# Gravitational wave imprints on spontaneous emission


**Jerzy Paczos**

Navdeep Arya, Sofia Qvarfort, Daniel Braun, and Magdalena Zych


RQI North, Naples 2025

# GWs and classical test masses

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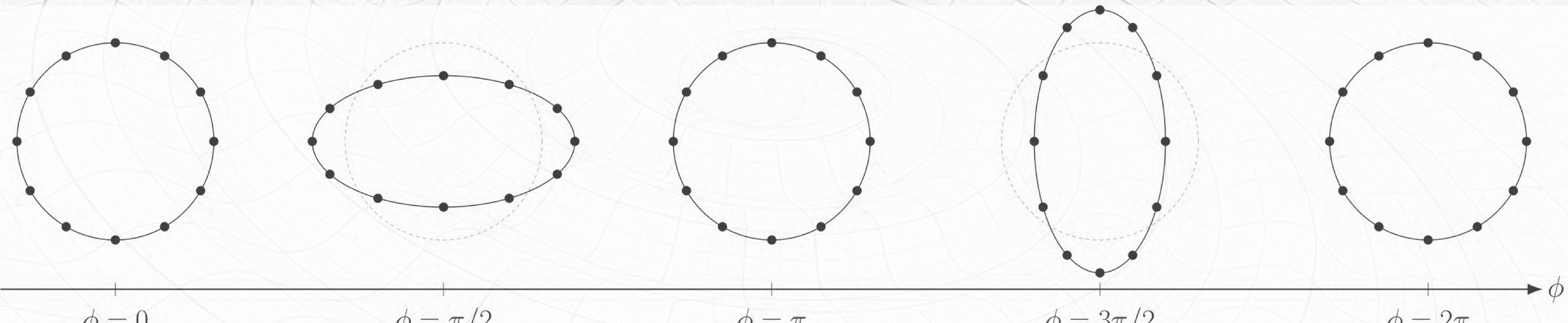

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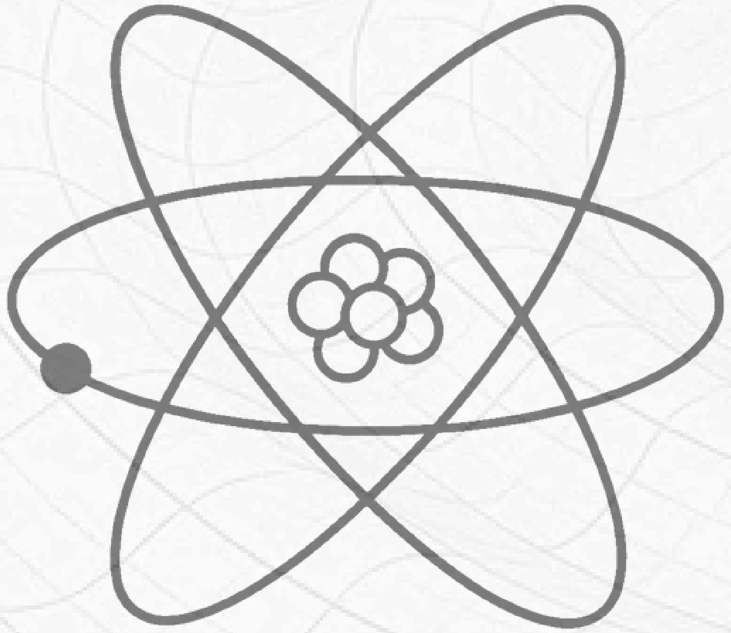
## Atoms



- energy shifts
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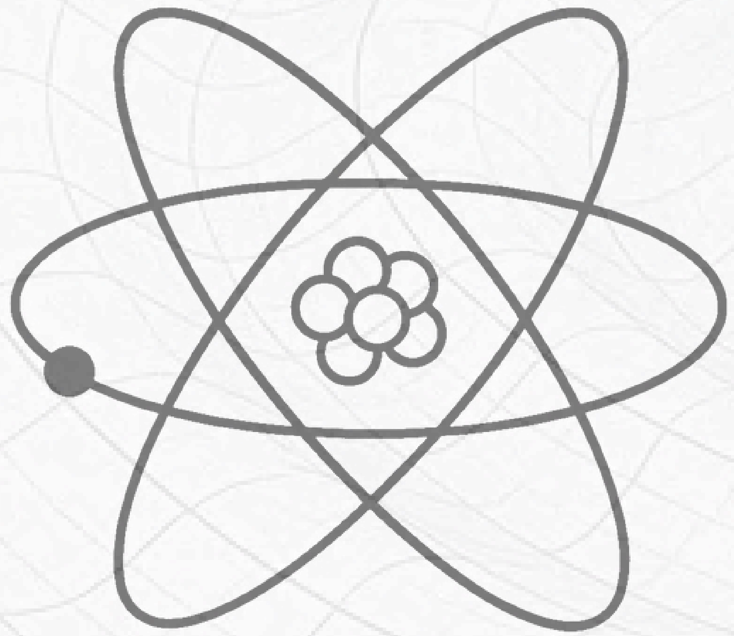

