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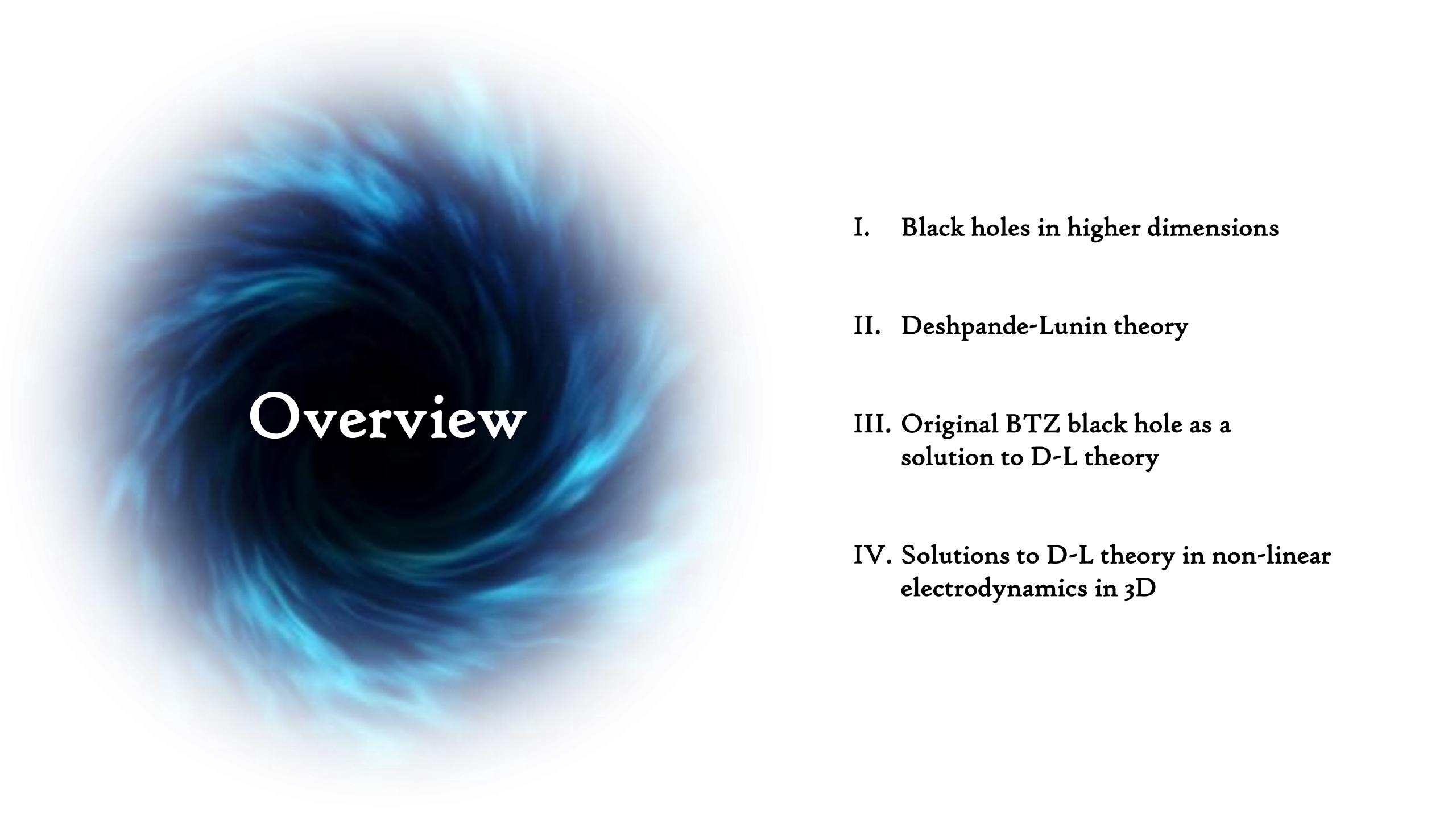
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New Interpretation of the Original BTZ Black Hole Spacetime

Robert B Mann

Brayden R Hull

T. Hale, B. R. Hull, D. Kubiznak, R. B. Mann, J. Mensiková, *Class. Quantum Grav.* **42** 09LT01 (2025)

A large, abstract image of a black hole's event horizon, rendered in shades of blue and white, serves as the background for the slide. It features a central dark region with concentric, swirling patterns of light blue and white, creating a sense of depth and motion.

