(Statistical Fluctuations in) the Causal Set-Continuum Correspondence

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Causal Set Theory

In causal set theory, the fundamental underlying structure of spacetime is proposed to be a causal set: a kind of partially ordered set. Bombelli, Lee, Meyer, Sorkin, 1987, *Space-Time as a Causal Set*, PRL. 59, 521.

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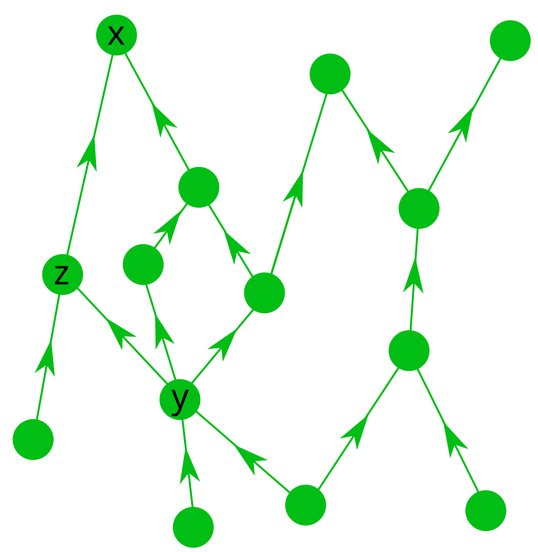
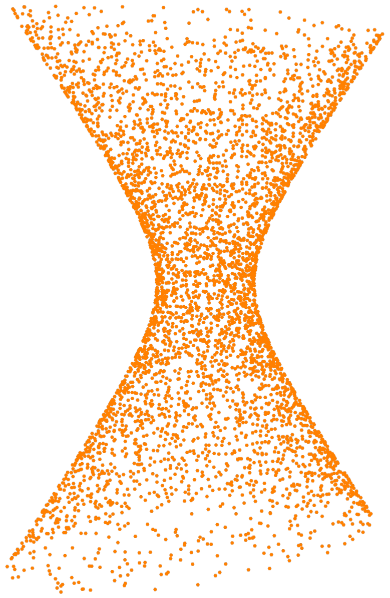


The set $\mathscr{C}$ is that of **spacetime elements** and the ordering relation $\preceq$ is the **causal precedence** relation.

The discreteness is expected to be around the **Planck scale**, and it is **Fundamental**.



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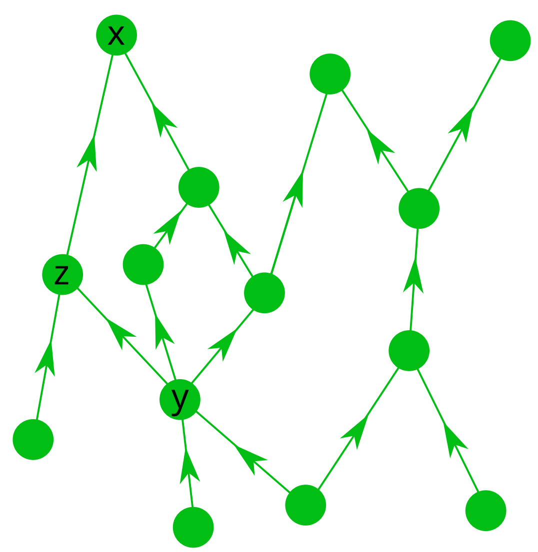

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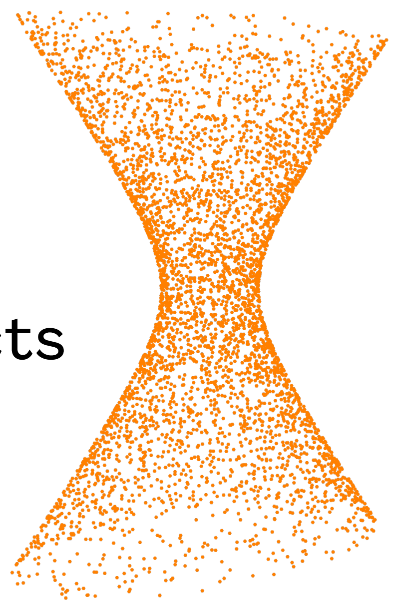
We know that we can recover all essential aspects of a continuous spacetime (**causal structure** + **volume**) from a causal set.



