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New Interpretation of the Original BTZ Black Hole Spacetime

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T. Hale, B. R. Hull, D. Kubiznak, R. B. Mann, J. Mensiková, Class. Quantum Grav. **42** *09LT01 (2025)*

Overview

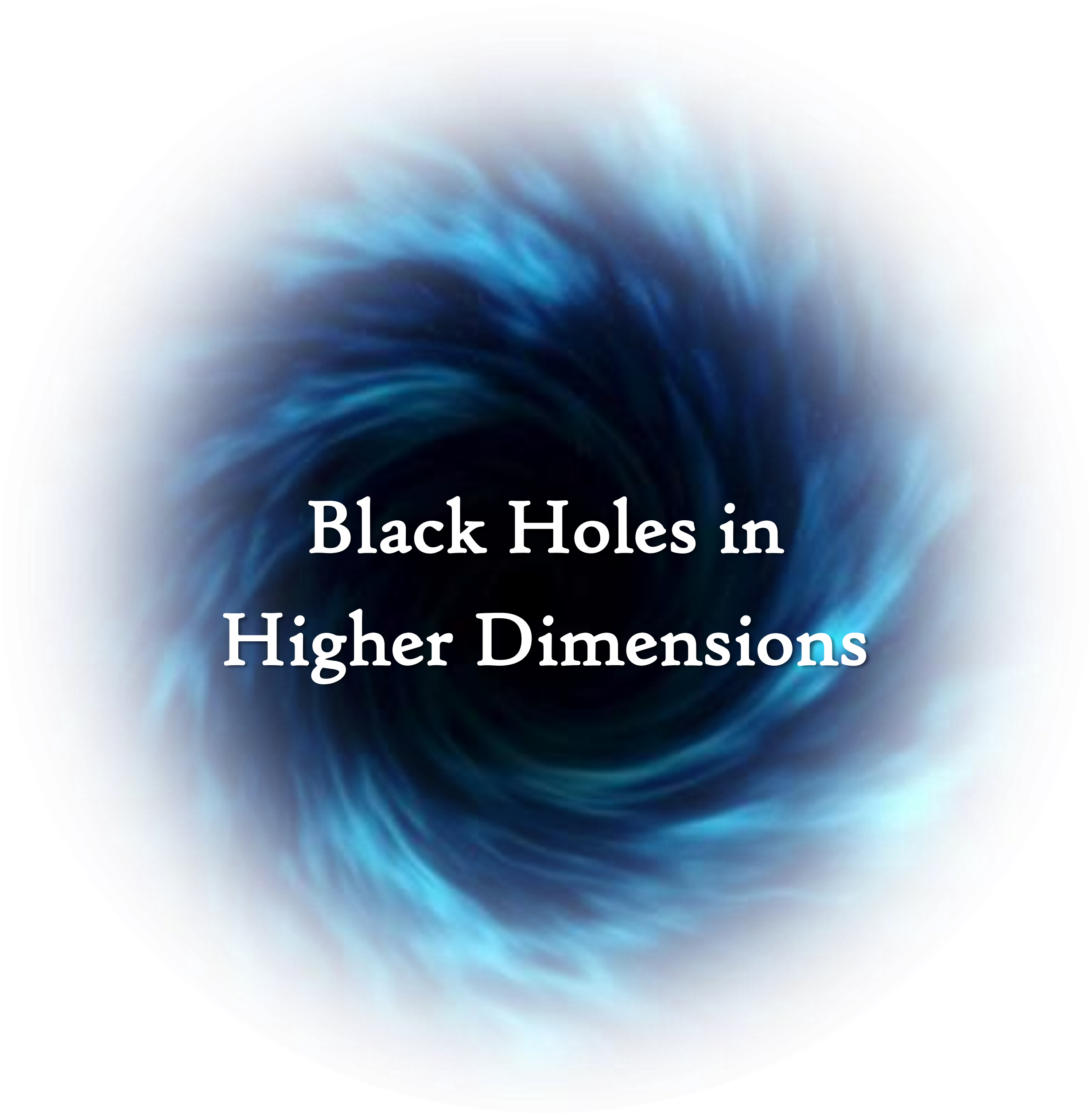
I. Black holes in higher dimensions

II. Deshpande-Lunin theory

III. Original BTZ black hole as a solution to D-L theory

IV. Solutions to D-L theory in non-linear electrodynamics in 3D

Black Holes in Higher Dimensions



Black Holes in Higher Dimensions

Parameters	4D	Higher Dimensions
M	Schwarzschild	Schwarzschild-Tangherlini
M, Q	Reissner-Nordström	Tangherlini-Reissner-Nordström
M, J	Kerr	Myers-Perry
M, Q, J	Kerr-Newman	???

- Attempts were made to employ topological Chern-Simons term into the action
- Unfortunately, general coupling does not yield analytic solutions

$$I_{\mathrm{CS}}^{(2n+1)} = \frac{\lambda}{4\pi} \int A \wedge \underbrace{F \wedge \cdots \wedge F}_{n\text{-times}}$$


Deshpande-Lunin
Theory

Deshpande-Lunin Theory

R. Deshpande, O. Lunin, JHEP 06 (2025) 066

$$I = \frac{1}{16\pi}\int (R - 2\Lambda + 4\mathcal{L}_{EM})\sqrt{-g}\,d^n x + \frac{\lambda}{4\pi}\int A \wedge H \wedge K$$

- Modifies the action by adding a topological term with a dimensionless coupling constant
- Employs two new non-dynamical fields
- Provides charged and rotating solutions in odd dimensions
- Does not affect energy-momentum tensor