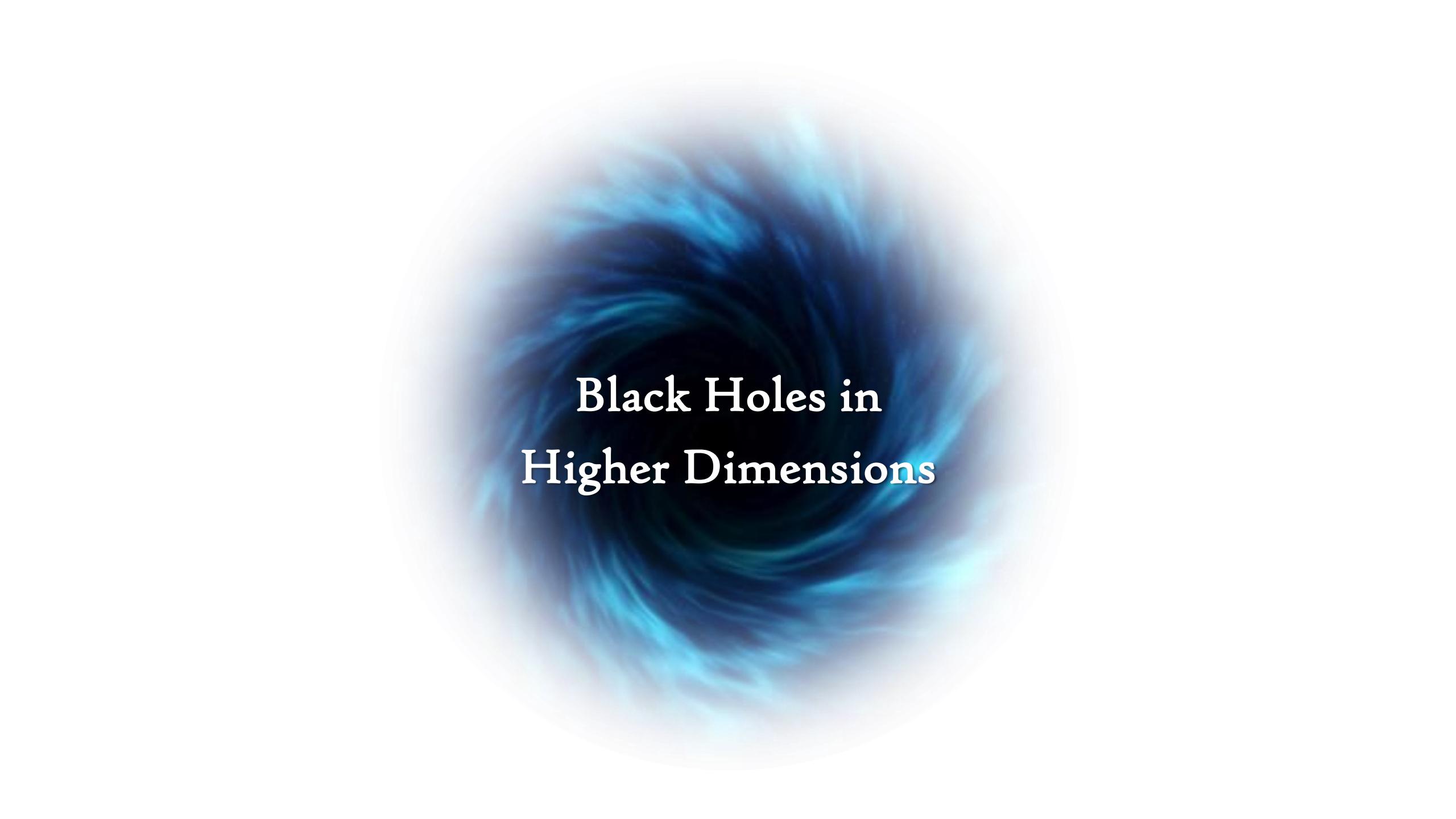
Overview

I. Black holes in higher dimensions

II. Deshpande-Lunin theory

III. Original BTZ black hole as a
solution to D-L theory

IV. Solutions to D-L theory in non-linear
electrodynamics in 3D



A black hole's event horizon, depicted as a dark, swirling center surrounded by a bright, glowing ring of light. The light is primarily blue and white, with some orange and yellow at the edges, creating a sense of depth and motion.

Black Holes in Higher Dimensions

Black Holes in Higher Dimensions

Parameters	4D	Higher Dimensions
M	Schwarzschild	Schwarzschild-Tangherlini
M, Q	Reissner-Nordström	Tangherlini-Reissner-Nordström
M, J	Kerr	Myers-Perry
M, Q, J	Kerr-Newman	???

- Attempts were made to employ topological Chern-Simons term into the action
- Unfortunately, general coupling does not yield analytic solutions

$$I_{\text{CS}}^{(2n+1)} = \frac{\lambda}{4\pi} \int A \wedge F \wedge \underbrace{\cdots \wedge F}_{n\text{-times}}$$



Deshpande-Lunin Theory

Deshpande-Lunin Theory

R. Deshpande, O. Lunin, JHEP 06 (2025) 066

$$I = \frac{1}{16\pi} \int (R - 2\Lambda + 4\mathcal{L}_{EM}) \sqrt{-g} d^n x + \frac{\lambda}{4\pi} \int A \wedge H \wedge K$$

- Modifies the action by adding a topological term with a dimensionless coupling constant
- Employs two new non-dynamical fields
- Provides charged and rotating solutions in odd dimensions
