Entanglement and CCR in QED scattering Processes

Bruno Micciola*

Dipartimento di Fisica E.R. Caianiello, Università di Salerno and INFN, Salerno, Italy

Fakultät für Mathematik, Universität Wien

*M. Blasone, G. Lambiase and B. M., Phys.Rev.D (2024).
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Summary

1. Entanglement in QED scattering processes

2. Complete Complementarity Relations in QED scattering processes

3. Entanglement saturation in quantum electrodynamics scattering processes

Motivations

- Investigation of fundamental properties of (elementary) particles via quantum correlations†
- Entanglement in relativistic systems as a possible resource in Quantum Information
- Entanglement in high energy processes as a probe for new physics beyond Standard Model

† G. Aad *et al.* [ATLAS], *Observation of quantum entanglement with top quarks at the ATLAS detector*, Nature **633** (2024), 542-547

Motivations: entanglement in scattering processes

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Entanglement in QED scattering processes

Entanglement generation in QED processes*

How entanglement (in helicity) is generated in the processes:

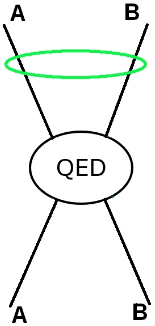
$e^- e^+ \to \mu^- \mu^+$

$e^- e^+ \to e^- e^+$

$e^- \mu^- \to e^- \mu^-$

$e^- \gamma \to e^- \gamma$

...



Initial and final helicity states:

$$|i\rangle = |R\rangle_A \otimes |L\rangle_B$$

$$|f\rangle = \sum_{r,s=R,L} \mathcal{M}(RL;rs)\,|r\rangle_A\,|s\rangle_B$$

*A. Cervera-Lierta et al., SciPost Phys. (2017).
S. Fedida, A. Serafini, Phys.Rev.D (2023).

Unitarity

- S matrix is unitary

- At tree level, unitarity is recovered by resorting by inclusion of 1-loop diagrams (optical theorem)†

- We consider a sharp momentum filtering on the final state ρ_f, corresponding to a POVM procedure: $M_{\bar{p}} = \sum_\eta |\bar{p}, \eta\rangle \langle \bar{p}, \eta|$.
Post-measurement state:

$$\tilde{\rho}_f = \frac{M_{\bar{p}} \rho_f M_{\bar{p}}}{Tr(M_{\bar{p}} \rho_f M_{\bar{p}})}$$

whose density matrix is provided in terms of S-matrix elements.

- The process $\rho_i \to \tilde{\rho}_f$ is described by a non-unitary map.

†K. Kowalska and E. M. Sessolo, JHEP (2024). S. Shivashankara and G. Gogliettino, Phys. Rev. D (2024)

Bhabha scattering in the COM reference frame

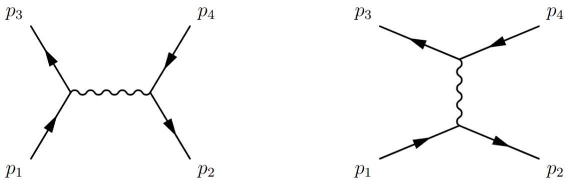


Figure 1: Feynman diagrams for Bhabha scattering.

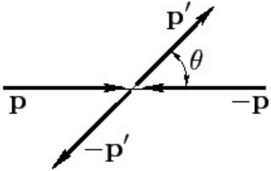


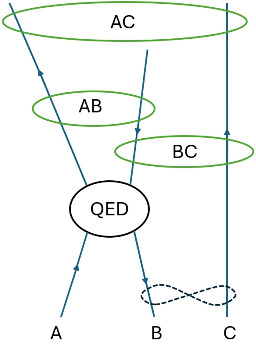
Figure 2: COM reference frame.

QED processes with spectator particles[‡]

Initial and final states

$$|i\rangle = |R\rangle_A \otimes \Big(\cos\eta\, |R\rangle_B\, |R\rangle_C + e^{i\beta} \sin\eta\, |L\rangle_B\, |L\rangle_C \Big)$$

$$|f\rangle = \sum_{r,s=R,L} \big[\cos\eta\, \mathcal{M}(RR;rs)\, |r\rangle_A\, |s\rangle_B\, |R\rangle_C\, + \\ e^{i\beta} \sin\eta\, \mathcal{M}(RL;rs)\, |r\rangle_A\, |s\rangle_B\, |L\rangle_C \big]$$



[‡] J.B. Araujo et al., Phys. Rev. D (2019).

