What process matrices tell us about quantum spacetime

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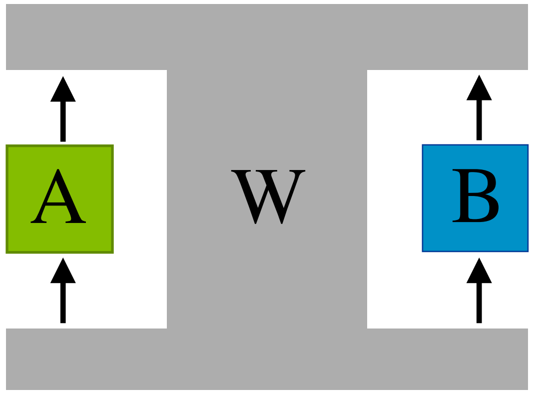
Indefinite Causal Structure

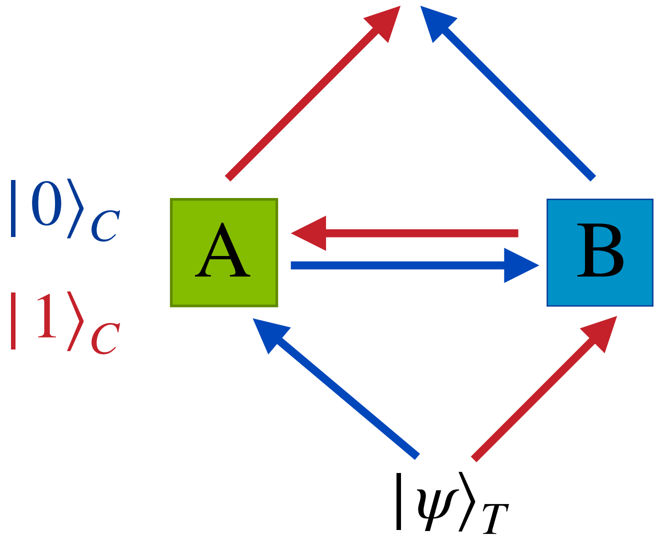
- Generalizes usual notion of causal structure
- Motivation: **Quantum uncertainties blur light cones** and cause-effect distinction
- **Operational** formulation of **events**:
    - A quantum system enters a lab of an agent A, B
    - Agent can apply arbitrary quantum instrument
    - Quantum system is sent out again

  Active agents instead of passive observers!



- Standard example: **Quantum Switch**

$$U_{\text{switch}} = |0\rangle\langle 0|_C \otimes BA_T + |1\rangle\langle 1|_C \otimes AB_T$$



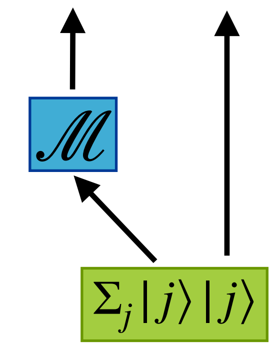
G. Chiribella, G. M. D'Ariano, P. Perinotti, and B. Valiron, PRA 88, 022318 (2013)

O. Oreshkov, F. Costa, and C. Brukner, Nat. Commun. 3, 1092 (2012)

Process matrices

M.-D. Choi, Linear Algebra and its Applications, 10, 285–290 (1975)

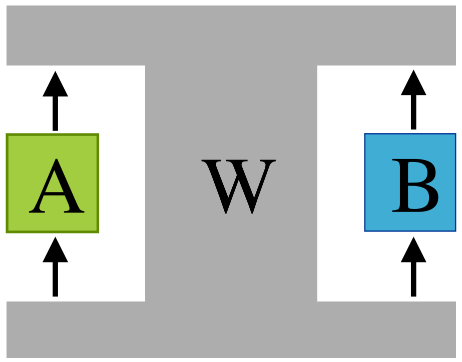
- **Choi** formalism to turn (probabilistic) quantum channels $\mathscr{M}$ into operators M



$$M := \sum_{j,k} (\mathscr{M} \otimes \mathrm{Id})\big(\, |j\rangle |j\rangle\langle k| \langle k| \,\big)$$

Allows to apply methods familiar from density matrices!

- Process matrix W encapsulates **causal structure/environment**

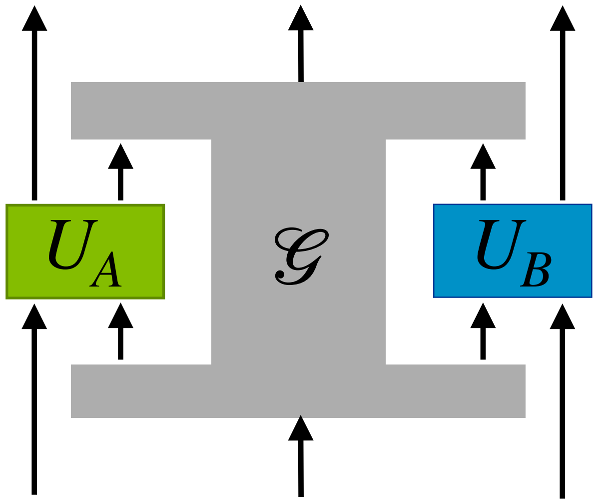


$$p(a, b \,|\, \{M_a\}_a, \{N_b\}_b, W) = \mathrm{Tr}[W \cdot (M_a \otimes N_b)]$$

Process matrix

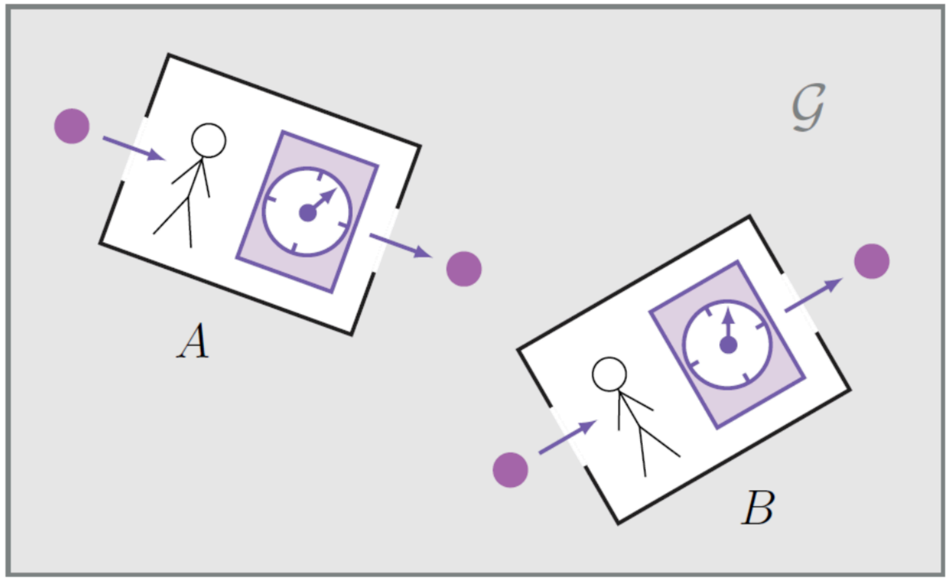
Choi operators of agents' instruments

- Only requirement: **Valid probabilities**
- Downside: Which W are **physical**?
- Purified/dilated process matrices: $\mathscr{G}(U_A, U_B, \ldots)$ unitary



G. Chiribella, G. M. D'Ariano, P. Perinotti, and B. Valiron, PRA 88, 022318 (2013)
M. Araújo, A. Feix, M. Navascués, Č. Brukner, Quantum 1, 10 (2017)
O. Oreshkov, F. Costa, and C. Brukner, Nat. Commun. 3, 1092 (2012)

