JSF Julian Schwinger Foundation

How to lose information with black holes: an update

Daniel Terno

School of Mathematical & Physical Sciences

MACQUARIE University
Astrophysics and Space Technologies Research Centre

Australian Government
Australian Research Council

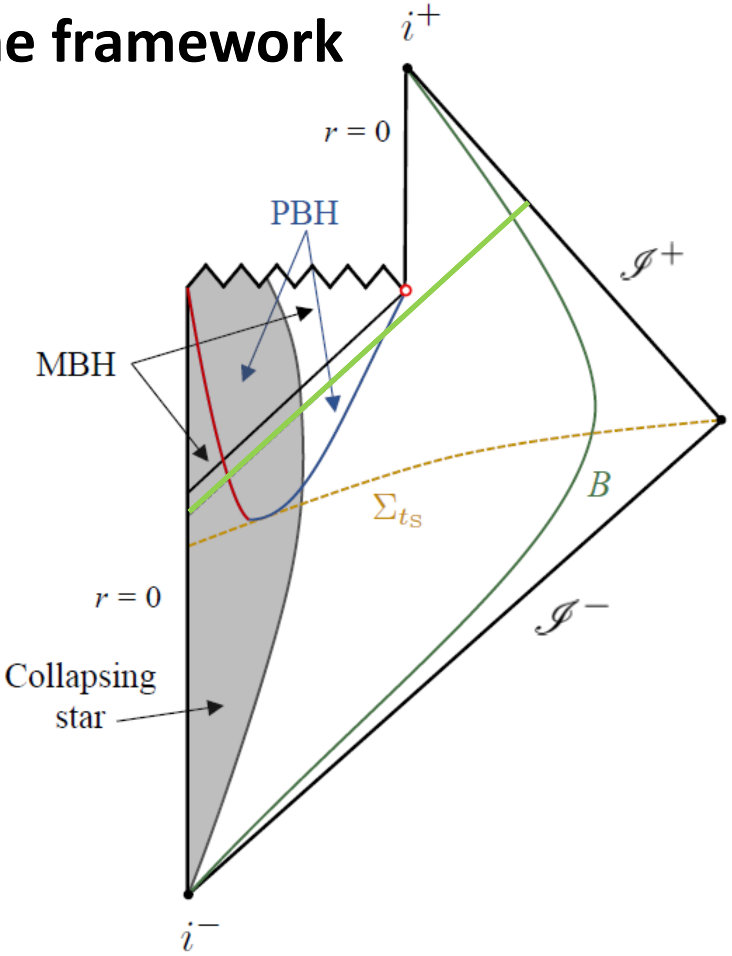
15th annual conference RQI(N)

LISA CONSORTIUM

RQi
Relativistic Quantum Information
COST Action 23115

info loss paradox/problem

the framework



Black hole evaporates (completely?) via the Hawking process within a finite time. If the correlations[§] between the inside and outside of the black hole are not restored during the evaporation process, then by the time that the black hole has evaporated, an initial pure state will have evolved to a mixed state, i.e., *``information'' will have been lost.* Wald, Living Rev. Rel. **4**, 6 (2001) .

[§] or worse --- entanglement

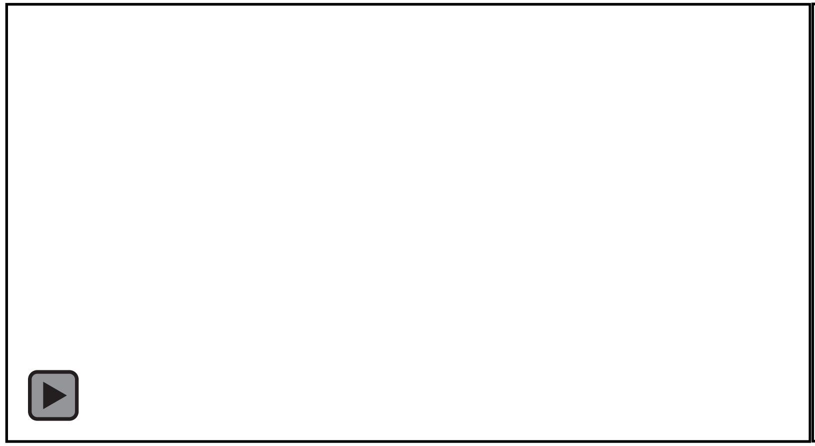
- Description of the process

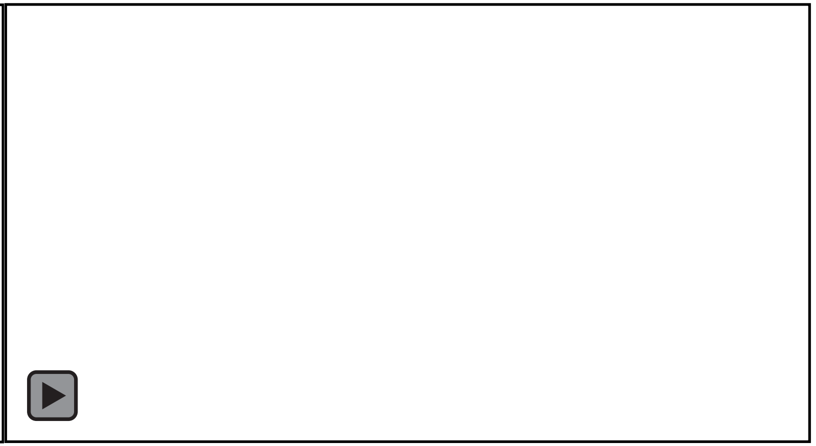
$$\$|\psi\rangle\langle\psi| = \rho$$

"Hawking superscattering operator"

- Dealing with the problem:

exorcism, denial, acceptance, embrace





outline

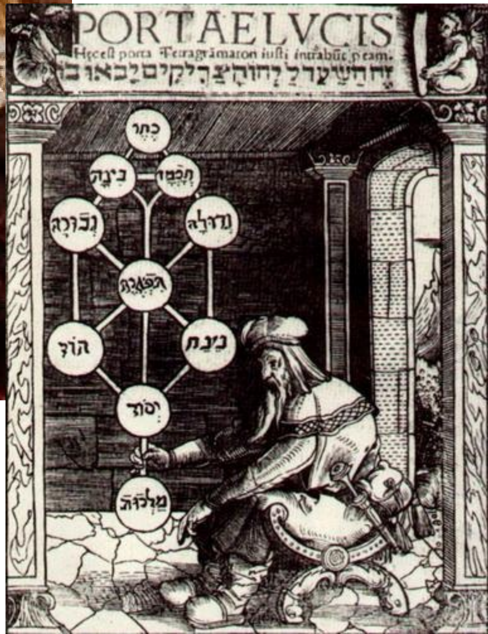
Paradoxes: logic & ingredients

What the black holes are and what they should have

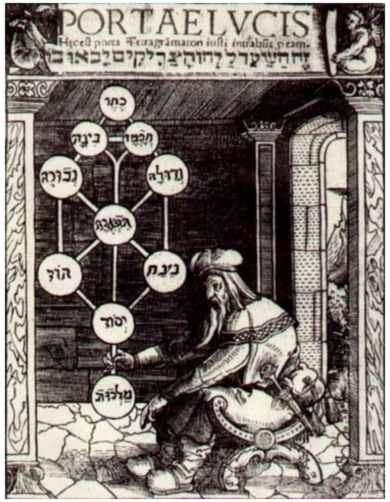
Implementation & consequences in spherical symmetry

Observations

Making of the paradox



motivation: a view from the foundations of QM



Ingredients of a paradox (general):

1. Classical ideas/assumptions/results
2. Quantum features/results
3. Combine and try to obtain probability distributions that satisfy all of (1) & (2)

Examples:

- EPR: Bell-CHSH; KS, wave-particle
- BH info loss, firewall

Ingredients (BH):

[2] Event horizon

[4] (pre-)Hawking radiation

Hayward, arXiv:gr-qc/0504037

Visser, PoS BHs, GRandStrings 2008 001, arXiv:0901.4365v3

resolve or aggravate

1966: "*event horizon*. . . is the boundary of the region from which particles or photons can escape to infinity. . . a black hole is a region. . . from which particles or photons cannot escape"

1976: "Because part of the information about the state of the system is lost down the hole, the final situation is represented by a density matrix rather than a pure quantum state"

1997: "Whereas Stephen Hawking and Kip Thorne firmly believe that information swallowed by a black hole is forever *hidden from the outside universe*, and can never be revealed even as the black hole evaporates and completely disappears"

2004: "Thus the total path integral is unitary and information is not lost in the formation and evaporation of black holes. The way the information gets out seems to be that a true *event horizon never forms*, just an apparent horizon"

The radical solution: no event horizon, no paradox.

No! Marolf, Rep. Prog. Phys. **80** (2017) 092001.

So what? Unruh and Wald, Rep. Prog. Phys. **80** (2017) 092002.

Raju,
Phys. Rep. **943**, 1 (2022).

Recent views

Almheiri et al,
Rev. Mod. Phys. **93**, 035002 (2021)

ultimum album ingredientia

[1] Finite time of formation & evaporation (distant observers)

Purpose: enable mortal observers to discuss; "in" and "out" setting

[2] Event horizon forms @ finite time of a distant observer

Purpose: observer-independent splitting

[3] "No drama at the horizon" [=weak cosmological censorship]

Purpose: standard (semiclassical) physics applies (most of the time)

[4] (nearly thermal) Hawking-like radiation

Purpose: generate high-entropy reduced state

Existence of black holes as a math question

black holes

defined

[35]

VII. *On the Means of diſcovering the Diſtance, Magnitude, &c. of the Fixed Stars, in conſequence of the Diminution of the Velocity of their Light, in caſe ſuch a Diminution ſhould be found to take place in any of them, and ſuch other Data ſhould be procured from Obſervations, as would be farther neceſſary for that Purpoſe. By the Rev.* John Michell, *B. D. F. R. S. In a Letter to* Henry Cavendiſh, *Eſq. F. R. S. and A. S.*

Read November 27, 1783.