M. Blasone, G. Lambiase and B. M., Phys. Rev. D 109 (2024).

Entanglement distribution in Bhabha scattering [§]

- We studied generation and distribution of entanglement in Bhabha scattering for the three bipartite channel AB, AC,BC.
- For incoming momenta of the order of the mass $\mu = \frac{|\vec{p}|}{m} \simeq 1$, non-trivial entanglement distribution in the three output channels.
- In the relativistic regime, analytic expressions for the concurrences:

$$\lim_{\mu\to\infty} C(\rho^f_{AB}) = \frac{2\sin^2\eta\sin^4(\theta/2)\cos^4(\theta/2)}{1-(1-\frac{1}{8}\sin^2\theta)\sin^2\theta\sin^2\eta}$$

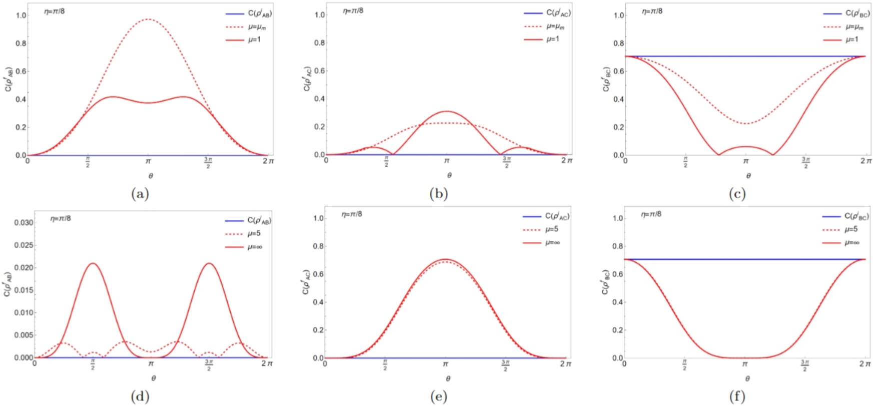
$$\lim_{\mu\to\infty} C(\rho^f_{AC}) = \frac{\sin(2\eta)\sin^4(\theta/2)}{1-(1-\frac{1}{8}\sin^2\theta)\sin^2\theta\sin^2\eta}$$

$$\lim_{\mu\to\infty} C(\rho^f_{BC}) = \frac{\sin(2\eta)\cos^4(\theta/2)}{1-(1-\frac{1}{8}\sin^2\theta)\sin^2\theta\sin^2\eta}$$

[§] M. Blasone, G. Lambiase and B. M., Phys. Rev. D 109 (2024).

Entanglement distribution ($\eta = \pi/8$)

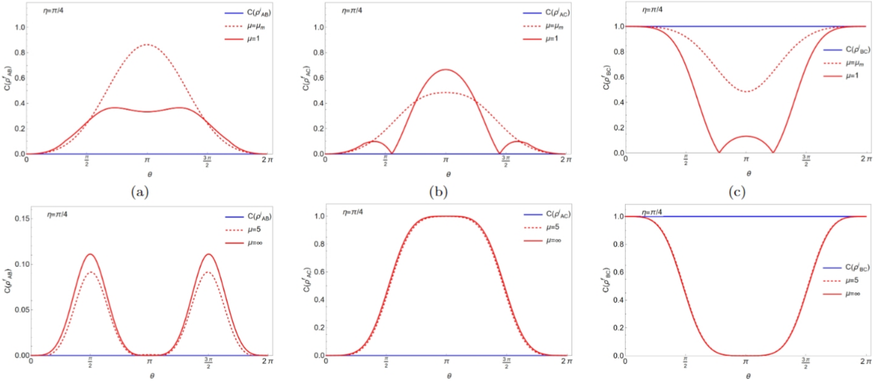
AB AC BC



$$\mu_m = \frac{1}{2}\sqrt{-3+\sqrt{17}}$$

Entanglement distribution ($\eta = \pi/4$)