- Adds a current on the RHS of electromagnetic equations

Equations for the form-fields

$$dA = F \qquad dB = H \qquad dC = K$$

$$F \wedge H = 0 \qquad F \wedge K = 0$$

$$\nabla_\mu F^{\mu\nu} = \mathcal{J}^\nu$$

$$\mathcal{J} = \lambda * (H \wedge K)$$


BTZ
Black Hole

Original BTZ Spacetime

M. Bañados, C. Teitelboim, J. Zanelli, Phys. Rev. Lett. ***69****, 1849 (1992)*

- Three-dimensional rotating black hole with negative cosmological constant and electric charge supported by a „static“ potential

$$ds^2 = -f dt^2 + \frac{dr^2}{f} + r^2(d\varphi + h dt)^2$$

$$f = \frac{r^2}{l^2} + \frac{j^2}{r^2} - m - 2Q^2 \ln\left(\frac{r}{r_0}\right), \qquad h = \frac{j}{r^2}, \qquad A = Q \ln\left(\frac{r}{r_0}\right) dt$$

$$I = \frac{1}{16\pi}\int (R - 2\Lambda + 4\mathcal{L}_{EM})\sqrt{-g}\, d^n x + \frac{\lambda}{4\pi}\int A \wedge H \wedge K$$

$$\nabla_\mu F^{\mu\nu} = \mathcal{J}^\nu \qquad dA = F \qquad F \wedge H = 0$$

$$\mathcal{J} = \lambda * (H \wedge K) \qquad dB = H \qquad F \wedge K = 0$$

$$dC = K$$

- It does not satisfy vacuum Maxwell equations…

  Correct charged solution was introduced in: *G. Clement, Phys. Lett. B* ***367*** *70 (1996)*

BUT

- **It is a solution to Deshpande-Lunin theory in three dimensions !**

$$B = \frac{t}{\lambda r}, \qquad H = \frac{dt}{\lambda r} - \frac{t dr}{\lambda r^2},$$

$$C = -\frac{2jQ}{r}, \qquad K = \frac{2jQ}{r^2} dr,$$

$$\mathcal{J} = -\frac{2jQ}{r^4}\partial_\varphi$$

Original BTZ Spacetime

It satisfies the equations of force-free electrodynamics.