42 Mr. Michell *on the Means of diſcovering the*

16. Hence, according to article 10, if the ſemi-diameter of a ſphære of the ſame denſity with the ſun were to exceed that of the ſun in the proportion of 500 to 1, a body falling from an infinite height towards it, would have acquired at its ſurface a greater velocity than that of light, and conſequently, ſuppoſing light to be attracted by the ſame force in proportion to its vis inertiæ, with other bodies, all light emitted from ſuch a body would be made to return towards it, by its own proper gravity.

nature astronomy

PERSPECTIVE

https://doi.org/10.1038/s41550-018-0602-1

The many definitions of a black hole

Erik Curiel[1,2,3]

Although black holes are objects of central importance across many fields of physics, there is no agreed upon definition for them, a fact that does not seem to be widely recognized. Physicists in different fields conceive of and reason about them in radically different, and often conflicting, ways. All those ways, however, seem sound in the relevant contexts. After examining and comparing many of the definitions used in practice, I consider the problems that the lack of a universally accepted definition leads to, and discuss whether one is in fact needed for progress in the physics of black holes. I conclude that, within reasonable bounds, the profusion of different definitions is in fact a virtue, making the investigation of black holes possible and fruitful in all the many different kinds of problems about them that physicists consider, although one must take care in trying to translate results between fields.

If names be not correct, language is not in accordance with the truth of things. If language be not in accordance with the truth of things, affairs cannot be carried on to success.

black holes

Hayward, gr-qc/0504037

Frolov, arXiv:/gr-qc1411.6981

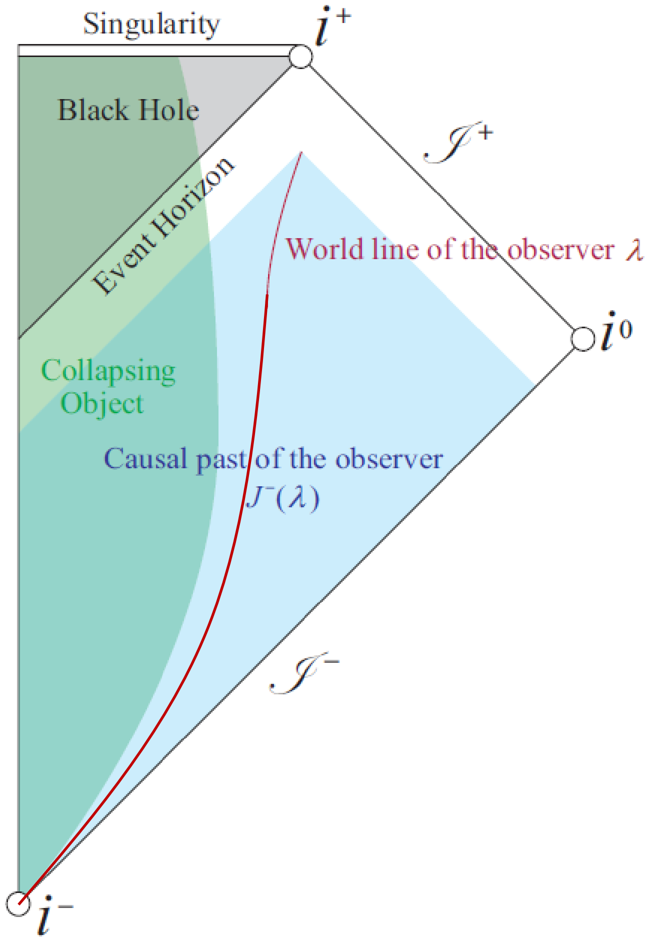
MBH

(M) black hole

$\mathcal{B} = \mathcal{M} - \mathrm{Past}(\mathscr{I}^+)$

singularity

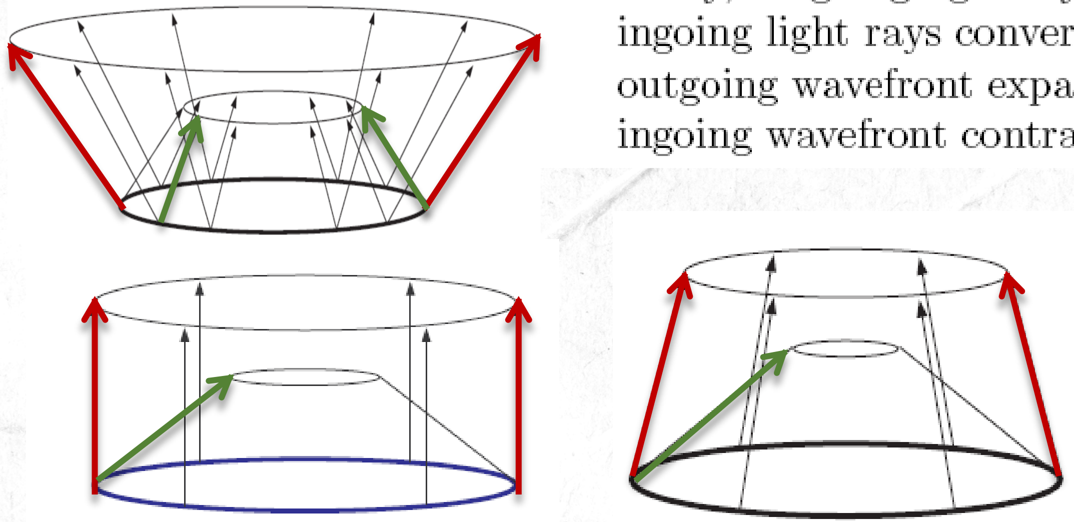
$r = 0$ 3D spacelike



ΦBH

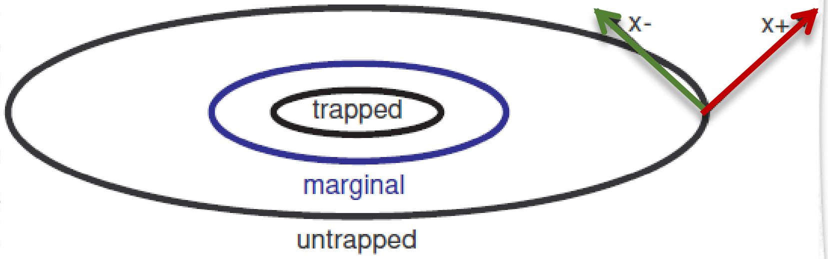
Area A, null coordinates $x^\pm$: $g^{-1}(dx^\pm, dx^\pm) = 0$, future-pointing, unique up to $x^\pm \mapsto \tilde{x}^\pm(x^\pm)$

Null expansions $\theta_\pm = \partial_\pm A/A$, $\partial_\pm = \partial/\partial x^\pm$.
Normally, outgoing light rays diverge, $\theta_+ > 0$, ingoing light rays converge, $\theta_- < 0$; outgoing wavefront expands, ingoing wavefront contracts:



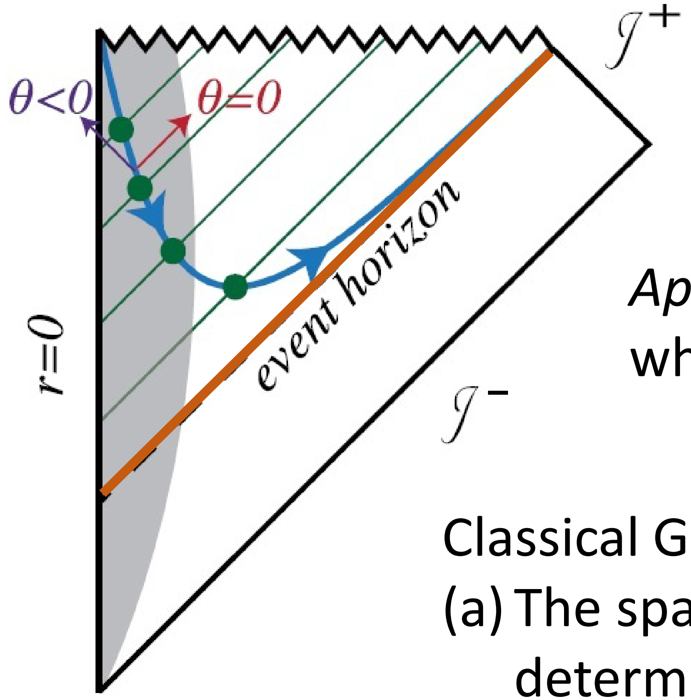
(Φ) black hole

$\mathfrak{B} = \{x \in \mathcal{M}, \theta^\pm(x) < 0\}$



black holes

apparent horizon



Event horizon is a teleological concept
It is not locally observable

Apparent horizon is related to $R_{\hat{\theta}\hat{\phi}\hat{\theta}\hat{\phi}}$
which is quasilocally measurable

Classical GR:

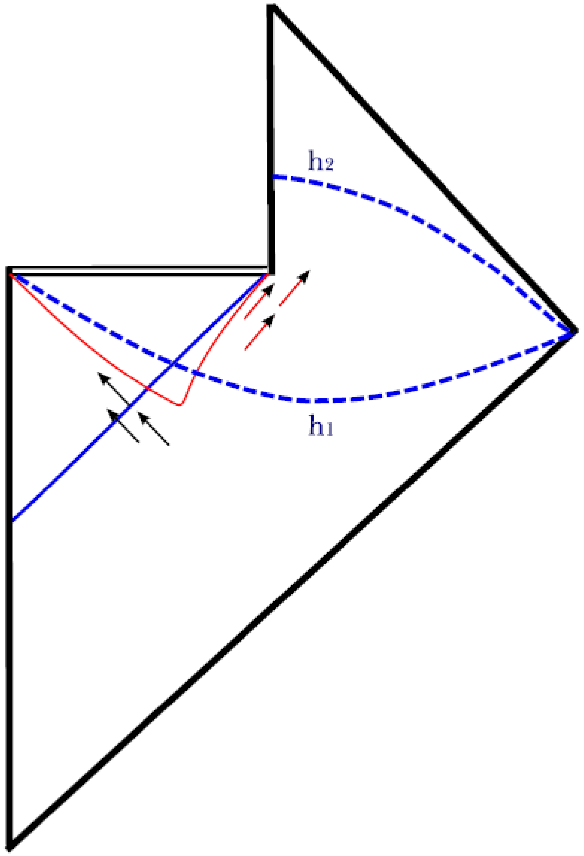
(a) The spacetime evolution is deterministic from the IC