Page-Wootters circuits for indefinite causal order



- Goal: Model time perception of agents probing quantum causal structure

- Each agent has a discrete quantum clock

- Modeled via history states: $$|\Psi\rangle\rangle = \sum_{t_{A_1}, t_{A_2}, \ldots} |\psi(t_{A_1}, t_{A_2}, \ldots)\rangle \otimes |t_{A_1}, t_{A_2}, \ldots\rangle$$

- Perspective of agent A_j : $$|\psi_{A_j}(t_{A_j})\rangle := N^{(j)}_{t_{A_j}} \cdot \langle t_{A_j} | \Psi\rangle\rangle$$

- Normalization operators due to discretization, and kinematical vs. physical Hilbert space

continuous analogue: E. Castro-Ruiz, F. Giacomini, A. Belenchia, and C. Brukner, *Quantum clocks and the temporal localisability of events in the presence of gravitating quantum systems*, Nature Commun. **11**, 2672 (2020), arXiv:1908.10165

Causal reference frame of agent X

$$|\Psi\rangle\rangle = \sum_{t_{A_1}, t_{A_2}, \ldots} |\psi(t_{A_1}, t_{A_2}, \ldots)\rangle \otimes |t_{A_1}, t_{A_2}, \ldots\rangle$$

- Perspective of agent A_j :

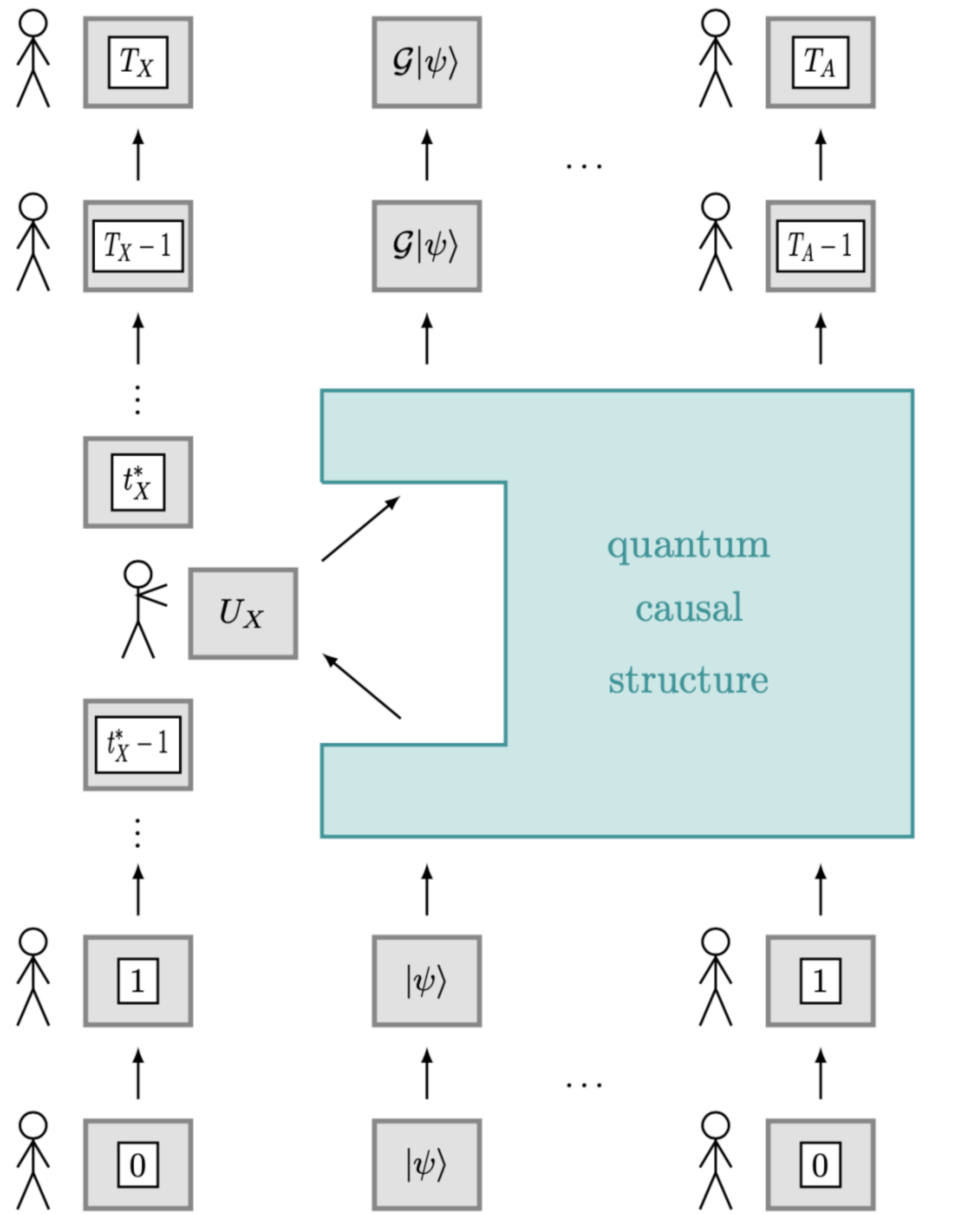
$$|\psi_{A_j}(t_{A_j})\rangle := N^{(j)}_{t_{A_j}} \cdot \langle t_{A_j} | \Psi\rangle\rangle$$

$$|\psi_{A_j}(t_{A_j}+1)\rangle = \mathcal{U}_{t_{A_j}}(t_{A_j}+1,\, t_{A_j})\,|\psi_{A_j}(t_{A_j})\rangle$$

Depends on U_{A_k} of other agents!

- Time of action $t^*_{A_j}$:

$$\mathcal{U}_{t_{A_j}}(t^*_{A_j}+1,\, t^*_{A_j}) = U_{A_j} \otimes \text{Rest}$$



Causal reference frames with time

$$\Big(\mathcal{G}(U_{A_1}, U_{A_2}, \ldots)\,|\psi\rangle\Big) \otimes |T_{A_2}, T_{A_3}, \ldots\rangle =$$

$$\mathcal{U}_{A_j}(T_{A_j},\, t^*_{A_j}) \cdot (U_{A_j} \otimes \mathrm{Rest}) \cdot \mathcal{U}_{A_j}(t^*_{A_j} - 1,\, 0)\,|\psi\rangle\,|0,0,\ldots\rangle$$

purified process matrix

causal future of A_j

action of A_j

causal past of A_j

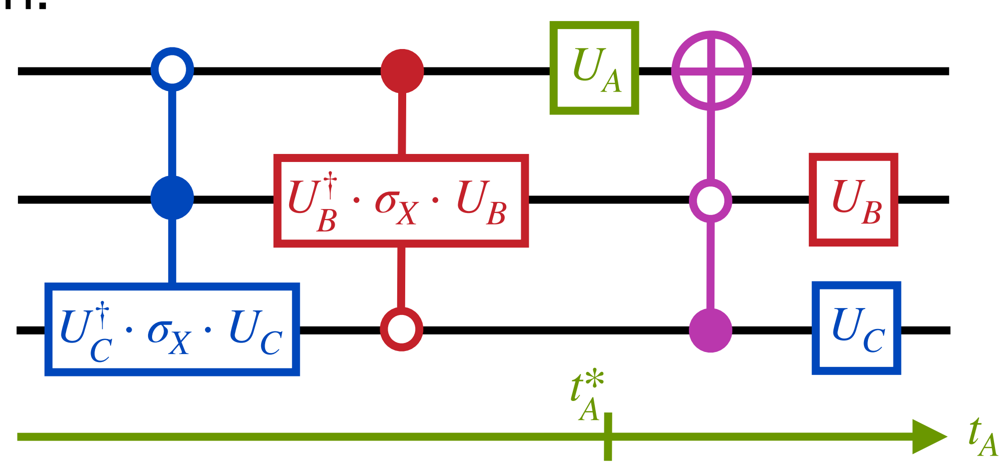
- Main postulate: Agents see global unitary time evolution for each discrete time step

  For all unitaries chosen by other agents! Mathematically restrictive!