Bombelli, Henson, Sorkin, Discreteness without symmetry breaking: a theorem*, Mod.Phys.Lett.A24:2579-2587, 2009.*

Naturally cures UV divergences. Causal sets are **frame independent**.

Zeeman, *Causality Implies the Lorentz Group*, J. Math. Phys. 5: 490-493 (1964). Hawking, King and McCarthy, *A New Topology for Curved Space-Time which Incorporates the Causal, Differential and Conformal Structures*, J. Math. Phys. 17: 174 (1976). Malament, *The Class of Continuous Timelike Curves Determines the Topology of Space-time*, J. Math. Phys 18: 1399 (1977)

Causal Set-Continuum Correspondence

Because causal sets are so different from the continuum, an active area of research is to recognize and understand how some continuum quantities that we are familiar with emerge from quantities more natural to causal sets, in the classical and semiclassical regime.

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Causal set quantities are **combinatorial** and causal set-continuum correspondence is **statistical**.

Consider an ensemble $\mathrm{Sp}[\mathscr{M}] = \{\mathscr{C}_1, \mathscr{C}_2, \dots\}$

Given a function $f : \mathrm{Sp}[\mathscr{M}] \longrightarrow \mathbb{R}$, its average over the ensemble of causal sets is $\langle f \rangle \equiv \dfrac{1}{|\mathrm{Sp}[\mathscr{M}]|} \sum_{\mathscr{C} \in \mathrm{Sp}[\mathscr{M}]} f(\mathscr{C})\,.$

Causal Set-Continuum Correspondence

Because causal sets are so different from the continuum, an active area of research is to recognize and understand how some continuum quantities that we are familiar with emerge from quantities more natural to causal sets, in the classical and semiclassical regime.

arXiv:gr-qc/0703099
arXiv:1403.1622
arXiv:1001.2725
arXiv:1305.2588

Causal set quantities are **combinatorial** and causal set-continuum correspondence is **statistical**.

If we fix an element $x \in \mathscr{C}$, at which we would like to know the value of $\Box\,\phi$, this will be:

$$B^{(d)}\phi(x) = \sum_{i=0}^{n_d} C_i^{(d)} \sum_{y \in \Diamond_i(x)} \phi(y)$$

$$\lim_{\ell \to 0} \langle B\,\phi(x) \rangle = \left(\Box - \frac{1}{2} R(x) \right) \phi(x)$$

action: $S = \sum_{x \in \mathscr{C}} B(-2)$

Statistical and finite discreteness deviations: rich source of phenomenology

Fluctuations and Correlations in Causal Set Theory

Statistical and finite discreteness deviations: rich source of phenomenology

Class.Quant.Grav. 42 (2025) 4, 045017, arXiv:2407.03395

Heidar Moradi,[a] **Yasaman K. Yazdi,**[b,c] **and Miguel Zilhão**[d]

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Abstract: We study the statistical fluctuations (such as the variance) of causal set quantities, with particular focus on the causal set action. To facilitate calculating such fluctuations, we develop tools to account for correlations between causal intervals with different cardinalities. We present a convenient decomposition of the fluctuations of the causal set action into contributions that depend on different kinds of correlations. This decomposition can be used in causal sets approximated by any spacetime manifold $\mathcal{M}$. Our work paves the way for investigating a number of interesting discreteness effects, such as certain aspects of the Everpresent Λ cosmological model.