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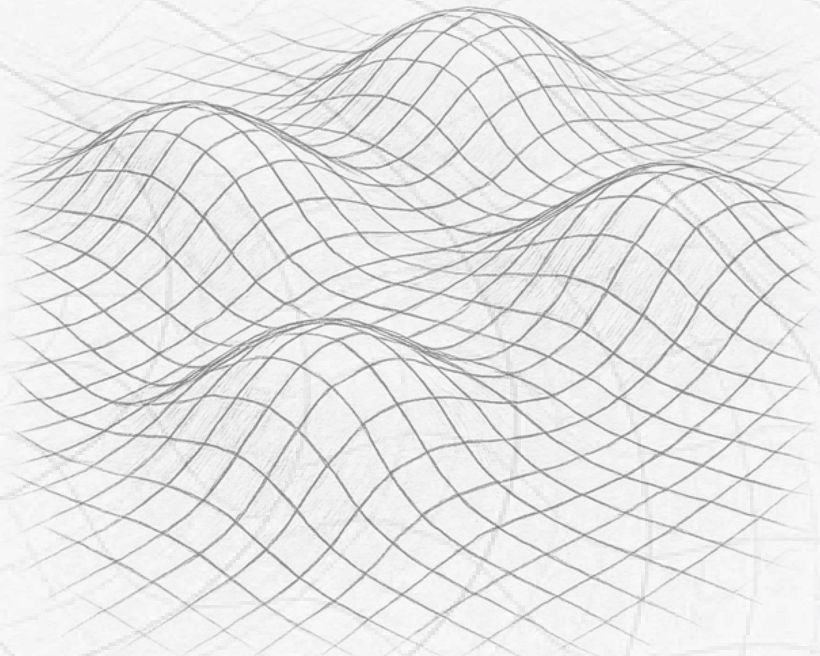
## Quantum fields



- particle production
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# GWs and spontaneous emission

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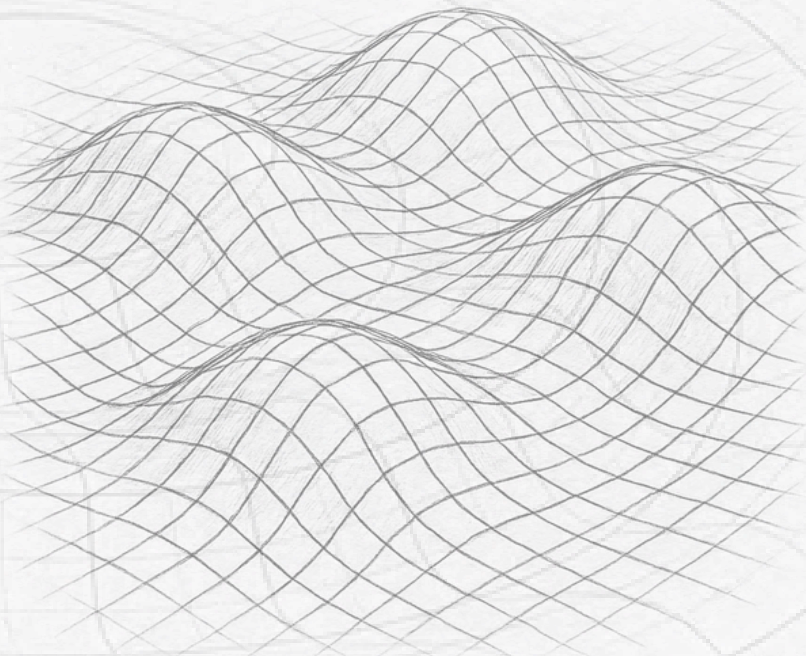