- Perspectival time evolution essentially affine-linear in actions of other agents:

  $\mathcal{U}_{A_1}(t,\, 0)\,|\psi\rangle\,|0,0,\ldots\rangle$ affine-linear in $U_{A_2}, U_{A_3}, \ldots$

- Application:



Causal reference frame decomposition of reversed Lugano process from Allard Guérin & Brukner (2018)

Invalid!

P. Allard Guérin and Č. Brukner, *Observer-dependent locality of quantum events*, New Journal of Physics **20**, 103031 (2018).

New directions for process matrices

Please tell me if you are interested in working on this!

Generalizability postulates

Central problem: Process matrices too permissive, need extra **physical** postulates!

Ongoing work with David Trillo from CUNEF university

- So far: Process matrix framework assumes **fixed dimensions** of agent operations
- **Quantum gravity setups** should generalize naturally do **probes of any dimension**
- Extension non-trivial: probe still in quantum gravity!
- Make this a **postulate/design principle**

- So far: Number of agents is fixed
- However, physical processes should allow to **insert more agents**
- Example for approach with causal reference frames:

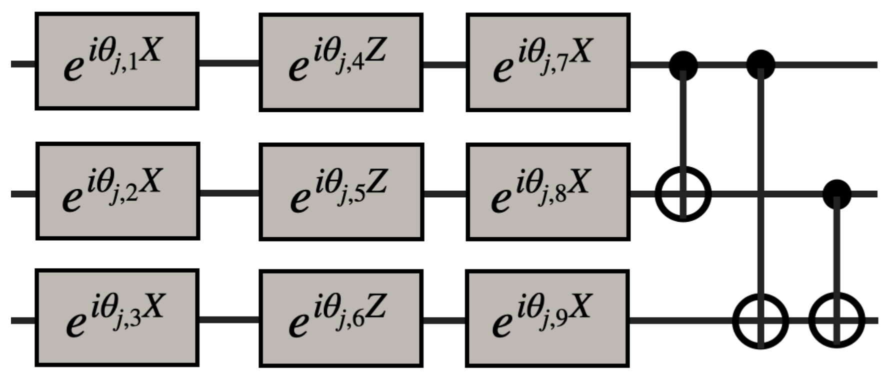
- **Postulate:** This should be a valid process too!

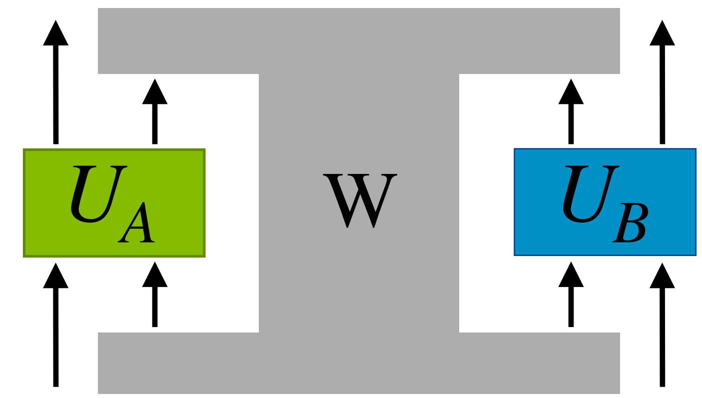
Variational Quantum Processes

- Finding optimal agent operations with quantum machine learning

$$U_A(\vec{\theta}) = U_A^{(D)}(\vec{\theta}) \cdot \ldots \cdot U_A^{(1)}(\vec{\theta})$$

$$U_A^{(\ell)}(\vec{\theta}) :=$$





- Can do the same for process matrix: $W \to W(\vec{\varphi})$

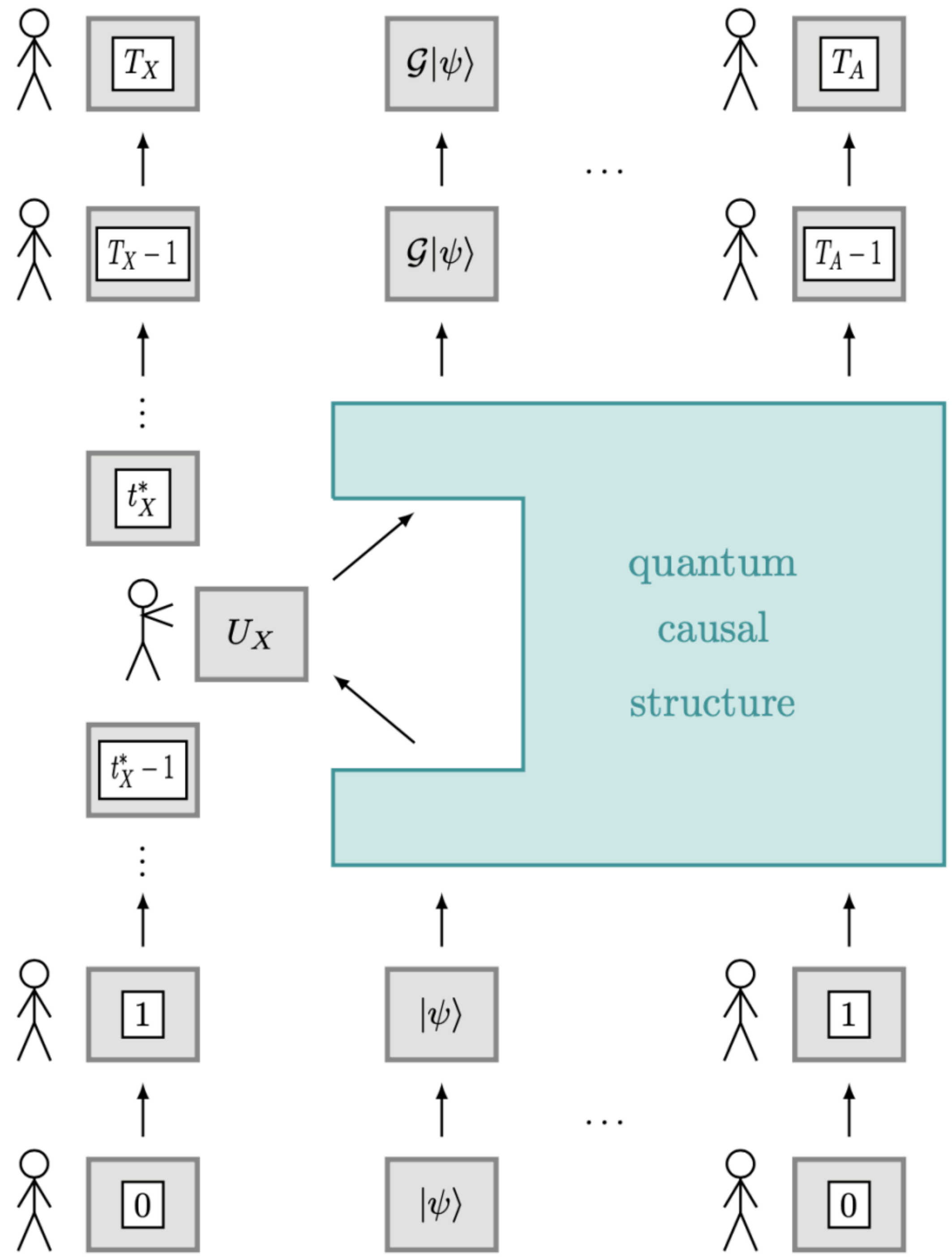
- Example application: Causal inequalities as loss function