- electromagnetic tensor is degenerate

$$\mathcal{P} = \frac{1}{2} F_{\mu\nu} * F^{\mu\nu} = 0$$

- characteristic force-free equations are satisfied

$$\left.\begin{aligned} 4\pi\nabla_\mu T^{\mu\nu} = F^{\nu}{}_{\alpha}\nabla_\mu F^{\mu\alpha} = 0 \\ \nabla_\mu F^{\mu\nu} = \mathcal{J}^{\nu} \end{aligned}\right\} F_{\mu\nu}\mathcal{J}^{\nu} = 0$$

- it is electrically dominated

$$\mathcal{S} = \frac{1}{2} F_{\mu\nu} F^{\mu\nu} = -\frac{Q^2}{r^2} = -E^2 < 0$$

First law and Smarr relation are satisfied.

$$dM = TdS + \phi dQ + \Omega dJ + VdP + \Pi_{r_0} dr_0$$

$$0 = TS - 2PV + \Omega J + \Pi_{r_0} r_0$$

$$M = \frac{m}{8}, \qquad \phi = -\frac{Q}{2}\ln\left(\frac{r_+}{r_0}\right),$$

$$S = \frac{\pi r_+}{2}, \qquad T = \frac{r_+^4 - j^2 l^2 - Q^2 l^2 r_+^2}{2\pi r_+^3 l^2},$$

$$J = \frac{j}{4}, \qquad \Omega = \frac{j}{r_+^2}, \qquad \Pi_{r_0} = \frac{Q^2}{4r_0},$$

$$V = \pi r_+^2, \qquad P = \frac{1}{8\pi l^2}$$

NLE
Solutions

Non-Linear Electrodynamics

$$I = \frac{1}{16\pi}\int (R - 2\Lambda + 4\mathcal{L}_{EM})\sqrt{-g}\,d^n x + \frac{\lambda}{4\pi}\int A \wedge H \wedge K$$

- Maxwell Lagrangian

$$\mathcal{L}_M = -\frac{\mathcal{S}}{2}$$

- Electromagnetic Tensor

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\mu A_\nu$$

- Energy-Momentum Tensor

$$T^{\mu\nu} = \frac{1}{4\pi}\left(F^{\mu\alpha}F^{\nu}{}_{\alpha} - \frac{1}{4}g^{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}\right)$$

- Equations of Motion

$$\nabla_\mu F^{\mu\nu} = \mathcal{J}^\nu$$

- NLE Lagrangian

$$\mathcal{L}_{NLE} = \mathcal{L}(\mathcal{S})$$

- Electromagnetic Tensor

$$D_{\mu\nu} = -2\frac{\partial\mathcal{L}}{\partial\mathcal{S}}F_{\mu\nu}$$

- Energy-Momentum Tensor

$$T^{\mu\nu} = -\frac{1}{4\pi}\left(2F^{\mu}{}_{\alpha}F^{\nu\alpha}\frac{\partial\mathcal{L}}{\partial\mathcal{S}} - \mathcal{L}g^{\mu\nu}\right)$$

- Equations of Motion

$$\nabla_\mu D^{\mu\nu} = \mathcal{J}^\nu$$

3D NLE Solutions in D-L Theory

- General solution:

$$\mathrm{d}s^2 = -f dt^2 + \frac{dr^2}{f} + r^2(d\varphi + h dt)^2$$

$$f = -m + \frac{j^2}{r^2} + \frac{r^2}{l^2} + \tilde{f} \qquad h = \frac{j}{r^2}$$

$$A = -\phi_0 dt$$

- Force-free electrodynamics:

$$\left.\begin{array}{r} \nabla_\mu T^{\mu\nu} = F^\nu{}_\alpha \nabla_\mu D^{\mu\alpha} = 0 \\ \nabla_\mu D^{\mu\nu} = \mathcal{J}^\nu \end{array}\right\} \; F^\nu{}_\alpha \mathcal{J}^\alpha = 0$$