- Does not affect energy-momentum tensor
- Adds a current on the RHS of electromagnetic equations

Equations for the form-fields

$$\begin{aligned} dA &= F & F \wedge H &= 0 \\ dB &= H & F \wedge K &= 0 \\ dC &= K \end{aligned}$$

$$\nabla_\mu F^{\mu\nu} = \mathcal{J}^\nu$$

$$\mathcal{J} = \lambda * (H \wedge K)$$

A black hole is centered in the frame, surrounded by a dynamic, swirling accretion disk. The disk is primarily composed of blue and white light, creating a sense of intense motion and energy. The black hole itself is a dark, central void.

BTZ
Black Hole

Original BTZ Spacetime

M. Bañados, C. Teitelboim, J. Zanelli, *Phys. Rev. Lett.* **69**, 1849 (1992)

- Three-dimensional rotating black hole with negative cosmological constant and electric charge supported by a „static“ potential

$$ds^2 = -fdt^2 + \frac{dr^2}{f} + r^2(d\varphi + hdt)^2$$

$$f = \frac{r^2}{l^2} + \frac{j^2}{r^2} - m - 2Q^2 \ln\left(\frac{r}{r_0}\right), \quad h = \frac{j}{r^2}, \quad A = Q \ln\left(\frac{r}{r_0}\right) dt$$

$$I = \frac{1}{16\pi} \int (R - 2\Lambda + 4\mathcal{L}_{EM}) \sqrt{-g} d^n x + \frac{\lambda}{4\pi} \int A \wedge H \wedge K$$

$\nabla_\mu F^{\mu\nu} = \mathcal{J}^\nu$	$dA = F$	$F \wedge H = 0$
$\mathcal{J} = \lambda * (H \wedge K)$	$dB = H$	$F \wedge K = 0$
	$dC = K$	

- It does not satisfy vacuum Maxwell equations...