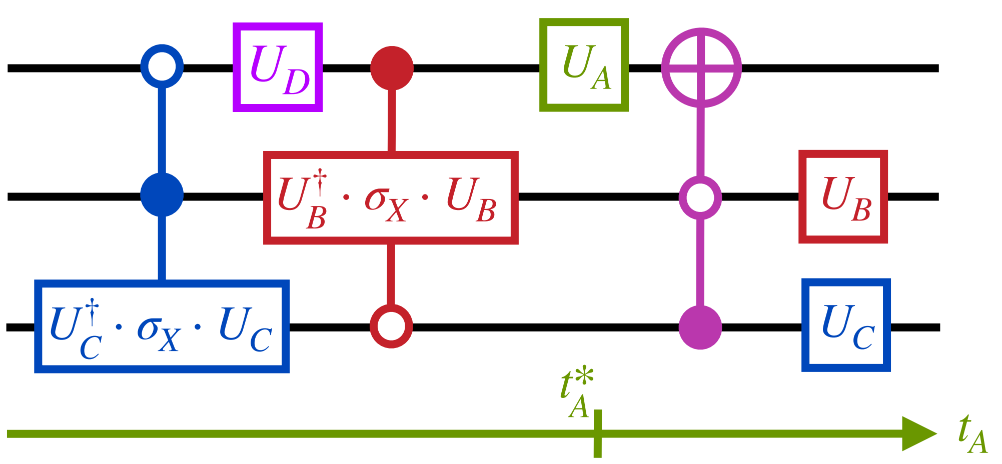
$$f(W, U_A, \ldots) \le C$$

$$L(\vec{\varphi}, \vec{\theta}, \ldots) := -f\big(\; W(\vec{\varphi}),\; U_A(\vec{\theta}), \ldots\big)$$

- Find strongest violation with gradient descent on $\vec{\varphi}, \vec{\theta}$!

Thanks for your attention!



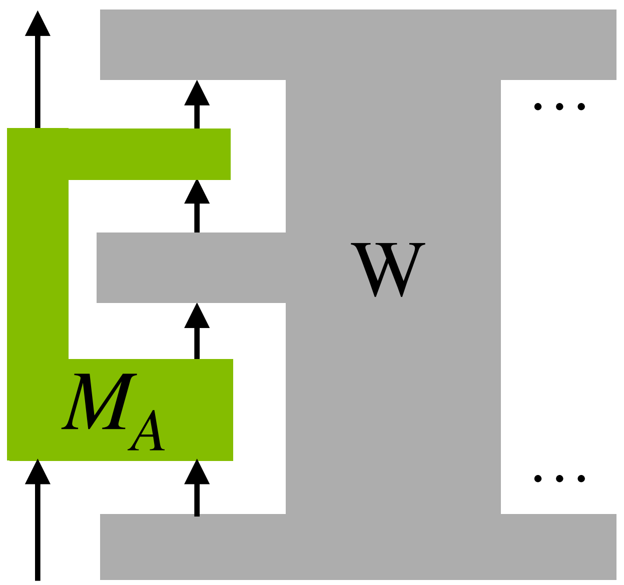


Multi-round process matrices

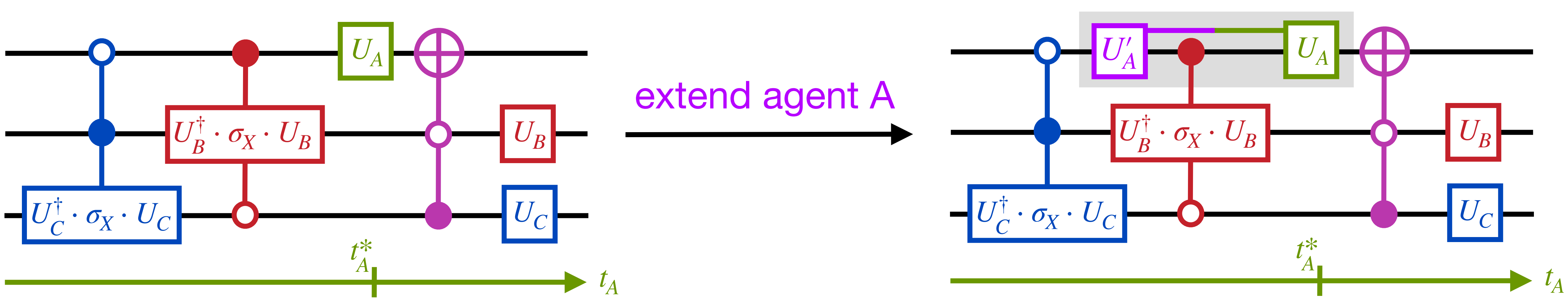
- Usually: Same agent at different times treated as **different agents**
- In particular:
  - Process matrix definition **does not acknowledge proper time**
  - Agent **cannot send itself a quantum signal** within its own lab
- Proposal by Hoffreumon & Oreshkov:

  Multi-round process matrices

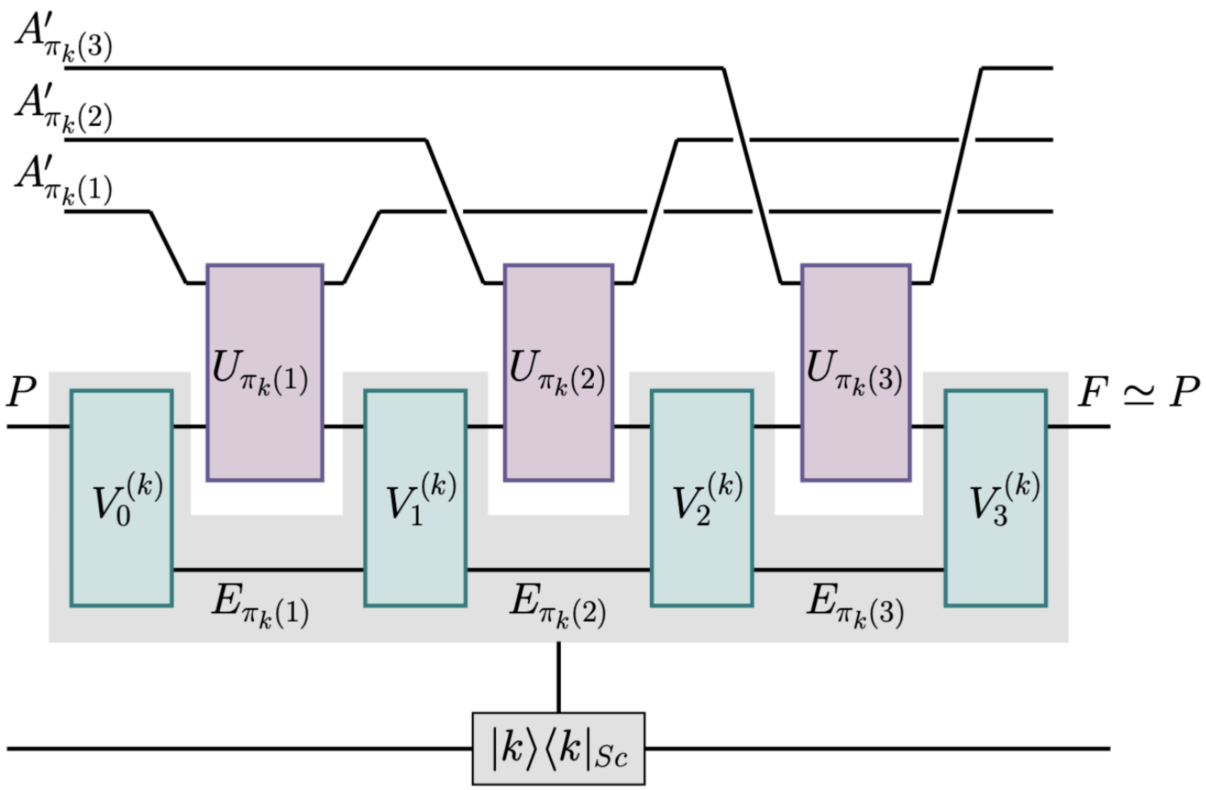
  T. Hoffreumon, O. Oreshkov, *The Multi-round Process Matrix*, Quantum **5**, 384 (2021)