AB AC BC

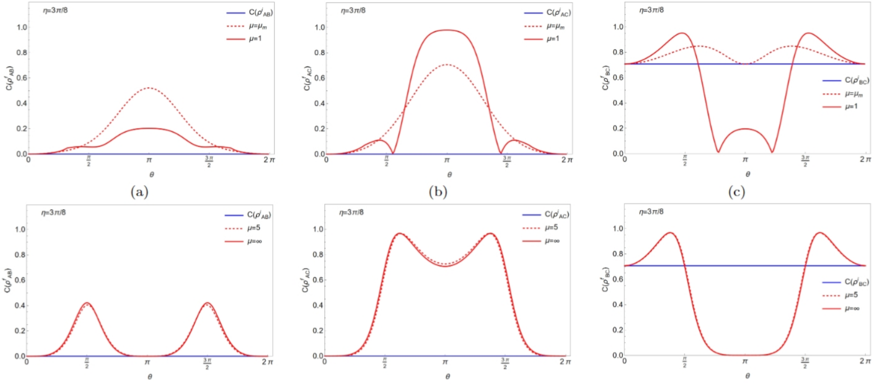


(a) (b) (c)

(d) (e) (f)

Entanglement distribution ($\eta = 3\pi/8$)

AB AC BC



(a) (b) (c)

(d) (e) (f)

Main results

- The distribution and generation of entanglement tend to concentrate in some bipartitions.

- Complete transfer from BC channel to the AC channel in $\theta = \pi$ in relativistic regime: interaction as a *quantum gate* $\Rightarrow$ possible application in quantum information protocols.

- For $\frac{\pi}{4} < \eta < \frac{3\pi}{4}$ the interaction generates entanglement in the BC channel, where it was present before the interaction.

Complete Complementarity Relations in QED scattering processes

Complementarity

Complementarity[¶]: the two aspect of duality, corpuscular and undulatory, are equally real but mutually exclusive.

The first quantitative version[‖] of the wave-particle duality was summarized by a simple complementarity relation

$$P^2 + V^2 \leq 1.$$

where P is the **predictability**, a measure of path information and V is the **visibility** of the interference pattern.

¶ N. Bohr, Nature (1928)

‖ W. K. Wootters, W. H. Zurek, Phys. Rev. D. (1979), B-G. Englert, Phys. Rev. Lett. (1996)

Complete Complementarity Relations

For bipartite pure states** one obtains a **triality relation** formed by two quantities generating *local*, single-partite realities and the correlation between subsystems, which generates an exclusive bipartite *non-local* reality:

$$P_k^2 + V_k^2 + C^2 = 1, \qquad k = A, B$$

where P_k and V_k are the predictability and visibility for the single-partite systems. C is a measure of entanglement.

**M. Jakob, J. A. Bergou, Opt. Commun. (2010)

CCR for general bipartite states

– General two qubit state

$$|\Theta\rangle = a\,|00\rangle + b\,|01\rangle + c\,|10\rangle + d\,|11\rangle,$$

- The three terms of the previous triality relation are:

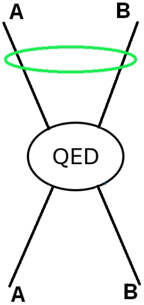
$$C(\Theta) = 2|ad - bc|$$

$$P_A = \Big|(|c|^2 + |d|^2) - (|a|^2 + |b|^2)\Big|; \quad P_B = \Big|(|b|^2 + |d|^2) - (|a|^2 + |c|^2)\Big|$$

$$V_A = 2\Big|ac^* + bd^*\Big| \; ; \quad V_B = 2\Big|ab^* + cd^*\Big|$$

CCR for Bhabha scattering: initial states

We analyzed C, P, V before and after the QED interaction for three different states:



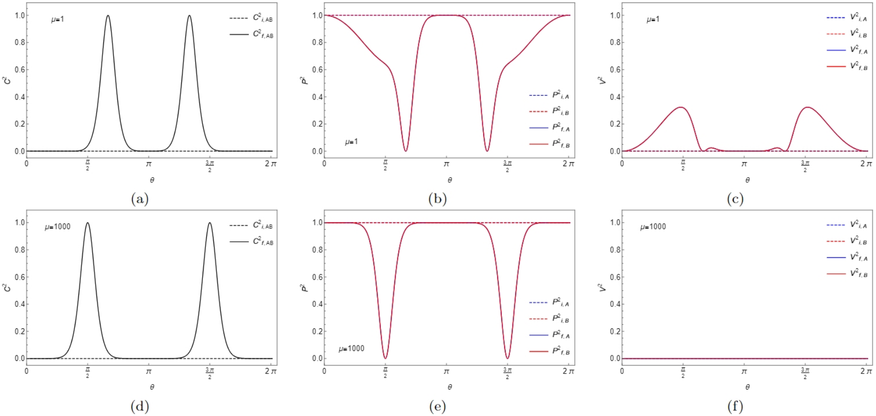
$$|i\rangle_I = |R\rangle_A |L\rangle_B$$