- Thermodynamics:

$$dM = TdS + \phi dQ + \Omega dJ + VdP + \cdots$$

$$0 = TS - 2PV + \Omega J + \cdots$$

$$M = \frac{m}{8}, \qquad T = \frac{f'(r_+)}{4\pi}, \qquad S = \frac{\pi r_+}{2},$$

$$\Omega = \frac{j}{r_+^2}, \qquad J = \frac{j}{4}, \qquad Q = \frac{1}{2\pi}\int * D,$$

$$\phi = \frac{1}{2}\phi_0(r_+), \qquad V = \pi r_+^2, \qquad P = \frac{1}{8\pi l^2}$$

Example: Regularized Conformal ED

- Lagrangian:

$$\mathcal{L}_{RC} = -2\beta\alpha^3\left(s + \frac{s^2}{2} + \ln(1-s)\right)$$

$$s \equiv \left(-\frac{\mathcal{S}}{\alpha^4}\right)^{\frac{1}{4}} \in (0,1) \qquad [\alpha^2] = [\beta^2] = \frac{1}{L}$$

- Metric functions:

$$f_{RC} = \frac{2\alpha Q^2}{\beta} - m - 4Q\alpha^2 r + \frac{r^2}{l^2} + \frac{j^2}{r^2} + 4\alpha^3\beta r^2 \ln\left(\frac{\alpha r + Q/\beta}{\alpha r}\right)$$

$$h_{RC} = \frac{j}{r^2}$$

$$A_{RC} = -\frac{\alpha Q^2}{\beta^2(\alpha r + Q/\beta)}dt$$

- Thermodynamics:

$$\delta M = T\delta S + \phi\delta Q + \Omega\delta J + V\delta P + \Pi_\alpha\delta\alpha + \Pi_\beta\delta\beta$$

$$0 = TS + \Omega J - 2PV - \frac{1}{2}\Pi_\alpha\alpha - \frac{1}{2}\Pi_\beta\beta$$

$$M = \frac{m}{8}, \quad S = \frac{\pi r_+}{2}, \quad V = \pi r_+^2, \quad P = \frac{1}{8\pi\ell^2},$$

$$T = \frac{f'_{\rm RC}(r_+)}{4\pi} = \frac{r_+}{2\pi\ell^2} - \frac{\alpha^2 Q(Q+2\alpha\beta r_+)}{\pi(Q+\alpha\beta r_+)} - \frac{j^2}{2\pi r_+^3} + \frac{2\alpha^3\beta r_+}{\pi}\log\left(\frac{Q+\alpha\beta r_+}{\alpha\beta r_+}\right),$$

$$\Omega = \frac{j}{r_+^2}, \quad J = \frac{j}{4}, \quad \phi = \frac{\alpha Q^2}{2(\beta Q + \alpha\beta^2 r_+)},$$

$$\Pi_\alpha = \frac{3}{2}\beta\alpha^2 r_+^2\log\left(\frac{\alpha\beta r_+ + Q}{\alpha\beta r_+}\right) + \frac{Q(Q^2 - 3Q\alpha\beta r_+ - 6\alpha^2\beta^2 r_+^2)}{4\beta(\alpha\beta r_+ + Q)},$$

$$\Pi_\beta = \frac{1}{2}\alpha^3 r_+^2\log\left(\frac{\alpha\beta r_+ + Q}{\alpha\beta r_+}\right) - \frac{Q\alpha(Q^2 + Q\alpha\beta r_+ + 2\alpha^2\beta^2 r_+^2)}{4\beta^2(\alpha\beta r_+ + Q)}$$

Summary

I. Deshpande-Lunin theory generates analytic solutions for spinning charged black holes in odd dimensions

II. Original BTZ black hole spacetime is a 3D solution to Deshpande-Lunin theory

III. Three-dimensional solutions employing non-linear electrodynamics can be constructed within D-L theory framework

IV. All 3D solutions satisfy equations of force-free electrodynamics

Thank you

for

your attention!