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- plane wave; traveling along $z$; amplitude $\mathcal{A} \ll 1$; frequency $\omega$
- $\mathrm{d}s^2 = -\mathrm{d}t^2 + \mathrm{d}z^2 + (1 + \mathcal{A}\cos[\omega(t-z)])\mathrm{d}x^2 + (1 - \mathcal{A}\cos[\omega(t-z)])\mathrm{d}y^2$

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- point-like two-level system: $|g\rangle$ and $|e\rangle$; energy gap $\omega_0$
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- massless, real, scalar
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**GW correction**

# Sidebands and directionality

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**Frequency dependence:**

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$\langle \tilde{n}_{\boldsymbol{k}}(t) \rangle \propto 1$ $\qquad$ $\langle \delta n_{\boldsymbol{k}}(t) \rangle \propto \cos^2(\theta/2)\cos(2\varphi)$

Directionality of the emission!

$$k = (|k| \sin\theta \cos\varphi, |k| \sin\theta \sin\varphi, |k| \cos\theta)$$

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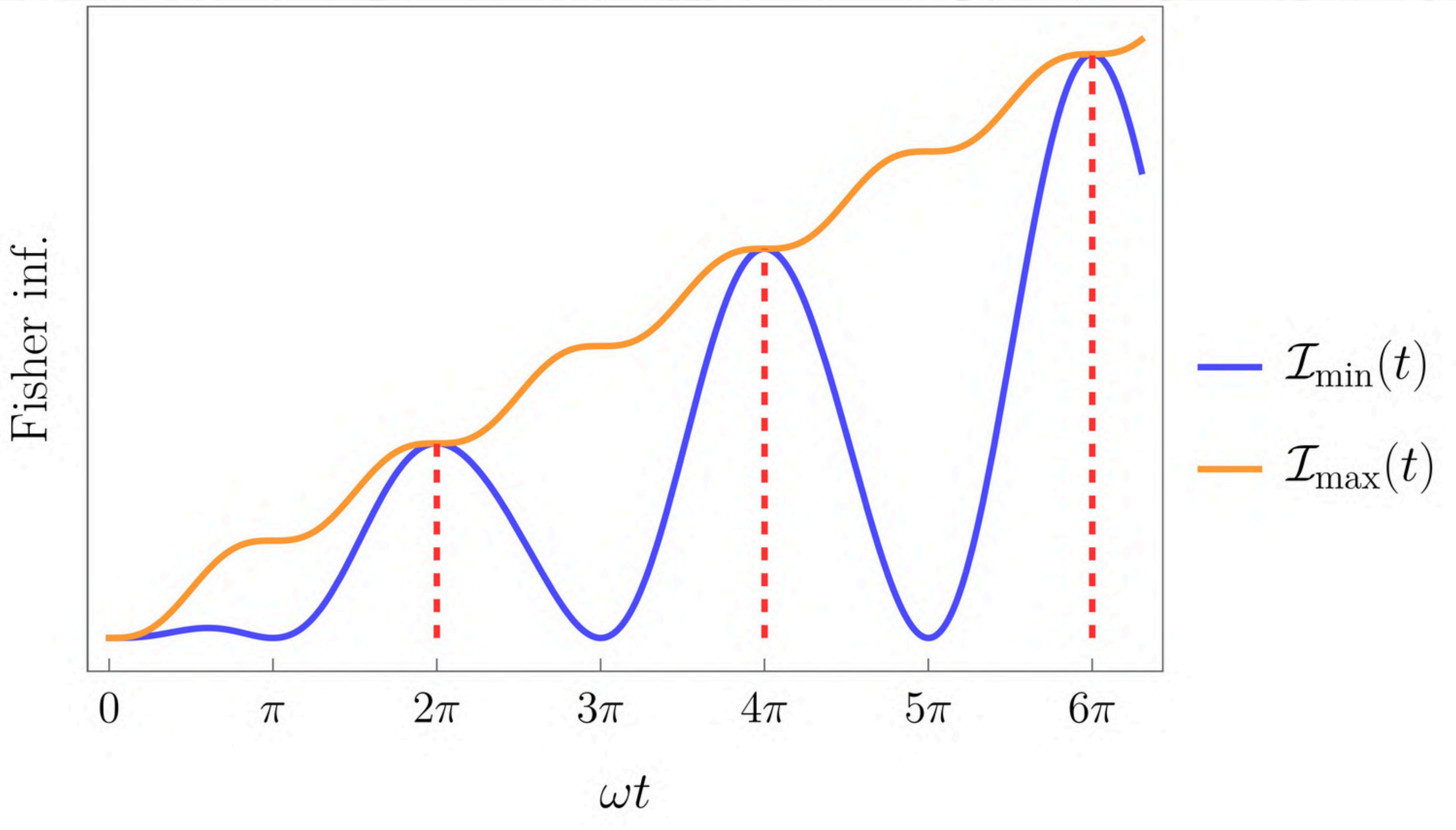