(b) NEC is satisfied

then $R_{\mu\nu}k^{\mu}k^{\nu} \geq 0$

a closed trapped surface cannot be seen from *the outside of BH.*

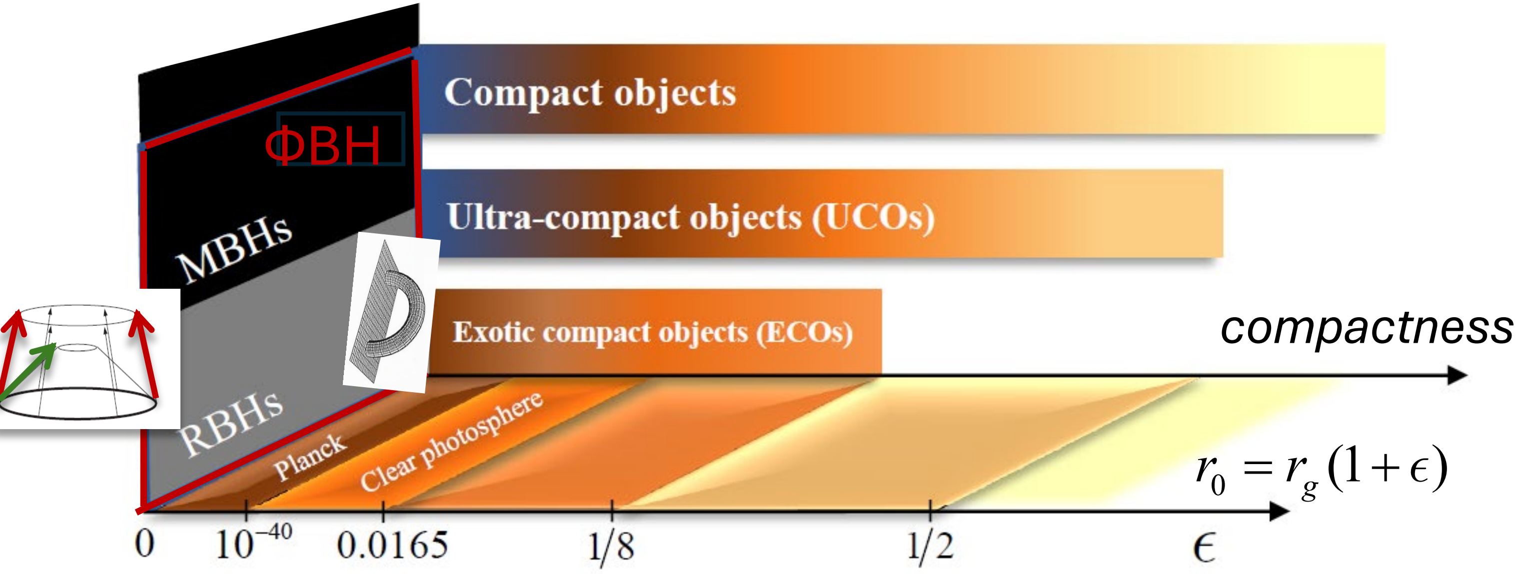
Hawking radiation:



ultra-compact objects

V. P. Frolov, arXiv:1411.6981 (2014)

the zoo of models & physical black holes



UCO: has a photosphere

BH: has a horizon

MBH: has an event horizon

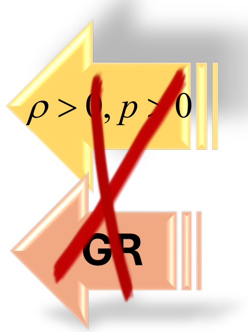
ΦBH: has a trapped region

ECO: non-BH UCO

Why ECOs are called exotic?

Buchdal's theorem

$\epsilon > 1/8$



Exotic matter

- Modified
- Semiclassical
- Quantum gravity

Cardoso and Pani, Nat. Astron. **1**, 586 (2017)

[1] Perpetual ongoing collapse, with an asymptotic horizon $\epsilon \to 0 \quad @t \to \infty$

[2] Formation of a transient or an asymptotic object, where the compactness reaches a minimum at some finite asymptotic [=distant observer] time $\epsilon \to \epsilon_{min}$

[3] Formation of a ΦBH with the apparent horizon in finite distant observer's time $\epsilon(t_f) = 0$

Mann, Murk, DRT, Int J Mod Phys D **31**, 2230015 (2022)

assumptions

1.The classical spacetime structure is still meaningful and is described by a metric $g_{\mu\nu}$.

2. Classical concepts, such as trajectory, event horizon or singularity can be used.

3. The metric is modified by quantum effects. The resulting curvature satisfies the semiclassical self-consistent equation

$$R_{\mu\nu} - \tfrac{1}{2} R g_{\mu\nu} = 8\pi \left\langle \hat{T}_{\mu\nu} \right\rangle_{\omega} \quad \leftarrow +E_{\mu\nu}$$

4. Dynamics of the collapsing matter is still described classically using the self-consistent metric: the total EMT

not assumed: global structure, singularity, types of fields, quantum state, presence of Hawking radiation

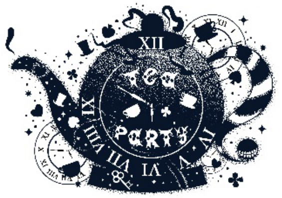
[2] [4]

Physical BH

(i) a light-trapping region forms at a **finite** time of a distant observer

(ii) curvature scalars [contractions of the Riemann tensor] are **finite** on the boundary of the trapped region

(iii) **consistent**: quantum energy inequalities are not obviously violated



[1]

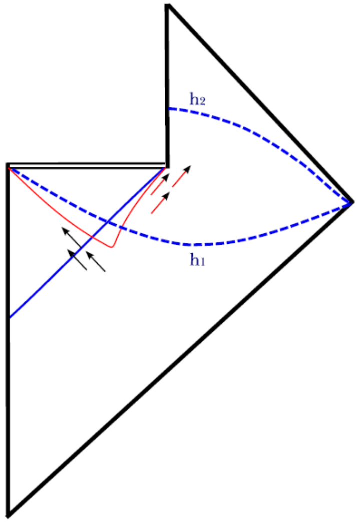
[3]

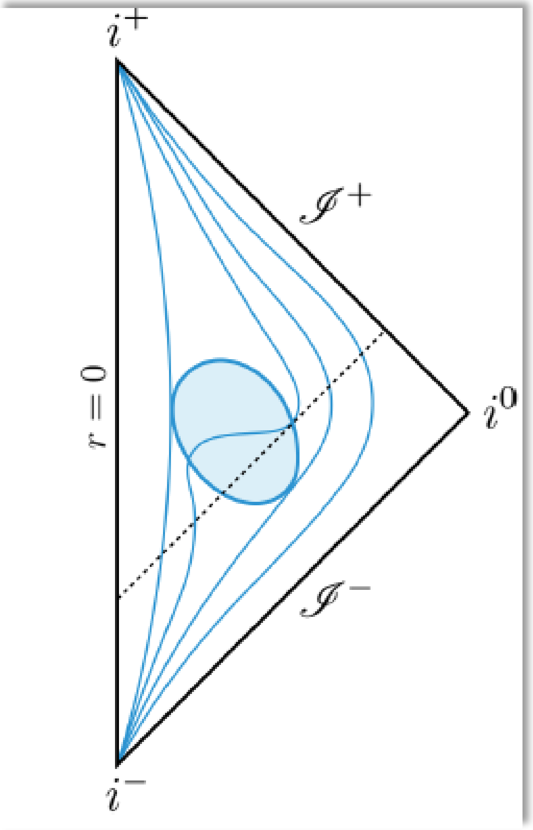
motivation for (i)

Sperical symmetry
(Asymptotic flatness)
Event horizon forms at finite time
Evaporation of BH
Regularity of the metric outside BH

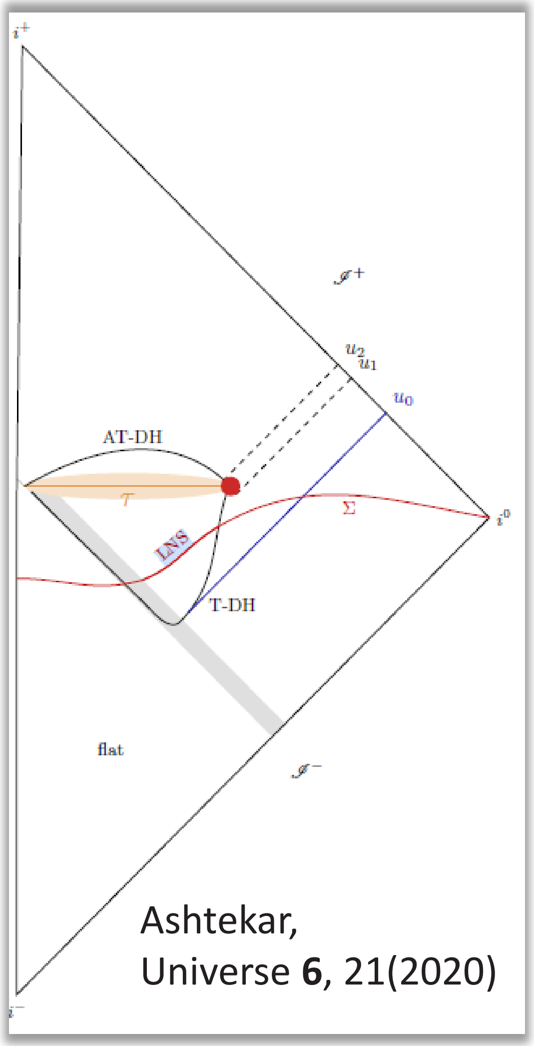
AH froms @ finite time t

Mann, Murk, DRT
Phys Rev D **105**, 124032 (2022)

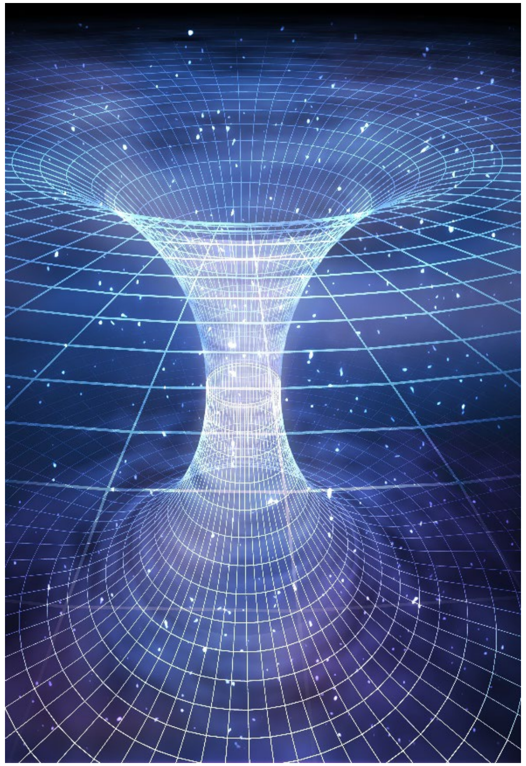




Carballo-Rubio, Di Filippo, Liberati, Visser, Phys Rev D **101**, 084047 (2020)