$$|i\rangle_{II} = \left(\cos\alpha\,|R\rangle_A + e^{i\xi}\sin\alpha\,|L\rangle_A\right) \otimes \left(\cos\beta\,|R\rangle_B + e^{i\eta}\sin\beta\,|L\rangle_B\right)$$

$$\begin{aligned} |i\rangle_{III} = & \cos\alpha\,|R\rangle_A |R\rangle_B + e^{i\xi}\sin\alpha\cos\beta\,|R\rangle_A |L\rangle_B \\ & + e^{i\eta}\sin\alpha\sin\beta\cos\chi\,|L\rangle_A |R\rangle_B + e^{i\tau}\sin\alpha\sin\beta\sin\chi\,|L\rangle_A |L\rangle_B \end{aligned}$$

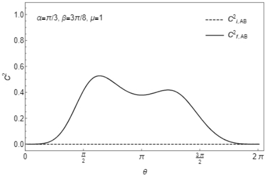
**M. B., S. De Siena, G. Lambiase, C. Matrella and B. Micciola, PRD (2025).

$$P_i = 1, \quad V_i = 0, \quad C = 0.$$



Examples: initial factorized state $|i\rangle_{II}$

$$P_i \neq 0, \quad V_i \neq 0, \quad C = 0.$$



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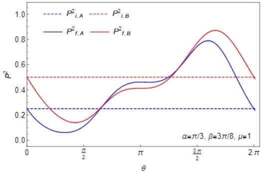

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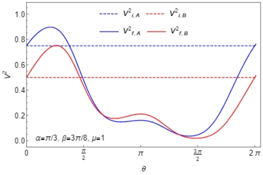

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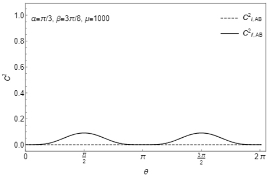

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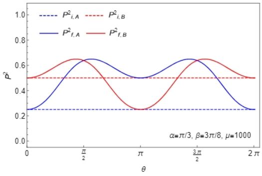

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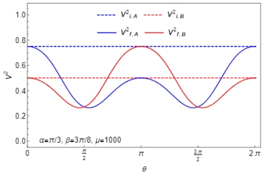
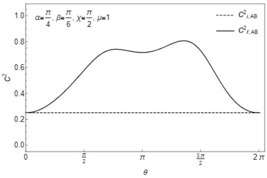


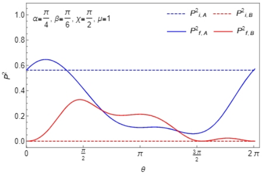
(f)

Examples: initial entangled states $|i\rangle_{III}$

Entanglophilus regime



(a)



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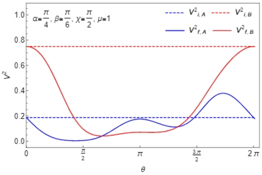

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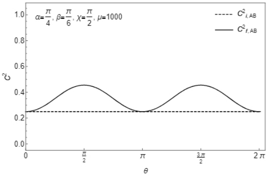

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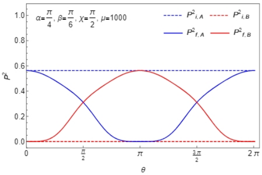

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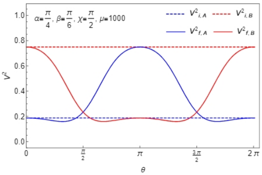
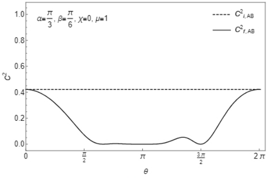


(f)

Examples: initial entangled states III

Entanglophobous regime



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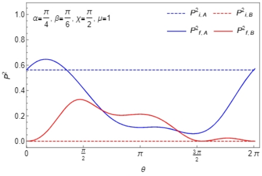