Correct charged solution was introduced in: G. Clement, *Phys. Lett. B* **367** 70 (1996)

BUT

- It is a solution to Deshpande-Lunin theory in three dimensions !

$$B = \frac{t}{\lambda r}, \quad H = \frac{dt}{\lambda r} - \frac{tdr}{\lambda r^2},$$

$$C = -\frac{2jQ}{r}, \quad K = \frac{2jQ}{r^2} dr,$$

$$\mathcal{J} = -\frac{2jQ}{r^4} \partial_\varphi$$

Original BTZ Spacetime

It satisfies the equations of force-free electrodynamics.

- electromagnetic tensor is degenerate

$$\mathcal{P} = \frac{1}{2} F_{\mu\nu} * F^{\mu\nu} = 0$$

- characteristic force-free equations are satisfied

$$\left. \begin{array}{l} 4\pi\nabla_\mu T^{\mu\nu} = F^\nu{}_\alpha \nabla_\mu F^{\mu\alpha} = 0 \\ \nabla_\mu F^{\mu\nu} = J^\nu \end{array} \right\} F_{\mu\nu} J^\nu = 0$$

- it is electrically dominated

$$S = \frac{1}{2} F_{\mu\nu} F^{\mu\nu} = -\frac{Q^2}{r^2} = -E^2 < 0$$

First law and Smarr relation are satisfied.

$$dM = TdS + \phi dQ + \Omega dJ + VdP + \Pi_{r_0} dr_0$$

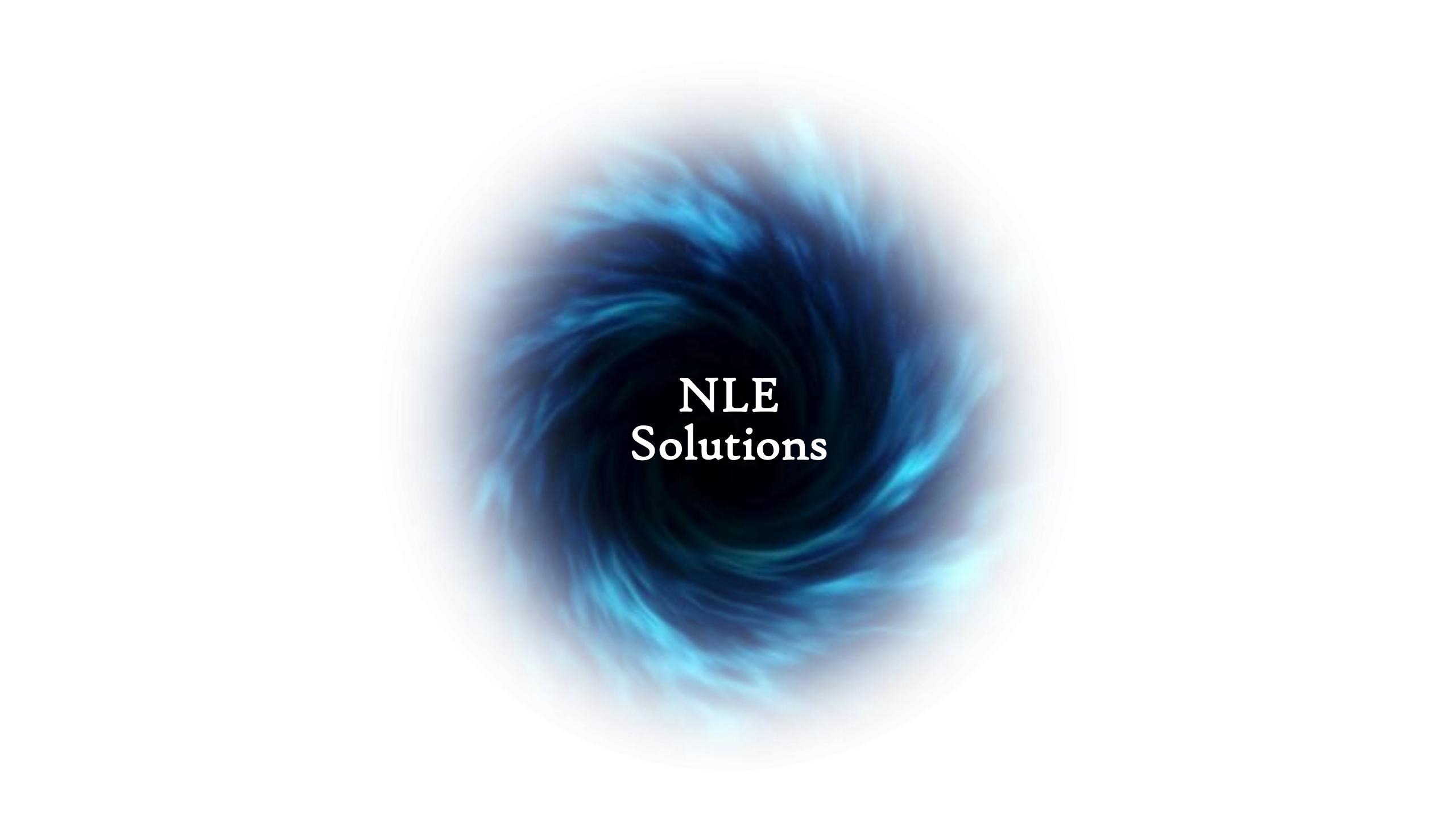
$$0 = TS - 2PV + \Omega J + \Pi_{r_0} r_0$$