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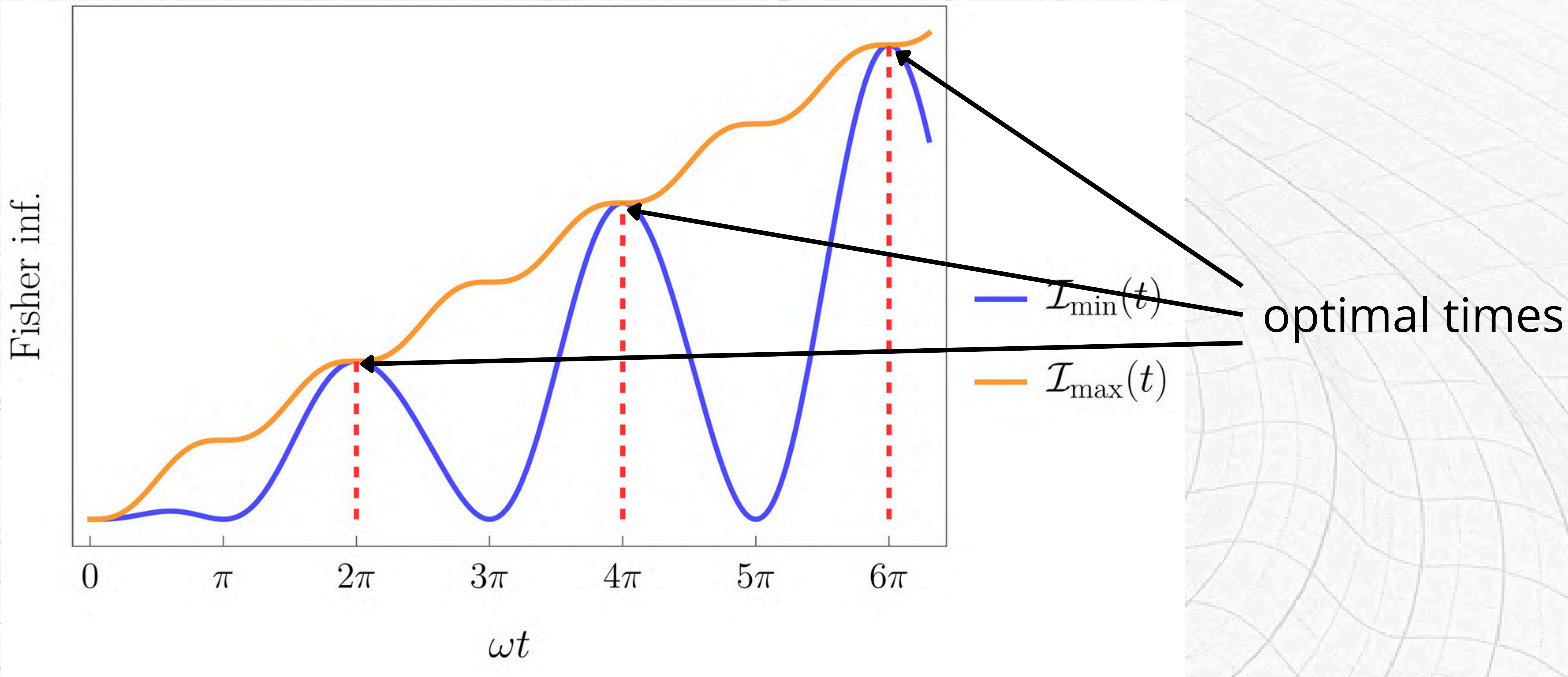