Ashtekar, Universe **6**, 21(2020)



Implications

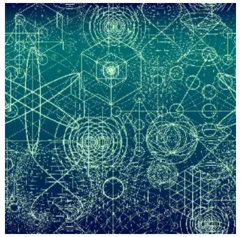
spherical symmetry: a preview

ΦBH: the process

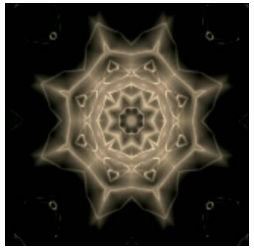
- ❑ Use Schwarzschild coordinates to extract the info from divergencies
- ❑ Pick the nice form of the Einstein equations. Demand existence of real solutions
- ❑ Use null coordinates to help classification

ΦBH (and white holes): the properties

- ❑ Finite infall time (according to a distant Bob)
- ❑ Zero angular momentum BH cannot grow & WH cannot shrink
- ❑ Collapse of a massive thin shell takes a finite time (according to Bob), but…



- ❑ Outer apparent horizon is always **timelike**
- ❑ Null energy condition is violated [in the vicinity of the outer horizon]
- ❑ Surface gravity generalisations: Kodama=good, peeling=bad
- ❑ Inner apparent horizon is timelike or null. The NEC is satisfied



- ❑ Parts of the popular RBH models do not work.
- ❑ Usual proofs of instability of RBH do not apply

physical black holes

Mann, Murk, DRT, Int J Mod Phys D **31**, 2230015 (2022)

assumptions

(1-4) $R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi \left\langle \hat{T}_{\mu\nu} \right\rangle_{\omega} + E_{\mu\nu}$

not assumed: type of the metric theory, global structure, singularity, types of fields, quantum state, presence of Hawking radiation

(*) Collapsing matter + excitations are the total EMT

$$ds^2 = -e^{2h} f dt^2 + f^{-1} dr^2 + r^2 d\Omega_2$$

spherical

$$l^{\mu}_{\text{out}} = \left(e^{-h}, f, 0, 0\right)$$

$$\theta^{(l)} = l^{\mu}_{\text{out};\mu} - \kappa = \frac{2f}{r}$$

$$r_g(t) = 2M\left(t, r_g(t)\right)$$

Apparent horizon at the Schwarzschild radius: the largest* root of $f=0$

ΦBH

i. light-trapping region forms at a **finite** time of a distant observer (Bob)

ii. curvature scalars [contractions of the Riemann tensor] are **finite** on the boundary of the trapped region

Logic

- use "bad" coordinates to express (i)
- require "self-renormalisation" to have (ii)

Curvature scalars

$$\mathrm{T} := T^{\mu}{}_{\mu}$$

$$\mathfrak{T} = T^{\mu\nu} T_{\mu\nu}$$

useful

$$\tau_t := e^{-2h} T_{tt}$$

$$\tau_t{}^r := e^{-h} T_t{}^r$$

$$\tau^r := T^{rr}$$

ΦBH structure

$$ds^2 = -e^{2h} f dt^2 + f^{-1} dr^2 + r^2 d\Omega_2$$

- circumference: $2\pi r$
- physical time at infinity: t

$$f = 1 - 2M(t,r)/r$$

$$2M(t,r) \equiv C(t,r)$$

MS invariant mass

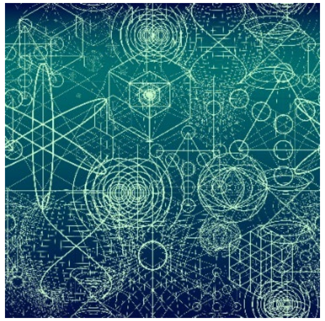
Schwarzschild radius

$$\max r_g = C(t, r_g)$$

Curvature scalars

$$\mathrm{T} := T^{\mu}{}_{\mu}$$

$$\mathfrak{T} = T^{\mu\nu} T_{\mu\nu}$$



$$\mathfrak{T} := \left((\tau^r)^2 + (\tau_t)^2 - 2(\tau_t{}^r)^2\right)/f^2$$

$$\mathrm{T} := (\tau^r - \tau_t)/f$$

+regular terms*

useful

$$\tau_t := e^{-2h} T_{tt}$$

$$\tau_t{}^r := e^{-h} T_t{}^r$$

$$\tau^r := T^{rr}$$

All three go to zero/ the same finite value/ diverge in the same way

Einstein equations

$$\partial_r C = 8\pi r^2 \tau_t / f,$$

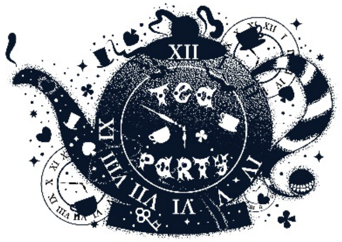
$$\partial_t C = 8\pi r^2 e^h \tau_t{}^r,$$

$$\partial_r h = 4\pi r \left(\tau_t + \tau^r\right) / f^2$$

$$\lim_{r \to r_g} \tau_a \sim \begin{cases} \pm \Upsilon^2 f^0 \\ \tau_a(t) f^k \end{cases}$$

k=0,1* [who]

$$C = r_g(t) + W(t,r)$$ ▶



$$\lim_{r \to r_g} \tau_t = \lim_{r \to r_g} \tau^r = -\Upsilon^2 \qquad k = 0$$

metrics

1. The limiting form (close apparent horizon) of dynamical metrics is almost uniquely defined (both *k*=0 and *k*=1).

$$x := r - r_g$$

$$C = r_g - 4\sqrt{\pi r_g^3}\Upsilon\sqrt{x} + \ldots \qquad h = -\frac{1}{2}\ln\frac{x}{\xi} + \ldots \quad \blacktriangleleft\; k=0$$

(dynamical BH/WoH; more static options exist)

$$k=1\;\blacktriangleright \quad C = r - c_{32}x^{3/2} + \ldots \qquad h = -\frac{3}{2}\ln\frac{x}{\xi} + \ldots$$

2. BH parameters are related via evaporation rate

$$\frac{dr_g}{dt} = -4\sqrt{\pi r_g \xi}\Upsilon$$

$$\frac{dr_g}{dt} = -\frac{c_{32}\xi^{\frac{3}{2}}}{r_g}$$

- ❑ No static *k*=0 solutions (their static limits belong to *k*=1)
- ❑ Vaidya metrics are *k*=0 solutions
- ❑ Reissner-Nordström, STU, static RBH are examples of *k*=1 solutions: $C = r_g + 8\pi r_g^2 \rho_g x + \ldots$
- ❑ Popular dynamic RBH models are *k*=0 solutions

spherical symmetry

metrics

3. Most convenient coordinates are retarded (u_-,r) for *white holes* and advanced (v_+,r) for *black holes*

$$dt = e^{-h}\left(e^{h_\pm}dv_\pm \mp f^{-1}dr\right)$$

$$ds^2 = -e^{2h_+} f dv^2 + e^{h_+} dvdr + r^2 d\Omega_2$$
$$= -e^{2h_-} f du^2 + e^{h_-} dudr + r^2 d\Omega_2$$
$$2M(t,r) \equiv C(t,r) \equiv C_-(u(t,r),r) \equiv \ldots$$

E.g, in (v,r) the metric is regular at $r_g \equiv r_+$ for $r_g' < 0$ and singular for $r_g' > 0$

$$\partial_v C_+ = 8\pi r^2 e^{h_+}(\theta_v + f\theta_{vr}) \qquad \theta_v := e^{-2h_+}\Theta_{vv} = \tau_t\,,$$
$$\partial_r C_+ = -8\pi r^2 \theta_{vr}\,, \qquad \theta_{vr} := e^{-h_+}\Theta_{vr} = ({\tau_t}^r - \tau_t)/f,$$
$$\partial_r h_+ = 4\pi r \theta_r\,. \qquad \theta_r := \Theta_{rr} = (\tau^r + \tau_t - 2{\tau_t}^r)/f^2$$

$r_g' < 0 \blacktriangleright (v,r)$	$r_g' > 0 \blacktriangleright (u,r)$
$\theta_{\text{in}} < 0, \theta_{\text{out}} < 0$	$\theta_{\text{in}} > 0, \theta_{\text{out}} > 0$
BH solutions	WH solutions

consequences

metric

4. Apparent horizon/ Anti-trapping horizon: timelike membrane

5. Finite infall time (and red-shift).

For the ingoing null rays

$$\frac{dt}{dr}\bigg|_{r_g} = \frac{1}{e^h f}\bigg|_{r_g} = \frac{1}{|r_g'|}$$

- DRT, Phys. Rev. D **100**, 124025 (2019) Murk and DRT, Phys. Rev. D **103**, 064082 (2021)

6. Energy density, pressure flux for a static observer are divergent (k=0)/finite (k=1) on the horizon

Energy momentum (k=0)

$$T_{\hat{a}\hat{b}} = -\frac{\Upsilon^2(t)}{f}\begin{pmatrix} 1 & \pm 1 \\ \pm 1 & 1 \end{pmatrix}$$

6½.The usual example: outgoing coordinates, Vaidya metric with C'<0 cannot describe the near-horizon region (with a finite formation time t)

$$C_+(v,r) = r_+(v) + w_1(v)y + \mathcal{O}(y^2)$$
$$h_+(v,r) = \chi_1(v)y + \mathcal{O}(y^2),$$

Bardeen, Phys. Rev. Lett. **46**, 382 (1981)...