Fluctuations and Correlations in Causal Set Theory

Statistical and finite discreteness deviations: rich source of phenomenology

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Heidar Moradi,[a] Yasaman K. Yazdi,[b,c] and Miguel Zilhão[d]

Integrations over three kinds of domains:

(1) $\mathscr{M}$,

(2) $\mathscr{J}^-(x)$,

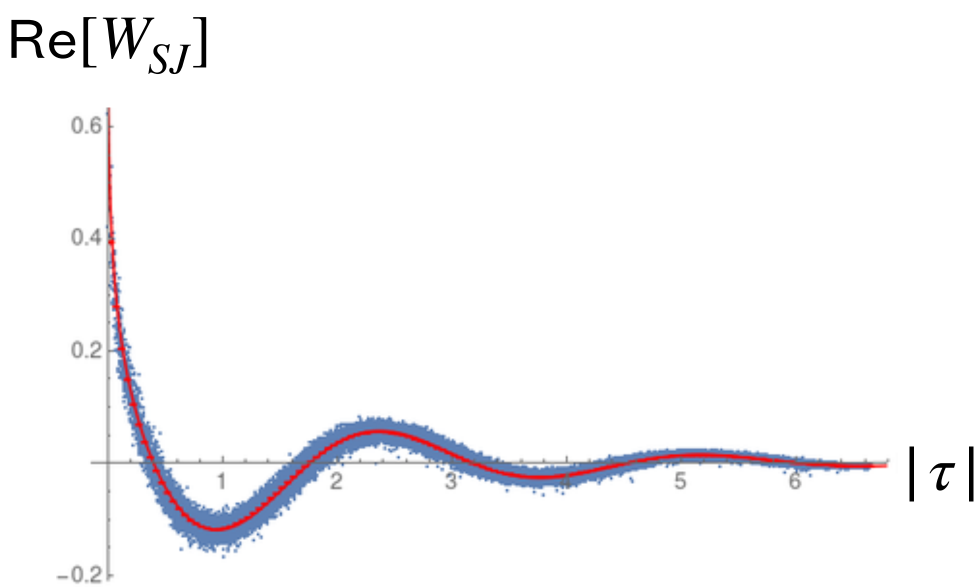
(3) $\mathscr{J}^-(x_1) \cap \mathscr{J}^-(x_2)$.

$\langle f \rangle$ and $\Delta f = \sqrt{\langle f^2 \rangle - \langle f \rangle^2}$

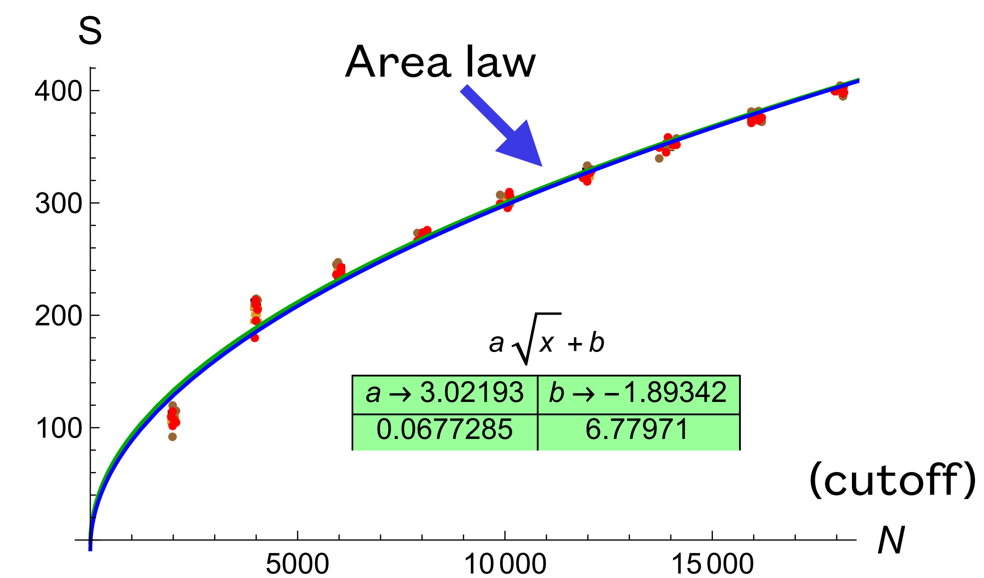
Abstract: We study the statistical fluctuations (such as the variance) of causal set quantities, with particular focus on the causal set action. To facilitate calculating such fluctuations, we develop tools to account for correlations between causal intervals with different cardinalities. We present a convenient decomposition of the fluctuations of the causal set action into contributions that depend on different kinds of correlations. This decomposition can be used in causal sets approximated by any spacetime manifold $\mathcal{M}$. Our work paves the way for investigating a number of interesting discreteness effects, such as certain aspects of the Everpresent Λ cosmological model.