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# More details:

arXiv:2506.13872

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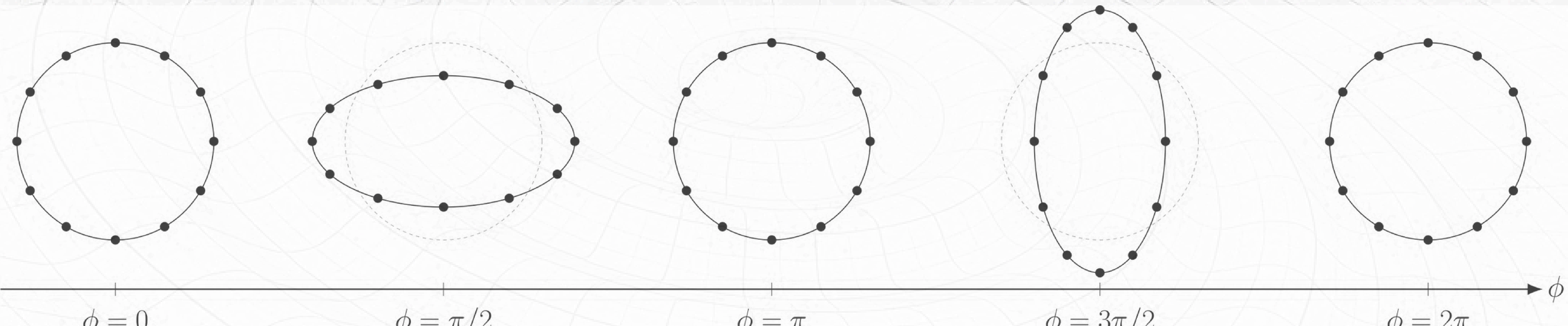

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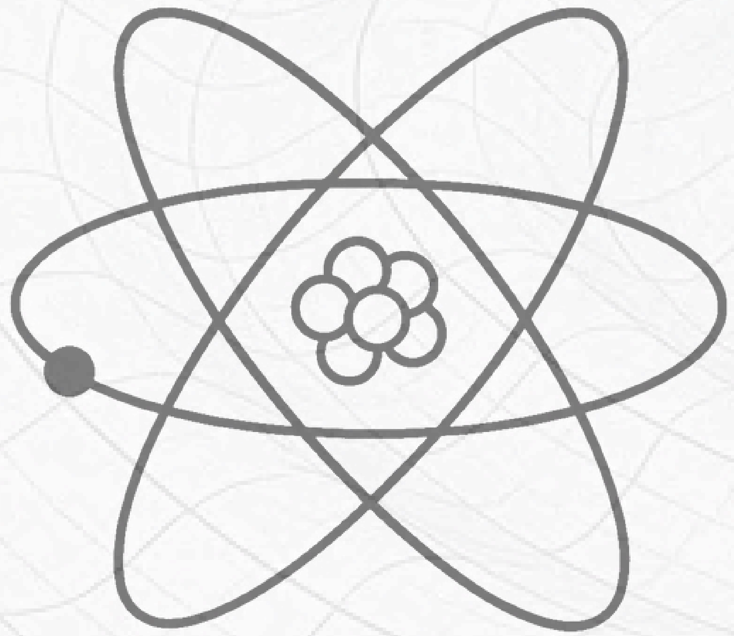

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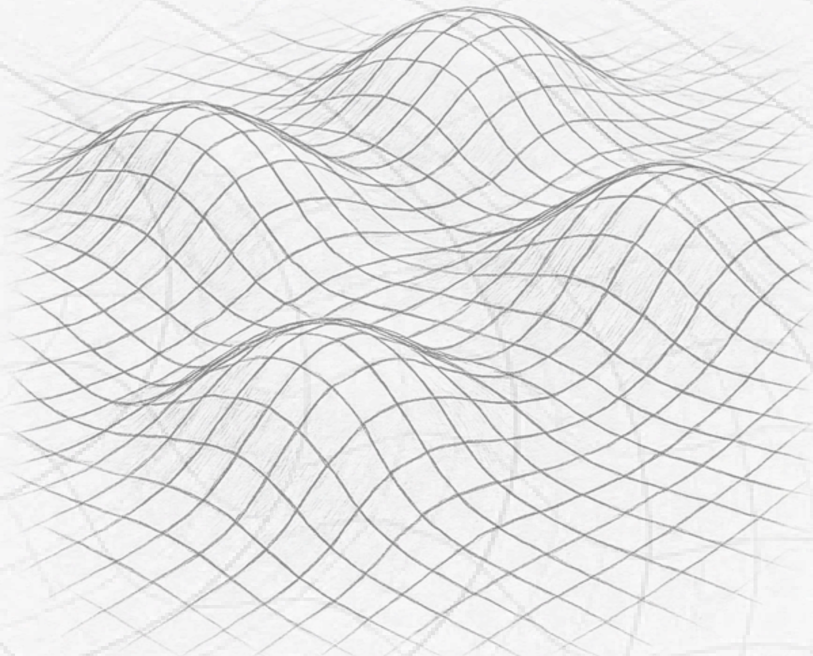