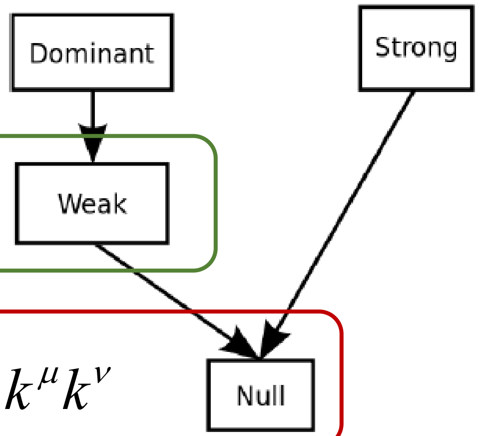
spherical symmetry

consequences

Energy conditions

$$T_{\mu\nu}u^{\mu}u^{\nu} \geq 0$$

$$T_{\mu\nu}k^{\mu}k^{\nu} \geq 0 \Leftrightarrow R_{\mu\nu}k^{\mu}k^{\nu}$$



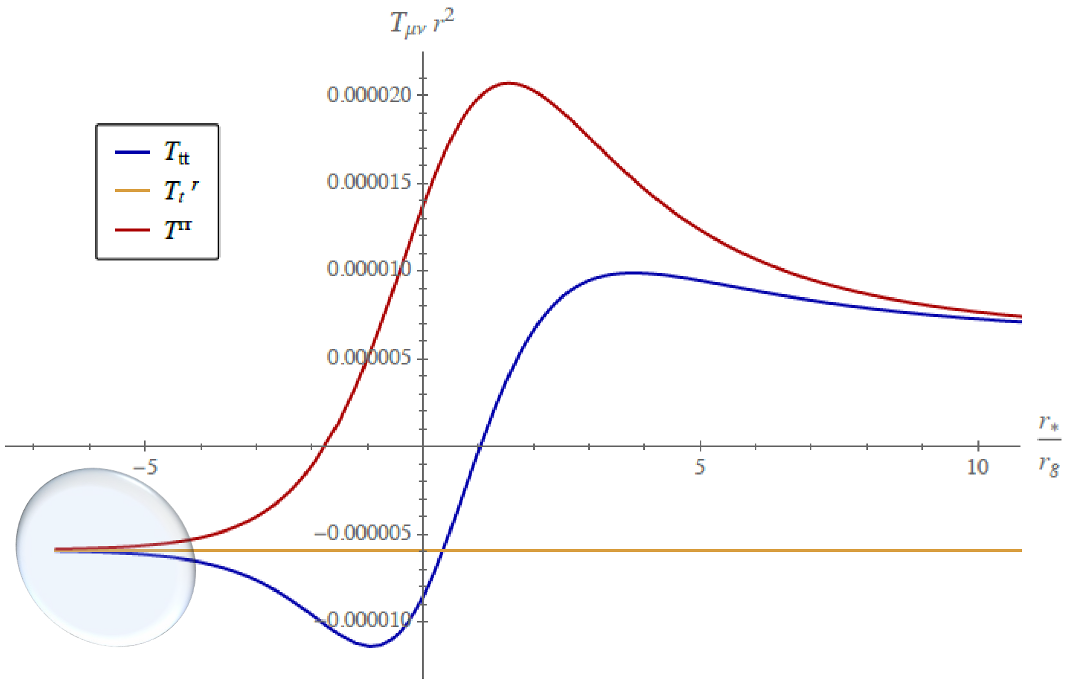
Solutions of the Einstein equations exist:
the NEC **must** be violated

$\mathrm{sgn}(T_{tt})$	$\mathrm{sgn}(T_t{}^r)$	Time-evolution of Vaidya mass function	Black/ White hole	NEC violation
$-$	$-$	$C'(v) < 0$	B	✓
$-$	$+$	$C'(u) > 0$	W	✓
				✗
				✗

Baccetti, Murk, Mann, and DRT, Phys. Rev. D 100, 064054 (2019)

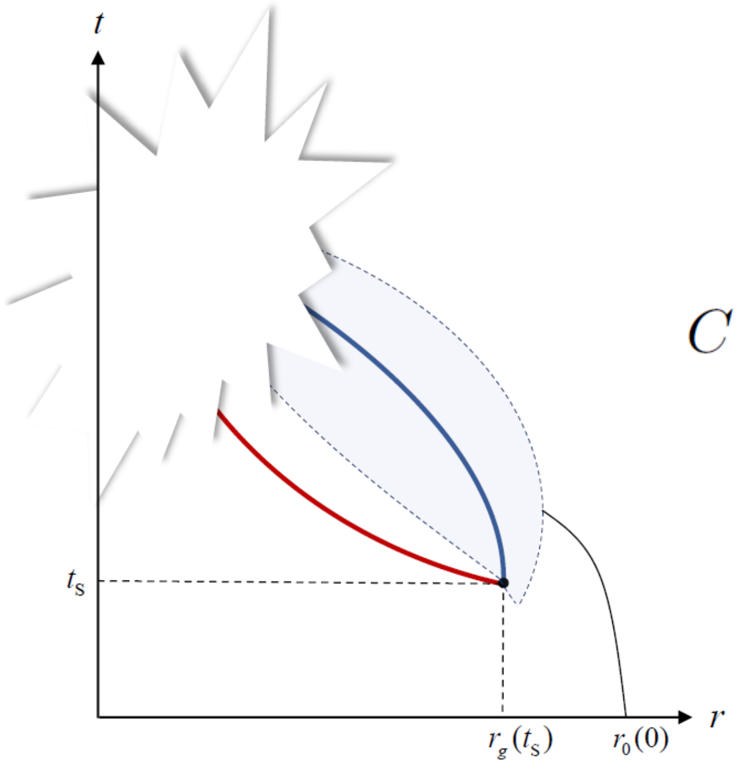
Energy momentum (k=0)

$$T_{\hat{a}\hat{b}} = -\frac{\Upsilon^2(t)}{f}\begin{pmatrix} 1 & \pm 1 \\ \pm 1 & 1 \end{pmatrix}$$



Levi and Ori, Phys. Rev. Lett. **117**, 231101 (2016)

BH formation

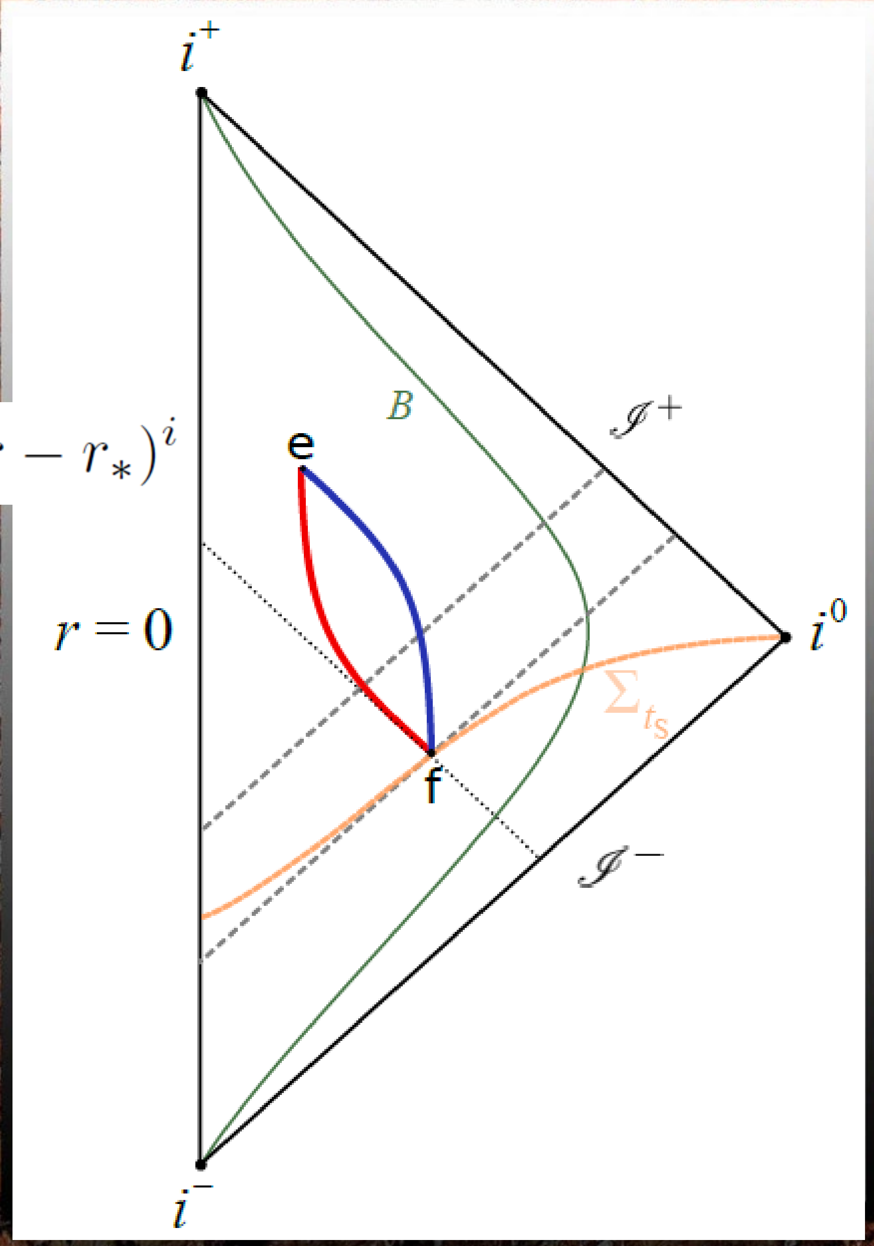


- A PBH forms as k=1 solution and then evolves as (evaporating) k=0 solution

$$C(v,r) = \Delta(v) + r_*(v) + \sum w_i(v)(r-r_*)^i$$

min gap C-r r of min C-r

Hence up to formation of the first marginally trapped surface w_1=1



At the formation: $\Delta(v_f)=0,\ r_+(v_f)=r_*(v_f)$

After formation: $\Delta=0$, but $r_+(v)$ is not @ min

Murk and DRT, Phys. Rev. D **104**, 064048 (2021)
Dahal, Simovic, Soranidis and DRT, Phys. Rev. D **108**, 104014 (2023).

Surface gravity

Surface gravity κ is:
(a) inaffinity of null geodesics *on the horizon*
(b) and the peeling off properties of null geodesics *near the horizon*

Interpretation: the force per unit mass as measured at infinity, to keep the observer stationary just outside the horizon (c)

Stationary Killing horizon: (a)=(b)=(c)

Schwarzschild: $\kappa = 1/4M = 1/2C$

Surface gravity plays a key role in BH thermo

NEC is true

- 0th law: surface gravity is constant on the horizon
- 1st law: $$dM = \frac{\kappa}{8\pi} dA + \omega_H dJ$$

NEC is false

Temperature

$$T = \frac{\kappa}{2\pi} \frac{\hbar c^3}{G k_B}$$

Surface gravity plays a key role in the Hawking radiation

surface gravity

@ outer apparent horizon

Peeling surface gravity

$$\kappa_{\text{peel}} = \frac{e^{h(t,r)}}{r}\left(1-\partial_r C(t,r)\right)\Big|_{r=r_g}$$

- Vanzo, Acquaviva, and Di Criscienzo,
Class. Quant. Grav. **28**, 183001 (2011).
- Cropp, Liberati,Visser,
Class. Quant. Grav. **30**, 125001 (2013)

$\kappa_{\text{peel}} = 0, \infty^{*}$ both k=0,1 solutions

Kodama surface gravity

$$\kappa_{\text{K}} = \frac{1}{2}\left(\frac{C(v,r)}{r^2} - \frac{\partial_r C(v,r)}{r}\right)\Big|_{r=r_g\equiv r_+}$$

- Hayward,
Class. Quant. Grav. **15**, 3147 (1996).