Quantum Fields on Causal Sets

- **The Sorkin-Johnston vacuum:** A unique, covariantly defined state in globally hyperbolic spacetimes; reflects the full geometry



- **Discreteness effects:** Quantum field correlation functions are UV finite; short distance modifications to some known expressions



- **Spacetime formulation of entropy:** Horizon entropy counted by covariant degrees of freedom.

- **Lorentzian Spectral Geometry:** geometric information in spectra of causal set operators

Quantum Fields on Causal Sets

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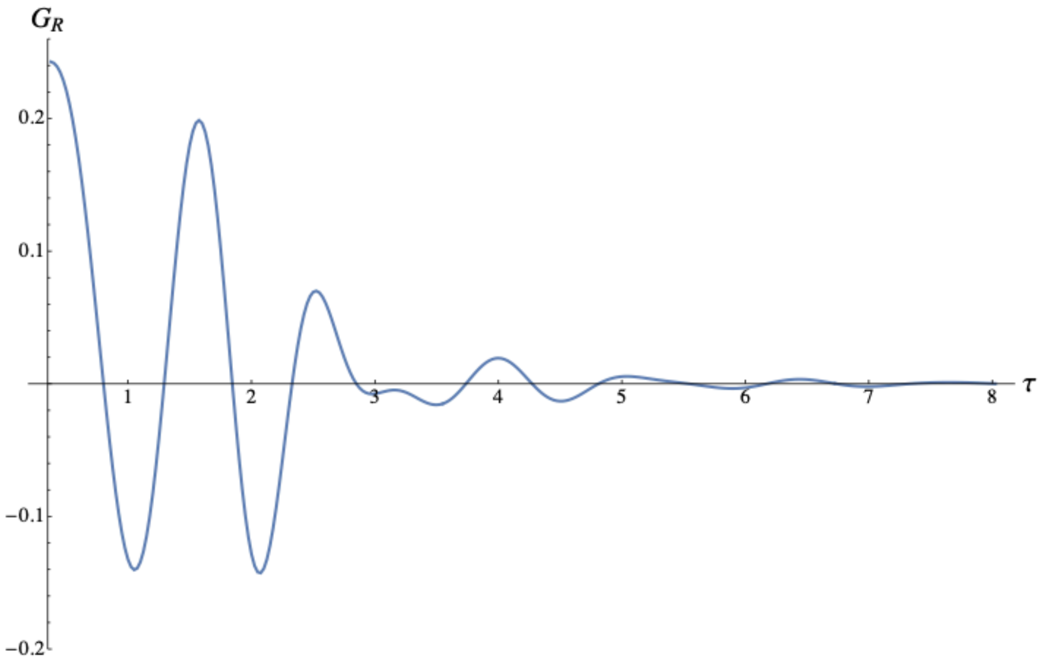
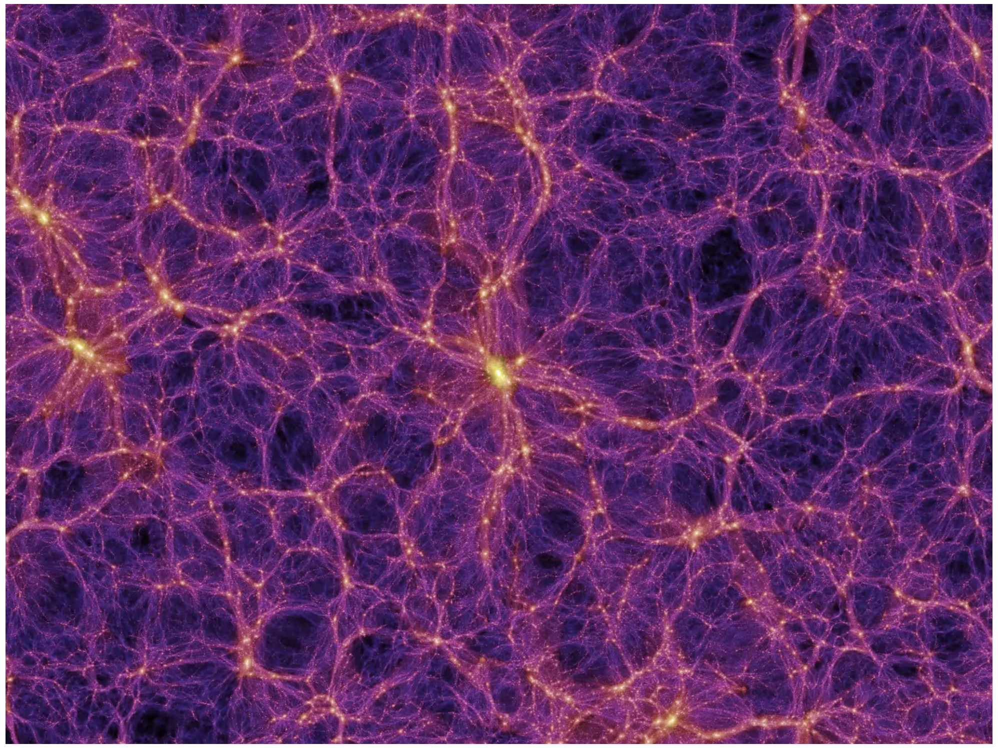
Nonlocal QFT