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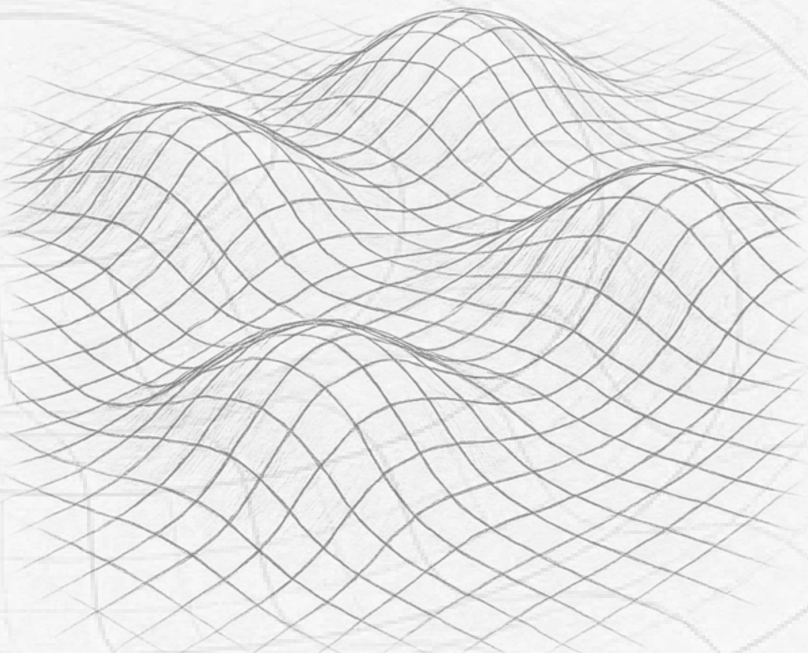

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**Flat spacetime contribution**

**GW correction**

# Spontaneous emission

**Flat spacetime contribution:**

$$\langle \tilde{n}_{\boldsymbol{k}}(t) \rangle = \frac{\varepsilon^2 t^2}{(2\pi)^3 8|k|} \mathrm{sinc}^2(\delta_k t/2) \qquad \delta_k \equiv |k| - \omega_0$$

**GW correction:**

$$\langle \delta n_{\boldsymbol{k}}(t) \rangle = \frac{\varepsilon^2 t^2}{(2\pi)^3 8|k|} \mathcal{A} \frac{|k|}{\omega} f(\delta_k, t) g(\theta, \varphi) \qquad k = (|k| \sin\theta \cos\varphi, |k| \sin\theta \sin\varphi, |k| \cos\theta)$$

$$f(\delta_k, t) = \mathrm{sinc}(\delta_k t/2) \cos(\omega t/2)(\mathrm{sinc}[(\delta_k - \omega)t/2] - \mathrm{sinc}[(\delta_k + \omega)t/2])$$

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# Sidebands and directionality

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$\langle \tilde{n}_{\boldsymbol{k}}(t) \rangle \propto 1$ $\langle \delta n_{\boldsymbol{k}}(t) \rangle \propto \cos^2(\theta/2)\cos(2\varphi)$

Directionality of the emission!

# Equivalence principle

The atom is point-like... Why is there any effect at all?

The atom-field system is extended.

**No problems with EP!**

The total emission rate remains unchanged.

**No information about GW in the atomic state!**

# GW detection

How well can we estimate the GW amplitude?

**Cramer-Rao bound:** $$\delta\mathcal{A} \geq \frac{1}{\sqrt{M\mathcal{I}(t)}}$$

$\delta\mathcal{A}$ - estimation uncertainty

$M$ - independent repetitions

$\mathcal{I}(t)$ - Fisher information; measurement-dependent

Measure $\langle n_{\boldsymbol{k}}(t) \rangle$ for all $\boldsymbol{k}$... What is the corresponding Fisher information?