$$M = \frac{m}{8}, \quad \phi = -\frac{Q}{2} \ln\left(\frac{r_+}{r_0}\right),$$

$$S = \frac{\pi r_+}{2}, \quad T = \frac{r_+^4 - j^2 l^2 - Q^2 l^2 r_+^2}{2\pi r_+^3 l^2},$$

$$J = \frac{j}{4}, \quad \Omega = \frac{j}{r_+^2}, \quad \Pi_{r_0} = \frac{Q^2}{4r_0},$$

$$V = \pi r_+^2, \quad P = \frac{1}{8\pi l^2}$$

A large, dark blue, swirling vortex centered on the page, set against a white background.

NLE
Solutions

Non-Linear Electrodynamics

$$I = \frac{1}{16\pi} \int (R - 2\Lambda + 4\mathcal{L}_{EM}) \sqrt{-g} d^n x + \frac{\lambda}{4\pi} \int A \wedge H \wedge K$$

- Maxwell Lagrangian

$$\mathcal{L}_M = -\frac{\mathcal{S}}{2}$$

- Electromagnetic Tensor

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

- Energy-Momentum Tensor

$$T^{\mu\nu} = \frac{1}{4\pi} \left(F^{\mu\alpha} F^\nu{}_\alpha - \frac{1}{4} g^{\mu\nu} F_{\alpha\beta} F^{\alpha\beta} \right)$$

- Equations of Motion

$$\nabla_\mu F^{\mu\nu} = \mathcal{J}^\nu$$

- NLE Lagrangian

$$\mathcal{L}_{NLE} = \mathcal{L}(\mathcal{S})$$

- Electromagnetic Tensor

$$D_{\mu\nu} = -2 \frac{\partial \mathcal{L}}{\partial \mathcal{S}} F_{\mu\nu}$$

- Energy-Momentum Tensor

$$T^{\mu\nu} = -\frac{1}{4\pi} \left(2F^\mu{}_\alpha F^{\nu\alpha} \frac{\partial \mathcal{L}}{\partial \mathcal{S}} - \mathcal{L} g^{\mu\nu} \right)$$

- Equations of Motion

$$\nabla_\mu D^{\mu\nu} = \mathcal{J}^\nu$$

3D NLE Solutions in D-L Theory

- General solution:

$$ds^2 = -fdt^2 + \frac{dr^2}{f} + r^2(d\varphi + hdt)^2$$

$$f = -m + \frac{j^2}{r^2} + \frac{r^2}{l^2} + \tilde{f} \quad h = \frac{j}{r^2}$$

$$A = -\phi_0 dt$$

- Force-free electrodynamics:

$$\left. \begin{aligned} \nabla_\mu T^{\mu\nu} &= F^\nu{}_\alpha \nabla_\mu D^{\mu\alpha} = 0 \\ \nabla_\mu D^{\mu\nu} &= J^\nu \end{aligned} \right\} F^\nu{}_\alpha J^\alpha = 0$$