- **Effects associated to the nonlocality scale**
- **Violations of Huygens' Principle**
- **Dark Matter Candidate**





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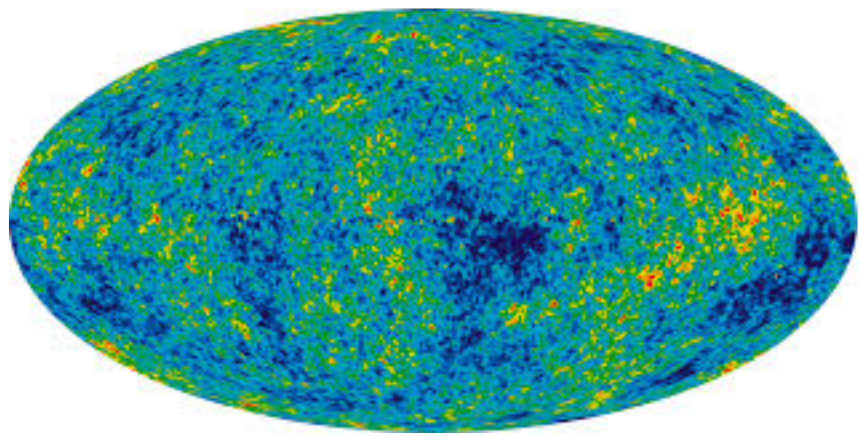
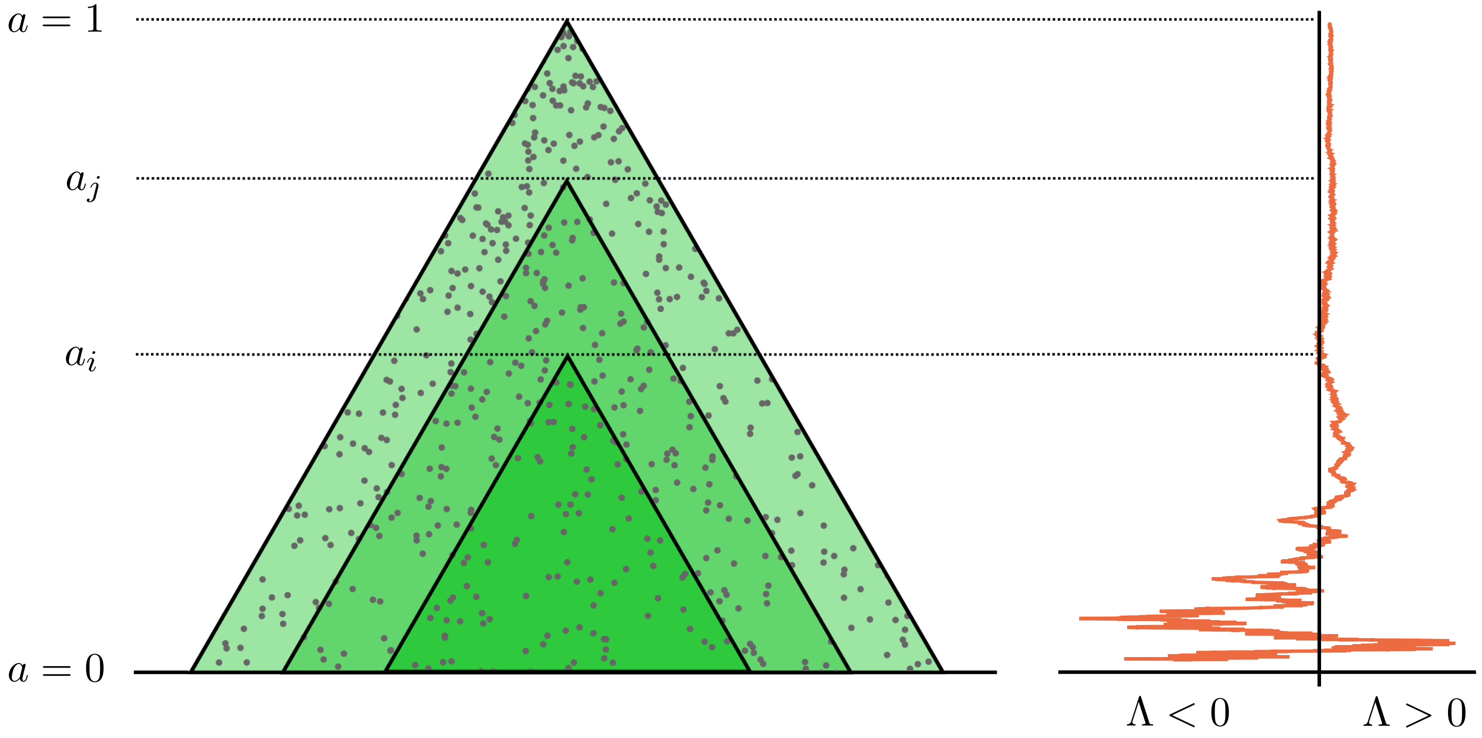
Everpresent Λ Cosmology

$$\Lambda \sim \frac{1}{\Delta N} = \frac{1}{\sqrt{V}} \sim H^2 \sim 10^{-122}$$





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Everpresent Λ Cosmology

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Thank You

Poisson Distribution

The best **number-volume correspondence** is achieved by the **Poisson distribution:**

$$P_N(V) = \frac{(\rho V)^N}{N!} e^{-\rho V}$$

$$\langle N \rangle = \rho V$$
$$\Delta N = \sqrt{\rho V}$$