\psi_{2s+1}(\boldsymbol{x}, t)$$

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$$\boldsymbol{v}_{\mathrm{D}} := i\,[H_{\mathrm{D}}, \boldsymbol{x}_{\mathrm{D}}] = \boldsymbol{\alpha}\,.$$

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Classically, however, we expect to obtain

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for the Hamiltonian $H = \sqrt{\boldsymbol{p}^2 + m^2}$. This is precisely the NW localization!

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