$C = r_+ + w_1(r-r_+) + \ldots$

$\kappa_{\text{K}} = 0$ for k=1 solution [@ formation!]

$\kappa_{\text{K}} \leq 1/2r_+$ for k=0 ($0<w_1<1$) solution

Mann, Murk and DRT,
Phys. Rev. D **105**, 124032 (2022)

universality
of BH dynamics

- If we want **the 1st law with AH**, then the metric is ``close'' to Vaidya

$$dM = \frac{\kappa}{8\pi} dA$$

$$\kappa_K = \frac{1}{2r_+} \Rightarrow w_1 = 0$$

- If Page's law is universal (=the same in the three coordinate systems)

$$r_*' = -\frac{\alpha}{r_*^2}$$

$$\Upsilon(t) = \sqrt{\frac{\Gamma_+(1-w_1)}{8\pi r_+^2}}$$

$$\xi(t) = \frac{r_g \Gamma^2}{2\Gamma_+}$$

$$\Upsilon = \frac{\sqrt{\alpha}}{2r_g^2}$$

$$\xi = \frac{\alpha}{2r_g}$$

a very short hair

$$l_\xi = \int_0^\xi \frac{dx}{\sqrt{f}} \sim \sqrt{\alpha}$$

ΦBH

in FRLW

- Schwarzschild coordinates
  [static patch of the de Sitter spacetime]

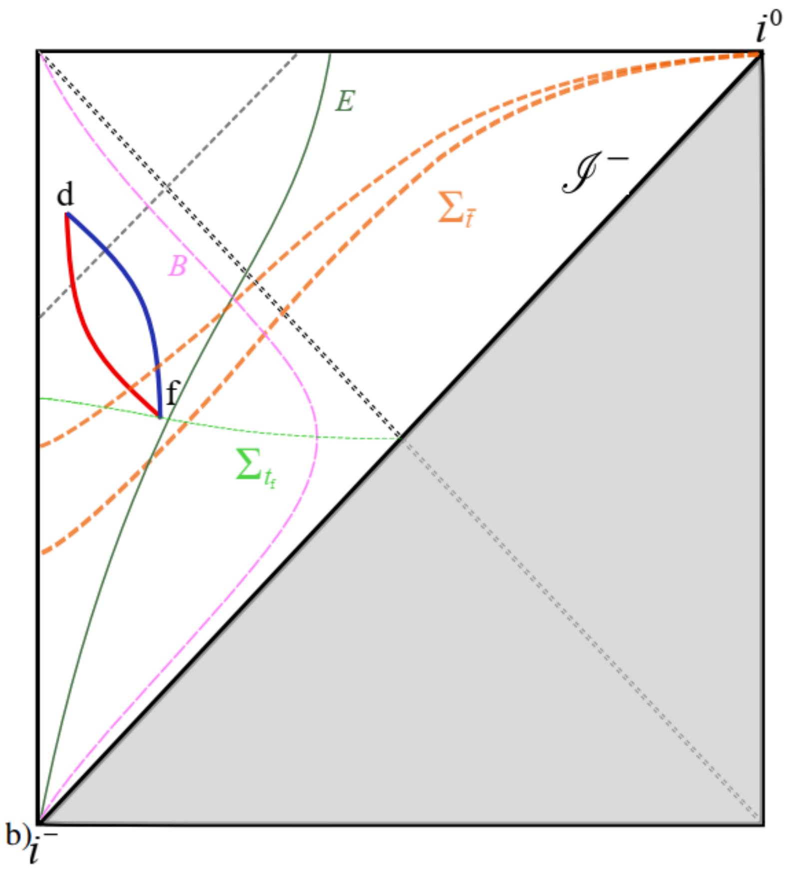
Einstein equations
$\partial_r C = 8\pi r^2 \tau_t / f + \Lambda r^2$
$\partial_t C = 8\pi r^2 e^h \tau_t{}^r,$
$\partial_r h = 4\pi r \left(\tau_t + \tau^r\right) / f^2$

❖ Example: de Sitter in static coordinates

$$ds^2 = -\left(1 - H^2 r^2\right) F^2 dt^2 + \frac{1}{1 - H^2 r^2} dr^2 + r^2 d\Omega_2$$

$$H := \frac{\dot{a}}{a} \qquad \Lambda = 3H^2$$

Dahal, Maharana, Simovic, Soranidis and DRT,
Phys. Rev. D **110**, 044032 (2024).



❖ Example: Vaydia-dS

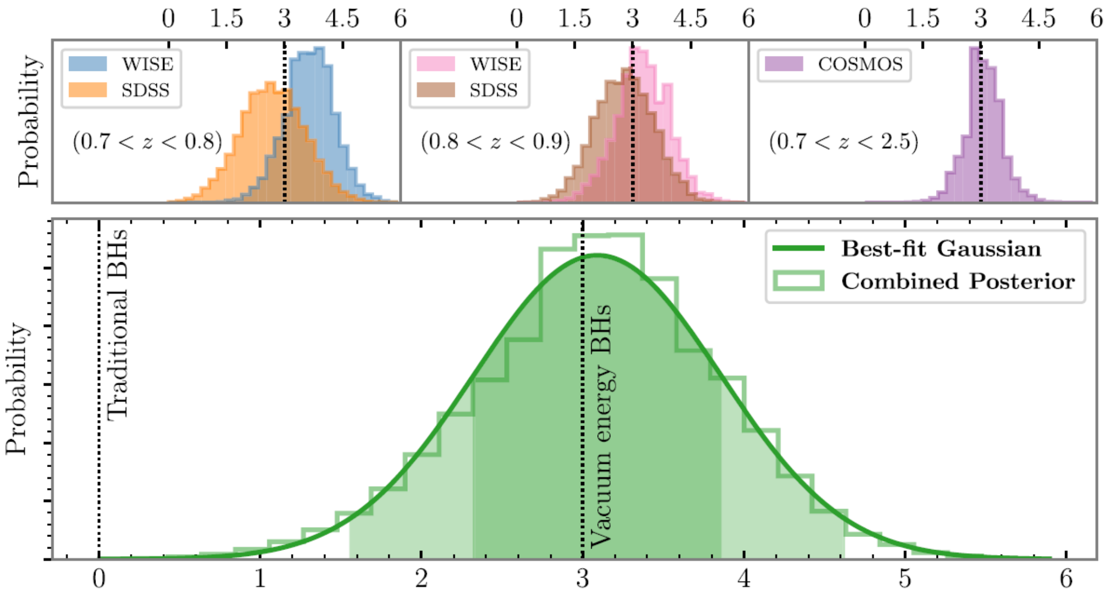
Mallett, Phys. Rev. D **31**, 416 (1985)
Phys. Rev. D **33**, 2201 (1986).

Connecting to observations

observational signatures

ΦBH in cosmology

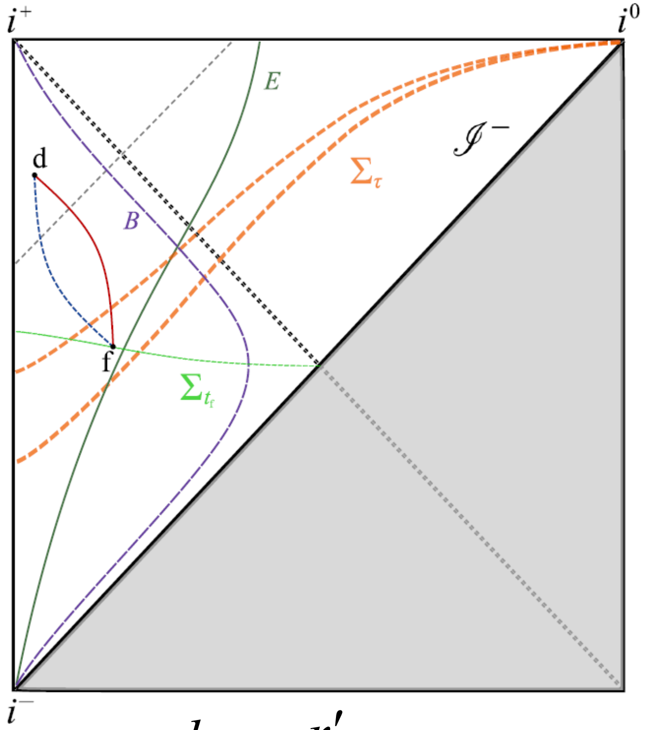
$$M(a) = M(a_{\rm in})\left(\frac{a}{a_{\rm in}}\right)^q$$



cosmological coupling q

Farrah, Croker, Zevin, Tarle *et al.*, Astrophys. J. Lett. **944**, L31 (2023).

$$q = 3 \Rightarrow p = -\rho$$



$$\frac{dr}{da} \approx \frac{r_g'}{aH} < 0 \sim 0(\text{now})$$

* distant observer is in the asymptotically de Sitter region but still far from the cosmological horizon

$$d\tau_{\rm comov} \approx dt_{\rm stat}$$

* From the definitions

$$\frac{dr_g}{da} = qa = \frac{\dot{r}_g}{\dot{a}} = \frac{\dot{r}_g}{aH}$$

DMSST, Phys. Rev D **110**, 044032 (2024)