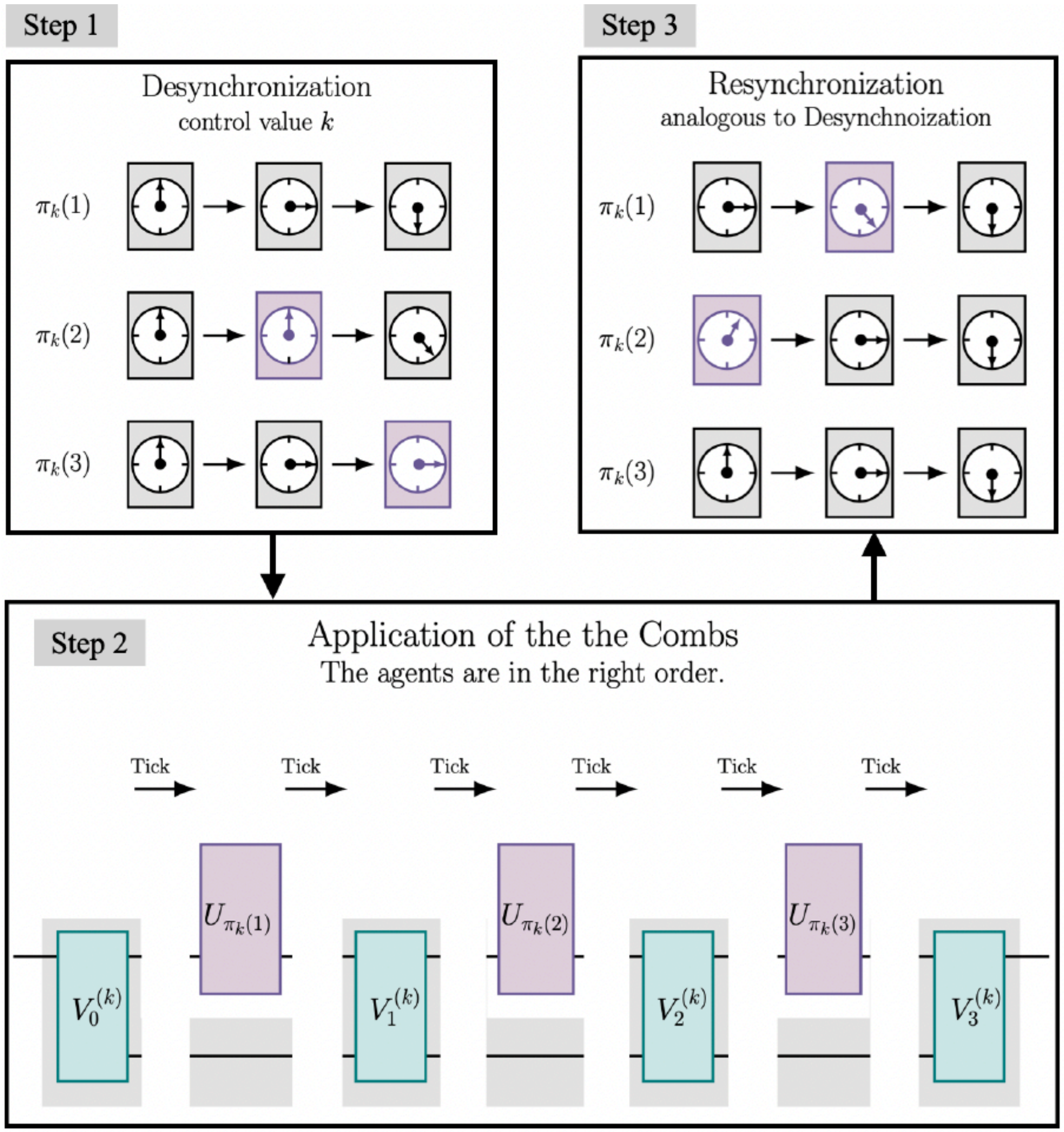


- Model agent at different times as **quantum comb**
- How does this restrict set of valid processes?
- Can combine with agent insertion postulate:

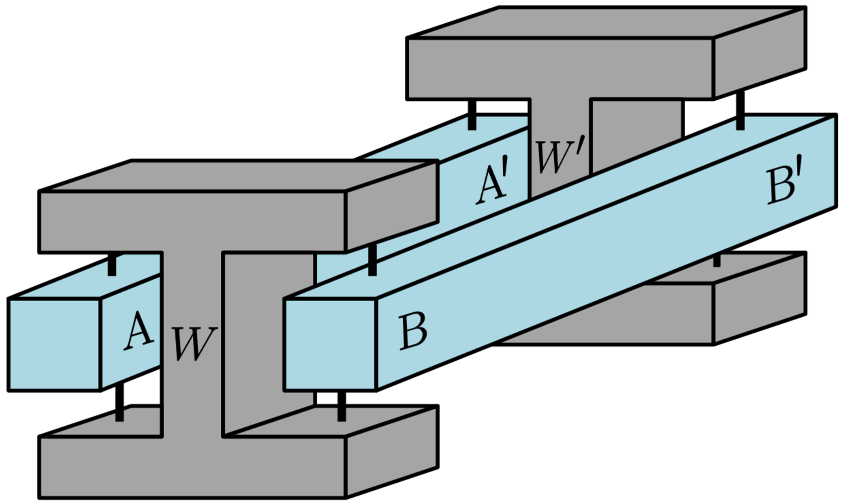


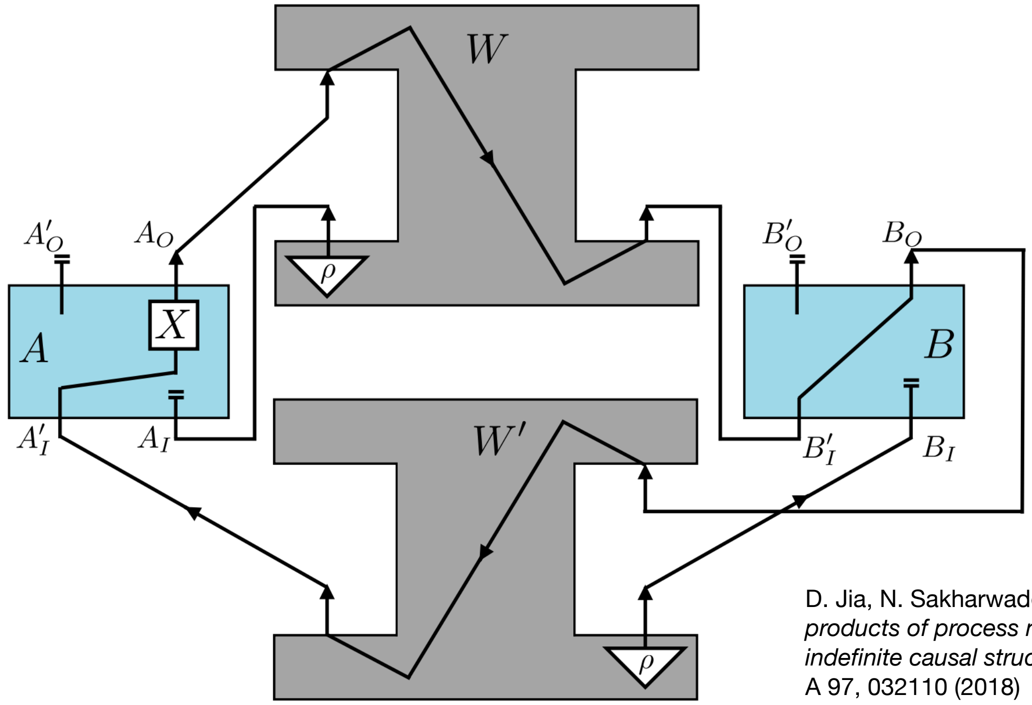
Arbitrary coherently controlled causal order





Parallel composition of process matrices





D. Jia, N. Sakharwade, *Tensor products of process matrices with indefinite causal structure*, Phys. Rev. A 97, 032110 (2018)

$\otimes$ has **grandfather paradox**, is there a **universal replacement** for $\otimes$?

No-Go Theorem:

W process matrix of agents (A,B), W' process matrix of (A',B')

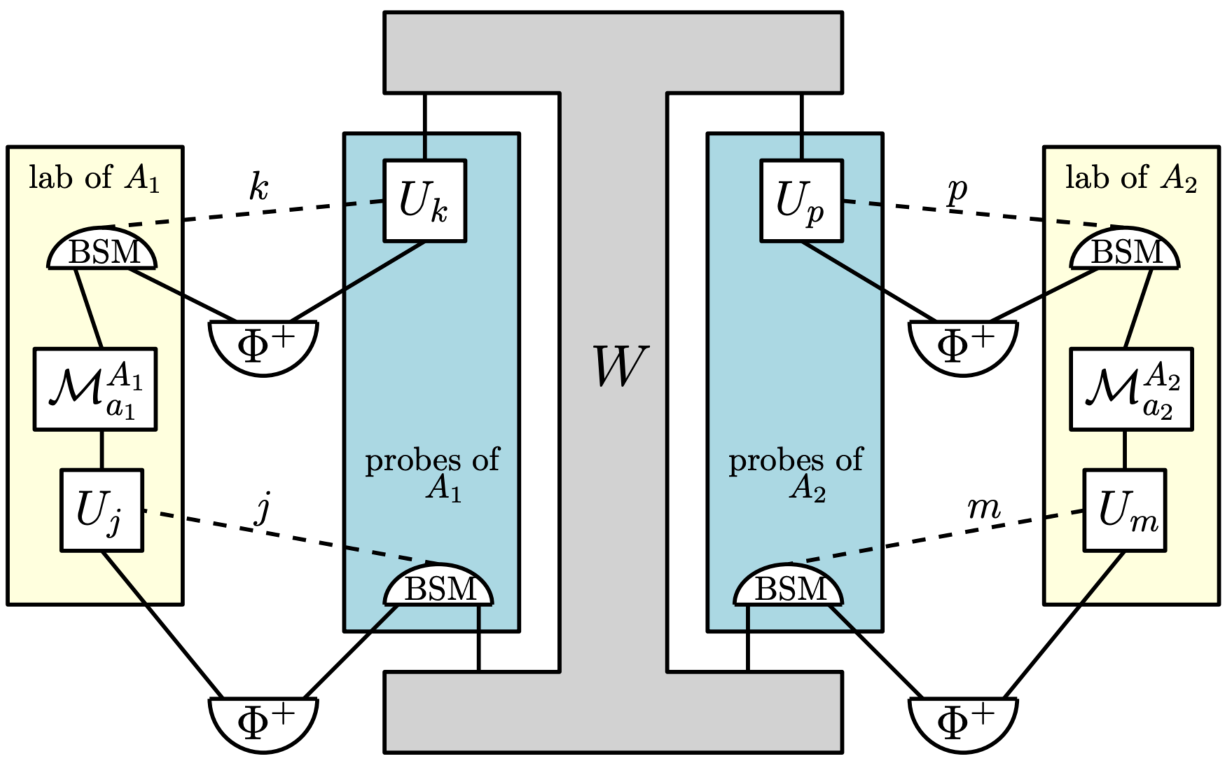
There is no function $\mu(\cdot\,,\cdot)$ such that:

- $\mu(W, W')$ **valid** process matrix of (AA', BB') for all W, W'
- μ is real-**bilinear**
- μ is $\otimes$ for W, W' with **same definite causal** order

Warm-up: Process teleportation

- **Motivation:**
  - Exotic natural processes might happen far away from laboratories
  - Process implementation might be too fragile to allow for arbitrary instruments
- **Process matrix picture:** It is enough to teleport the inputs and output!

- Ideal, deterministic teleportation:



- **Crucial problem:**

outside agents ⇄ inside probes (classical communication)