# Fisher information

**Bounds:** $$\mathcal{I}_{\min}(t) = \frac{\bar{n}(t)}{3}\left(\frac{\omega_0}{\omega}\right)^2 \cos^2(\omega t/2)[1 - \mathrm{sinc}(\omega t)]$$

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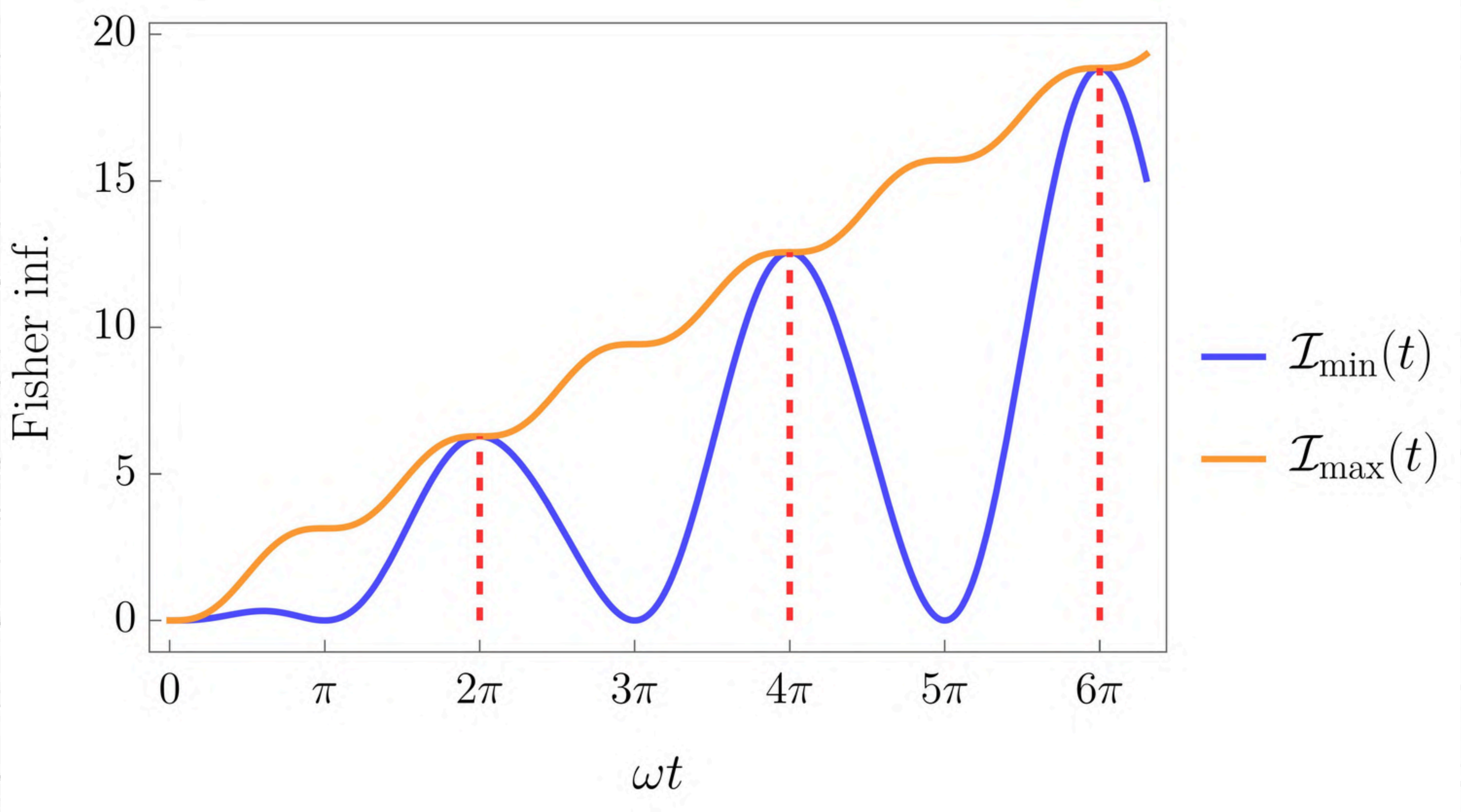