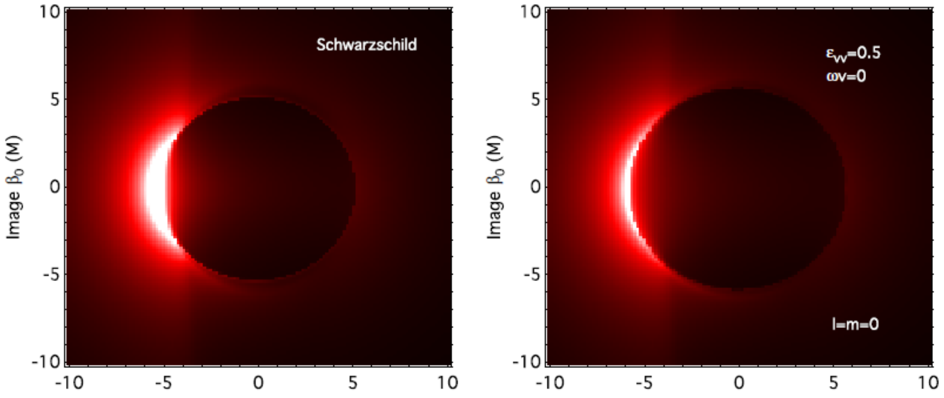
meaning

IF the observations are correct, then ABH are not PBH

ultra-compact objects

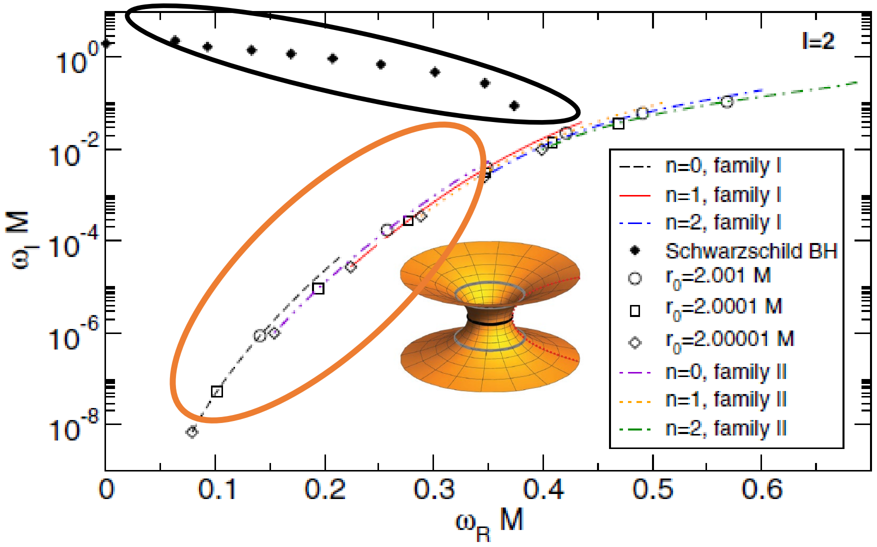
OBSERVATIONS?

how to distinguish



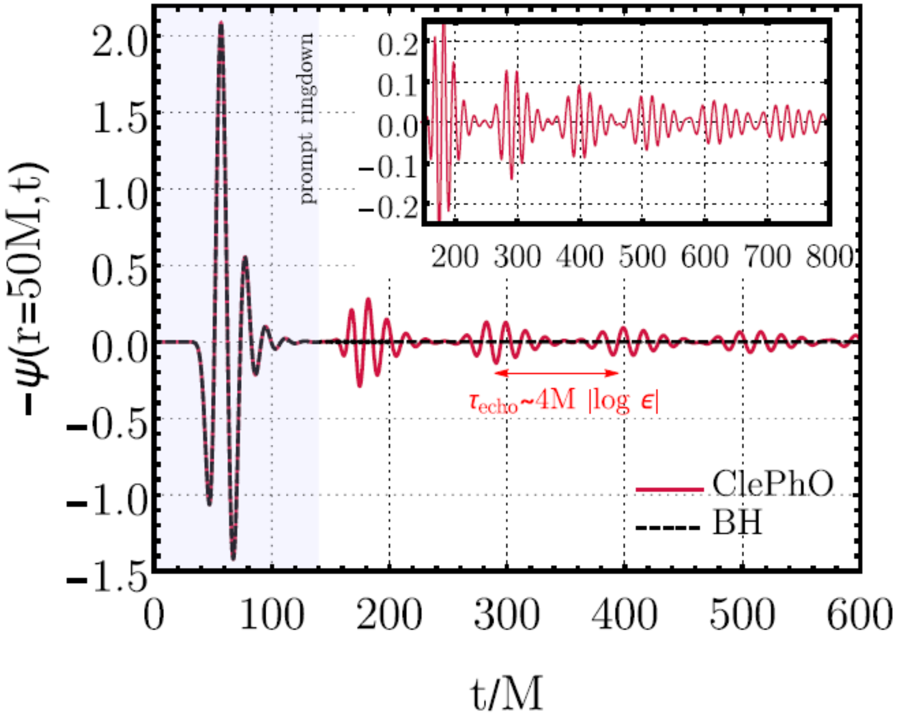
Light ring*:*
Schwarzschild vs ECO

Giddings and Psaltis,
Phys. Rev. D **97**, 084035 (2018)



QNM*: Schwarzschild vs wormholes*

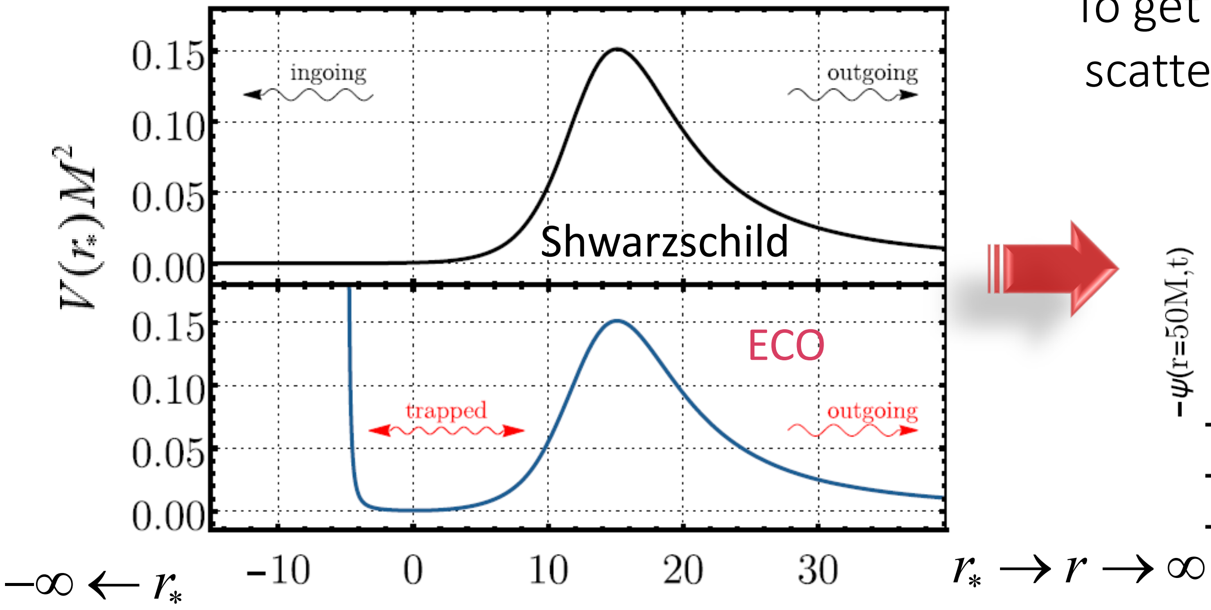
Cardoso, Franzin, and Pani,
PRL **116**, 171101 (2016)



ECO vs BH

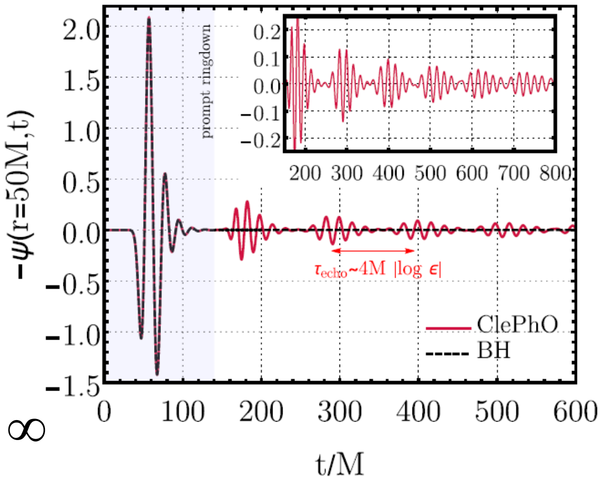
OBSERVATIONS?

will be there a smoking gun?

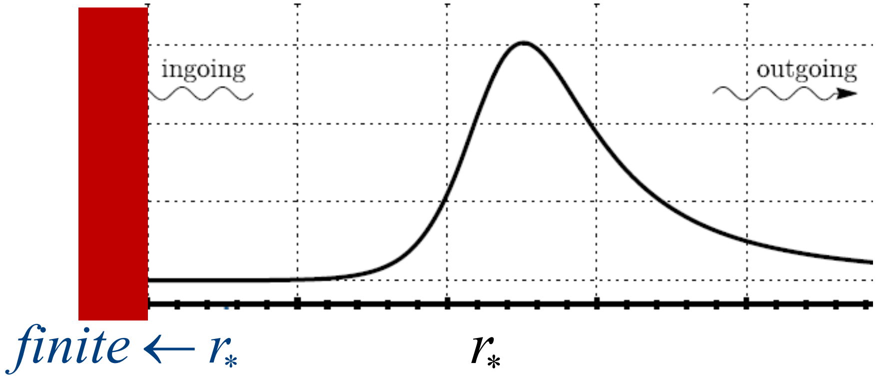


tortoise coordinate ▶ r_*

To get to the QNM, waveforms:
scattering in the effective potential



Cardoso and Pani,
Liv. Rev. Rel. **22**, 4 (2019)



A missing piece

ΦBH (k=0) give yet another potential:

what are the QNM & waveforms?

ECO vs MBH vs ΦBH

HOW?

will be there a smoking gun?

- The main problem: we know/parameterize the near-horizon geometry, but need more
- Link the near & far regions via Padé-like expansion

Rezzolla and A. Zhidenko,
Phys. Rev. D **90**, 084009 (2014).

- **Trial**: k=0 static metrics

Idea: combine near (ΦBH) and far (observations+ theory) for each of the metric functions expansions into a single approximant.

Maharana, Soranidis, Simovic, DRT,
Phys. Rev. D **111**, 104063 (2025).
Simovic and DRT,
Phys. Rev. D **110**, 084025 (2024).