- Thermodynamics:

$$dM = TdS + \phi dQ + \Omega dJ + VdP + \dots$$

$$0 = TS - 2PV + \Omega J + \dots$$

$$M = \frac{m}{8}, \quad T = \frac{f'(r_+)}{4\pi}, \quad S = \frac{\pi r_+}{2},$$

$$\Omega = \frac{j}{r_+^2}, \quad J = \frac{j}{4}, \quad Q = \frac{1}{2\pi} \int *D,$$

$$\phi = \frac{1}{2}\phi_0(r_+), \quad V = \pi r_+^2, \quad P = \frac{1}{8\pi l^2}$$

Example: Regularized Conformal ED

- Lagrangian:

$$\mathcal{L}_{RC} = -2\beta\alpha^3 \left(s + \frac{s^2}{2} + \ln(1-s) \right)$$

$$s \equiv \left(-\frac{\mathcal{S}}{\alpha^4} \right)^{\frac{1}{4}} \in (0,1) \quad [\alpha^2] = [\beta^2] = \frac{1}{L}$$

- Metric functions:

$$f_{RC} = \frac{2\alpha Q^2}{\beta} - m - 4Q\alpha^2 r + \frac{r^2}{l^2} + \frac{j^2}{r^2} + 4\alpha^3\beta r^2 \ln\left(\frac{\alpha r + Q/\beta}{\alpha r}\right)$$

$$h_{RC} = \frac{j}{r^2}$$

$$A_{RC} = -\frac{\alpha Q^2}{\beta^2(\alpha r + Q/\beta)} dt$$

- Thermodynamics:

$$\begin{aligned} \delta M &= T\delta S + \phi\delta Q + \Omega\delta J + V\delta P + \Pi_\alpha\delta\alpha + \Pi_\beta\delta\beta \\ 0 &= TS + \Omega J - 2PV - \frac{1}{2}\Pi_\alpha\alpha - \frac{1}{2}\Pi_\beta\beta \end{aligned}$$

$$M = \frac{m}{8}, \quad S = \frac{\pi r_+}{2}, \quad V = \pi r_+^2, \quad P = \frac{1}{8\pi\ell^2},$$