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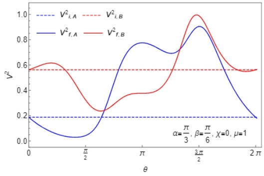

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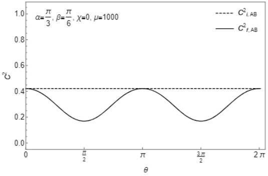

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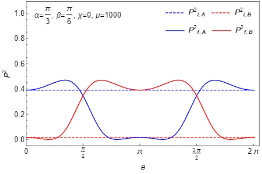

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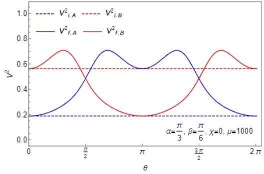
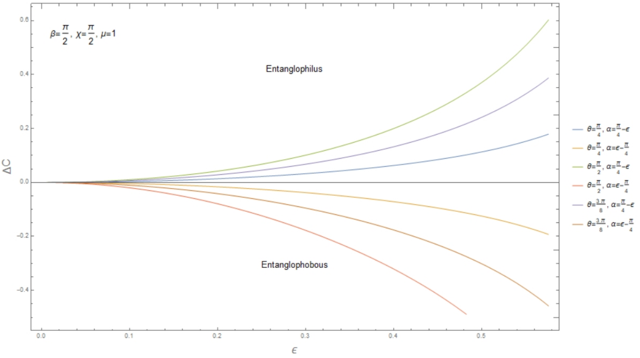


(f)

Entanglophilus, Entanglophobous and Mixed regimes

Entanglophilus, *Entanglophobous* from Bell states

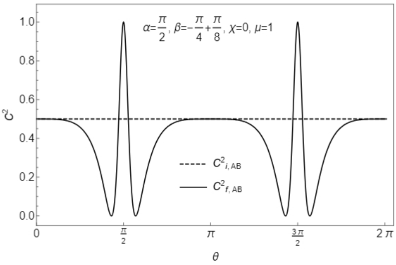
$$\Delta C \equiv (C_f - C_i)/C_i \qquad \Phi^{\pm}_{\alpha} = \cos\alpha\,|RR\rangle \pm \sin\alpha\,|LL\rangle \qquad \Phi^{\pm} = \frac{1}{\sqrt{2}}(|RR\rangle \pm |LL\rangle)$$



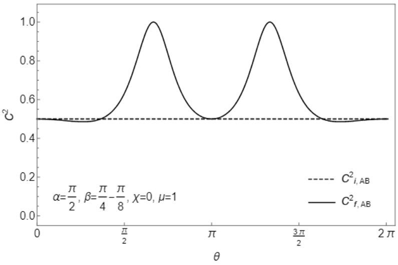
Entanglophilus, Entanglophobous and Mixed regimes

Mixed regimes from Bell states

$$\Psi^{\pm}_{\alpha} = \cos\alpha\,|RL\rangle \pm \sin\alpha\,|LR\rangle \qquad \Psi^{\pm} = \frac{1}{\sqrt{2}}(|RL\rangle \pm |LR\rangle)$$



(a)



(b)

Maximal entanglement conservation

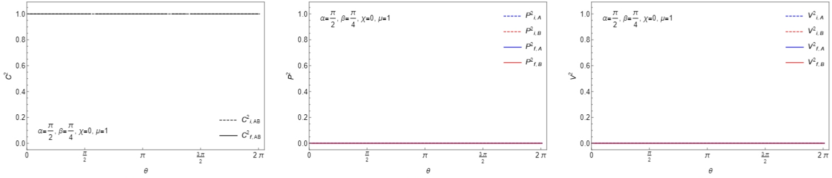
$$P_i = 0 \qquad V_i = 0 \qquad C = 1$$

Processes preserving maximal entanglement:

- $e^- e^+ \to e^- e^+$
- $e^- e^- \to e^- e^-$
- $e^- e^+ \to \mu^- \mu^+$
- $e^- \mu^- \to e^- \mu^-$

Example: Bhabha scattering

$$\Phi^+ \to \Phi^+$$
$$\Phi^- \to \cos s_1 \Phi^- + \sin s_1 \Psi^+$$
$$\Psi^+ \to \cos s_2 \Phi^- + \sin s_2 \Psi^+$$
$$\Psi^- \to \Psi^-$$