Outside agents need to act "while" their socket in W is active

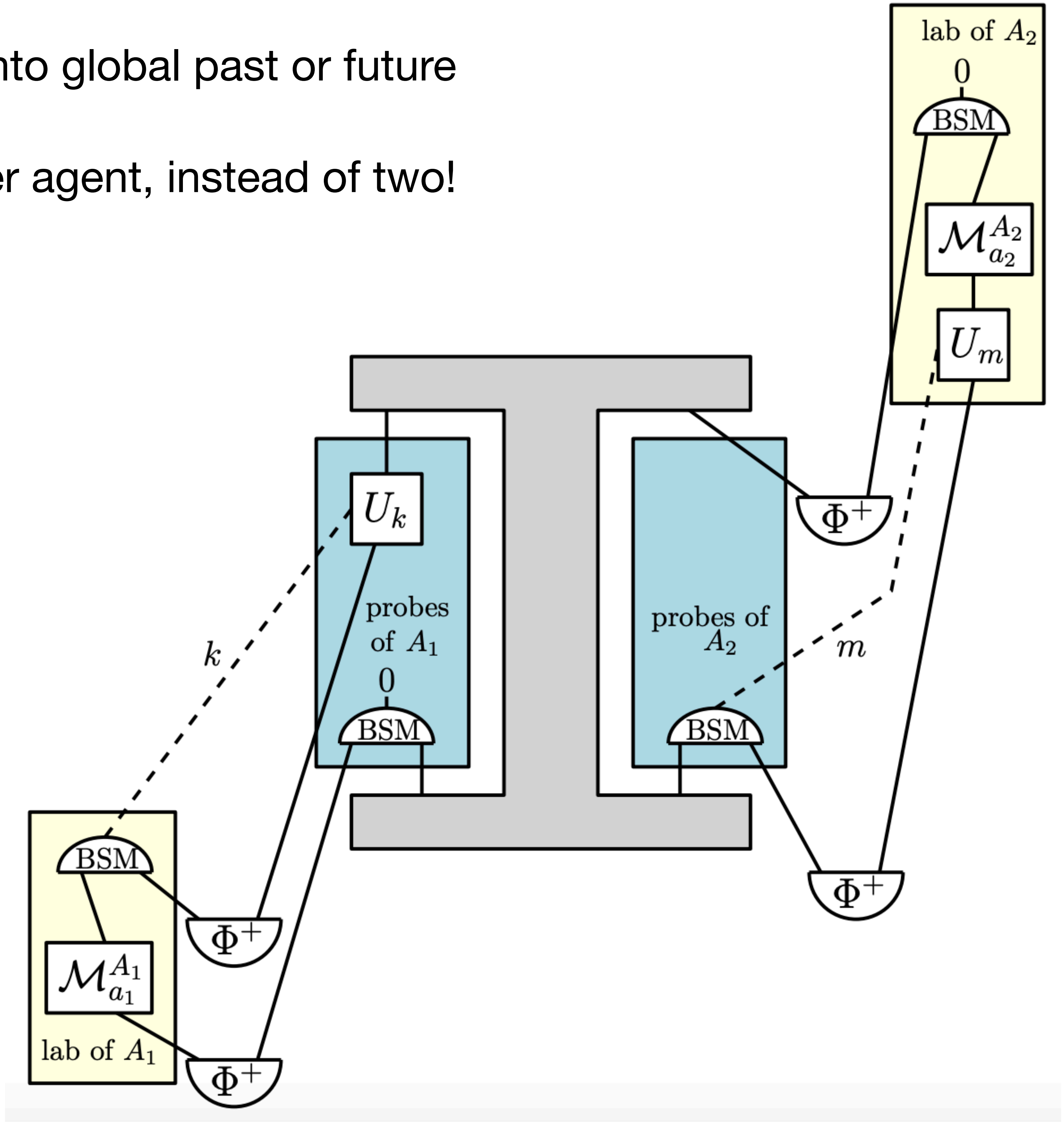
If probes are in an exotic indefinite causal structure, also agents need to be!

- Can replace classical communication with post-selection

But this discards many runs of the experiment…

Partially post-selected teleportation

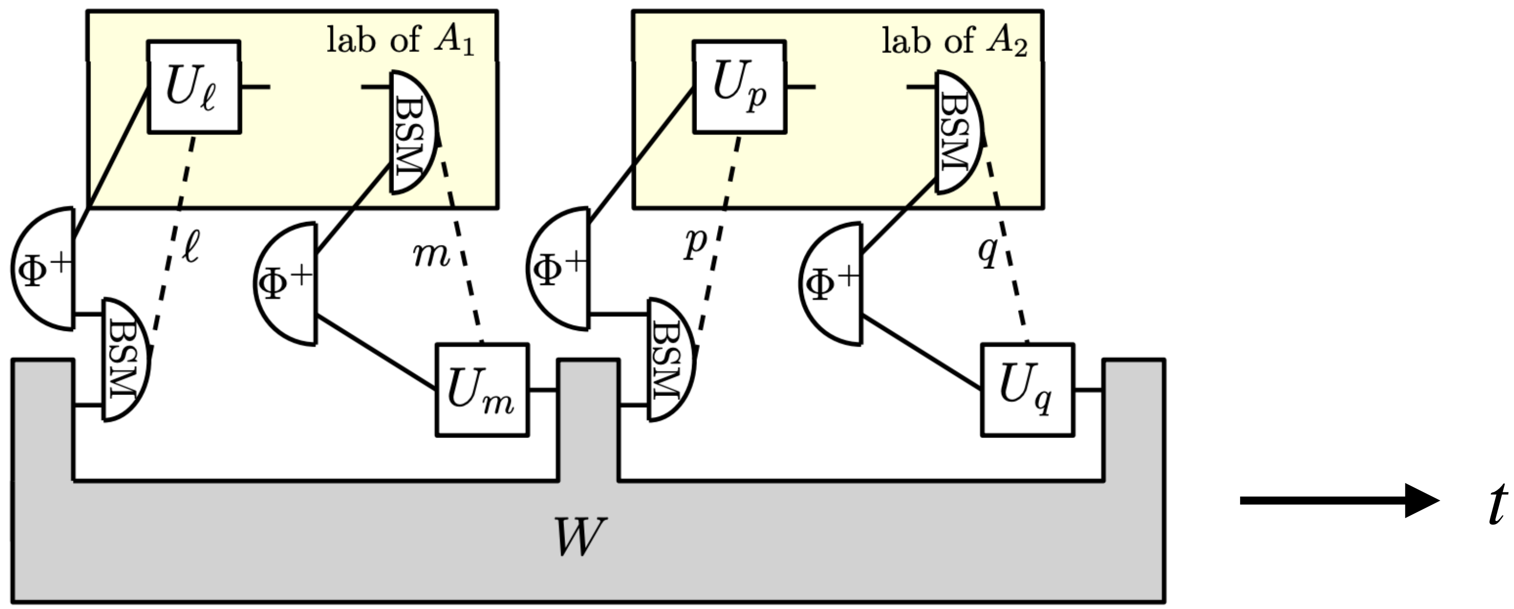
- Compromise: Put agents into global past or future
- Only one post-selection per agent, instead of two!