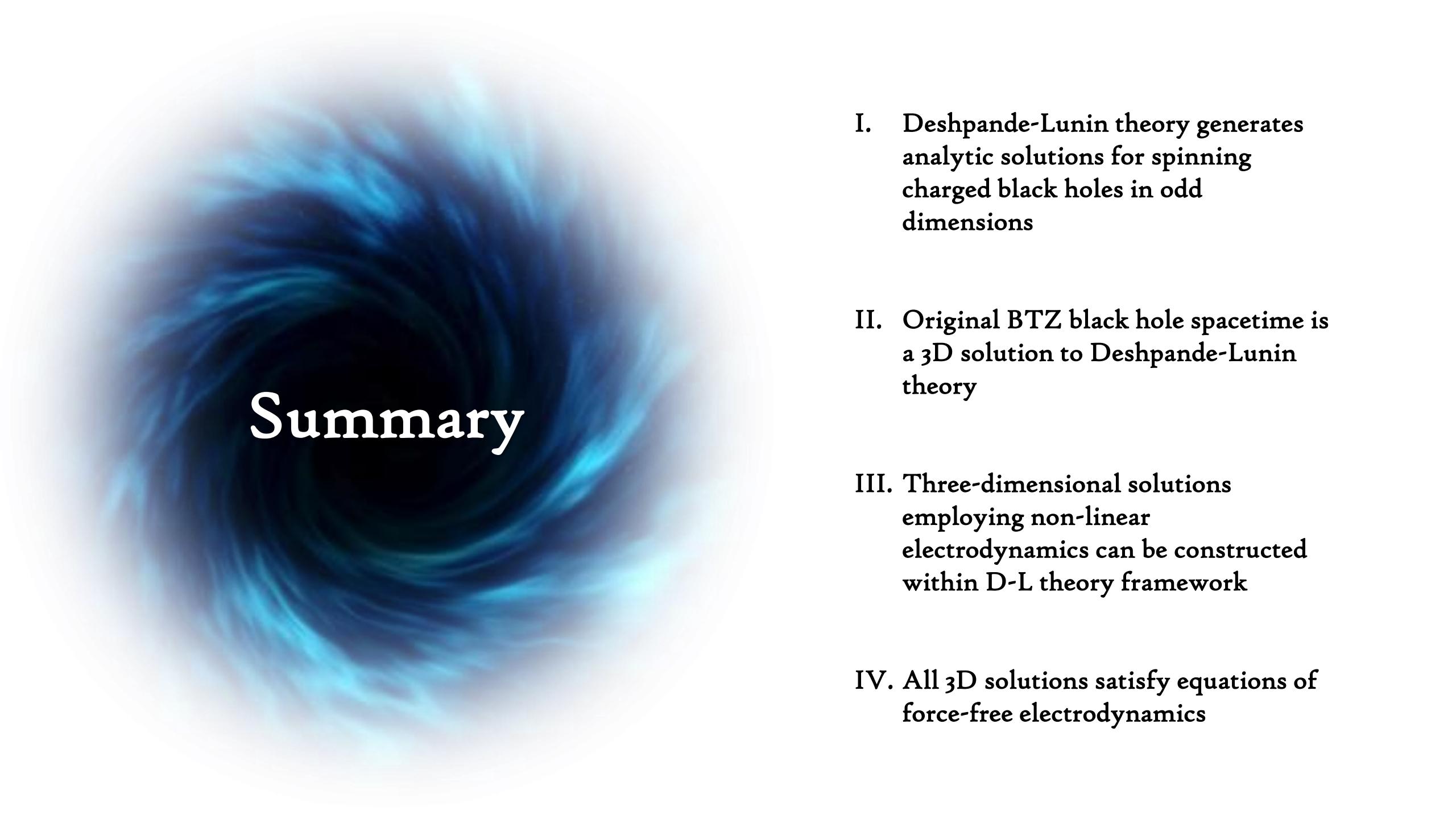
$$T = \frac{f'_{RC}(r_+)}{4\pi} = \frac{r_+}{2\pi\ell^2} - \frac{\alpha^2 Q(Q + 2\alpha\beta r_+)}{\pi(Q + \alpha\beta r_+)}$$

$$-\frac{j^2}{2\pi r_+^3} + \frac{2\alpha^3\beta r_+}{\pi} \log\left(\frac{Q + \alpha\beta r_+}{\alpha\beta r_+}\right),$$

$$\Omega = \frac{j}{r_+^2}, \quad J = \frac{j}{4}, \quad \phi = \frac{\alpha Q^2}{2(\beta Q + \alpha\beta^2 r_+)},$$

$$\Pi_\alpha = \frac{3}{2}\beta\alpha^2 r_+^2 \log\left(\frac{\alpha\beta r_+ + Q}{\alpha\beta r_+}\right) + \frac{Q(Q^2 - 3Q\alpha\beta r_+ - 6\alpha^2\beta^2 r_+^2)}{4\beta(\alpha\beta r_+ + Q)},$$

$$\Pi_\beta = \frac{1}{2}\alpha^3 r_+^2 \log\left(\frac{\alpha\beta r_+ + Q}{\alpha\beta r_+}\right) - \frac{Q\alpha(Q^2 + Q\alpha\beta r_+ + 2\alpha^2\beta^2 r_+^2)}{4\beta^2(\alpha\beta r_+ + Q)}$$



Summary

- I. Deshpande-Lunin theory generates analytic solutions for spinning charged black holes in odd dimensions
- II. Original BTZ black hole spacetime is a 3D solution to Deshpande-Lunin theory
- III. Three-dimensional solutions employing non-linear electrodynamics can be constructed within D-L theory framework
- IV. All 3D solutions satisfy equations of force-free electrodynamics



**Thank you
for
your attention!**

The End