(a) (b) (c)

QED scattering with photons

- $e^- e^+ \to 2\gamma$ **: maximal entanglement is not conserved for all the initial maximally entangled states**
- $e^- \gamma \to e^- \gamma$ **: maximal entanglement is never achieved**

	$e^- e^+ \to \gamma\gamma$			Compton	
In. State	Fin. State	Conc.	In. State	Fin. State	Conc.
Φ^+	$\Phi-$	1	Φ^+	GC	< 1
Φ^-	$\cos r\, \Phi^+ + \sin r\, \Psi^+$	$\|\sin(2\, r)\|$	Φ^-	GC	< 1
Ψ^+	Ψ^+	1	Ψ^+	GC	< 1
Ψ^-	Ψ^-	1	Ψ^-	GC	< 1

Entanglement saturation in quantum electrodynamics scattering processes

Quantum maps and invariant sets[††]

For QED $2 \to 2$ scattering processes involving only fermions, maximal entanglement is completely preserved, independently of θ, μ, and of how many scattering events are performed.

$$\mathbf{M} = \begin{pmatrix} \mathcal{M}(RR;RR) & \mathcal{M}(RL;RR) & \mathcal{M}(LR;RR) & \mathcal{M}(LL;RR) \\ \mathcal{M}(RR;RL) & \mathcal{M}(RL;RL) & \mathcal{M}(LR;RL) & \mathcal{M}(LL;RL) \\ \mathcal{M}(RR;LR) & \mathcal{M}(RL;LR) & \mathcal{M}(LR;LR) & \mathcal{M}(LR;LL) \\ \mathcal{M}(RR;LL) & \mathcal{M}(RL;LL) & \mathcal{M}(LR;LL) & \mathcal{M}(LL;LL) \end{pmatrix}$$

$$|f\rangle = \widetilde{\mathbf{M}}\,|i\rangle \doteq \mathcal{N}^{-1}\mathbf{M}\,|i\rangle$$

The quantum map $\widetilde{\mathbf{M}}$ is in general non-unitary. The set of maximally entangled states is an invariant set under its action.

[††] M. B., S. De Siena, G. Lambiase, C. Matrella and B. Micciola, Chaos, Solitons & Fractals (2025)

Entanglement saturation

By iterating this map, i.e. applying $\mathbf{M}^n$ to a general initial state, we observe three relevant properties:

- The set of the maximally entangled states is an invariant set for the maps defined by the scattering matrices $\mathbf{M}$.
- The powers $\mathbf{M}^n$ hold the same form of the original matrices for any value of n (self-similarity by raising to a generic power).
- The infinite iteration of the map on an initial state converges to a maximally entangled state, ensuring entanglement saturation.

Entanglement saturation in Bhabha scattering

The Bhabha process that is described by the map

$$\mathbf{M} = \begin{pmatrix} A & -B & -B & D \\ B & E & F & -B \\ B & F & E & -B \\ D & B & B & A \end{pmatrix}.$$

with $A = \mathcal{M}(RR;RR), B = \mathcal{M}(RR;RL), D = \mathcal{M}(RR;LL), E = \mathcal{M}(RL;RL), F = \mathcal{M}(RL;LR)$.

After iteration, we obtain

$$\mathbf{M}^n = \begin{pmatrix} A_n & -B_n & -B_n & D_n \\ B_n & E_n & F_n & -B_n \\ B_n & F_n & E_n & -B_n \\ D_n & B_n & B_n & A_n \end{pmatrix},$$

where any element in this matrix is expressed in terms of those of $\mathbf{M}$.

Entanglement saturation mechanism (Bhabha scattering)

- The Bhabha matrix has four eigenvectors $|\lambda_i\rangle$ and eigenvalues λ_i.

In a particular case, one obtains:

$|\lambda_1\rangle = \Phi^+,\ |\lambda_2\rangle = \Psi^-,\ |\lambda_3\rangle = \cos\delta_3\Phi^- + \sin\delta_3\Psi^+,\ |\lambda_4\rangle = \cos\delta_4\Phi^- + \sin\delta_4\Psi^+,$

δ_3, δ_4 functions of the matrix elements $A, ..., F$.