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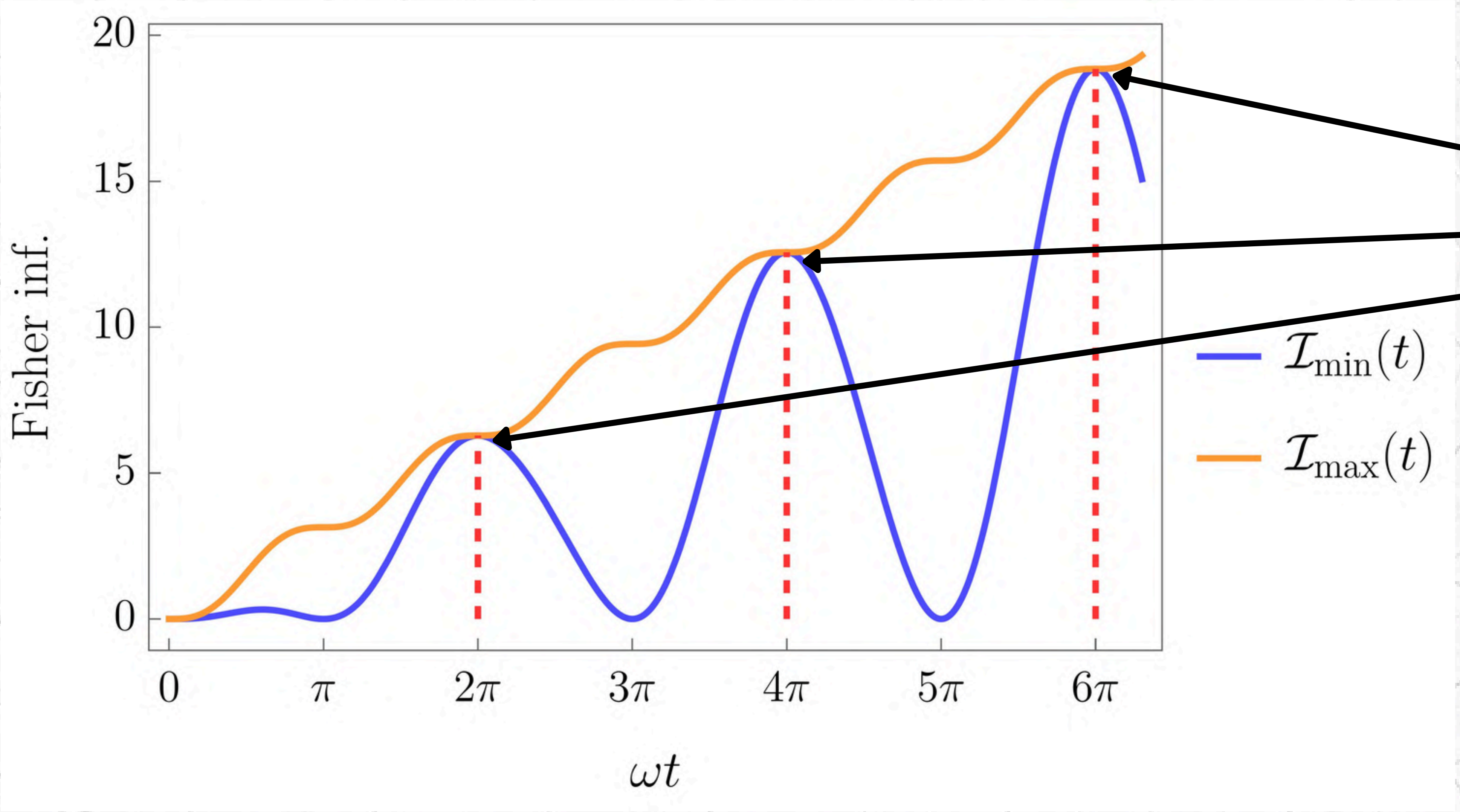


optimal times

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$M$ - independent repetitions (number of atoms)

Detection possible (in principle) if $\delta\mathcal{A} \leq \mathcal{A} \implies 1 \leq \mathcal{A}\sqrt{M\mathcal{I}(t)}$

$$t = \frac{2\pi m}{\omega},\ m \in \mathbb{N}: \qquad \mathcal{I}(t) = \frac{\bar{n}(t)}{3}\left(\frac{\omega_0}{\omega}\right)^2$$

$$\bar{n}(t) \lesssim 1 \implies (\mathcal{A}\omega_0/\omega)^{-2} \leq M$$

$$\mathcal{A} \sim 10^{-21}, \qquad \omega_0 \sim 10^{14}\ \mathrm{Hz}$$

Lower LIGO limit: $\omega \sim 10\ \mathrm{Hz} \implies M \geq 10^{16}$

Strontium linewidth: $\omega \sim 10^{-3}\ \mathrm{Hz} \implies M \geq 10^{8}$

# Summary

Gravitational waves induce sidebands and directionality of the emission.

The effect is consistent with the equivalence principle, even though the atom is point-like.

It might be detectable - the requirements are not daunting.

# Outlook

A more realistic model of atom-light interaction is needed for accurate experimental predictions.

An analysis of noise is required.

The method is general. Can we use spontaneous emission to probe the geometry of spacetime?