Example: n=3 approximation can accommodate 5 near and far coefficients

$$f_3(r) = 1 - \cfrac{1}{1 + G_1 x + \cfrac{F_2 x}{1+G_2 x + \cfrac{F_3 x}{1+G_3 x}}}.$$

$$f_{r_g} = \sum_{k\geq 1} \alpha_k \frac{x^k}{r_{r_g}^k}$$

$$f_3(r) = \frac{A_3 r^3 + A_2 r^2 + A_1 r + A_0}{A_3 r^3 + B_2 r^2 + B_1 r + B_0}$$

$$f_\infty = 1 - \sum_{k\geq 1} \beta_k \frac{r_g^k}{r^k}$$

(Tested on Reissner-Nordström, Bardeen, hayward BH)

ECO vs MBH vs ΦBH

DIFFICULTIES

Padé poles

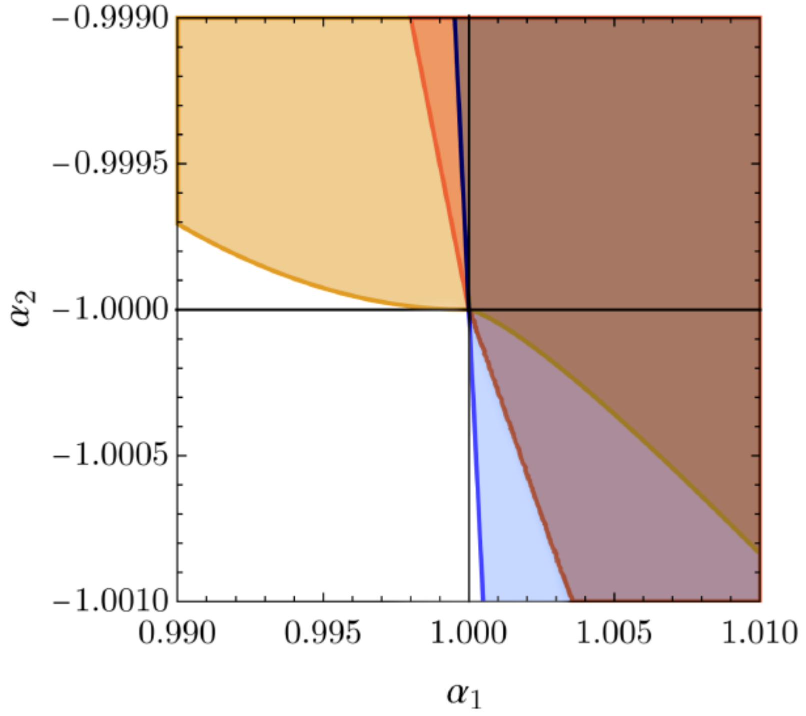


FIG. 1. Parameter space (α_1, α_2) covered by the two-point Padé approximations f_{S3} (blue) and the approximations with $n = 4$ (orange) and $n = 5$ (yellow) of Ref. [15]. In all expansions we set $\beta_1 = 1$. For the expansions $n = 4$ and $n = 5$ we take the Schwarzschild values of the higher order coefficients, $\alpha_3 = 1, \alpha_4 = -1$. Poles in the respective function f develop on the interval $r \in [r_g, \infty)$ for coefficients lying in the shaded region.

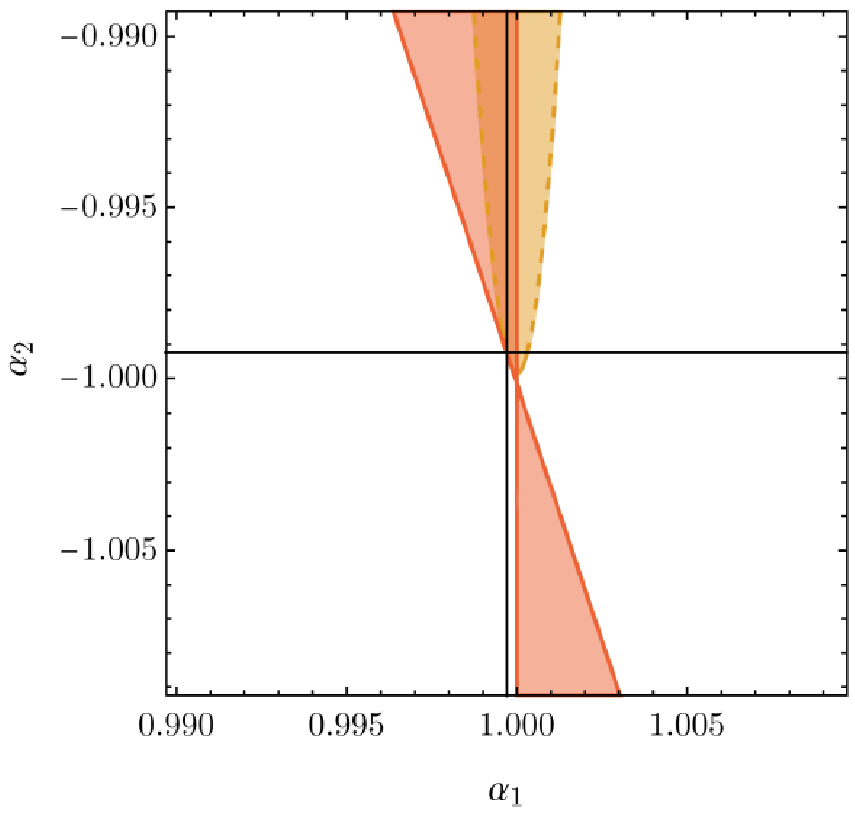
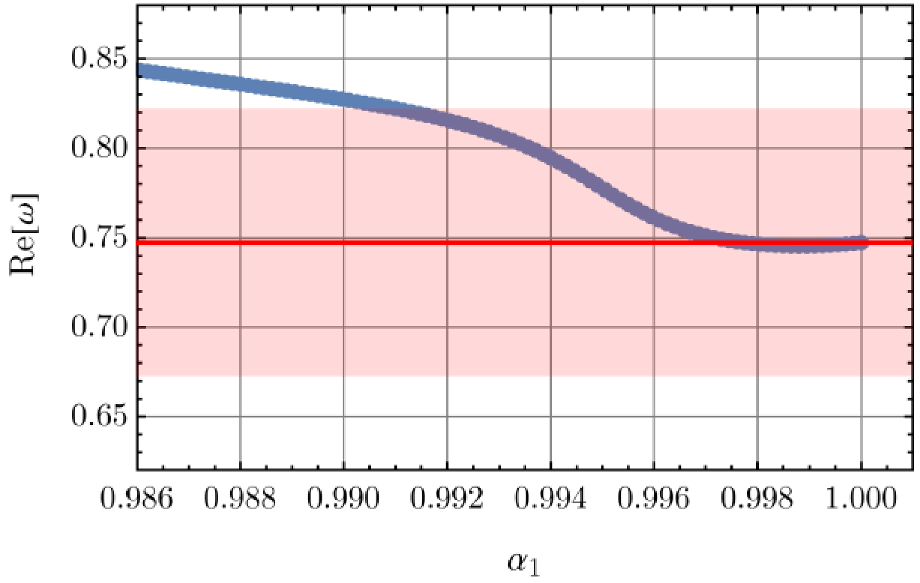
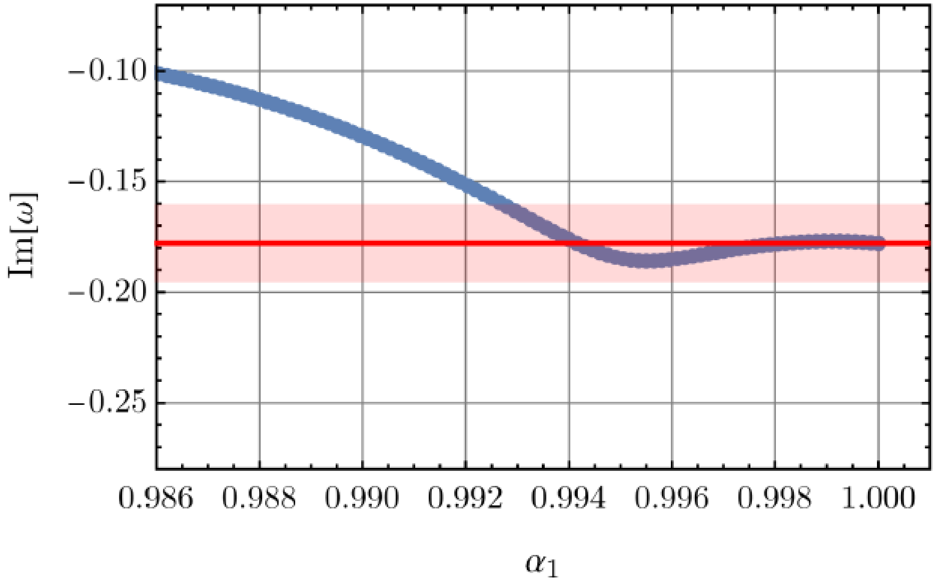


FIG. 3. Domains of validity for the lowest order M fraction (yellow) and RZ (red) approximations of the Bardeen metric. Poles are present for the respective metrics in the shaded region. β_1 is fixed to its Bardeen value.

QNM & LR





- Agreement 10^{-4} to 10^{-3} with the most precise results for the Schwarzschild, Bardeen, RN quasi-normal mode frequencies [when n=3 parameters are matched]
- Location of the light rings matches using only α_1 and α_2

Example (the worst): Bardeen

$$f(r) = 1 - \frac{2Mr^2}{(q^2 + r^2)^{3/2}}$$

original

$$f(r) = 1 - \frac{r^2(q^2 + r_g^2)^{3/2}}{r_g^2(q^2 + r^2)^{3/2}}$$

rewritten

$$\rho_B = \frac{3}{2} + \frac{7q^2}{12r_g^2} + \mathcal{O}(q^4)$$

exact

$$\rho_3 = \frac{3}{2} + \frac{3q^2}{4r_g^2} + \mathcal{O}(q^4)$$

approximate

Summary

- ❑ BH require as exotic matter as wormholes & ECOs
- ❑ Some of the popular models of regular BH are wrong
- ❑ Once a spherical ΦBH is formed, it stops growing
- ❑ Finite redshift (but very big)
- ❑ Collapse (for distant observers) happens in finite time, but… possibly not yet
- ❑ It is thennot clear that info loss paradox can be formulated
- ❑ Observations: cosmo, QNM, light rings…

https://www.mq.edu.au/research/phd-and-research-degrees/how-to-apply/scholarship-opportunities/scholarship-search/international-scholarship-round

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