Any initial state can be written in terms of the $|\lambda_i\rangle$. For example:

$$|RL\rangle = \frac{1}{\sqrt{2}}(\Psi^+ + \Psi^-) = \frac{1}{\sqrt{2}}[\Psi^- + c_3' |\lambda_3\rangle + c_4' |\lambda_4\rangle],$$

with $c_3' \cos\delta_3 + c_4' \cos\delta_4 = 0$ and $c_3' \sin\delta_3 + c_4' \sin\delta_4 = 1$.

The action of $\mathbf{M}^n$ gives

$$\begin{aligned}\mathcal{N}_n^{-1}\mathbf{M}^n |RL\rangle &= \frac{1}{\sqrt{2}}\mathcal{N}_n^{-1}\Big[\cos\alpha\,\lambda_1^n\Phi^+ + \sin\alpha\,\lambda_2^n\Psi^- \\ &\quad +(\cos\alpha\, c_3 + \sin\alpha\, c_3')\lambda_3^n |\lambda_3\rangle + (\cos\alpha\, c_4 + \sin\alpha\, c_4')\lambda_4^n |\lambda_4\rangle\Big]\end{aligned}$$

with $\mathcal{N}_n$ a normalization factor.

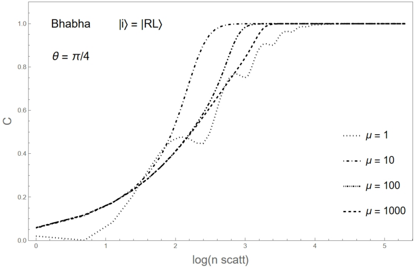
- For $n \to \infty$, the eigenvector with dominant eigenvalue will survive.

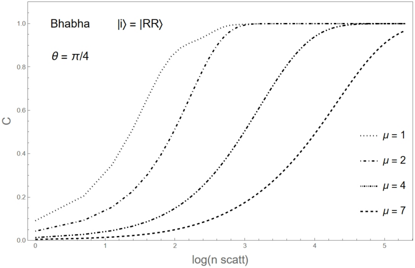
Entanglement saturation

SCATTERING PROCESS	INITIAL STATE(S)	REGIME	ASYMPTOTIC STATE
Bhabha	$\|RL\rangle$	u. r.	Ψ^+
Bhabha	$\|RL\rangle$	n. r.	$\cos s_1 \Phi^- + \sin s_1 \Psi^+$
Bhabha	$\|RR\rangle$	n. r.	Φ^+
Møller	$\|RL\rangle$	u. r.	Ψ^-
Møller	$\|RR\rangle$	n. r.	Φ^-
Møller	$\|RL\rangle$	n. r.	$\cos s_3 \Phi^+ + \sin s_3 \Psi^-$

Entanglement saturation: Bhabha scattering

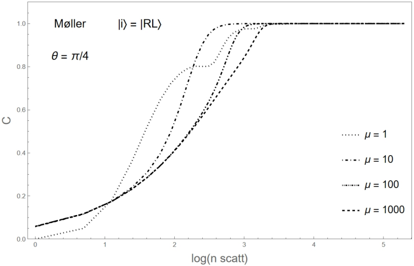
Entanglement saturation in Bhabha scattering at $\theta = \pi/4$ with initial states RR and LL

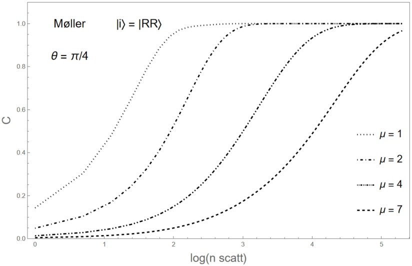




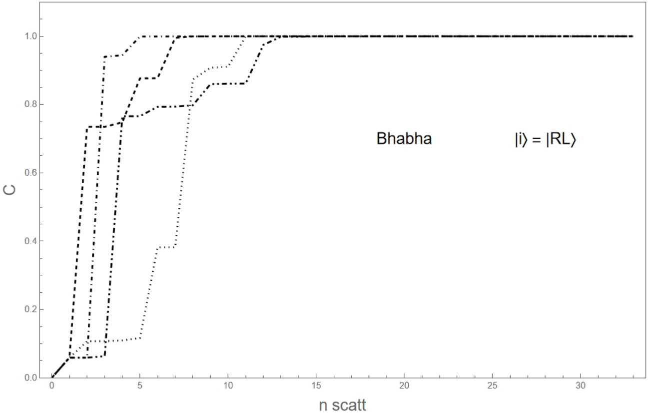
Entanglement saturation: Møller scattering