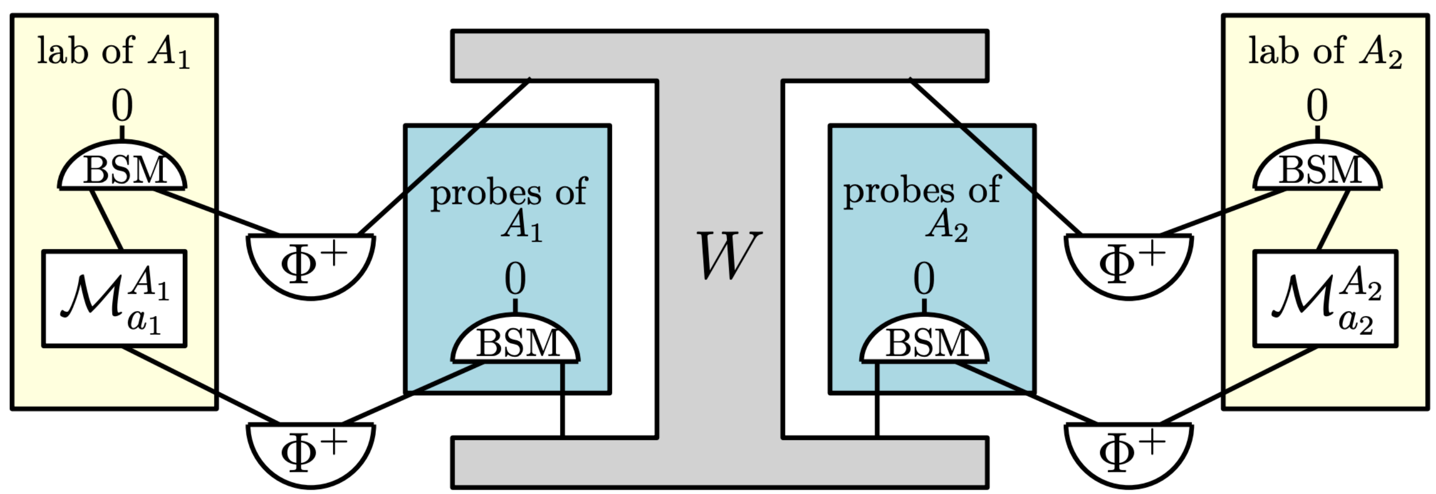
Ways around the problem

- **Definite causal order or causally separable:**

Just put the outside agents into the right order



- **Avoid classical communication via post-selection:**

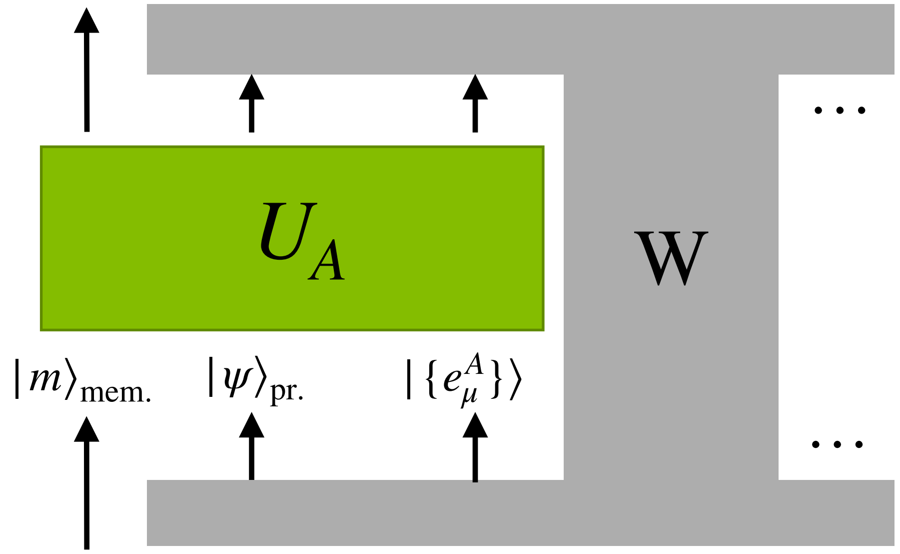


Works for all processes, but low success probability

What measurement does an agent actually perform?

- How do (quantum) relativistic effects change agent measurements?

  time dilation, spinning, …



- Measurement setup of agent:
  - Quantum memory $|m\rangle_{\text{memory}}$
  - Triad $|\{e_\mu^A\}\rangle$
  - Probe system $|\psi\rangle_{\text{probe}}$

- Agent can only use **things inside** the lab to **define measurements**

  In particular: triad e_μ^A $\quad$ e_μ^A embeds **agent's choice into spacetime**

- Agent treats inside of lab non-relativistically

  "everything is slow in the elevator"

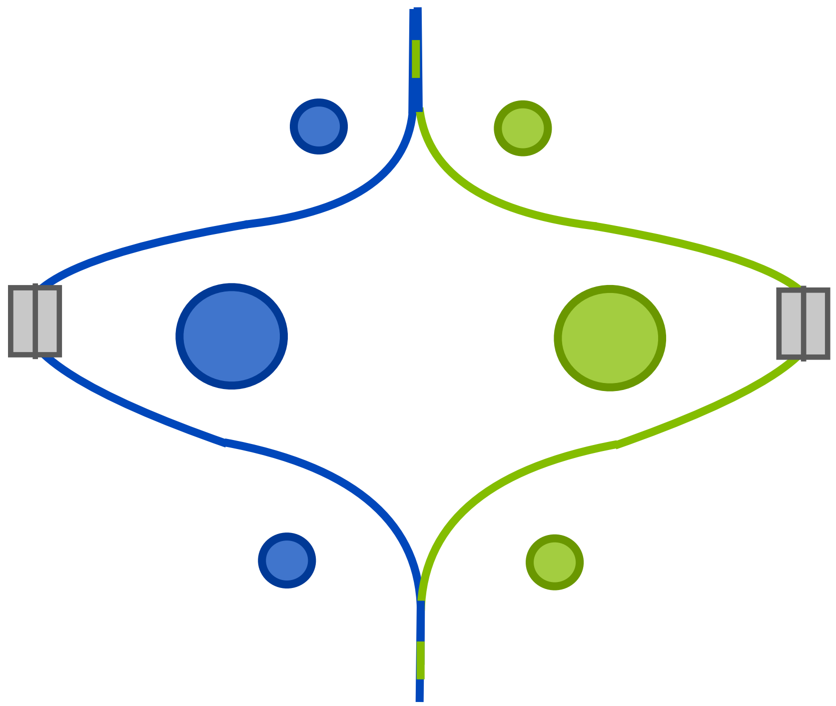
- Agent **implements** measurement **with respect to triad**

  For qubit:
  - Expand in operator basis σ^A
  - Combined transformation law:

$$|e_\mu^A\rangle \mapsto |R^A{}_B e_\nu^B\rangle \qquad \sigma^A \mapsto D(R)\cdot\sigma^A\cdot D(R)^{-1}$$

Superpositions of spacetimes

- Paradigm of superpositions of metrics $\quad |\psi\rangle = |g_{\mu\nu}\rangle + |g'_{\mu\nu}\rangle$



- Make operational sense by demanding ***asymptotic classicality:***

$$g_{\mu\nu}(x)\,,\, g'_{\mu\nu}(x) \;\to\; g^{\text{class.}}_{\mu\nu}(x) \quad \text{for} \quad x \to \infty$$

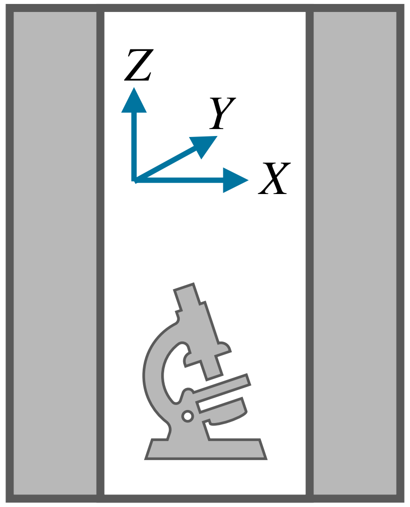
  - Sort of like *asymptotically free Hilbert spaces* in QFT scattering
  - Initial state preparation and final measurement happen in asymptotic regions!

- Like in interferometers:

  Require that asymptotically, world lines of same agent overlap

Which measurement does an agent **actually** perform?

- Commonly, measurements only treated **abstractly**
  - Some operator in an **algebra of observables**
  - Which one is it?
- Agent is in a **sealed lab** "elevator"
- Agent treats inside as **non-curved** "Einstein elevator"



- Agent can only use **things inside** the lab to **define measurements**

  In particular: triad e_μ^A e_μ^A embeds **agent's choice into spacetime**

- Agent **implements** measurement **with respect to triad**

  Agent **knows A-indices**, but does **not know μ-indices**

Partially post-selected teleportation

- **Idea:** Put outside agents in global past or future
  - One teleportation post-selected
  - Other one deterministic with one-way communication
- Agents see **statistics of indefinite causal structure**, despite their **own definite causal situation**
- **Proof**, category theory style (formally via link product):

  Push boxes along wires into the blue labs.
  In lab, we know that teleportation works.
- **Important:** Extended scenario is still a valid process

  Otherwise, agents/probes could implement other instruments and induce **logical inconsistencies**