Entanglement saturation in Møller scattering at $\theta = \pi/4$ with initial states RR and LL

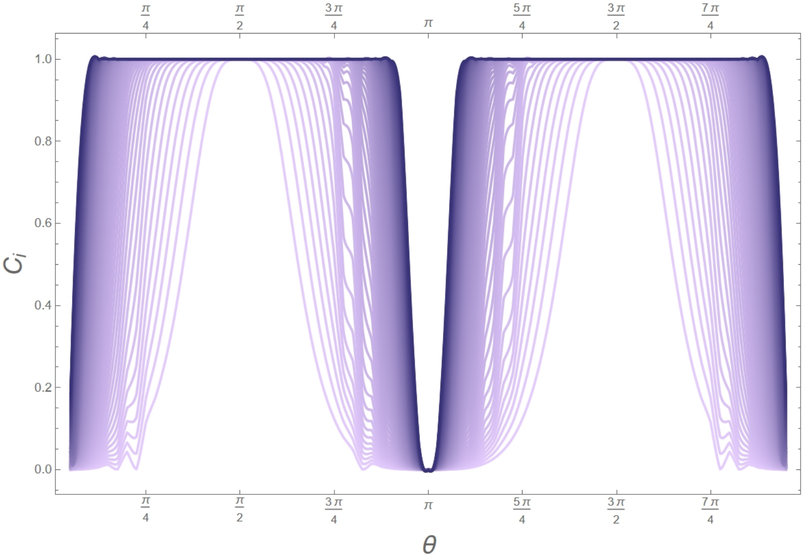




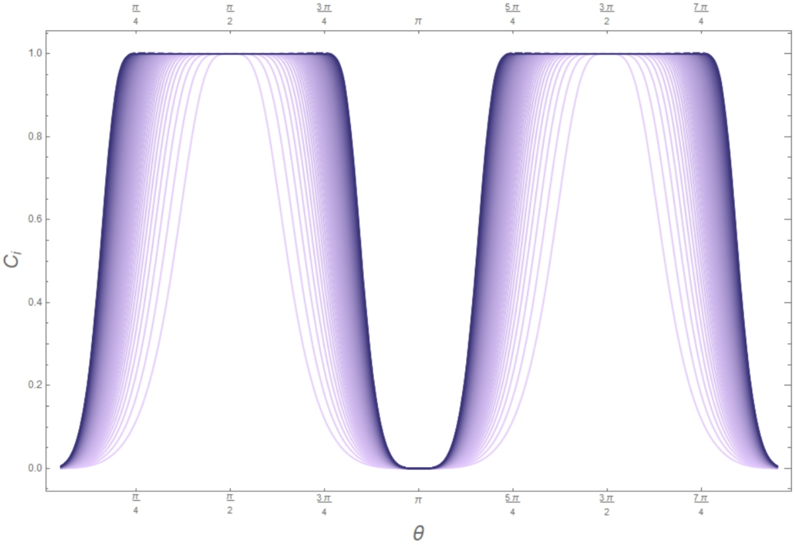
Entanglement saturation: Bhabha random filtering



Entanglement saturation: Bhabha scattering in low energy regime



Entanglement saturation: Bhabha scattering in high energy regime



Conclusions

- We have studied entanglement generation and distribution in QED processes, after a POVM (filtering) procedure, with and without entangled spectator particle;

- With entangled spectator particle we observe a non trivial distribution of initial entanglement in the various bipartitions;

- For $\theta = \pi$ and $\mu = \infty$ complete transfer of entanglement from one channel to another (quantum gate);

Conclusions

- CCR give a complete characterization of entanglement generation in QED s
- We found regimes in which entanglement increases/decreases with respect to its initial value;
- Remarkably, maximal entanglement (Bell states) is totally preserved for processes that involve massive fermions;
- The action of QED scattering processes, for $2 \to 2$ fermions, are described by quantum maps whose iterations produces a saturation of entanglement.

Outlook

- Extension of the analysis to other interactions;
- 1-loop corrections;
- Analysis of the same processes in other reference frames;
- Relation between conservation/saturation of entanglement and symmetries;
- Possible implementations of our results in quantum information protocols;
- Possible identification of new physics BSM.